Parton Transverse Momentum and Orbital Angular Momentum

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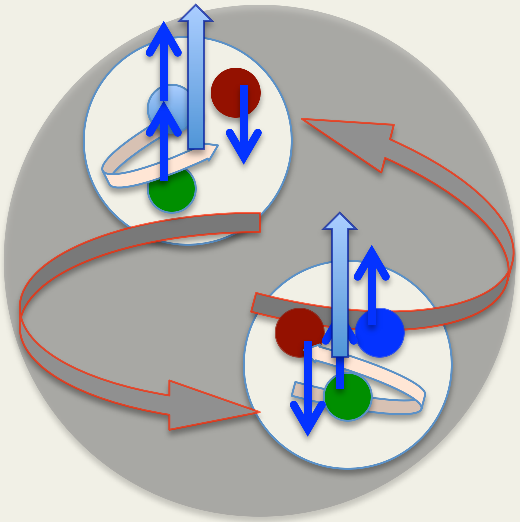

ECT* Workshop “Parton Transverse Momentum Distributions at Large x”, Trento, April 11-15, 2016

The end of the nucleon-spin crisis

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(Dated: March 21, 2016)

… already 1 citation

The end of WHAT nucleon-spin crisis?

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(Dated: April 6, 2016)

Maybe it is not a spin crisis…but there is a spin deficit…

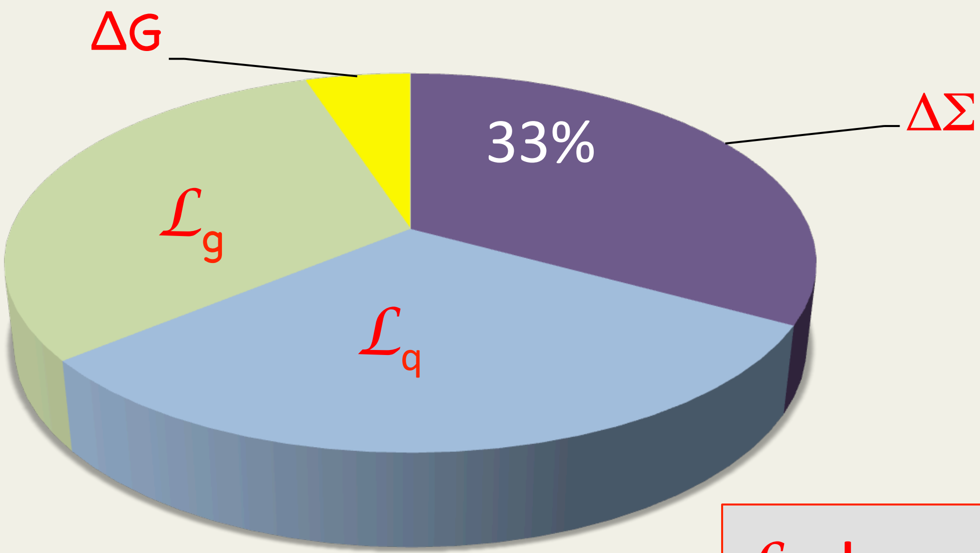
The spin crisis in a cartoon

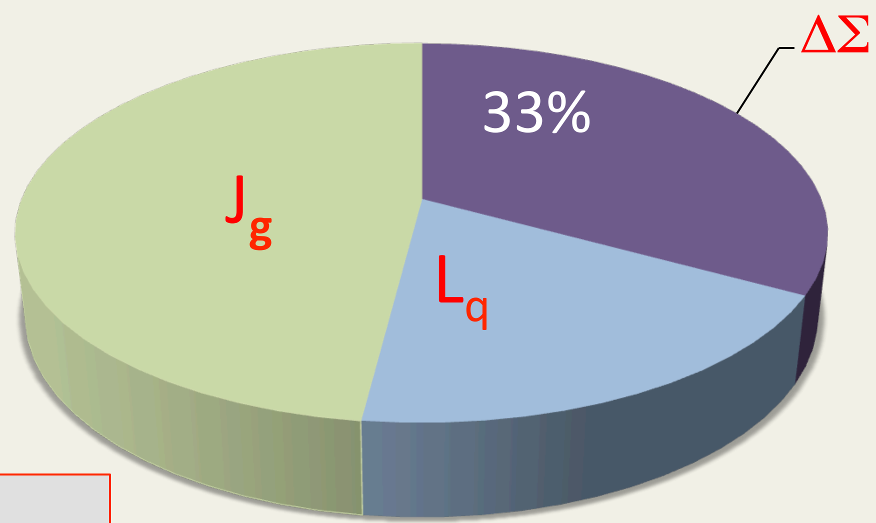
Jaffe Manohar

$$\boxed{\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \mathcal{L}_q + \Delta G + \mathcal{L}_g}$$

Ji

$$\boxed{\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g}$$

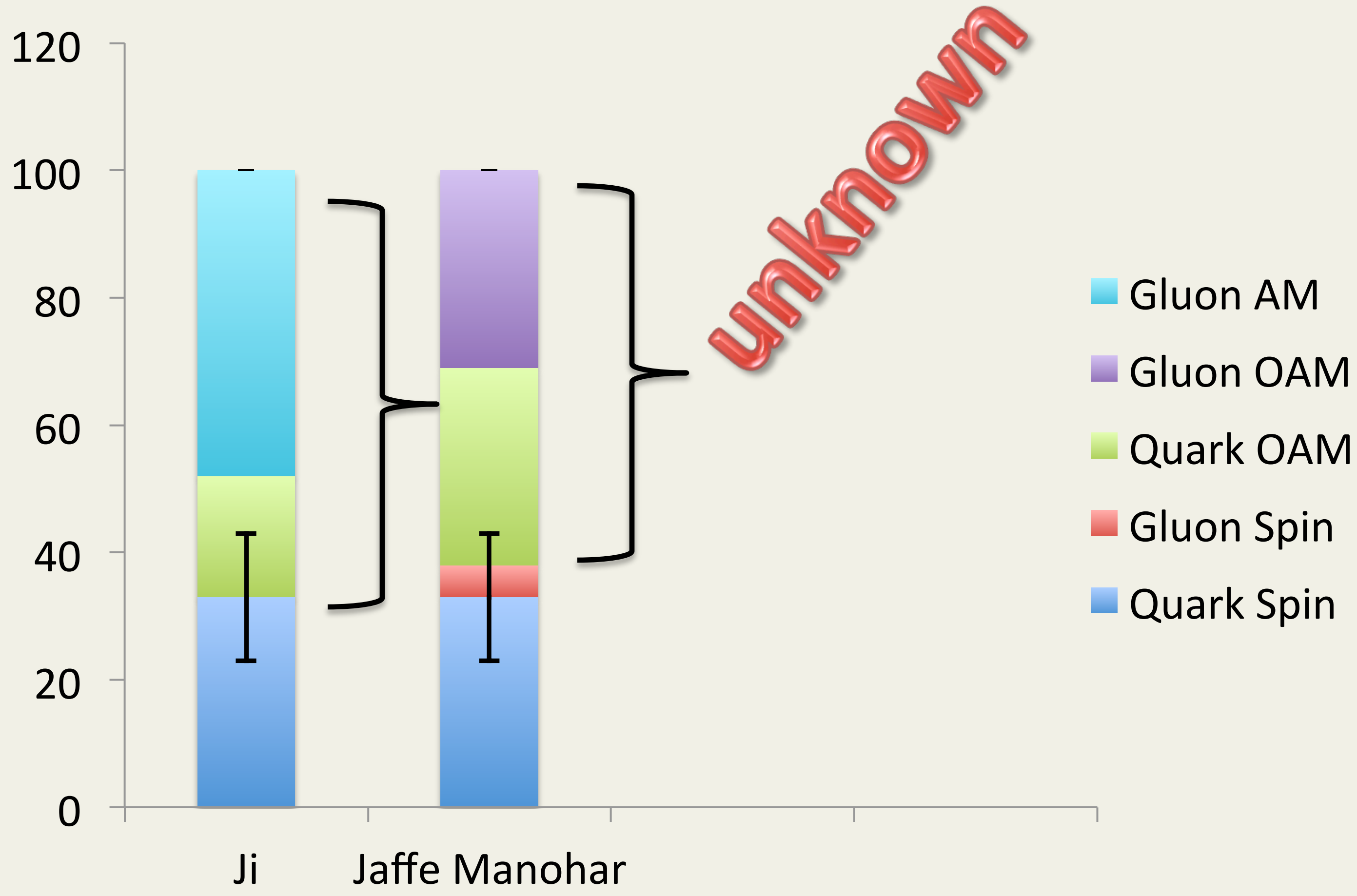




$\mathcal{L}_q \neq L_q$

$J_g \neq \mathcal{L}_g + \Delta G$

Angular Momentum Budget



- 1990 Jaffe and Manohar give for the first time a field theoretical description of the quark and gluon orbital angular momentum through its relation to the QCD energy momentum tensor➔ L_q, L_g

The QCD Energy Momentum Tensor

Energy density (mass)

Momentum density

Shear stress

Momentum density

T^{00}	T^{01}	T^{02}	T^{03}
T^{10}	T^{11}	T^{12}	T^{13}
T^{20}	T^{21}	T^{22}	T^{23}
T^{30}	T^{31}	T^{32}	T^{33}

Pressure

$$\boxed{T^{\mu\nu} = \frac{1}{4} i q \bar{\psi}\left(\gamma^{\mu}\overleftrightarrow{D}^{\nu} + \gamma^{\nu}\overleftrightarrow{D}^{\mu}\right)\psi + Tr\left\{F^{\mu\alpha}F^{\nu}_{\alpha} - \frac{1}{2} g^{\mu\nu} F^{2}\right\}} \rightarrow \boxed{M^{\mu\nu\lambda} = x^{\nu} T^{\mu\lambda} - x^{\lambda} T^{\mu\nu}}$$

Angular Momentum density

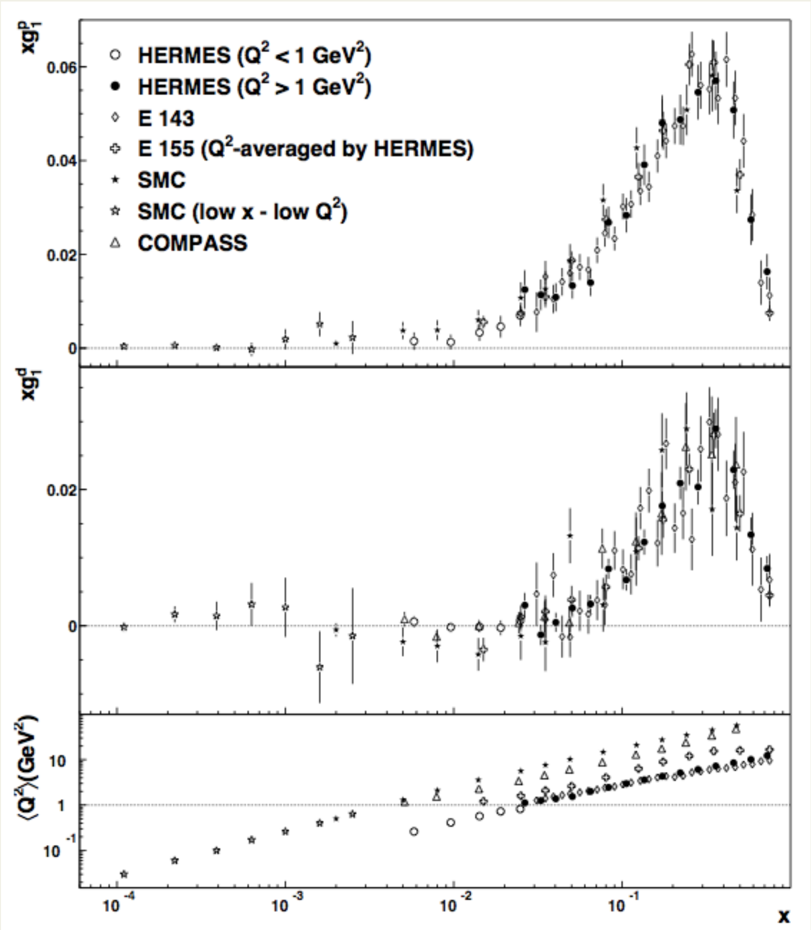
$$M^{+12} = \psi^\dagger \sigma^{12}\psi + \psi^\dagger\left[\vec{x}\times(-i\partial)\right]^3\psi + Tr(\varepsilon^{+-ij}F^{+j}A^j) + 2iTrF^{+j}(\vec{x}\times\partial)A^j$$

$\Delta\Sigma$ $\mathcal{L}_q$ ΔG $\mathcal{L}_g$

*Chen, Goldman et al., are consistent with this definition (see K.F.Liu et al.)

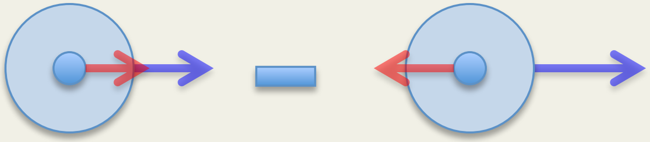
ΔΣ

$$\langle p,S|\bar{\psi}\gamma^{o}\sigma_{12}\psi|p,S\rangle \to \langle p,S_{\parallel}|\bar{\psi}\gamma^{+}\gamma_{5}\psi|p,S_{\parallel}\rangle$$



$$g_1(x) = \int \frac{dz^-}{4\pi} e^{ixz^-P^+} \langle p,S_{\parallel}|\bar{\psi}(0)\gamma^{+}\gamma_{5}\psi(z)|p,S_{\parallel}\rangle$$

$$g_1(x) = \frac{1}{2}\Sigma_q e_q^2(\Delta q(x) + \Delta\bar{q}(x)) = \frac{1}{2}\Delta\Sigma$$



Helicity Distribution

- 1990 Jaffe and Manohar discuss for the first time a field theoretical description of the quark gluon orbital angular momentum through its relation to the QCD energy momentum tensor→ L_q, L_g

- 1997 Ji and Radyushkin, independently, introduce GPDs as a tool to access angular momentum in electroproduction experiments → J_q, J_g

Jaffe Manohar:

$$M^{+12} = \psi^{\dagger}\sigma^{12}\psi + \psi^{\dagger}\left[\vec{x}\times(-i\partial)\right]^{3}\psi + Tr(\varepsilon^{+-ij}F^{+j}A^{j}) + 2i\,Tr F^{+j}(\vec{x}\times\partial)A^{j}$$

$\Delta\Sigma$ $\mathcal{L}_q$ ΔG $\mathcal{L}_g$

Ji:

$$M^{+12} = \psi^{\dagger}\sigma^{12}\psi + \psi^{\dagger}\left[\vec{x}\times\left(-i\vec{D}\right)\right]^{3}\psi + \left[\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right]^{3}$$

$\Delta\Sigma$ L_q J_g

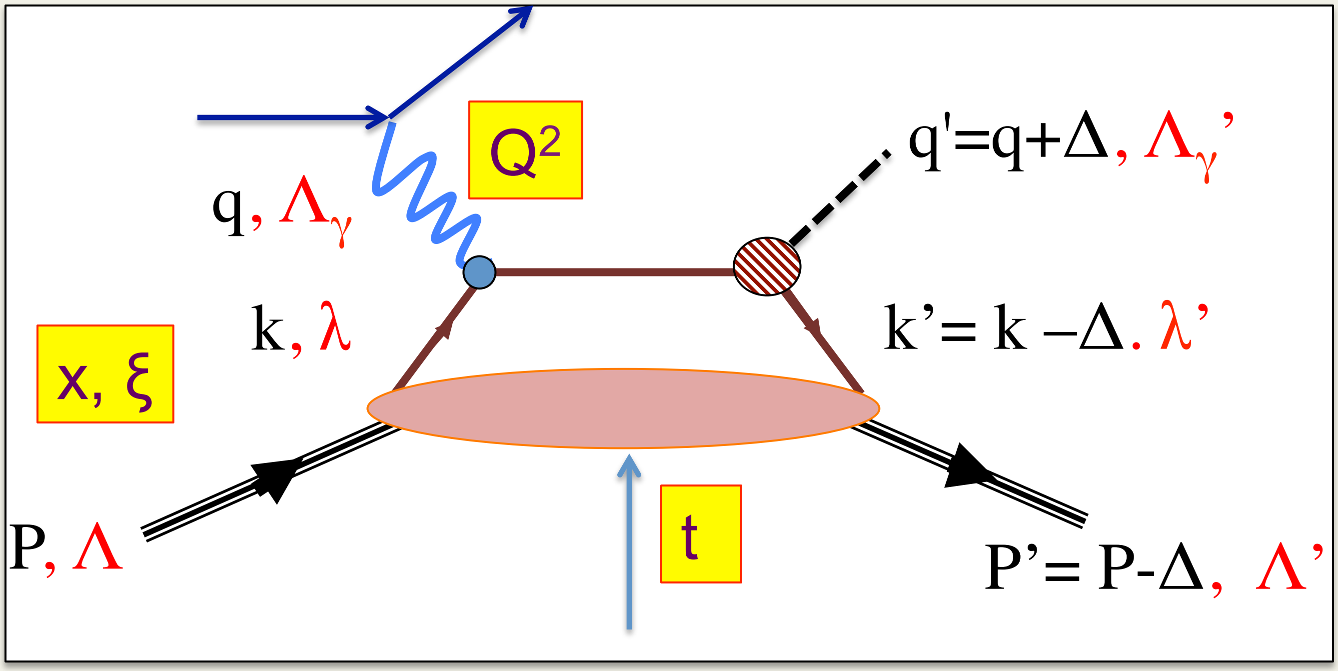
*Chen, Goldman et al., are consistent with this definition (see K.F.Liu et al.)

$$\boxed{\frac{1}{2}\sum_{i=q,g}\int dx\, x\left(H_i + E_i\right) = J_q + J_g = \frac{1}{2}}$$

Ji (1997)

J is measured through DVCS and DVMP (and TCS, DDVCS, …)

$$\gamma^* p \to \gamma(M) p'$$

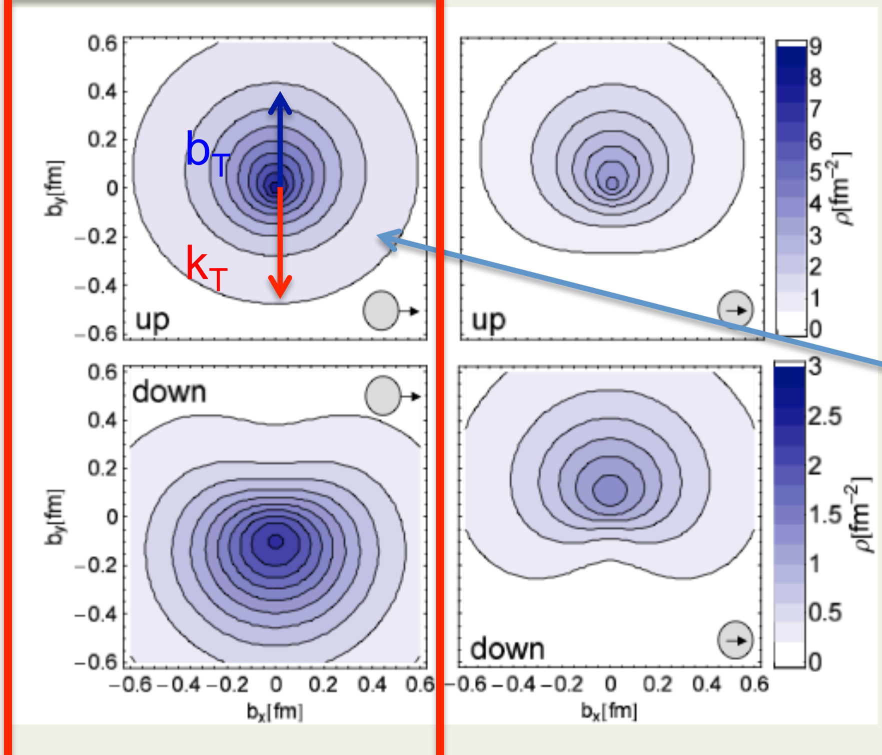


One needs to measure both H and E similarly to how we measure both the Dirac and Pauli form factors to obtain the anomalous magnetic moment: indirect signal of the existence of OAM

$$F^{[\gamma^+]}_{\Lambda\Lambda'} = \frac{1}{2P^+}\bar{U}(p,\Lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+j}\Delta_j}{2M}E(x,\xi,t)\right]U(p,\Lambda)$$

P. Haegler et al.

S_T



E $\rightarrow$ $\boxed{\sigma^{+j}\Delta_j \Rightarrow \vec{S}_T \times \vec{\Delta}}$

Lensing Mechanism (Burkardt)

b_T corresponds to net k_T in the opposite direction (attractive color force due to FSI)

Helicity Structure of E

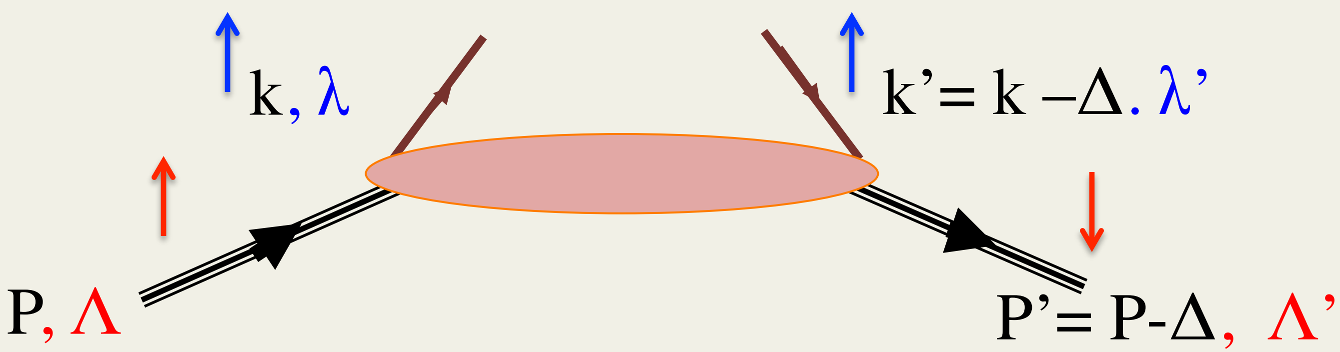
$$\left(A_{++,++}+A_{+-,+-}+A_{-+,-+}+A_{--,--}\right)+\left(A_{++,-+}+A_{+-,--}-A_{--,+-}-A_{-+,++}\right)$$ Helicity basis (flip)

$$\left(A^X_{++,++}+A^X_{+-,+-}+A^X_{-+,-+}+A^X_{--,--}\right)+\left(A^X_{++,++}+A^X_{+-,+-}-A^X_{-+,-+}-A^X_{--,--}\right)$$ Transversity basis (non flip)

$$\approx H-i\Delta_2 E$$

The GPD E is diagonal in the transversity basis: <u>it measures J and not L, but a change of one unit of L,</u> because of the spin flip

$$S_z=-1/2\rightarrow 1/2 \Rightarrow \Delta L_z=1$$ at fixed J



Brodsky and Drell '80s, Belitsky, Ji and Yuan, '90's

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OAM is measured through a Wigner Distribution

OAM represents the correlation between the position and momentum of the quarks and gluons

$$\mathcal{L}_q, L_q = \int dx\, d^2b\, d^2k_T \left(\vec{b}_T \times \vec{k}_T\right)_3 \mathcal{W}^{\mathcal{U}}\left(x, \vec{b}_T, \vec{k}_T\right)$$

Hatta (2011)
Lorce, Pasquini (2011)

Generalized Transverse Momentum Distribution (GTMD)

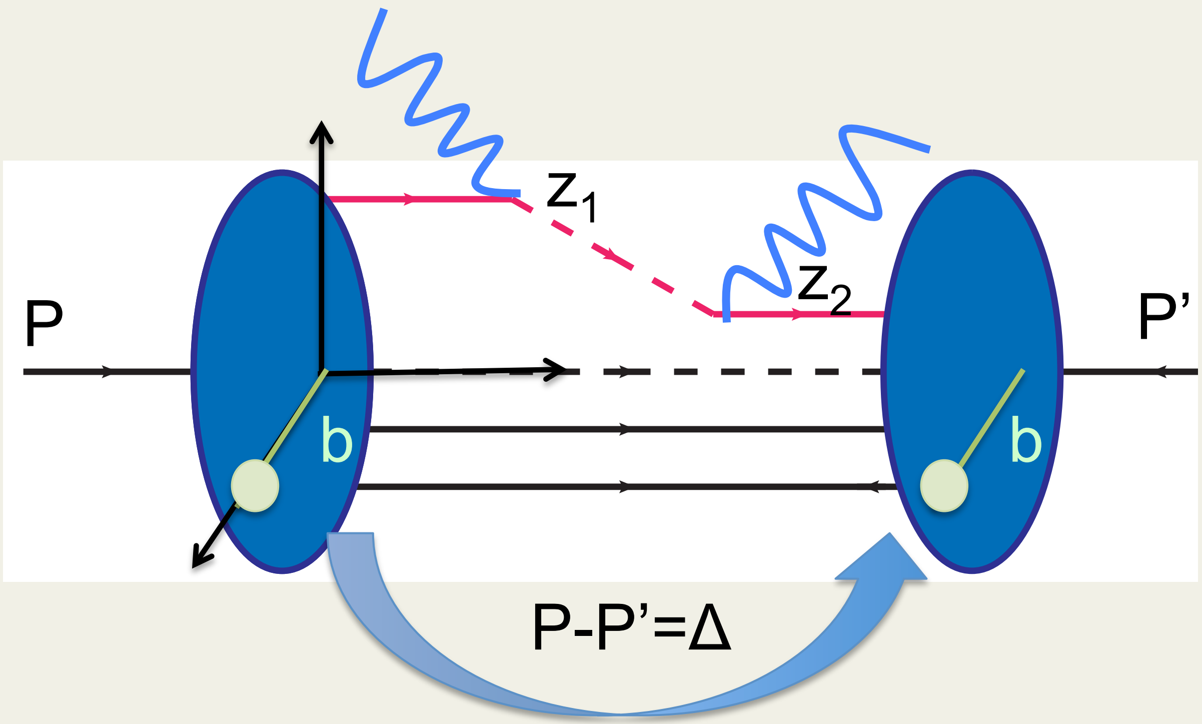
Wigner Distribution

$$\mathcal{W}^{\mathcal{U}}\left(x, \vec{b}_T, \vec{k}_T\right) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\Delta_T \cdot b_T} \int \frac{d^2 z_T d^2 z^-}{(2\pi)^3} e^{i(xP^+ z^- - k_T \cdot z_T)} \left\langle P', \Lambda' \middle| \bar{\psi}(0)\gamma^+ \mathcal{U}(0,\infty \mid n)\psi(z) \middle| P, \Lambda \right\rangle \Big|_{z^+=0}$$

Hatta (2011)
Burkardt(2013)

Wilson-line gauge link, "n" defines the path along which we evaluate the vector potential

Space-time picture of the unintegrated, off-forward correlation function



$$\langle P-\Delta,\Lambda' \mid \bar{q}(0)\gamma^{+}\mathcal{U}(0,z)q(z^{-}) \mid P,\Lambda\rangle_{\mathbf{z}_T=0}$$

The Wigner distribution defining OAM through a (b_T x k_T) term is the Fourier transform of a GTMD called F_{14} with coefficient (Δ_T x k_T)

(Lorce, Pasquini, 2011; Meissner, Metz, Schlegel, 2009)

$$W^{\gamma^+}_{\Lambda\Lambda'} = \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\gamma^+ F_{11} + \frac{i\sigma^{i+}\Delta^i_T}{2M}(2F_{13}-F_{11}) + \frac{i\sigma^{i+}\bar{k}^i_T}{2M}(2F_{12}) + \frac{i\sigma^{ij}\bar{k}^i_T\Delta^j_T}{M^2}F_{14}\right]U(p,\Lambda)$$

$$= \delta_{\Lambda,\Lambda'}F_{11} + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\Delta_1 - i\Delta_2}{2M}(2F_{13}-F_{11}) + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\bar{k}_1 - i\bar{k}_2}{2M}(2F_{12}) + \delta_{\Lambda,\Lambda'}i\Lambda\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14}$$

$\boxed{\text{helicity non-flip}}$

$$L_q = -\int_{-1}^{1} dx \int d^2 k_T \, \frac{k_T^2}{M^2} F_{14}(x, 0, k_T, 0, 0) = -F_{14}^{(1)}$$

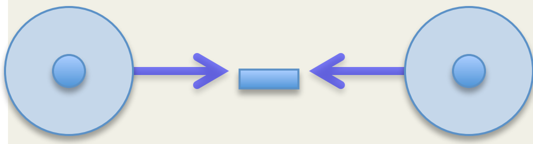
Lorce&Pasquini
Hatta
Meissner, Metz and Schlegel

Can be observed in A_{UL} asymmetry

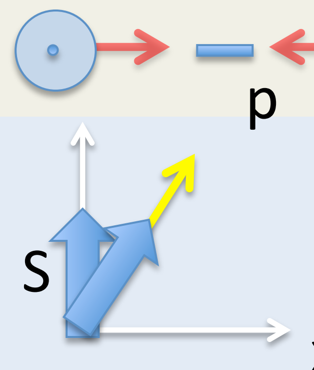
A.Courtoy, G.Goldstein, O.Gonzalez Hernandez, S.Liuti and A.Rajan, PLB 731(2014)

$$i\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14} = \underline{A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--}}$$

$$-i\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}G_{11} = A_{++,++} - A_{+-,+-} + A_{-+,-+} - A_{--,--}$$



UL correlation



F_{14} is Parity even: its matrix element transforms like:

$$S_z = p_z \Lambda$$

it can therefore in principle represent OAM ➔ there is no Parity violation at this level

However, because F_{14} transforms opposite to helicity, one cannot find a combination of twist 2 helicity amplitudes to measure it if we stick to one hadronic plane!!

- 1990 Jaffe and Manohar discuss for the first time a field theoretical description of the quark gluon orbital angular momentum through its relation to the QCD energy momentum tensor → L_q, L_g

- 1997 Ji and Radyushkin, independently, introduce GPDs as tool to access angular momentum in electroproduction experiments → J_q, J_g

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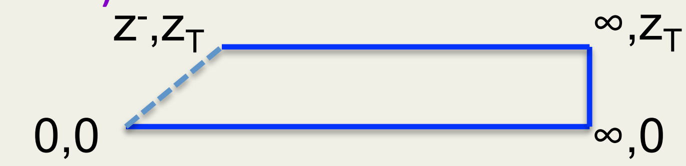
Insert gauge links (Hatta Yoshida, 2012; Burkardt, 2013)

$\mathcal{L}_q^{\mathrm{JM}}$ ➔ "staple" gauge link,

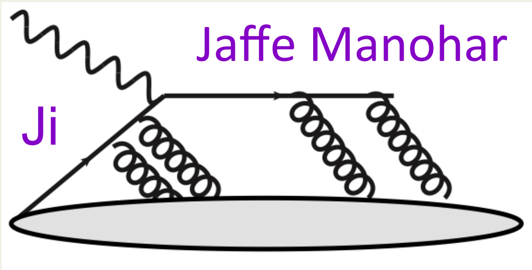


L_q^{Ji} ➔ "straight" gauge link

The difference of the two gives a <u>torque,</u>

$$\boxed{\mathcal{L}_q^{JM} - L_q^{Ji} = \int \frac{d^2 z_T dz^-}{(2\pi)^3} \langle P',\Lambda' | \bar{\psi}(z)\gamma^+(-g)\int_{z^-}^{\infty} dy^- U \underbrace{\left[z_1 G^{+2}(y^-) - z_2 G^{+1}(y^-) \right]} U \psi(z) | P,\Lambda\rangle \Big|_{z^+=0}}$$

$\vec{\tau} = \vec{r} \times \vec{F}$ ➔ "Chromodynamic torque"*



*a Qiu-Sterman term type term analogous to f_{1T}^{perp}

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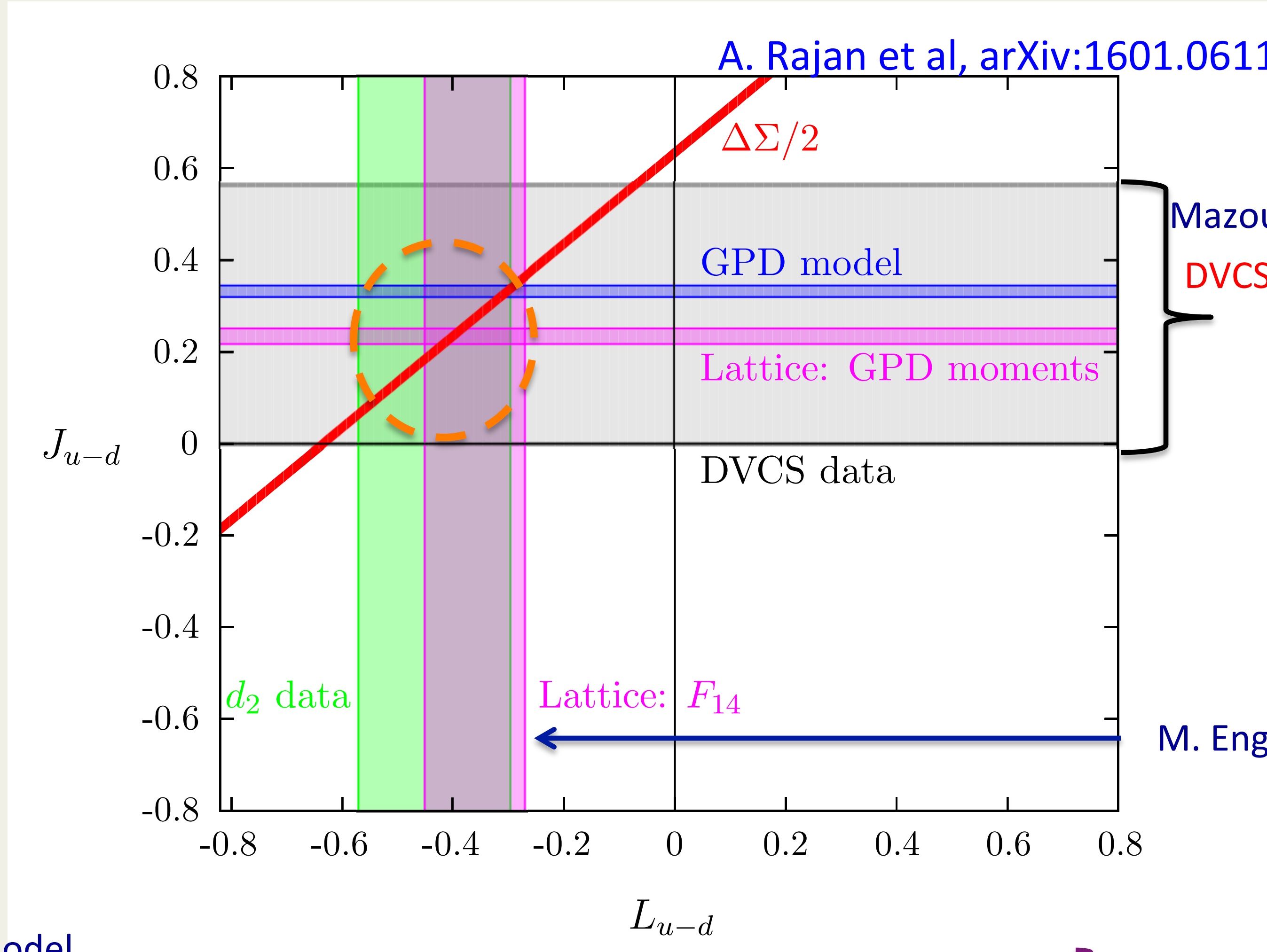
- 2012 Burkardt connects the Jaffe Manohar and Ji orbital angular momenta
- 2016 Rajan, Courtoy, Engelhardt, S.L. connect GTMD to twist 3 GPD

Ji's Sum Rule: $J_q = L_q + \frac{1}{2}\Delta\Sigma_q$ ➔ three independently measured quantities



Mazous et al. PRL (2007)

DVCS + VGG

M. Engelhardt

GPD model

- O. Gonzalez Hernandez et al., Phys. Rev. C88; arXiv:1206.1876
- M. Diehl and P. Kroll, Eur. Phys. J. C73; arXiv:1302.4604

Using flavor separated nucleon form factors from Jlab

$$\boxed{\Phi^{\mathcal{U}} = \int \frac{d^4 z}{(2\pi)^4} e^{i(k\cdot z)} \langle P',\Lambda' | \bar{\psi}(0)\gamma^+ \mathcal{U}(0,\infty | n)\psi(z) | P,\Lambda\rangle}$$

The completely unintegrated correlation function can be parametrized in terms of invariant functions A_i (Goeke, Metz and Schlegel (2005), Meissner, Metz and Schlegel (2009))

$\boxed{\tilde{\Phi}^{\mathcal{U}} = \int dk^- \Phi^{\mathcal{U}}}$ is parametrized in terms of GTMDs

One finds the following relations

$$\frac{k_T\cdot\Delta_T}{\Delta_T^2}F_{12} + F_{13} = 2P^+ \int dk^- \left(\frac{k_T\cdot\Delta_T}{\Delta_T^2}A_5 + A_6 - \frac{xP^2 - k\cdot P}{M^2}(A_8 + xA_9) \right)$$

$$F_{14} = 2P^+ \int dk^- \,(A_8 + xA_9)$$

$$\frac{k_T\cdot\Delta_T}{\Delta_T^2}F_{27} + F_{28} = 2P^+ \int dk^- \left(\frac{k_T\cdot\Delta_T}{\Delta_T^2}A_5 + A_6 + \frac{1}{M^2}\left(\frac{(k_T\cdot\Delta_T)^2}{\Delta_T^2} - k_T^2 \right) A_9 \right)$$

Combining integrals over k_T of these relations, one arrives at the Lorentz Invariance Relation (LIR)

$$\frac{d}{dx} \int d^2 k_T \, \frac{k_T^2}{M^2} \, F_{14} = \widetilde{E}_{2T} + H + E$$

k_T moment of a GTMD

twist 3 GPD

(we have kept to a straight gauge link)

Twist three Generalized Parton Distributions

$$W^{\gamma^i}_{\Lambda'\Lambda} = \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\frac{\Delta^i_T}{M}G_1 + \frac{i\sigma^{ji}\Delta_j}{M}G_2 + \frac{Mi\sigma^{i+}}{P^+}G_4 + \frac{\Delta^i_T}{P^+}\gamma^+ G_3\right]U(p,\Lambda),$$

Polyakov et al. (2000), Meissner, Metz and Schlegel (2009), Hatta&Yoshida (2012)

$$G_2 \rightarrow \tilde{E}_{2T} + H + E$$

Polyakov et al. Meissner, Metz and Schlegel, JHEP(2009)

Sum rules relating GTMD and twist 3 GPD descriptions

Lorentz Invariant Relation (LIR) A. Rajan, A. Courtoy, M. Engelhardt, S. L.

$$F_{14}^{(1)} = -\int_x^1 dy\, (\tilde{E}_{2T} + H + E) \;\; \Rightarrow -L_q = \int_0^1 dx F_{14}^{(1)} = \int_0^1 dx x G_2$$

Equations of Motion (EoM) relation

$$x(\widetilde{E}_{2T} + H + E) = \quad x\left[(H+E) - \int_x^1 \frac{dy}{y}(H+E) - \frac{1}{x}\widetilde{H} + \int_x^1 \frac{dy}{y^2}\,\widetilde{H} \right] + G^{(3)}$$

L J -S

In Ji's picture F_{14} and $\bar{E}_{2T}$ give us similar information on OAM!

Genuine twist three term: accounting for gauge links

$$\boxed{\begin{aligned} G^{(3)} &= -\widetilde{\mathcal{M}} + x\int_x^1 \frac{dy}{y^2}\widetilde{\mathcal{M}} \\ \widetilde{\mathcal{M}} &= 2M\frac{\Delta_T^i}{\Delta_T^2}\int d^2k_T\left[\mathcal{M}^i_{++} - \mathcal{M}^i_{--}\right].\end{aligned}}$$

$$\mathcal{M}^i_{\Lambda\Lambda'} = \frac{1}{4}\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T}$$

$$\langle p', \Lambda' \mid \overline{\psi}(-z/2)\boxed{\left[(\overrightarrow{\partial} - ig\not{A})\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\partial} + ig\not{A})\Big|_{z/2}\right]}\psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

For a straight link:

$$[\ldots]_{z\to 0} = 0$$

For a staple link:

$$[\ldots]_{z\to 0} = 2i\epsilon_{ij}\int_0^1 ds\,\mathcal{U}_{0\to s}v^-\gamma^+F^{+j}(z+sv)\mathcal{U}_{s\to 0}$$

✓ Similar to the Wandzura Wilczek relation for g_2

$$\underbrace{g_2(x)}_{t=3} = \underbrace{-g_1(x) + \int_x^1 \frac{dy}{y} g_1(x)}_{g_2^{WW} \rightarrow \tau = 2} + \underbrace{\left[\bar{g}_2^{tw3} - \int_x^1 \frac{dy}{y} \bar{g}_2^{tw3} \right]}_{\tau = 3}$$

genuine
twist three

How does one measure OAM?

How does one access information on Wigner distributions?

Parity predicts a lack of UL correlation at twist 2

TMDs

	U	T_x	T_y	L
U	f_1	$-i\frac{k_y}{M}\, h_1^\perp$	$i\frac{k_x}{M}\, h_1^\perp$	✖
T_x	$-i\frac{k_y}{M}\, f_{1T}$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2}\, h_{1T}^\perp$	$\frac{k_x k_y}{M^2}\, h_{1T}^\perp$	$\frac{k_x}{M}\, g_{1T}$
T_y	$i\frac{k_x}{M}\, f_{1T}$	$\frac{k_x k_y}{M^2}\, h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2}\, h_{1T}^\perp$	$\frac{k_y}{M}\, g_{1T}$
L	✖	$\frac{k_x}{M}\, h_{1L}^\perp$	$\frac{k_y}{M}\, h_{1L}^\perp$	g_{1L}

GPDs

	U	T_x	T_y	L
U	H	$i\,\frac{\Delta_y}{2M}\,(2\tilde{H}_T + E_T)$	$-i\,\frac{\Delta_x}{2M}\,(2\tilde{H}_T + E_T)$	✖
T_x	$i\,\frac{\Delta_y}{2M}\,E$	$H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2}\,\tilde{H}_T$	$-\frac{\Delta_x \Delta_y}{2M^2}\,\tilde{H}_T$	
T_y	$-i\,\frac{\Delta_x}{2M}\,E$	$-\frac{\Delta_x \Delta_y}{2M^2}\,\tilde{H}_T$	$H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2}\,\tilde{H}_T$	
L	✖			$\tilde{H}$

✓The combination of helicity amplitudes will cancel unless they are imaginary:

$$\boxed{A_{++,++} = A^*_{--,--}\ ;\ A_{+-,+-} = A^*_{-+,-+}}$$

✓ If the scattering is in one single hadronic plane there can be no relative phase between helicity amps

✓Different from GPD E where the amplitude's phase is a consequence of helicity flip and off-forward spinor rotation

Observability: Cross section and asymmetries

$$\frac{d^4\sigma}{dx_{Bj}dyd\phi dt} = \Gamma\Big\{\Big[F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{2\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi} + h\,\boxed{\sqrt{2\epsilon(1-\epsilon)}\,\sin\phi F_{LU}^{\sin\phi}}\Big]$$
$$+\; S_{||}\Big[\boxed{\sqrt{2\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi}} + h\,\boxed{\Big(\sqrt{1-\epsilon^2}\,F_{LL} + \sqrt{2\epsilon(1-\epsilon)}\,\cos\phi\,F_{LL}^{\cos\phi}\Big)}\Big]$$
$$+\; S_\perp\Big[\sin(\phi-\phi_S)\Big(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\Big) + \epsilon\Big(\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)}\Big)$$
$$+\; \sqrt{2\epsilon(1+\epsilon)}\Big(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\Big)\Big]$$
$$+\; S_\perp h\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\Big(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big)\Big]\Big\}$$

$$A_{LU} = \sqrt{\epsilon(1-\epsilon)}\,\frac{F_{LU}^{\sin\phi}}{F_{UU,T}+\epsilon F_{UU,L}}$$

$$A_{UL} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}}{F_{UU,T}+\epsilon\, F_{UU,L}} + \frac{\epsilon\sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T}+\epsilon\, F_{UU,L}}$$

Twist 3!

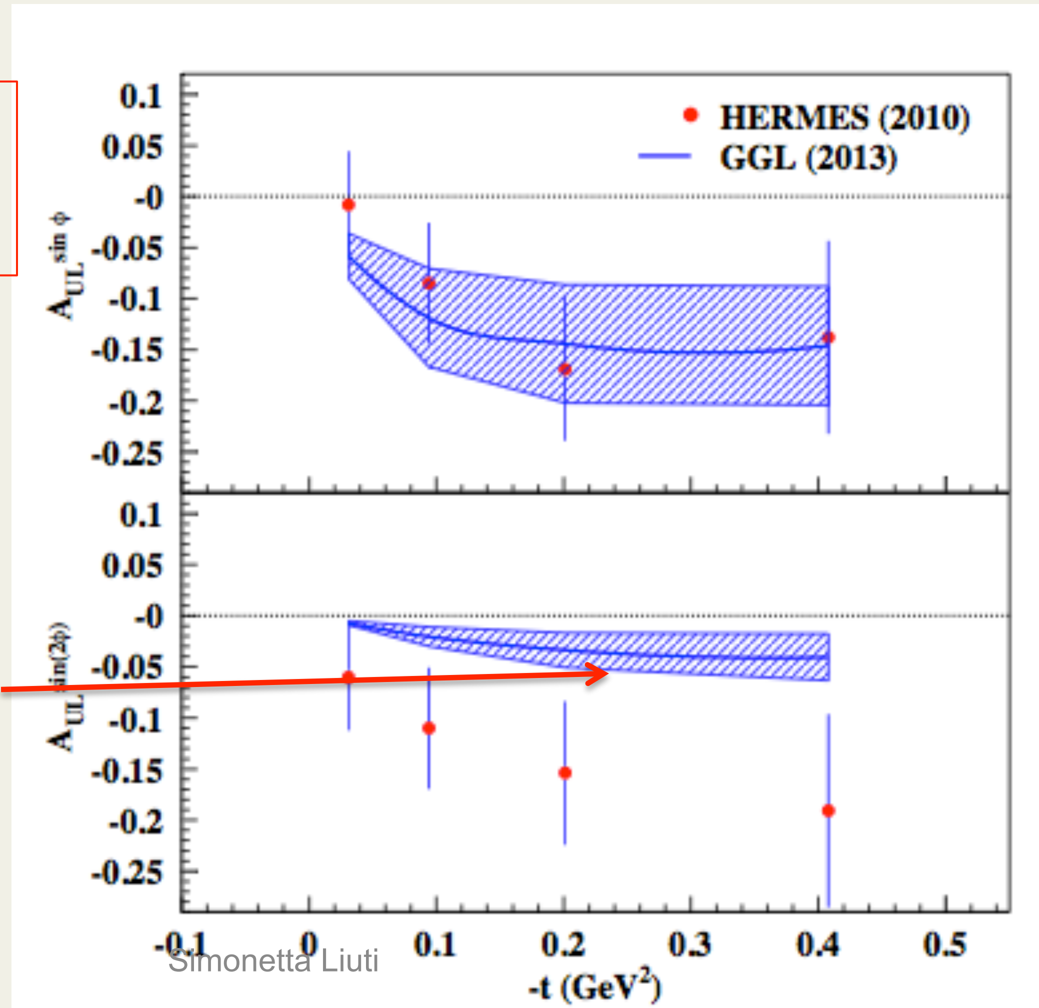
$$A_{LL} = \frac{N^{\rightarrow}_{s_z=+} - N^{\rightarrow}_{s_z=-} + N^{\leftarrow}_{s_z=+} - N^{\leftarrow}_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{1-\epsilon^2}\;F_{LL}}{F_{UU,T}+\epsilon F_{UU,L}} + \frac{\sqrt{\epsilon(1-\epsilon)}\,\cos\phi\,F_{LL}^{\cos\phi}}{F_{UU,T}+\epsilon F_{UU,L}}$$

Direct measurement of OAM via G_2: DVCS on a longitudinally polarized target

$$A_{UL,L} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{2\epsilon(\epsilon+1)}\,\sin\phi\, F_{UL}^{\sin\phi}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\epsilon \sin 2\phi\, \boxed{F_{UL}^{\sin 2\phi}}}{F_{UU,T} + \epsilon F_{UU,L}}$$

Hall B at 6 GeV does not have enough bins in ϕ
(Avakian, Pisano)

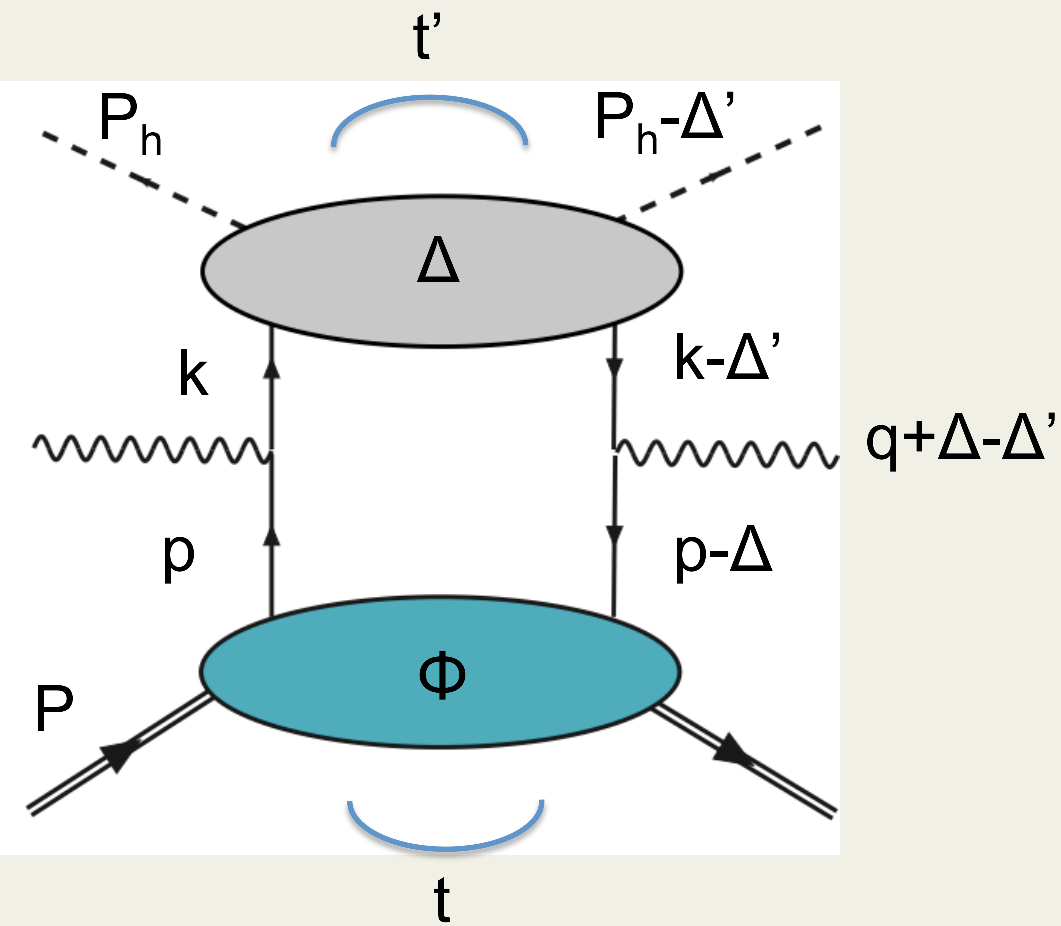
WW, small ξ



GTMDs from “off-forward SIDIS”

F_{14}

ep⟼e’p’ γ π π



t,t’,P_h^2<Q^2

$$g_{\Lambda'_\gamma,\Lambda'_N,0;\Lambda_\gamma,\Lambda_N,0} = \sum_{\lambda,\lambda'} \tilde{g}^{\lambda'\lambda}_{\Lambda'_\gamma\Lambda_\gamma} \otimes \underbrace{A_{\Lambda'_N,\lambda',\Lambda_N,\lambda}(x,\xi,t)}_{\Phi} \otimes \underbrace{F^{\pi_1}_{\lambda 0}(z) F^{\pi_2}_{\lambda' 0}(v)}_{\Delta}$$

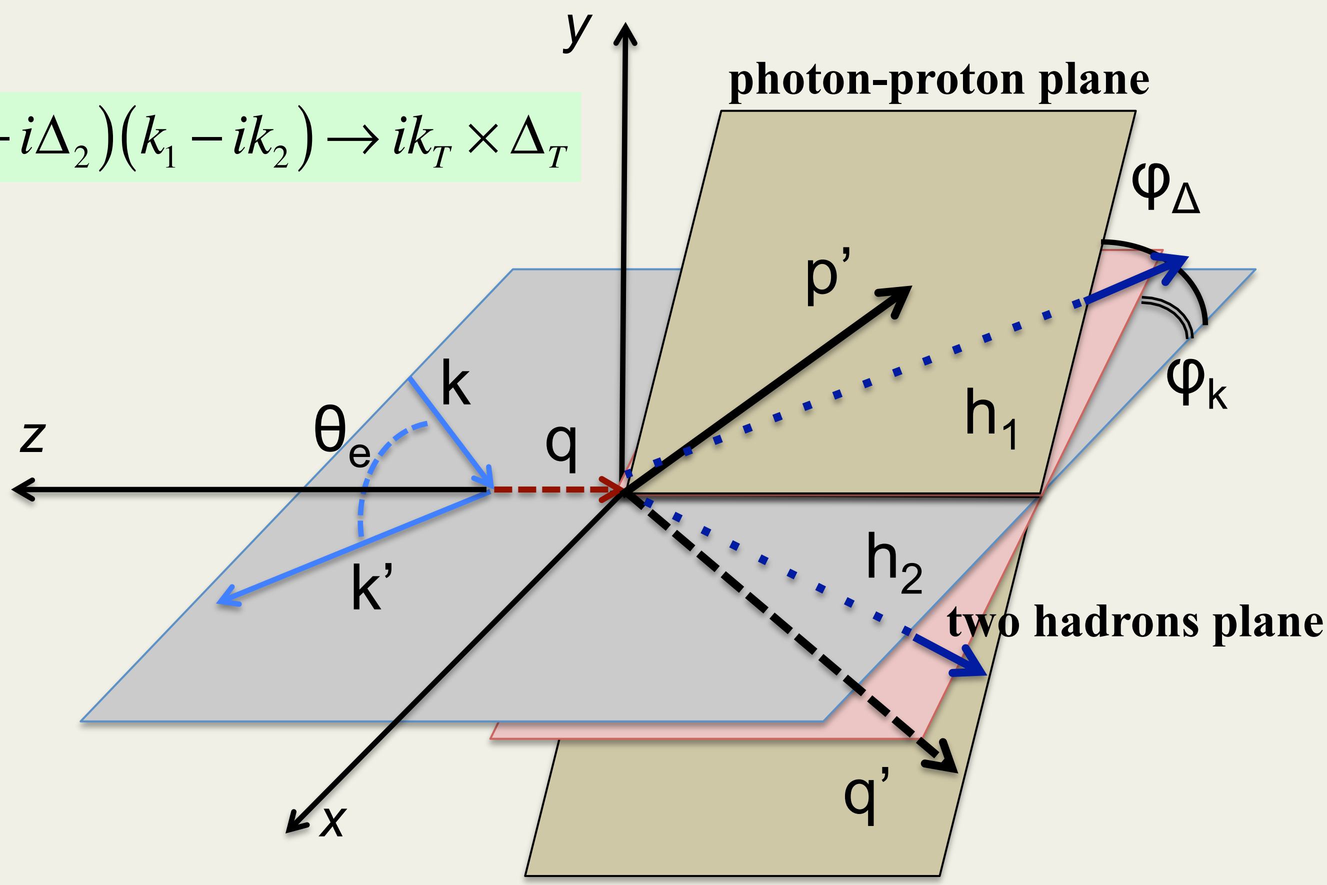
Off forward SIDIS

- To measure F_{14} one has to be in a frame where the reaction cannot be viewed as a two-body quark-proton scattering

- In the CoM the amplitudes are imaginary ➔UL connection goes to 0

- The way to accomplish this is to define two planes

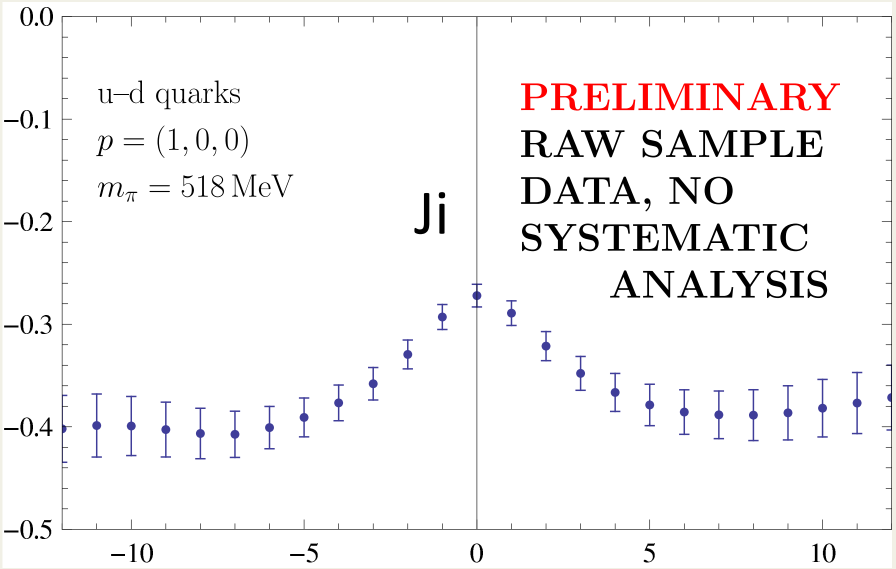
$$|\Delta|e^{i\varphi_\Delta}|k_T|e^{i\varphi_k}(\Delta_1 - i\Delta_2)(k_1 - ik_2) \rightarrow ik_T \times \Delta_T$$



F_{14} is not at reach experimentally but it can be calculated on the lattice: first direct determination of OAM on lattice (M. Engelhardt)

Quark orbital angular momentum in units of the number of valence quarks

$$\frac{L_z}{n} = \frac{2\epsilon_{ij}\frac{\partial}{\partial b_{T,i}}\frac{\partial}{\partial q_{T,j}} \langle P,S \mid \overline{\psi}(0)\gamma^+\mathcal{U}[0,b]\psi(b) \mid P',S\rangle\big|_{b^+=b^-=0,\; q_T=0,\; b_T\to 0}}{\langle P,S \mid \overline{\psi}(0)\gamma^+\mathcal{U}[0,b]\psi(b) \mid P',S\rangle\big|_{b^+=b^-=0,\; q_T=0,\; b_T\to 0}}$$



Jaffe Manohar

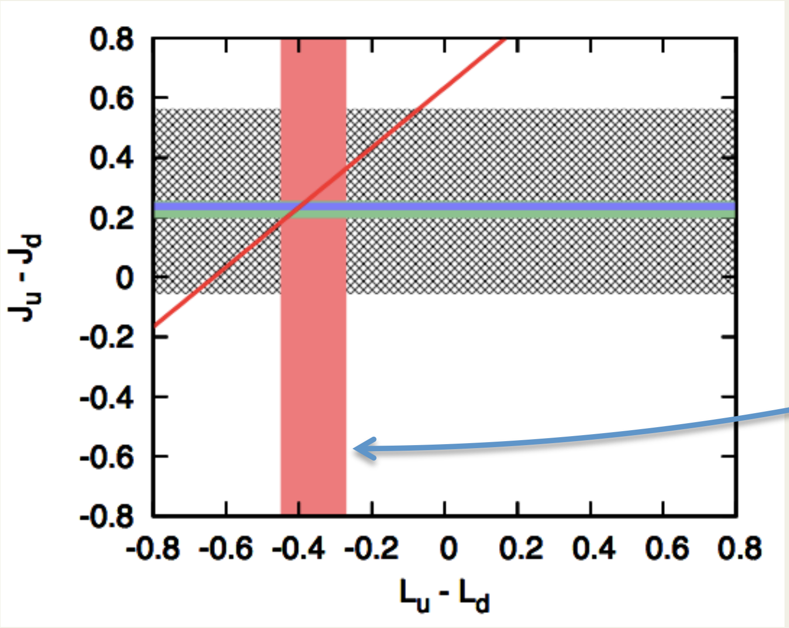
Connection with $g_{2,}$, d_2

$$\underbrace{g_2(x)}_{t=3} = \underbrace{-g_1(x) + \int_x^1 \frac{dy}{y} g_1(x)}_{g_2^{WW} \to \tau=2} + \underbrace{\left[\bar{g}_2^{tw3} - \int_x^1 \frac{dy}{y} \bar{g}_2^{tw3}\right]}_{\tau=3}$$

$$d_2 = 2\int dx\, x^2 g_1(x) + 3\int dx\, x^2 g_2(x)$$

$$d_2 = 2\int dx\, x^2\,(H(x) + E(x)) + 3\int dx\, x^2\,\tilde{E}_{2T}(x)$$

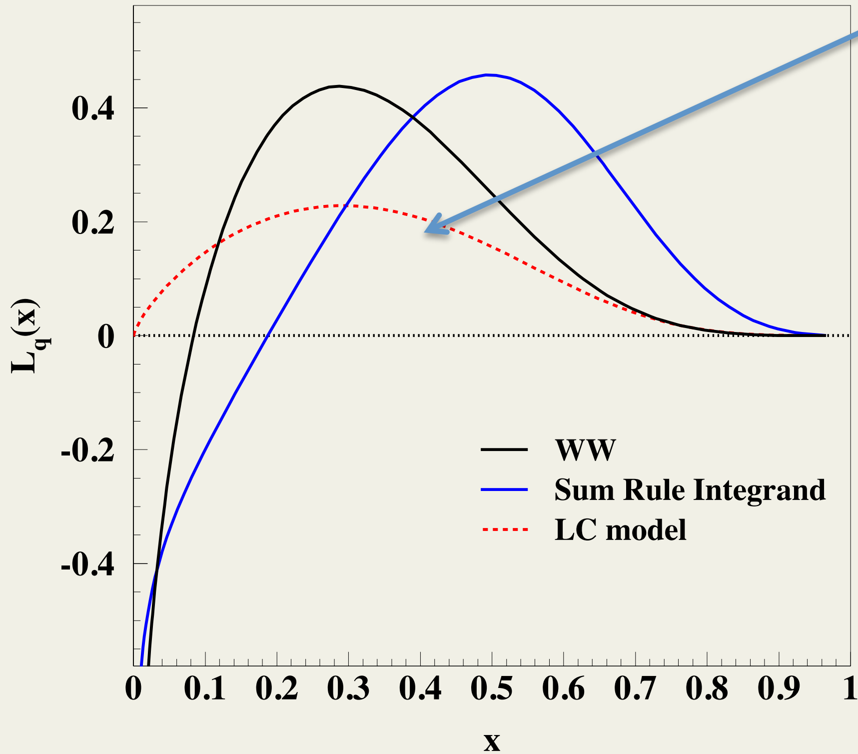
d_2 measurements provide an independent normalization



x dependence

$$L_q(x,0,0) = x\int_x^1 \frac{dy}{y}(H_q(y,0,0)+E_q(y,0,0)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y,0,0),$$

$\neq F_{14}!$

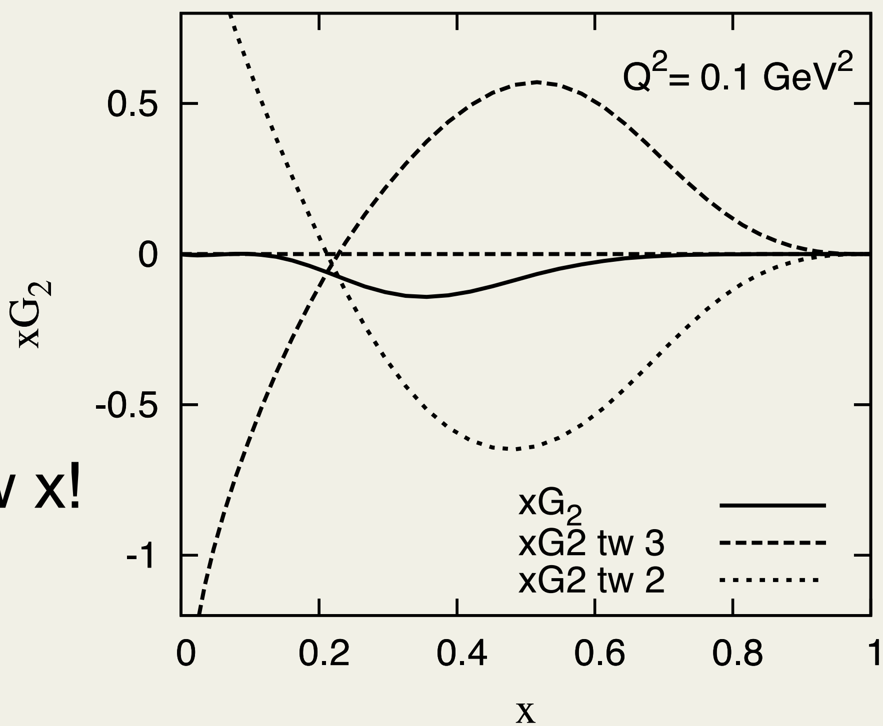


u quark

Low x!

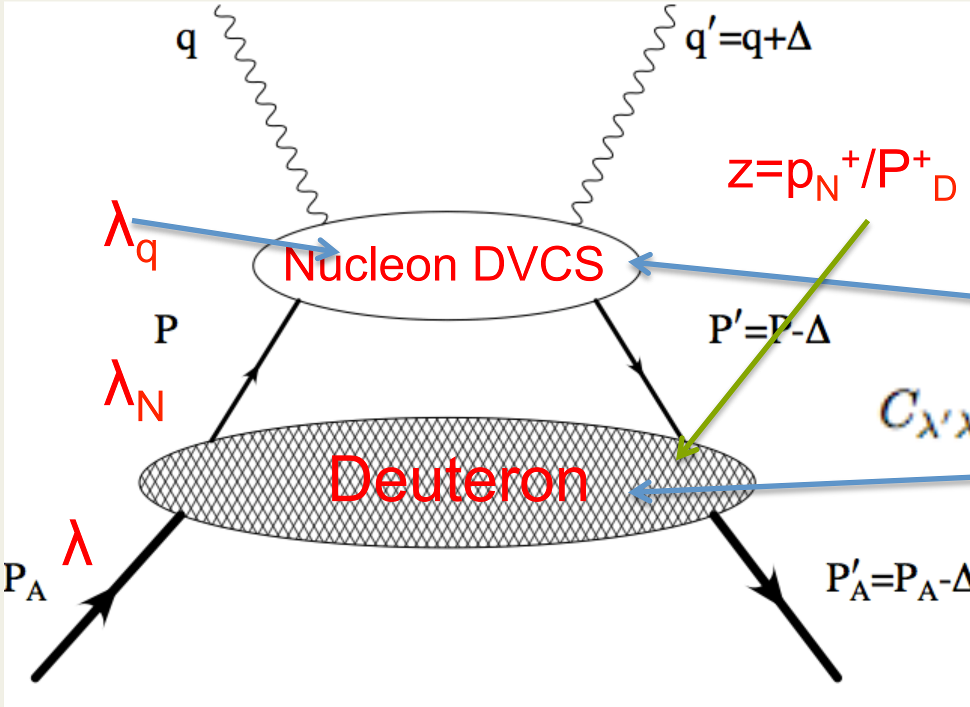
GPDs calculated in
Reggeized diquark model
GGL PRD (2010),
O. Gonzalez et al, PRC(2013)

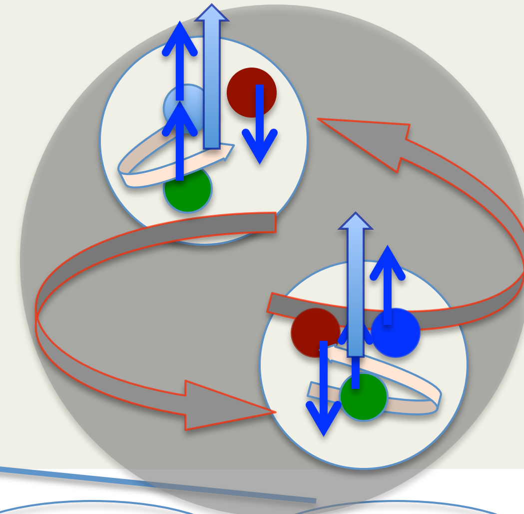
A. Rajan et al.



low x!

Sum Rules in nuclei: Deuteron





$$C_{\Lambda'\lambda'_q,\lambda\lambda_q} = \sum_{\lambda_N,\lambda'_N} B_{\Lambda'\lambda'_N,\lambda\lambda_N} \otimes A_{\lambda'_N\lambda'_q,\lambda_N\lambda_q}$$

$$J_q = \frac{1}{2}\int dx\, x\, [H_q(x,0,0) + E_q(x,0,0)]\,,$$

Nucleon

$\rightarrow$

$$J_q = \frac{1}{2}\int dx\, x\, H_2^q(x,0,0),$$

Deuteron

$F_1+F_2= G_M$

G_M

Deuteron Angular Momentum Sum Rule

$$\frac{1}{2}\mathcal{G}_5^{q,g} = \frac{1}{2}\int dx\; xH_2(x,0,0) = J_z^{q,g}$$

S. Taneja, K. Kathuria, S. Liuti, G. Goldstein, PRD86 (2012)

4He: Spin 0

Interference between Bethe Heitler process and DVCS measurable in BSA

(Mohammed Hattawy's talk)

(Muller-Kirchner)

$$c_0^{\mathcal{I}} = -8(2-y)\frac{t}{Q^2}F_A\,\Re e\{\mathcal{H}_A\} \times \left\{(2-x_A)(1-y)-(1-x_A)(2-y)^2\left(1-\frac{t_{min}}{Q^2}\right)\right\},$$

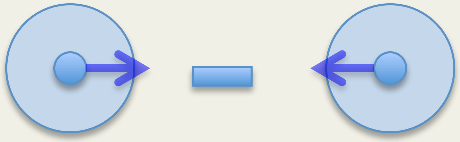
$$c_1^{\mathcal{I}} = 8K(2y-y^2-2)F_A\,\Re e\{\mathcal{H}_A\},$$

$$s_1^{\mathcal{I}} = 8Ky(2-y)F_A\,\Im m\{\mathcal{H}_A\}.$$

1 twist two chiral even GPD, H_A

1 twist two chiral odd GPD, H_T^A

2 twist three GPDs which measure spin orbit correlation



A few observations

- ✓ The new expression relates a GTMD (F_{14}) with a twist 3 GPD (G_2), intrinsic k_T enters even if integrated over ➔ it establishes a connection between transverse spatial and momentum dependences

- ✓ Similar to the Sivers effect but here the function vanishes for a straight link, only staple links are probed

- ✓ A unique setup to study/test transverse momentum dependence and related effects, evolution of twist 3 and kT moment of GTMD, renormalization issues…

- ✓ A unique handle on quark-gluon interactions through the explicit appearance of the quark-gluon-quark correlator in the sum rule

- ✓ The role of partonic k_T and off-shellness, k^2 is manifest.

- ✓ Similarities with g_2

Conclusions and Outlook

The connection we established through the new sum rules between (G)TMDs and GPDs, opens many interesting avenues:

- It allows us to study in detail the role of quark-gluon correlations, and of transverse momentum or off-shellness
- OAM was obtained so far by subtraction (also in lattice). We can now both calculate OAM on the lattice (GTMD) and validate this through measurements (twist 3 GPD)
- The difference between JM and Ji sheds light on the working of the quark-gluon correlations (twist analysis)
- Many more interesting new connections: with transverse spin (Sivers effect, transverse spin, nuclei, spin orbit-term) and axial vector sector (g_2)

Test renormalization issues, evolution etc…

Back Up

Outline

- ✓ Definitions J, L and S in QCD
  - ✓ J Ji Sum Rule, Energy Momentum Tensor (EMT), proton anomalous magnetic moment, accessing parton dynamics through GPDs
  - ✓ S does not enter directly EMT: accessing parton dynamics through Helicity Distribution
  - ✓ L does not enter directly EMT: accessing parton dynamics through k_T moment of Generalized TMD and twist 3 GPD
- ✓ Jaffe Manohar and Ji decompositions: importance of FSI and gluons, where are the gluons coming from (Burkardt relation)
- ✓ Different types of sum rules: quark sector in Ji and Jaffe Manohar
- ✓ How to measure them: what spin correlations are they sensitive to, longitudinal polarization, connection to g_2/d_2
- ✓ Recommendations for EIC: angular momentum and low x physics, extending the DVCS/DVMP framework to EIC kinematics, more complex exclusive measurements

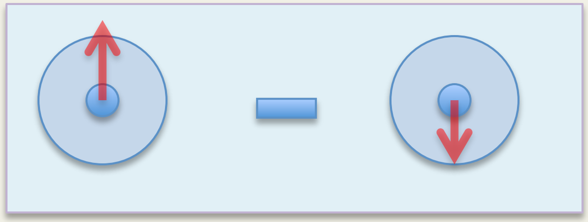
Twist 3 decomposition of hadronic tensor in various notations

Polyakov et al. [13]	$2G_1$	G_2	G_3	G_4
Meissner et al. [3]	$2\widetilde{H}_{2T}$	$\widetilde{E}_{2T}$	E_{2T}	H_{2T}
Belitsky et al. [16]	E^3_+	$\widetilde{H}^3_-$	$H^3_+ + E^3_+$	$\frac{1}{\xi}\widetilde{E}^3_-$

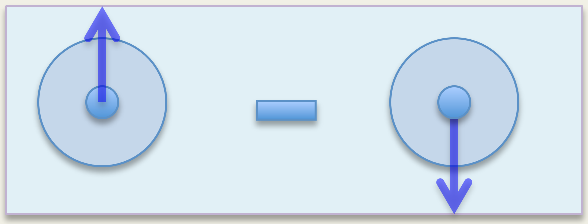
TABLE I: Comparison of notations for different twist 3 GPDs.

Courtoy et al, PLB (2014)arXiv:1310.5157

$h_1^{\perp}$



$f_{1T}^{\perp}$



Ah ha!
This is the same argument that allows us to observe the T-odd TMDs by understanding the role of the gauge links

$f_{1T}^{\perp}$

S.J. Brodsky et al. / Physics Letters B 530 (2002) 99–107

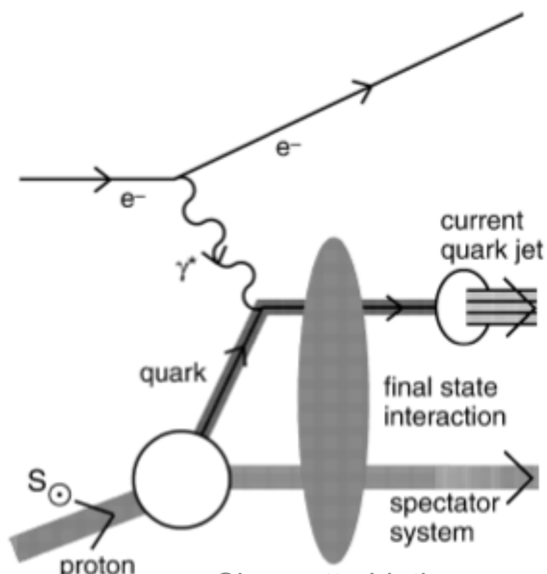


Fig. 1. The final-state interaction in the semi-inclusive deep inelastic lepton scattering $\ell p^{\uparrow} \to \ell' \pi X$.

$$
\begin{aligned}
G_1 \;\propto\; & (g'_V g_V + g'_A g_A) \otimes \underline{(A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,+-})} && g_1 \\
& + (g'_V g_A + g'_A g_V) \otimes \underline{(A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,+-})} && F_{14}
\end{aligned}
$$

parity odd

$$
\begin{aligned}
A_1 \;\propto\; & (g'_V g_V + g'_A g_A) \otimes (A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,+-}) && F_{14} \\
& + (g'_V g_A + g'_A g_V) \otimes (A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,+-}), && g_1
\end{aligned}
$$

F_{14} is the parity odd contribution to g_1 and the parity even contribution to A_1!