Tensor charge from lattice QCD

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CaSToRC

Outline

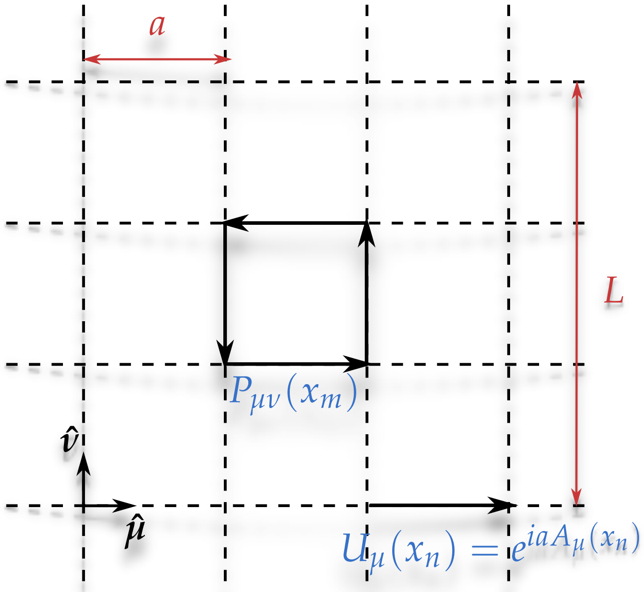
- ★ **Short introduction to lattice calculations**
  - Challenges and current landscape
- ★ **Nucleon structure on the lattice**
  - Benchmark quantities
  - Axial charge
  - Scalar matrix elements
- ★ **Nucleon tensor matrix elements**
  - Nucleon tensor charge
  - Tensor/transversity generalized form factors
- ★ **Summary and outlook**

Lattice QCD – *ab initio* simulation of QCD

– Freedom in choice of:
  – quark masses (*heavier is cheaper*)
  – lattice spacing *a* (*larger is cheaper*)
  – lattice volume $L^3 \times T$ (*smaller is cheaper*)
– Choice of discretisation scheme

  e.g. Clover, Twisted Mass, Staggered, Overlap, Domain Wall

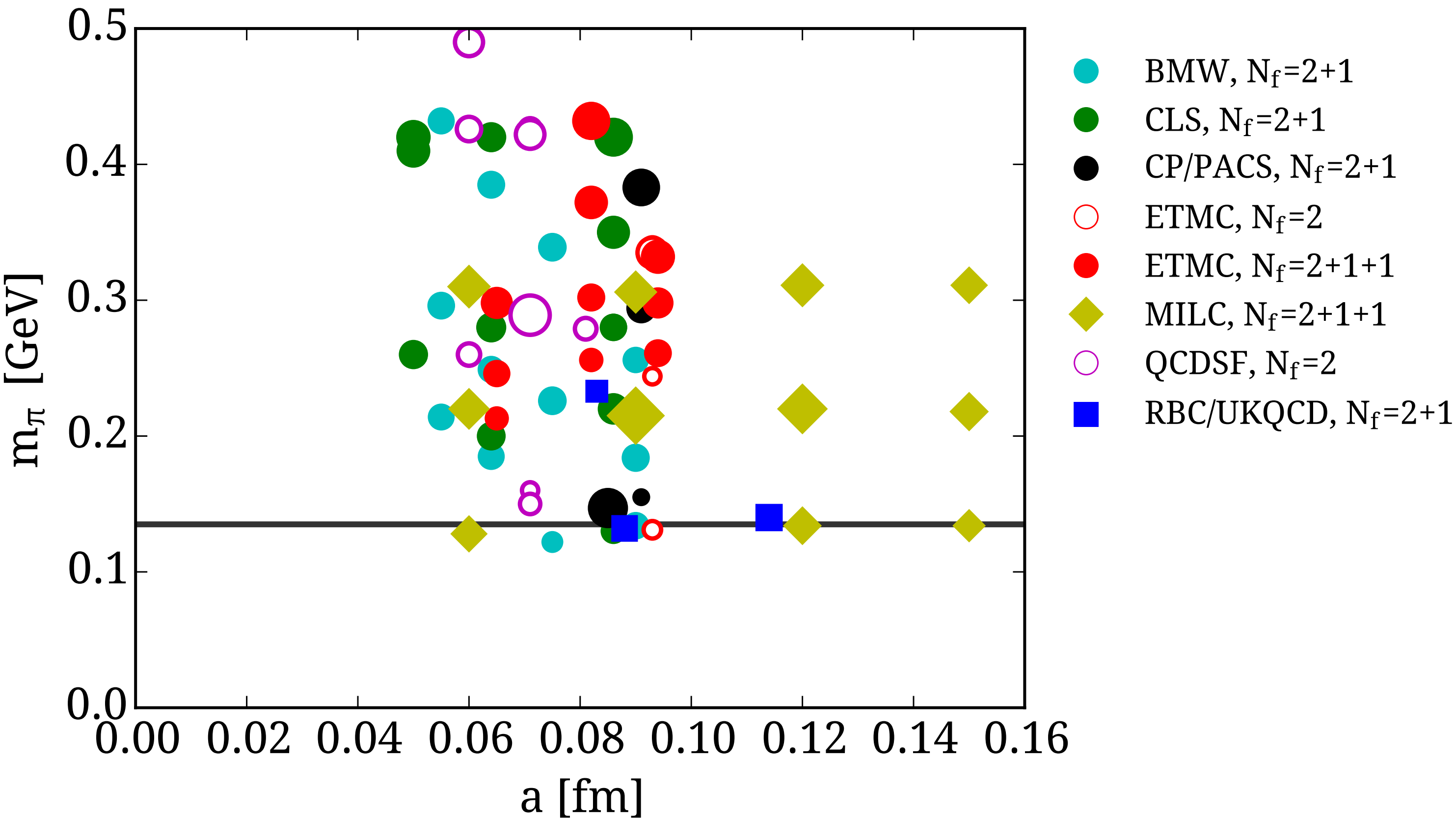
  Trade – offs and advantages for each differ



Eventually, all schemes must agree:

– At the continuum limit: $a \to 0$
– At infinite volume limit $L \to \infty$
– At physical quark mass

Simulations landscape

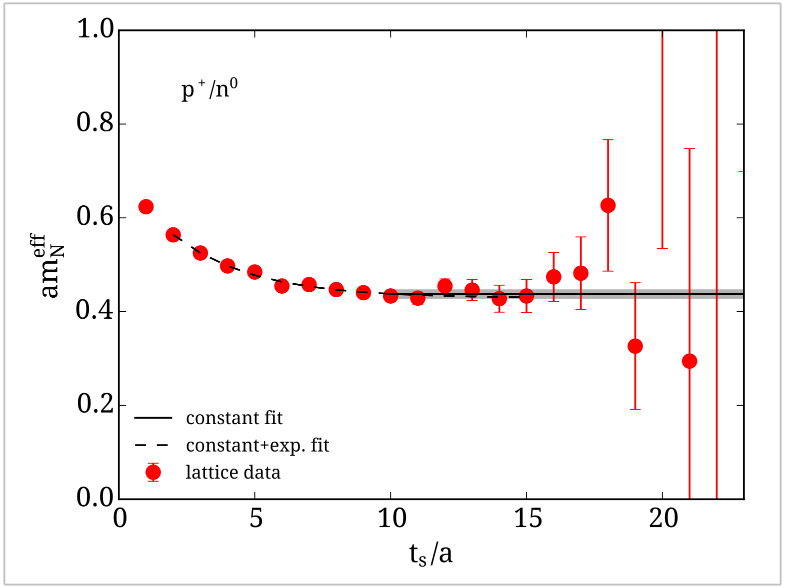


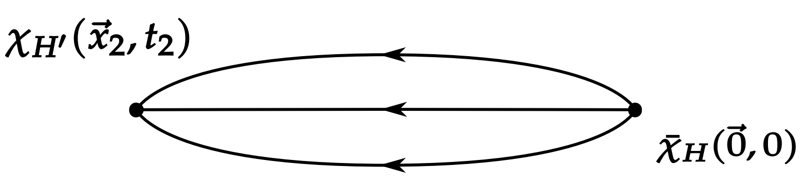
Selected lattice simulation points used for hadron structure

- Multiple collabs. simulating at physical pion mass
- Size of points indicates $m_\pi L$

Sources of uncertainty

- **Statistical error:** $\frac{1}{\sqrt{N}}$, with MC samples
- **Correlation functions:** exponentially decay with time-separation
- **Disconnected contributions:** stochastic error





- Systematic uncertainties
  - Extrapolations a, L, m_π
  - Contamination from higher energy states

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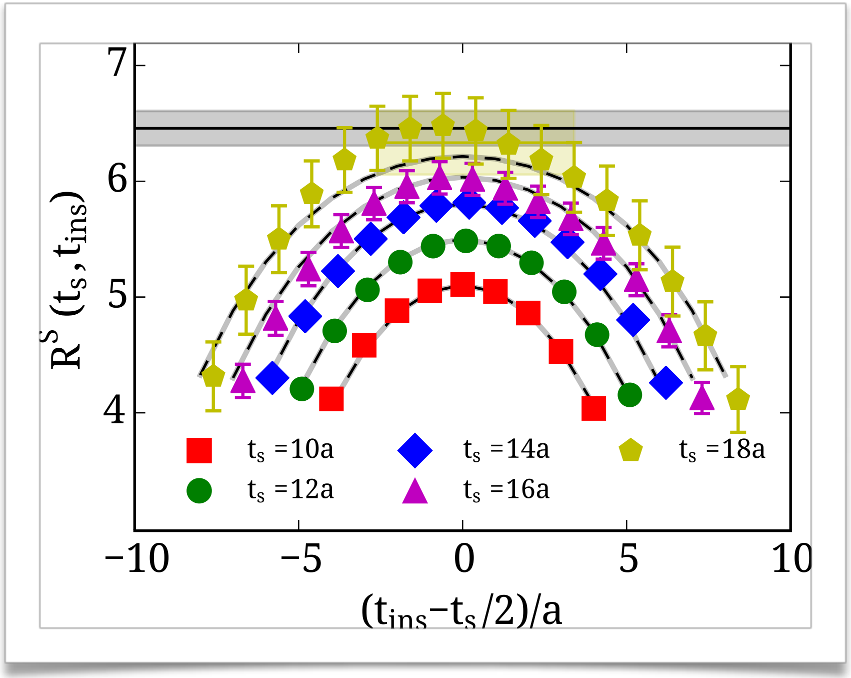

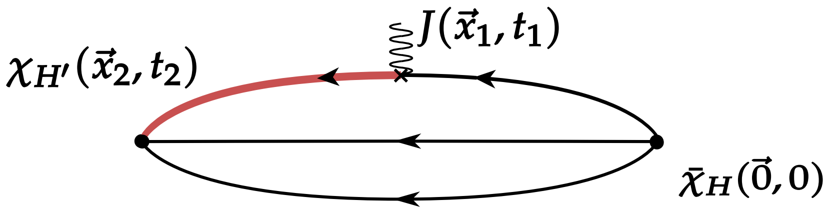

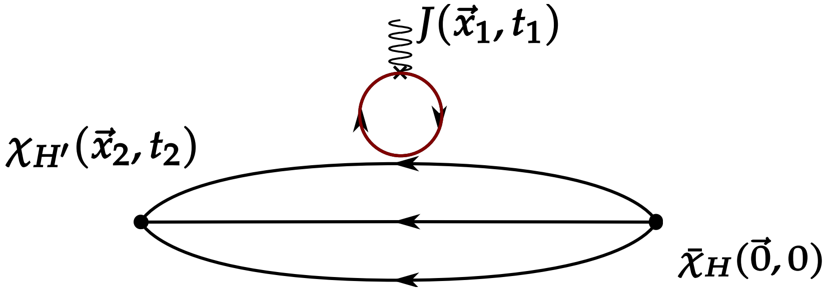

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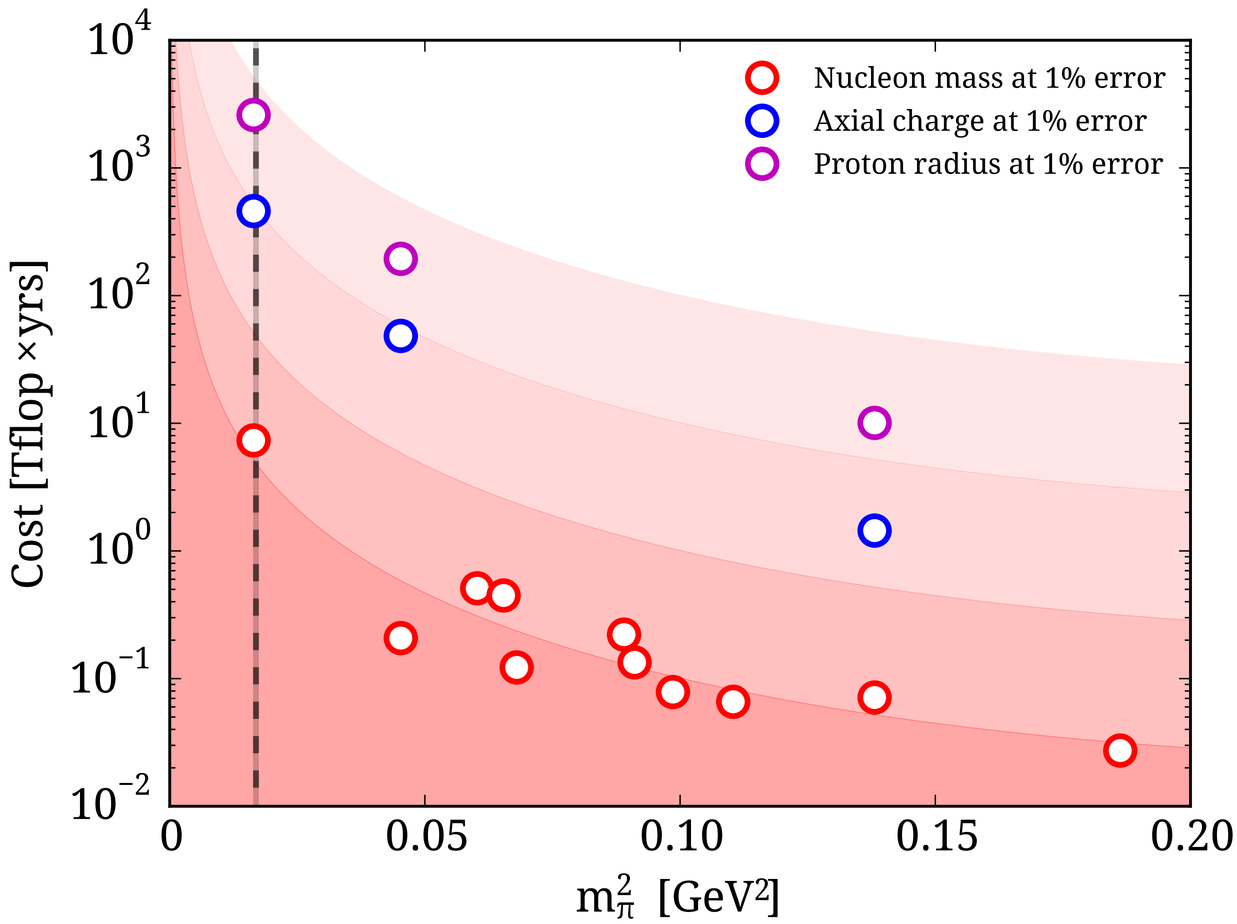
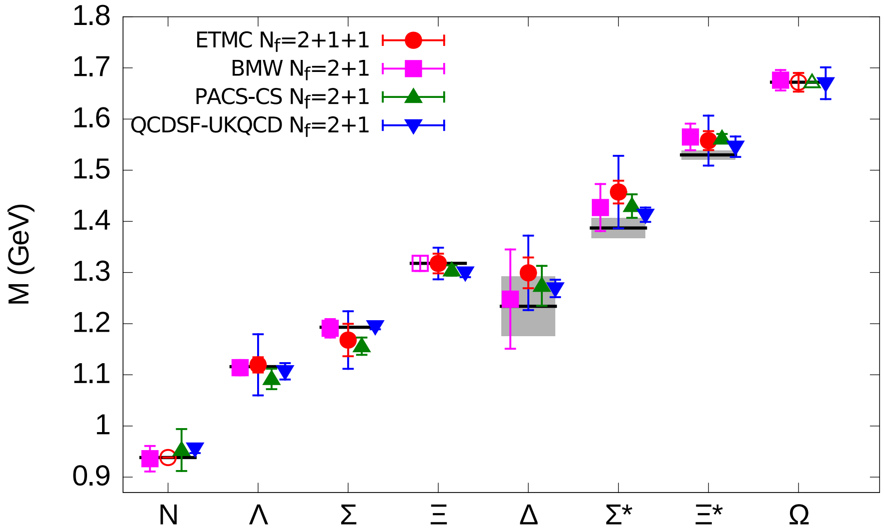
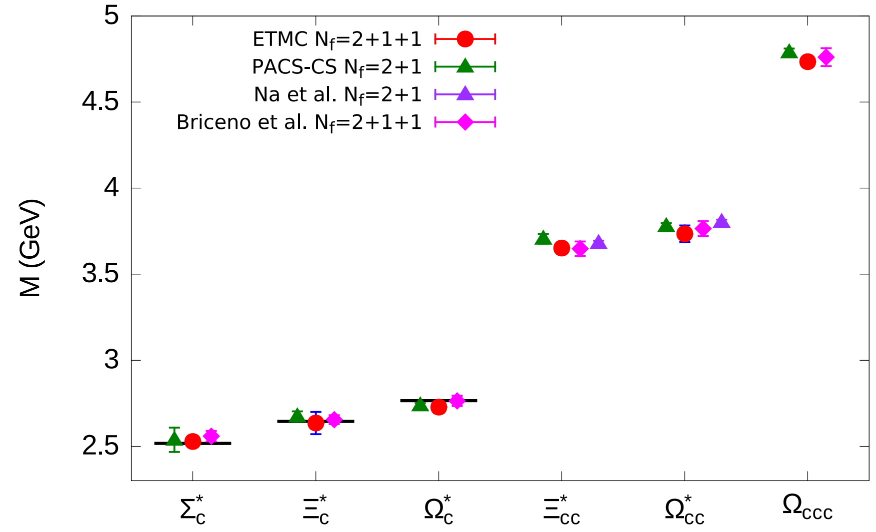
Multi-petascale to exa-scale requirements



Indicative computer time requirements for nucleon structure



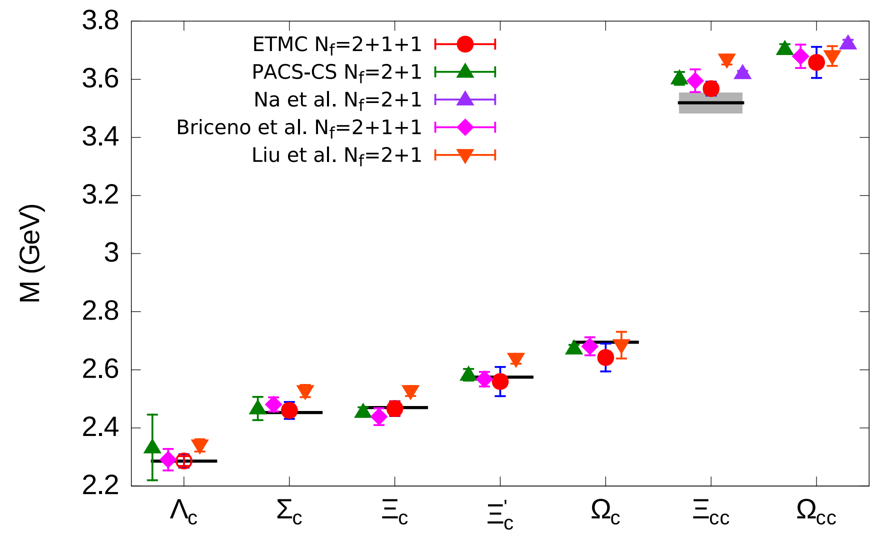


Reproduction of light baryon masses

- Agreement between lattice discretisation schemes
- Reproduction of experiment

Prediction of yet-to-be-observed charmed baryons

- Confidence through agreement between lattice schemes
- Nucleon structure…

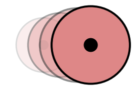


Nucleon structure on the lattice

– **Lattice:** moments of GPDs are readily accessible

Unpolarised

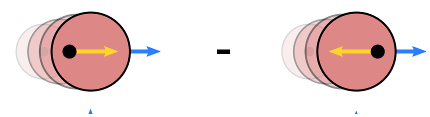
$$\mathcal{O}_{\boxed{V}}^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\boxed{\gamma}^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$



$$\langle 1\rangle_{u-d} = g_V\,,\quad \langle x\rangle_{u-d},\quad \ldots$$

Helicity

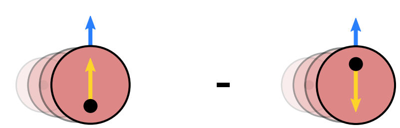
$$\mathcal{O}_{\boxed{A}}^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\boxed{\gamma_5\gamma}^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$



$$\langle 1\rangle_{\Delta u-\Delta d} = g_A,\quad \langle x\rangle_{\Delta u-\Delta d},\quad \ldots$$

Transverse

$$\mathcal{O}_{\boxed{T}}^{\nu\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\boxed{\sigma}^{\nu\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$



$$\langle 1\rangle_{\delta u-\delta d} = g_T,\quad \langle x\rangle_{\delta u-\delta d},\quad \ldots$$

Nucleon structure on the lattice

- **Benchmark** by calculating quantities well known experimentally
- Confidence in **prediction** of less well known quantities

Benchmark with:

- Axial charge
- Momentum fraction
- EM form-factors

Predict:

- Scalar charge/σ-terms
- Tensor charge

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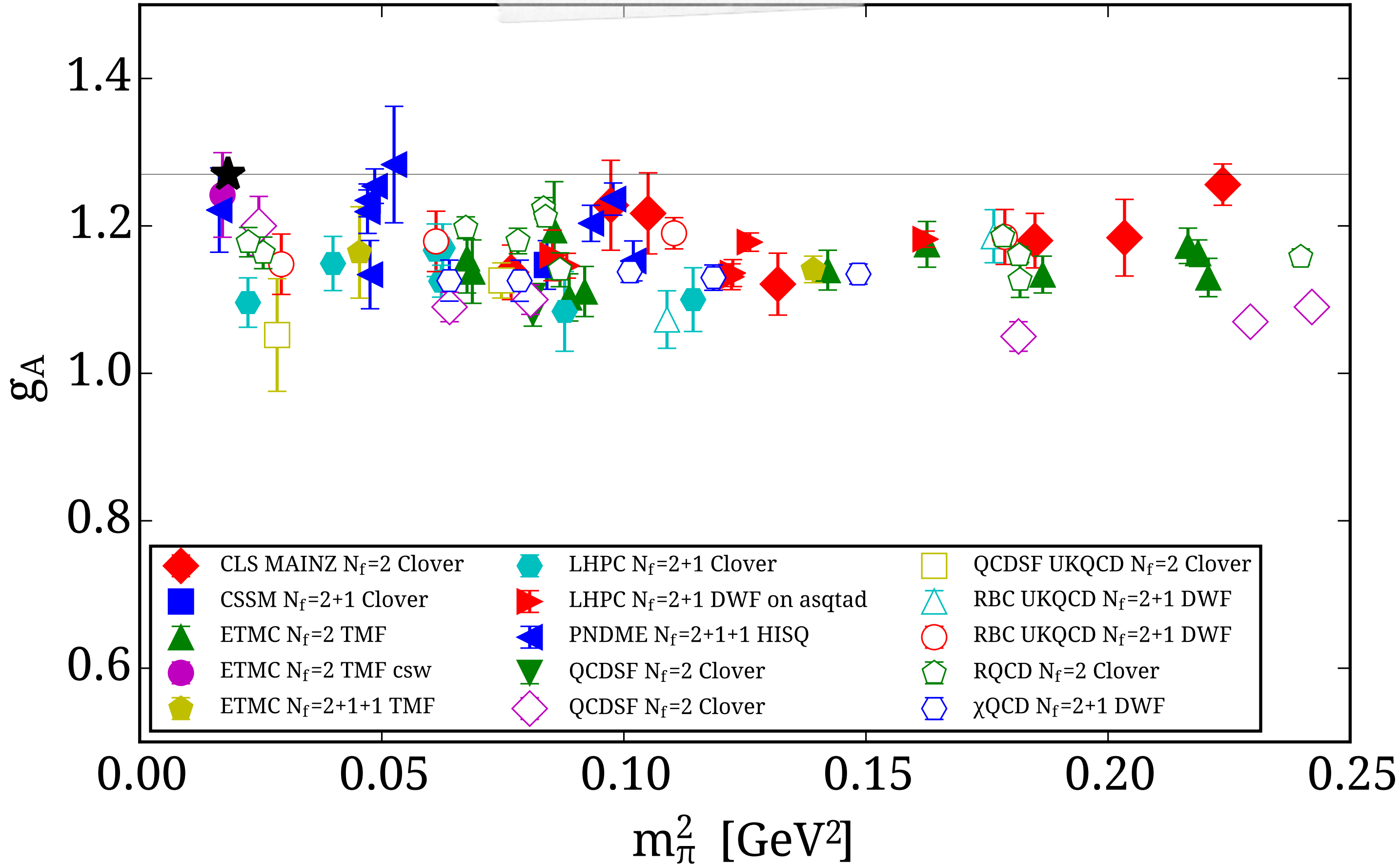
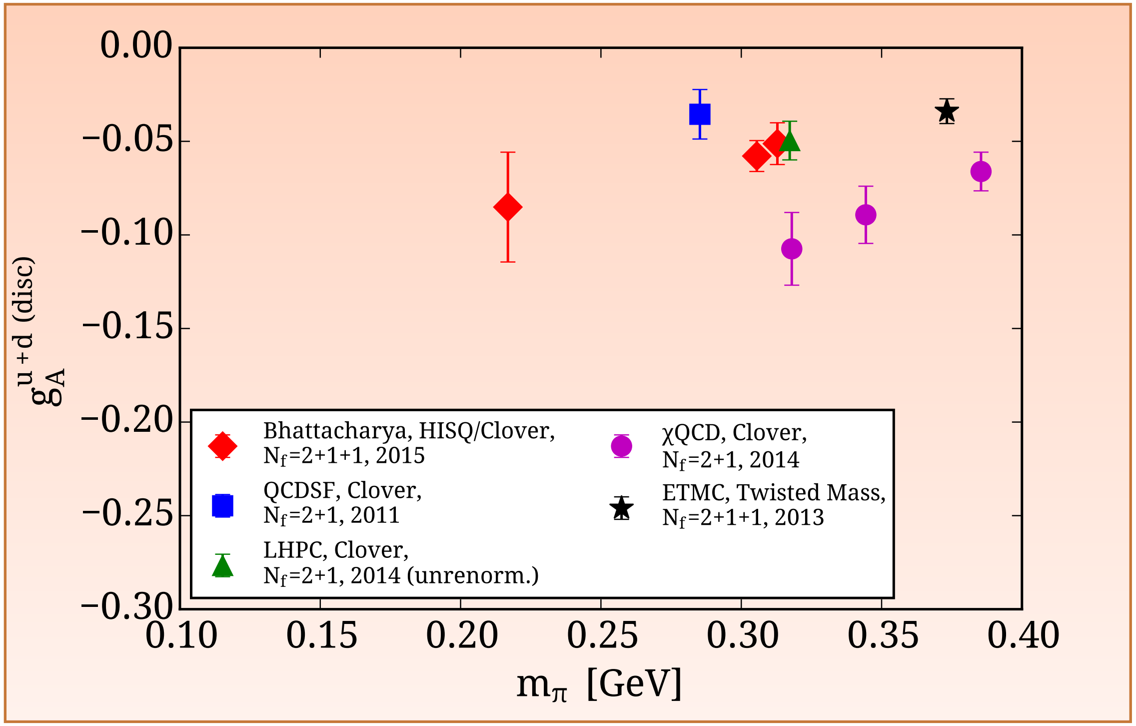
Benchmark

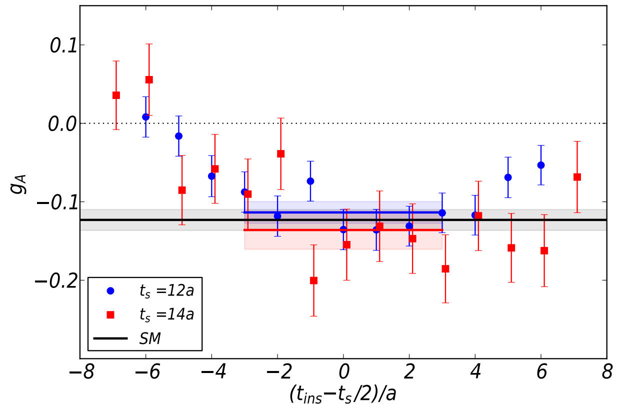


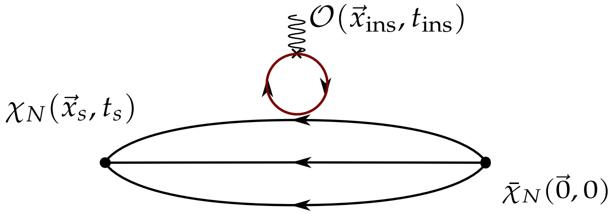
Axial charge

- Agreement towards experiment
- Simulations very close to or at the physical quark mass

$$\bar{u}\gamma_5\gamma_\mu u + \bar{d}\gamma_5\gamma_\mu d$$



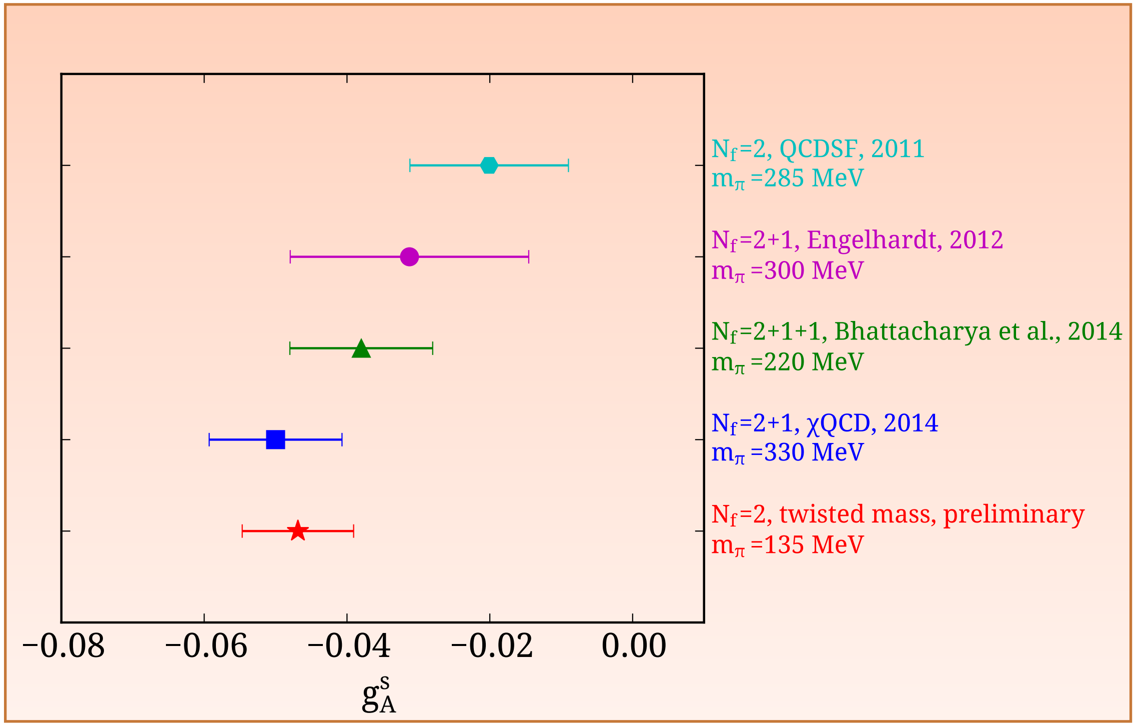


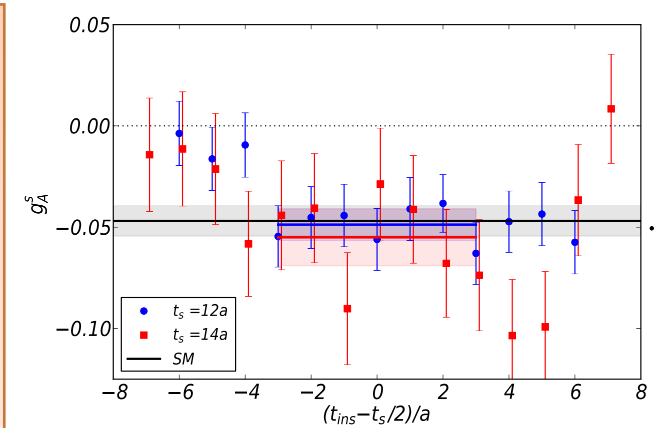


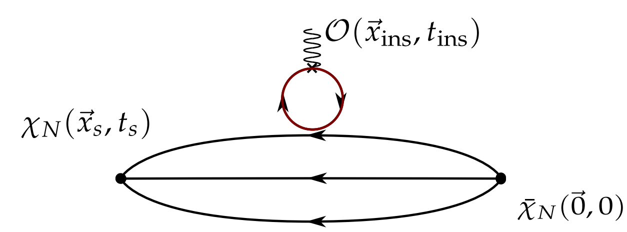
Axial charge – light disconnected

- Required for individual u- and d- contributions
- Requires dedicated calculations for “disconnected quark loop”
- Large statistical fluctuations in correlation functions
- Sign is negative: brings connected result down
- About 10% of connected value

$$\bar{s}\gamma_5\gamma_\mu s$$



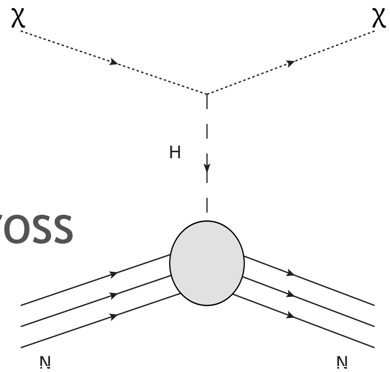




Axial charge — strange contribution

- Contribution exclusively by "disconnected quark loop"

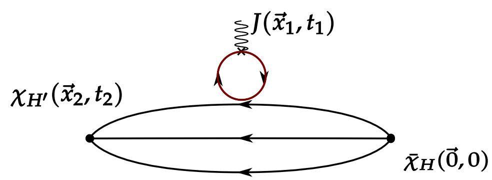
Nucleon sigma — terms

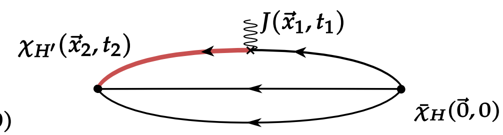


- Pion nucleon σ-term: $\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle$
- Strange σ-term: $\sigma_s = m_s\langle N|\bar{s}s|N\rangle$
- Enter super-symmetric candidate particle scattering cross sections with nucleon (e.g. neutralino through Higgs)

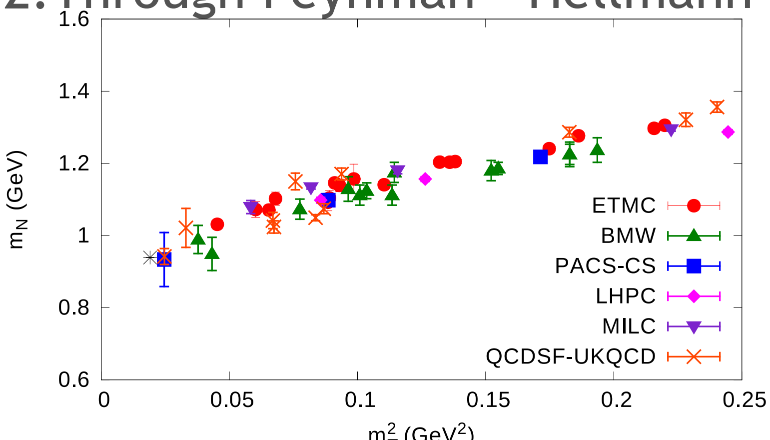
1. Direct calculation of matrix elements

Involves disconnected contributions



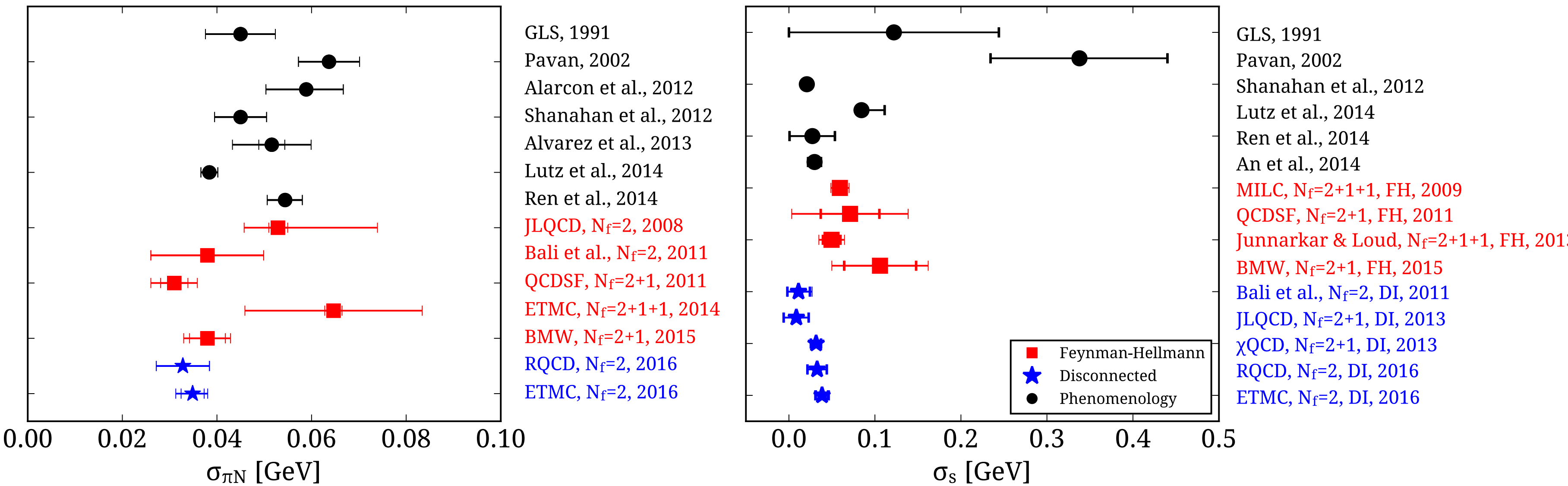


2. Through Feynman - Hellmann theorem: $\sigma_{\pi N} = m_{ud}\dfrac{\partial m_N}{\partial m_{ud}}$ $\sigma_s = m_s\dfrac{\partial m_N}{\partial m_s}$



- Reliance on effective theories for dependence on m_π
- Weak dependence on m_s

Nucleon sigma — terms



- More results coming from the lattice using direct evaluation of the matrix element
- First results from simulations directly at the physical point

Tensor charge

Tensor matrix element:

$$\langle N(p',s')|\bar{q} i\sigma_{\mu\nu} q|N(p,s)\rangle = \bar{u}(p',s') K^{\mu\nu} u(p,s)$$

$$K^{\mu\nu} = i\sigma^{\mu\nu} \boxed{A_{T10}(q^2)} + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N} \boxed{B_{T10}(q^2)} + \frac{\bar{P}^{[\mu}\Delta^{\nu]}}{m_N^2} \boxed{\tilde{A}_{T10}(q^2)}$$

$[\mu\nu]$: Antisymmetrize $\quad \Delta = p' - p \quad \bar{P} = \frac{p'+p}{2} \quad q^2 = \Delta^2$

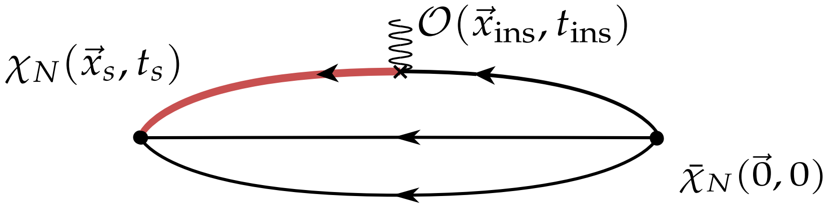
At zero momentum transfer:

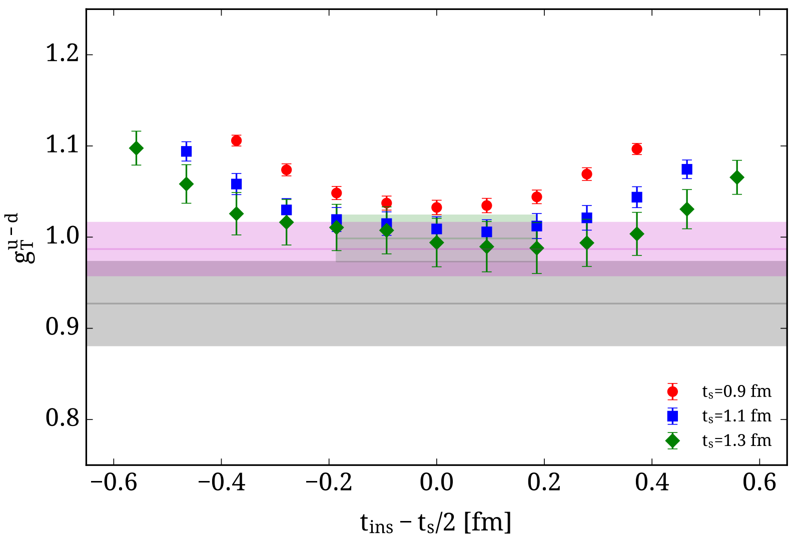
$$K^{\mu\nu} \to A_{T10}(0) = \langle 1 \rangle_{\delta q} = g_T^q$$

Isovector: $\bar{u} i\sigma^{\mu\nu} u - \bar{d} i\sigma^{\mu\nu} d \to g_T^{u-d}$, only connected contributions

Isoscalar: $\bar{u} i\sigma^{\mu\nu} u + \bar{d} i\sigma^{\mu\nu} d \to g_T^{u+d}$, contributions from disconnected fermion loops

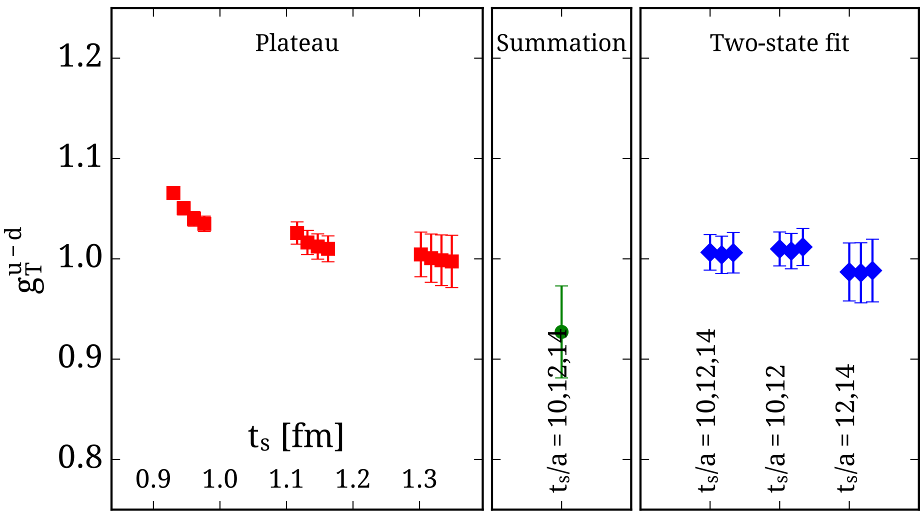
Tensor charge





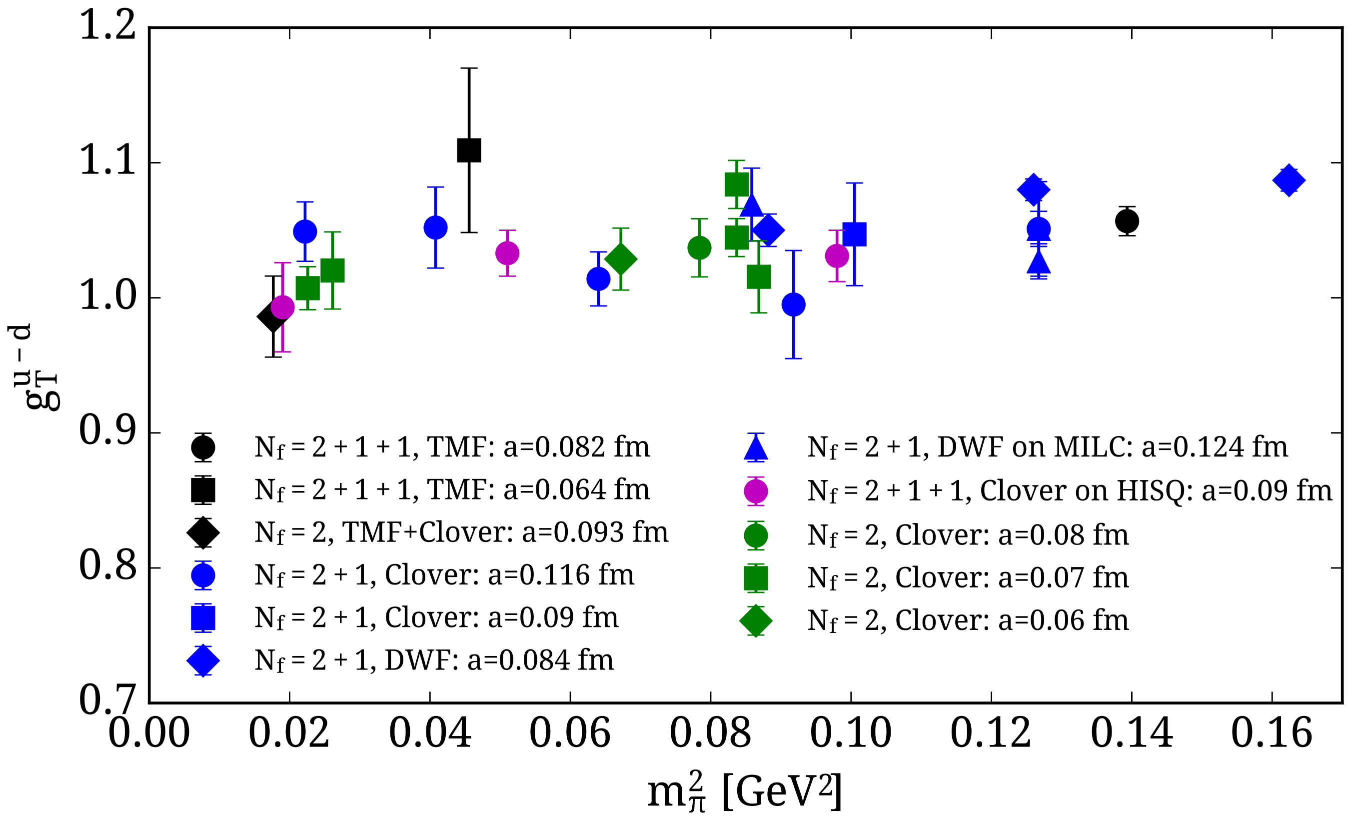
Isovector tensor charge

- Excited states increase value
- Agreement between methods at t_s~1.3 fm



m_π=135 MeV, Twisted Mass:
Phys.Rev. **D92** (2015) 114513

Tensor charge



ETMC, Phys. Rev. **D92** (2015) 114513

LHPC, Phys. Rev. **D86**, (2012) 114509

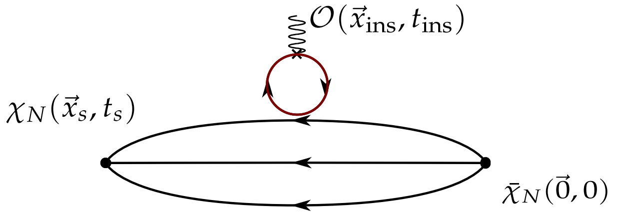
PNDME, Phys.Rev. **D92** (2015) 094511

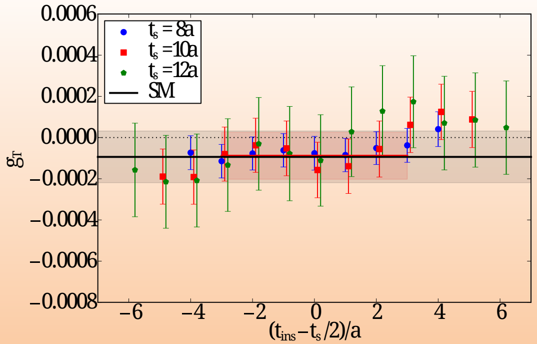
RQCD, Phys. Rev. **D91** (2015) 054501

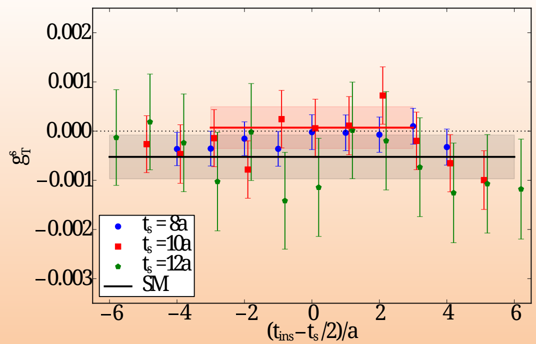
General agreement between lattice formulations

Tensor charge

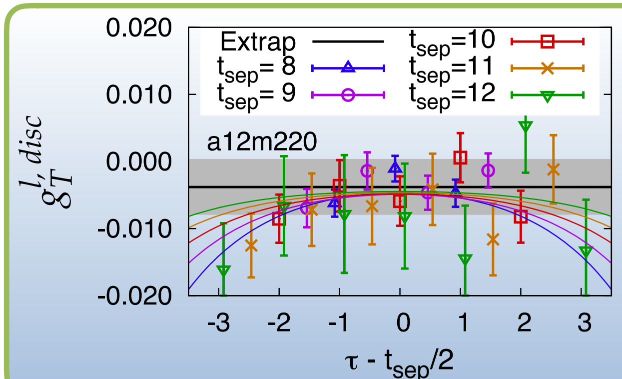
Disconnected contribution

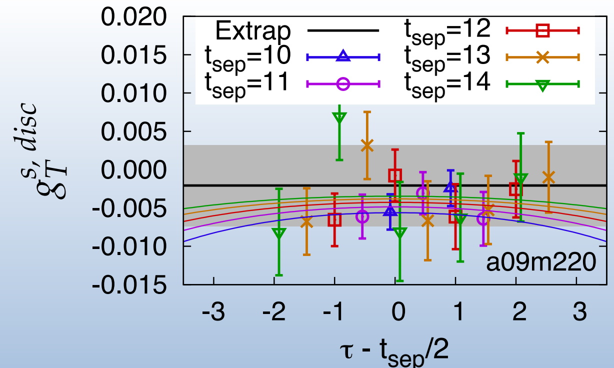






Twisted Mass
m_π=135 MeV,
PoS LATTICE 2015

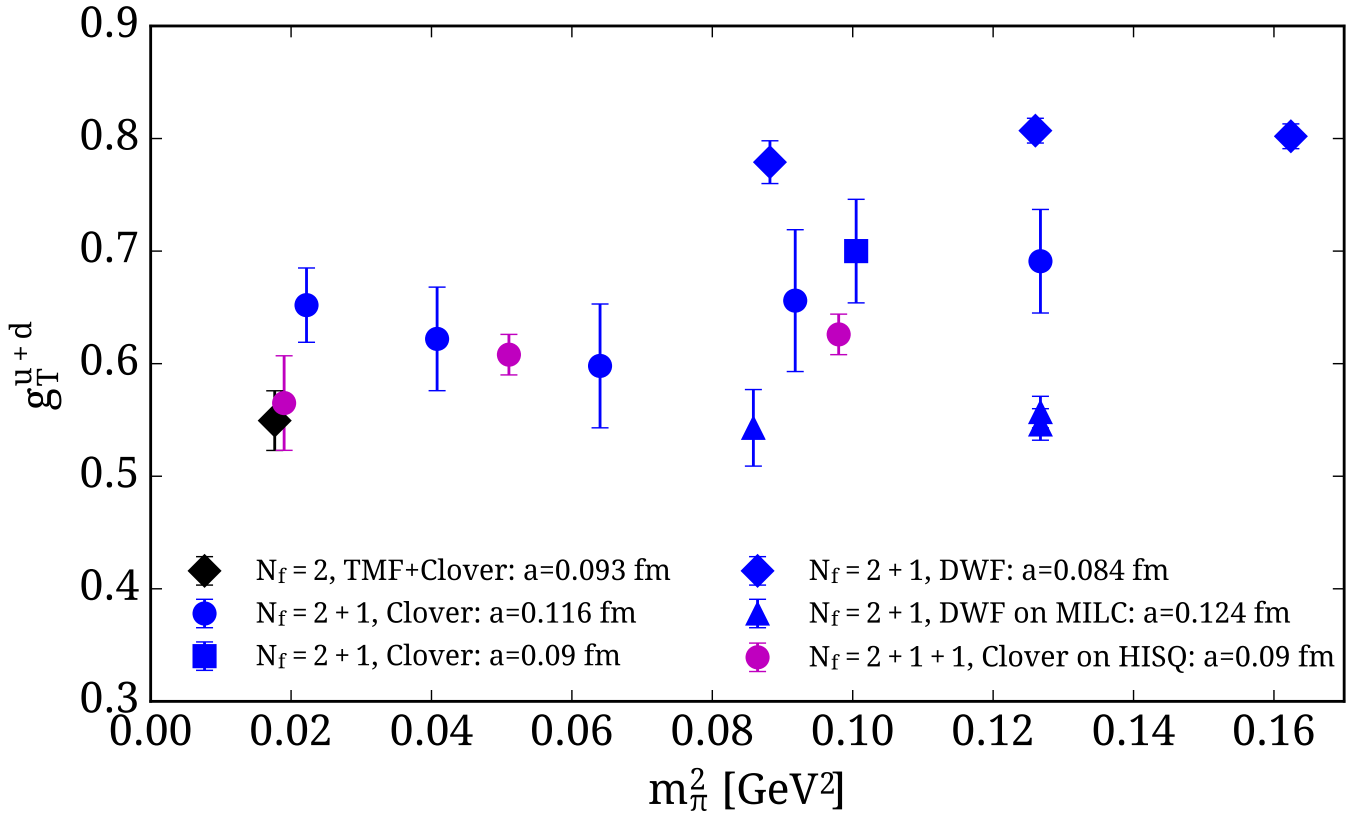




Clover on HISQ
m_π=220 MeV,
PRD92, (2015) 094511

Two recent lattice calculations: consistent with zero

Tensor charge

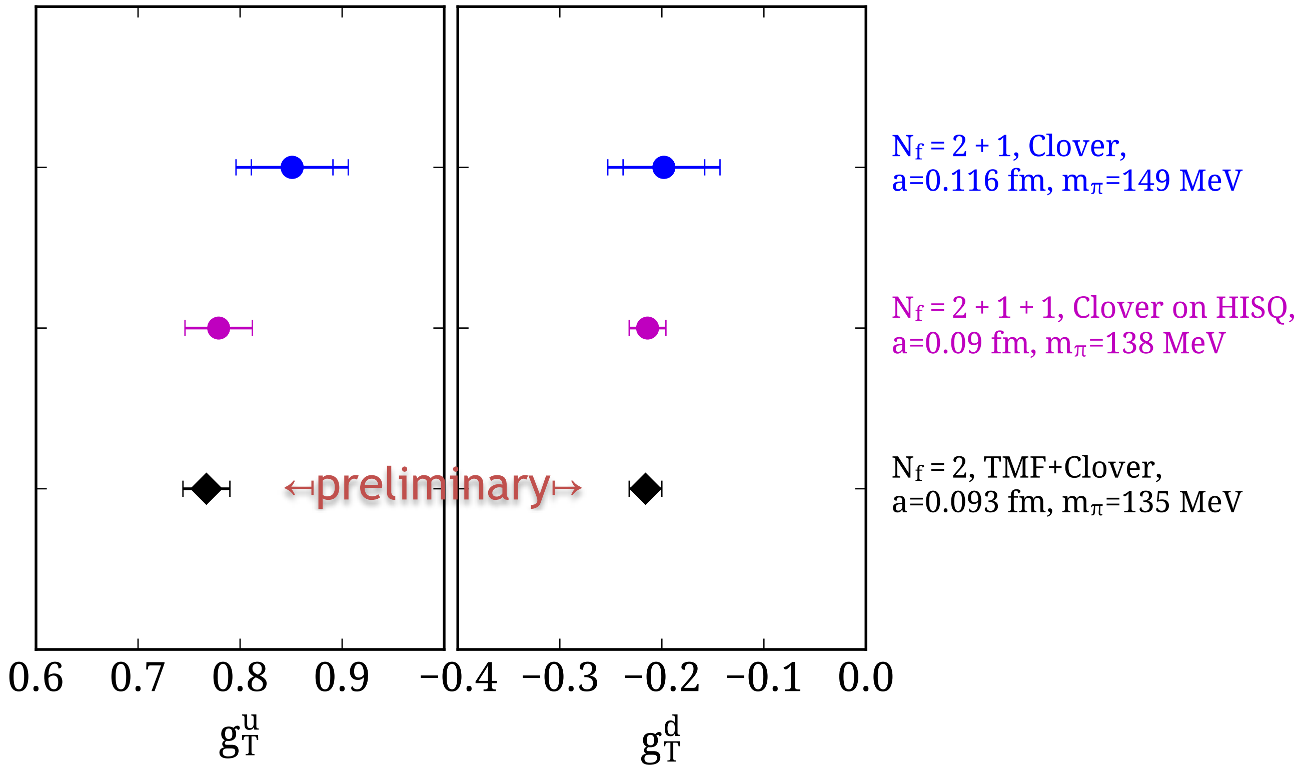


ETMC, Phys. Rev. **D92** (2015) 114513

LHPC, Phys. Rev. **D86**, (2012) 114509

PNDME, Phys.Rev. **D92** (2015) 094511

Tensor charge



- Assuming negligible contribution from disconnected loops
- Consistency between lattice formulations
- Expect future calculations with: $a \to 0$, $L \to \infty$ limits

Tensor form factors

$$\Gamma^{\beta\alpha}\langle\chi_N^\alpha(p')|\mathcal{O}^{\mu\nu}|\chi_N^\beta(p)\rangle \to \mathrm{Tr}[\Gamma\frac{-i\not{p}'+m_N}{m_N}K^{\mu\nu}\frac{-i\not{p}+m_N}{m_N}]$$

$$K^{\mu\nu} = i\sigma^{\mu\nu}\boxed{A_{T10}(q^2)} + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}\boxed{B_{T10}(q^2)} + \frac{\bar{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\boxed{\tilde{A}_{T10}(q^2)}$$

With Γ projects the nucleon's polarization. Construct different linear combinations of form factors, e.g.:

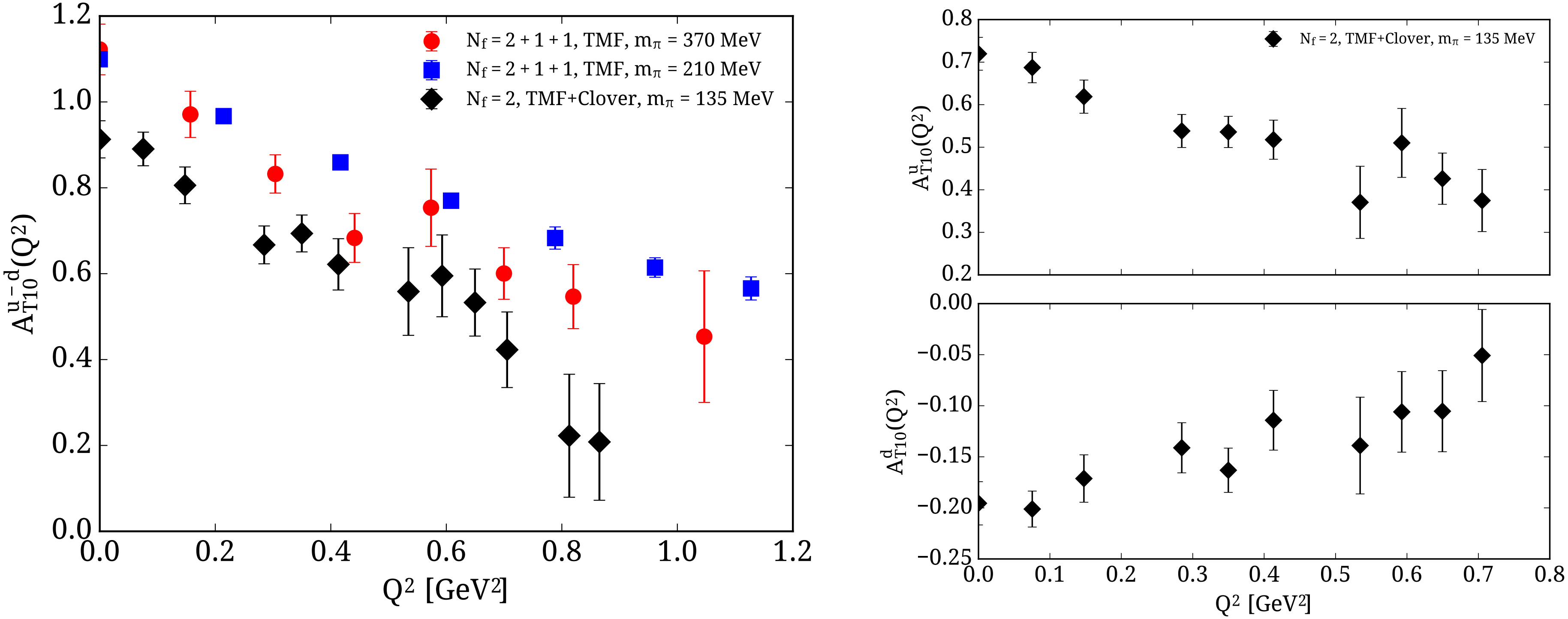
$$\Pi^{0j}(\Gamma_k) \to \frac{\epsilon^{ijk}p_i}{2m_N}[\boxed{A_{T10}(q^2)} - \boxed{B_{T10}(q^2)}\frac{E(q^2)-m_N}{2m_N}]$$

$$\Pi^{ij}(\Gamma_k) \to \epsilon^{ijk}[\frac{E(q^2)+m_N}{2m_N}\boxed{A_{T10}(q^2)} - \boxed{B_{T10}(q^2)}\frac{p_i^2+p_j^2}{4m_N^2}]$$

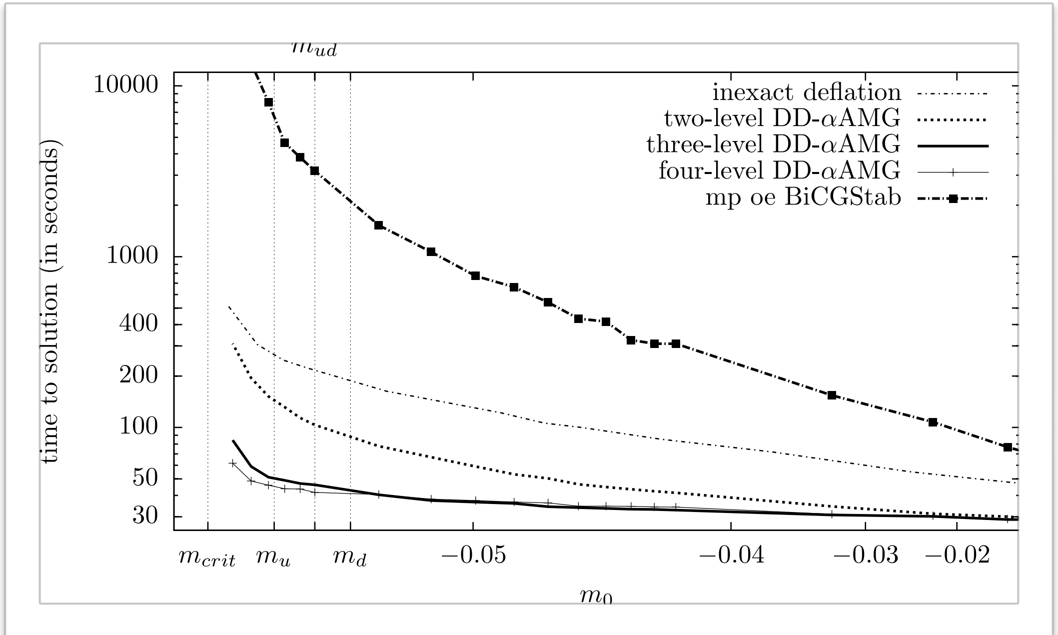
with: $p'=0,\ \ p=-q,\ \ \Gamma_k = \frac{1+\gamma_0}{4}i\gamma_5\gamma_k$

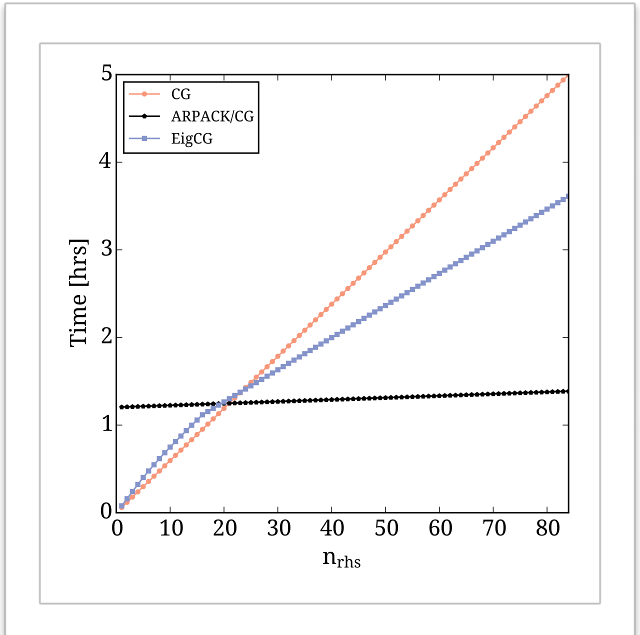
lead to an overdetermined set of equations for form-factors

Tensor form factor



- PoS LATTICE 2013 (2014) 294
- Decreasing values with $m_\pi \to m_\pi^{\mathrm{phys}}$
- Excited state investigation available at $m_\pi = 135 \text{ MeV}$





Algebraic multi-grid: A. Frommer et al. SIAM J. Sci. Comput. **36** (2014) A1581-A1608

Exact eigenvalue deflation

At physical quark masses

- Mathematical algorithms for alleviating “critical slowing down”
- Multiple right-hand-side methods for efficient multiplication of statistics

Summary and outlook

★ **Lattice QCD in new era**

- Physical pion mass simulations from a number of collaborations
- Other systematic uncertainties coming under control

★ **Confidence on results for less well-known quantities: g_T, tensor form factors**

- Three recent lattice calculations consistent
- Two with thorough investigation of excited state contaminations
- Valence-dominated: disconnected fermion-loop contribution very small (if at all)
- Up- and down-quark contributions to g_T at physical point

★ **What to expect**

- Continuum and infinite volume limits
- Longer term: effects of isospin breaking