TMD to Twist-3 Approach

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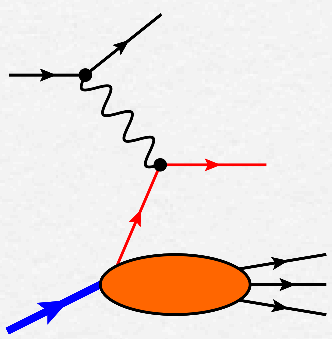
in collaboration with K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak
based on Phys. Rev. D 93, 054024 (2016)

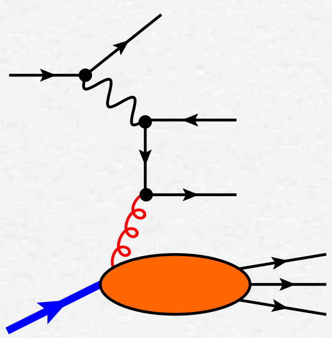
"Parton TMDs at large x",
ECT*, Trento / Italy, April 13, 2016

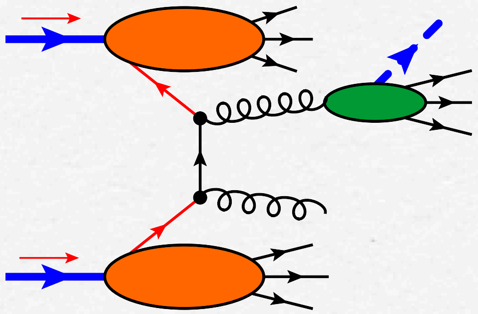
TMD vs. collinear factorization

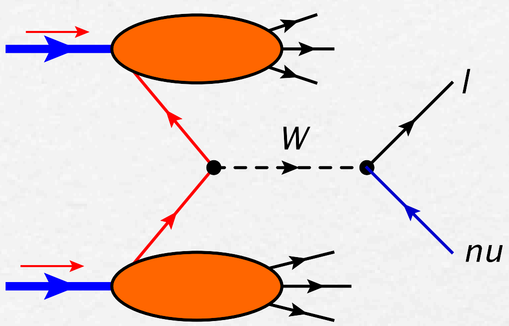
Collinear factorization in pQCD

- applicable to one-scale processes, e.g. 1-particle inclusive processes









$$\frac{d\sigma}{dx\,dQ^2}$$

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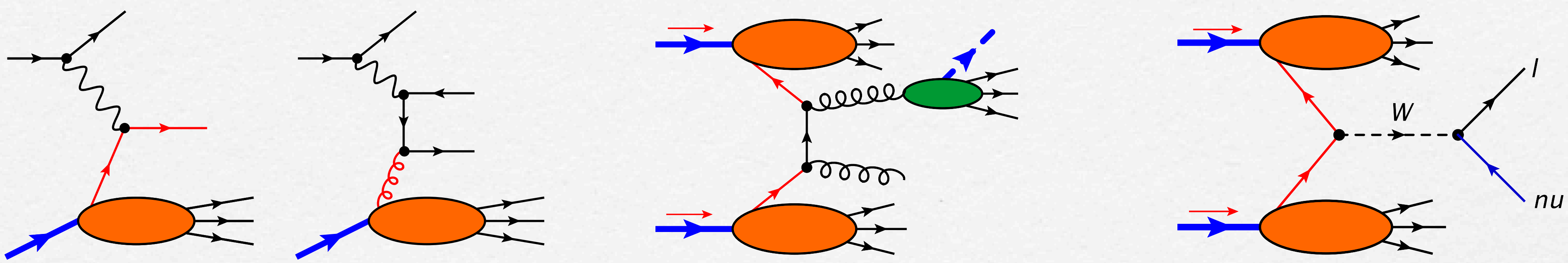

$$\frac{d\sigma}{dx\, dQ^2}$$

- Cross sections at high energies ➙ (hard part) × (soft parts)

- hard part ➙ pQCD (NLO, NNLO,...) ; soft parts ➙ universal, 1-dim

collinear parton distributions

$$q(x,\mu),\ \Delta q(x,\mu),\ \delta q(x,\mu) \qquad G(x,\mu),\ \Delta G(x,\mu)$$

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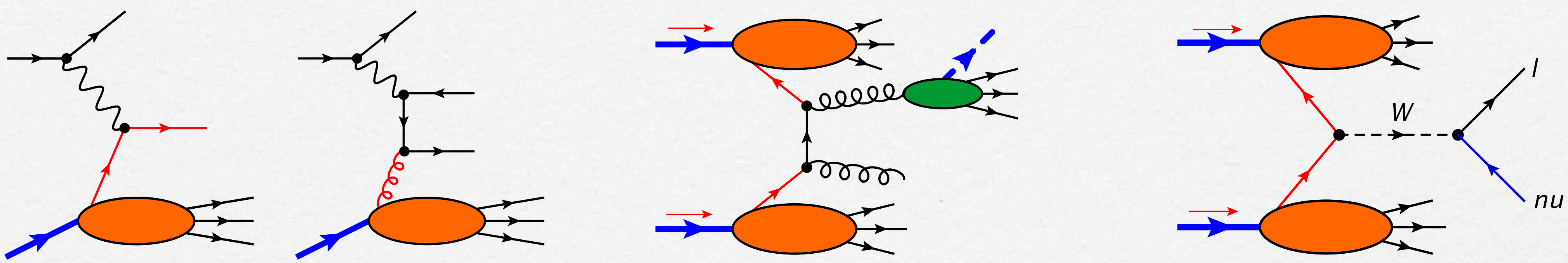

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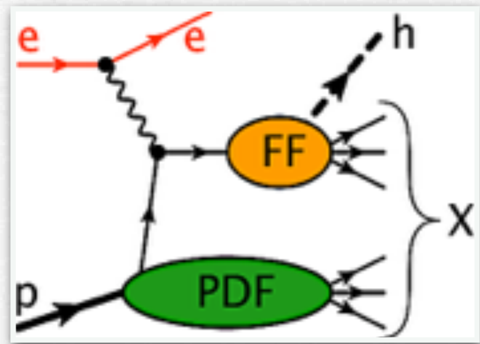
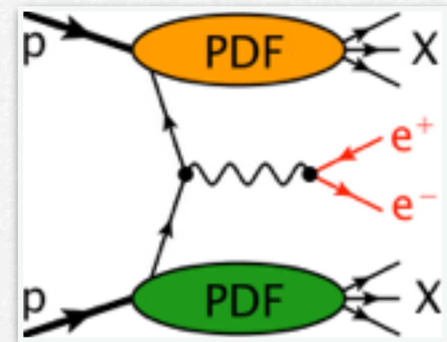
- Transverse Single-Spin Asymmetries → Twist-3 approach

- Collinear factorization: 2 (or more...)-particle inclusive processes

SIDIS





Drell-Yan

two scales: **hard scale** Q + **final state transverse momentum** q_T

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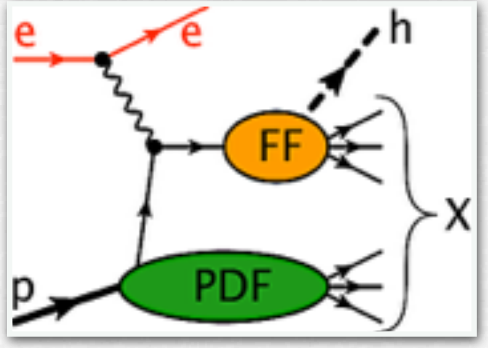

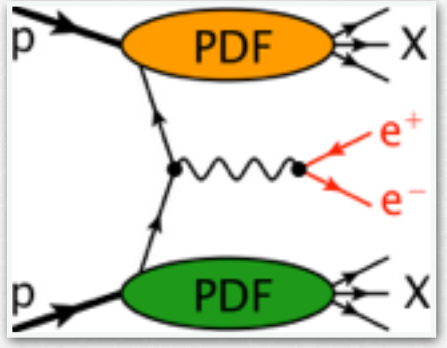

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→ integrated observables $$\int d^2 q_T \, w(q_T) \frac{d\sigma}{dx\, dQ^2\, dq_T} \equiv \langle w(q_T) \rangle$$

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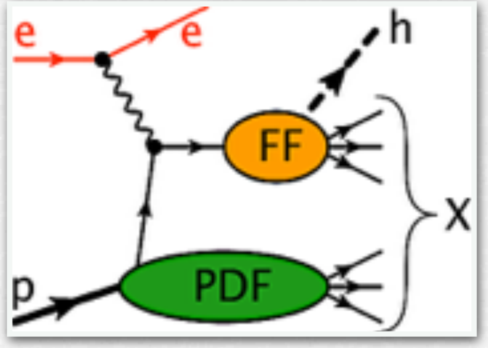

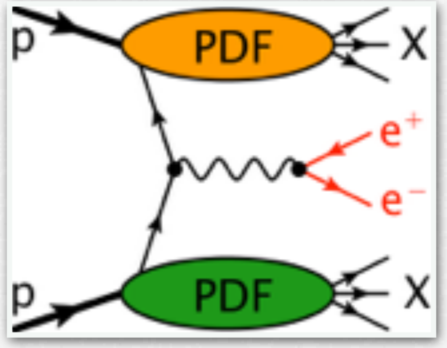

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one scale ➙ collinear factorization ok,

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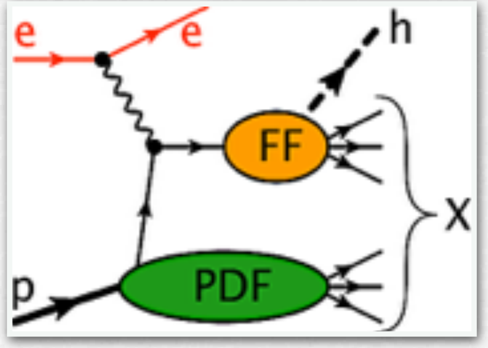
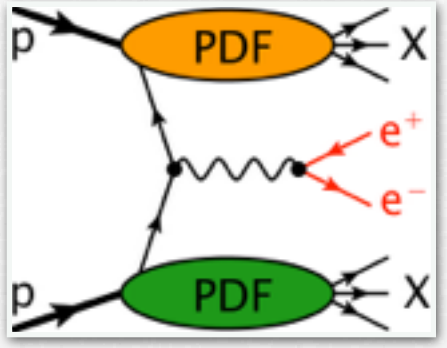
SIDIS   Drell-Yan

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$$\frac{d\sigma}{dq_T}(\Lambda_{\rm QCD} \ll q_T \ll Q)$$

large logs in the hard part (gluon radiation) $\log^n(q_T/Q)$
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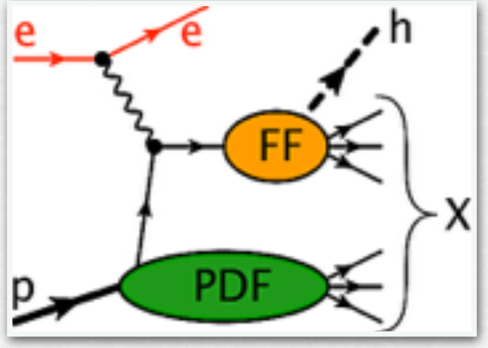
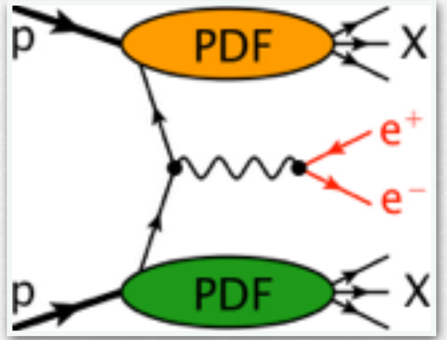
SIDIS   Drell-Yan

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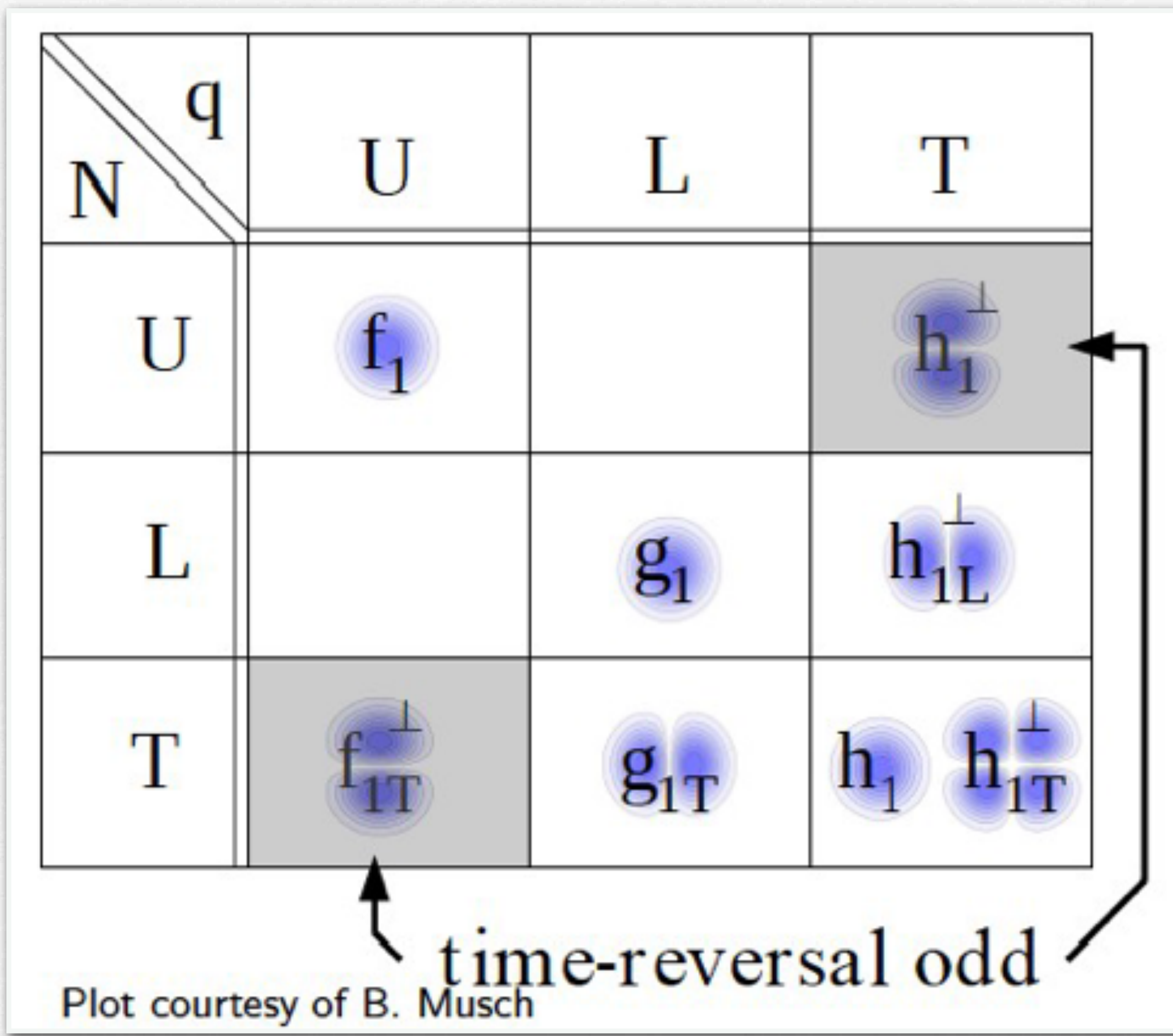
→ **Transverse momentum dependent (TMD) factorization!**

(Naive) definition of the TMD parton distributions

$$\Phi^{[\Gamma]}(x, k_T) = \int \frac{dz^- d^2 z_T}{2(2\pi)^3} \mathrm{e}^{ik\cdot z} \langle P, S|\, \bar{q}(0)\, \Gamma\, \mathcal{W}[0, z]\, q(z)\, |P, S\rangle \Big|_{z^+=0}$$



Plot courtesy of B. Musch

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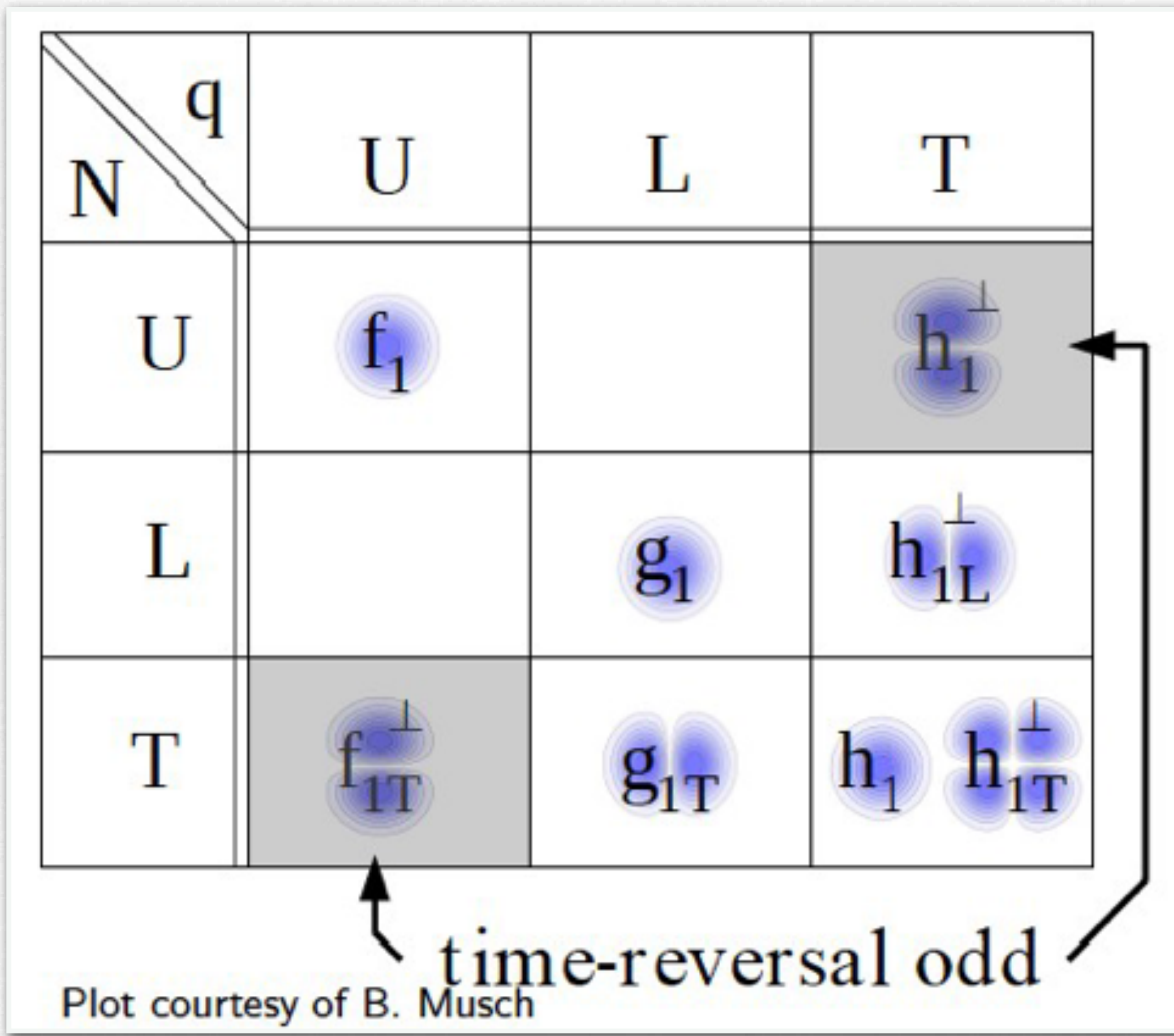

Color gauge Invariance

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N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

time-reversal odd

Plot courtesy of B. Musch

Color gauge Invariance

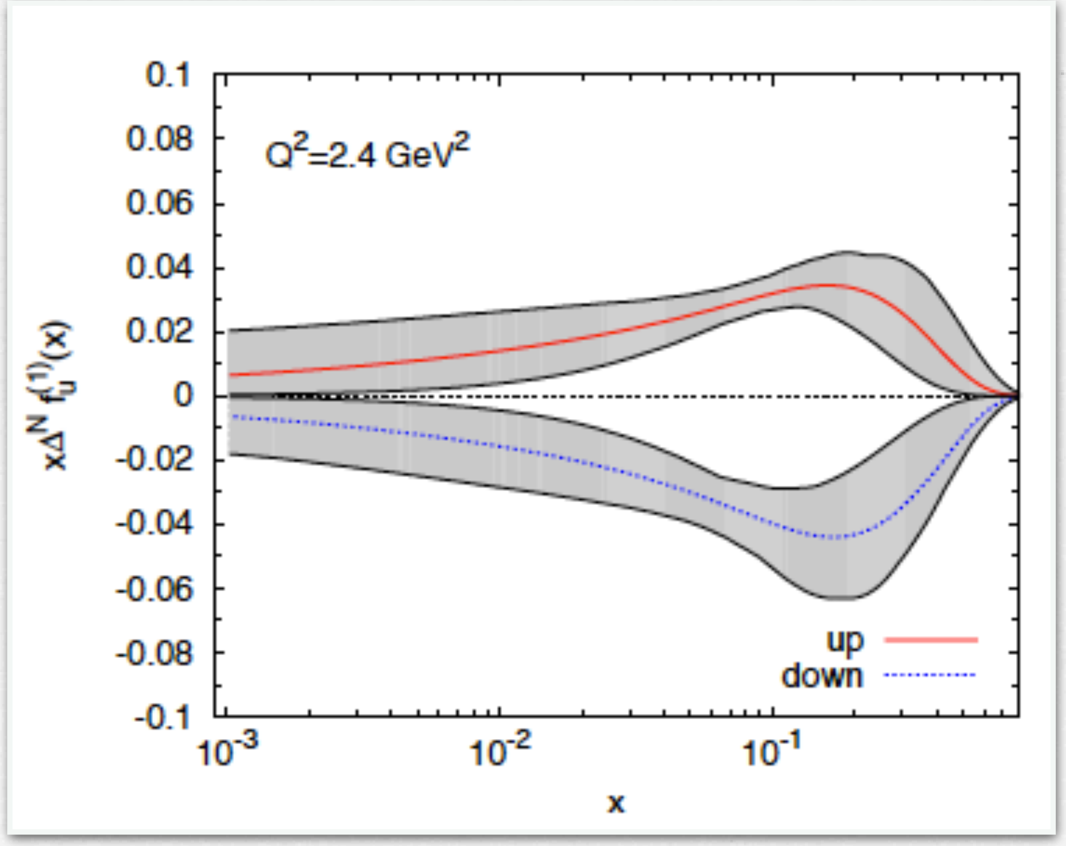
vanish without Wilson line

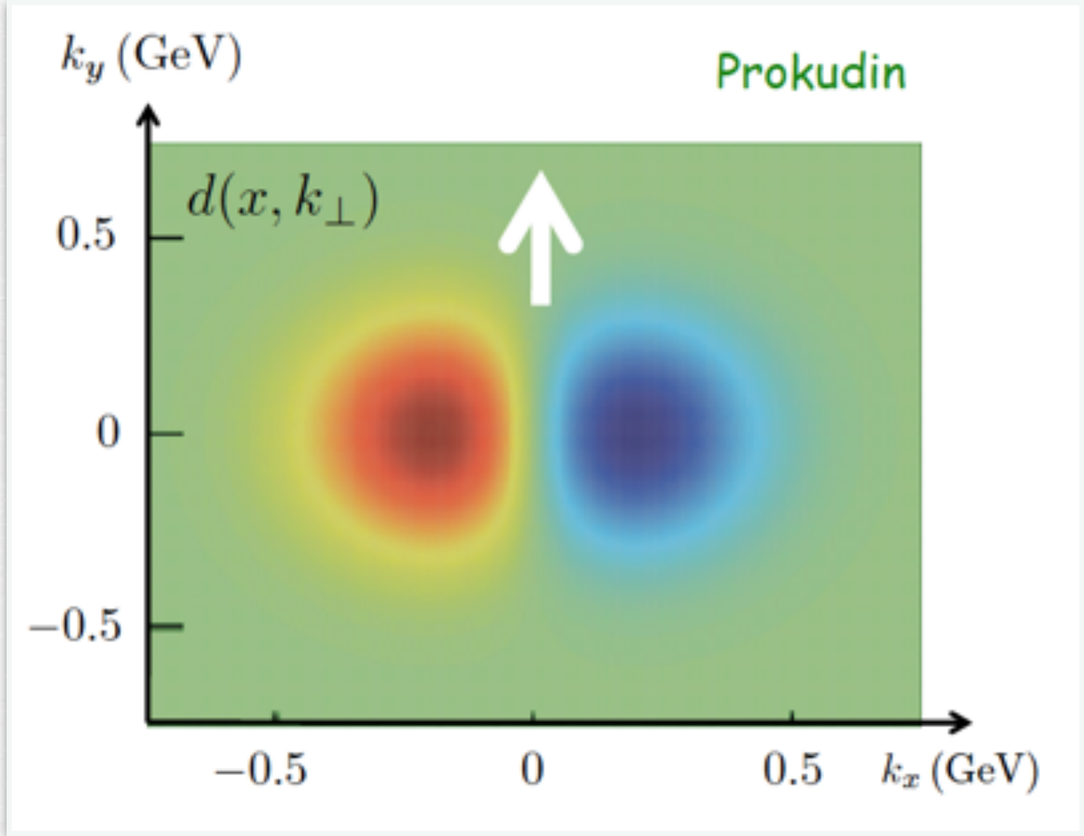
Sivers - Effect in Semi-Inclusive DIS (HERMES, COMPASS, JLab)

$$A_{UT}^{\text{Siv}} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

$$\propto \sin(\phi_h - \phi_s) \int d^2k_T d^2p_T \; \delta^{(2)}(k_T - p_T - P_{h\perp}/z_h) \; \frac{P_{h\perp}\cdot k_T}{|P_{h\perp}|M} \; f_{1T}^{\perp q}(x_B, k_T^2) \; D_1^q(z_h, p_T^2)$$

Sivers function

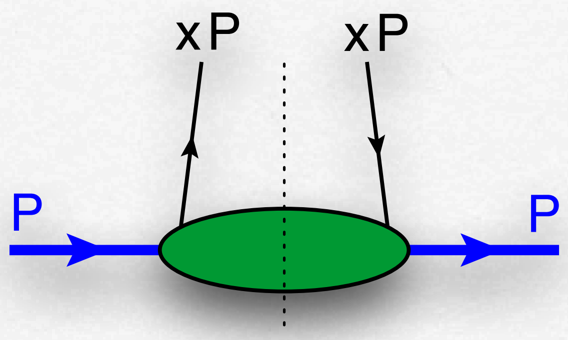




[Anselmino et al., PRD(2012)]

Twist-3 approach to Transverse Spin effects in single-inclusive processes

Transverse Spin Matrix Elements



1) Transversity distribution

$$h_1^q(x) = \int_{-\infty}^{\infty} \frac{d\lambda}{4\pi}\, \mathrm{e}^{i\lambda x (P\cdot n)}\, \langle P, S_T|\, \bar{q}(0)\, \not{S}_T \not{n} \gamma_5\, [0;\lambda n]\, q(\lambda n)\, |P, S_T\rangle$$

<u>"chiral-odd" structure</u>

→ in combination with other chiral-odd objects (Collins effect)

→ extractions from Collins effect, Dihadron fragmentation

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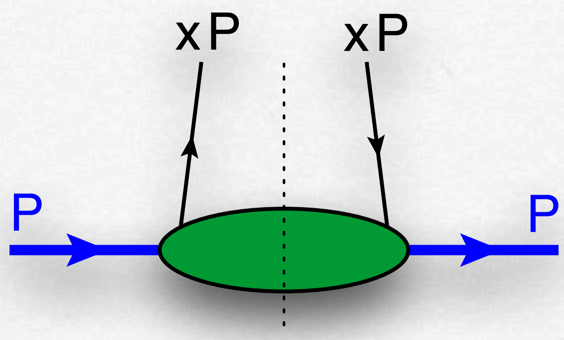

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2) 'intrinsic twist-3' → g_T

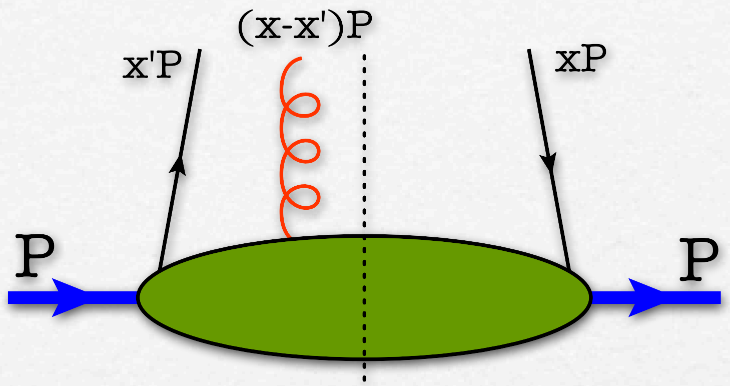
$$g_T^q(x) = -\frac{(P\cdot n)}{M} \int_{-\infty}^{\infty} \frac{d\lambda}{4\pi}\, e^{i\lambda x(P\cdot n)}\, \langle P, S_T|\, \bar{q}(0)\, \not{S}_T \gamma_5\, [0;\lambda n]\, q(\lambda n)\, |P, S_T\rangle$$

Twist - 3 characteristics hidden in Dirac structure

3) Three-parton correlation functions

'dynamical twist-3'

$$\Phi_F^\alpha(x,x') \sim \langle P,S|\,\bar{q}(0)\,gF^{n\alpha}(\mu n)\,q(\lambda n)\,|P,S\rangle$$
$$\Longrightarrow\; F_{FT}(x,x')\,,\; G_{FT}(x,x')\,,\; H_{FL}(x,x')\,,\; H_{FU}(x,x')$$



2-dimensional support, much richer parton dynamics

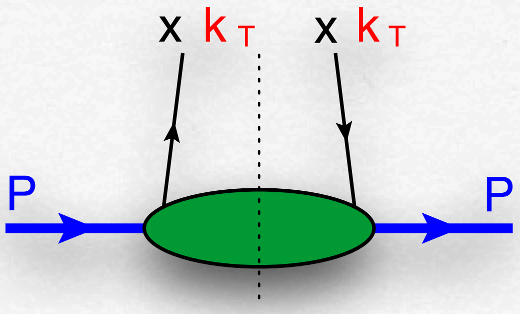
observables beyond leading twist → test of factorization in pQCD

→ so far: only "diagonal support" $F_{FT}(x,x)$ constraint by data

→ color Lorentz force on struck quark [Burkardt]

$$F^y = [\vec{E} + \vec{v} \times \vec{B}]^y \propto \int dx \int dx' \;\; F_{FT}(x,x')$$

4) 'Transverse Parton Momenta k_T'



'Sivers function'

$$f_{1T}^{\perp,(1)}(x) = \int d^2k_T \; \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x, k_T^2)$$

'wormgear function'

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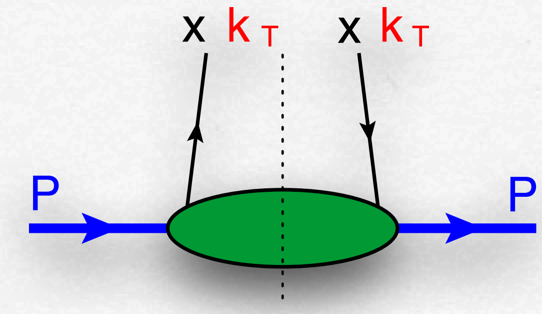

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Relations in collinear factorization

Gluonic Poles:

$$f_{1T}^{\perp(1)}(x) = \pm\pi F_{FT}(x, x)$$

$$h_1^{\perp(1)}(x) = \pm\pi H_{FU}(x, x)$$

'QCD - equation of motion':

$$g_{1T}^{(1)}(x) = x\, g_T(x) - \frac{m_q}{M} h_1(x) + \int_{-1}^{1} dx' \, \frac{F_{FT}(x, x') - G_{FT}(x, x')}{x - x'}$$

+ Lorentz - Invariance relations

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[Kanazawa, Koike, Metz, Pitonyak, MS, PRD 93, 054024 (2016)]

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x) - 2 \int_{-1}^{1} dx' \, \frac{G_{FT}(x,x')}{(x-x')^2}$$

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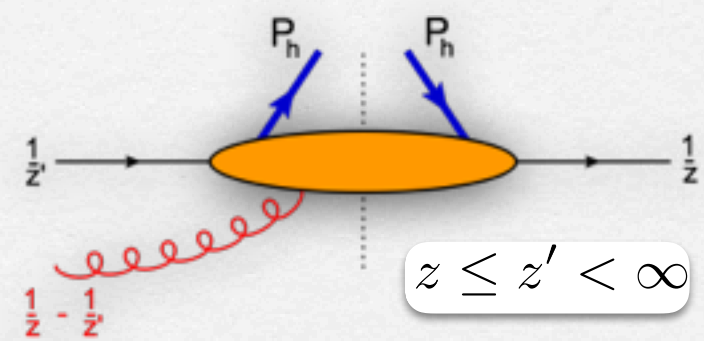
Twist-3 Fragmentation Functions:

$$\Delta_F^{\alpha}(z, z') \sim \langle 0 | \, q(\lambda m) \, gF^{m\alpha}(\mu m) | P_h, S_h; X \rangle \langle P_h, S_h; X | \, \bar{q}(0) \, | 0 \rangle$$

$$\Longrightarrow \hat{H}_{FU}(z, z') \, , \, \hat{F}_{FT}(z, z') \, , \, \hat{G}_{FT}(z, z') \, , \, \hat{H}_{FL}(z, z')$$



$z \leq z' < \infty$

complex functions: $FF(z, z) = 0$ $FF(z, 0) = 0$ $\frac{\partial}{\partial z'} FF(z, z')\Big|_{z'=z} = 0$

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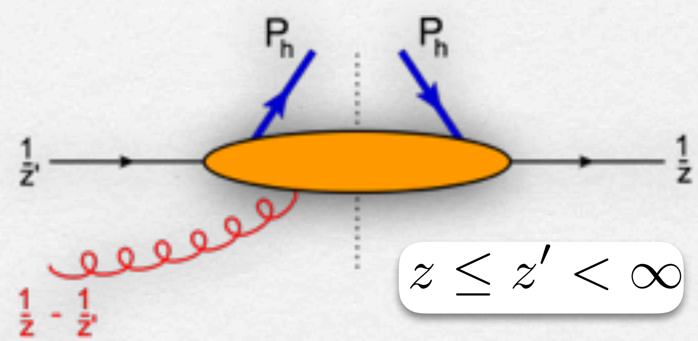

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QCD - equation of motion

$$H_1^{\perp(1)}(z) = -\frac{H(z)}{2z} + \int_z^{\infty} \frac{dz'}{z'^2} \frac{\Im[\hat{H}_{FU}(z,z')]}{1/z - 1/z'}$$

Lorentz - Invariance relations for FFs

$$\frac{H(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1)}(z) - \frac{2}{z}\int_z^{\infty} \frac{dz'}{z'^2} \frac{\Im[\hat{H}_{FU}(z,z')]}{(1/z - 1/z')^2}$$

QCD Equation of Motion + Lorentz-invariance relation:

Two equations, three functions → eliminate 'intrinsic & kinematical twist-3'

$$g_T(x) = \int_x^1 \frac{dy}{y}\, g_1(y) + \frac{m_q}{M}\left(\frac{1}{x}\, h_1(x) - \int_x^1 \frac{dy}{y^2}\, h_1(y)\right)$$
$$+ \int_x^1 \frac{dy}{y^2} \int_{-1}^1 dz \left[\frac{(1-y\delta(y-x))\, F_{FT}(y,z)}{y-z} - \frac{(3y-z-y(y-z)\delta(y-x))\, G_{FT}(y,z)}{(y-z)^2}\right]$$

Wandzura-Wilczek term

$$g_{1T}^{(1)}(x) = x\left(\int_x^1 \frac{dy}{y}\, g_1(y) - \frac{m_q}{M}\int_x^1 \frac{dy}{y^2}\, h_1(y) + \int_x^1 \frac{dy}{y^2}\int_{-1}^1 dz \left[\frac{F_{FT}(y,z)}{y-z} - \frac{(3y-z)\, G_{FT}(y,z)}{(y-z)^2}\right]\right)$$

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Fragmentation functions:

$$H(z) = 2\int_z^1 \frac{dy}{y}\int_y^\infty \frac{d\bar{y}}{\bar{y}^2}\left[\frac{(2(2/y-1/\bar{y})+(1/y)(1/y-1/\bar{y})\,\delta(1/y-1/z))\,\Im[\hat{H}_{FU}](y,\bar{y})}{(1/y-1/\bar{y})^2}\right]$$

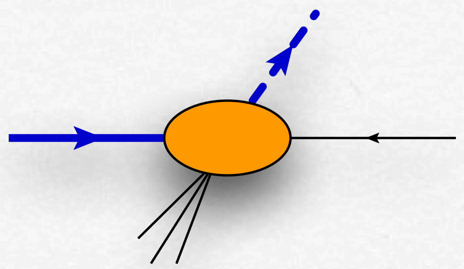
$$H_1^{\perp(1)}(z) = -\frac{2}{z}\int_z^1 \frac{dy}{y}\int_y^\infty \frac{d\bar{y}}{\bar{y}^2}\left[\frac{(2/y-1/\bar{y})\,\Im[\hat{H}_{FU}](y,\bar{y})}{(1/y-1/\bar{y})^2}\right]$$

also for polarized Fragmentation Functions (Λ - Production)…

Why are Lorentz-Invariance Relations important?

Example: Single-Inclusive Hadron Production in e-p collisions [e(l) + N(P) → h(P_h) + X]

theoretically simple, measured at HERMES, JLab 6



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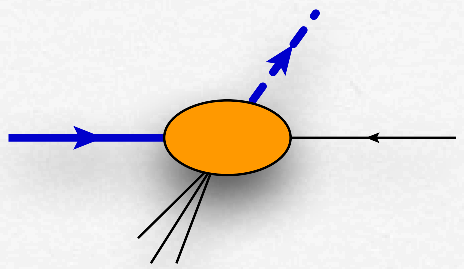

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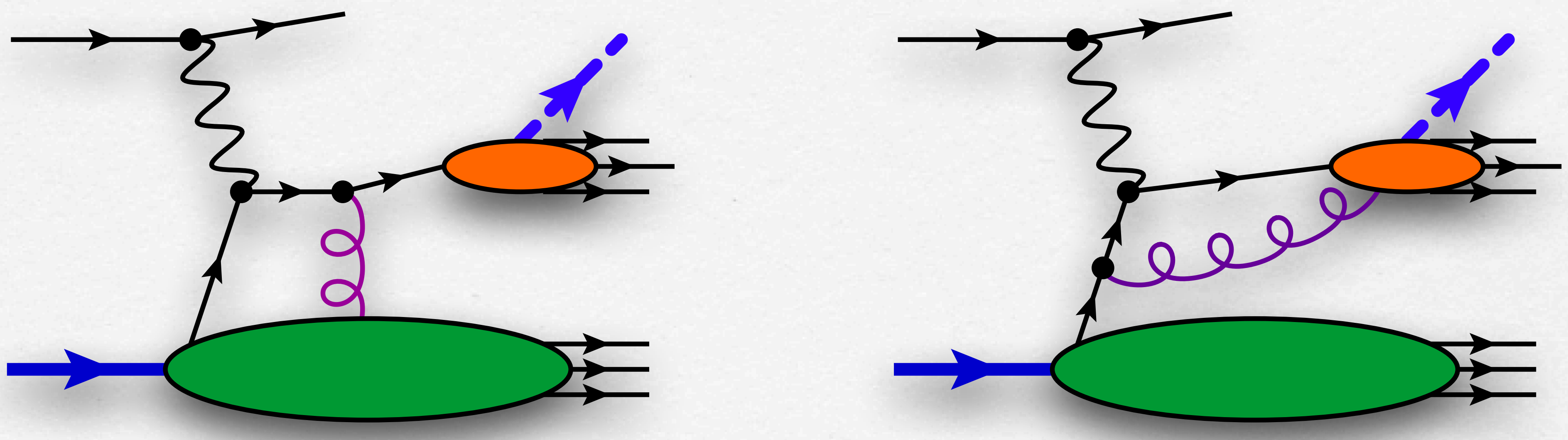
Transverse single spin asymmetry A_{UT} (LO):



$$A_{UT} \sim \left[\left(1 - x\frac{\mathrm{d}}{\mathrm{d}x}\right)F_{FT}(x,x)\otimes D_1(z)\otimes\hat{\sigma}_1\right]$$
$$+h_1(x)\otimes\left[H_1^{\perp(1)}(z)\otimes\hat{\sigma}_2 + \left(z\frac{\mathrm{d}}{\mathrm{d}z}\right)H_1^{\perp(1)}(z)\otimes\hat{\sigma}_3 + H(z)\otimes\hat{\sigma}_4 + \Im[\hat{H}_{FU}]\otimes\hat{\sigma}_5\right]$$

Partonic Coefficients differ in various frames (!?)

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S_{P\perp}^{\mu}\,P_{h\perp}^{\nu}\sum_q e_q^2\int_{z_{\min}}^{1}\frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$
$$+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\left\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right]\right.$$
$$\left.\left.+\frac{1}{z}H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^{\infty}\frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\right\}\right\}$$

e-p c.m. **frame**

$$\xi_z = z\left(\frac{1}{z}-\frac{1}{z_1}\right)$$

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$$\left.+M_h\,h_1^q(x)\left\{\frac{H^{h/q}(z)}{z}\left[\frac{\hat{s}(\hat{s}-\hat{u})}{\hat{t}^3}\right] + \hat{H}^{h/q}(z)\left[\frac{\hat{s}^2}{\hat{t}^3}\right] + z\frac{d\hat{H}^{h/q}(z)}{dz}\left[\frac{\hat{s}\hat{u}}{\hat{t}^3}\right]\right\}\right\}$$

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p-π c.m. frame

QCD - EoM already implemented, σ_i Lorentz invariant

Origin of the discrepancy:

Different choice of light-cone vector m in different frames

e-N c.m. frame:

$$m^\mu = \frac{1}{2E_h^2}\left(E_h, -\vec{P}_h\right)$$

N-π c.m. frame:

$$m^\mu = \frac{1}{2E_h}\left(1,0,0,1\right)$$

Twist-3 partonic coefficients do depend on m!

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Frame independent ansatz

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Re-evaluation of SSA with general light-cone vector:

$$A_{UT} = \ldots + \left[\frac{H(z)}{z} + \left(1 - z\frac{d}{dz}\right) H_1^{\perp(1)}(z) + \frac{2}{z}\int_z^\infty \frac{dz'}{z'^2}\, \frac{\Im[\hat{H}_{FU}](z,z')}{(1/z-1/z')^2}\right] (\eta\, a + \eta_\epsilon\, b)$$

= 0, LIR

Lorentz - Invariance relations remove spurious dependence on the choice of light cone vectors!

Summary & Conclusions

- Twist-3 formalism $\Longleftrightarrow$
  'intrinsic', 'kinematical' & 'dynamical' twist-3 contributions to transverse spin observables
- Relations through QCD equation of motion & Lorentz - Invariance relations
- Write 'intrinsic' & 'kinematical' distributions/FFs in terms of Quark-Gluon-Quark Correlations
- Lorentz - Invariance Relations remove dependence on the choice of light-cone vectors
- Conjecture: General behaviour, removal valid to all orders!

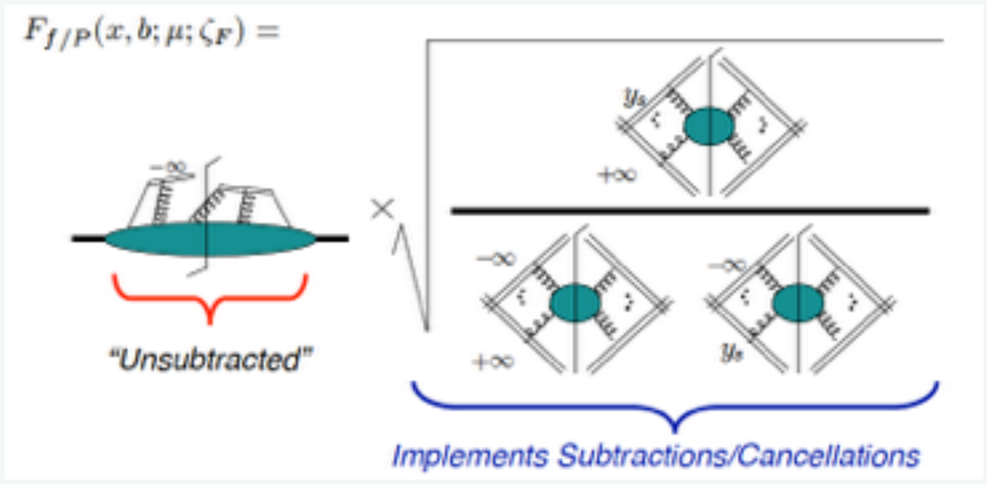
Relations in TMD factorization: Evolution

[Aybat, Collins, Qiu, Rogers; Collins' "Foundations of pQCD"; Ji, Ma, Yuan; Echevarria, Idilbi, Schimemi]

<u>Exact TMD definition beyond tree-level:</u>

1) Wilson lines are off the light cone
   - ➙ ξ regulates light cone divergences
   - ➙ "unsubtracted " TMD

2) "Soft factors" implemented



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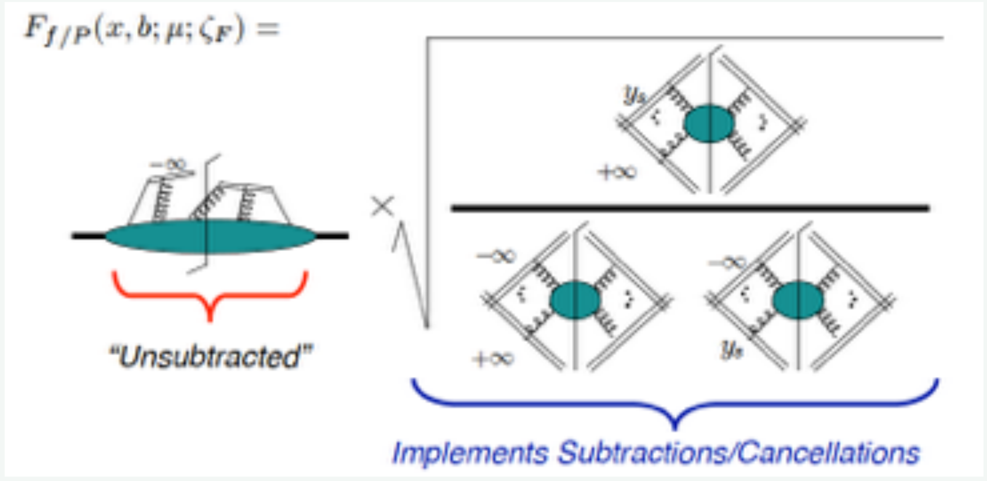

Solution of CS - evolution equation

$$f_1^q(x, \vec{b}_T^2; \mu; \xi) = \sum_{q'} \left(\tilde{C}_{qq'} \otimes q(x) \right) \Big|_{\mu \propto 1/b_*} e^{S_{\rm pert}(b_*)} \Big|_{\mu \propto 1/b_*} e^{g_q(x, b_T) + \frac{1}{2} g_K(b_T) \ln \frac{\xi}{\xi_0}}$$

TMD at large k_T perturbative Sudakov factor non-perturbative input

$$\frac{\partial}{\partial b_T} f_{1T}^{\perp}(x, b_T; \mu, \xi) \propto \left[\int_x^1 dz\, dz'\, \tilde{C}(z, z', \mu_b)\, F_{FT}(z, z', \mu_b) \right] e^{(S_{\rm pert} + S_{\rm non-pert})(b_T)}$$

F_{FT} drives x-dependence of the Sivers function