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EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS
TRENTO, ITALY
Institutional Member of the European Expert Committee NUPECC

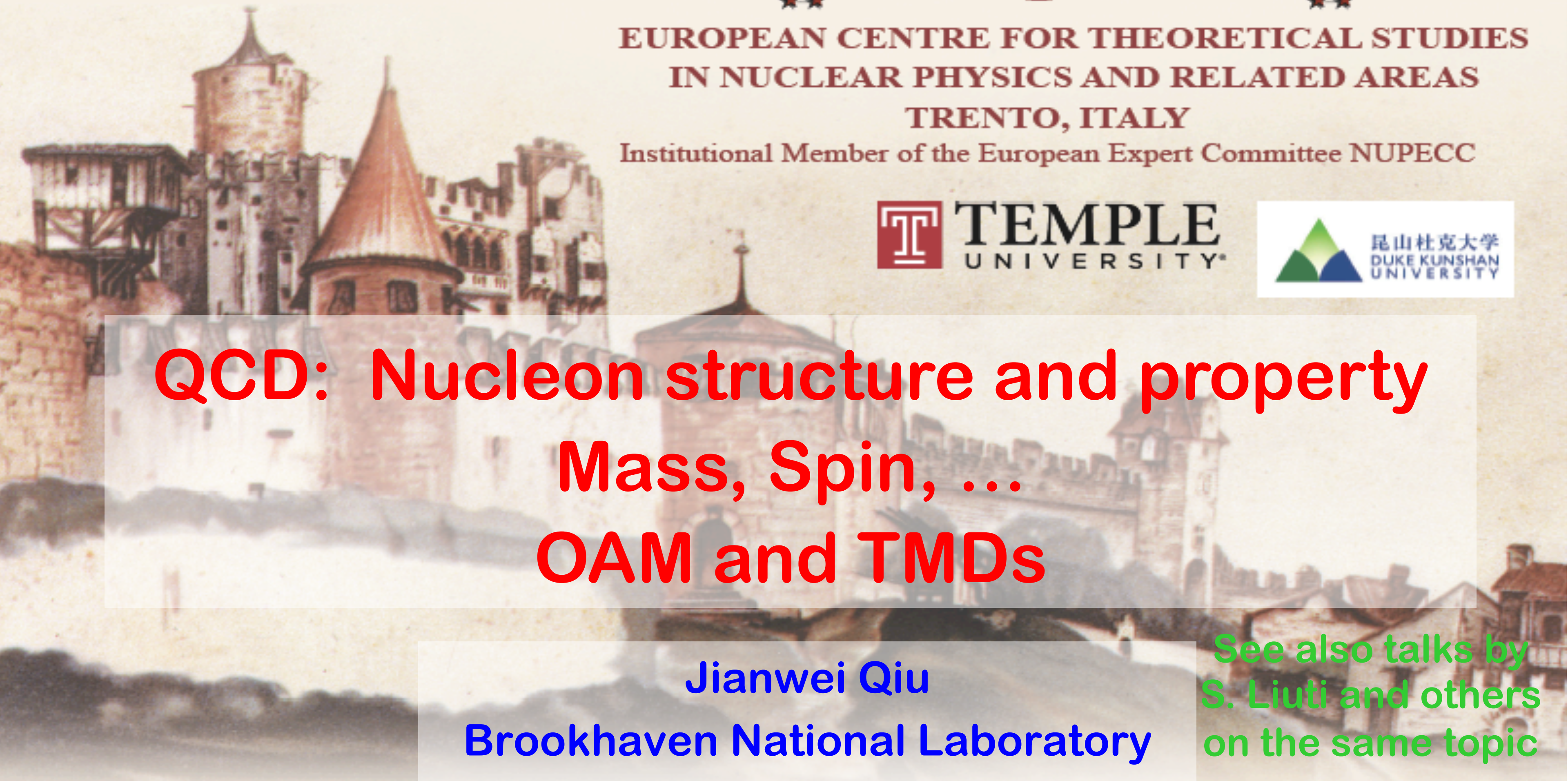
TEMPLE UNIVERSITY

昆山杜克大学 DUKE KUNSHAN UNIVERSITY

QCD: Nucleon structure and property
Mass, Spin, …
OAM and TMDs

Jianwei Qiu
Brookhaven National Laboratory

See also talks by
S. Liuti and others
on the same topic

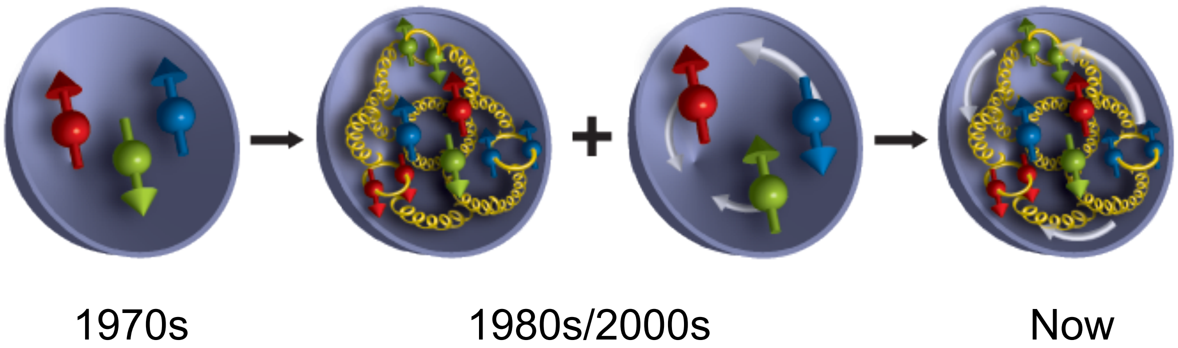


Castello di Trento ("Trint"), watercolor 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495). British Museum, London

Partons Transverse Momentum Distribution
at Large x: A Window into Partons Dynamics in
Nucleon Structure within QCD
Trento, April 11-15, 2016

Nucleon – building block of the visible world

- ❑ Our understanding of the nucleon evolves



Nucleon is a strongly interacting, relativistic bound state of quarks and gluons

- ❑ "Big" question:

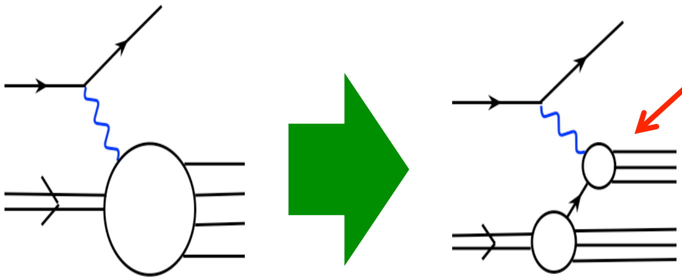
How does the nucleon property: mass, spin, … is determined by the nucleon's partonic structure and dynamics?

- ❑ Challenge:

No modern detector can see quarks and gluons in isolation!

Connecting the nucleon to partons

- ❑ **Necessary condition to "see" partons:**
  - – Scattering with large momentum transfer(s)



Sensitive to partonic dynamics

$+\ldots$ (Diagrams with more active partons from each hadron!)

Connection between hadron and parton

Connecting the nucleon to partons

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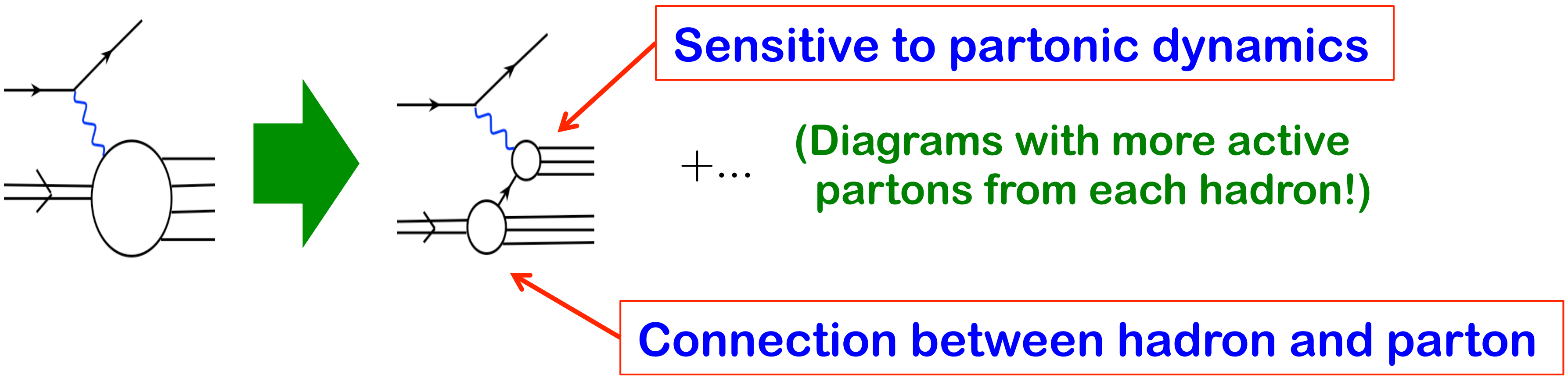


- **QCD factorization – Approximation!**

Connecting hadron to parton via hadronic matrix elements:

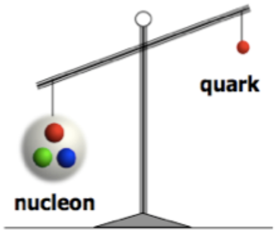
$$\langle p,s|\mathcal{O}(\psi,A^\alpha)|p,s\rangle\ :\quad \langle p,s|\overline{\psi}(0)\gamma^+\psi(y)|p,s\rangle,\ \ \langle p,s|F^{+\alpha}(0)F^{+\beta}(y)|p,s\rangle(-g_{\alpha\beta})$$

so as off-diagonal matrix elements, …

with pQCD calculable coefficients – short-distance parton dynamics

Hadron mass

- Nucleon mass – dominates the mass of visible world:



m_q ~ 10 MeV

m_N ~ 1000 MeV

Current quark mass $\sim 1\%$ proton's mass

Higgs mechanism is not enough!!!

- How does QCD generate hadron mass?

Hadron mass

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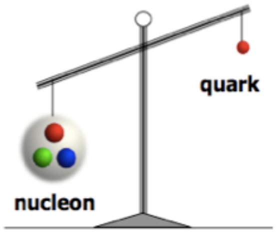

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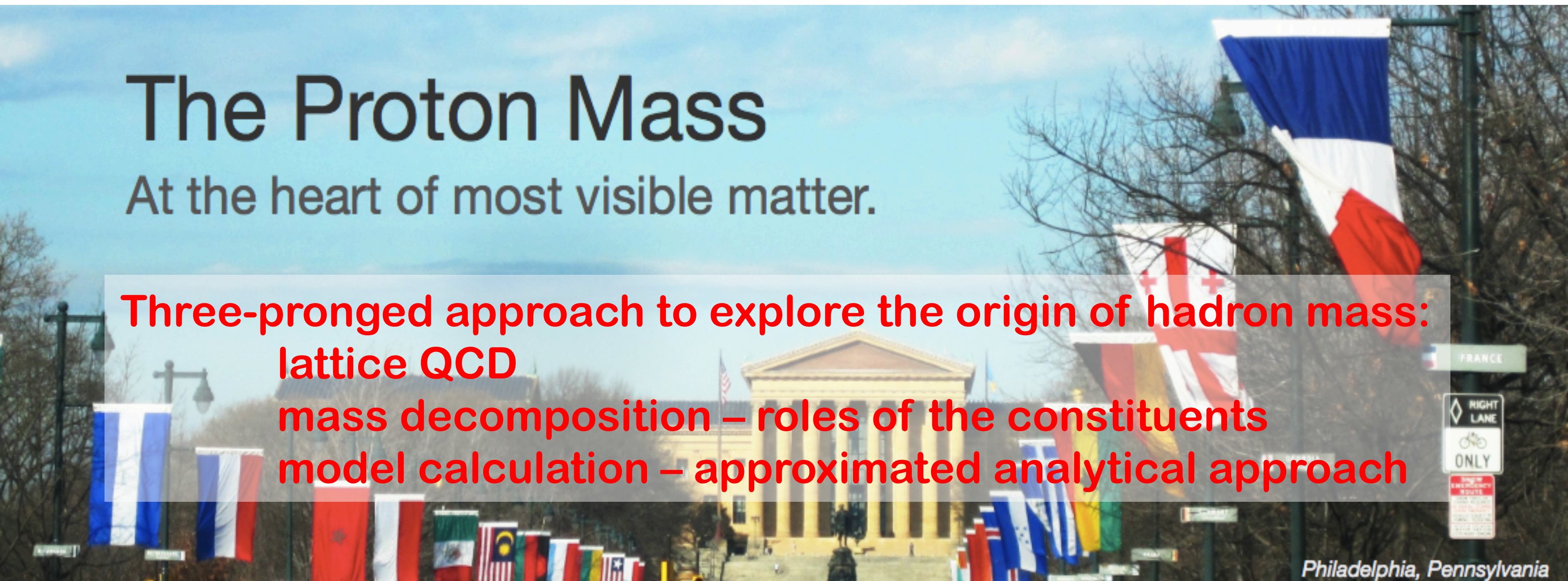


Three-pronged approach to explore the origin of hadron mass:

lattice QCD

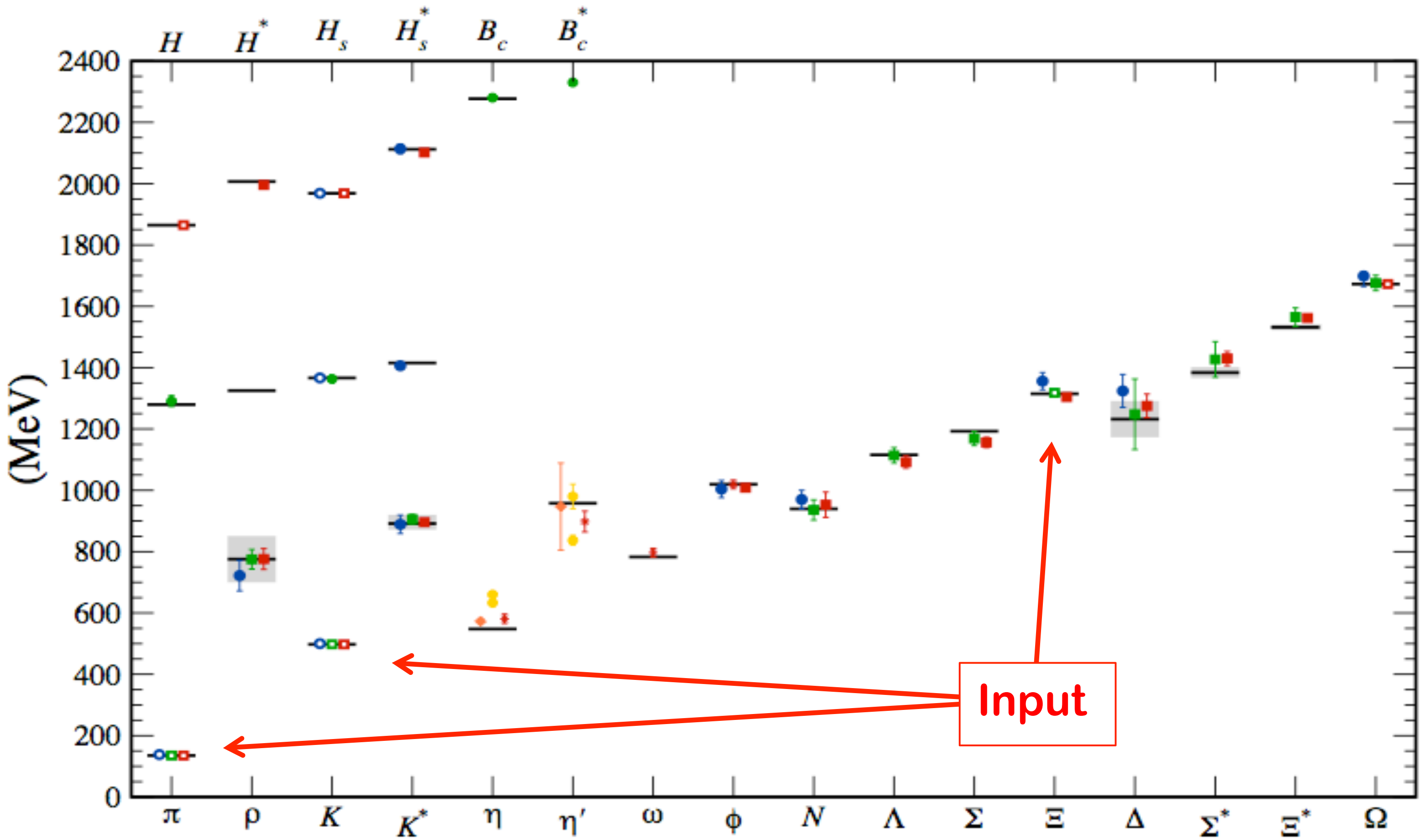
mass decomposition – roles of the constituents

model calculation – approximated analytical approach

https://phys.cst.temple.edu/meziani/proton-mass-workshop-2016/

Hadron mass

❑ Hadron mass from Lattice QCD calculation:



How does QCD generate this? The role of quarks vs that of gluons?

Not to discuss BSE and approximated analytical approaches in this talk – Cloet's talk

Hadron mass

❑ **How do quarks and gluons contribute to the hadron mass?**

✧ **QCD energy-momentum tensor in terms of quarks and gluons**

$$T^{\mu\nu} = \frac{1}{2}\,\overline{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi + \frac{1}{4}\,g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}$$

✧ **Its hadronic matrix element with zero momentum transfer:**

$$\langle p|\,T^{\mu\nu}\,|p\rangle \propto p^\mu p^\nu \quad \Rightarrow \quad \langle p|\,T^{\mu\nu}\,|p\rangle(g_{\mu\nu}) \propto p^\mu p^\nu(g_{\mu\nu}) = m^2$$

✧ **Invariant hadron mass (in any frame):**

$$m^2 \propto \langle p|T^{\alpha}{}_{\alpha}\,|p\rangle$$

with
$$T^{\alpha}{}_{\alpha} = \underbrace{\frac{\beta(g)}{2g}F^{\mu\nu,a}F^{a}_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s} m_q(1+\gamma_m)\overline{\psi}_q\psi_q$$

$$\beta(g) = -(11 - 2n_f/3)\,g^3/(4\pi)^2 + \ldots$$

$\Rightarrow$ **At the chiral limit, the entire mass is from gluons!**

Kharzeev @ Temple workshop

Hadron mass

- How do quarks and gluons contribute to the hadron mass?
  - QCD energy-momentum tensor in terms of quarks and gluons

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  - Ji's decomposition – hadron's rest frame:

X. Ji, PRL (1995)

$$m = \frac{\langle p|\int d^3x\, T^{00}\, |p\rangle}{\langle p|p\rangle} = H_q + H_m + H_g + H_a$$

Mass type	H_i	M_i	$m_s \to 0$ (MeV)	$m_s \to \infty$ (MeV)
Quark energy	$\psi^\dagger(-i\mathbf{D}\cdot\boldsymbol{\alpha})\psi$	$3(a-b)/4$	270	300
Quark mass	$\overline{\psi} m \psi$	b	160	110
Gluon energy	$\frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2)$	$3(1-a)/4$	320	320
Trace anomaly	$\frac{9\alpha_s}{16\pi}(\mathbf{E}^2-\mathbf{B}^2)$	$(1-b)/4$	190	210

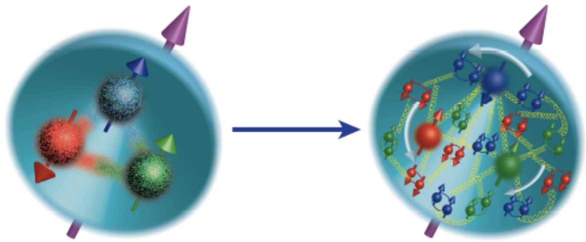
$$a(\mu^2) = \sum_f \int_0^1 x[q_f(x,\mu^2) + \overline{q}_f(x,\mu^2)]dx$$

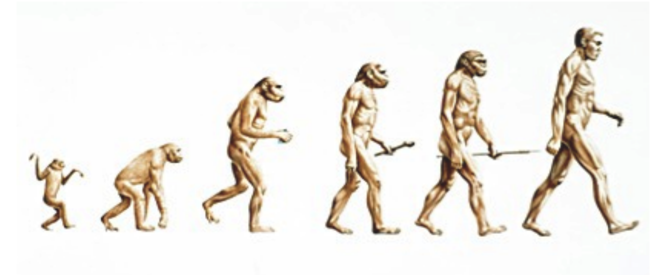
$$bM = \langle P|m_u\overline{u}u + m_d\overline{d}d|P\rangle + \langle P|m_s\overline{s}s|P\rangle$$

K.F. Liu's talk with updated numbers

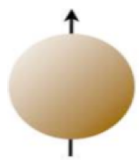
Hadron spin

- Proton's spin:



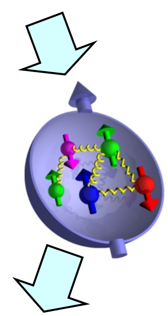


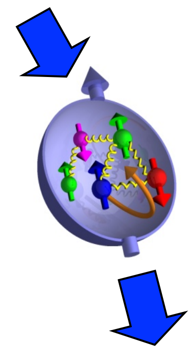
- Current understanding:



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

Proton Spin





Quark helicity
Best known

$$\frac{1}{2}\int dx \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$

$$\sim 30\%$$

Spin "puzzle"

Gluon helicity
Start to know

$$\Delta G = \int dx \Delta g(x)$$

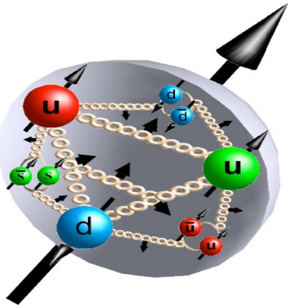
$$\sim 20\% (\text{with RHIC data})$$

Orbital Angular Momentum of quarks and gluons
Little known

If we do not understand proton spin, we do not understand QCD

Hadron spin

- How does QCD generate the proton spin?



Known from QCD

$$S(\mu) = \sum_f \langle P, S | \hat{J}^z_f(\mu) | P, S \rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)$$

From QCD, But, unknown

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma} \gamma_5 \psi_q + \psi_q^\dagger (\vec{x} \times (-i\vec{D}))\, \psi_q \right] \qquad \vec{J}_g = \int d^3x \left[\vec{x} \times (\vec{E} \times \vec{B}) \right]$$

- Asymptotic limit:

X. Ji, 2005

$$J_q(\mu \to \infty) \Rightarrow \frac{1}{2} \frac{3N_f}{16 + 3N_f} \sim \frac{1}{4} \qquad J_g(\mu \to \infty) \Rightarrow \frac{1}{2} \frac{16}{16 + 3N_f} \sim \frac{1}{4}$$

Hadron spin

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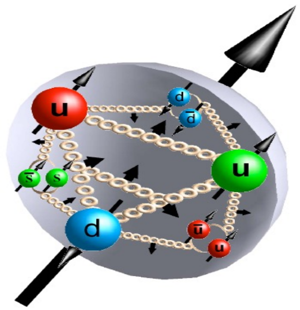

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Spin sum rule – not unique!

$$S(\mu) = \frac{1}{2}\Sigma(\mu) + L_q(\mu) + \Delta G(\mu) + [\overbrace{J_g(\mu) - \Delta G(\mu)}^{L_g(Q^2)}]$$

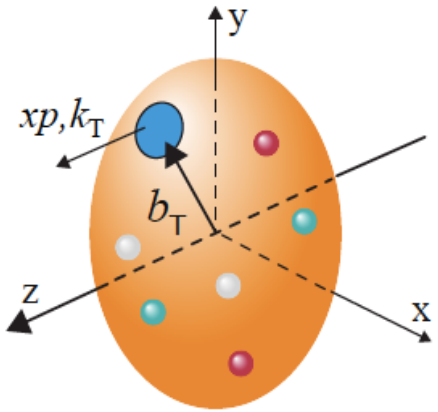
Intrinsic from partons' spin: $\Sigma(Q^2) = \sum_q \left[\Delta q(Q^2) + \Delta \bar{q}(Q^2) \right], \quad \Delta G(Q^2)$

dynamical from partons' motion: $L_q(Q^2), \quad L_g(Q^2)$

- Matrix elements of quark and gluon fields are NOT physical observables!
- Infinite possibilities of decompositions – connection to observables?

Observables – High energy scatterings

❑ **High energy scattering with a large momentum transfer:**



- ✧ **Momentum scale of the hard probe:**

$$Q \gg 1/R \sim \Lambda_{\rm QCD} \sim 1/{\rm fm}$$

- ✧ **Combined motion ~ 1/R**
  is too week to be sensitive to the hard probe

- ✧ **Collinear factorization – integrated into PDFs, …**

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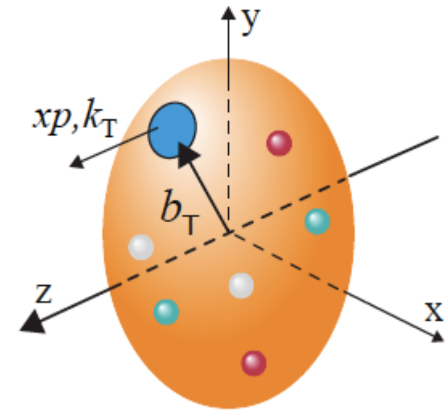
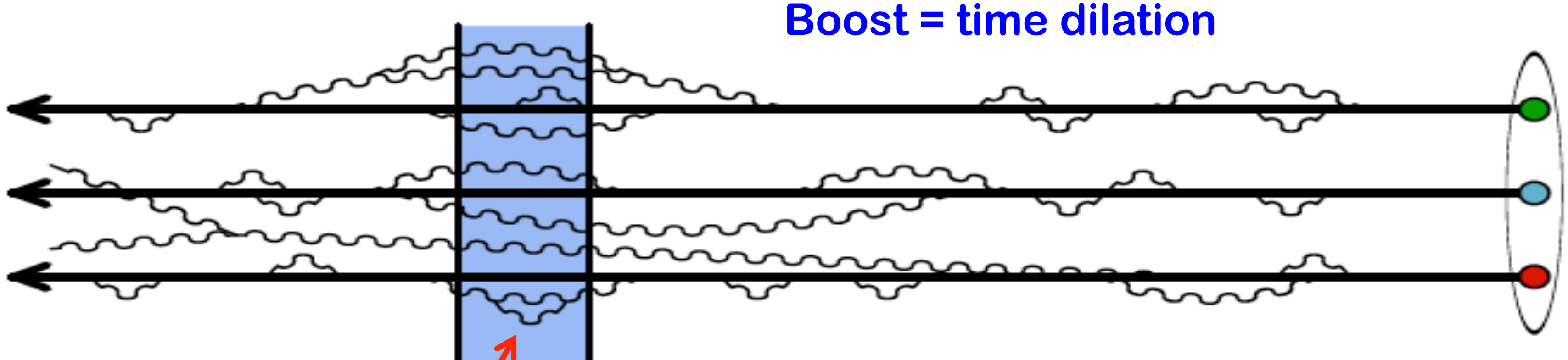


- **High energy probes "see" the boosted partonic structure:**

Boost = time dilation



Momentum fraction x

Hard probe: t ~ 1/Q < 1/10 fm

Observables – High energy scatterings

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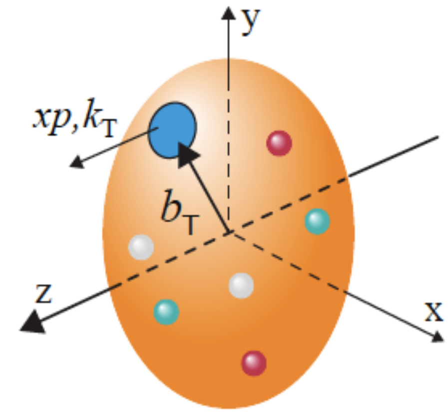

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✧ **Collinear factorization – integrated into PDFs, …**

➡ ***Need scattering with two momentum scales observed!***

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

✧ **Hard scale** Q_1 **localizes the probe to see the quark or gluon d.o.f.**

✧ **"Soft" scale** Q_2 **could be sensitive to the confined motion**

❑ Observables break the proton:

✧ **Such as SIDIS, low p_T Drell-Yan, …**

✧ **TMD factorization: partons' confined motion is encoded into TMDs**

Observables – High energy scatterings

- **High energy scattering with a large momentum transfer:**
  - **Momentum scale of the hard probe:**

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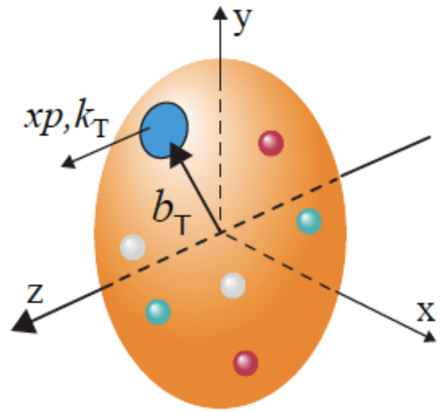
  - **Collinear factorization – integrated into PDFs, …**



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  - **Hard scale Q_1 localizes the probe to see the quark or gluon d.o.f.**
  - **"Soft" scale Q_2 could be sensitive to the confined motion**

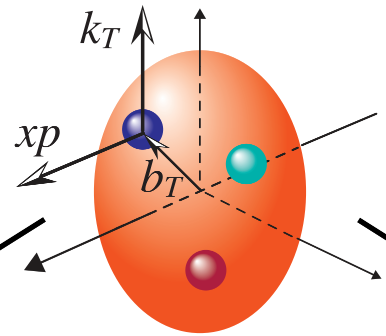
- **Observables without breaking the proton:**
  - **Such as the exclusive DIS, DVCS, diffractive scattering, …**
  - **GPD factorization: partons' spatial imaging is encoded into GPDs**

Unified view of nucleon structure

- **Wigner distributions:**

Momentum Space

Coordinate Space



TMDs $\int d^2b_T$ $\int d^2k_T$ *GPDs*

Confined motion $f(x,k_T)$ $\boxed{\text{Two-scales observables}}$ $f(x,b_T)$ Spatial distribution

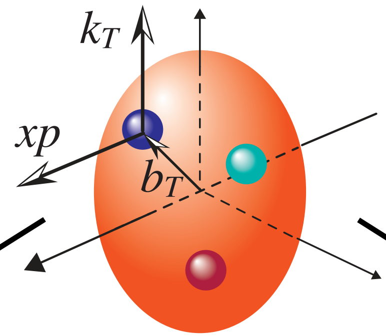
Unified view of nucleon structure

❑ **Wigner distributions:**

Momentum Space

TMDs

Confined motion



$\int d^2b_T$ $\qquad$ $\int d^2k_T$

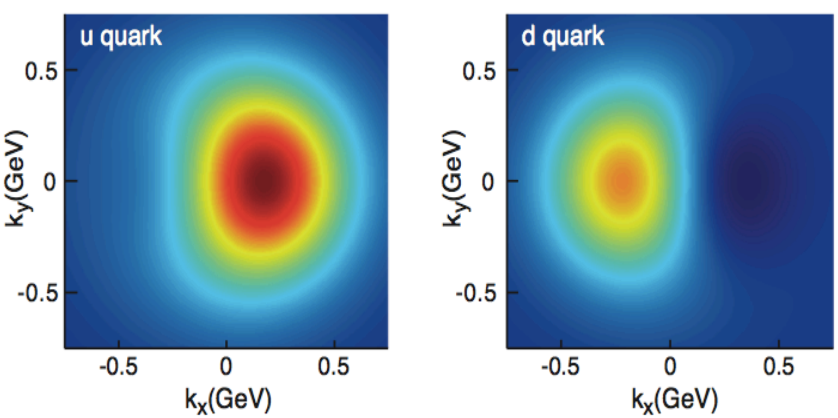
$f(x,k_T)$ **Two-scales observables** $f(x,b_T)$

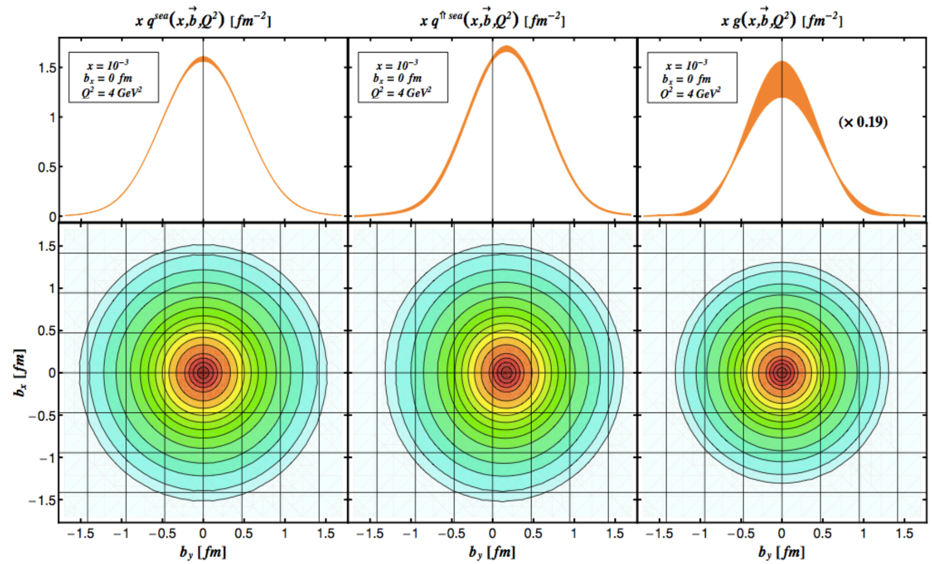
Coordinate Space

GPDs

Spatial distribution

Sivers Functions





Position *r* x Momentum *p* → Orbital Motion of Partons

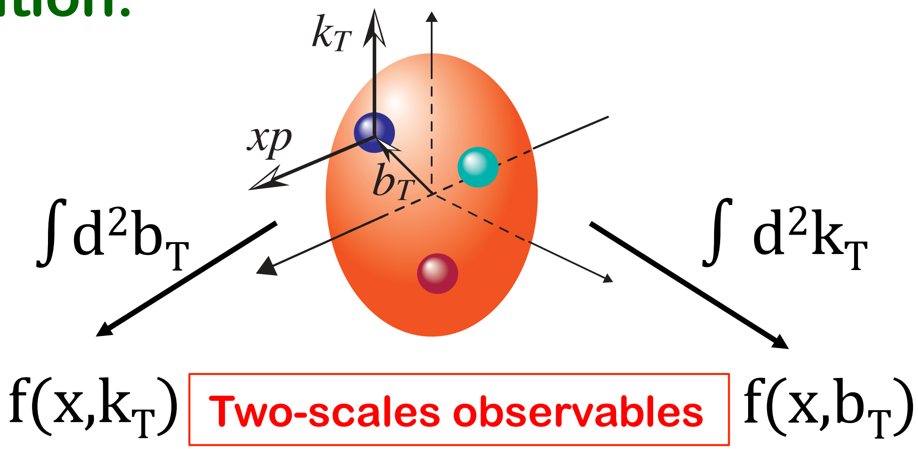
Unified view of nucleon structure

❑ Wigner distribution:

Momentum Space

TMDs

Confined motion



Coordinate Space

GPDs

Spatial distribution

❑ Note:

- ✧ Partons' confined motion and their spatial distribution are unique – the consequence of QCD
- ✧ But, the TMDs and GPDs that represent them are not unique!
  - – Depending on the definition of the Wigner distribution and QCD factorization to link them to physical observables

Position r x Momentum p → Orbital Motion of Partons

Orbital angular momentum

OAM: Correlation between parton's position and its motion
– in an averaged (or probability) sense

- **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

- **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

- **Difference between them:**

Hatta, Lorce, Pasquini, …

- ✧ compensated by difference between gluon OAM density
- ✧ represented by different choice of gauge link for OAM Wagner distribution

$$\mathcal{L}_q^3 \left\{L_q^3\right\} = \int dx\, d^2b\, d^2k_T \left[\vec{b} \times \vec{k}_T\right]^3 \mathcal{W}_q(x,\vec{b},\vec{k}_T) \left\{W_q(x,\vec{b},\vec{k}_T)\right\}$$

with

$$\mathcal{W}_q \left\{W_q\right\}(x,\vec{b},\vec{k}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}} \int \frac{dy^- d^2y_T}{(2\pi)^3} e^{i(xP^+y^- - \vec{k}_T\cdot\vec{y}_T)}$$

$$\times \langle P'|\, \overline{\psi}_q(0)\, \frac{\gamma^+}{2}\, \underbrace{\Phi^{\mathrm{JM}\{\mathrm{Ji}\}}(0,y)}_{\text{Gauge link}}\, \psi(y)\, |P\rangle_{y^+=0}$$

JM: "staple" gauge link
Ji: straight gauge link

between 0 and $y=(y^+=0, y^-, y_T)$

Orbital angular momentum

OAM: Correlation between parton's position and its motion – in an averaged (or probability) sense

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❑ **Difference between them:**

Hatta, Yoshida, Burkardt, Meissner, Metz, Schlegel, …

✧ generated by a "torque" of color Lorentz force

$$\mathcal{L}_q^3 - L_q^3 \propto \int \frac{dy^- d^2 y_T}{(2\pi)^3} \langle P'|\overline{\psi}_q(0) \frac{\gamma^+}{2} \int_{y^-}^{\infty} dz^- \Phi(0, z^-)$$
$$\times \underbrace{\sum_{i,j=1,2} \left[\epsilon^{3ij} y_T^i F^{+j}(z^-)\right]}_{\text{“Chromodynamic torque”}} \Phi(z^-, y)\psi(y)|P\rangle_{y^+=0}$$

Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of g_2

Summary on mass and spin decomposition

- **The "big" question:**

  If there are infinite possibilities, why bother and what do we learn?

- **The "origin" of the difficulty/confusion:**

  QCD is a gauge theory: a pure quark field in one gauge is a superposition of quarks and gluons in another gauge

- **The fact:**

  None of the items in all spin decompositions are direct physical observables, unlike cross sections, asymmetries, …

- **Ambiguity in interpretation – two old examples:**

  ✧ **Factorization scheme:**

  $$F_2(x,Q^2) = \sum_{q,\bar{q}} C_q^{\mathrm{DIS}}(x, Q^2/\mu^2) \otimes q^{\mathrm{DIS}}(x,\mu^2)$$

  No glue contribution to F_2?

  ✧ **Anomaly contribution to longitudinal polarization:**

  $$g_1(x,Q^2) = \sum_{q,\bar{q}} \widetilde{C}_q^{\mathrm{ANO}} \otimes \Delta q^{\mathrm{ANO}} + \widetilde{C}_g^{\mathrm{ANO}} \otimes \Delta G^{\mathrm{ANO}}$$

  $$\Rightarrow \quad \Delta\Sigma \longrightarrow \Delta\Sigma^{\mathrm{ANO}} - \frac{n_f \alpha_s}{2\pi} \Delta G^{\mathrm{ANO}}$$

  Larger quark helicity?

Summary on mass and spin decomposition

- **Key for a good decomposition – sum rule:**
  - **Every term can be related to a physical observable with controllable approximation – "independently measurable"**

    DIS scheme is ok for F2, but, less effective for other observables

    Additional symmetry constraints, leading to "better" decomposition?

  - **Natural physical interpretation for each term – "hadron structure"**
  - **Hopefully, calculable in lattice QCD – "numbers w/o distributions"**

The most important task is,

Finding the connection to physical observables!

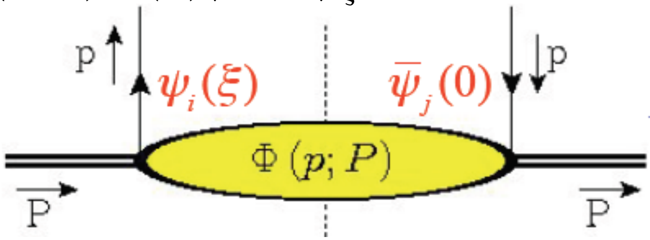
See talks by Liuti and others on the measurability

Questions/issues for TMDs

❑ Non-perturbative definition:

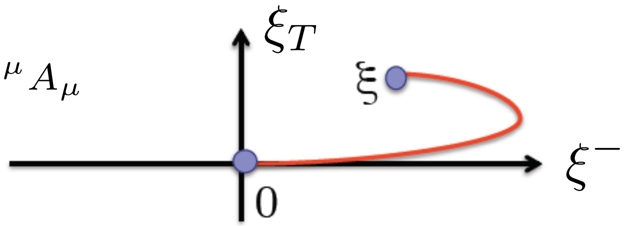
✧ In terms of matrix elements of parton correlators:

$$\Phi^{[U]}(x, p_T; n) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{i\,p\cdot\xi} \langle P, S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P, S\rangle_{\xi^+=0}$$



✧ Depends on the choice of the gauge link:

$$U(0,\xi) = e^{-ig\int_0^\xi ds^\mu A_\mu}$$



✧ Decomposes into a list of TMDs:

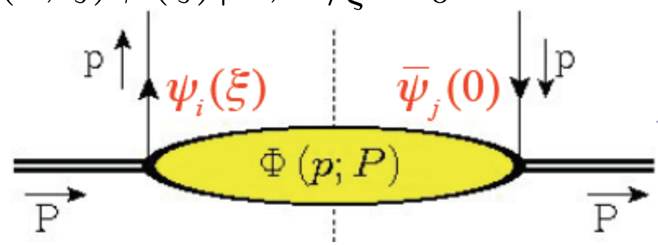
$$\Phi^{[U]}(x, p_T; n) = \Big\{ f_1^{[U]}(x, p_T^2) - f_{1T}^{\perp[U]}(x, p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x, p_T)\gamma_5 + h_{1T}^{[U]}(x, p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x, p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + ih_1^{\perp[U]}(x, p_T^2)\,\frac{\not{p}_T}{M} \Big\}\frac{\not{P}}{2},$$

Questions/issues for TMDs

❑ Non-perturbative definition:

✧ In terms of matrix elements of parton correlators:

$$\Phi^{[U]}(x, p_T; n) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{i\,p\cdot\xi}\, \langle P, S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P,S\rangle_{\xi^+=0}$$



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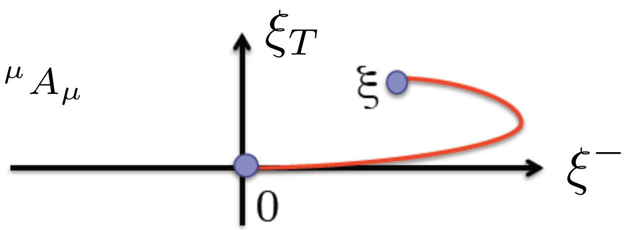

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✧ IF we knew proton wave function, this definition gives "unique" TMDs!

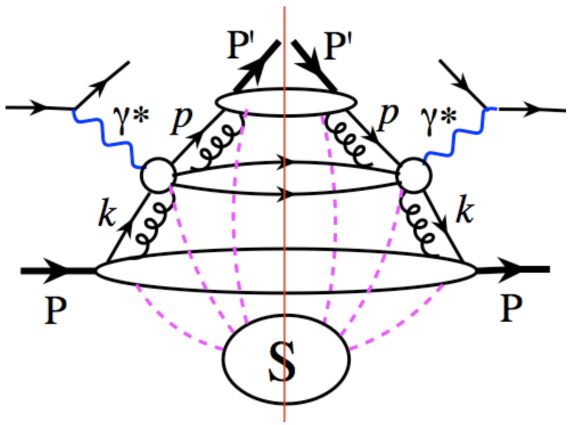
But, we do NOT know proton wave function (may calculate it using BSE?)

TMDs defined in this way are NOT direct physical observables!

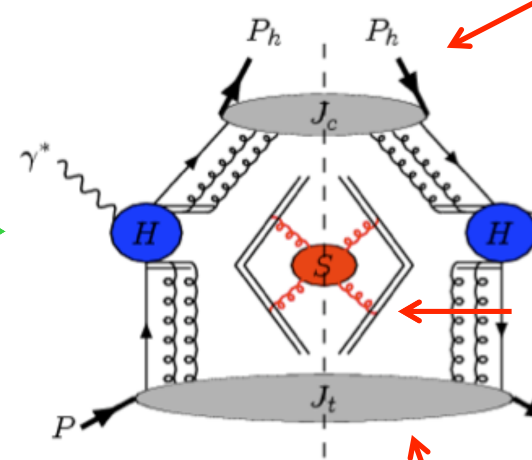
Questions/issues for TMDs

❑ **Perturbative definition – in terms of TMD factorization:**

SIDIS as an example:



TMD fragmentation



$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

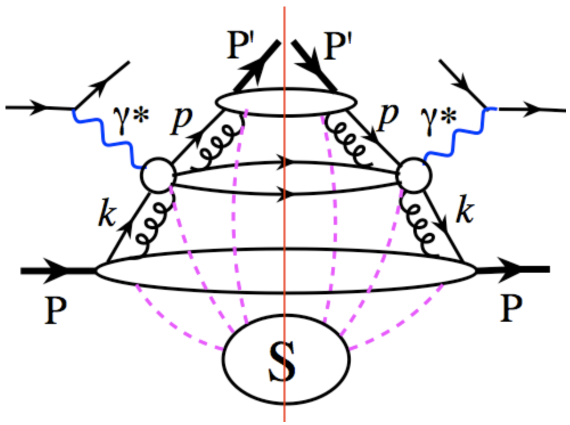
Soft factors

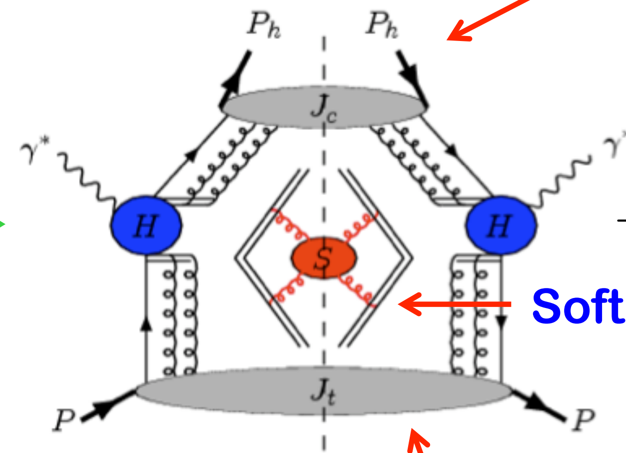
TMD parton distribution

Definitions of TMDs

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:





TMD fragmentation

$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

Soft factors

TMD parton distribution

❑ Low P_{hT} – TMD factorization:

$$\sigma_{\mathrm{SIDIS}}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

❑ High P_{hT} – Collinear factorization:

$$\sigma_{\mathrm{SIDIS}}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q,P_{h\perp},\alpha_s)\otimes\phi_f\otimes D_{f\to h}+\mathcal{O}\left(\frac{1}{P_{h\perp}},\frac{1}{Q}\right)$$

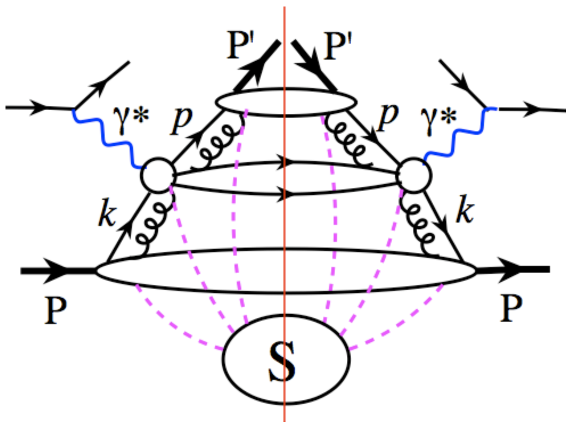
❑ P_{hT} Integrated - Collinear factorization:

$$\sigma_{\mathrm{SIDIS}}(Q,x_B,z_h)=\tilde{H}(Q,\alpha_s)\otimes\phi_f\otimes D_{f\to h}+\mathcal{O}\left(\frac{1}{Q}\right)$$

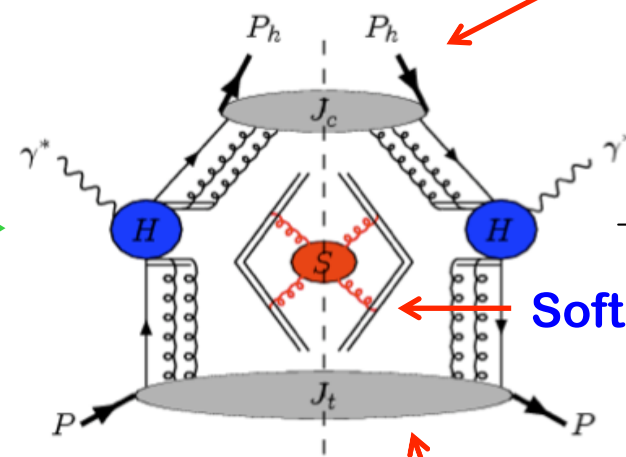
Definitions of TMDs

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:



TMD fragmentation



$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

Soft factors

TMD parton distribution

❑ Extraction of TMDs:

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

TMDs are extracted by fitting DATA using the factorization formula (approximation) and the perturbatively calculated $\hat{H}(Q;\mu)$.

Extracted TMDs are valid only when the <p^2> << Q^2

See also talks by Rogers, …

Evolution equations for TMDs

J.C. Collins, in his book on QCD

- **TMDs in the b-space:**

$$\tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F) = \tilde{F}^{\mathrm{unsub}}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; y_P - (-\infty)) \sqrt{\frac{\tilde{S}_{(0)}(\mathrm{b_T}; +\infty, y_s)}{\tilde{S}_{(0)}(\mathrm{b_T}; +\infty, -\infty)\tilde{S}_{(0)}(\mathrm{b_T}; y_s, -\infty)}} Z_F Z_2$$

- **Collins-Soper equation:**

Renormalization of the soft-factor

$$\frac{\partial \tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu) \tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F)$$

$$\zeta_F = M_P^2 x^2 e^{2(y_P - y_s)}$$

Introduced to regulate the rapidity divergence of TMDs

$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_s} \ln \left(\frac{\tilde{S}(b_T; y_s, -\infty)}{\tilde{S}(b_T; +\infty, y_s)} \right)$$

- **RG equations:**

Wave function Renormalization

$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

Evolution equations are only valid when $b_T << 1/\Lambda_{QCD}$!

$$\frac{d\tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F)}{d \ln \mu} = \gamma_F(g(\mu); \zeta_F/\mu^2) \tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F).$$

- **Momentum space TMDs:**

Need information at large b_T for all scale μ !

$$F_{f/P^\uparrow}(x, \mathrm{k_T}, S; \mu, \zeta_F) = \frac{1}{(2\pi)^2} \int d^2\mathrm{b}_T \, e^{i\mathrm{k}_T \cdot \mathrm{b}_T} \, \tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu, \zeta_F)$$

Evolution equations for Sivers function

Aybat, Collins, Qiu, Rogers, 2011

- **Sivers function:**

$$F_{f/P^\uparrow}(x,k_T,S;\mu,\zeta_F) = F_{f/P}(x,k_T;\mu,\zeta_F) - F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F)\frac{\epsilon_{ij}k_T^i S^j}{M_p}$$

- **Collins-Soper equation:**

Its derivative obeys the CS equation

$$\frac{\partial \ln \tilde{F}'^{\perp f}_{1T}(x,b_T;\mu,\zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T;\mu)$$

$$\tilde{F}'^{\perp f}_{1T}(x,b_T;\mu,\zeta_F) \equiv \frac{\partial \tilde{F}^{\perp f}_{1T}(x,b_T;\mu,\zeta_F)}{\partial b_T}$$

- **RG equations:**

$$\frac{d\tilde{F}'^{\perp f}_{1T}(x,b_T;\mu,\zeta_F)}{d\ln\mu} = \gamma_F(g(\mu);\zeta_F/\mu^2)\tilde{F}'^{\perp f}_{1T}(x,b_T;\mu,\zeta_F)$$

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \quad \Longrightarrow \quad \frac{\partial\gamma_F(g(\mu);\zeta_F/\mu^2)}{\partial\ln\sqrt{\zeta_F}} = -\gamma_K(g(\mu)),$$

- **Sivers function in momentum space:**

$$F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F) = \frac{-1}{2\pi k_T}\int_0^\infty db_T\, b_T\, J_1(k_T b_T)\tilde{F}'^{\perp f}_{1T}(x,b_T;\mu,\zeta_F)$$

JI, Ma, Yuan, 2004
Idilbi, et al, 2004,
Boer, 2001, 2009,
Kang, Xiao, Yuan, 2011
Aybat, Prokudin, Rogers, 2012
Idilbi, et al, 2012,
Sun, Yuan 2013, …

Extrapolation to large b_T

Aybat and Rogers, arXiv:1101.5057
Collins and Rogers, arXiv:1412.3820

- CSS b*-prescription:

$$\tilde{F}_{f/P}(x, \mathrm{b}_T; Q, Q^2) = \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b)}^{\mathrm{AA}}$$

$$\times \overbrace{\times \exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\}}^{\mathrm{BB}}$$

$$\times \boxed{\overbrace{\exp\left\{ g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \right\}}^{\mathrm{CC}}}$$

Nonperturbative "form factor"

$$b_* = \frac{b_\mathrm{T}}{\sqrt{1 + b_\mathrm{T}^2 / b_\mathrm{max}^2}} \qquad \text{with } b_\mathrm{max} \sim 1/2 \ \mathrm{GeV}^{-1}$$

- Nonperturbative fitting functions

Various fits correspond to different choices for $g_{f/P}(x, b_T)$ and $g_K(b_T)$
e.g.

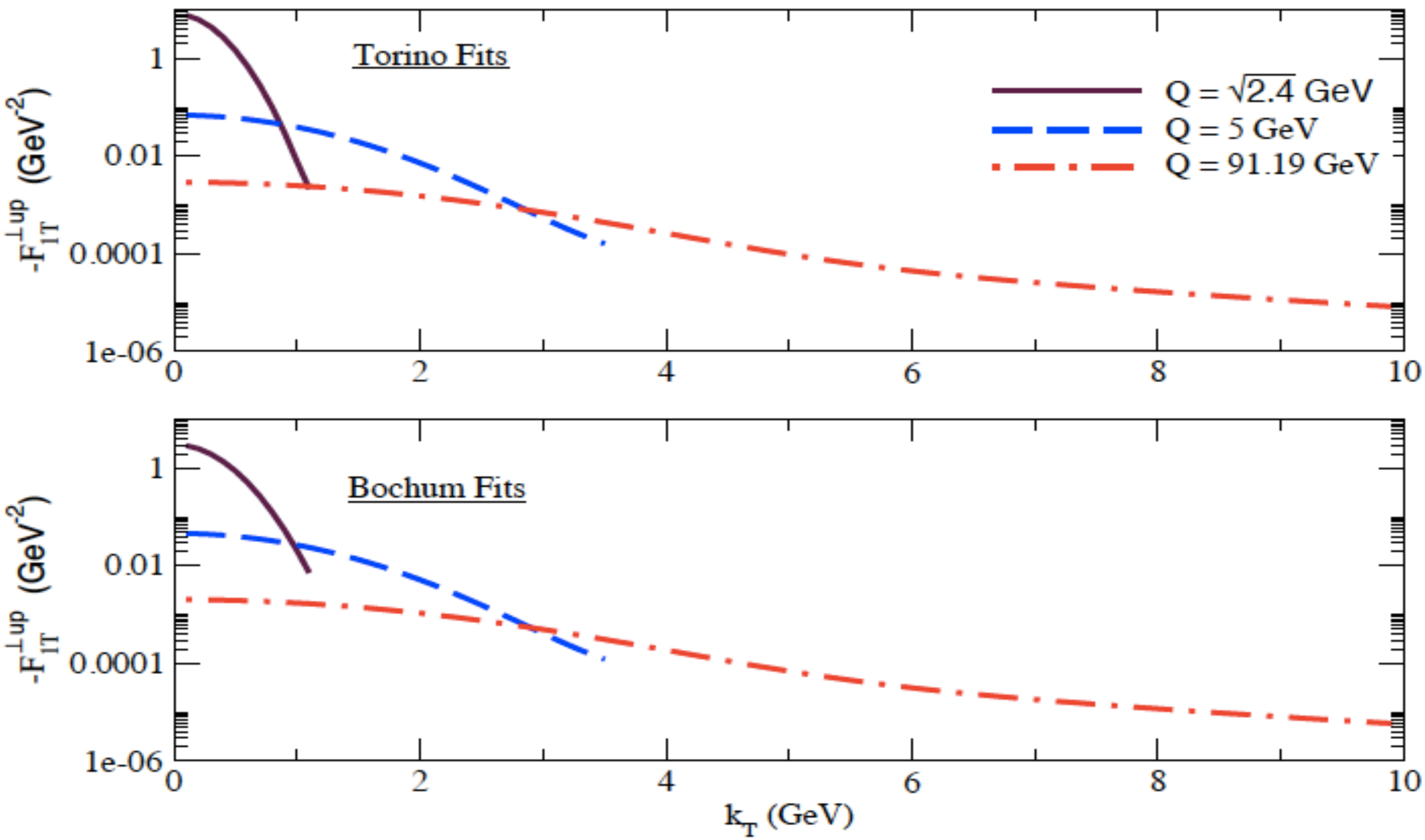
$$g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \equiv -\left[g_1 + g_2 \ln \frac{Q}{2Q_0} + g_1 g_3 \ln(10x) \right] b_T^2$$

Different choice of g2 & b_* could lead to different over all Q-dependence!

Evolution of Sivers function

Aybat, Collins, Qiu, Rogers, 2011

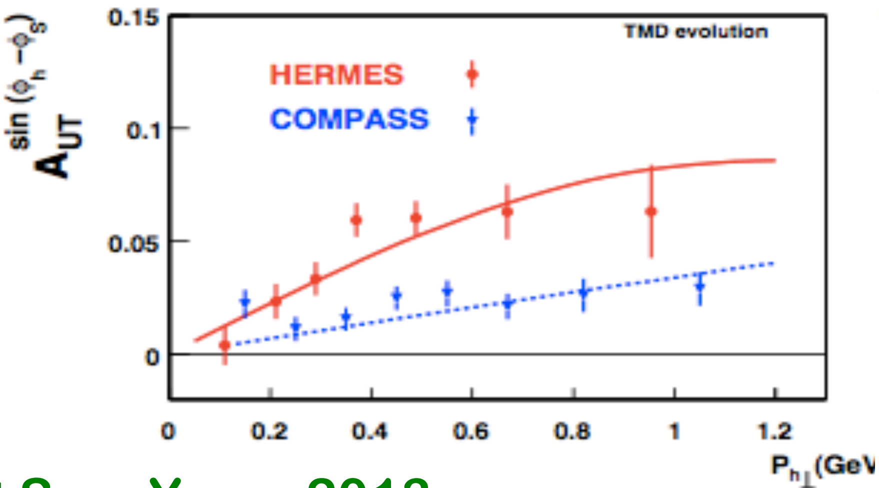
❑ Up quark Sivers function:

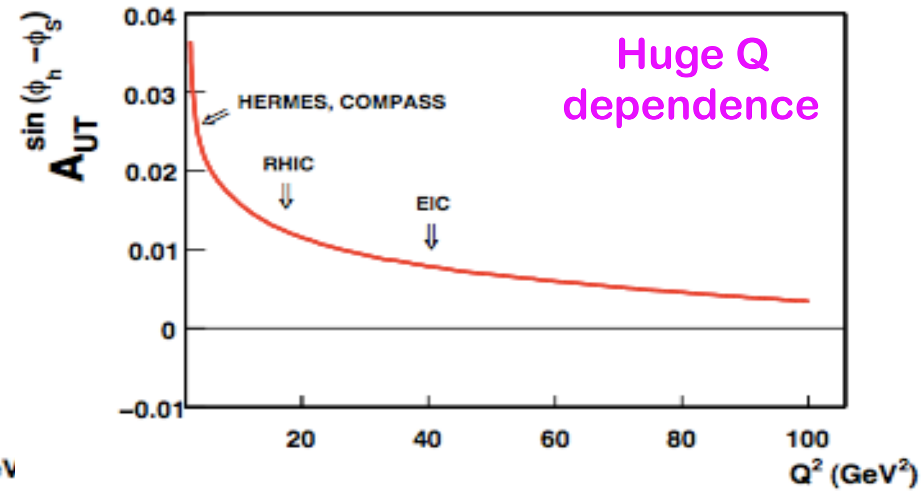


Very significant growth in the width of transverse momentum

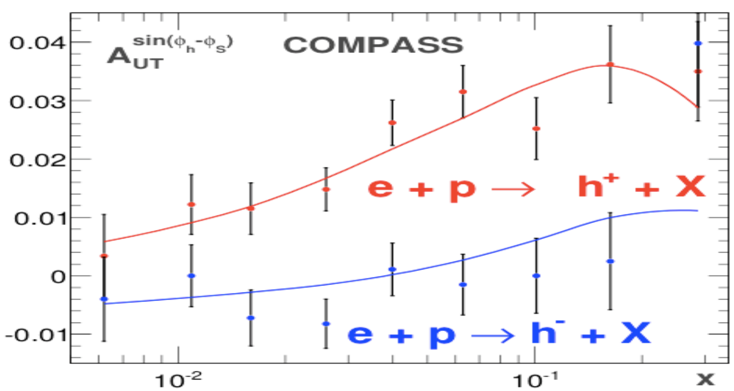
Different fits – different Q-dependence

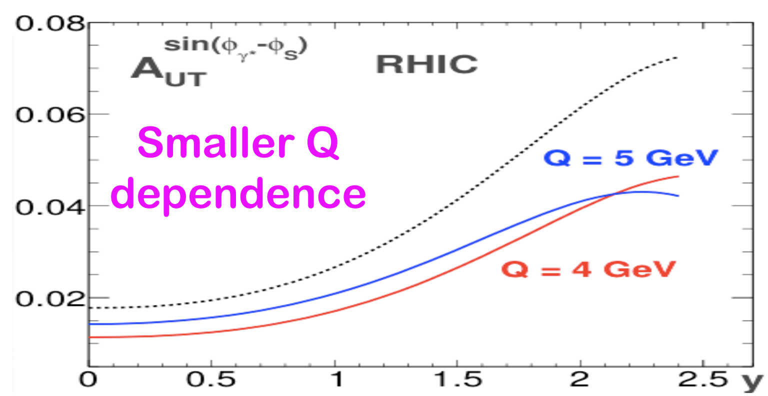
- Aybat, Prokudin, Rogers, 2012:





- Sun, Yuan, 2013:



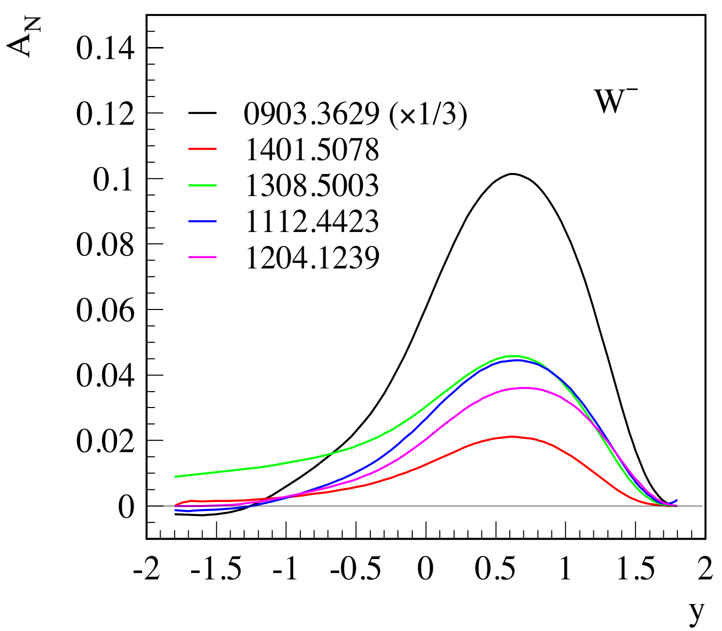


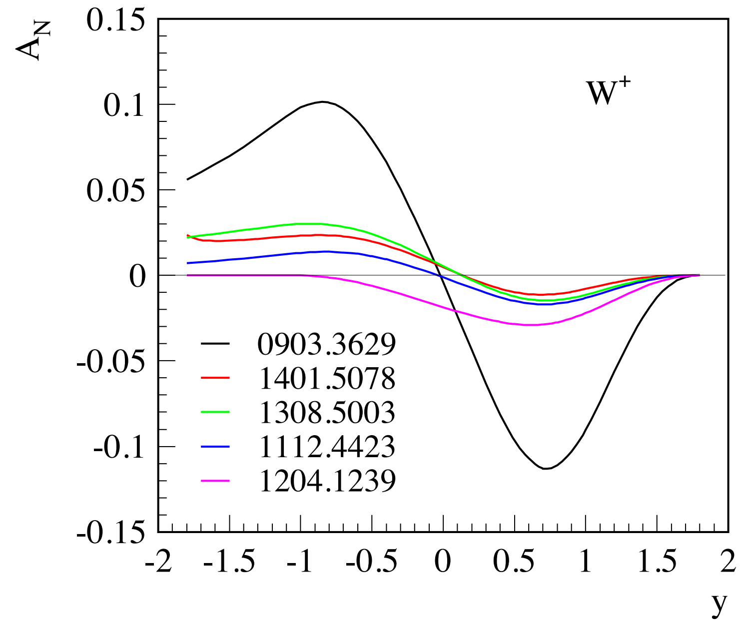
No disagreement on evolution equations!

Issues: Extrapolation to non-perturbative large b-region
Choice of the Q-dependent "form factor"

"Predictions" for A_N of W-production at RHIC?

- **Sivers Effect:**
  - ✧ **Quantum correlation between the spin direction of colliding hadron and the preference of motion direction of its confined partons**
  - ✧ **QCD Prediction: Sign change of Sivers function from SIDIS and DY**
- **Current "prediction" and uncertainty of QCD evolution:**





TMD collaboration proposal: Lattice, theory & Phenomenology

RHIC is the excellent and unique facility to test this (W/Z – DY)!

What happened?

- Sivers function:

Differ from PDFs!

$$F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F) = \frac{-1}{2\pi k_T}\int_0^\infty db_T\, b_T\, J_1(k_T b_T)\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)$$

Need non-perturbative large b_T information for any value of Q! $Q = \mu$

- What is the "correct" Q-dependence of the large b_T tail?

$$\tilde{F}_{f/P}(x,\mathrm{b}_T;Q,Q^2) = \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}}\tilde{C}_{f/j}(x/\hat{x},b_*;\mu_b^2,\mu_b,g(\mu_b))f_{j/P}(\hat{x},\mu_b)}^{\mathrm{AA}}$$

$$\times \overbrace{\times \exp\left\{\ln\frac{Q}{\mu_b}\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(g(\mu'))\right]\right\}}^{\mathrm{BB}}$$

$$\times \boxed{\overbrace{\exp\left\{g_{f/P}(x,b_T) + g_K(b_T)\ln\frac{Q}{Q_0}\right\}}^{\mathrm{CC}}}$$

Nonperturbative "form factor"

$$g_{f/P}(x,b_T) + g_K(b_T)\ln\frac{Q}{Q_0} \equiv -\left[g_1 + g_2\ln\frac{Q}{2Q_0} + g_1 g_3\ln(10x)\right]b_T^2$$

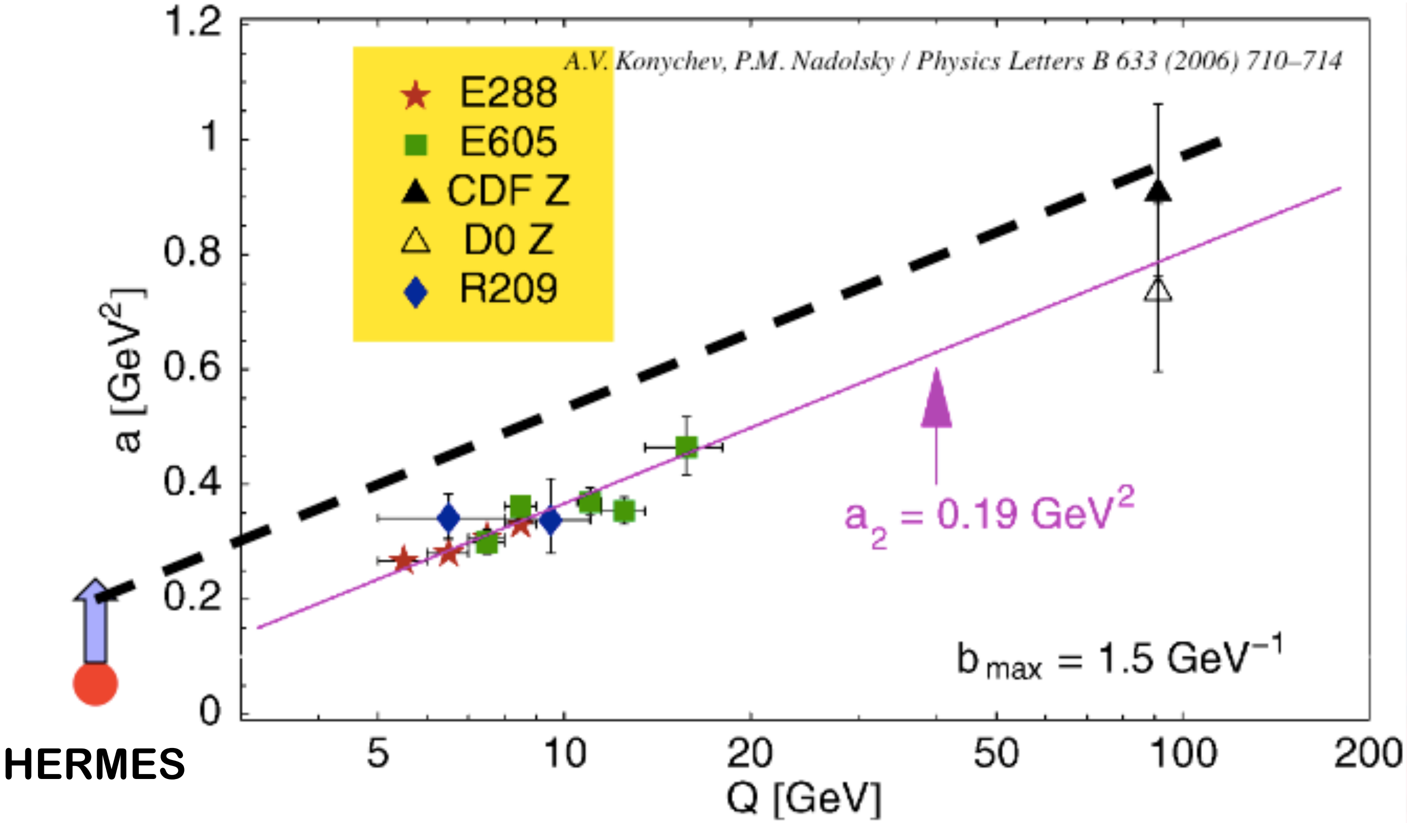
Is the log(Q) dependence sufficient? Choice of g_2 & b_* affects Q-dep.

The "form factor" and b_* change perturbative results at small b_T!

Q-dependence of the "form" factor

Konychev, Nadolsky, 2006

- Q-dependence of the "form factor" :



$$\mathcal{F}^{\rm NP}(b,Q) = a(Q^2)\, b^2$$

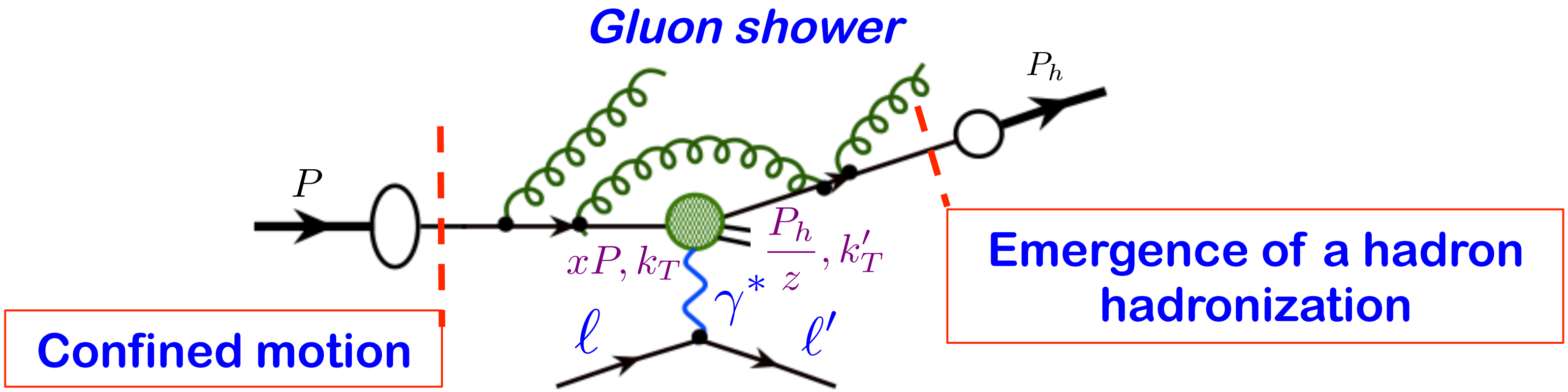
At Q ~ 1 GeV, ln(Q/Q_0) term may not be the dominant one!

$$\mathcal{F}^{\rm NP} \approx b^2(a_1 + a_2 \ln(Q/Q_0) + a_3 \ln(x_A x_B) + \ldots) + \ldots$$

Power correction? **($Q_0/Q)^n$-term?** ***Better fits for HERMES data?***

Parton k_T at the hard collision

❑ **Sources of parton k_T at the hard collision:**



❑ **Large k_T generated by the shower (caused by the collision):**

✧ Q^2-dependence – linear evolution equation of TMDs in b-space

✧ The evolution kernels are perturbative at small b, but, not large b

➡ The nonperturbative inputs at large b could impact TMDs at all Q^2

❑ **Challenge: to extract the "true" parton's confined motion:**

✧ Separation of perturbative shower contribution from nonperturbative hadron structure – not as simple as PDFs

What controls the b-space distribution?

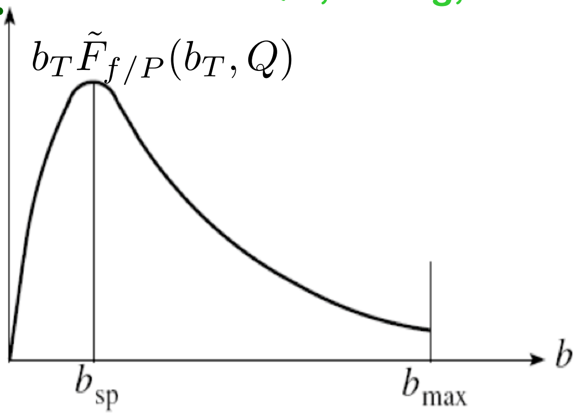
Qiu, Zhang, 2001

- Features of perturbative calculation at small-b:

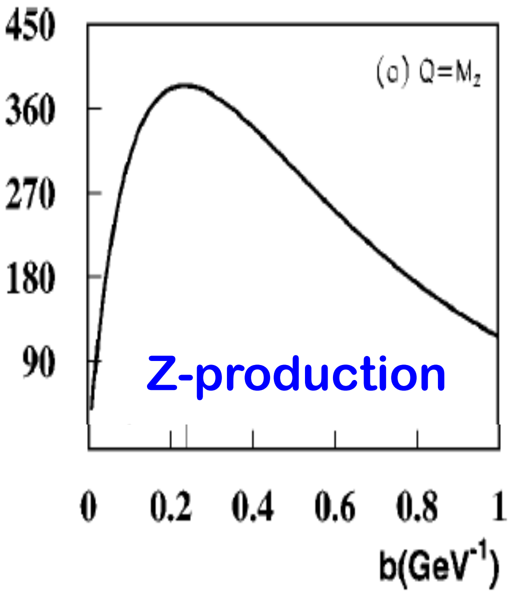
  – Sudakov form factor $\rightarrow$ $b_{sp} \propto (\frac{\Lambda_{QCD}}{Q})^{\lambda}, \lambda \sim 0.5$

  – evolution of $f_{a/A}$ and $D_{c\rightarrow h}$ also moves b_{sp}

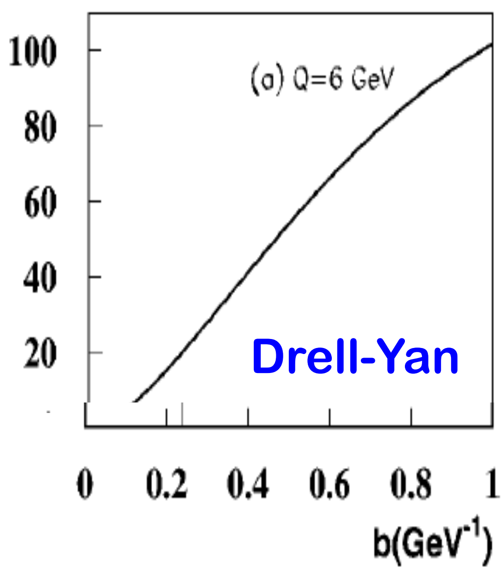
  smaller $\xi \Rightarrow \mu\frac{\partial}{\partial\mu} f_{a/A}(\xi) > 0 \Rightarrow$ lower b_{sp}



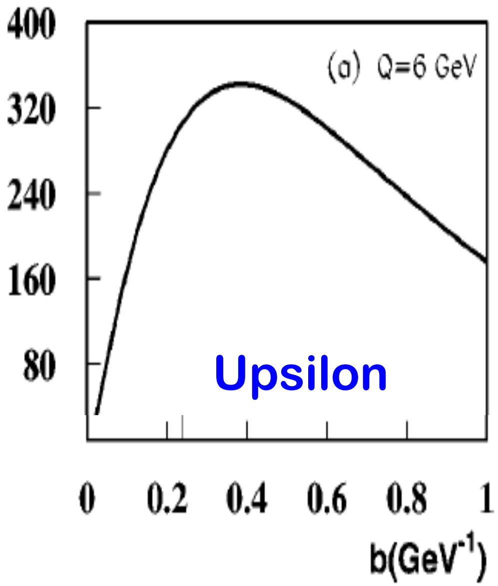
- b-space distribution, and its Q and √s dependence:



$\sqrt{s} = 1.8$ TeV



$\sqrt{s} = 27.4$GeV



$\sqrt{s} = 1.8$ TeV

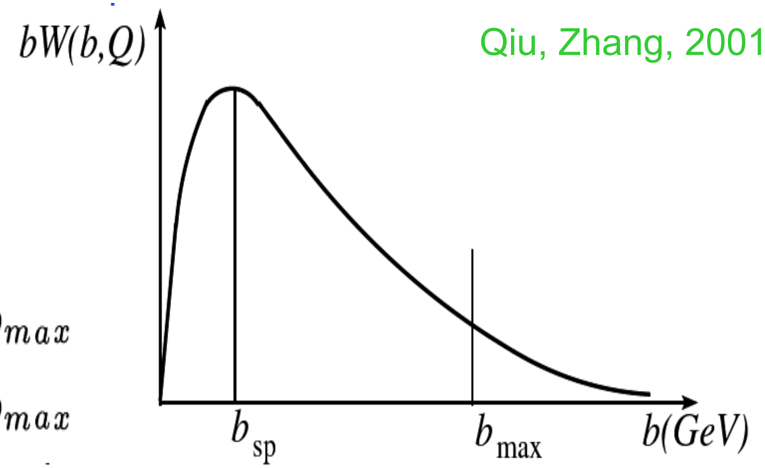
Extrapolation to large b_T

❑ Another approach:

Qiu, Zhang, 2001

$$\frac{d\sigma^{\rm resum}_{AB\to Z}}{dq_T^2} \propto \int_0^\infty db\, J_0(q_T b)\, b\, W(b,Q)$$

$$W = \begin{cases} W^{\rm pert}(b,x,z,Q) & b \le b_{max} \\ W^{\rm pert}(b_{max},x,z,Q)\, F^{NP}(b,Q;b_{max}) & b > b_{max} \end{cases}$$



$$W^{\rm pert}(b,x,z,Q) = \sum_j e_j^2 \left[f_{a/A}\otimes C^{in}_{a\to j}\right]\left[C^{out}_{j\to c}\otimes D_{b\to h}\right] \times e^{-S(b,Q)}$$

$$F^{NP}_{QZ}(b,Q;b_{max}) = \exp\left\{ -\ln\left(\frac{Q^2 b_{max}^2}{c^2}\right)\left[g_1\left((b^2)^\alpha - (b_{max}^2)^\alpha\right) + g_2\left(b^2 - b_{max}^2\right)\right] - \bar{g}_2\left(b^2 - b_{max}^2\right)\right\}$$

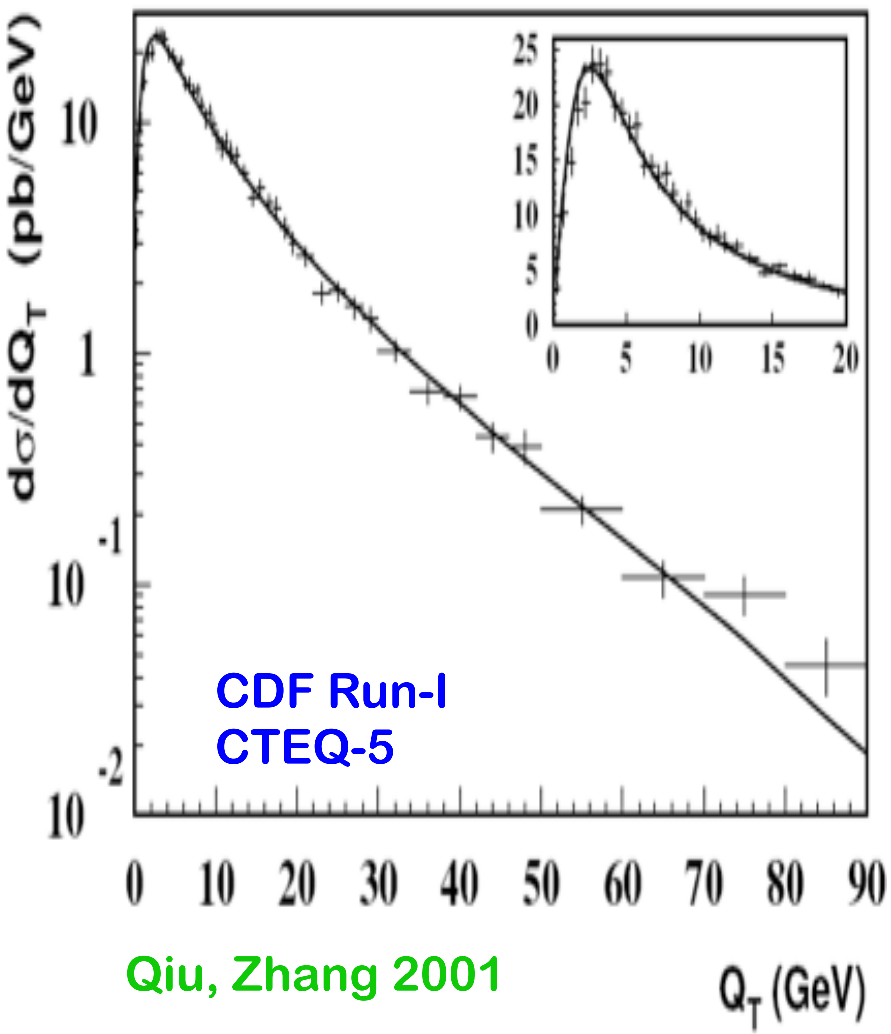
Leading twist

Intrinsic power corrections

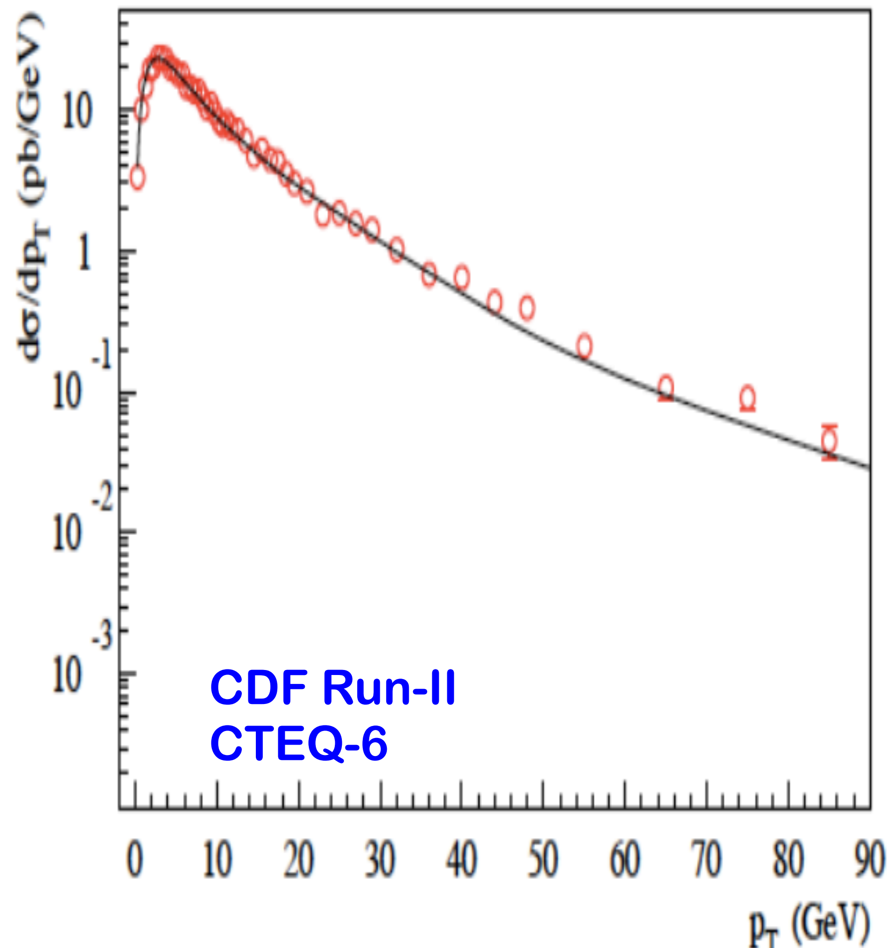
Dynamical power corrections

All parameters, $\alpha, g_1, g_2,$ are fixed by the continuity of the "W" and its derivatives at b_{max} – excellent predictive power for observables with the saddle point at small enough b_{sp}

Phenomenology – Z^0 at Tevatron



Qiu, Zhang 2001

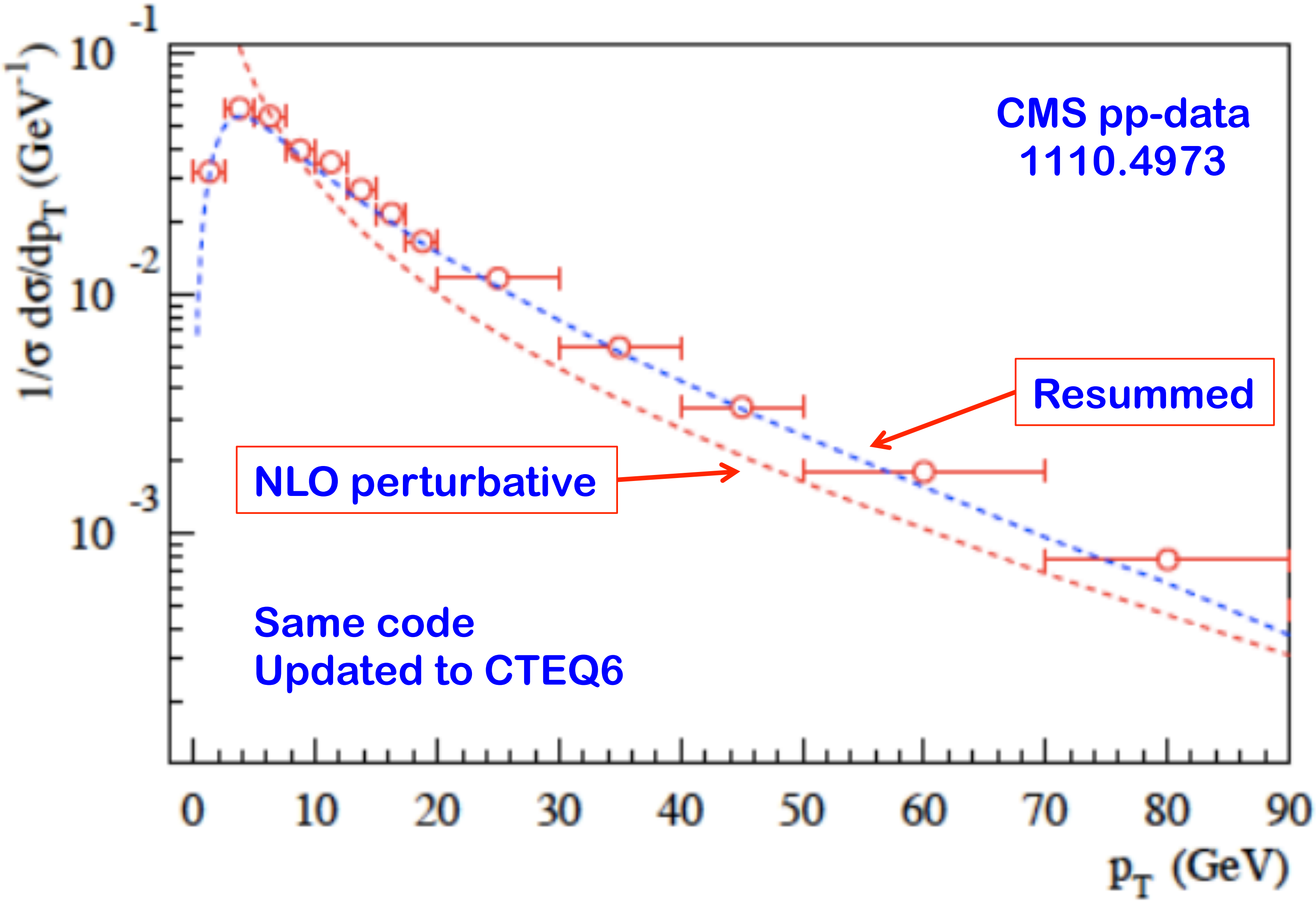


Kang, Qiu 2012

No free fitting parameter!

Phenomenology – Z^0 at the LHC

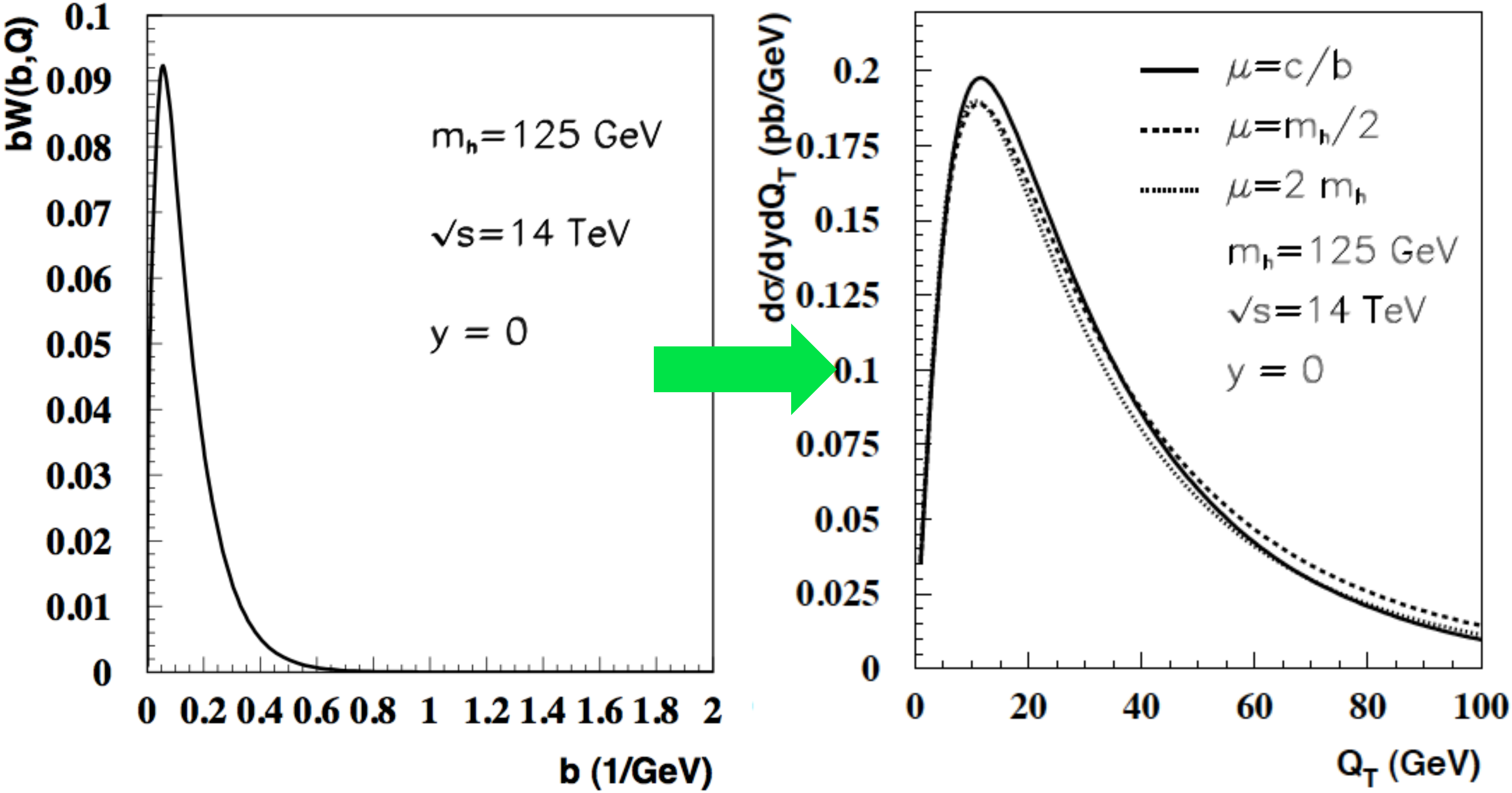
Kang, Qiu, 2012



Effectively no non-perturbative uncertainty!

Phenomenology – Higgs

Berger, Qiu, 2003



Effectively no non-perturbative uncertainty!

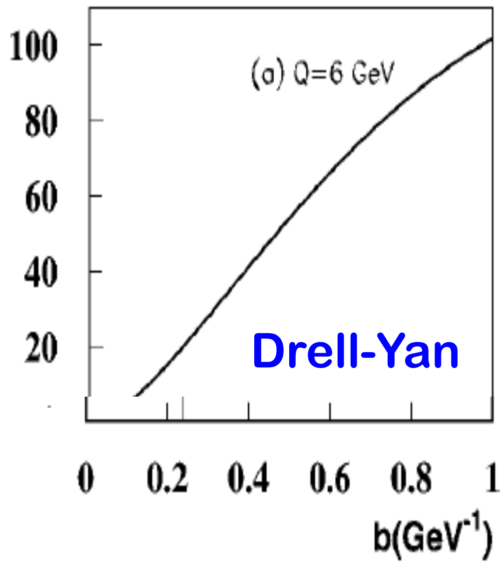
Observables sensitive to the large b_T

❑ Saddle point is in nonperturbative regime:

Qiu, Zhang, 2001

Low energy Drell-Yan and low energy SIDIS

$$\sqrt{s} = 27.4\mathrm{GeV}$$



b-space distribution is dominated by large b_T region

❑ Possible solution:

Kang, Qiu in preparation

- ✧ Bessel function help suppress the large b_T contribution
- ✧ Preserve pQCD calculation at small b_T
- ✧ Simple logarithmic Q-dependence of the form factor is not sufficient
- ✧ Observation:
  - Large b_T – small k_T – active parton is nearly collinear
  - Develop a better extrapolation by resummation of power corrections

Proposal from Collins and Roger

- "Resummed" large b_T behavior:

Collins and Rogers, arXiv:1412.3820

$$\tilde{F}_{f/P}(x, b_T; Q, Q^2) = \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b)}^{\text{AA}}$$

$$\times \overbrace{\times \exp\left\{ \ln\frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln\frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\}}^{\text{BB}}$$

$$\times \boxed{\overbrace{\exp\left\{ g_{f/P}(x, b_T) + g_K(b_T) \ln\frac{Q}{Q_0} \right\}}^{\text{CC}}}$$

Nonperturbative "form factor"

$$g_{f/P}(x, b_T) + g_K(b_T) \ln\frac{Q}{Q_0} \equiv -\left[g_1 + g_2 \ln\frac{Q}{2Q_0} + g_1 g_3 \ln(10x) \right] b_T^2$$

$$\Longrightarrow \quad g_K(b_\mathrm{T}; b_\mathrm{max}) = g_0(b_\mathrm{max}) \left(1 - \exp\left[-\frac{C_F \alpha_s(\mu_{b_*}) b_\mathrm{T}^2}{\pi g_0(b_\mathrm{max})\, b_\mathrm{max}^2} \right] \right)$$

$$g_0(b_\mathrm{max}) = g_0(b_{\mathrm{max},0}) + \frac{2C_F}{\pi} \int_{C_1/b_{\mathrm{max},0}}^{C_1/b_\mathrm{max}} \frac{d\mu'}{\mu'} \alpha_s(\mu')$$

$$\Longrightarrow \quad \frac{C_F}{\pi} \frac{b_\mathrm{T}^2}{b_\mathrm{max}^2} \alpha_s(\mu_{b_*}) + \mathcal{O}(b_T^4)$$

Summary

- Mass and spin decompositions are valuable if individual terms can be measured independently with controllable approximations

- OAMs, TMDs and GPDs are NOT direct physical observables – could be defined differently

  Relevant definition arises from the approximation used in deriving the factorization!

- The evolution equation of the TMDs is the consequence of the factorization, defined in b-space

- Knowledge of nonperturbative inputs at large b is crucial in determining the TMDs from fitting the data

Thank you!