TMD Factorization in Semi-Inclusive Hard Processes

Old Dominion University and
Jefferson Laboratory

Ted Rogers

Based on current work with
J. Collins, L. Gamberg, A. Prokudin, N. Sato and B. Wang

April 11, 2016

Interest in Semi-Inclusive Processes

- Large Q: probe quark & gluon degrees of freedom

- Additional structures accessible
  - TMD PDFs
  - TMD fragmentation functions
  - Spin dependent TMD functions

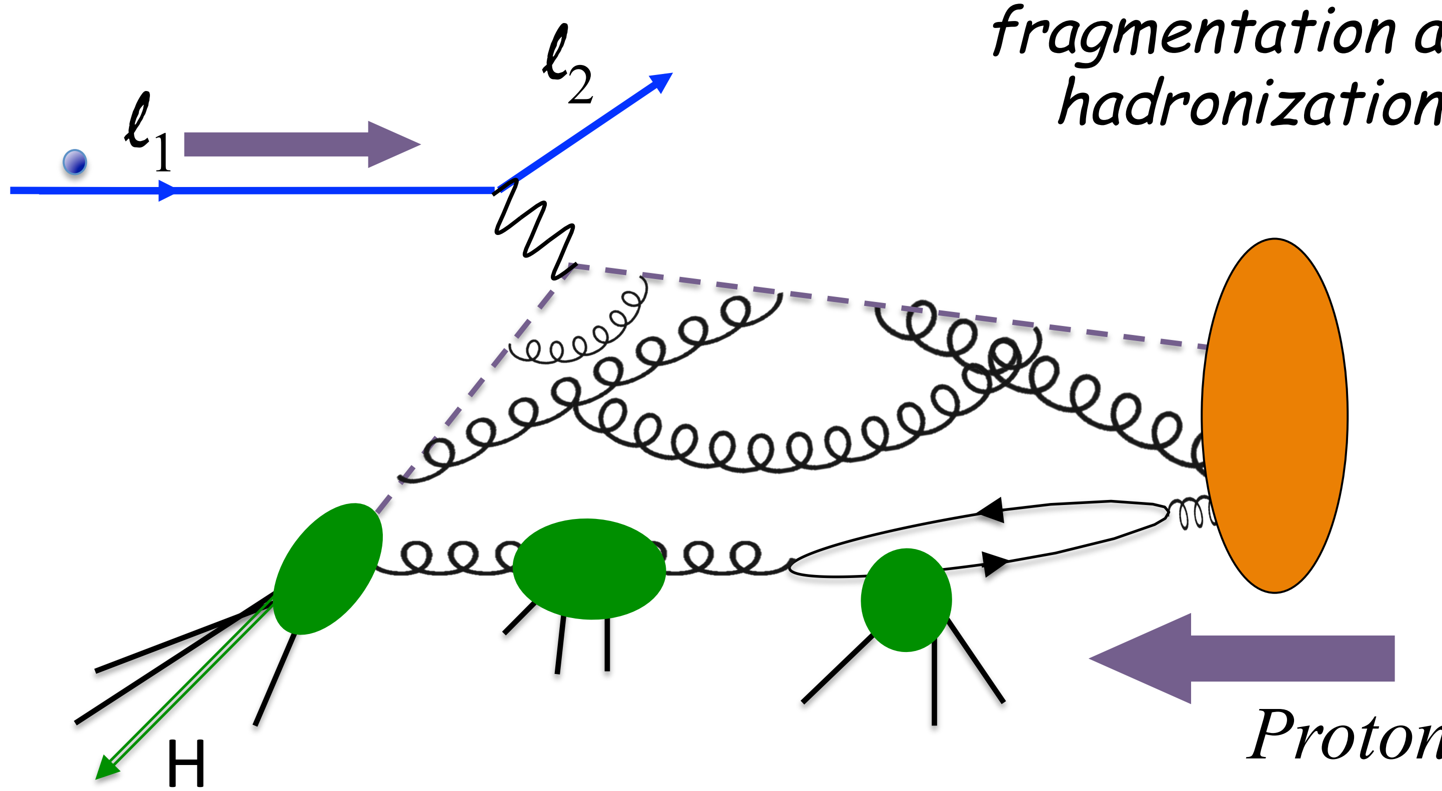
- Evolution
  - Relate different physical observables

- Careful account of factorization is essential
  - Calculations involves interplay of different kinds of physics

Transverse Momentum in Semi-Inclusive DIS

Open up the black boxes

Slide from Nov. ECT workshop on fragmentation and hadronization*



Fragmentation Function: dependence on Q^2, z

Outline

- Not in this talk (but important):
  - Precise definitions of TMD correlation functions.
  - Detailed treatment of evolution.

- In this talk:
  - Matching all regions of q_T:
    - Incorporate directly at the level of factorization formalization rather than at implementation.
  - Relationship between integrated and collinear cross sections.
  - W + Y formalization
    - What has usually been done
    - What we do

Notation

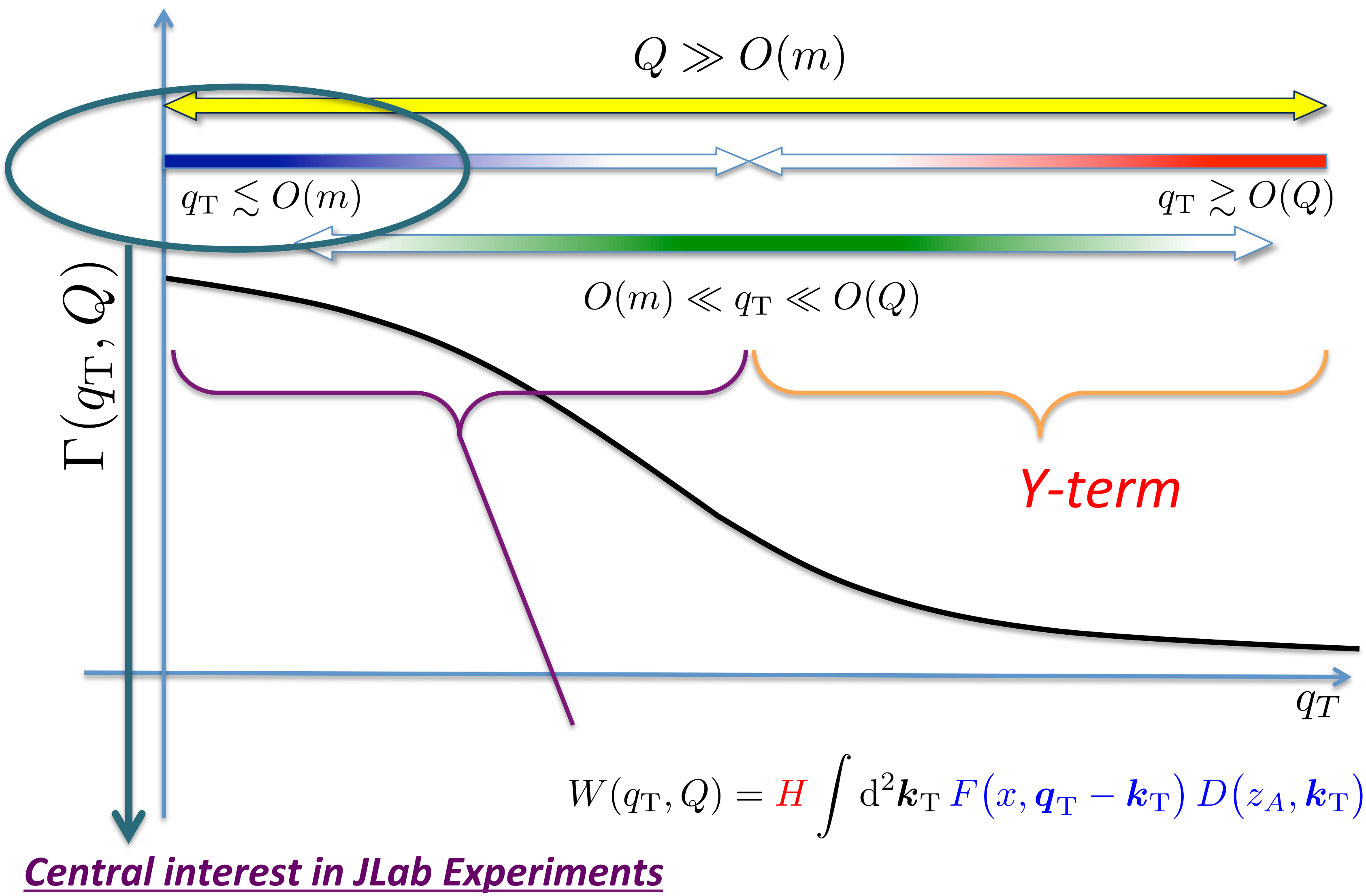
- Cross Section (unpolarized)

$$\Gamma(q_\mathrm{T}, Q) = \frac{\mathrm{d}\,\sigma}{\mathrm{d}^2\boldsymbol{q}_\mathrm{T}\,\mathrm{d}Q\cdots}$$

- Hadronic mass scale: m
- Errors: $O\left(\frac{m}{Q}\right)$
- Coordinate space cutoff: $b_*(b_\mathrm{T}) \equiv \sqrt{\frac{b_\mathrm{T}^2}{1 + b_\mathrm{T}^2/b_\mathrm{max}^2}}$
- Scales:

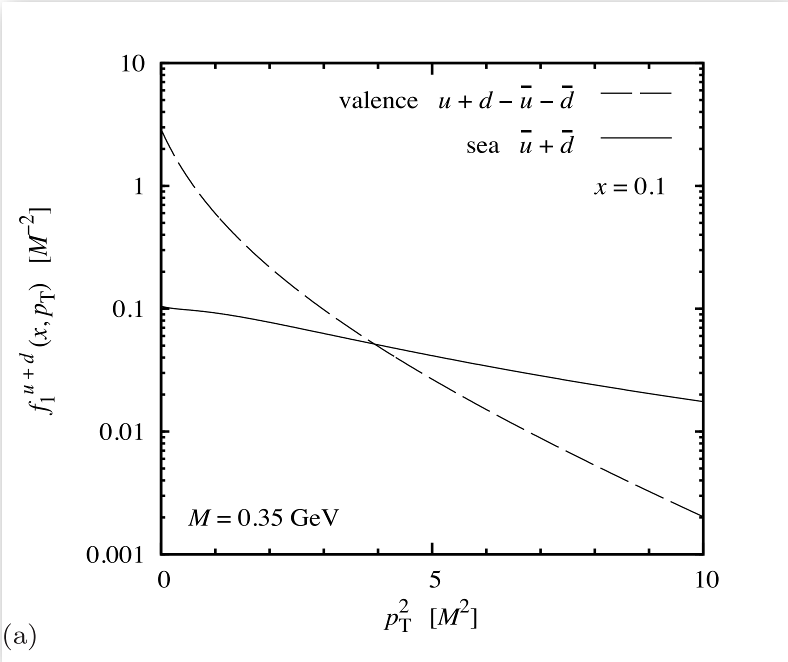
$$\mu_b \equiv C_1/b_\mathrm{T}\,, \qquad \mu_{b_*} \equiv C_1/b_*\,, \qquad \mu_Q \equiv C_2 Q\,,$$

Regions of Transverse Momentum

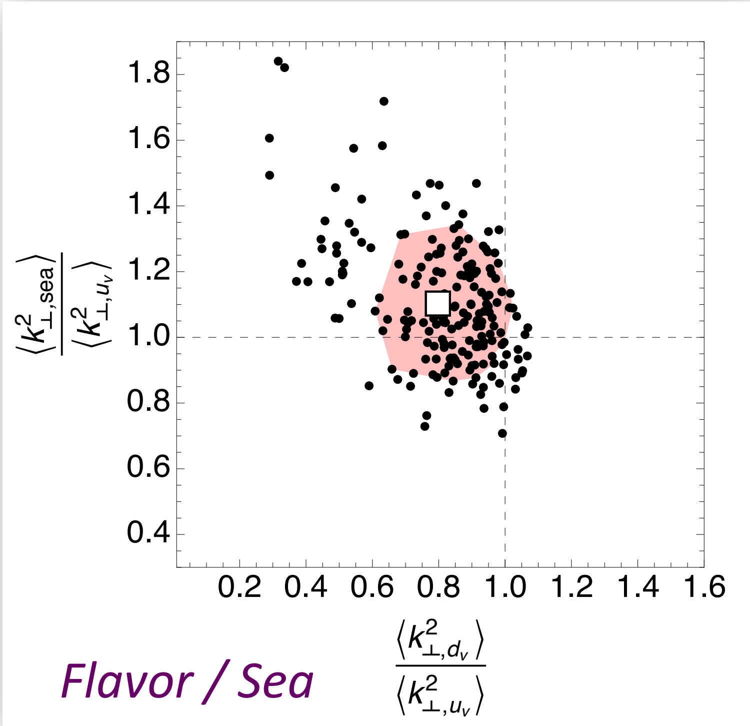


Example: Sea vs. Valence Quark TMD PDFs

Theory:
(Schweitzer, Strikman, Weiss, (2013))



Phenomenology:
(Signori, Bacchetta, Radici, Schnell (2013))



Flavor / Sea

Transverse Momentum Dependent Factorization

- Incorporate all processes.
  - SIDIS, DY, e^+e^-, different target, different final states…
  - Unpolarized cross sections, spin asymmetries…

$$d\sigma_{\mathrm{SIDIS}} = \sum_f \mathcal{H}_{f,\mathrm{SIDIS}}(Q) \otimes F_{f/H_1}(x, k_{1T}, Q) \otimes D_{H_2/f}(z, k_{2T}, Q) \qquad + Y_{\mathrm{SIDIS}}$$

$$d\sigma_{\mathrm{DY}} = \sum_f \mathcal{H}_{f,\mathrm{DY}}(Q) \otimes F_{f/H_1}(x_1, k_{1T}, Q) \otimes F_{\bar{f}/H_2}(x_2, k_{2T}, Q) \qquad + Y_{\mathrm{Drell-Yan}}$$

$$d\sigma_{\mathrm{e^+e^-}} = \sum_f \mathcal{H}_{f,\mathrm{e^+e^-}}(Q) \otimes D_{H_1/\bar{f}}(z_1, k_{1T}, Q) \otimes D_{H_2/f}(z_2, k_{2T}, Q) \qquad + Y_{\mathrm{e^+e^-}}$$

Transverse Momentum Dependent Factorization in SIDIS

$$\Gamma(q_{\rm T},Q) = H(\mu_Q,Q) \int {\rm d}^2\boldsymbol{k}_{\rm T}\, F(x,\boldsymbol{q}_{\rm T}-\boldsymbol{k}_{\rm T};Q^2,\mu_Q)\, D(z,\boldsymbol{k}_{\rm T};Q,\mu_Q) \quad + \quad Y(q_{\rm T},Q)$$

<u>The W term</u>

Good approximation

If

$$q_{\rm T} \ll Q$$

<u>The Y term</u>

Good approximation

If

$$q_{\rm T} \gg m$$

$$+\,O(m/Q)$$

(Collins-Soper-Sterman: 1981-1985)

(Meng, Olness, Soper: 1992,1996)

(J.C. Collins: (Foundations of Perturbative QCD, 2011), Chaps. 10,13,14)

The W-term

Transverse Coordinate Space

$$W(q_{\mathrm{T}}, Q) = \int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i \boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \tilde{W}(b_{\mathrm{T}}, Q)$$

$$\tilde{W}(b_{\mathrm{T}}, Q) = H(\mu_Q, Q) \tilde{F}_{j/A}\left(x_A, \boldsymbol{b}_{\mathrm{T}}; Q^2, \mu_Q\right) \tilde{D}_{B/j}\left(z_B, \boldsymbol{b}_{\mathrm{T}}; Q^2, \mu_Q\right)$$

Transverse Coordinate Space

$$W(q_{\mathrm{T}}, Q) = \int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \, \tilde{W}(b_{\mathrm{T}}, Q)$$

$$H(\mu_Q, Q) \tilde{F}_{j/A}(x_A, \boldsymbol{b}_{\mathrm{T}}; Q_0^2, \mu_{Q_0}) \, \tilde{D}_{B/j}(z_B, \boldsymbol{b}_{\mathrm{T}}; Q_0^2, \mu_{Q_0})$$

$$\times \exp\left\{ \int_{\mu_{Q_0}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'} \left[2\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{(\mu')^2} \gamma_K(\alpha_s(\mu')) \right] \right\}$$

$$\times \exp\left\{ \left[-g_K(b_{\mathrm{T}}; b_{\mathrm{max}}) + \tilde{K}(b_*; \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_{Q_0}} \frac{d\mu'}{\mu'} \gamma_K(\alpha_s(\mu')) \right] \ln\left(\frac{Q^2}{Q_0^2} \right) \right\}$$

Transverse Coordinate Space

$$W(q_{\mathrm{T}}, Q) = \int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i \boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \, \tilde{W}(b_{\mathrm{T}}, Q)$$

$$\int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i \boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \tilde{W}^{\mathrm{OPE}}(b_*(b_{\mathrm{T}}), Q) \tilde{W}_{\mathrm{NP}}(b_{\mathrm{T}}, Q)$$

$$\tilde{W}(b_{\mathrm{T}}, Q) = \tilde{W}^{\mathrm{OPE}}(b_*(b_{\mathrm{T}}), Q) + O\left((b_{\mathrm{T}} m)^p\right)$$

$$\mu_{b_*} \equiv C_1 / b_*$$

Evolve TMD functions again

Transverse Coordinate Space

$$\tilde{W}^{\rm OPE}(b_*(b_{\rm T}),Q) \;\equiv\; H(\mu_Q,Q)\sum_{j'i'}\int_{x_A}^{1}\frac{d\hat{x}}{\hat{x}}\tilde{C}^{\rm PDF}_{j/j'}(x_A/\hat{x},b_*(b_{\rm T});\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))f_{j'/A}(\hat{x};\mu_{b_*})\times$$
$$\times\int_{z_B}^{1}\frac{d\hat{z}}{\hat{z}^3}\tilde{C}^{\rm FF}_{i'/j}(z_B/\hat{z},b_*(b_{\rm T});\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))d_{B/i'}(\hat{z};\mu_{b_*})\times$$
$$\times\;\exp\left\{\ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*(b_{\rm T});\mu_{b_*})+\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1)-\ln\frac{Q^2}{\mu'^2}\gamma_K(\alpha_s(\mu'))\right]\right\}$$

$$\tilde{W}_{\rm NP}(b_{\rm T},Q)\;\equiv$$

$$e^{-g_A(x_A,b_{\rm T};b_{\rm max})-g_B(z_B,b_{\rm T};b_{\rm max})-2g_K(b_{\rm T};b_{\rm max})\ln(Q/Q_0)}\equiv\frac{\tilde{W}(b_{\rm T},Q)}{\tilde{W}^{\rm OPE}(b_*(b_{\rm T}),Q)}$$

Transverse Coordinate Space

$$W(q_\mathrm{T}, Q) = \int \frac{\mathrm{d}^2 \boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i \boldsymbol{q}_\mathrm{T} \cdot \boldsymbol{b}_\mathrm{T}} \, \tilde{W}(b_\mathrm{T}, Q)$$

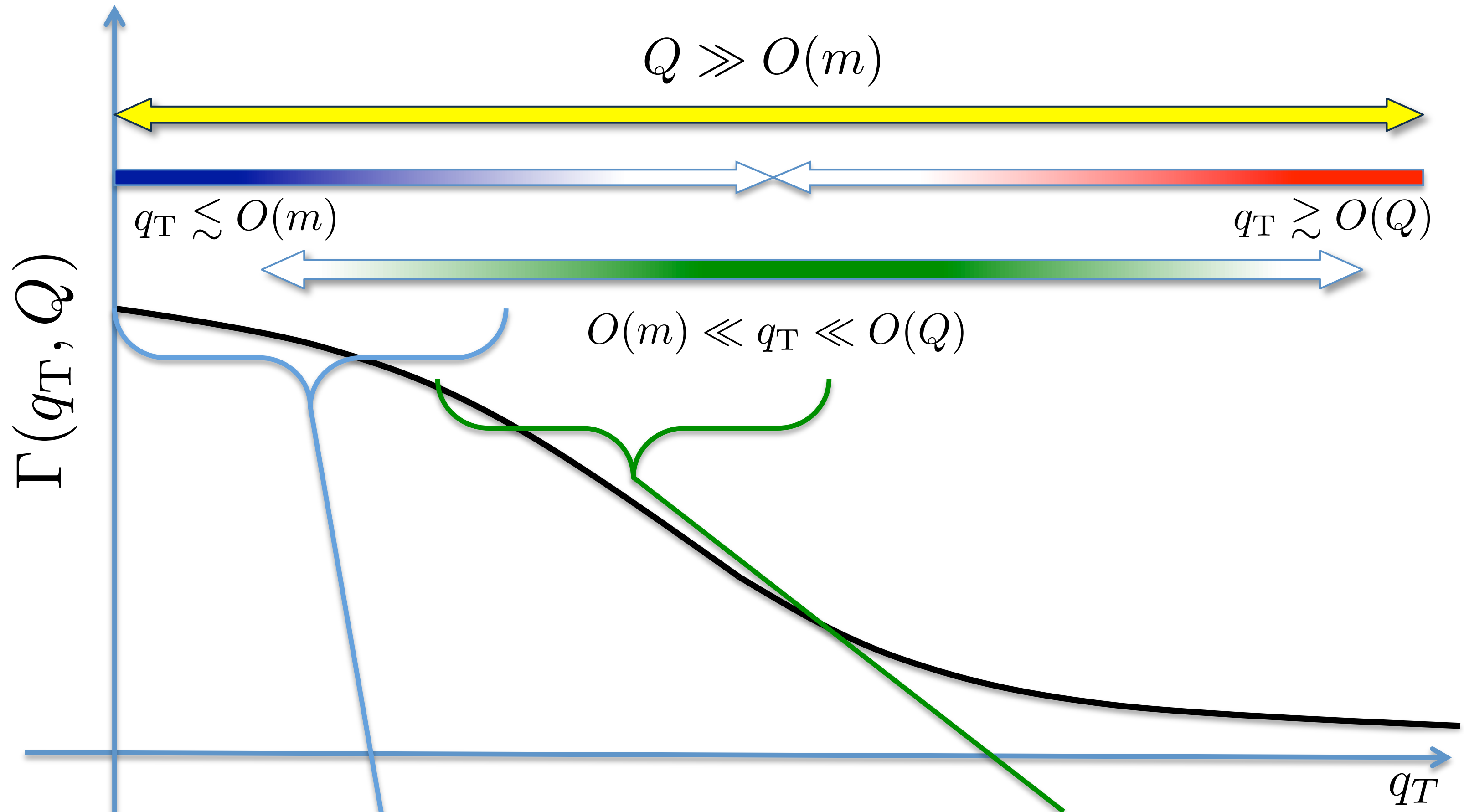
$$H(\mu_Q, Q) \sum_{j'i'} \int_{x_A}^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}^{\mathrm{PDF}}_{j/j'}(x_A/\hat{x}, b_*(b_\mathrm{T}); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) f_{j'/A}(\hat{x}; \mu_{b_*}) \times$$

$$\times \int_{z_B}^1 \frac{d\hat{z}}{\hat{z}^3} \tilde{C}^{\mathrm{FF}}_{i'/j}(z_B/\hat{z}, b_*(b_\mathrm{T}); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) d_{B/i'}(\hat{z}; \mu_{b_*}) \times$$

$$\times \exp\left\{ \ln \frac{Q^2}{\mu_{b_*}^2} \tilde{K}(b_*(b_\mathrm{T}); \mu_{b_*}) + \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_K(\alpha_s(\mu')) \right] \right\}$$

$$\times \exp\left\{ -g_A(x_A, b_\mathrm{T}; b_{\max}) - g_B(z_B, b_\mathrm{T}; b_{\max}) - 2 g_K(b_\mathrm{T}; b_{\max}) \ln\left(\frac{Q}{Q_0} \right) \right\}$$

All Transverse Momenta



$$W(q_\mathrm{T}, Q) = \left[H \int \mathrm{d}^2 \boldsymbol{k}_\mathrm{T} \, F(x, \boldsymbol{q}_\mathrm{T} - \boldsymbol{k}_\mathrm{T}) \, D(z_A, \boldsymbol{k}_\mathrm{T}) \right]_\mathrm{OPE}$$

$$W(q_\mathrm{T}, Q) = \left[H \int \mathrm{d}^2 \boldsymbol{k}_\mathrm{T} \, F(x, \boldsymbol{q}_\mathrm{T} - \boldsymbol{k}_\mathrm{T}) \, D(z_A, \boldsymbol{k}_\mathrm{T}) \right]_\mathrm{NP}$$

The Y-term

q_T Regions

- "TMD part" / W-term (good q_T << Q approximation)

$$\text{Approx. 1}\left[\quad \Gamma(q_{\rm T},Q) \quad\right] = \Gamma(q_{\rm T},Q) + O\left(\frac{q_{\rm T}}{Q}\right)^{a}\Gamma(q_{\rm T},Q) + O\left(\frac{m}{Q}\right)^{a'}\Gamma(q_{\rm T},Q) + \cdots$$

$$+ \; O\left(\frac{q_{\rm T}}{Q}\right)^{a+1} + \cdots + \; O\left(\frac{m}{Q}\right)^{a'+1} + \cdots$$

- "Fixed Order" (good q_T >> m approximation)

$$\text{Approx. 2}\left[\quad \Gamma(q_{\rm T},Q) \quad\right] = \Gamma(q_{\rm T},Q) + O\left(\frac{m}{q_{\rm T}}\right)^{b}\Gamma(q_{\rm T},Q) + \cdots$$

$$+ \; O\left(\frac{m}{q_{\rm T}}\right)^{b+1} + \cdots$$

- W-term <u>error</u>:

$$\Gamma(q_{\rm T},Q) - \text{Approx. 1}\left[\quad \Gamma(q_{\rm T},Q) \quad\right] = O\left(\frac{q_{\rm T}}{Q}\right)^{a}\Gamma(q_{\rm T},Q) + O\left(\frac{m}{Q}\right)^{a'}\Gamma(q_{\rm T},Q) + \cdots$$

$$+ \; O\left(\frac{q_{\rm T}}{Q}\right)^{a+1} + \cdots + \; O\left(\frac{m}{Q}\right)^{a'+1} + \cdots$$

q_T Regions

- W-term approximation with error

$$\Gamma(q_T,Q) = \text{Approx. } 1\left[\quad \Gamma(q_T,Q) \quad\right] + \left(\Gamma(q_T,Q) - \text{Approx. } 1\left[\quad \Gamma(q_T,Q) \quad\right]\right)$$

- Y-term Correction

$$\text{Approx. } 2\left[\quad \Gamma(q_T,Q) - \text{Approx. } 1\left[\quad \Gamma(q_T,Q) \quad\right] \quad\right]$$

$$= \Gamma(q_T,Q) - \text{Approx. } 1\left[\quad \Gamma(q_T,Q) \quad\right]$$

$$+ \left(O\left(\frac{m}{q_T}\right)^b + \cdots\right)\left(\quad \Gamma(q_T,Q) - \text{Approx. } 1\left[\quad \Gamma(q_T,Q) \quad\right] \quad\right)$$

$$= \Gamma(q_T,Q) - \text{Approx. } 1\left[\quad \Gamma(q_T,Q) \quad\right]$$

$$+ \left(O\left(\frac{m}{q_T}\right)^b + \cdots\right)\left(O\left(\frac{q_T}{Q}\right)^a \Gamma(q_T,Q) + O\left(\frac{m}{Q}\right)^{a'} \Gamma(q_T,Q) + \cdots\right)$$

$$= \Gamma(q_T,Q) - \text{Approx. } 1\left[\quad \Gamma(q_T,Q) \quad\right] + O\left(\frac{m}{Q}\right)^{\min(a,a',b)} \Gamma(q_T,Q)$$

q_T Regions

- Total:

$$\Gamma(m \lesssim q_T \lesssim Q, Q) = \underbrace{W(q_T, Q) + Y(q_T, Q)} + O\left(\frac{m}{Q}\right)^c \Gamma(q_T, Q)$$

$$\underbrace{\text{Approx. 1}\left[\ \Gamma(q_T, Q)\ \right]}_{\textit{W-term}} \underbrace{\underbrace{+ \text{Approx. 2}\left[\ \Gamma(q_T, Q)\ \right]}_{\textit{Fixed Order Term}} \underbrace{- \text{Approx. 2 Approx. 1}\left[\ \Gamma(q_T, Q)\ \right]}_{\textit{Asymptotic Term}}}_{\textit{Y-term}}$$

q_T Regions

- What about $q_T < m$ and $q_T > Q$?

- What about inclusive integral?

Goal:

Generalize/Improve
W+Y
Formalization

Semi-Inclusive to Inclusive

- Parton Model W-term:

$$W(q_{\mathrm{T}}, Q) = H \int \mathrm{d}^2\boldsymbol{k}_{\mathrm{T}}\, F\big(x, \boldsymbol{q}_{\mathrm{T}} - \boldsymbol{k}_{\mathrm{T}}\big)\, D\big(z_A, \boldsymbol{k}_{\mathrm{T}}\big)$$

$$\int \mathrm{d}^2 q_{\mathrm{T}}\, W(q_{\mathrm{T}}, Q) = H\, f(x) d(z)$$

- CSS / TMD factorization formalism W-term:

$$W(q_{\mathrm{T}}, Q) = H(\mu_Q, Q) \int \mathrm{d}^2\boldsymbol{k}_{\mathrm{T}}\, F\big(x, \boldsymbol{q}_{\mathrm{T}} - \boldsymbol{k}_{\mathrm{T}}; Q^2, \mu_Q\big)\, D\big(z, \boldsymbol{k}_{\mathrm{T}}; Q, \mu_Q\big)$$

$$\int \mathrm{d}^2 q_{\mathrm{T}}\, W(q_{\mathrm{T}}, Q) = 0$$

Very Large and Very Small q_T

a) W-term: **good approximation for $q_T/Q \rightarrow 0$**

b) Fixed order term: **good for $q_T \rightarrow O(Q)$**

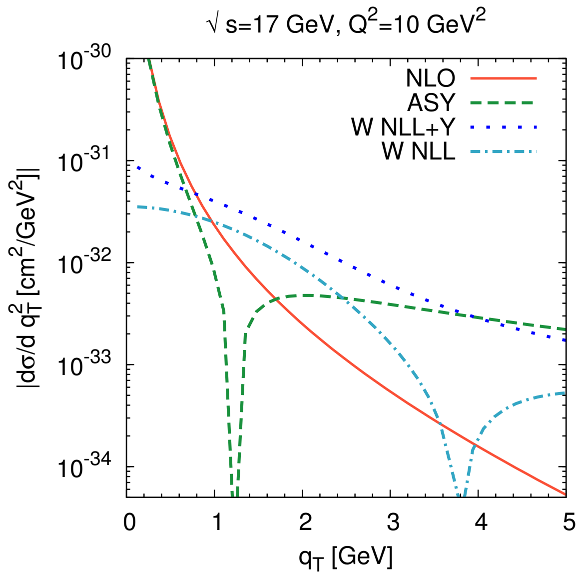
c) W + Y is good for $O(m) << q_T << O(Q)$

- Bad for $q_T < O(m)$, $q_T > O(Q)$
- But no problem in principle: Recall a) & b)
- Switch off W+Y below some min q_T and above some max q_T/Q

d) Need to simultaneously drop powers of m/q_T & q_T/Q

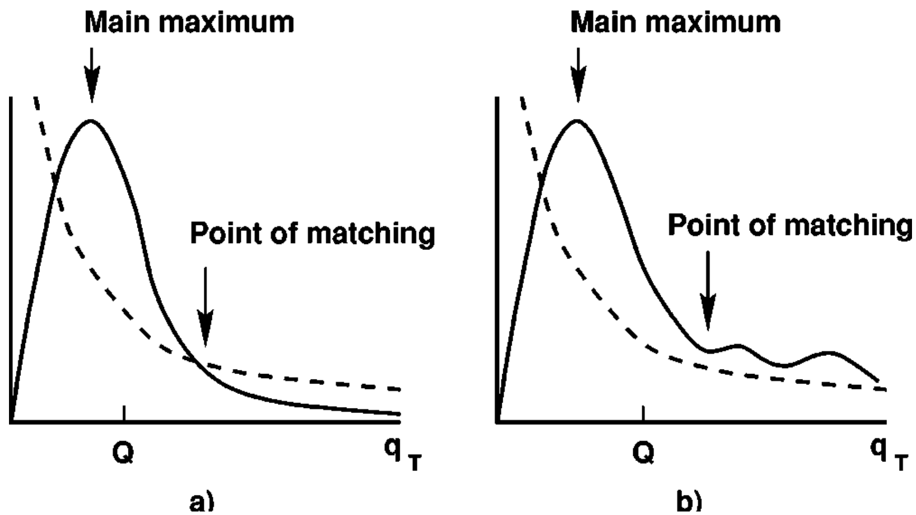
Very Large and Very Small q_T

Direct implementations



M. Boglione et al, JHEP 02, 095 (2015)

Matching W+Y to Fixed Order



*P. Nadolsky et al, Phys. Rev.**D61**, 014003 (1999)*

d) Culprit: W and Y are used far outside their domains of accuracy; default treatments of errors not necessarily optimal.

Generalized W+Y Formalization

- <u>Requirements</u>

1) $q_T << Q$ approximation shouldn't contribute for $q_T > Q$.
   $q_T >> m$ approximation shouldn't contribute for $q_T < m$.
   Guzzi,, Nadolsky, Wang (2014)

2) Integrated TMD formalism should match collinear formalism.
   "Unitarity": *Bozzi, Catani, de Florian, Grazzini (2006)*

3) Integrated TMD parton model should match collinear parton model.

4) Should recover standard W+Y treatment for $Q \rightarrow \infty$ and for $m << q_T << Q$.

Generalized W-term

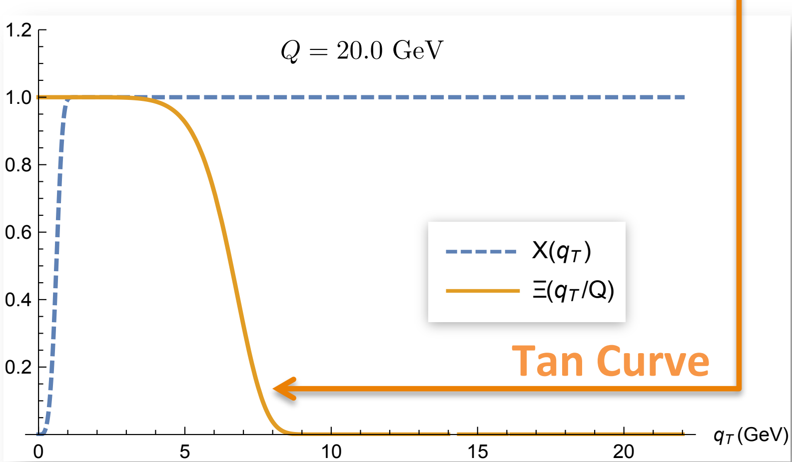
- Basic

$$W(q_{\rm T}, Q) = \int \frac{d^2 \boldsymbol{b}_{\rm T}}{(2\pi)^2} e^{i\boldsymbol{q}_{\rm T}\cdot \boldsymbol{b}_{\rm T}} \tilde{W}(b_{\rm T}, Q)$$

- Generalized

Called F(Q/q_T) in Collins Textbook (2011)

$$W_{\rm New}(q_{\rm T}, Q; \eta, C_5) = \Xi\left(\frac{q_{\rm T}}{Q}, \eta\right) \int \frac{d^2 \boldsymbol{b}_{\rm T}}{(2\pi)^2} e^{i\boldsymbol{q}_{\rm T}\cdot \boldsymbol{b}_{\rm T}} \tilde{W}(b_c(b_{\rm T}), Q)$$



$$b_c(b_{\rm T}) = \sqrt{b_{\rm T}^2 + C_1^2/\mu_{\rm max}^2}$$

$$\mu_{\rm max} = C_5 \mu_Q / b_0$$

$$b_0 \equiv 2\exp(-\gamma_E)$$

Addresses point 3.) and part of point 1.)

Notation

- Cross Section (unpolarized)

$$\Gamma(q_{\rm T}, Q) = \frac{\mathrm{d}\,\sigma}{\mathrm{d}^2\boldsymbol{q}_{\rm T}\,\mathrm{d}Q\cdots}$$

- Hadronic mass scale: m

- Errors: $O\left(\frac{m}{Q}\right)$

- Coordinate space cutoff: $b_*(b_{\rm T}) \equiv \sqrt{\frac{b_{\rm T}^2}{1 + b_{\rm T}^2/b_{\rm max}^2}}$

- Scales:

$$\mu_b \equiv C_1/b_{\rm T}\,, \qquad \mu_{b_*} \equiv C_1/b_*\,, \qquad \mu_Q \equiv C_2 Q\,,$$

Generalized W-term

- Ordinary steps still apply:

$$W_{\text{New}}(q_{\text{T}}, Q; \eta, C_5) = \Xi\left(\frac{q_{\text{T}}}{Q}, \eta\right) \int \frac{\text{d}^2 \boldsymbol{b}_{\text{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\text{T}} \cdot \boldsymbol{b}_{\text{T}}} \tilde{W}^{\text{OPE}}(b_*(b_c(b_{\text{T}})), Q) \tilde{W}_{\text{NP}}(b_c(b_{\text{T}}), Q)$$

– $$b_*(b_c(b_{\text{T}})) = \sqrt{\frac{b_{\text{T}}^2 + C_1^2/\mu_{\text{max}}^2}{1 + b_{\text{T}}^2/b_{\text{max}}^2 + C_1^2/(\mu_{\text{max}}^2 b_{\text{max}}^2)}}$$

$$\bar{\mu} \equiv \frac{C_1}{b_*(b_c(b_{\text{T}}))}$$

Use in OPE

– $$b_{\text{min}} \equiv b_*(b_c(0)) = \frac{C_1}{\mu_{\text{max}}} \sqrt{\frac{1}{1 + C_1^2/(\mu_{\text{max}}^2 b_{\text{max}}^2)}} \approx \frac{C_1}{\mu_{\text{max}}} \sim \frac{1}{Q}$$

See also Boer, den Dunnen (2014)

– $$b_*(b_c(b_{\text{T}})) \longrightarrow \begin{cases} b_{\text{min}} & b_{\text{T}} \ll b_{\text{min}} \\ b_{\text{T}} & b_{\text{min}} \ll b_{\text{T}} \ll b_{\text{max}} \\ b_{\text{max}} & b_{\text{T}} \gg b_{\text{max}} \end{cases}$$

Transverse Coordinate Space

$$W(q_{\mathrm{T}}, Q) = \int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i \boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \tilde{W}(b_{\mathrm{T}}, Q)$$

$$\int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i \boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \tilde{W}^{\mathrm{OPE}}(b_*(b_{\mathrm{T}}), Q) \tilde{W}_{\mathrm{NP}}(b_{\mathrm{T}}, Q)$$

$$\tilde{W}(b_{\mathrm{T}}, Q) = \tilde{W}^{\mathrm{OPE}}(b_*(b_{\mathrm{T}}), Q) + O\left((b_{\mathrm{T}} m)^p\right)$$

$$\mu_{b_*} \equiv C_1 / b_*$$

Evolve TMD functions again

Transverse Coordinate Space

$$\int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T} \tilde{W}^{OPE}(b_*(b_c(b_T)), Q)\tilde{W}_{NP}(b_c(b_T), Q)$$

$$\tilde{W}(b_c(b_T), Q) = \tilde{W}^{OPE}(b_*(b_c(b_T)), Q) + O\left((b_c(b_T)m)^p\right)$$

Evolve TMD functions again

$$\bar{\mu} \equiv \frac{C_1}{b_*(b_c(b_T))}$$

Transverse Coordinate Space

$$\tilde{W}(b_{\rm T}, Q)$$

$$H(\mu_Q, Q) \sum_{j'i'} \int_{x_A}^{1} \frac{d\hat{x}}{\hat{x}} \tilde{C}^{\rm PDF}_{j/j'}(x_A/\hat{x}, b_*(b_{\rm T}); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) f_{j'/A}(\hat{x}; \mu_{b_*}) \times$$

$$\times \int_{z_B}^{1} \frac{d\hat{z}}{\hat{z}^3} \tilde{C}^{\rm FF}_{i'/j}(z_B/\hat{z}, b_*(b_{\rm T}); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) d_{B/i'}(\hat{z}; \mu_{b_*}) \times$$

$$\times \exp\left\{ \ln\frac{Q^2}{\mu_{b_*}^2} \tilde{K}(b_*(b_{\rm T}); \mu_{b_*}) + \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q^2}{\mu'^2} \gamma_K(\alpha_s(\mu')) \right] \right\}$$

$$\times \exp\left\{ -g_A(x_A, b_{\rm T}; b_{\rm max}) - g_B(z_B, b_{\rm T}; b_{\rm max}) - 2g_K(b_{\rm T}; b_{\rm max}) \ln\left(\frac{Q}{Q_0}\right) \right\}$$

Transverse Coordinate Space

$$\tilde{W}(b_{\rm T}, Q) \longrightarrow \tilde{W}(b_c(b_{\rm T}), Q)$$

$$H(\mu_Q, Q) \sum_{j'i'} \int_{x_A}^{1} \frac{d\hat{x}}{\hat{x}} \tilde{C}^{\rm PDF}_{j/j'}(x_A/\hat{x}, b_*(b_c(b_{\rm T})); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) f_{j'/A}(\hat{x}; \bar{\mu}) \times$$

$$\times \int_{z_B}^{1} \frac{d\hat{z}}{\hat{z}^3} \tilde{C}^{\rm FF}_{i'/j}(z_B/\hat{z}, b_*(b_c(b_{\rm T})); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) d_{B/i'}(\hat{z}; \bar{\mu}) \times$$

$$\times \exp\left\{ \ln \frac{Q^2}{\bar{\mu}^2} \tilde{K}(b_*(b_c(b_{\rm T})); \bar{\mu}) + \int_{\bar{\mu}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_K(\alpha_s(\mu')) \right] \right\}$$

$$\times \exp\left\{ -g_A(x_A, b_c(b_{\rm T}); b_{\rm max}) - g_B(z_B, b_c(b_{\rm T}); b_{\rm max}) - 2g_K(b_c(b_{\rm T}); b_{\rm max}) \ln\left(\frac{Q}{Q_0}\right) \right\}$$

Generalized W-term

- Ordinary steps still apply:

$$W_{\text{New}}(q_{\text{T}}, Q; \eta, C_5) = \Xi\left(\frac{q_{\text{T}}}{Q}, \eta\right) \int \frac{\mathrm{d}^2\boldsymbol{b}_{\text{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\text{T}}\cdot\boldsymbol{b}_{\text{T}}} \tilde{W}^{\text{OPE}}(b_*(b_c(b_{\text{T}})), Q) \tilde{W}_{\text{NP}}(b_c(b_{\text{T}}), Q)$$

– $$b_*(b_c(b_{\text{T}})) = \sqrt{\frac{b_{\text{T}}^2 + C_1^2/\mu_{\max}^2}{1 + b_{\text{T}}^2/b_{\max}^2 + C_1^2/(\mu_{\max}^2 b_{\max}^2)}}$$

$$\bar{\mu} \equiv \frac{C_1}{b_*(b_c(b_{\text{T}}))}$$

Use in OPE

– $$b_{\min} \equiv b_*(b_c(0)) = \frac{C_1}{\mu_{\max}} \sqrt{\frac{1}{1 + C_1^2/(\mu_{\max}^2 b_{\max}^2)}} \approx \frac{C_1}{\mu_{\max}} \sim \frac{1}{Q}$$

– $$b_*(b_c(b_{\text{T}})) \longrightarrow \begin{cases} b_{\min} & b_{\text{T}} \ll b_{\min} \\ b_{\text{T}} & b_{\min} \ll b_{\text{T}} \ll b_{\max} \\ b_{\max} & b_{\text{T}} \gg b_{\max} \end{cases}$$

Relationship to Inclusive (Integrated) Cross Section

- Recall:

$$\int \mathrm{d}^2 q_{\mathrm{T}}\, W(q_{\mathrm{T}}, Q) = 0$$

$$\mu_c \equiv \lim_{b_{\mathrm{T}} \to 0} \bar{\mu} \approx \frac{C_1}{b_{\min}} \sqrt{1 + \frac{b_{\min}^2}{b_{\max}^2}}$$

- Modified:

$$\int \mathrm{d}^2 \boldsymbol{q}_{\mathrm{T}}\, W_{\mathrm{New}}^{(0)}(q_{\mathrm{T}}, Q) = H_0 f(x_A; \mu_c) d(z_B; \mu_c) + O\left(\left(\frac{b_{\min}^2}{b_{\max}^2} \right)^p \right)$$

$$= H_0 f(x_A; \mu_c) d(z_B; \mu_c) + O\left(\left(\frac{1}{Q^2 b_{\max}^2} \right)^p \right)$$

Generalized Y-term

- Standard steps apply to deriving a modified Y-term.

$$\text{Approx. } 2 \left[\quad \Gamma(q_{\rm T}, Q) - \text{Approx. } 1 \left[\quad \Gamma(q_{\rm T}, Q) \quad \right] \quad \right]$$

$$\Longrightarrow \text{Approx. } 2 \left[\quad \Gamma(q_{\rm T}, Q) - \underbrace{\overline{\text{Approx. } 1} \left[\quad \Gamma(q_{\rm T}, Q) \quad \right]}_{\text{Use } b_T \rightarrow b_c(b_T)} \quad \right]$$

Generalized W+Y Formalization

- <u>Requirements</u>

1) $q_T << Q$ approximation shouldn't contribute for $q_T > Q$.
   $q_T >> m$ approximation shouldn't contribute for $q_T < m$.
   Guzzi,, Nadolsky, Wang (2014)

2) Integrated TMD formalism should match collinear formalism.
   "Unitarity": *Bozzi, Catani, de Florian, Grazzini (2006)*

3) Integrated TMD parton model should match collinear parton model.

4) Should recover standard W+Y treatment for $Q \rightarrow \infty$ and for $m << q_T << Q$.

Generalized Y-term

- Standard steps apply to deriving a modified Y-term.

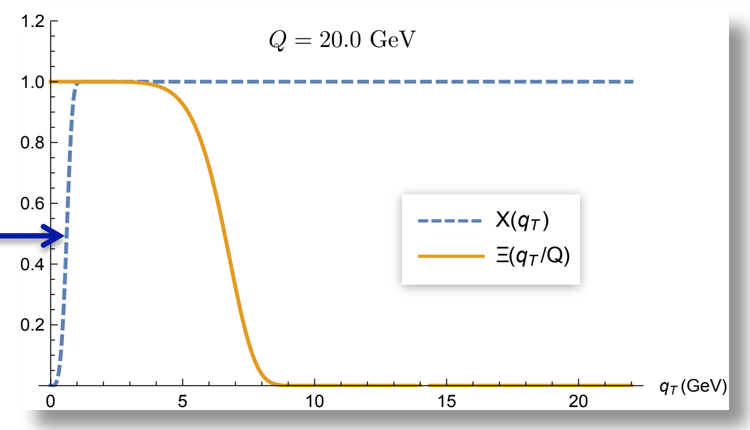
$$\text{Approx. 2}\left[\quad \Gamma(q_\mathrm{T},Q) - \text{Approx. 1}\left[\quad \Gamma(q_\mathrm{T},Q) \quad\right]\quad\right]$$

$$\Longrightarrow \text{Approx. 2}\left[\quad \Gamma(q_\mathrm{T},Q) - \overline{\text{Approx. 1}}\left[\quad \Gamma(q_\mathrm{T},Q) \quad\right]\quad\right]$$

$$\Longrightarrow X(q_\mathrm{T}/\lambda)\text{Approx. 2}\left[\quad \Gamma(q_\mathrm{T},Q) - \overline{\text{Approx. 1}}\left[\quad \Gamma(q_\mathrm{T},Q) \quad\right]\quad\right]$$

λ = hadronic mass scale (ResBos ≈ 0.5-1.5 GeV)

Blue Curve



λ Corresponds to Q_T^{min} in CSS, 1985

q_T Regions

- Total:

$$\Gamma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c \Gamma(q_T, Q)$$

$$\text{Approx. 1}\left[\ \Gamma(q_T, Q)\ \right] + \text{Approx. 2}\left[\ \Gamma(q_T, Q)\ \right] - \text{Approx. 2 Approx. 1}\left[\ \Gamma(q_T, Q)\ \right]$$

W-term ***Fixed Order Term*** ***Asymptotic Term***

Y-term

q_T Regions

- Total:

$$\Gamma(q_T,Q) = W_{\rm New}(q_T,Q;\eta,C_5) + Y_{\rm New}(q_T,Q;\eta,C_5) + O\left(\frac{m}{Q}\right)^c \Gamma(q_T,Q)$$

$$\overline{\text{Approx.}}\ 1\left[\ \Gamma(q_T,Q)\ \right] + \text{Approx. } 2\left[\ \Gamma(q_T,Q)\ \right] - \text{Approx. } 2\ \overline{\text{Approx.}}\ 1\left[\ \Gamma(q_T,Q)\ \right]$$

New W-term ***Fixed Order Term*** ***New Asymptotic Term***

New Y-term

Generalized Asymptotic Term

- Evaluation of logarithms.
  - $$\text{Approx. 2 Approx. 1}\left[\quad \Gamma(q_{\mathrm{T}}, Q) \quad\right]$$
    $$\alpha_s(\mu_Q)^m \ln^n\left(\frac{\mu_Q^2 b_{\mathrm{T}}^2}{b_0^2}\right)$$
-
  - $$\text{Approx. 2 }\overline{\text{Approx.}}\text{ 1}\left[\quad \Gamma(q_{\mathrm{T}}, Q) \quad\right]$$
    $$\alpha_s(\mu)^m \ln^n\left(\frac{\mu^2 b_*(b_c(b_{\mathrm{T}}))^2}{b_0^2}\right) \to$$
    $$\alpha_s(\mu_Q)^m \ln^n\left(\frac{\mu_Q^2 b_{\mathrm{T}}^2}{b_0^2} + \frac{C_1^2}{C_5^2}\right)$$

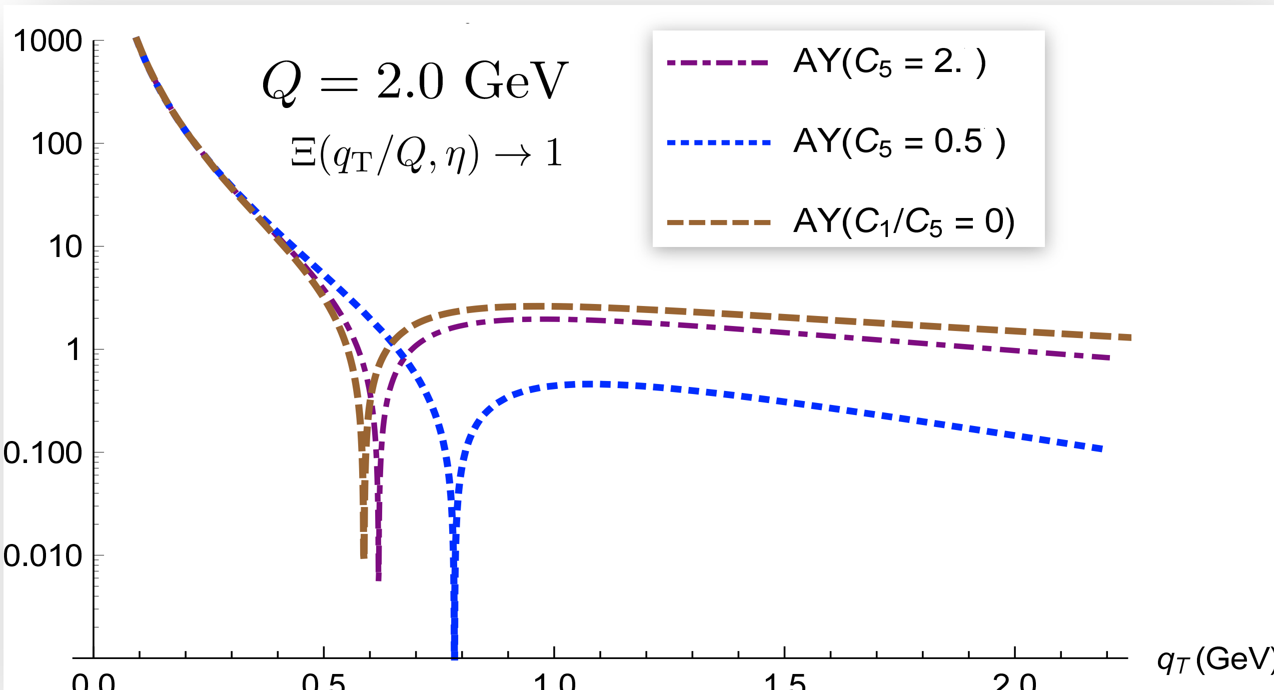
Generalized Asymptotic Term

- Transverse momentum space version of logarithms.

Coordinate Space	Momentum Space
Standard Logarithms $\alpha_s(\mu_Q)^m \ln^n\left(\frac{\mu_Q^2 b_T^2}{b_0^2}\right)$ →	$\frac{1}{q_T^2}\,,\ \frac{1}{q_T^2}\ln\left(\frac{Q^2}{q_T^2}\right)\,,\ \ldots$
After Modification: $\alpha_s(\mu_Q)^m \ln^n\left(\frac{\mu_Q^2 b_T^2}{b_0^2}+\frac{C_1^2}{C_5^2}\right)$ →	*(Bozzi-Catani-de Florian-Grazzini: ($C_1 = C_5$))* Nucl. Phys. B737, 73 (2006), App. A $\frac{C_1 b_0}{q_T \mu_Q C_5} K_1\left(\frac{C_1 q_T b_0}{C_5 \mu_Q}\right)\,,\ \ldots$

Generalized Asymptotic Term

<u>Normalized Absolute Value of Asymptotic Term</u>



$C_1/C_5 \to 0$ *(Collins, Soper, Sterman (1985), ResBos, etc...)*

$C_1/C_5 \to 1$ *(Parisi, Petronzio (1979), Bozzi-Catani-de Florian-Grazzini ...)*

Summary

- Control over Y-term is important for studies of intrinsic transverse momentum.
- Improvements over past approaches possible, especially when considering relationship between differential and inclusive quantities.
- Outlook
  - Improvements in purely collinear factorization treatment needed?
  - Extend to polarization observables.
  - Phenomenology: See N. Sato talk Thursday.