New statistical PDF, TMD and all that...

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Outline

- Basic procedure to construct the statistical polarized parton distributions
- Essential features from unpolarized and polarized Deep Inelastic Scattering data
- New results using a much broader DIS data set:
  Find a new gluon helicity distribution (to be confirmed)
- Predictions for hadron colliders up to LHC energy:
  The structure of the nucleon light sea a new challenge
  Cross sections and Helicity asymmetries for single-jet and $W^{\pm}$ production
- Transverse momentum dependence (TMD) extention:
  Transverse energy sum rule. Gaussian shape with no x, k_T factorization
  Melosh-Wigner effects mainly in low x, Q^2 region
  Double helicity asymmetry in SIDIS
- Conclusions

Selected references for PDF

- A Statistical Approach for Polarized Parton Distributions
  Euro. Phys. J. C23, 487 (2002)
- The Statistical Parton Distributions: status and prospects
  Euro. Phys. J. C41, 327 (2005)
- $W^{\pm}$ bosons production in the quantum statistical parton distributions approach
  Phys. Lett. B726, 296 (2013)
- Statistical description of the proton spin with a large gluon helicity distribution
  Phys. Lett. B740, 168 (2015)
- New developments in the statistical approach of parton distributions: tests and predictions up to LHC energies
  Nucl. Phys. A941, 307 (2015)
- The Drell-Yan process as a testing ground for parton distributions up to LHC
  Nucl. Phys. A948, 63 (2016)

References for TMD

- The extension to the transverse momentum of the statistical parton distributions
  Mod. Phys. Letters A21, 143 (2006)
- Semiinclusive DIS cross sections and spin asymmetries in the quantum statistical parton distributions approach, Phys. Rev. D83, 074008 (2011)
- The transverse momentum dependent statistical parton distributions revisited
  Int. Journal of Mod. Phys. A28, 1350026 (2013)

Hadron production using statistical models is an old story

- E. Fermi, Phys. Rev. 92, 452 (1953)
- I. Ya. Pomeranchuk, Izv. Dokl. Akad. Nauk Ser.Fiz. 78, 889 (1951)
- L.D. Landau, Izv. Akad. Nauk Ser. Fiz. 17, 51 (1953)
- R. Hagedorn, Supple. al Nuovo Cimento III, 147 (1965)
- R. Hagedorn, Nuovo Cimento 35, 395 (1965)
- R. Hagedorn, Nuovo Cimento A 56, 1027 (1968)

Our motivation and goals

- Will propose a quantum statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.
- Will incorporate some well known phenomenological facts and some QCD features

Our motivation and goals

- Will propose a quantum statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.
- Will incorporate some well known phenomenological facts and some QCD features
- Will parametrize our PDF in terms of a rather small number of physical parameters, at variance with standard polynomial type parametrizations
- Will be able to construct simultaneously unpolarized and polarized PDF:
  A UNIQUE CASE ON THE MARKET!
- Will be able to describe physical observables both in DIS and hadronic collisions
- Will make some very specific challenging predictions, from the behavior of unpolarized and polarized PDF, either in the sea quark region or in the valence region
- Will also consider the case of the elusive polarized gluon distribution

Basic procedure

Use a simple description of the PDF, at input scale Q_0^2, proportional to $[\exp[(x - X_{0p})/\bar{x}] \pm 1]^{-1}$, *plus* sign for quarks and antiquarks, corresponds to a Fermi-Dirac distribution and *minus* sign for gluons, corresponds to a Bose-Einstein distribution. X_{0p} is a constant which plays the role of the *thermodynamical potential* of the parton *p* and $\bar{x}$ is the *universal temperature*, which is the same for all partons.

NOTE: x is indeed the natural variable, since all the sum rules we will use are expressed in terms of x

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From the chiral structure of QCD, we have two important properties, allowing to RELATE quark and antiquark distributions and to RESTRICT the gluon distribution:
- Potential of a quark q^h of helicity *h* is opposite to the potential of the corresponding antiquark $\bar{q}^{-h}$ of helicity *-h*, $X_{0q}^h = -X_{0\bar{q}}^{-h}$.
- Potential of the gluon G is zero, $X_{0G} = 0$.

The polarized PDF $q^{\pm}(x, Q_0^2)$ ***at initial scale*** Q_0^2

For light quarks $q = u, d$ of helicity $h = \pm$, we take

$$xq^{(h)}(x, Q_0^2) = \frac{A X_{0q}^h x^b}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A} x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} ,$$

consequently for antiquarks of helicity $h = \mp$

$$x\bar{q}^{(-h)}(x, Q_0^2) = \frac{\bar{A} (X_{0q}^h)^{-1} x^{\bar{b}}}{\exp[(x + X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A} x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} .$$

Note: $q = q^+ + q^-$ and $\Delta q = q^+ - q^-$ (idem for $\bar{q}$).
Extra term is absent in Δq and q_v also in $u - d$ or $\bar{u} - \bar{d}$.
The additional factors X_{0q}^h and $(X_{0q}^h)^{-1}$ are coming from TMD (see below)

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For strange quarks and antiquarks, s and $\bar{s}$, use the same procedure which leads to $xs(x, Q_0^2) \neq x\bar{s}(x, Q_0^2)$ and $x\Delta s(x, Q_0^2) \neq x\Delta\bar{s}(x, Q_0^2)$ (Phys. Lett. B648, 39 (2007)).

For gluons we use a Bose-Einstein expression given by $xG(x, Q_0^2) = \frac{A_G x^{b_G}}{\exp(x/\bar{x}) - 1}$, with a vanishing potential and the same temperature $\bar{x}$. For the polarized gluon distribution $x\Delta G(x, Q_0^2)$ we take a similar expression at initial scale (positive for all x)

Essential features from the DIS data

From well established features of u and d extracted from DIS data, we anticipate some simple relations between the potentials:

- $u(x)$ dominates over $d(x)$, so we should have $X^+_{0u} + X^-_{0u} > X^+_{0d} + X^-_{0d}$
- $\Delta u(x) > 0$, therefore $X^+_{0u} > X^-_{0u}$
- $\Delta d(x) < 0$, therefore $X^-_{0d} > X^+_{0d}$.

Essential features from the DIS data

From well established features of u and d extracted from DIS data, we anticipate some simple relations between the potentials:

- $u(x)$ dominates over $d(x)$, so we should have $X_{0u}^{+} + X_{0u}^{-} > X_{0d}^{+} + X_{0d}^{-}$
- $\Delta u(x) > 0$, therefore $X_{0u}^{+} > X_{0u}^{-}$
- $\Delta d(x) < 0$, therefore $X_{0d}^{-} > X_{0d}^{+}$.

So we expect X_{0u}^{+} to be the largest potential and X_{0d}^{+} the smallest one. In fact, from our fit we have obtained the following ordering

$$X_{0u}^{+} > X_{0d}^{-} \sim X_{0u}^{-} > X_{0d}^{+} \; .$$

This ordering has important consequences for $\bar{u}$ and $\bar{d}$, namely

Essential features from DIS data

- $\bar{d}(x) > \bar{u}(x)$, flavor symmetry breaking expected from Pauli exclusion principle. This was already confirmed by the violation of the Gottfried sum rule (NMC).
- $\Delta\bar{u}(x) > 0$ and $\Delta\bar{d}(x) < 0$, a PREDICTION from 2002, in agreement with polarized DIS (see below) and has been more precisely checked at RHIC-BNL from $W^{\pm}$ production, already in active running phase (see below).

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- Note that since $u^{-}(x) \sim d^{-}(x)$, it follows that $\bar{u}^{+}(x) \sim \bar{d}^{+}(x)$, so we have

$$\Delta\bar{u}(x) - \Delta\bar{d}(x) \sim \bar{d}(x) - \bar{u}(x) \; ,$$

i.e. the flavor symmetry breaking is almost the same for unpolarized and polarized distributions ($\Delta\bar{u}$ and $\Delta\bar{d}$ contribute to about 10% to the Bjorken sum rule).

Very few free parameters

By performing a NLO QCD evolution of these PDF, we were able to obtain a good description of a large set of very precise data on $F_2^p(x,Q^2), F_2^n(x,Q^2), xF_3^{\nu N}(x,Q^2)$ and $g_1^{p,d,n}(x,Q^2)$, in correspondance with TEN free parameters for the light quark sector with some physical significance:

* the four potentials $X_{0u}^+,\ X_{0u}^-,\ X_{0d}^-,\ X_{0d}^+$,

* the universal temperature $\bar{x}$,

* and $b,\ \bar{b},\ \tilde{b},\ b_G,\ \tilde{A}$.

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* the universal temperature $\bar{x}$,
* and $b,\ \bar{b},\ \tilde{b},\ b_G,\ \tilde{A}$.

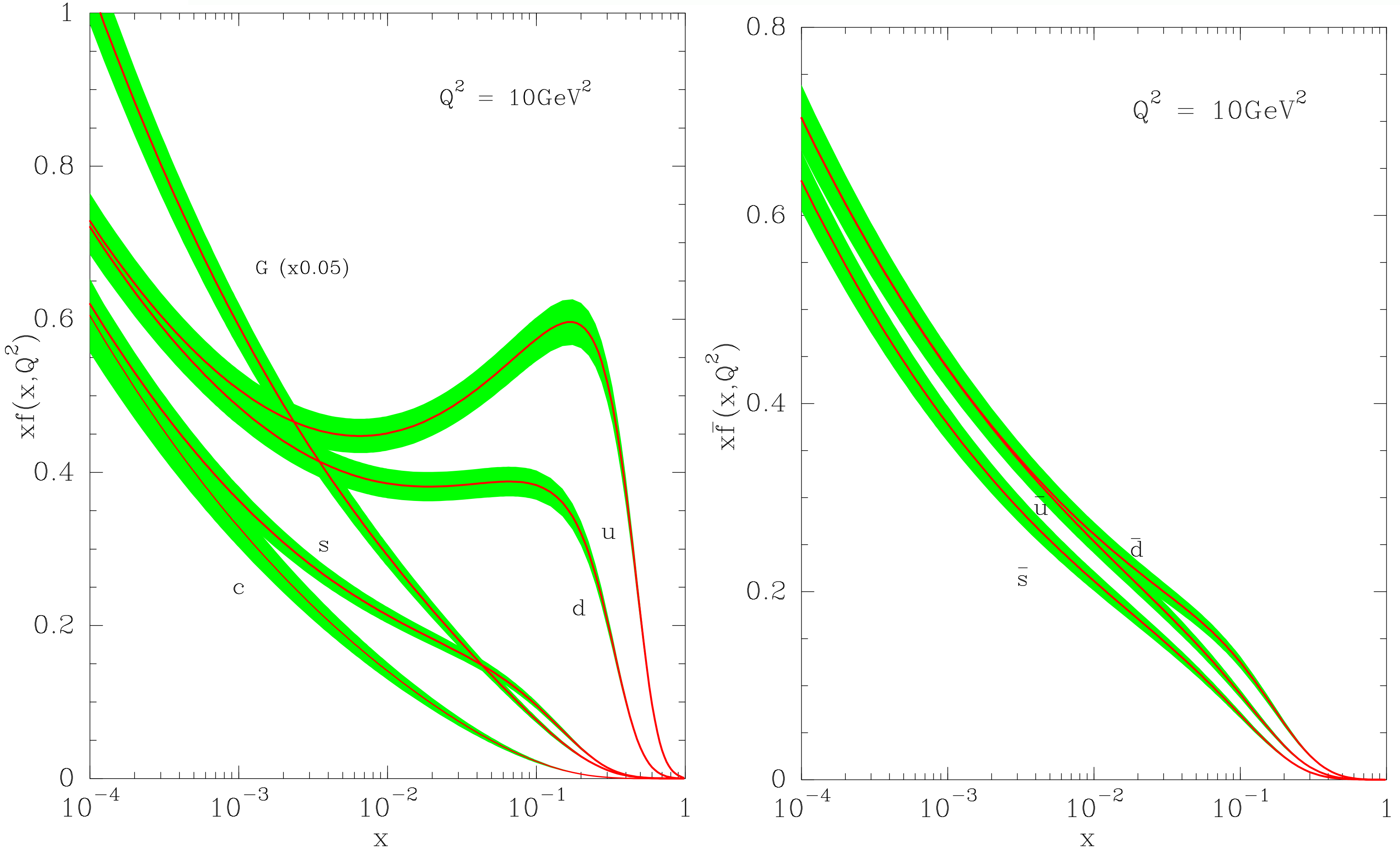
We also have three additional parameters, $A,\ \bar{A},\ A_G$, which are fixed by 3 normalization conditions .

$$u - \bar{u} = 2, \ \ d - \bar{d} = 1$$

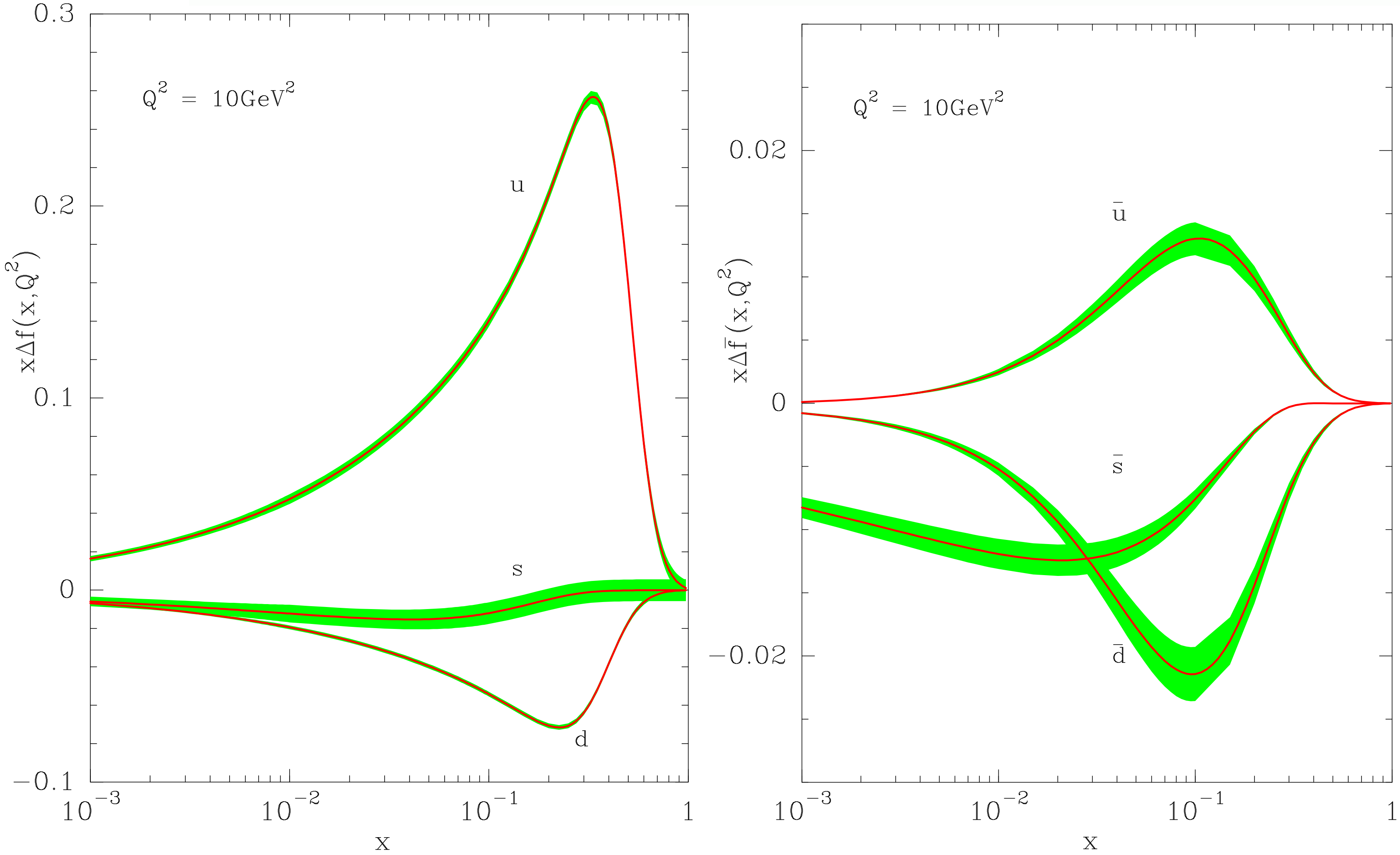
and the momentum sum rule.

There are several additional parameters to describe the strange quark-antiquark sector and for the gluon polarization. We use the constraint $s - \bar{s} = 0$.
We note that potentials become smaller for heaviest quarks and since $X_{0s}^- > X_{0s}^+$, we will have $\Delta s < 0$ like for d-quarks.

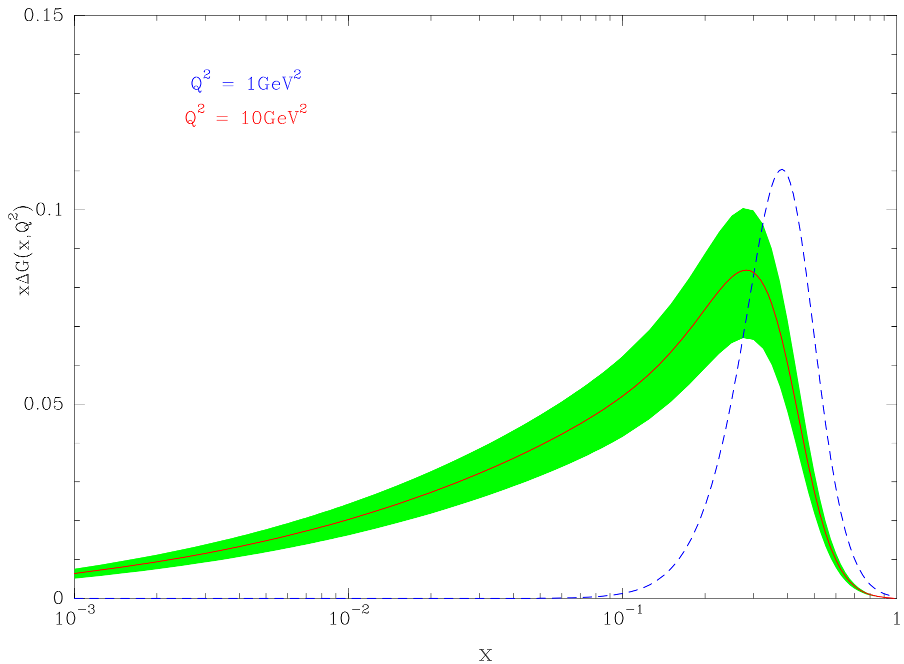
A global view of the unpolarized parton distributions

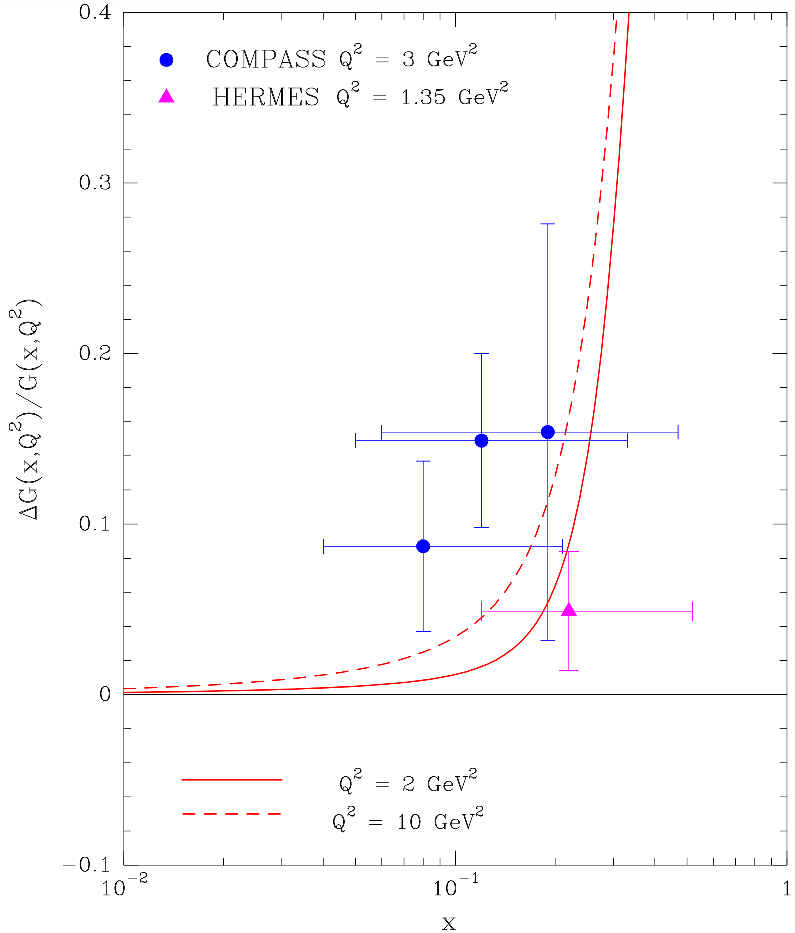


A global view of the quark (antiquark) helicity distributions



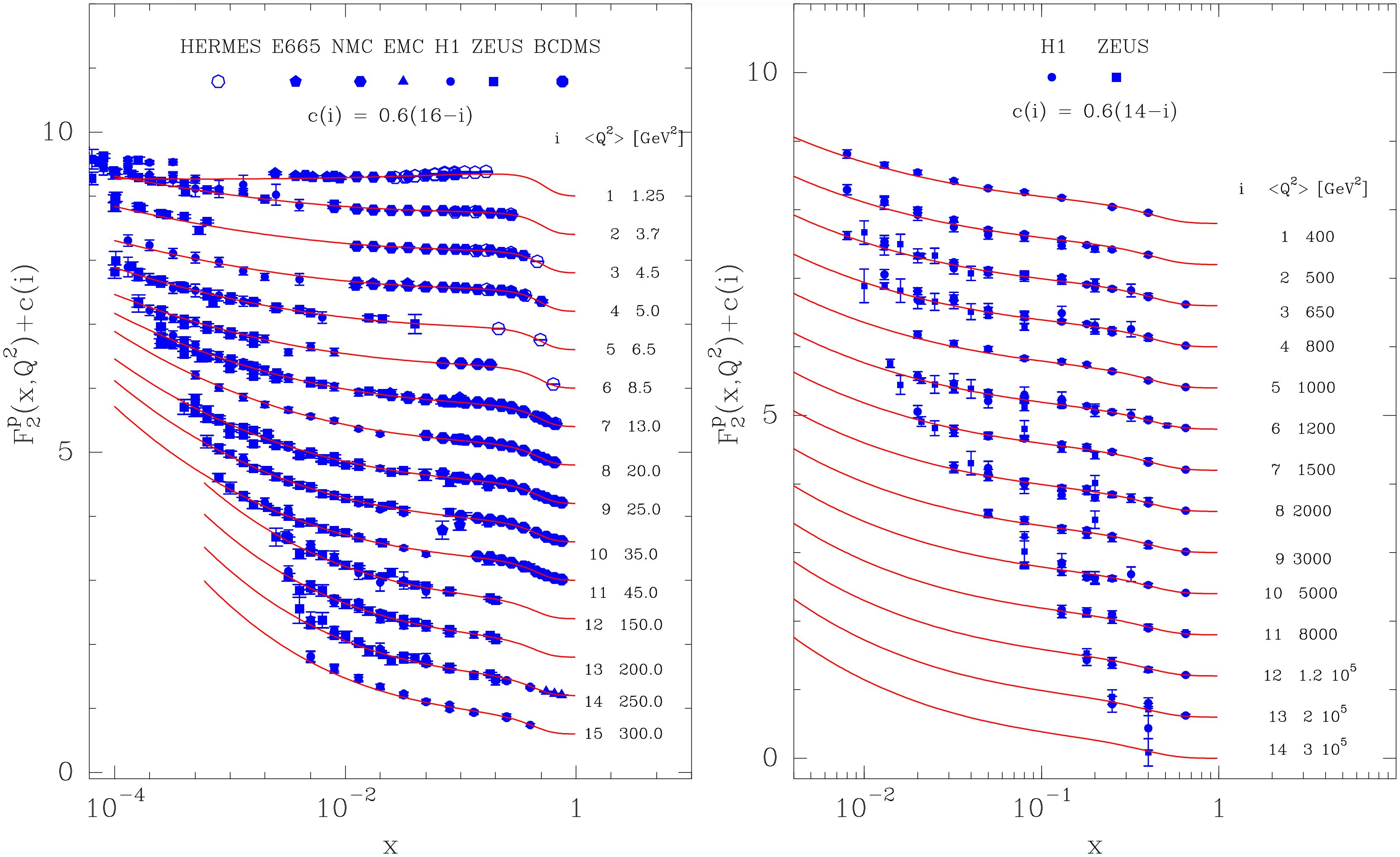
The resulting gluon helicity distribution



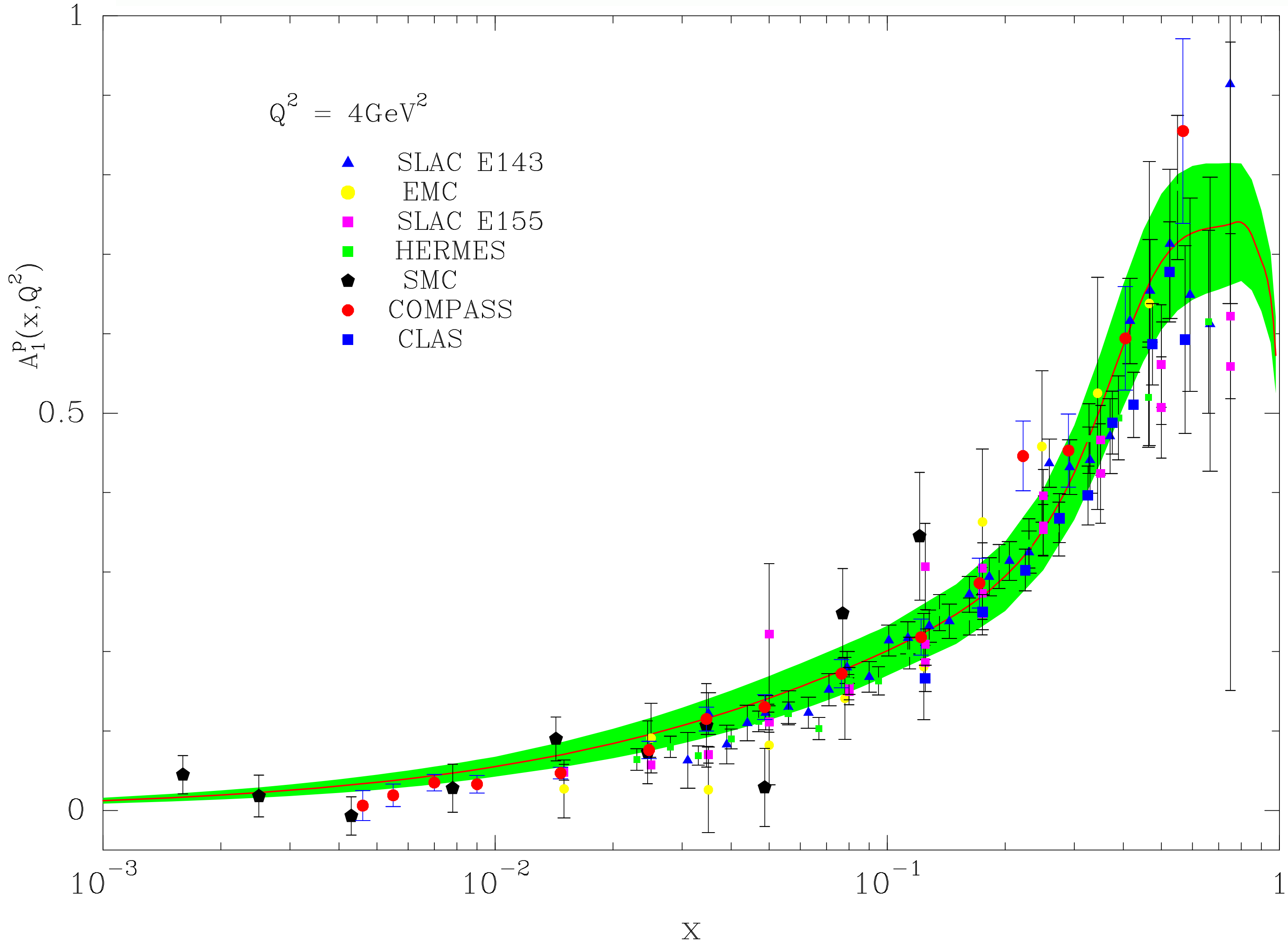


It is concentrated in the medium x-region. We show a comparison with COMPASS data STAR and PHENIX at BNL-RHIC can check it

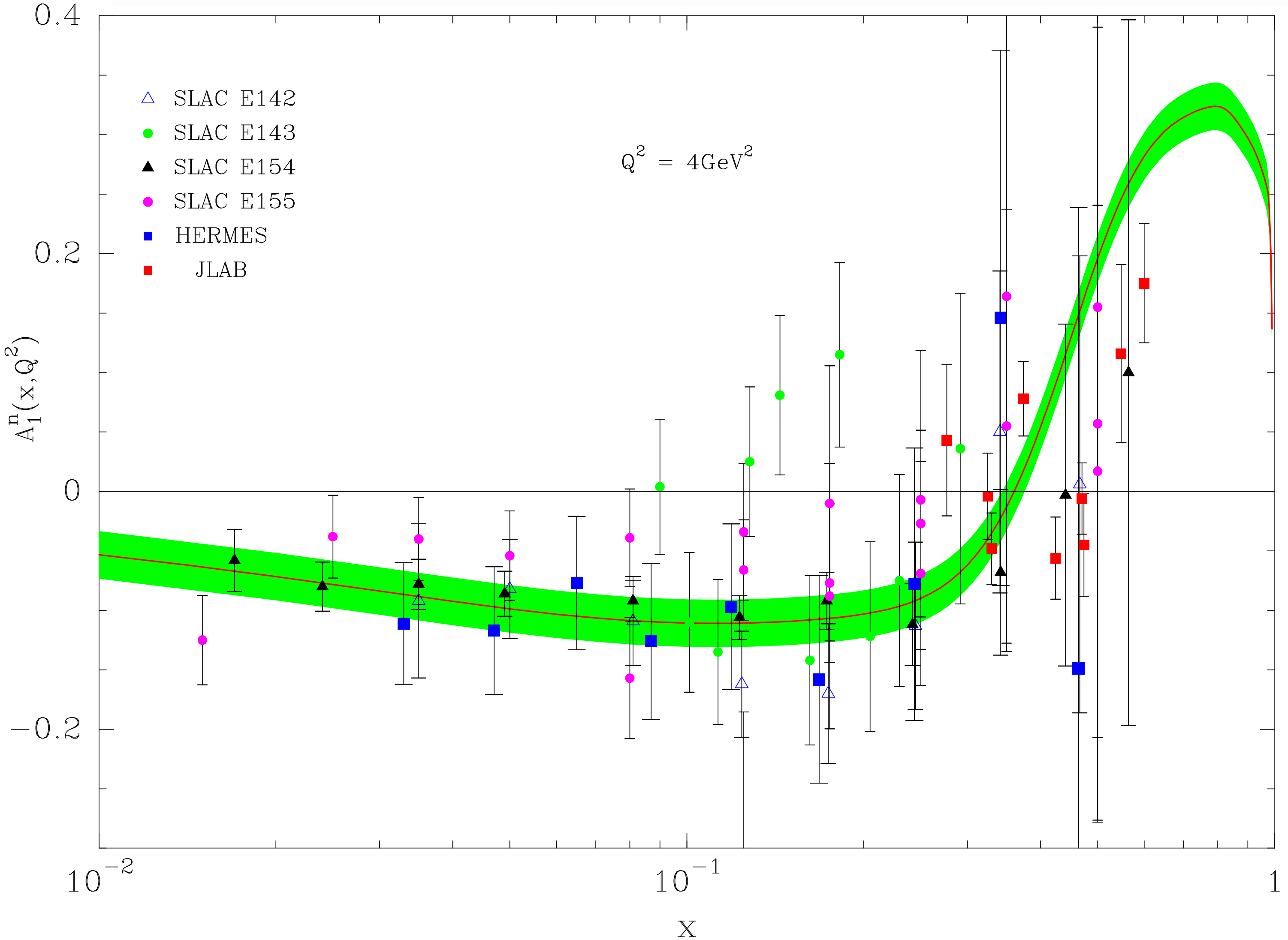
A compilation of data on $F_2^p(x, Q^2)$ *in DIS*



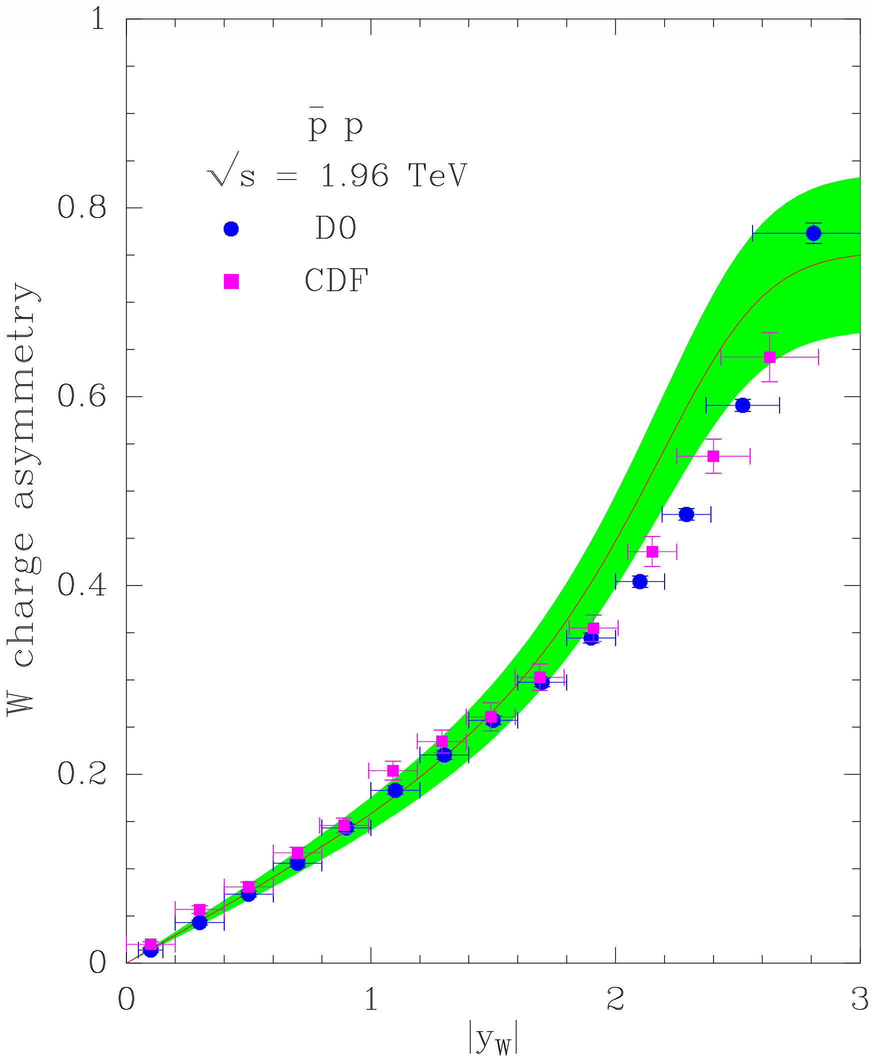
A compilation of data on $A_1^p(x, Q^2)$ in DIS



A compilation of data on $A_1^n(x, Q^2)$ in DIS

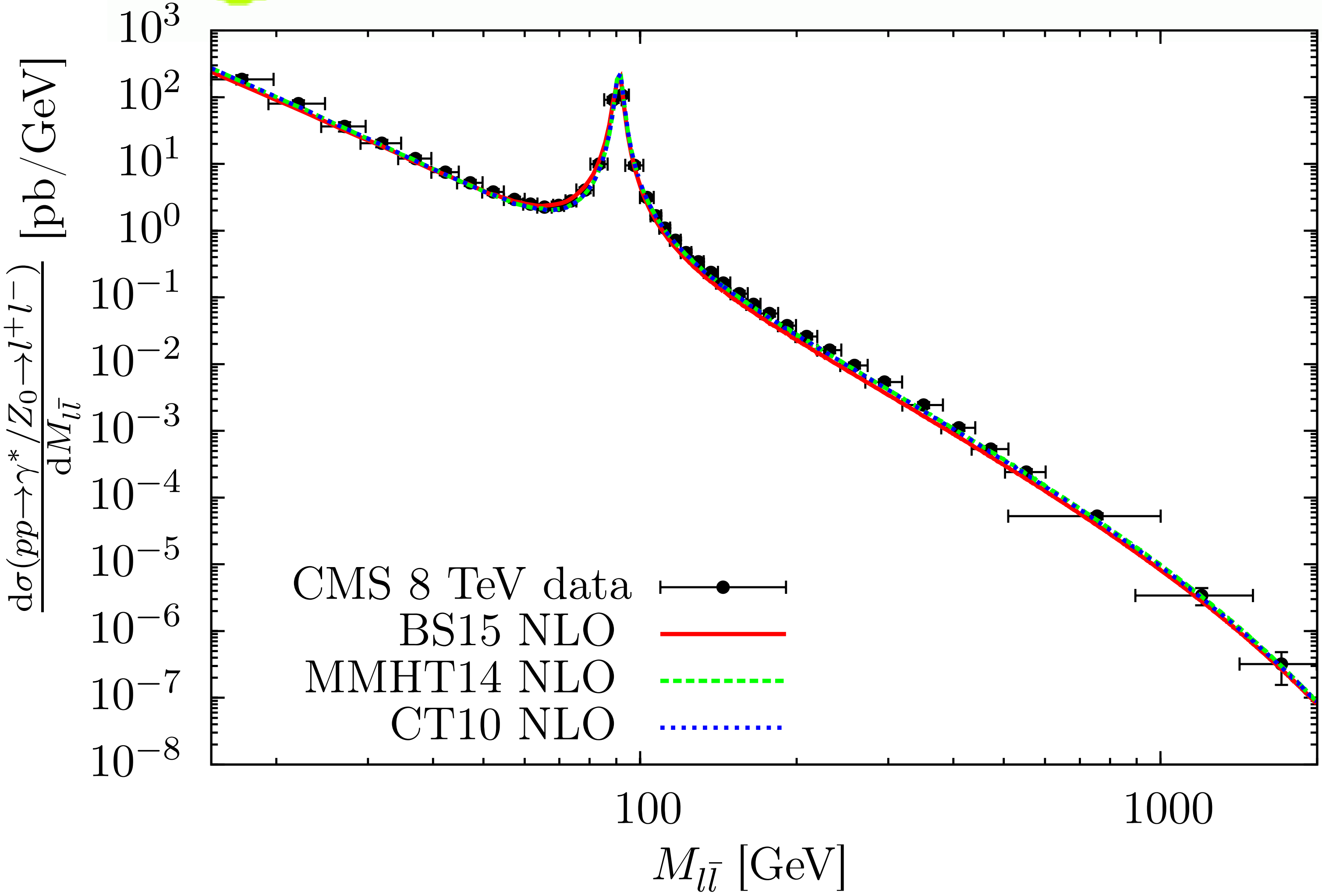


The predicted charge asymmetry



It is sensitive to the ratio d/u

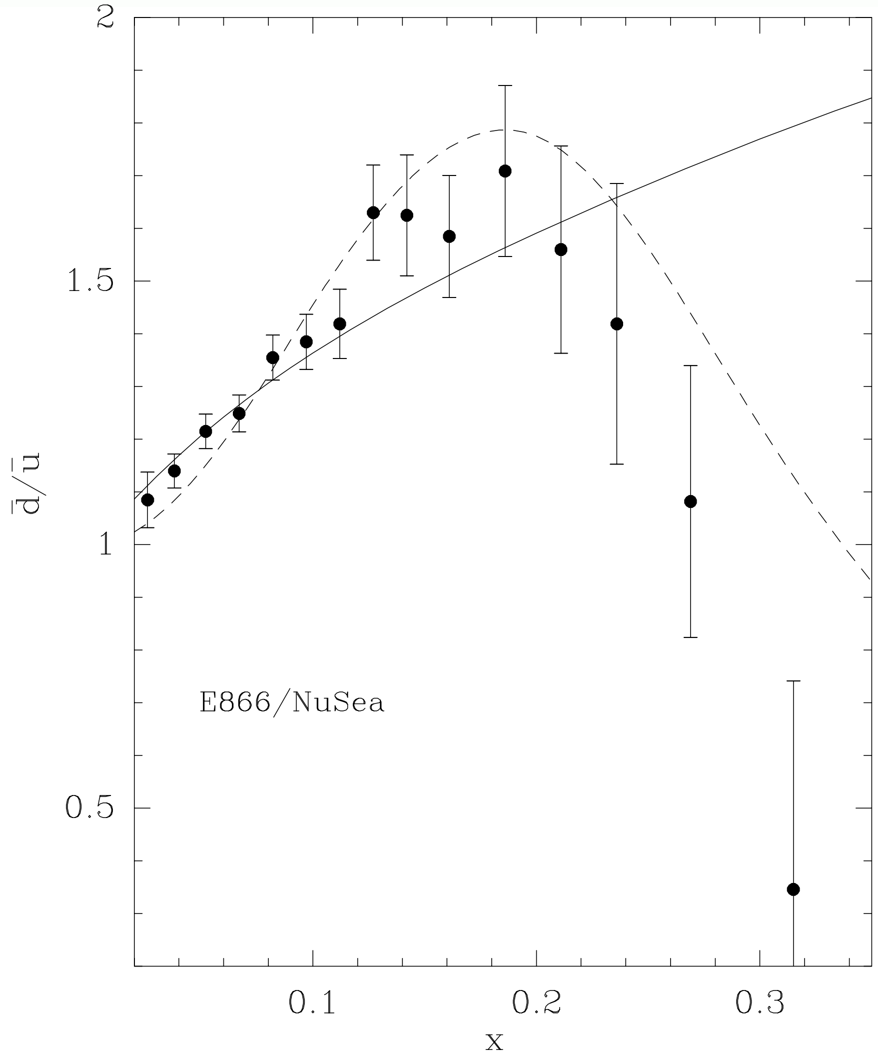
A remarkable simple process: Drell-Yan



Excellent agreement at LHC up to very high dimuon masses

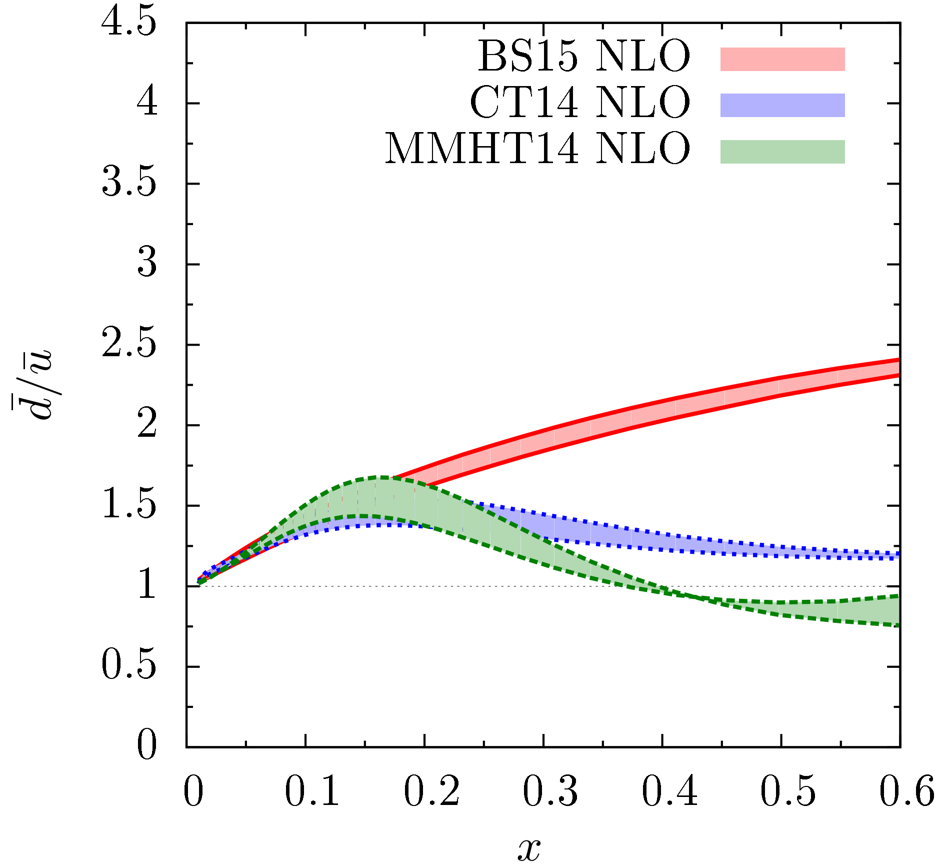
No way to discriminate between different PDF sets

Important issue: $\bar{d}/\bar{u}$ ***at large*** x ***and high*** Q^2



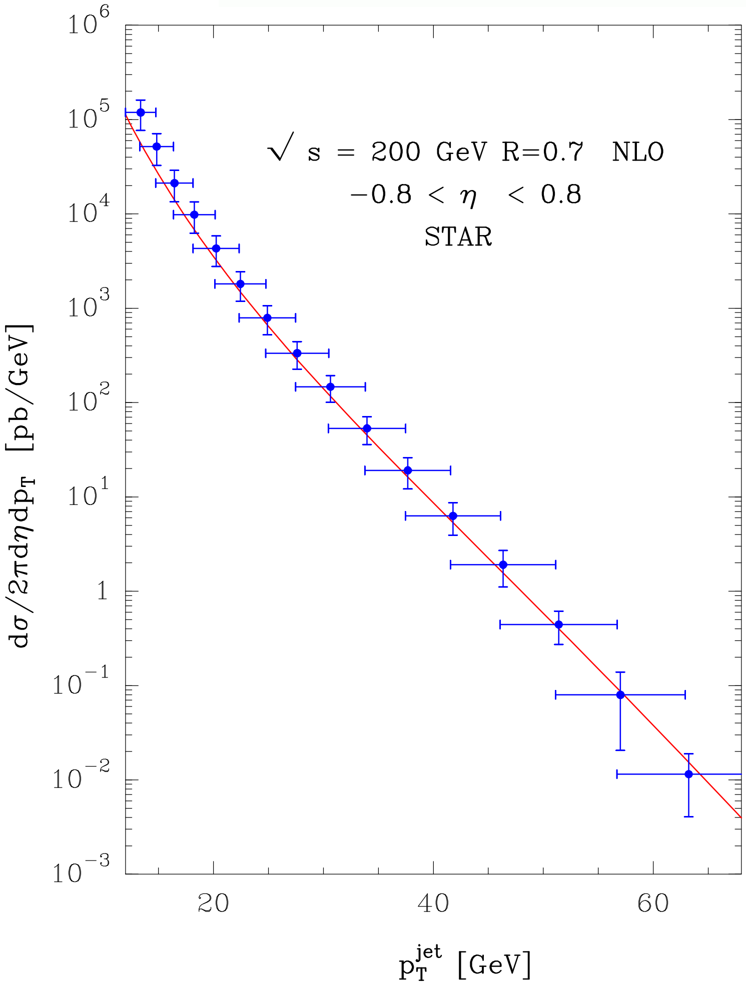
We look forward to the results of E906 at FNAL

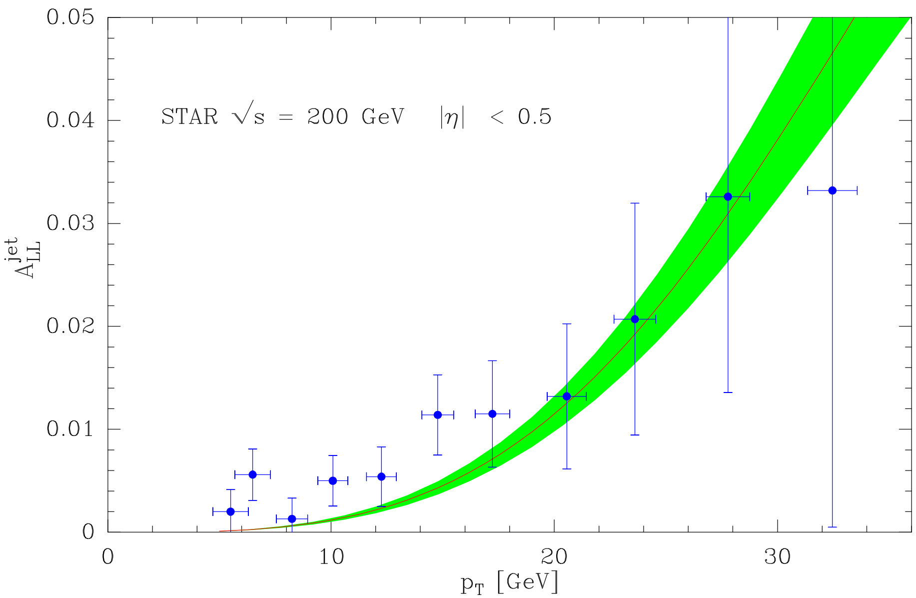
Important issue: $\bar{d}/\bar{u}$ ***at large*** x ***and high*** Q^2



Ratio of $W^{\pm}$ cross sections: Another possible way to access it

Single-jet production at RHIC: cross section and double helicity asymmetry





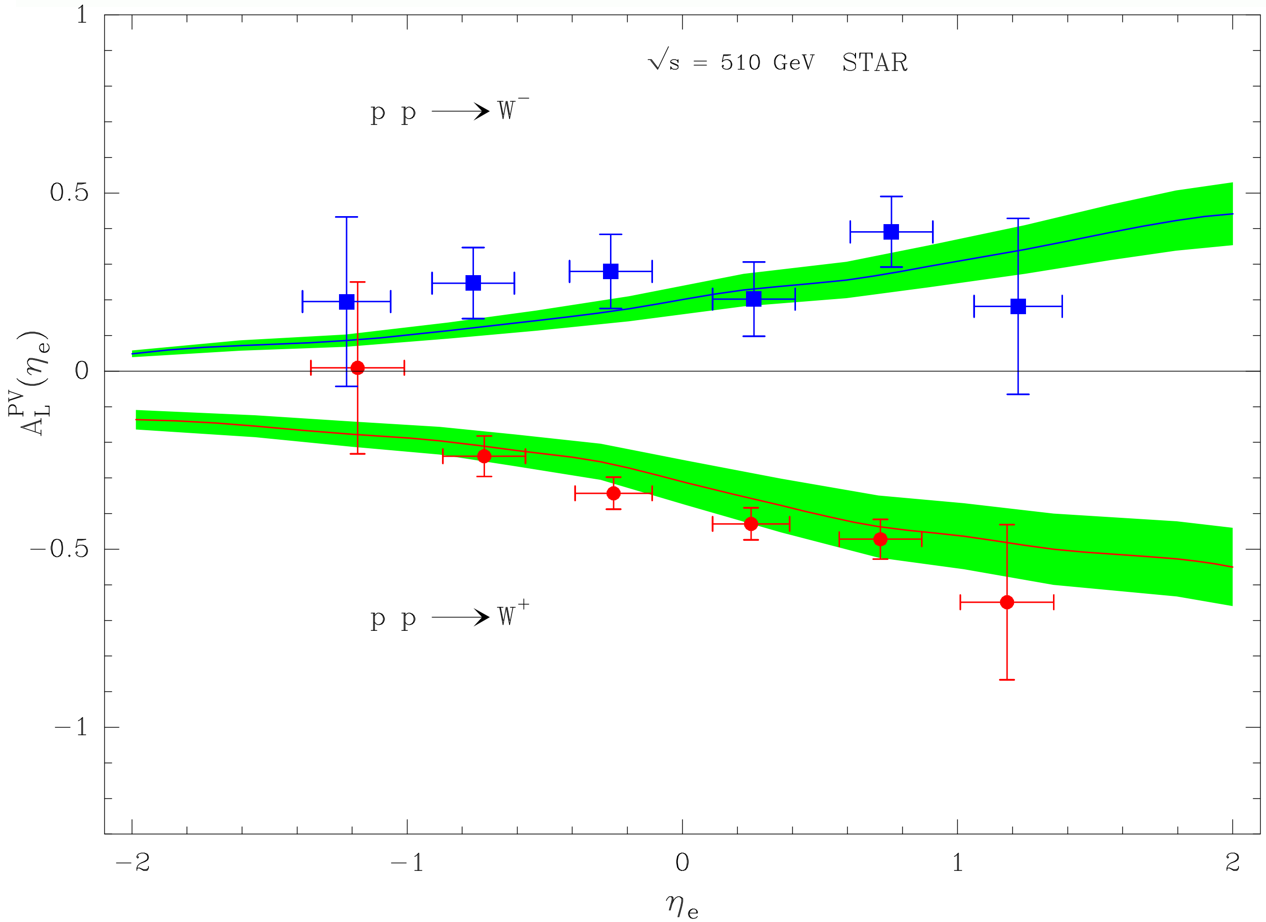
Helicity asymmetry in $W^{\pm}$ production at BNL-RHIC

Consider the processes $\overrightarrow{p}p \to W^{\pm} + X \to e^{\pm} + X$, where the arrow denotes a longitudinally polarized proton and the outgoing $e^{\pm}$ have been produced by the leptonic decay of the $W^{\pm}$ boson. The helicity asymmetry is defined as $A_L = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-}$.
Here σ_h denotes the cross section where the initial proton has helicity h.
For W^- production, the numerator of the asymmetry is found to be proportional to

$$\Delta\bar{u}(x_1, M_W^2)d(x_2, M_W^2)(1-\cos\theta)^2 - \Delta d(x_1, M_W^2)\bar{u}(x_2, M_W^2)(1+\cos\theta)^2 \ ,$$

where θ is the polar angle of the electron in the $c.m.s.$, with $\theta = 0$ in the forward direction of the polarized parton. The denominator of the asymmetry has a similar form, with a plus sign between the two terms of the above expression. For W^+ production, the asymmetry is obtained by interchanging the quark flavors ($u \leftrightarrow d$).
We first show below the results of the calculations of the helicity asymmetries, versus the charged-lepton pseudo-rapidity and for a clear interpretation some explanations are required. At high negative η_e, one has $x_2 >> x_1$ and $\theta >> \pi/2$, so the first term above dominates and the asymmetry generated by the W^- production is driven by $\Delta\bar{u}(x_1)/\bar{u}(x_1)$, for medium values of x_1. Similarly for high positive η_e, the second term dominates and now the asymmetry is driven by $-\Delta d(x_1)/d(x_1)$, for large values of x_1. So we have a clear separation between these two contributions.

The parity-violating helicity asymmetry for $W^{\pm}$ production



Statistical prediction compared with STAR data (2014)

Transverse momentum dependence (TMD) of the PDF

How to introduce the TMD of the PDF ?

There are several possibilities

- Assume factorization and simple Gaussian behavior for the PDF

$$q(x,k_T) = q(x)\frac{1}{\pi\mu_0^2}\exp[-k_T^2/\mu_0^2]\ ,$$

and also for the fragmentation function

$$D(z,q_T) = D(z)\frac{1}{\pi\mu_D^2}\exp[-q_T^2/\mu_D^2]\ .$$

A naive assumption which has no theoretical justification.
Cannot be valid for ALL x-values

- No factorization: The statistical distributions for quarks and antiquarks

TMD in the statistical approach

The parton distributions $p_i(x, k_T^2)$ of momentum k_T, must obey the momentum sum rule

$$\sum_i \int_0^1 dx \int x p_i(x, k_T^2) dk_T^2 = 1 \ ,$$

and also the transverse energy sum rule

$$\sum_i \int_0^1 dx \int p_i(x, k_T^2) \frac{k_T^2}{x} dk_T^2 = M^2 \ .$$

From the general method of statistical thermodynamics we are led to put $p_i(x, k_T^2)$ in correspondance with the following expression

$$\exp(\frac{-x}{\bar{x}} + \frac{-k_T^2}{x\mu^2}) \ ,$$

where μ^2 is a parameter interpreted as the transverse temperature.

So we have now the main ingredients for the extension to the TMD of the statistical PDF.

We obtain in a natural way the Gaussian shape with NO x, k_T factorization

TMD in the statistical approach

The quantum statistical distributions for quarks and antiquarks read in this case

$$xq^h(x,k_T^2) = \frac{F(x)}{\exp(x - X_{0q}^h)/\bar{x} + 1} \frac{1}{\exp(k_T^2/x\mu^2 - Y_{0q}^h) + 1} \ ,$$

$$x\bar{q}^h(x,k_T^2) = \frac{\bar{F}(x)}{\exp(x + X_{0q}^{-h})/\bar{x} + 1} \frac{1}{\exp(k_T^2/x\mu^2 + Y_{0q}^{-h}) + 1},$$

where

$$F(x) = \frac{Ax^{b-1}X_{0q}^h}{\ln(1+\exp Y_{0q}^h)\mu^2} = \frac{Ax^{b-1}}{k\mu^2} \ ,$$

because Y_{0q}^h are the thermodynamical potentials chosen such that $\ln(1+\exp Y_{0q}^h) = kX_{0q}^h$, in order to recover the factors X_{0q}^h, introduced earlier. Similarly for $\bar{q}$ we have $\bar{F}(x) = \bar{A}x^{2b-1}/k\mu^2$. This determination of the 4 potentials Y_{0q}^h can be achieved with the choice $k = 2.83$. Finally μ^2 will be determined by the transverse energy sum rule and one finds $\mu^2 = 0.110\text{GeV}^2$.

Physical interpretation of $\bar{x}$ and μ^2

The basic statistical weight of a quark can be written in the form $\exp\left[(E_q - V)/T\right]$, where E_q is the quark energy in the nucleon rest frame.

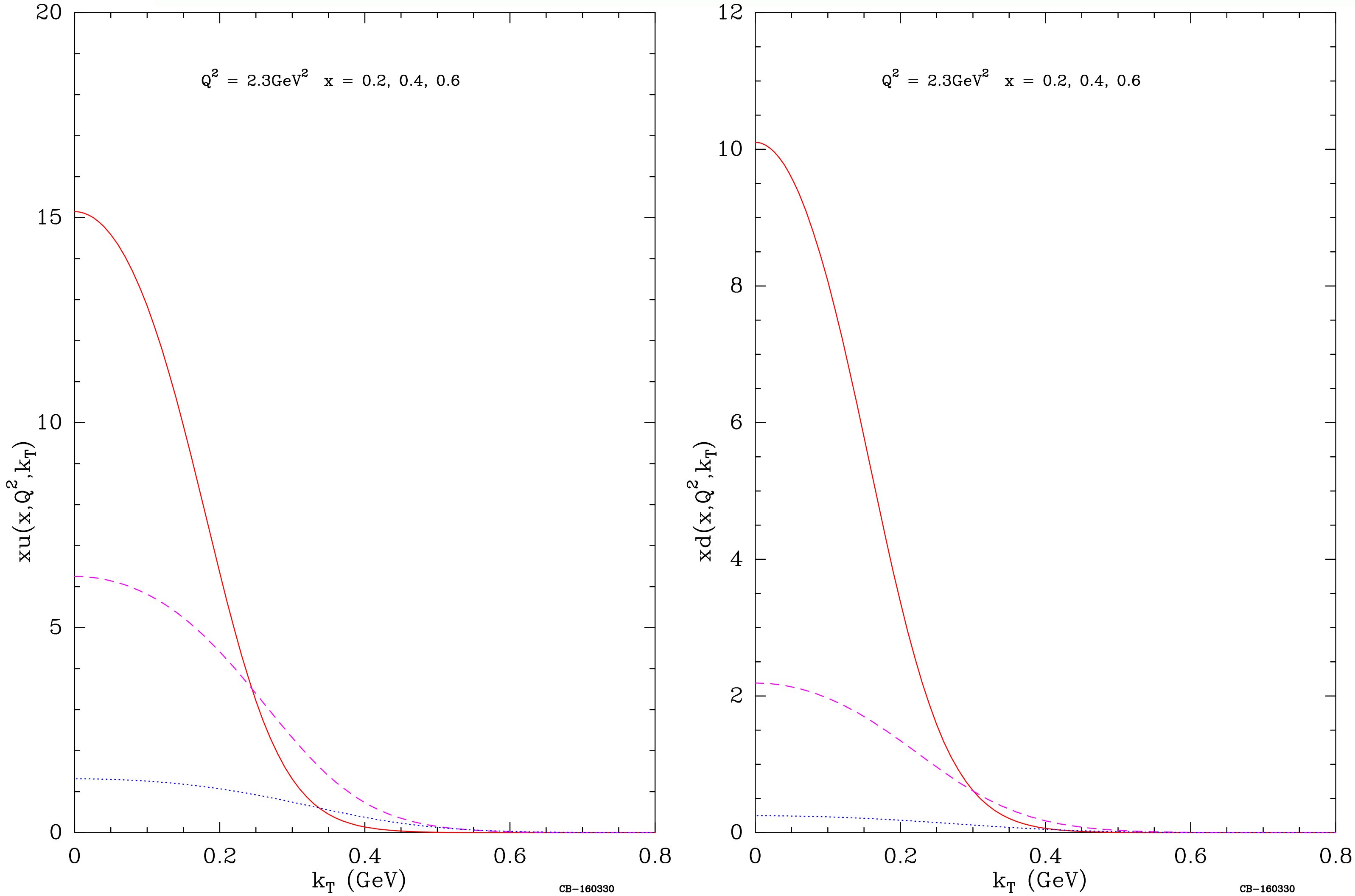
So $\bar{x}$ is related to the longitudinal temperature T_l according to $T_l = M\bar{x}/2$, where M is the nucleon mass.

Since we found $\bar{x} = 0.090$, one has $T_l = 42$ MeV.

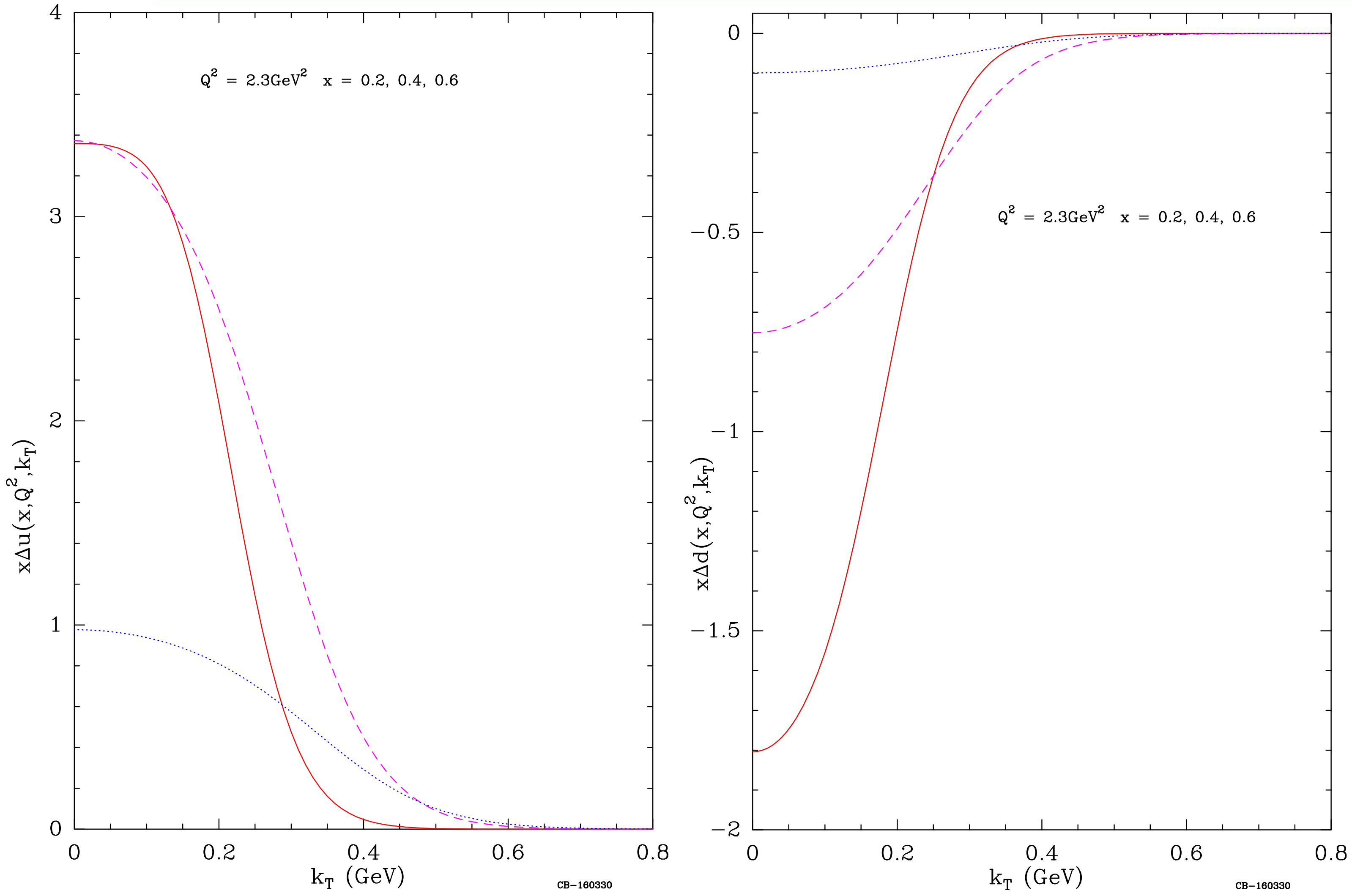
Similarly the transverse temperature T_t is related to μ according to $T_t = \mu\sqrt{\bar{x}}/2$.

Since we found $\mu^2 = 0.110$GeV^2, one has $T_t = 50$ MeV.

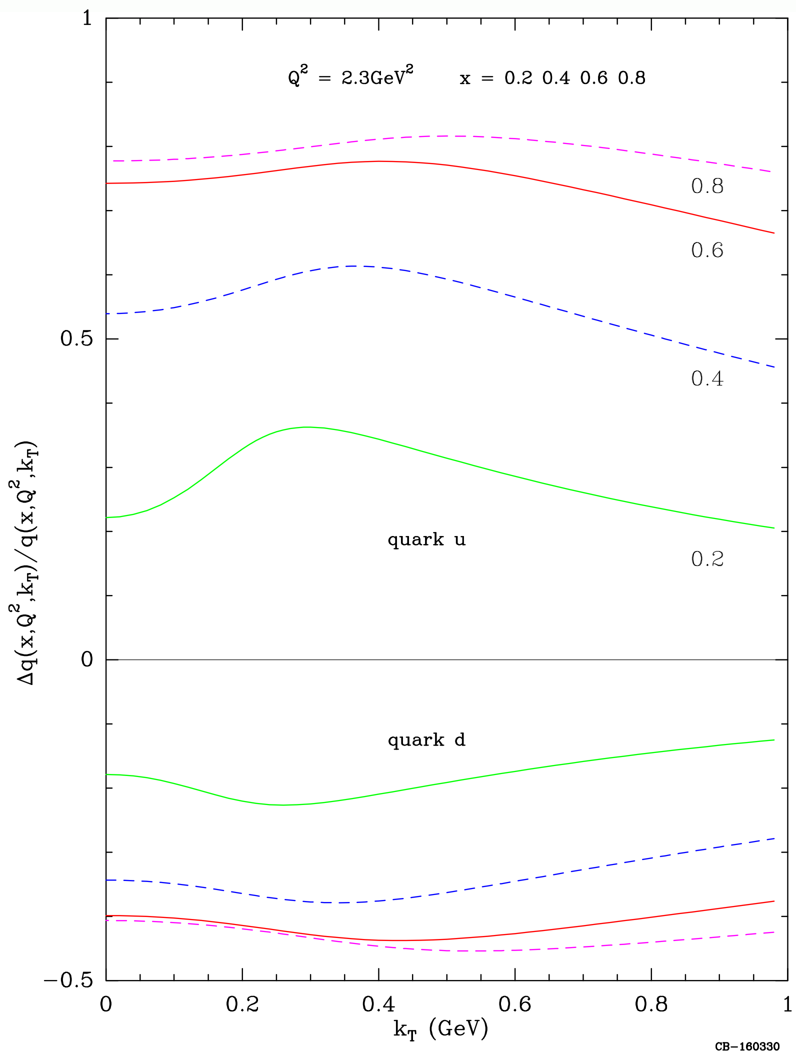
Predicted TMD quark distributions



Predicted TMD quark helicity distributions



Predicted TMD ratios $\Delta q/q$



Rather flat in k_T and increasing xith x

Melosh-Wigner effects

So far in all our quark or antiquark TMD distributions, the label "'h"' stands for the helicity along the longitudinal momentum and not along the direction of the momentum, as normally defined for a genuine helicity. The basic effect of a transverse momentum $k_T \neq 0$ is the Melosh-Wigner rotation, which mixes the components $q^{\pm}$ in the following way

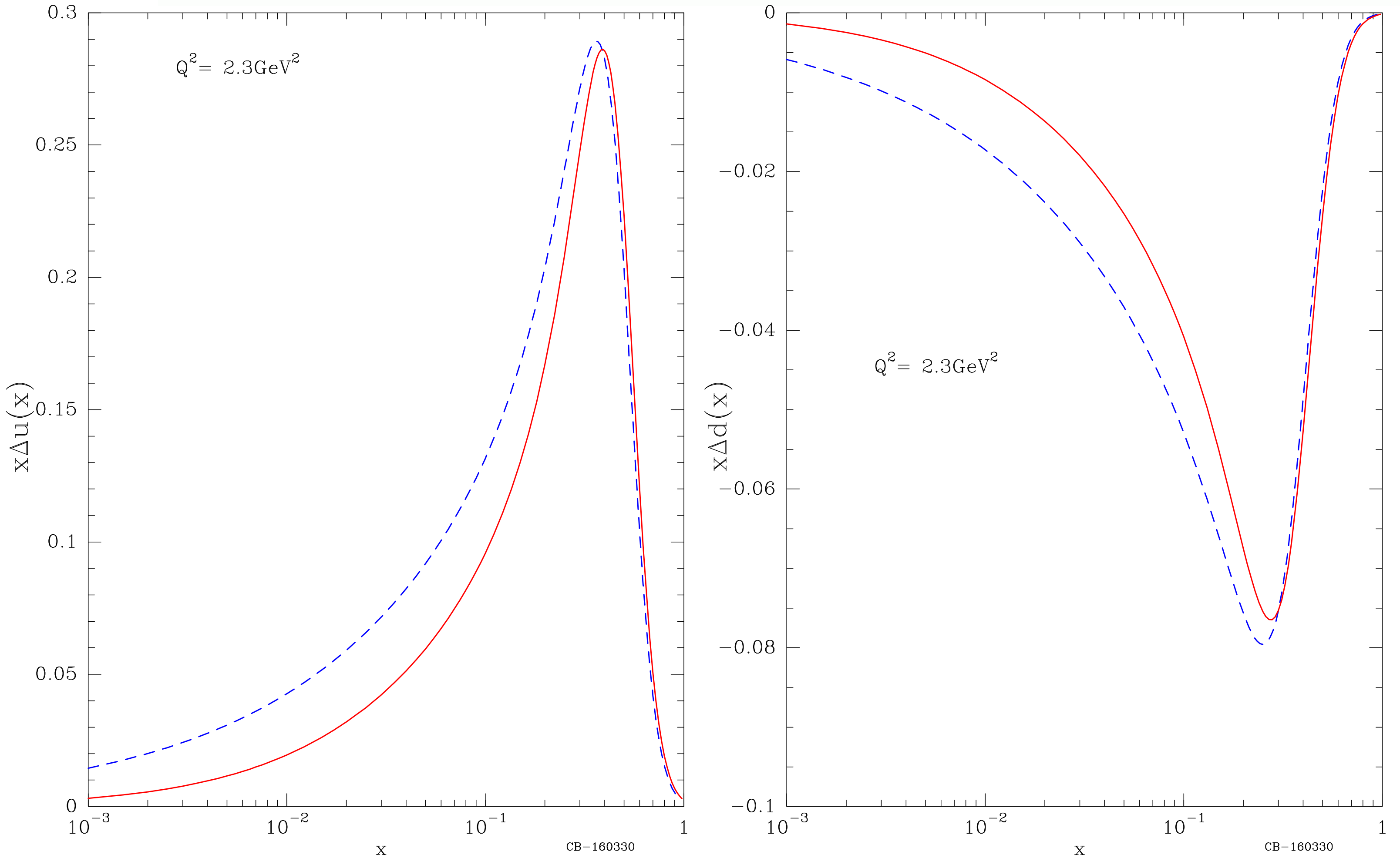
$$q^{+MW} = \cos^2\theta\, q^+ + \sin^2\theta\, q^- \quad \text{and} \quad q^{-MW} = \cos^2\theta\, q^- + \sin^2\theta\, q^+ \ ,$$

where, for massless partons, $\theta = \arctan\left(\frac{k_T}{p_0+p_z}\right)$, with $p_0 = \sqrt{k_T^2 + p_z^2}$.

It vanishes when either $k_T = 0$ or p_z goes to infinity.

Consequently $q = q^+ + q^-$ remains unchanged since $q^{MW} = q$,
whereas we have $\Delta q^{MW} = (\cos^2\theta - \sin^2\theta)\Delta q$.

Melosh-Wigner effect



The effect is relevant for small Q^2 and mainly in the low x region

Double helicity asymmetry A_{LL} *in SIDIS*

Consider the polarized SIDIS, $\ell N \to \ell' H X$ in the simple quark-parton model. According to the standard notations for DIS variables, ℓ and ℓ' are, respectively, the four-momenta of the initial and the final state leptons, $q = \ell - \ell'$ is the exchanged virtual photon momentum, P is the target nucleon momentum, P_H is the final hadron momentum, $Q^2 = -q^2, \quad x = Q^2/2P\cdot q, \quad y = P\cdot q/P\cdot \ell, \; z = P\cdot P_H/P,$ $Q^2 = xy(s - M^2)$ and $s = (\ell + P)^2$. We work in a frame with the z-axis along the virtual photon momentum direction and the x-axis in the lepton scattering plane, with positive direction chosen along the lepton transverse momentum.
The produced hadron has transverse momentum p_T.
The cross section for SIDIS of longitudinally polarized leptons off a longitudinally polarized target can be written as:

$$\frac{d^5\sigma^{\overset{\rightarrow}{\Leftarrow}}}{dx\,dy\,dz\,d^2p_T} = \frac{2\alpha^2}{xy^2 s}\;\{\mathcal{H}_1 + \lambda\, S_L \mathcal{H}_2\}\;,$$

where the arrows indicate the direction of the lepton ($\rightarrow$) and target nucleon ($\Leftarrow$) polarizations, with respect to the lepton momentum; λ, and S_L are the magnitudes of the longitudinal beam polarization and the longitudinal target polarization, respectively.

Double helicity asymmetry A_{LL} in SIDIS

$$\mathcal{H}_1(p_T) = \sum_q e_q^2 \int d^2k_T q(x, k_T)\, \pi y^2\, \frac{\hat{s}^2 + \hat{u}^2}{Q^4}\, D_q^h(z, q_T),$$

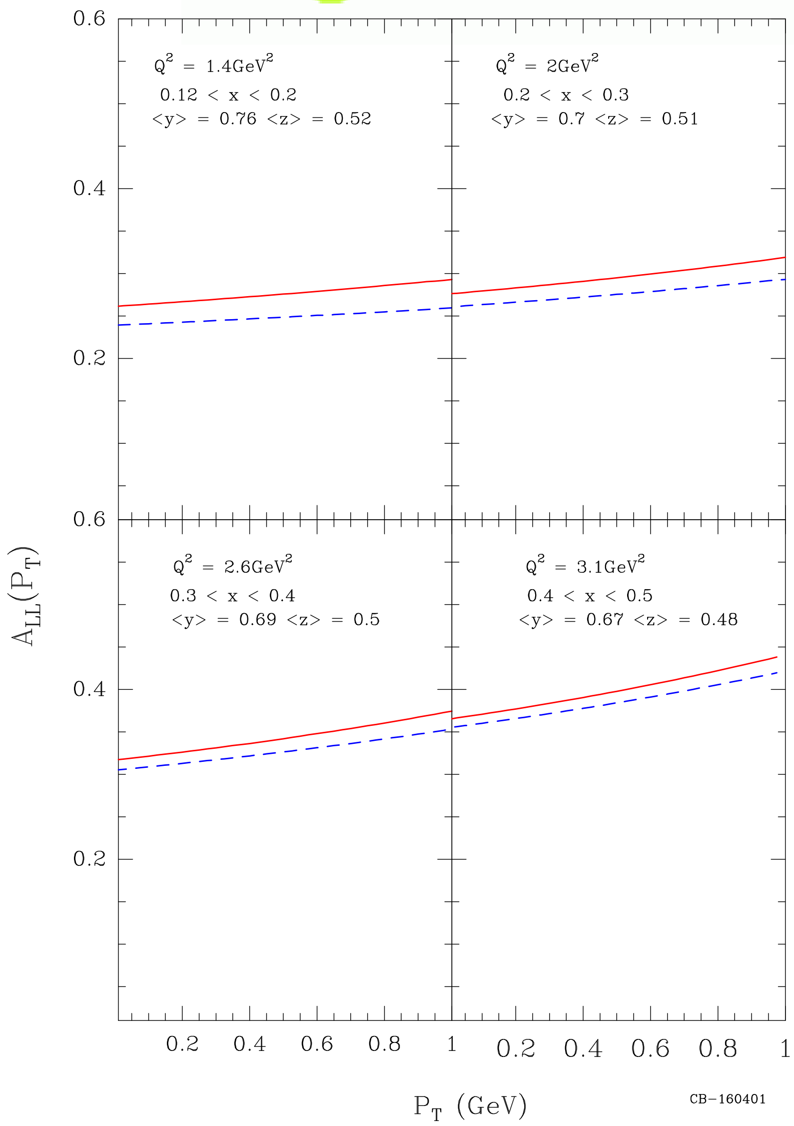
$$\mathcal{H}_2(p_T) = \sum_q e_q^2 \int d^2k_T \Delta q'(x, k_T)\, \pi y^2\, \frac{\hat{s}^2 - \hat{u}^2}{Q^4}\, D_q^h(z, q_T),$$

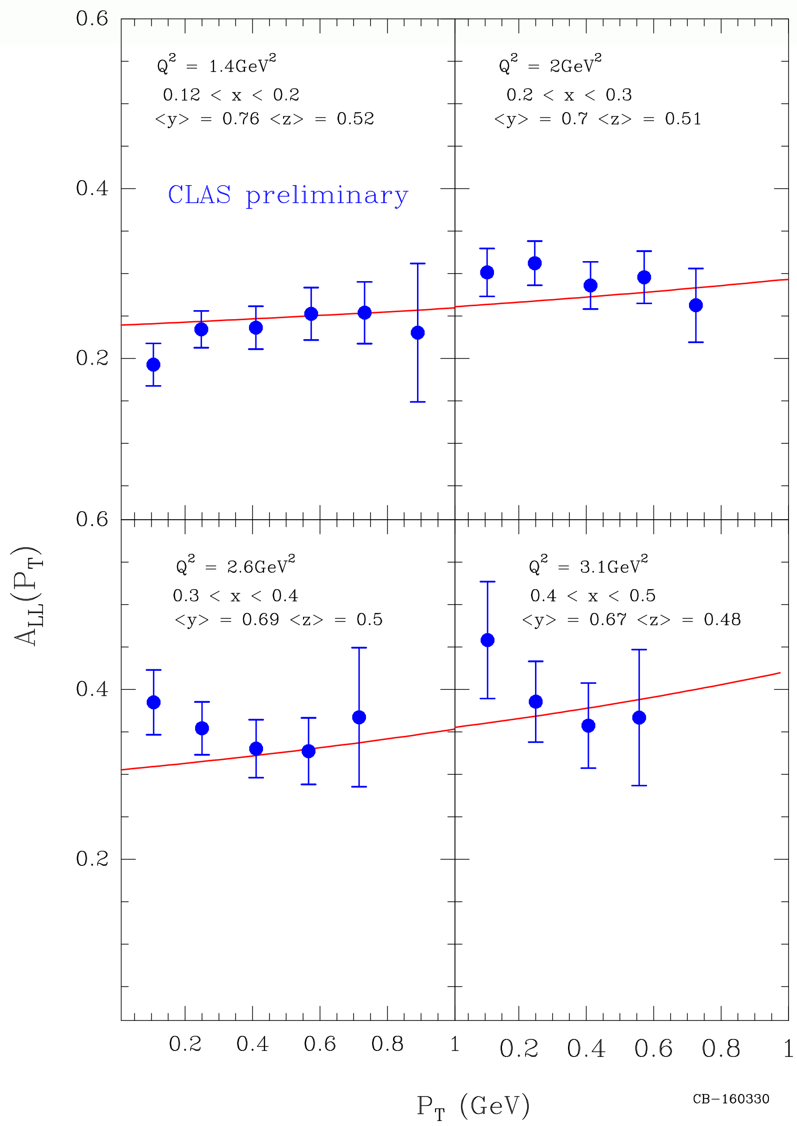
where $p_T = q_T + zk_T$ and q_T is the intrinsic transverse momentum of the hadron H with respect to the fragmenting quark direction. Here $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the Mandelstam variables for the subprocess $\ell q \to \ell q$. These two contributions give, respectively, the unpolarized cross section and the numerator of the double helicity asymmetry A_{LL}

$$\frac{d^5\sigma}{dx\, dy\, dz\, d^2p_T} = \frac{2\alpha^2}{x\, y^2 s}\, \mathcal{H}_1 \qquad \frac{d^5\sigma^{++}}{dx\, dy\, dz\, d^2p_T} - \frac{d^5\sigma^{+-}}{dx\, dy\, dz\, d^2p_T} = \frac{4\alpha^2}{x\, y^2 s}\, \mathcal{H}_2\ ,$$

where $+, -$ stand for helicity states. So we simply have $A_{LL} = 2\mathcal{H}_2/\mathcal{H}_1$.
We take for $D_q^h(z, q_T)$, the standard factorized Gaussian model, since we have not yet generalized our statistical approach to the TMD fragmentation functions.

Double helicity asymmetry A_{LL} in SIDIS





Conclusions

- A new set of PDF is constructed in the framework of a statistical approach of the nucleon.
- All unpolarized and polarized distributions depend upon a small number of free parameters, with some physical meaning.
- New tests against experiments in particular, for unpolarized and polarized sea distributions, are very satisfactory.
- Gluon helicity distribution is concentrated in the medium x-region.
A real challenge
- Another challenge is the ratio $\bar{d}/\bar{u}$ in the high x-region.
- This statistical approach has a good predictive power up to LHC energies
- Extension to TMD has been achieved and must be checked more accurately together with Melosh-Wigner effects in the low x-region