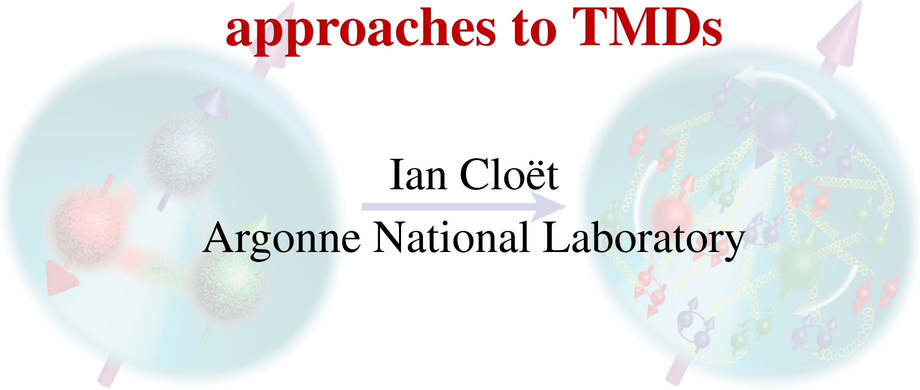
Dyson-Schwinger Equation approaches to TMDs



Ian Cloët

Argonne National Laboratory

Parton transverse momentum distributions at large x:
a window into parton dynamics in nucleon structure within QCD

ECT*, 11–25 April 2016

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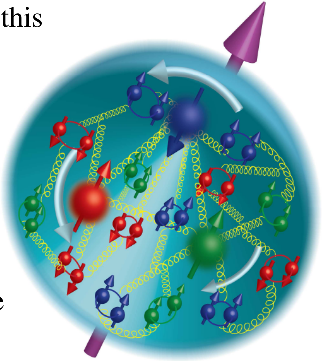
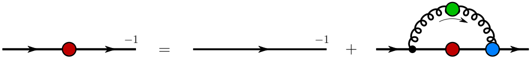
QCD: The Unifying Challenge

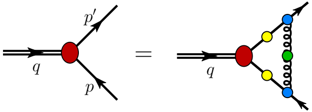
- Understanding QCD means to chart and compute this distribution of matter and energy within hadrons and nuclei; together with the complementary process of fragmentation functions
  - but *a priori* have no idea what QCD can produce



- Solving QCD explain how massless gluons and light quarks form hadrons & thereby explain the origin of $\sim 98\%$ of the mass in the visible universe
  - must understand the emergent phenomena of *confinement* and *dynamical chiral symmetry breaking*
  - *best promise for progress is a strong interplay between experiment and theory*

- *In the DSEs an understanding of QCD is gained by exposing the properties and behaviour of its dressed propagators, dressed vertices and interaction kernels*





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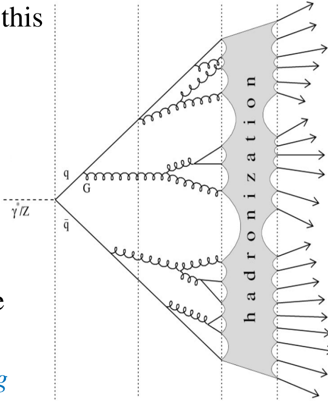

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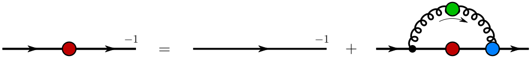
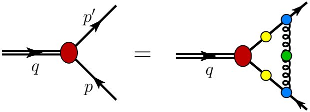
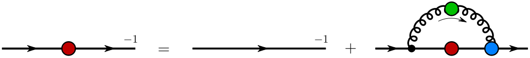




- The equations of motion of QCD $\Longleftrightarrow$ QCD's Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - must implement a symmetry preserving truncation
- Most important DSE is QCD's gap equation $\Longrightarrow$ *dressed quark propagator*

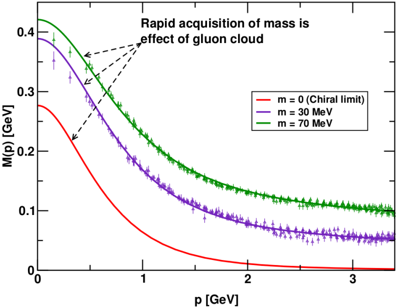


  - ingredients – *dressed gluon propagator & dressed quark-gluon vertex*

$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

- $S(p)$ has correct perturbative limit
- $M(p^2)$ exhibits dynamical mass generation $\Longleftrightarrow$ DCSB
- $S(p)$ has complex conjugate poles
  - no real mass shell $\Longleftrightarrow$ confinement

[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]



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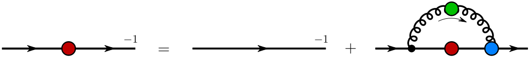

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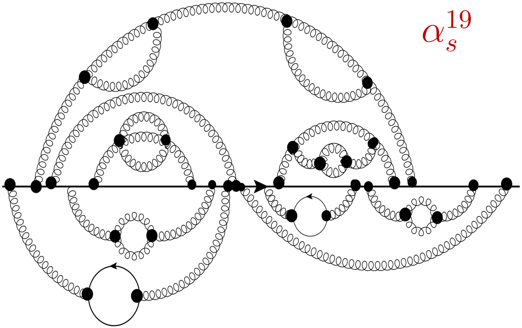
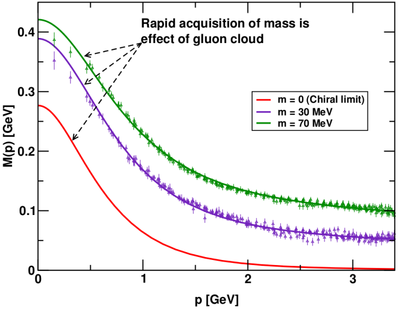


α_s^{19}

[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]



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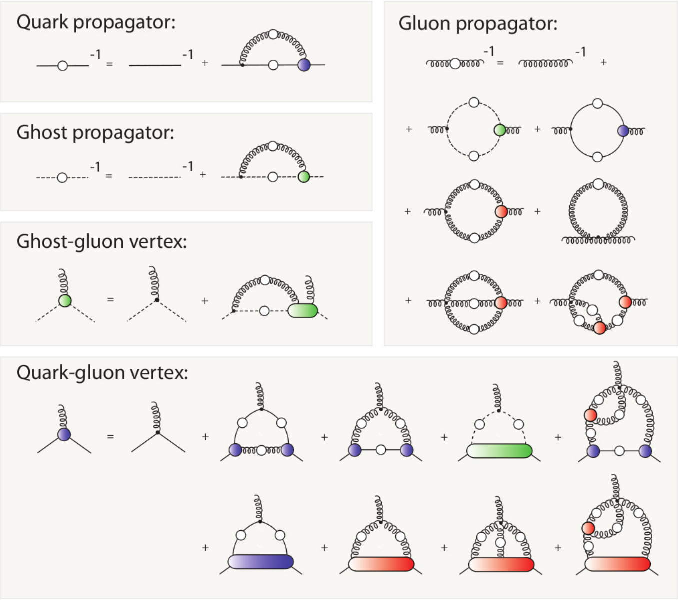
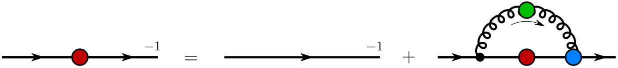


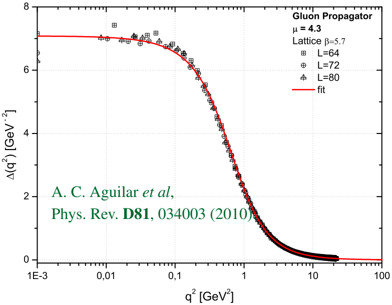
ETC!



- Not possible to solve tower of equations – start with gap equation
  - need ansatz for *dressed gluon propagator* × *dressed quark-gluon vertex*
  - truncation must preserve symmetries, e.g., electromagnetic current, chiral

$$D^{\mu\nu}(p) = \left(\delta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Delta(q^2) + \xi\,\frac{q^\mu q^\nu}{q^4}$$

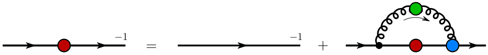
$$\Gamma^{a,\mu}_{gqq}(p',p) = \frac{\lambda^a}{2}\sum_{i=1}^{12}\Lambda_i^\mu\; f_i(p'^2,p^2,q^2) = \frac{\lambda^a}{2}\left[\Gamma_L^\mu(p',p) + \Gamma_T^\mu(p',p)\right]$$

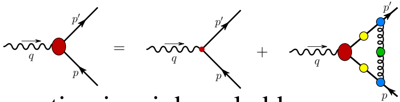


- usually choose Landau gauge $\xi = 0$

- Therefore both gluons and quarks posses dynamically generated masses
  - *QCD dynamically generates its own infrared cutoffs*

Rainbow Ladder Truncation





- The most common symmetry preserving truncation is rainbow-ladder

$$\frac{1}{4\pi}\, g^2\, D_{\mu\nu}(p-k)\,\Gamma_\nu(p,k) \quad \longrightarrow \quad \alpha_{\text{eff}}(p-k)\, D^{\text{free}}_{\mu\nu}(p-k)\,\gamma_\nu$$

- Need model for $\alpha_{\text{eff}}(k^2)$ – must agree with perturbative QCD for large k^2
  - Maris–Tandy model is historically the most successful example [PRC 60, 055214 (1999)]

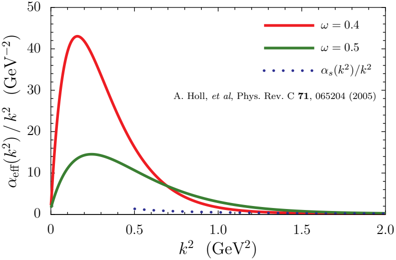
$$\alpha_{\text{eff}}(k^2) = \frac{\pi\, D}{\omega^6}\, k^4\, e^{-k^2/\omega^2} + \frac{24\,\pi}{25}\left(1 - e^{-k^2/\mu^2}\right)\ln^{-1}\!\left[e^2 - 1 + \left(1 + k^2/\Lambda^2_{\text{QCD}}\right)^2\right]$$

- Satisfies vector & axial-vector WTIs

$$q_\mu\, \Gamma^\mu_{\gamma qq}(p',p) = \hat{Q}_q\left[S_q^{-1}(p') - S_q^{-1}(p)\right]$$

[em current conservation]

$$q_\mu\, \Gamma_5^{\mu,i}(p',p) = S^{-1}(p')\,\gamma_5\, t_i + t_i\,\gamma_5\, S^{-1}(p) + 2\, m\, \Gamma^i_\pi(p',p) \qquad \text{[DCSB]}$$



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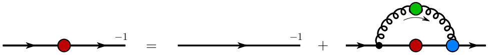

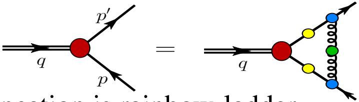

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- Need model for $\alpha_{\text{eff}}(k^2)$ – must agree with perturbative QCD for large k^2
  - Qin–Chang model is a modern update [PRC 84, 042202 (2011)]

$$\alpha_{\text{eff}}(k^2) = \frac{\pi\, D}{\omega^4}\, k^2\, e^{-k^2/\omega^2} + \frac{24\,\pi}{25}\left(1 - e^{-k^2/\mu^2}\right)\ln^{-1}\!\left[e^2 - 1 + \left(1 + k^2/\Lambda^2_{\text{QCD}}\right)^2\right]$$

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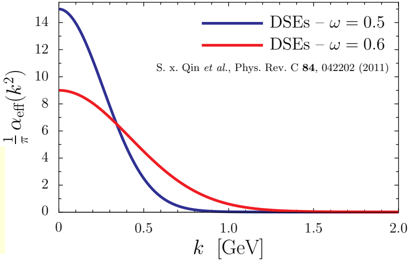
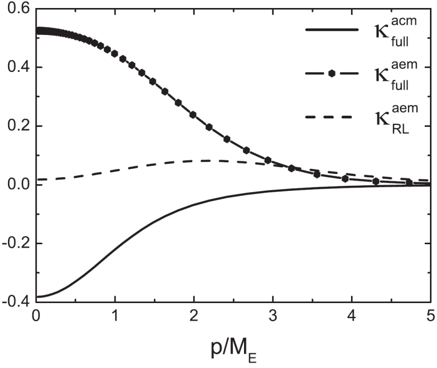


- Include *"anomalous chromomagnetic"* term in quark-gluon vertex

$$\frac{1}{4\pi} g^2 D_{\mu\nu}(\ell)\Gamma_\nu(p',p) \rightarrow \alpha_{\text{eff}}(\ell) D^{\text{free}}_{\mu\nu}(\ell)[\gamma_\nu + i\sigma^{\mu\nu} q_\nu \tau_5(p',p) + \ldots]$$

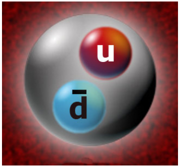
- In chiral limit *anomalous chromomagnetic* term can only appear through DCSB – since it is not chirally symmetric
- Expect strong gluon dressing to produce non-trivial structure for a dressed quark
  - recall dressing produces – from massless quark – a $M \sim 400\,\text{MeV}$ dressed quark
  - dressed quarks likely contain large amounts of orbital angular momentum
- Large anomalous chromomagnetic moment in the quark-gluon vertex – *produces a large quark anomalous electromagnetic moment*
  - *dressed quarks are not point particles!*

[L. Chang, Y. -X. Liu, C. D. Roberts, PRL **106**, 072001 (2011)]

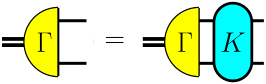


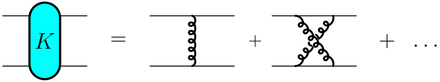
The Pion in QCD

- Today the pion is understood as both a bound state of a *dressed-quark* and a *dressed-antiquark* in QFT and the Goldstone mode associated with DCSB in QCD



- In QFT a two-body bound state (e.g. a pion or rho) is described by the BSE:





- For the pion the solution has the general form

$$\Gamma_\pi(p,k) = \gamma_5 \Big[E_\pi(p,k) + \not{p}\, F_\pi(p,k) + \not{k}\, k \cdot p\, \mathcal{G}(p,k) + \sigma^{\mu\nu} k_\mu p_\nu\, \mathcal{H}(p,k) \Big]$$

  - the kernel must yield a solution that encapsulates the consequences of dynamical chiral symmetry breaking, e.g., in chiral limit $m_\pi = 0$ & also $m_\pi^2 \propto m_u + m_d$

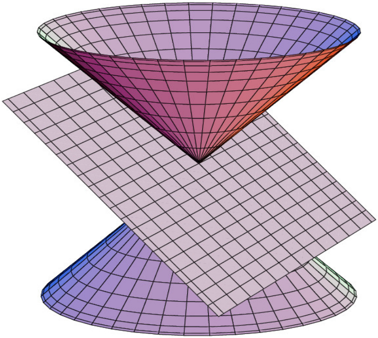
- DSCB implies, e.g., a Goldberger-Treiman-like relation for the pion:

$$f_\pi\, E_\pi(p=0,k^2) = B(k^2) \qquad \text{recall} \qquad S(p)^{-1} = \not{p}\, A(k^2) + B(k^2)$$

  - *DCSB implies intimate connection between 1-body and 2-body problems*

Light-Front Wave Functions

- In equal-time quantization a hadron wave function is a frame dependent concept
  - boost operators are dynamical, that is, they are interaction dependent

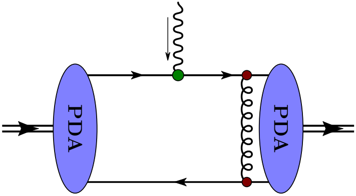


- In high energy scattering experiments particles move at near speed of light
  - natural to quantize a theory at equal light-front time: $\tau = (t+z)/\sqrt{2}$
- Light-front quantization $\Longrightarrow$ light-front WFs; many remarkable properties:
  - frame-independent; probability interpretation – as close as QFT gets to QM
  - boosts are kinematical – *not dynamical*
- With the LFWFs many observables can be straightforwardly determined – so far we have focused on the parton distribution amplitudes (PDAs):

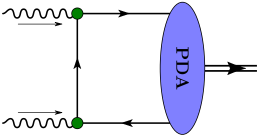
$$\varphi(x) = \int d^2\vec{k}_\perp \, \psi(x, \vec{k}_\perp)$$

Pion's Parton Distribution Amplitude

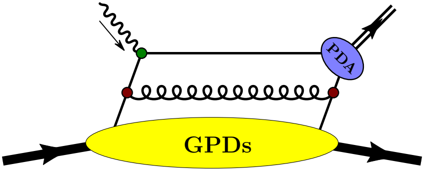
- pion's PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state*
  - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2

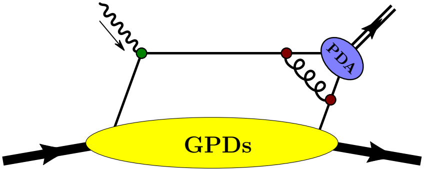


$$Q^2 \, F_\pi(Q^2) \to 16\pi \, f_\pi^2 \, \alpha_s(Q^2)$$



$$Q^2 \, F_{\gamma^*\gamma\pi}(Q^2) \to 2 \, f_\pi$$





- PDAs enter numerous hard exclusive scattering processes

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  - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2
- The pion's PDA is defined by

$$f_\pi\, \varphi_\pi(x) = Z_2 \int \frac{d^4k}{(2\pi)^2}\, \delta\left(k^+ - x\, p^+\right) \mathrm{Tr}\left[\gamma^+\gamma_5\; S(k)\, \Gamma_\pi(k,p)\, S(k-p)\right]$$

- $S(k)\, \Gamma_\pi(k,p)\, S(k-p)$ is the pion's Bethe-Salpeter wave function
  - in the non-relativistic limit it corresponds to the Schrodinger wave function
- $\varphi_\pi(x)$: is the axial-vector projection of the pion's Bethe-Salpeter wave function onto the light-front [at twist-2 also pseudoscalar projection]
- Pion PDA is an essentially nonperturbative quantity whose asymptotic form is known; in this regime governs, e.g., Q^2 dependence of pion form factor

$$Q^2\, F_\pi(Q^2) \overset{Q^2\to\infty}{\longrightarrow} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2) \qquad \Longleftrightarrow \qquad \varphi_\pi^{\mathrm{asy}}(x) = 6\, x\, (1-x)$$

QCD Evolution & Asymptotic PDA

- ERBL (Q^2) evolution for pion PDA [c.f. DGLAP equations for PDFs]

$$\mu \frac{d}{d\mu}\,\varphi(x,\mu) = \int_0^1 dy\, V(x,y)\,\varphi(y,\mu)$$

- This evolution equation has a solution of the form

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1 + \sum\nolimits_{n=2,\,4,\dots} a_n^{3/2}(Q^2)\, C_n^{3/2}(2x-1)\right]$$

  - $\alpha = 3/2$ because in $Q^2 \to \infty$ limit QCD is invariant under the collinear conformal group $SL(2;\mathbb{R})$
  - Gegenbauer-$\alpha = 3/2$ polynomials are irreducible representations $SL(2;\mathbb{R})$

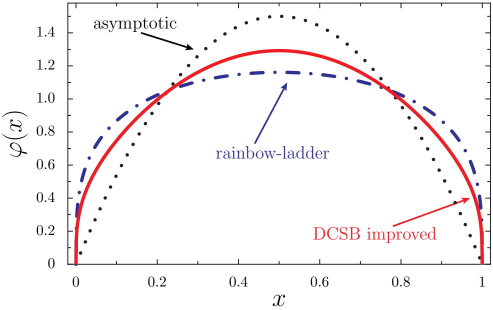
- The coefficients of the Gegenbauer polynomials, $a_n^{3/2}(Q^2)$, evolve logarithmically to zero as $Q^2 \to \infty$: $\varphi_\pi(x) \to \varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$

- At what scales is this a good approximation to the pion PDA?

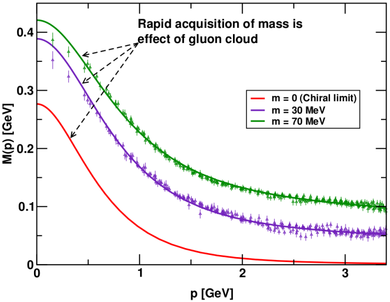
- E.g., AdS/QCD find $\varphi_\pi(x) \sim x^{1/2}\,(1-x)^{1/2}$ at $Q^2 = 1\,\text{GeV}^2$; expansion in terms of $C_n^{3/2}(2x-1)$ convergences slowly: $a_{32}^{3/2} /\, a_2^{3/2} \sim 10\,\%$

Pion PDA from the DSEs

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)]



[C.D. Roberts, Prog. Part. Nucl. Phys. 61 50 (2008)]



- Both DSE results, each using a different Bethe-Salpeter kernel, exhibit a pronounced broadening compared with the asymptotic pion PDA
  - scale of calculation is given by renormalization point $\zeta = 2\,\text{GeV}$
- A realization of DCSB on the light-front
- As we shall see the dilation of pion's PDA will influence the Q^2 evolution of the pion's electromagnetic form factor

Pion PDA from lattice QCD

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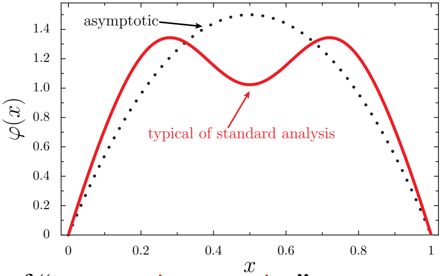
- Lattice QCD can only determine one non-trivial moment

$$\int_0^1 dx\,(2\,x-1)^2\varphi_\pi(x) = 0.27 \pm 0.04$$

[V. Braun *et al.*, Phys. Rev. D **74**, 074501 (2006)]

  - scale is $Q^2 = 4\,\text{GeV}^2$



- Standard practice to fit first coefficient of "*asymptotic expansion*" to moment

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1 + \sum_{n=2,\,4,\dots} a_n^{3/2}(Q^2)\,C_n^{3/2}(2x-1)\right]$$

  - however this expansion is guaranteed to converge rapidly only when $Q^2 \to \infty$
  - this procedure results in a *double-humped* pion PDA

- Advocate using a *generalized expansion*

$$\varphi_\pi(x,Q^2) = N_\alpha\,x^\alpha(1-x)^\alpha\left[1 + \sum_{n=2,\,4,\dots} a_n^{\alpha+1/2}(Q^2)\,C_n^{\alpha+1/2}(2x-1)\right]$$

- Find $\varphi_\pi \simeq x^\alpha(1-x)^\alpha,\ \alpha = 0.35^{+0.32}_{-0.24}$; good agreement with DSE: $\alpha \sim 0.52$

Pion PDA from lattice QCD

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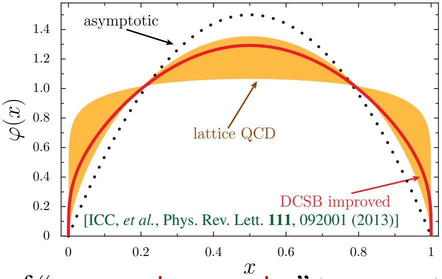

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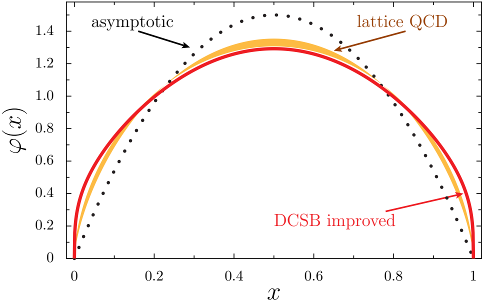
Updated Pion PDA from lattice QCD

- Generalized expansion

$$\varphi_\pi(x) = N_\alpha\, x^\alpha (1-x)^\alpha \left[1 + \sum a_n^{\alpha+}(Q^2)\, C_n^{\alpha+}(2x-1)\right]$$



- Updated lattice QCD moment: [V. Braun *et al.*, arXiv:1503.03656 [hep-lat]]

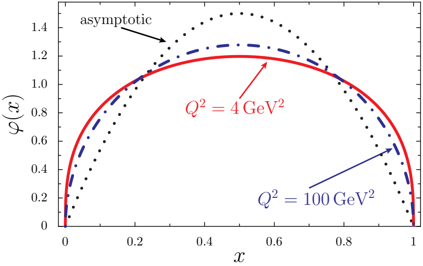
$$\int_0^1 dx\,(2\,x-1)^2 \varphi_\pi(x) = 0.2361\,(41)\,(39)\,(?)$$

- DSE prediction:

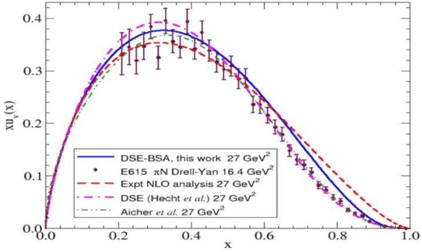
$$\int_0^1 dx\,(2\,x-1)^2 \varphi_\pi(x) = 0.251$$

When is the Pion's PDA Asymptotic

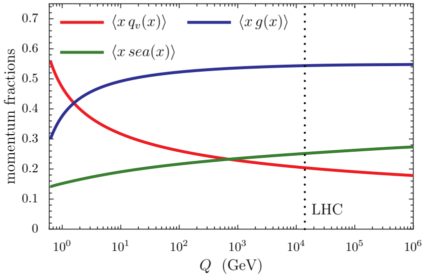
[I. C. Cloët, *et al.*, Phys. Rev. Lett. **111**, 092001 (2013)]

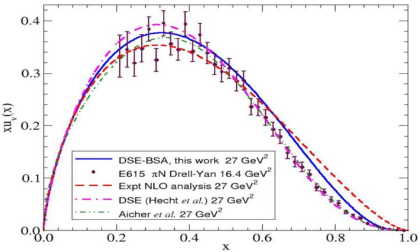


[T. Nguyen, *et al.*, Phys. Rev. C **83**, 062201 (2011)]



- Under leading order Q^2 evolution the pion PDA remains broad to well above $Q^2 > 100\,\text{GeV}^2$, compared with $\varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$
- *Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors*
- Importantly, $\varphi_\pi^{\text{asy}}(x)$ is only guaranteed be an accurate approximation to $\varphi_\pi(x)$ when pion valence quark PDF satisfies: $q_v^\pi(x) \sim \delta(x)$
- This is far from valid at forseeable energy scales





- LO QCD evolution of momentum fraction carried by valence quarks

$$\langle x\, q_v(x)\rangle\,(Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{qq}^{(0)2}/(2\beta_0)} \langle x\, q_v(x)\rangle\,(Q_0^2) \quad \text{where} \quad \frac{\gamma_{qq}^{(0)2}}{2\beta_0} > 0$$

  - therefore, as $Q^2 \to \infty$ we have $\langle x\, q_v(x)\rangle \to 0$ implies $q_v(x) \propto \delta(x)$

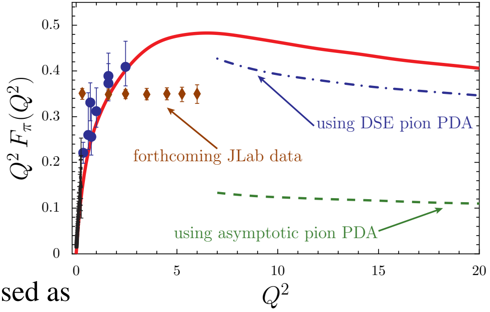
- At LHC energies valence quarks still carry 20% of pion momentum
  - the gluon distribution saturates at $\langle x\, g(x)\rangle \sim 55\%$

- *Asymptotia is a long way away!*

Pion Elastic Form Factor

- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2 F_\pi(Q^2)$ at $Q^2 \approx 6\,\text{GeV}^2$
  - magnitude of this product is determined by strength of DCSB at all accessible scales

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]



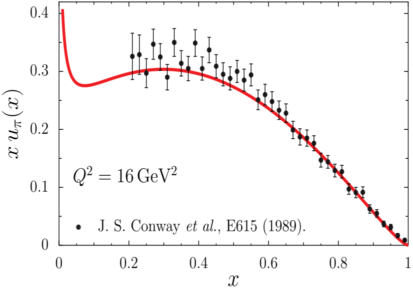
- The QCD prediction can be expressed as

$$Q^2 F_\pi(Q^2) \overset{Q^2 \gg \Lambda^2_{\text{QCD}}}{\sim} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_\pi(x)$$

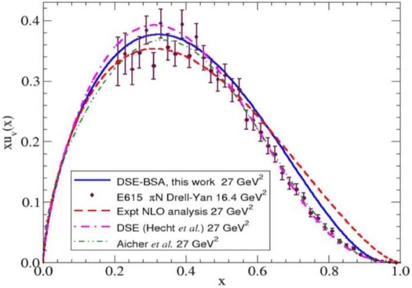
- Within DSEs there is consistency between the direct pion form factor calculation and that obtained using the DSE pion PDA
  - 15% disagreement explained by higher order/higher-twist corrections
- *We predict that QCD power law behaviour – with QCD's scaling law violations – sets in at* $Q^2 \sim 8\,\text{GeV}^2$

Pion PDF

[J. S. Conway et al. (E615 Collaboration), Phys. Rev. D 39, 92 (1989)]

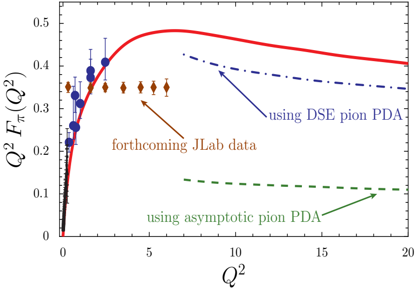


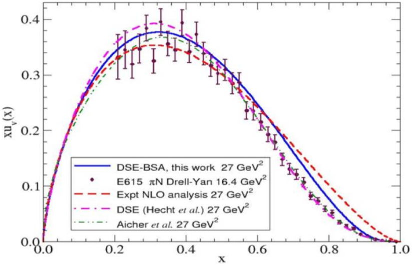
[M. Aicher, *et al.*, Phys. Rev. Lett. **105**, 252003 (2010)]



- Need for *QCD-based* calculation is emphasized by story of pion's valence quark distribution function:
  - 1989: $u_v^\pi \overset{x\to 1}{\sim} (1-x)^1$ – inferred from LO-Drell-Yan & disagrees with QCD
  - 2001: Dyson-Schwinger Equations (DSEs) predicts $u_v^\pi \overset{x\to 1}{\sim} (1-x)^{2+\gamma}$ – argues that distribution inferred from data can't be correct
  - 2010: new NLO reanalysis – including soft-gluon resummation – inferred distribution agrees with DSE-QCD
- *Potentially important ramifications for nucleon PDF studies!*

A Unified Picture of the Pion





- A single framework has provided a unified picture of the pion, that is, its valence PDF, form factor and PDA
- Surprisingly much of this physics is encapsulated in a simple algebraic model:

$$S(p) = \left[-i\not{p} + M\right]\left[p^2 + M^2\right]^{-1},$$

$$\Gamma_\pi(p,k) = i\gamma_5 \frac{3\,M^3}{4\,f_\pi}\int_{-1}^{1} dz\,\left(1 - z^2\right)\left[k_+^2 + M^2\right]^{-1}$$

- Can easily apply this model to the unpolarized pion TMD

Unpolarized pion TMD – early DSE result

- To determine the TMD use the light-front formalism – pion has two LFWFs

$$\left[\Psi_{\uparrow\downarrow}(x,\vec{k}_T^2);\ k^j\,\Psi_{\uparrow\uparrow}(x,\vec{k}_T^2)\right] = \frac{1}{2p^+}\int\frac{dk^-}{2\pi}\,\mathrm{Tr}\left[\gamma^+\gamma_5\,\chi(p,k);\ i\sigma^{+j}\gamma_5\,\chi(p,k)\right]$$

  - $\chi(p,k)$ is the pion's Bethe-Salpeter wavefunction
  - pion TMD given by a linear combination of the square of these LFWFs

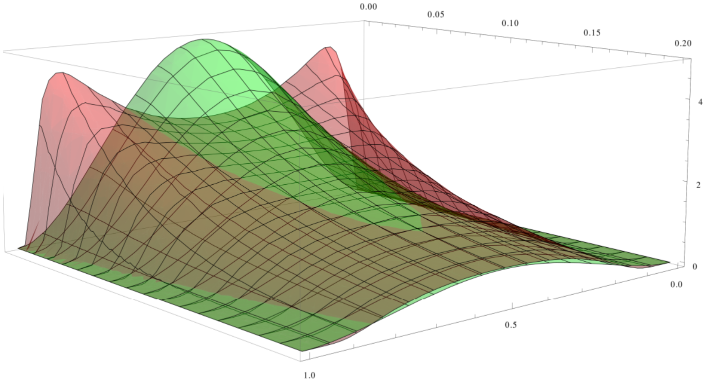
- We obtain

$$\Psi_{\uparrow\downarrow}(x,\vec{k}_T^2) = \frac{2\,M^3\,x(1-x)}{[\vec{k}_T^2+M^2]^2}, \qquad \Psi_{\uparrow\uparrow}(x,\vec{k}_T^2) = \frac{4i\,M^2\,x(1-x)}{[\vec{k}_T^2+M^2]^2}$$

  - these LFWFs factorize in x and $\vec{k}_T^2$; however expect dependence like $\vec{k}_T^2/[x(1-x)]$ the light-front kinetic energy for massless quarks; also issues with momentum conservation

- Nevertheless this simple model reproduces many pion properties
  - to obtain correct $x, \vec{k}_T^2$ dependence likely need a more sophisticated interaction
  - clear example where interplay between experiment and theory can expose the nature of the dressed interactions in QCD

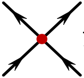
DSEs Results



- Comparsion of our results [green] with those from *Pasquini and Schweitzer, PRD* **90** *014050 (2014)* [red]
- Each model gives a similar PDF but a different TMD, near $k_T^2 = 0$ and at large k_T^2 one behaves as a Gasussian and our result as a power law in k_T^2
- Illustration of the potential for TMDs to differentiate between different frameworks and thereby expose quark-gluon dynamics in QCD

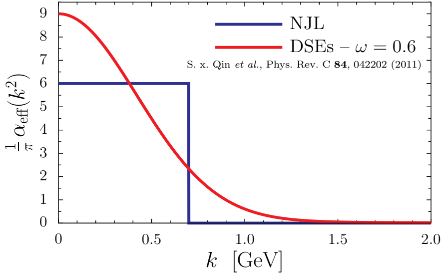
Nambu–Jona-Lasinio model

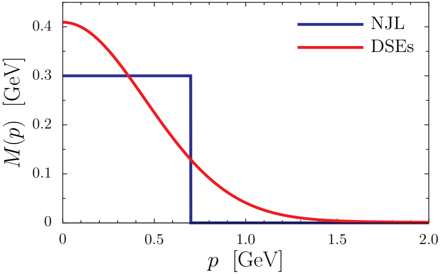
Continuum QCD *"integrate out gluons"* →



$\frac{1}{m_G^2}\;\Theta(\Lambda^2 - k^2)$

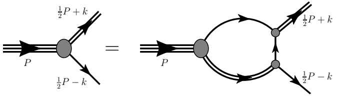
- this is just a modern interpretation of the Nambu–Jona-Lasinio (NJL) model
- model is a Lagrangian based covariant QFT which exhibits dynamical chiral symmetry breaking & it elements can be QCD motivated via the DSEs

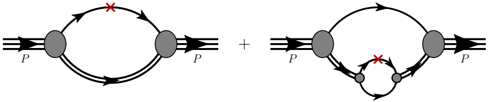




- The NJL model is very successful - provides a good description of numerous hadron properties: form factors, PDFs, in-medium properties, etc
  - however the NJL model has no direct link to QCD
  - in general NJL has no confinement – but can be implemented with proper-time RS

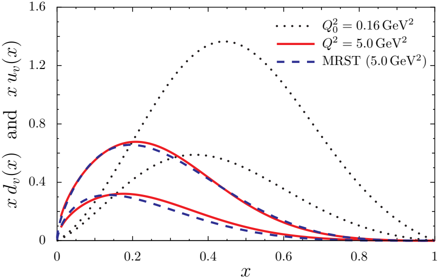
- Nucleon = quark+diquark
- PDFs given by Feynman diagrams: $\langle \gamma^+ \rangle$

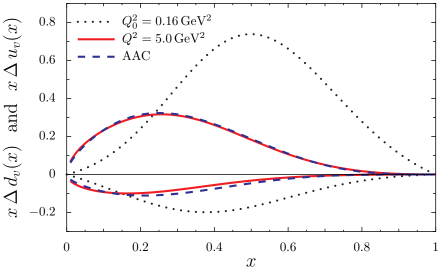




- Covariant, correct support; satisfies sum rules, Soffer bound & positivity

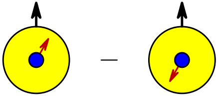
$$\langle q(x) - \bar{q}(x) \rangle = N_q, \quad \langle x\,u(x) + x\,d(x) + \ldots \rangle = 1, \quad |\Delta q(x)|\,,\ |\Delta_T q(x)| \leqslant q(x)$$





[ICC, W. Bentz and A. W. Thomas, Phys. Lett. B **621**, 246 (2005)]

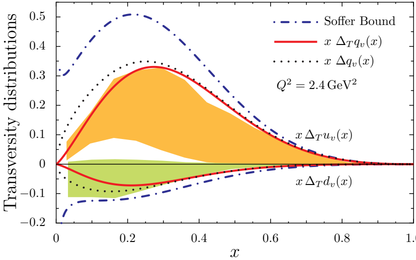
Nucleon transversity quark distributions

$\Delta_T q(x) =$  $-$

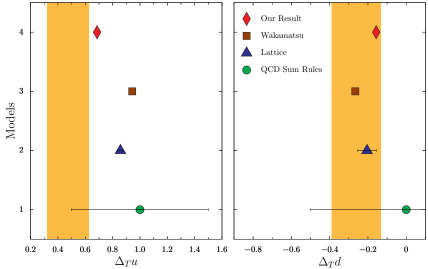
- Sum rule gives tensor charge

$$g_T = \int dx\,[\Delta_T u(x) - \Delta_T d(x)]$$

- quarks in eigenstates of $\gamma^\perp \gamma_5$
- Non-relativistically: $\Delta_T q(x) = \Delta q(x)$ – a measure of relativistic effects
- Helicity conservation: no mixing bet'n $\Delta_T q$ & $\Delta_T g$: $J \leqslant \frac{1}{2} \Rightarrow \Delta_T g(x) = 0$
- Therefore for the nucleon $\Delta_T q(x)$ is valence quark dominated
- At model scale we find: $g_T = 1.28$ *compare* $g_A = 1.267$ (input)



[ICC, W. Bentz and A. W. Thomas, Phys. Lett. B **659**, 214 (2008)]



[M. Anselmino *et al*, Nucl. Phys. Proc. Suppl. **191**, 98 (2009)]

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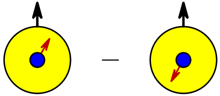
$\Delta_T q(x) =$ 

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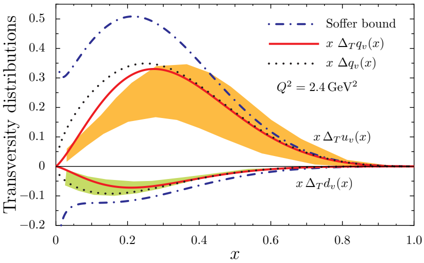
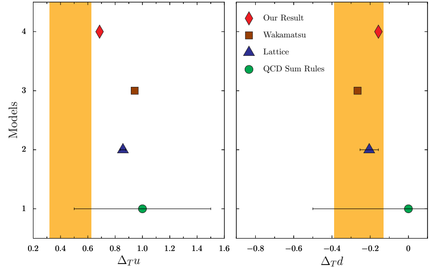
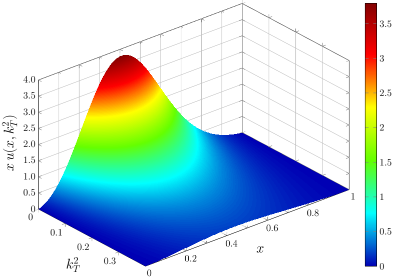


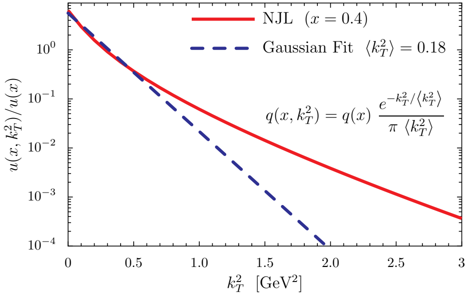
[ICC, W. Bentz and A. W. Thomas, Phys. Lett. B **659**, 214 (2008)]



[M. Anselmino *et al*, Nucl. Phys. Proc. Suppl. **191**, 98 (2009)]

Transverse Momentum Dependent PDFs



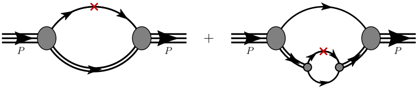


- So far only considered the simplest spin-averaged TMDs – $q(x,k_T^2)$
- Rigorously included diquark correlations in TMD calculation

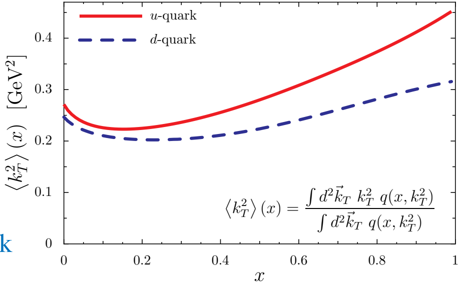
$$q_{D/N}(x,k_T^2) = \int_0^1 dy \int_0^1 dz \int d^2\vec{q}_\perp \int d^2\vec{\ell}_\perp \; \delta(x-yz)\;\delta(\vec{\ell}_\perp - \vec{k}_\perp - z\vec{q}_\perp)\; f_{D/N}(y,\vec{q}_\perp)\; f_{q/D}(z,\vec{\ell}_\perp)$$

- Scalar diquark correlations greatly increase $\langle k_T^2\rangle$

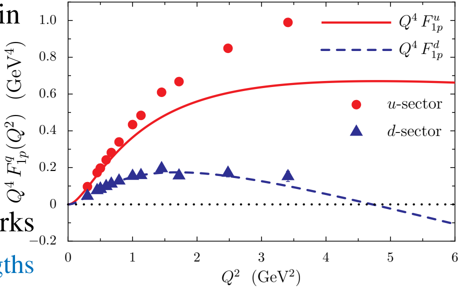
$$\langle k_T^2\rangle_u^{Q^2=Q_0^2} = 0.43\,\mathrm{GeV}^2 \qquad \langle k_T^2\rangle = 0.31\,\mathrm{GeV}^2\ \text{[HERMES]},\quad 0.41\,\mathrm{GeV}^2\ \text{[EMC]}$$



- Scalar diquark correlations give sizable flavour dependence in $\langle k_T^2 \rangle$
  - 70% of proton (uud) WF contains a scalar diquark $[ud]$; $M_s \simeq 650\,\text{MeV}$, with $M \simeq 400\,\text{MeV}$ difficult for d-quark to be at large x



- Scalar diquark correlations also explain the very different scaling behaviour of the quark sector form factors
  - $u[ud]$ diquark $\Longrightarrow$ extra $1/Q^2$ for d
- Zero in F_{1p}^d a result of interference between scalar and axial-vector diquarks
  - location of zero indicates relative strengths – correlated with d/u ratio as $x \to 1$



[ICC, Bentz, Thomas, PRC **90**, 045202 (2014)]

Conclusion

- Using the DSEs we find that DCSB drives numerous effects in QCD, e.g., hadron masses, confinement and many aspects of hadron structure
  - e.g. broading of pion PDA and maximum of $Q^2\, F_\pi(Q^2)$ directly related to DCSB
- Have made significant advances in understanding the pion form factor, PDF & PDA
  - TMDs and quark fragmentation into pions an important next step
- Have rigorously included scalar and axial-vector diquark correlations in a calculation of the nucleon TMDs
  - results in a dramatic increase in $\langle k_T^2 \rangle$ and a significant flavour dependence of the TMDs