Transverse-momentum resummation

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Milan University & INFN Milan

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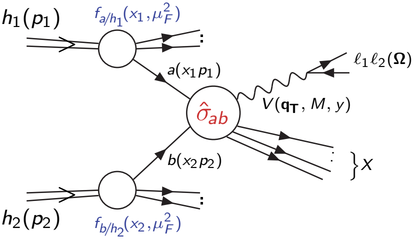
Drell–Yan $\mathbf{q_T}$ distribution



$$\mathbf{h_1(p_1) + h_2(p_2) \;\rightarrow\; V + X \;\rightarrow\; \ell_1 \ell_2 + X}$$

where $\quad V = Z^0/\gamma^*, W^\pm$

QCD *collinear* factorization formula:

$$\frac{d\sigma}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\; f_{a/h_1}(x_1, \mu_F^2)\; f_{b/h_2}(x_2, \mu_F^2)\; \frac{d\hat{\sigma}_{ab}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\mathbf{\Omega}}(\hat{s}; \alpha_S, \mu_R^2, \mu_F^2).$$

Fixed-order perturbative expansion not reliable for $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \overset{q_T \ll M}{\sim} 1 + \alpha_S \left[c_{12} \ln^2 \frac{M^2}{q_T^2} + c_{11} \ln \frac{M^2}{q_T^2} + c_{10} \right] + \cdots$$

$\alpha_S \ln(M^2/q_T^2) \gg 1$: need for resummation of large logs.

$$\frac{d\sigma}{dq_T^2} = \frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2};$$

$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}^{(fin)}}{d\bar{q}_T^2} \overset{q_T \to 0}{=} 0$$

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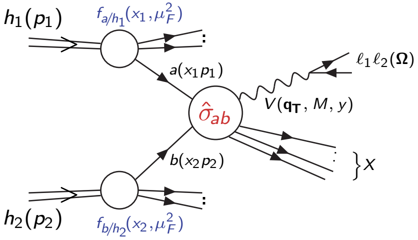

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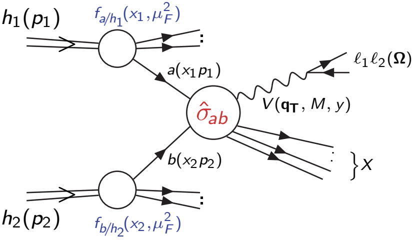

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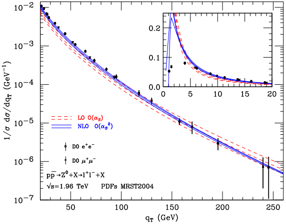

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State of the art: q_T resummation

- Method to resum large q_T logarithms is known [Dokshitzer,Diakonov,Troian('78)], [Parisi,Petronzio('79)],[Curci,Greco,Srivastava('79)],[Kodaira,Trentadue('82)], [Collins,Soper('81,'82)],[Collins,Soper,Sterman('85)],[Catani,de Florian, Grazzini('01)],[Bozzi et al.('06,'08)],[Catani,Grazzini('11)],[Catani et al.('13)].
- Phenomenological studies [Altarelli et al.('84)],[ResBos:Balazs et al.('95,'97)], [Guzzi et al.('13)],[Ellis et al.('97,'98)],[Qiu et al.('01)],[Kulesza et al. ('01,'02)],[Berger et al.('02,'03)],[Landry et al.('03)],[Banfi et al.('12)].
- Results for q_T resummation by using Soft Collinear Effective Theory methods and transverse-momentum dependent (TMD) factorization [Gao et al.('05)],[Idilbi et al.('05)],[Mantry,Petriello('10,'11)],[Becher et al.('11)],[Echevarria et al. ('12,'13,'15)],[Chiu et al.('12)], [Roger,Mulders('10)],[Collins('11)], [Collins, Rogers('13)],[D'Alesio et al.('14)].
- Effective q_T-resummation can be obtained with Parton Shower algorithms. QCD/EW DY corrections implemented in POWHEG [Barze et al.('12,'13)]. Results for NNLO DY predictions matched with PS obtained [Hoeche,Li,Prestel('14)], [Karlberg,Re,Zanderighi('14)], [Alioli,Bauer,Berggren,Tackmann,Walsh('14)].

Soft gluon exponentiation

Sudakov resummation feasible when:
dynamics AND kinematics factorize
$\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions.

$$dw_n(q_1,\ldots,q_n) \simeq \frac{1}{n!}\prod_{i=1}^{n} dw_i(q_i)$$

- Kinematics factorization: not valid in general. For q_T distribution of DY process it holds in the impact parameter space (Fourier transform).

$$\int d^2\mathbf{q_T}\ \exp(-i\mathbf{b}\cdot\mathbf{q_T})\,\delta^{(2)}\left(\mathbf{q_T}-\sum_{j=1}^{n}\mathbf{q_{T_j}}\right) = \exp(-i\mathbf{b}\cdot\sum_{j=1}^{n}\mathbf{q_{T_j}}) = \prod_{j=1}^{n}\exp(-i\mathbf{b}\cdot\mathbf{q_{T_j}})\,.$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space.

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Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \to F$.

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$$\times \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c};a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)\, f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) \;,$$

[Collins,Soper,Sterman('85)],

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$$S_q(M, b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \; \ln\frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} \;.$$

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$b_0 = 2e^{-\gamma_E}$ ($\gamma_E = 0.57\dots$), $\quad x_{1,2} = \frac{M}{\sqrt{s}}\, e^{\pm y}$, $\quad L \equiv \ln Mb$

$$S_q(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2))\, \ln\frac{M^2}{q^2} + B_q(\alpha_S(q^2))\right]\right\} \;.$$

$$\left[H^F C_1 C_2\right]_{q\bar{q};a_1a_2} = H_q^F(x_1p_1, x_2p_2; \mathbf{\Omega}; \alpha_S(M^2))\;\; C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\;\; C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2)) \;,$$

$$A_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)}, \quad B_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_q^{F\,(n)}, \qquad C_{qa}(z;\alpha_S) = \delta_{qa}\;\delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n C_{qa}^{(n)}(z)\,.$$

LL($\sim\alpha_S^n L^{n+1}$): $A_q^{(1)}$; NLL($\sim\alpha_S^n L^n$): $A_q^{(2)}, B_q^{(1)}, H_q^{F\,(1)}, C_{qa}^{(1)}$; NNLL($\sim\alpha_S^n L^{n-1}$): $A_q^{(3)}, B_q^{(2)}, H_q^{F\,(2)}, C_{qa}^{(2)}$

q_T resummation: $q\bar{q}$-annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \to F$.

$$\frac{d\sigma_F^{(res)}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \frac{M^2}{s} \sum_{c=q,\bar{q}} \left[d\sigma^{(0)}_{c\bar{c},F} \right] \int \frac{d^2\mathbf{b}}{(2\pi)^2}\; e^{i\mathbf{b}\cdot\mathbf{q_T}}\; S_q(M,b)$$

$$\times \sum_{a_1,a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c};a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)\, f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) \;,$$

[Collins,Soper,Sterman('85)],
[Catani,de Florian,Grazzini('01)]

$b_0 = 2e^{-\gamma_E}$ $(\gamma_E = 0.57\dots)$, $\quad x_{1,2} = \frac{M}{\sqrt{s}}\, e^{\pm y}$, $\quad L \equiv \ln Mb$

$$S_q(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \ln\frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} \;.$$

$$\left[H^F C_1 C_2 \right]_{q\bar{q};a_1 a_2} = H_q^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\;\; C_{q a_1}(z_1;\alpha_S(b_0^2/b^2))\;\; C_{\bar{q}\, a_2}(z_2;\alpha_S(b_0^2/b^2)) \;,$$

$$A_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)}, \quad B_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_q^{F\,(n)}, \qquad C_{qa}(z;\alpha_S) = \delta_{qa}\;\delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n C_{qa}^{(n)}(z)\,.$$

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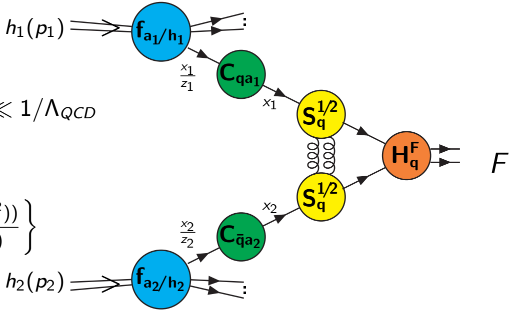
Transverse-momentum resummation formula



$$M \gg \Lambda_{QCD}\ ,\quad b \gg 1/M\ ,\quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times\ \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\ \beta(\alpha_S(q^2))\ \frac{d\ln C(\alpha_S(q^2))}{d\ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \frac{M^2}{s}\left[d\sigma^{(0)}_{q\bar{q},F}\right] H_q^F(x_1p_1, x_2p_2; \mathbf{\Omega}; \alpha_S(M^2)) \sum_{a_1,a_2}\int \frac{d^2\mathbf{b}}{(2\pi)^2}\ e^{i\mathbf{b}\cdot\mathbf{q_T}}\ S_q(M,b)$$
$$\times \int_{x_1}^1 \frac{dz_1}{z_1}\ C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2}\ C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\ f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z}\ \sqrt{S_q(M,b)}\, C_{q_f\,a}(z;\alpha_S(b_0^2/b^2))\ f_{a/h}(x/z, b_0^2/b^2)$$

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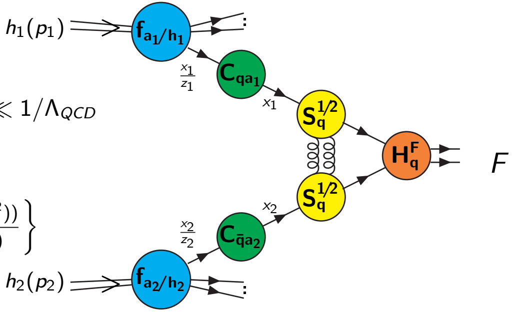

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$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}\; dM^2\; dy\; d\mathbf{\Omega}} = \frac{M^2}{s} \left[d\sigma^{(0)}_{q\bar{q},F} \right] H_q^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2)) \sum_{a_1, a_2} \int \frac{d^2\mathbf{b}}{(2\pi)^2} \; e^{i\mathbf{b}\cdot\mathbf{q_T}} \; S_q(M,b)$$

$$\times \int_{x_1}^{1} \frac{dz_1}{z_1} \; C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) \; f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^{1} \frac{dz_2}{z_2} \; C_{\bar{q}\,a_2}(z_2; \alpha_S(b_0^2/b^2)) \; f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

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$\mathbf{q_T}$ resummation: gluon fusion processes

In processes initiated at Born level by the gluon fusion channel ($gg \to F$), collinear radiation from gluons leads to spin and azimuthal correlations [Catani,Grazzini('11)].

$$\left[H^F C_1 C_2\right]_{gg;a_1 a_2} = H^F_{g;\mu_1\nu_1,\mu_2\nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))$$
$$\times \; C^{\mu_1\nu_1}_{ga_1}(z_1; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2))\, C^{\mu_2\nu_2}_{ga_2}(z_2; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)) \,.$$

where $\quad H_g^{F\,\mu_1\nu_1,\mu_2\nu_2}(\alpha_S) = \sum_{n=0}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n H_g^{F(n)\mu_1\nu_1,\mu_2\nu_2} \,,$

$$C^{\mu\nu}_{ga}(z; p_1, p_2, \mathbf{b}; \alpha_S) = d^{\,\mu\nu}(p_1, p_2)\; C_{ga}(z;\alpha_S) + D^{\,\mu\,\nu}(p_1, p_2; \mathbf{b})\; G_{ga}(z;\alpha_S) \;\;,$$

$$d^{\,\mu\nu}(p_1,p_2) = -\,g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1\cdot p_2} \;\;, \quad D^{\,\mu\nu}(p_1,p_2;\mathbf{b}) = d^{\,\mu\nu}(p_1,p_2) - 2\,\frac{b^\mu\, b^\nu}{\mathbf{b}^2} \;\;,$$

$$C_{ga}(z;\alpha_S) = \delta_{ga}\;\;\delta(1-z) + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n C^{(n)}_{ga}(z)\,, \quad G_{ga}(z;\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n G^{(n)}_{ga}(z) \;\;.$$

- Unlike $q\bar{q}$ annih. $\left[H^F C_1 C_2\right]$ does depend on the azimuthal angle $\phi(\mathbf{b})$, this leads to azimuthal correlations with respect to the azimuthal angle $\phi(\mathbf{q_T})$ (consistent with [Mulders,Rodrigues('00)],[Henneman et al.('02)]).
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Universality in q_T resummation

The resummation formula is invariant under the *resummation scheme* transformations [Catani,de Florian,Grazzini('01)] (for $h_c(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n h_c^{(n)}$):

$$
\begin{aligned}
H_c^F(\alpha_S) &\rightarrow H_c^F(\alpha_S) \left[h_c(\alpha_S) \right]^{-1} , \\
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C_{cb}(z, \alpha_S) &\rightarrow C_{cb}(z, \alpha_S) \left[h_c(\alpha_S) \right]^{1/2} .
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$$

- This implies that H_c^F, S_c (B_c) and C_{cb} not unambiguously computable separately.
- Resummation scheme: define H_c^F (or C_{ab}) for *single* processes (one for $q\bar{q} \to F$ one for $gg \to F$) and unambiguously determine the process-dependent H_c^F and the universal (process-independent) S_c and C_{ab} for any other process.
- *DY/H resummation scheme:* $H_q^{DY}(\alpha_S) \equiv 1$, $H_g^H(\alpha_S) \equiv 1$.
  Hard resummation scheme: $C_{ab}^{(n)}(z)$ for $n \geq 1$ do not contain any $\delta(1-z)$ term (other than plus distributions).
- $H_c^F(\alpha_S) = 1$ (i.e. $h_c(\alpha_S) = H_c^F(\alpha_S)$) *does not* correspond to a resummation scheme (S_c^F and C_{ab}^F would be process dependent, [de Florian,Grazzini('00)]).

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Hard-collinear coefficients at NNLO

- Resummation coefficients in Sudakov form factor known since some time up to $\mathcal{O}(\alpha_S^2)$ ($A_c^{(1,2)}$, $B_c^{(1,2)}$), $A_c^{(3)}$ calculated more recently [Becher,Neubert('11)]
- Explicit NNLO *analytic* calculations of the q_T cross section (at small-q_T): *(i)* SM Higgs boson production [Catani,Grazzini('07,'12)] and *(ii)* DY process [Catani,Cieri,de Florian,G.F.,Grazzini('09,'12)].
- These calculations provide complete knowledge of the process-independent *collinear* coeff. $C_{ca}(z,\alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ ($G_{ga}(z,\alpha_S)$ up to $\mathcal{O}(\alpha_S)$), and of the *hard-virtual* factor $H_c^F(\alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ for DY/H processes. In the *hard scheme*:

$$C_{qq}^{(1)}(z) = \frac{C_F}{2}(1-z)\,,\; C_{gq}^{(1)}(z) = \frac{C_F}{2}z\,,\; C_{qg}^{(1)}(z) = \frac{z}{2}(1-z)\,,$$

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- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
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renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \tilde{I}_c^{(n)}(\epsilon),$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

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- E.g. diphoton production: we rederived the result for $H_q^{\gamma\gamma\,(1)}$ [Balazs et al.('98)] and (using the two-loop amplitudes [Anastasiou et al.('02)]) we obtained the $H_q^{\gamma\gamma\,(2)}$ [Catani,Cieri,de Florian,GF,Grazzini('12)]

$$H_q^{\gamma\gamma(1)} = \frac{C_F}{2}\left\{(\pi^2-7) + \frac{\left((1-v)^2+1\right)\ln^2(1-v) + v(v+2)\ln(1-v) + (v^2+1)\ln^2 v + (1-v)(3-v)\ln v}{(1-v)^2+v^2}\right\} .$$

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The q_T resummation formalism

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
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- Perturbative unitarity constraint:

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$$\ln\left(Q^2 b^2\right) \to \widetilde{L} \equiv \ln\left(Q^2 b^2 + 1\right)$$

  - avoids unjustified higher-order contributions in the small-b region.
  - recover *exactly* the total cross-section (upon integration on q_T)

The q_T resummation formalism

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\,\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
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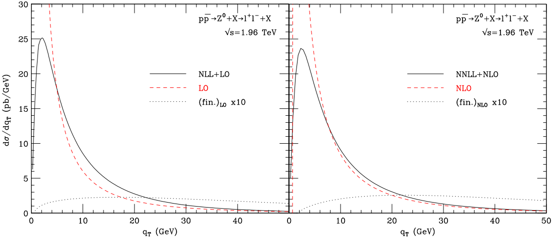
Matching with fixed-order results



- To obtain a uniform accuracy over the range $q_T \ll M$ up to $q_T \sim M$, *resummed* and *fixed-order* components have to be consistently matched $\frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2}$,

$$\left[\frac{d\hat{\sigma}_{ab}^{(fin.)}}{dq_T^2}\right]_{f.o.} = \left[\frac{d\hat{\sigma}_{ab}}{dq_T^2}\right]_{f.o.} - \left[\frac{d\hat{\sigma}_{ab}^{(res.)}}{dq_T^2}\right]_{f.o.}$$

- Finite NLO component contribution is: $\lesssim 1\%$ near the peak, $\sim 8\%$ at $q_T \sim 20\,GeV$, $\sim 60\%$ at $q_T \sim 50\,GeV$.
- Integral of the matched curve reproduce the total cross section to better 1% (check of the code).

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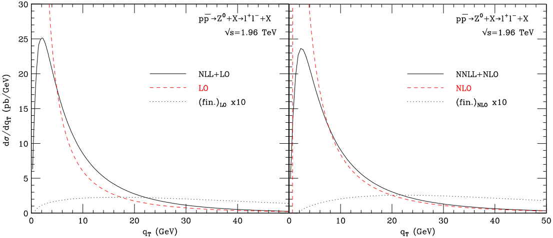

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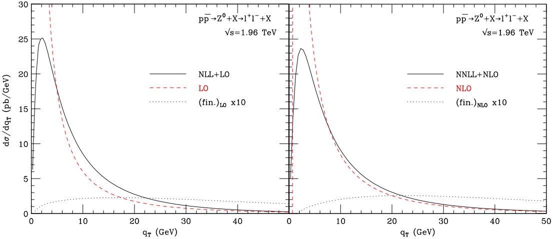

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q_T resummation at full NNLL

- q_T resummation performed for Drell–Yan process up to NNLL+NNLO by using the formalism developed in [Catani,de Florian,Grazzini('01)], [Bozzi,Catani,de Florian,Grazzini('06,'08)]. We have included
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DYqT: computes resummed q_T spectrum, inclusive over other kinematical variables [Bozzi,Catani,de Florian,G.F.,Grazzini('09,'11)]

http://pcteserver.mi.infn.it/~ferrera/dyqt.html

DYRes: computes resummed q_T spectrum and related distributions, it retains full kinematics of the vector boson and of its leptonic decay products (possible to apply arbitrary cuts on these variables, and to plot the corresponding distributions) [Catani,de Florian,G.F.,Grazzini('15)]

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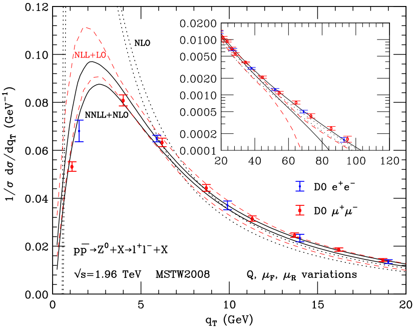
DYqT results: q_T spectrum of Z boson at the Tevatron



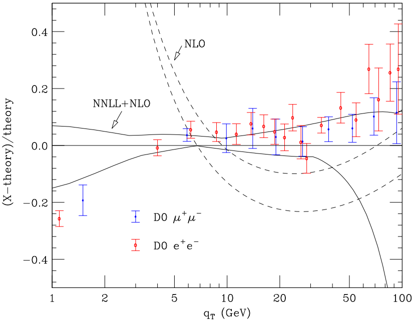
D0 data for the Z q_T spectrum compared with perturbative results.

- Uncertainty bands obtained varying μ_R, μ_F, Q independently:

$$\frac{1}{2} \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$$

- Significant reduction of scale dependence from NLL to NNLL for all q_T.
- Good convergence of resummed results: NNLL and NLL bands overlap (contrary to the fixed-order case).
- Good agreement between data and resummed predictions (without any model for non-perturbative effects).
  The perturbative uncertainty of the NNLL results is comparable with the experimental errors.

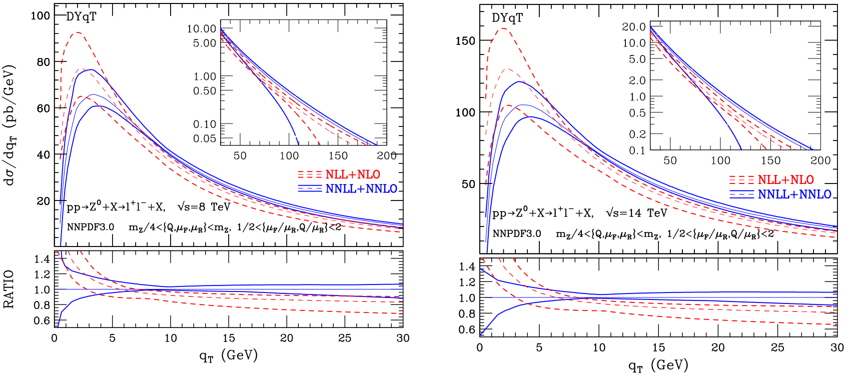
DYqT results: q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum: Fractional difference with respect to the reference result: NNLL, $\mu_R = \mu_F = 2Q = m_Z$.

- NNLL scale dependence is $\pm 6\%$ at the peak, $\pm 5\%$ at $q_T = 10\,GeV$ and $\pm 12\%$ at $q_T = 50\,GeV$. For $q_T \geq 60\,GeV$ the resummed result looses predictivity.
- At large values of q_T, the NLO and NNLL bands overlap.
  At intermediate values of transverse momenta the scale variation bands do not overlap.
- The resummation improves the agreement of the NLO results with the data.
  In the small-q_T region, the NLO result is theoretically unreliable and the NLO band deviates from the NNLL band.

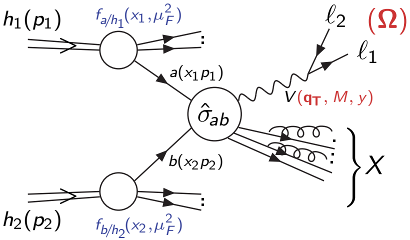
DYqT results: q_T spectrum of Z boson at the LHC



NLL+NLO and NNLL+NNLO bands for Z q_T spectrum at the LHC at $\sqrt{s} = 8$ TeV (left) and $\sqrt{s} = 14$ TeV (right).

Lower panel: ratio of the NLL+NLO and NNLL+NNLO results with respect to the NNLL+NNLO result at $\mu_F = \mu_R = Q = m_Z/2$.

DYRes: q_T resummation and leptonic decay



- Experiments have finite acceptance: important to provide exclusive theoretical predictions.
- Analytic resummation formalism inclusive over soft-gluon emission: not possible to apply selection cuts on final state partons.

- We have included the full dependence on vector boson and its decay products variables: possible to apply cuts on these variables.
- To construct the "finite" part we rely on the fully-differential NNLO result from the code DYNNLO [Catani,Cieri,de Florian,G.F.,Grazzini('09)].
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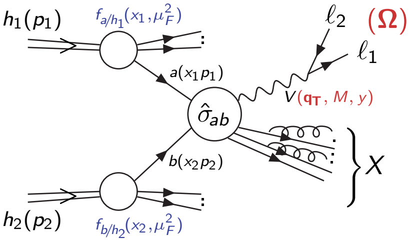

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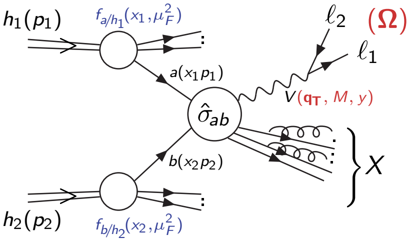

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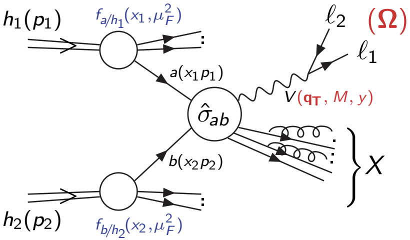

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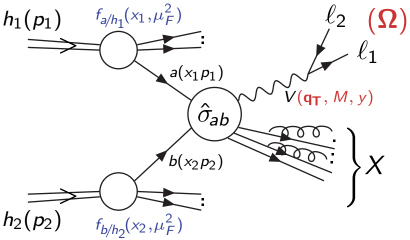

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$\mathbf{q_T}$ recoil and lepton angular distribution

- The dependence of the resummed cross section on the leptonic variable $\mathbf{\Omega}$ is

$$\frac{d\hat{\sigma}^{(0)}}{d\mathbf{\Omega}} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}/M; M^2, \mathbf{\Omega}) \;\;, \;\; \text{with} \;\; \int d\mathbf{\Omega}\, F(\mathbf{q_T}/M; \mathbf{\Omega}) = 1 \; .$$

  the q_T dependence arise as a *dynamical* q_T-recoil of the vector boson due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

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- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

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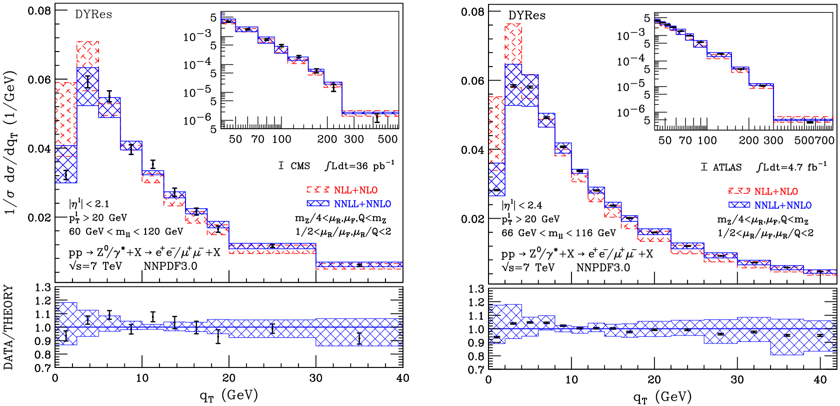
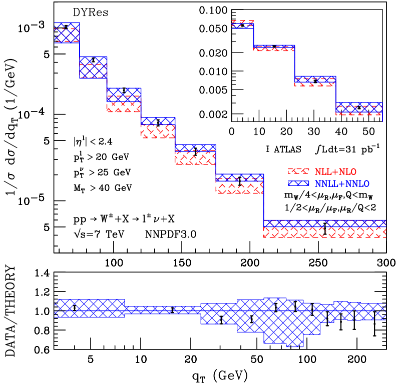
DYRes **results:** q_T spectrum of Z boson at the LHC



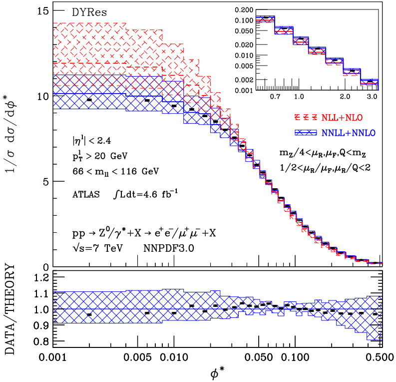
NLL+NLO and NNLL+NNLO bands for Z/γ^* q_T spectrum compared with CMS (left) and ATLAS (right) data.
Lower panel: ratio with respect to the NNLL+NNLO central value.

Program performances: for high statistic runs (i.e. few per mille accuracy on cross sections) *on a single CPU*: $\sim$ 1day at full NLL, $\sim$ 3days at full NNLL.

DYRes results: q_T spectrum of W and ϕ^* spectrum of Z boson at the LHC



NLL+NLO and NNLL+NNLO bands for $W^{\pm}$ q_T spectrum compared with ATLAS data.
Lower panel: ratio with respect to the NNLL+NNLO central value.



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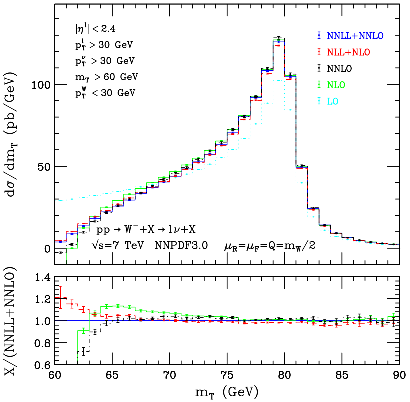
DYRes **results:** lepton kinematical distributions from W decay



Effect of q_T resummation on the transverse mass (m_T) for W^- production at the LHC. NLL+NLO and NNLL+NNLO results compared with LO, NLO and NNLO results. Lower panel: ratio between various results and NNLL+NNLO result.

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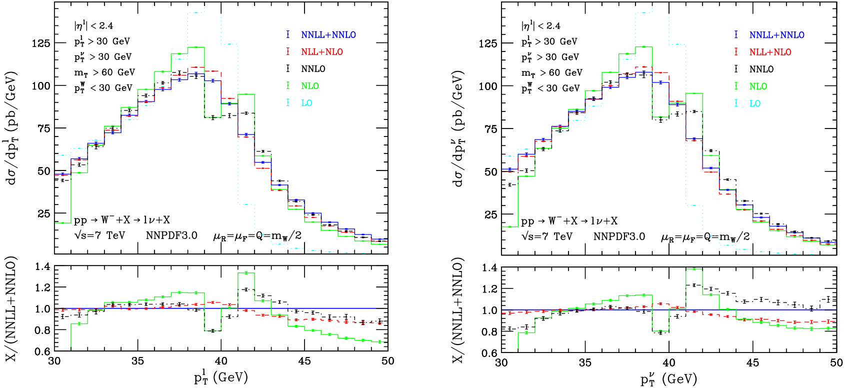
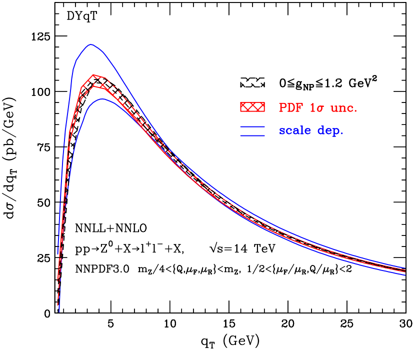


Effect of q_T resummation on lepton p_T (left) and missing p_T distribution for W^- production at the LHC. NLL+NLO and NNLL+NNLO results compared with LO, NLO and NNLO results.

Lower panel: ratio between various results and NNLL+NNLO result.

PDF uncertainties and NP effects



NNLL+NNLO result for Z q_T spectrum at the LHC at $\sqrt{s} = 14$ TeV. Perturbative scale dependence, PDF uncertainties and impact of NP effects.

- PDF uncertainty is smaller than the scale uncertainty and it is approximately independent on q_T (around the 3% level).
- Non perturbative *intrinsic* k_T effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$ with $0 < g_{NP} < 1.2\, GeV^2$:

$$S_q(M, b) \quad \to \quad S_q(M, b)\, S_{NP}$$

- NP effects increase the hardness of the q_T spectrum at small values of q_T. Non trivial interplay of perturbative and NP effects (higher-order contributions at small q_T can be mimicked by NP effects).
- NNLL+NNLO result with NP effects very close to perturbative result except for $q_T < 3 GeV$ (i.e. below the peak).

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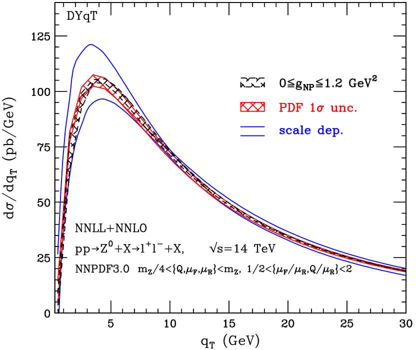

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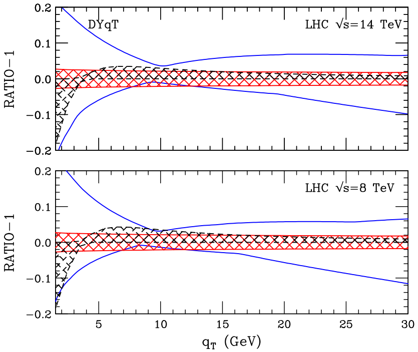

NNLL+NNLO result for Z q_T spectrum at the LHC at $\sqrt{s} = 14$ TeV (up) $\sqrt{s} = 8$ TeV (down). Perturbative scale dependence, PDF uncertainties and impact of NP effects normalized to central NNLL+NNLO prediction.

- PDF uncertainty is smaller than the scale uncertainty and it is approximately independent on q_T (around the 3% level).
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Conclusions

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- General procedure to treat the q_T recoil in q_T resummed calculations introduced.
- A public version of the DYRes (and DYqT) code is available:
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Back up slides

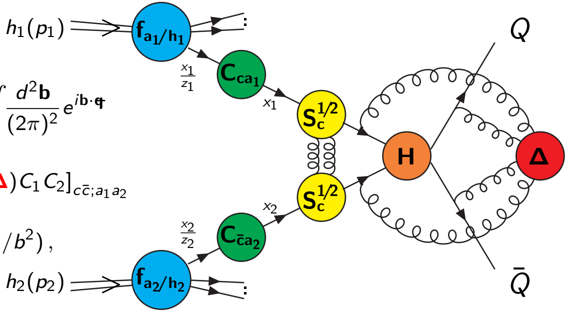
q_T resummation for heavy-quark hadroproduction

[Catani,Grazzini,Torre('14)]

$$\frac{d\sigma^{(res)}}{d^2\mathbf{q_T}\, dM^2 dy d\mathbf{\Omega}} = \frac{M^2}{s} \sum_{c=q,\bar{q},g} \left[d\sigma^{(0)}_{c\bar{c}}\right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q_T}}$$

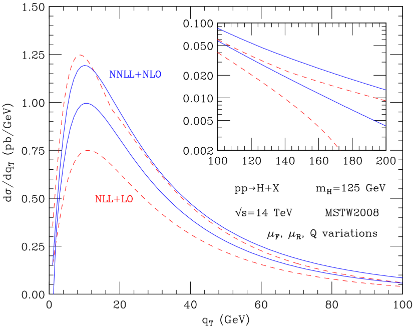
$$\times\ S_c(M,b) \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \left[(\mathbf{H}\mathbf{\Delta}) C_1 C_2\right]_{c\bar{c};a_1 a_2}$$

$$\times\ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)\,,$$



- Main difference with colourless case: soft factor (colour matrix) $\mathbf{\Delta}(\mathbf{b}, M; \mathbf{\Omega})$ which embodies soft (wide-angle) emissions from $Q\bar{Q}$ and from initial/final-state interferences (no collinear emission from heavy-quarks). Its contribution starts at NLL.
- Soft radiation produce colour-dependent azimuthal correlations at small-q_T entangled with the azimuthal dependence due to gluonic collinear radiation.
- Explicit results for coefficients obtained up NLO and NNLL accuracy.
- Soft-factor $\mathbf{\Delta}(\mathbf{b}, M; \mathbf{\Omega})$ consistent with breakdown (in weak form) of TMD factorization (additional process-dependent non-perturbative factor needed) [Collins,Qiu('07)].

HqT **results:** q_T spectrum of H boson at the LHC $\sqrt{s} = 14$ TeV



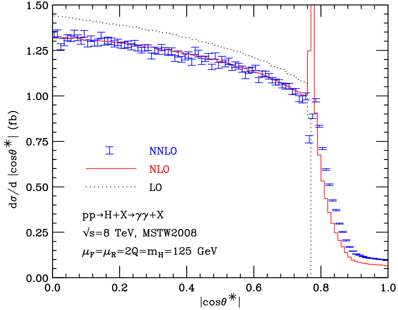
Higgs q_T spectrum for $m_H = 125\ GeV$ at LHC.

- Uncertainty bands obtained as before:

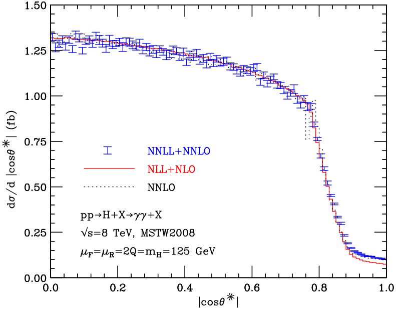
$1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$

- Significant reduction of scale dependence from NLL+LO to NNLL+NLO for all q_T.
- Good convergence of resummed results: NNLL+NLO and NLL+LO bands overlap (contrary to the fixed-order case).

HRES results: q_T-resummation with H boson decay

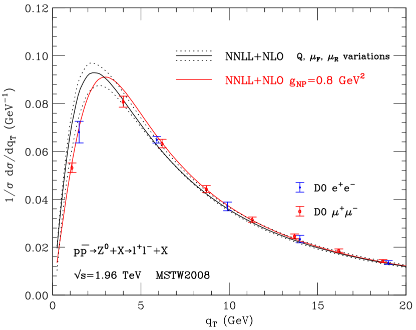


Fixed order results for $|\cos\theta^*| = \sqrt{1 - 4p_{T,\gamma}^2/m_H^2}$ distribution at the LHC.



Resummed results for $|\cos\theta^*| = \sqrt{1 - 4p_{T,\gamma}^2/m_H^2}$ distribution at the LHC.

Non perturbative intrinsic k_T effects



D0 data for the Z q_T spectrum.

- Up to now result in a complete perturbative framework (plus PDFs).
- Non perturbative *intrinsic* k_T effects can be parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$:

$$S_c(\alpha_S, \widetilde{L}) \rightarrow S_c(\alpha_S, \widetilde{L})\ S_{NP}$$

$$g_{NP} \simeq 0.8\ GeV^2$$ [Kulesza et al.('02)]

- With NP effects the q_T spectrum is harder. Quantitative impact of intrinsic k_T effects is comparable with perturbative uncertainties and with non perturbative effects from PDFs.

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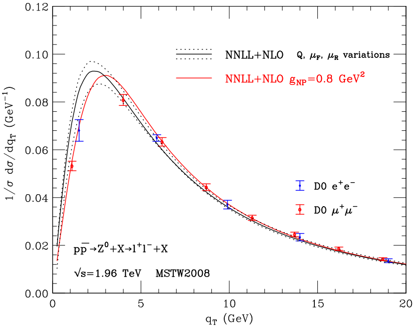

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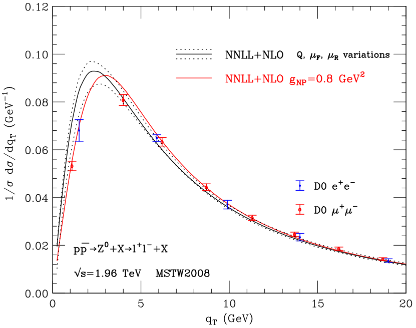

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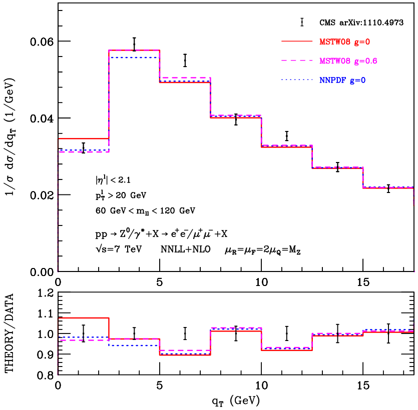

CMS data for the Z q_T spectrum.

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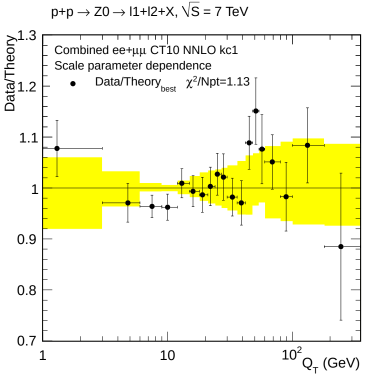
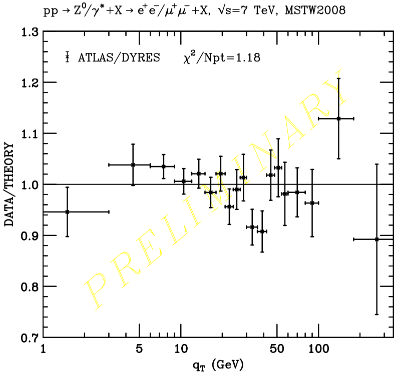
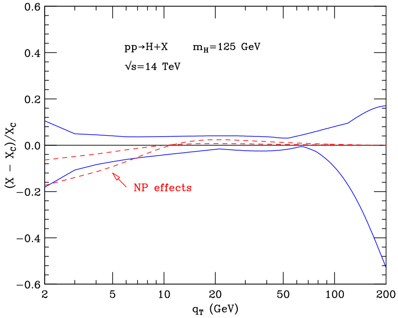




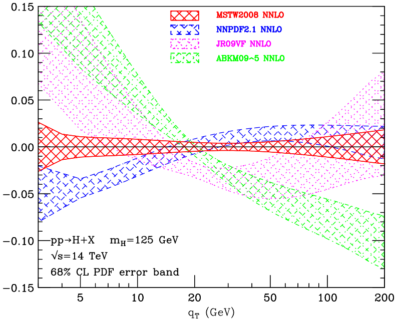
ATLAS ('11) data for the Z q_T spectrum compared with ResBos predictions with a Non Perturbative smearing parameter $g_{NP} = 1.1\,GeV^2$ [Guzzi,Nadolsky,Wang('13)].

ATLAS ('11) data for the Z q_T spectrum compared with DYRES predictions without Non Perturbative smearing ($g_{NP} = 0$).

Non perturbative intrinsic k_T effects

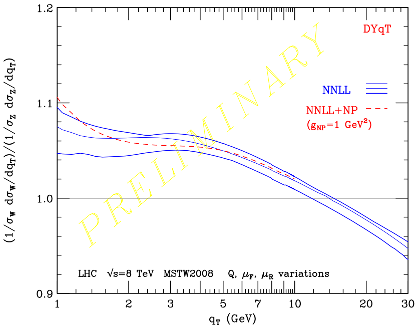


Uncertainties in the normalized q_T spectrum of the Higgs boson at the LHC. NNLL+NLO uncertainty bands (solid) compared to an estimate of NP effects with smearing parameter $g_{NP} = 1.67 - 5.64\ GeV^2$ (dashed).



The q_T spectrum has a strong sensitivity from collinear PDFs (especially from the gluon density).

W/Z ratio: the q_T spectrum



DYqT resummed predictions for the ratio of W/Z normalized q_T spectra.

- The use of the W/Z ratio observables substantially reduces both the experimental and theoretical systematic uncertainties [Giele,Keller('97)].
- Resummed perturbative prediction for

$$\frac{\frac{1}{\sigma_W}\frac{d\sigma_W}{dq_T}}{\frac{1}{\sigma_Z}\frac{d\sigma_Z}{dq_T}}(\mu_R, \mu_F, Q)$$

  with the customary scale variation.
- NNLL perturbative uncertainty band very small: 2-5% for $1 < q_T < 2$ GeV, 1.5-2% for $2 < q_T < 30$ GeV.
- Non perturbative effects within 1% for $1.5 < q_T < 5$ GeV and negligible for $q_T > 5$ GeV.