

# Potential of EDM Measurement in Figure-8 Ring

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# Spin Dynamics

- Generalized Thomas-BMT equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S},$$

Thomas precession

$$\vec{\Omega} = -\frac{e}{\gamma mc} \left[ (1+G)\vec{B}_{rest} + G_d \vec{E}_{rest} + \frac{\gamma-1}{\gamma v^2} \vec{v} \times \vec{E}_{rest} \right]$$

Spin rotation due to MDM

$$\vec{\mu} = (1+G) \frac{e\hbar}{mc} \vec{S}$$

Spin rotation due to EDM

$$\vec{d} = G_d \frac{e\hbar}{mc} \vec{S}$$

- Thomas-BMT expressed through fields in the laboratory frame

$$\vec{\Omega} = -\frac{e}{\gamma m} \left[ (1+G)\vec{B}_{\parallel} + (1+\gamma G)\vec{B}_{\perp} + \gamma \left( G + \frac{1}{\gamma+1} \right) \vec{E} \times \vec{v} + G_d \vec{E}_{\parallel} + \gamma G_d (\vec{E}_{\perp} + \vec{v} \times \vec{B}) \right]$$

# Spin Dynamics

- Spin equation in the frame aligned with the particle velocity

$$\frac{d\vec{S}}{d\theta} = \vec{W} \times \vec{S}, \quad \vec{W}_{\perp} = \gamma G(\vec{\tau} \times \vec{\tau}') - \gamma v G_d \vec{\tau}' + (1+G) \frac{\vec{\tau} \times \vec{D}}{\gamma v} \quad \vec{W}_{\parallel} = \left( (1+G)H_{\parallel} + G_d D_{\parallel} \right) \vec{\tau}$$

$$\vec{\tau} = \vec{v}/v$$

$$\vec{\tau}' = d\tau/d\theta$$

$$\vec{H} = \vec{B}/B_0$$

$$\vec{D} = \vec{E}/B_0$$

$$B_0 = -pc/eR$$

- Spin precession in the frame aligned with the particle velocity expressed through lab-frame fields

$$\vec{\Omega} - \vec{\tau} \times \frac{d\vec{\tau}}{dt} = -\frac{e}{\gamma m} \left[ (1+G)\vec{B}_{\parallel} + \gamma G\vec{B}_{\perp} + \gamma v \left( G - \frac{1}{\gamma^2 v^2} \right) \vec{E} \times \vec{v} + G_d \vec{E}_{\parallel} + \gamma G_d (\vec{E}_{\perp} + \vec{v} \times \vec{B}) \right]$$

where

$$\frac{d\vec{\tau}}{dt} = \frac{e}{\gamma m} \left( \frac{\vec{E}_{\perp}}{v} + \vec{\tau} \times \vec{B} \right)$$

# EDM Effect

- Current state of EDM data
  - Direct measurement only for neutron,  $\text{EDM} < 3 \cdot 10^{-26} \text{ e}\cdot\text{cm}$
  - Proton EDM deduced from atomic EDM limit,  $\text{EDM} < 7.9 \cdot 10^{-25} \text{ e}\cdot\text{cm}$
  - No measurement for deuteron or other nuclei
- Expected proton EDM values
  - Standard model,  $\text{EDM} < 3 \cdot 10^{-31} \text{ e}\cdot\text{cm} \Rightarrow G_d \sim 5 \cdot 10^{-17}$
  - Extensions of Standard model, EDM as large as  $3 \cdot 10^{-24} \text{ e}\cdot\text{cm} \Rightarrow G_d \sim 5 \cdot 10^{-11}$
- MDM spin transparency: spin effect of fields vanishes without EDM

$$\left\langle G\vec{H}_\perp + \left( G - \frac{1}{\gamma^2 v^2} \right) \vec{D} \times \vec{v} \right\rangle = 0 \quad \langle H_\parallel \rangle = 0 \quad \vec{W}_\perp = -\gamma v G_d \vec{\tau}' \quad \vec{W}_\parallel = G_d D_\parallel \vec{\tau}$$

- Existing approaches
  - Purely electric ring at a magic energy (BNL), does not work for  $G < 0$

$$p^* = \gamma v mc = mc / \sqrt{G}$$

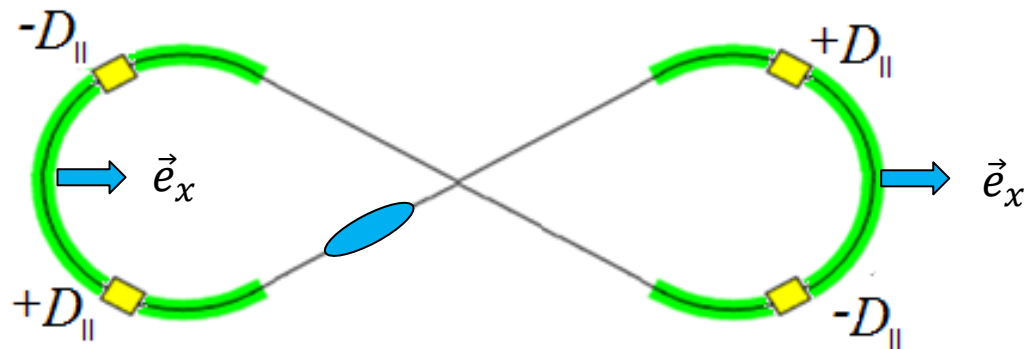
- Combination of magnetic and electric field (COSY)



# Figure-8 Ring

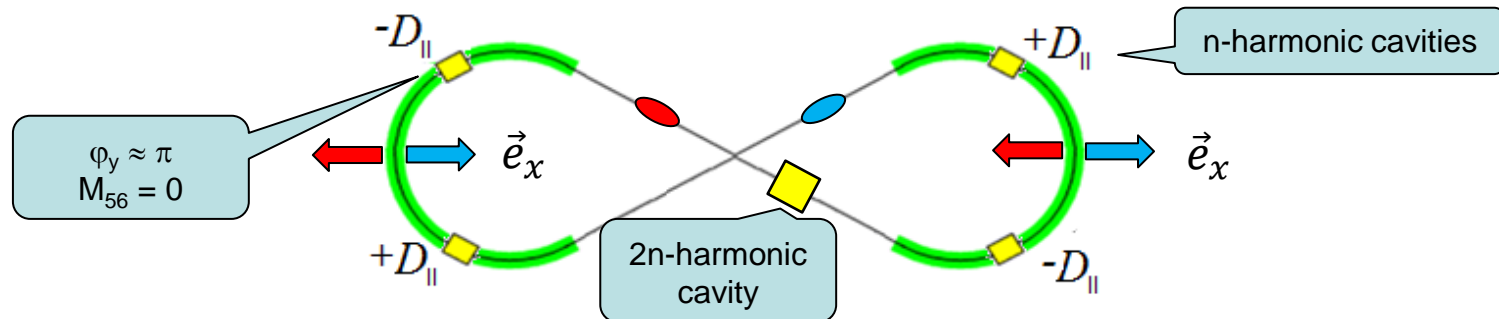
- Potentially new opportunities for EDM measurements
- By the basic design, both MDM and EDM spin transparent
- Longitudinal RF electric fields introduced for EDM measurement
  - RF provides additional flexibility for enhancing EDM measurements
- EDM measurement in a figure-8 ring
  1. Calibration of spin transparent regime without electric fields
  2. Turn on electric fields in symmetric pairs

$$\nu = \frac{2}{\pi} \varphi_D \sin \frac{\varphi_y}{2}, \quad \varphi_D = G_d D_{\parallel} L_{\parallel}, \quad \varphi_y = \gamma G \alpha_y$$



# RF Electric Fields in Figure-8 Ring

- $2n$  bunches with alternating polarizations
  - Similar to having two beams for systematics suppression
- $2n$ -harmonic cavity for stabilization of longitudinal motion
- Two pairs of  $n$ -harmonic cavities for EDM effect
  - Effects of magnetic fields cancel (ignoring the difference in off-momentum closed orbits)
  - EDM effects add up for the opposite polarity bunches
  - If start vertically polarized monitor buildup of transverse polarization
- Control of remaining systematic effects
  - Symmetry
  - Calibration
  - Closed orbit monitoring and control
  - RF field focusing can be compensated by overtone resonators



# Zero-Integer Spin Resonance & Spin Stability Criterion

- The total zero-integer spin resonance strength

$$W_0 = W_{coherent} + W_{incoherent}, \quad W_{incoherent} \ll W_{coherent}$$

is composed of

- coherent part  $W_{coherent}$  due to closed orbit excursions
- incoherent part  $W_{incoherent}$  due to transverse and longitudinal emittances
- The coherent part can be compensated by small static magnetic fields
- Spin stability criterion
  - the spin tune induced by the EDM must significantly exceed the incoherent part

$$\nu \gg W_{incoherent}$$

- Estimates of the incoherent part of the resonance strength

- Random simple lattice of figure 8

$$W_{incoherent} \sim 10^{-7} - 10^{-5} \text{ for protons,} \quad W_{incoherent} \sim 10^{-9} - 10^{-7} \text{ for deuterons}$$

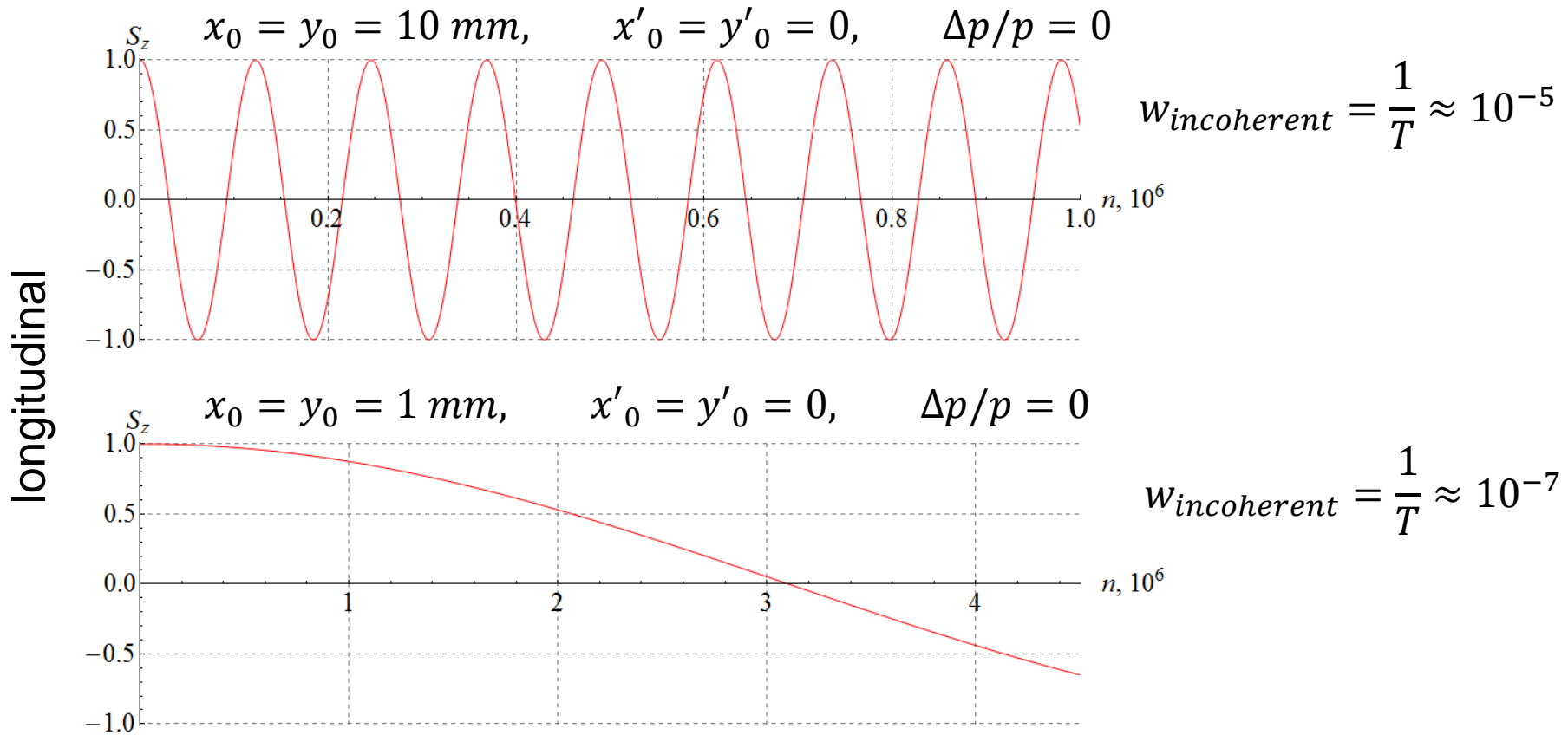
- Lattice with special symmetries of betatron motion

$$W_{incoherent} \sim 10^{-14} - 10^{-10} \text{ for protons,} \quad W_{incoherent} \sim 10^{-18} - 10^{-14} \text{ for deuterons}$$



# Incoherent Part for Protons

- Ideal figure-8, fixed  $p = 0.785 \text{ GeV}/c$ ,  $V = 6 \text{ kV}$
- **Stable polarization direction vertical as expected!**
- **Scales as a square of the betatron amplitude**



Here and later all spin and orbital tracking done using *Zgoubi* by Francois Meot

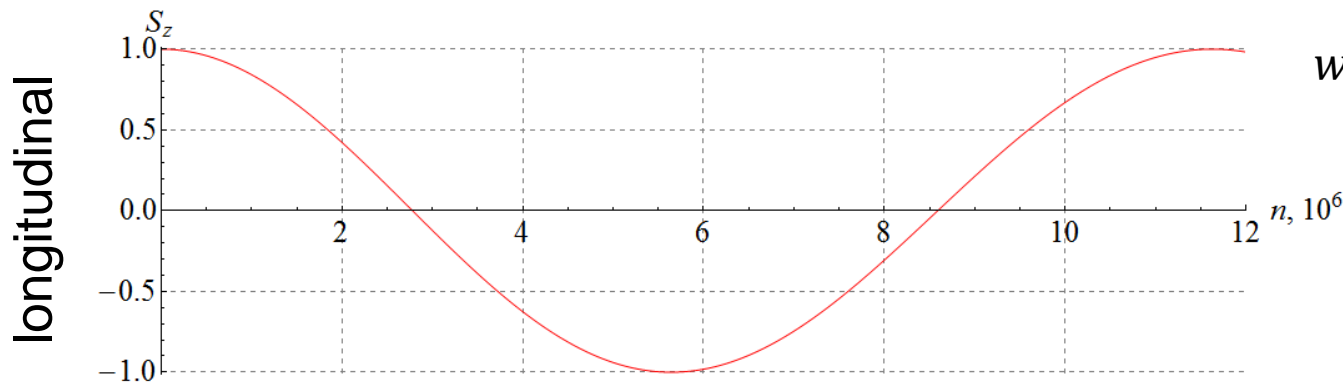




# Incoherent Part for Deuterons

- Ideal figure-8, fixed  $p = 0.785 \text{ GeV}/c$
- **Stable polarization again vertical**
- **Original strength estimates are correct**
- **Transverse size reduction (cooling) is highly beneficial**

$$x_0 = y_0 = 10 \text{ mm}, \quad x'_0 = y'_0 = 0, \quad \Delta p/p = 0$$



$$W_{incoherent} = \frac{1}{T} \approx 10^{-7}$$

?

$$x_0 = y_0 = 1 \text{ mm}, \quad x'_0 = y'_0 = 0, \quad \Delta p/p = 0$$

$$W_{incoherent} \approx 10^{-9}$$

# Time Estimate of EDM Experiment

- In the scheme with two pairs of RF resonators

$$G_d = \frac{L}{c\tau} \frac{\gamma mc^2}{4eE_{\parallel}L_{\parallel}} \Psi_d$$

- Example for deuterons

$$\Psi_d = 10^{-2}, \quad \tau = 10^4 \text{ s}, \quad L = 300 \text{ m}, \quad eE_{\parallel}L_{\parallel} = 20 \text{ MeV}, \quad \gamma mc^2 = 2 \text{ GeV}$$

$$G_d = 2.5 \cdot 10^{-11} \Rightarrow \text{deuteron EDM} = 2.5 \cdot 10^{-25} \text{ e} \cdot \text{cm}$$

# Conclusions

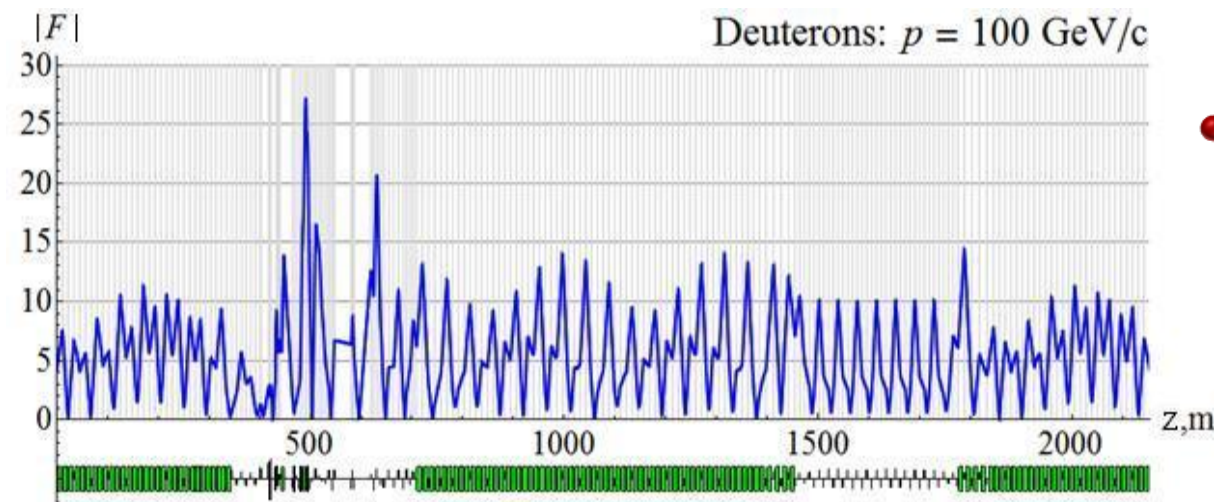
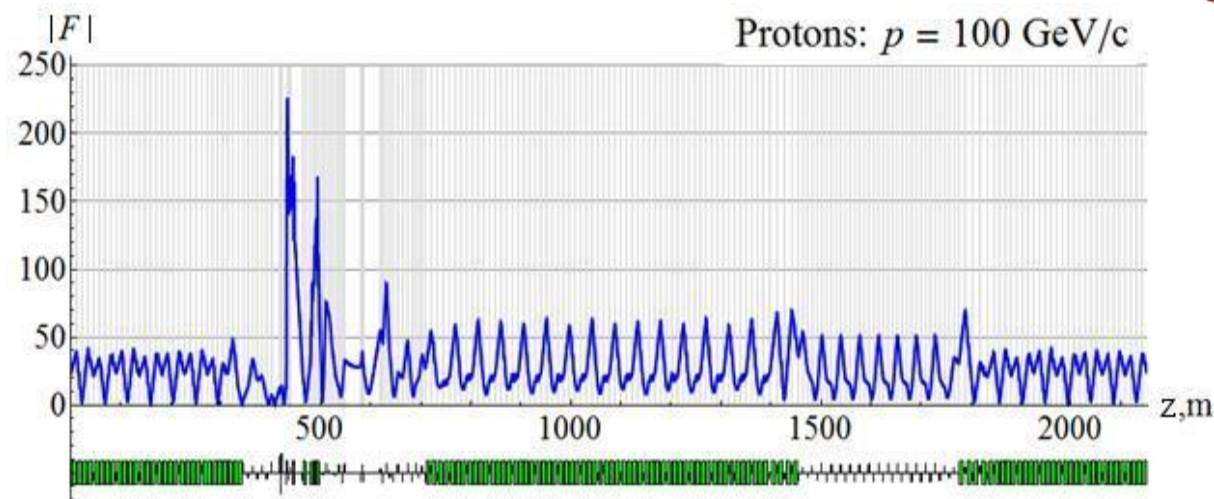
- Properties and features of figure-8 rings beneficial for EDM experiments
  1. Automatic compensation of MDM and EDM effects on the spin due to basic lattice
  2. EDM can be measured using longitudinal electric fields, which do not bend the orbit
  3. Compensation of the coherent part of the zero-integer resonance strength allows one to reduce a “real” lattice to an “ideal” one **without** implementation errors
  4. By choosing a special symmetry of the ring optical elements, one can significantly reduce the part of the resonance strength depending on the beam emittances
  5. Use of electron cooling allows for additional reduction of the incoherent part of the resonance strength
  6. Figure-8 rings allow measurements of EDM of any particle species at any energy
  7. Figure 8 is especially efficient for EDM measurements with deuteron beams



# Backup Slides



# Spin Response Function of Collider Ring



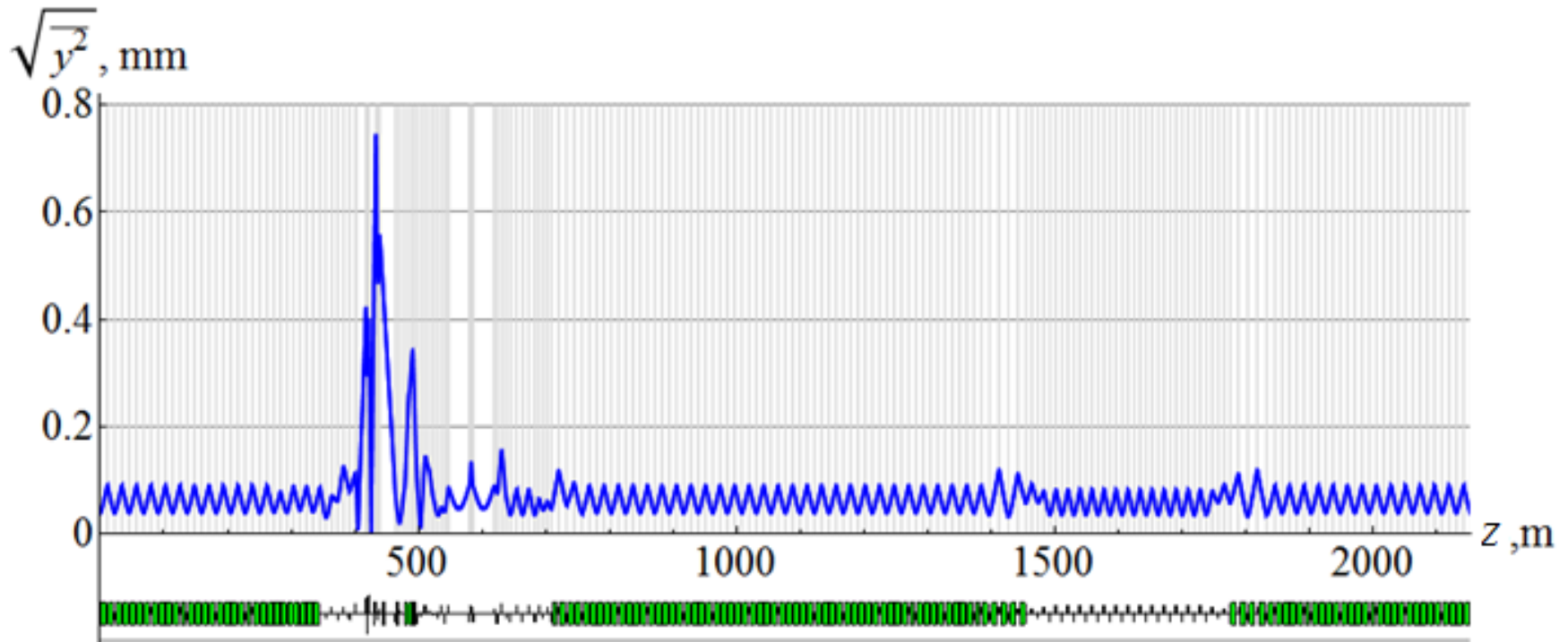
- The coherent part of zero-integer spin resonance induced by perturbing radial field  $h_x(\theta)$  can be calculated using periodical response function  $F(\theta)$ :

$$w_{\text{coherent}} = \gamma G \langle h_x(\theta) F(\theta) \rangle$$

- $F(\theta)$  is calculated from linear optics
- Perturbing radial fields arise, for example, due to dipole roll errors, vertical quadrupole misalignments, etc. Such perturbing fields result in vertical closed orbit distortion

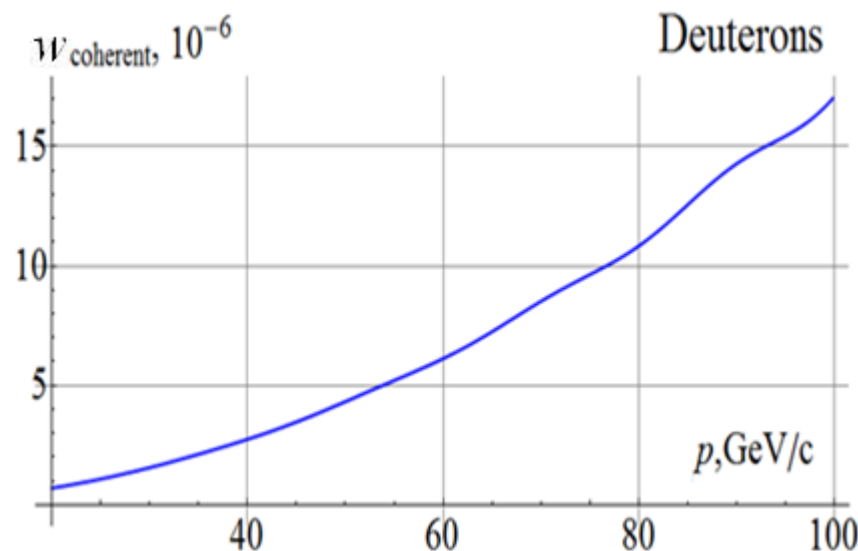
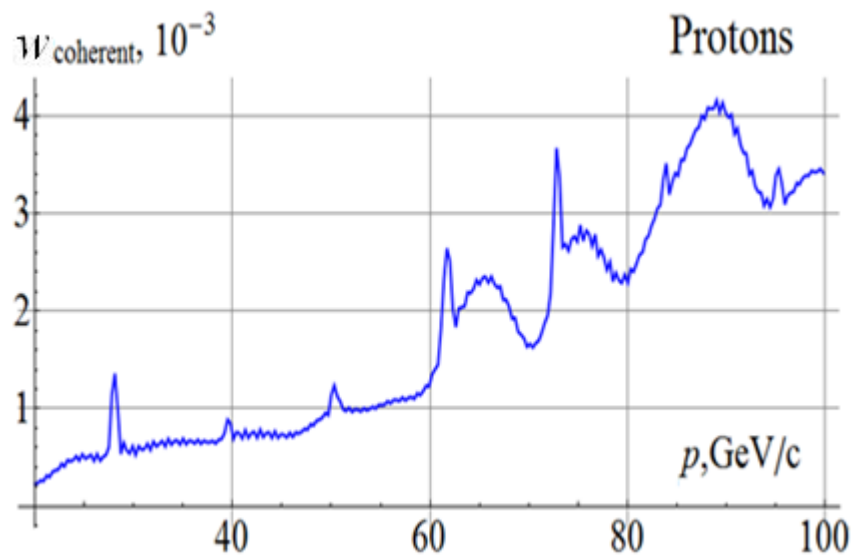
# RMS Vertical Closed Orbit Distortion

- Assuming certain rms vertical closed orbit distortion, we statistically calculate vertical orbit offset in the quadrupoles and resonance strength
- rms vertical closed orbit distortion for random misalignment of all quads
  - Assuming that orbit distortion in the arcs  $< 100 \mu\text{m}$



# Coherent Part of Resonance Strength

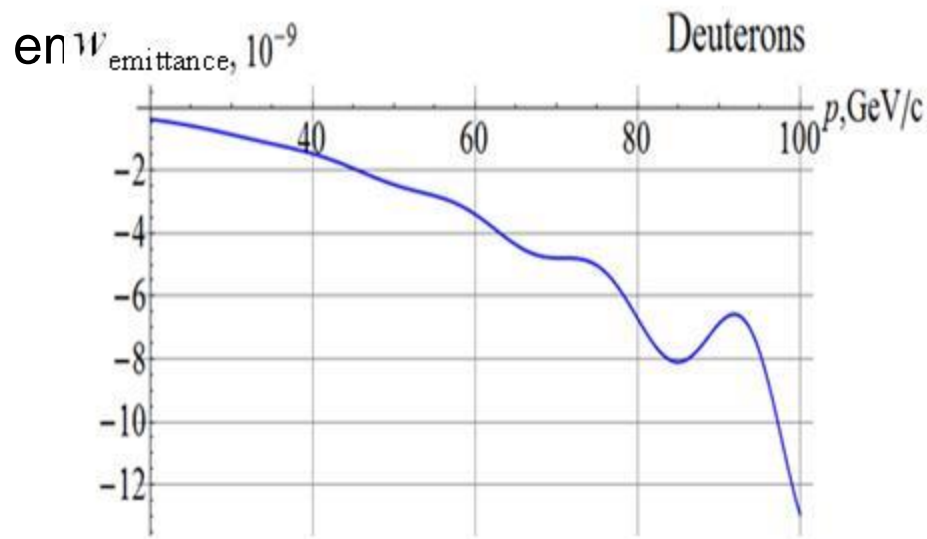
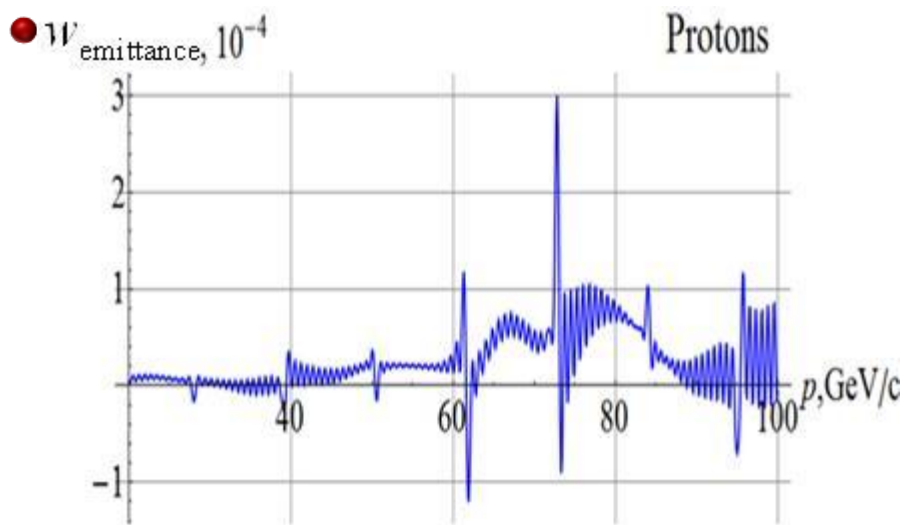
- $2\text{ T} \times 2\text{ m}$  control solenoids allow setting  $\nu_p = 10^{-2}$  and  $\nu_d = 10^{-4}$ 
  - Sufficient for polarization stabilization and control



- The coherent part of the zero-integer spin resonance strength is determined by closed orbit distortions.
- Orbit distortion does not exceed  $\pm 100\ \mu\text{m}$

# Incoherent Part of Resonance Strength

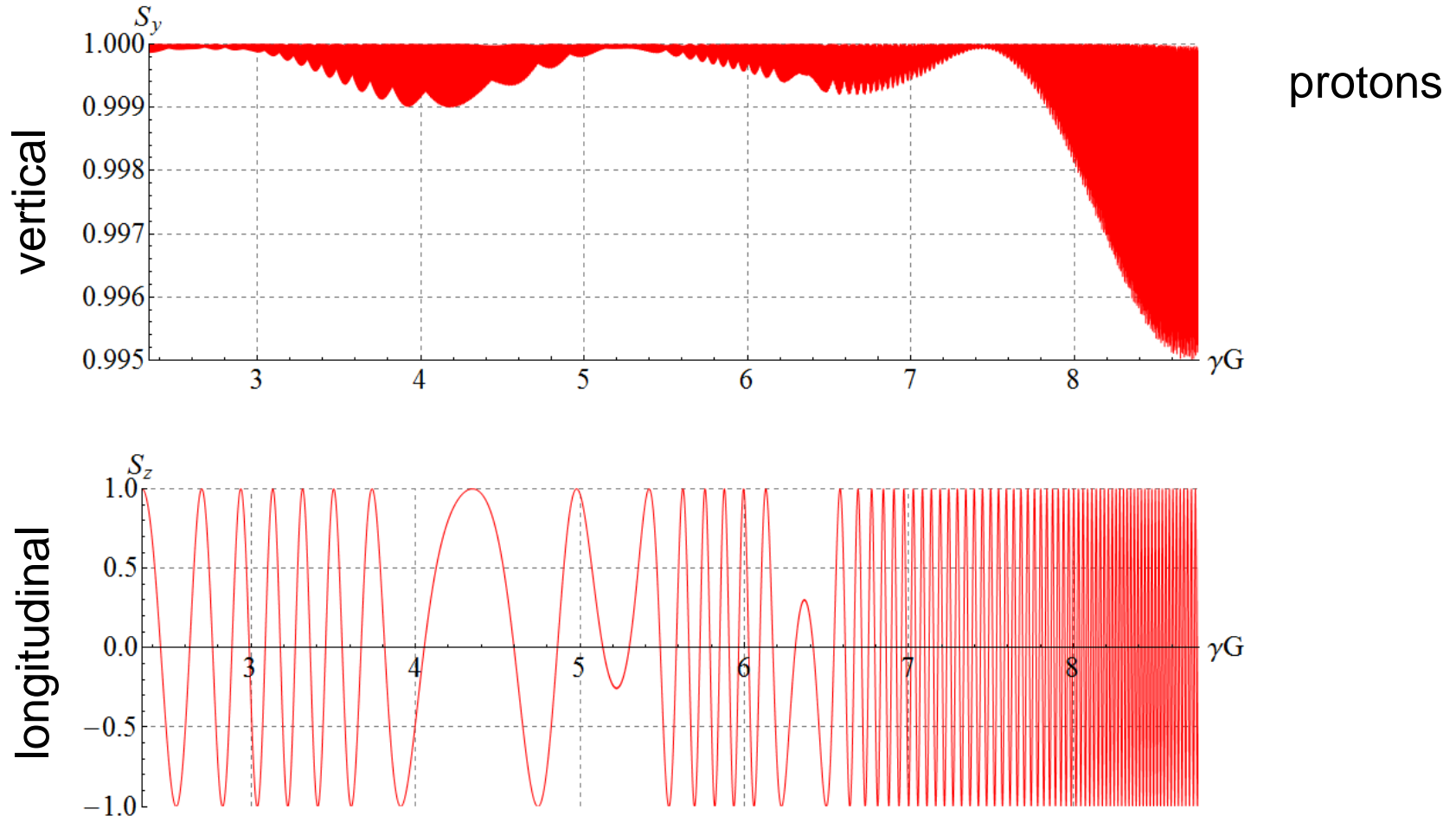
- Effect of higher-order resonances due to incoherent tune spread requires study similarly to the case of two Siberian snakes
- The incoherent part of the zero-integer spin resonance strength is determined by emittances of betatron and synchrotron oscillations and depends on magnetic lattice
- In an ideal lattice, the incoherent part is determined primarily by the emittance of vertical betatron oscillations
- Incoherent part of the spin field is vertical in the second order





# Incoherent Part of Resonance Strength

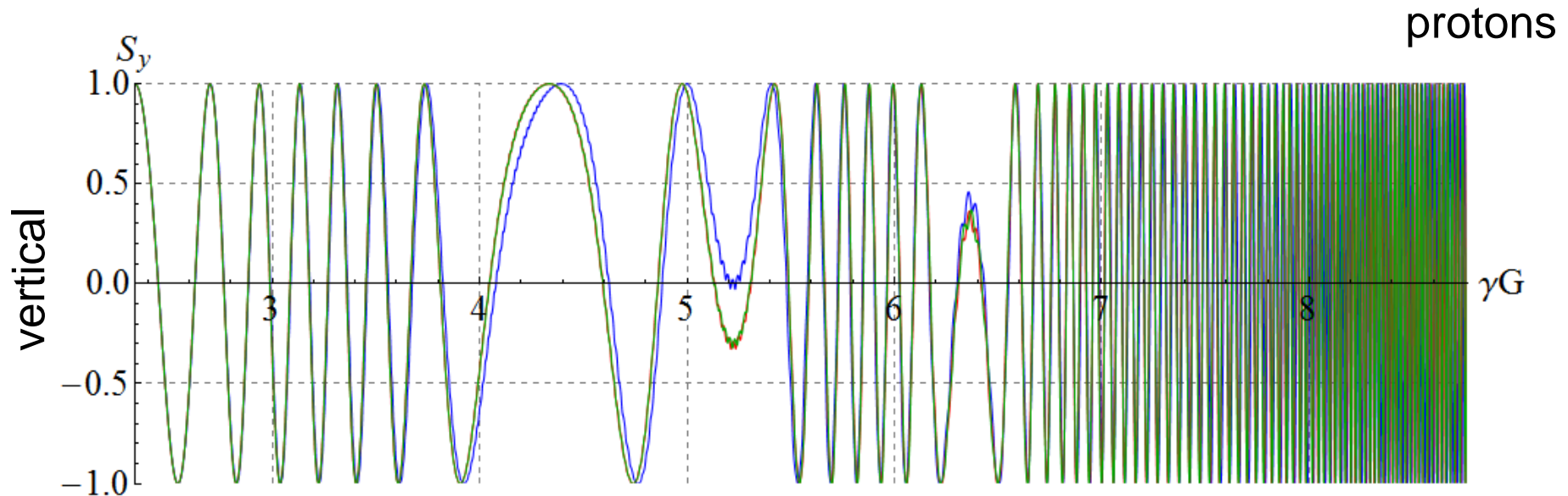
- Ideal figure-8 booster, on-momentum particles with 1 cm offsets in x and y
- **Stable polarization direction vertical as expected!**



Here and later all spin and orbital tracking done using *Zgoubi* by Francois Meot

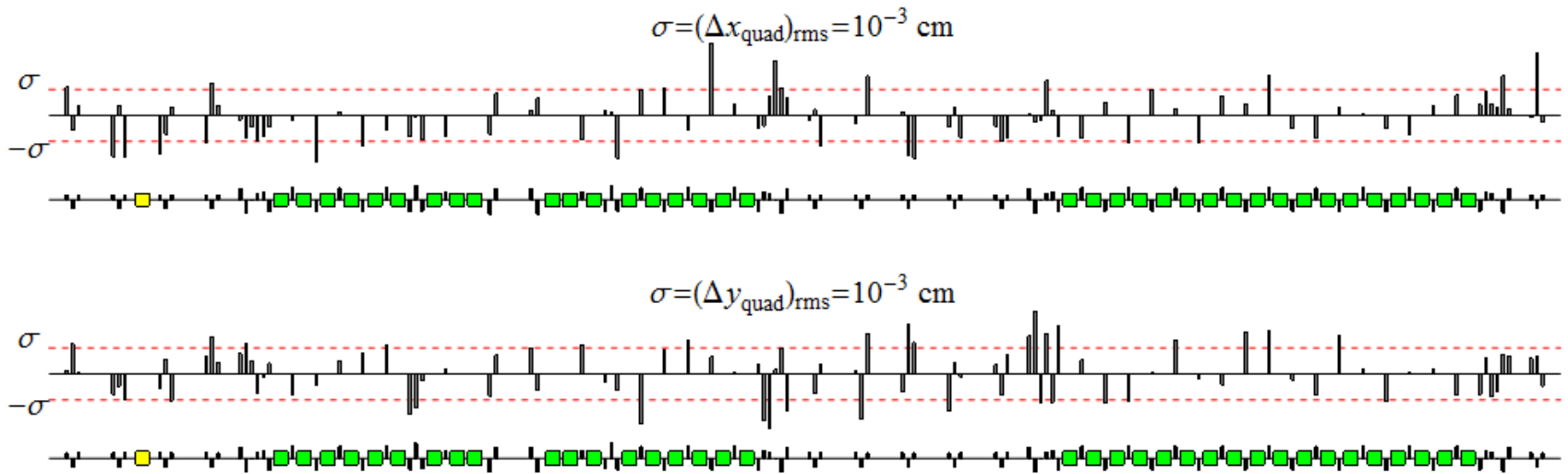
# Synchrotron Motion Effect

- Ideal figure-8 booster
- Three particles uniformly distributed on a longitudinal phase-space ellipse with  $\Delta p/p = 5 \times 10^{-4}$ , each particle has initial 1 cm offset in x and y
- **Almost no momentum dependence!**



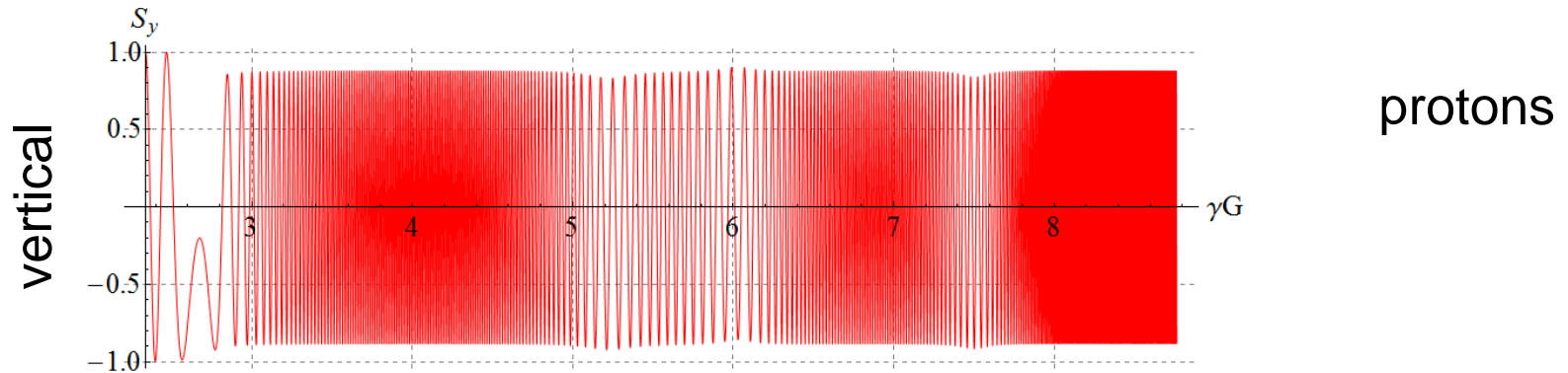
# Coherent Part of Resonance Strength

- Primary contribution to the coherent part comes from transverse quadrupole shifts

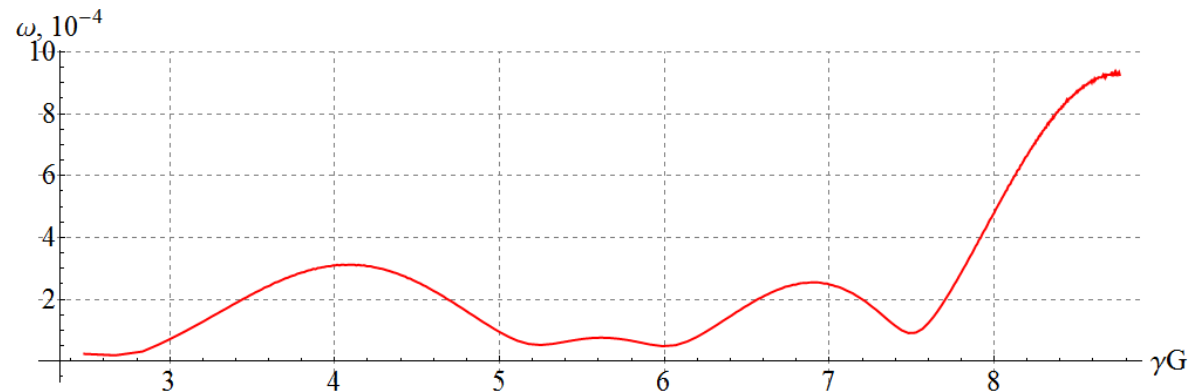


# Coherent Part of Resonance Strength

- Stable spin direction due to coherent part lies in the horizontal plane
- On-momentum particle launched along CO

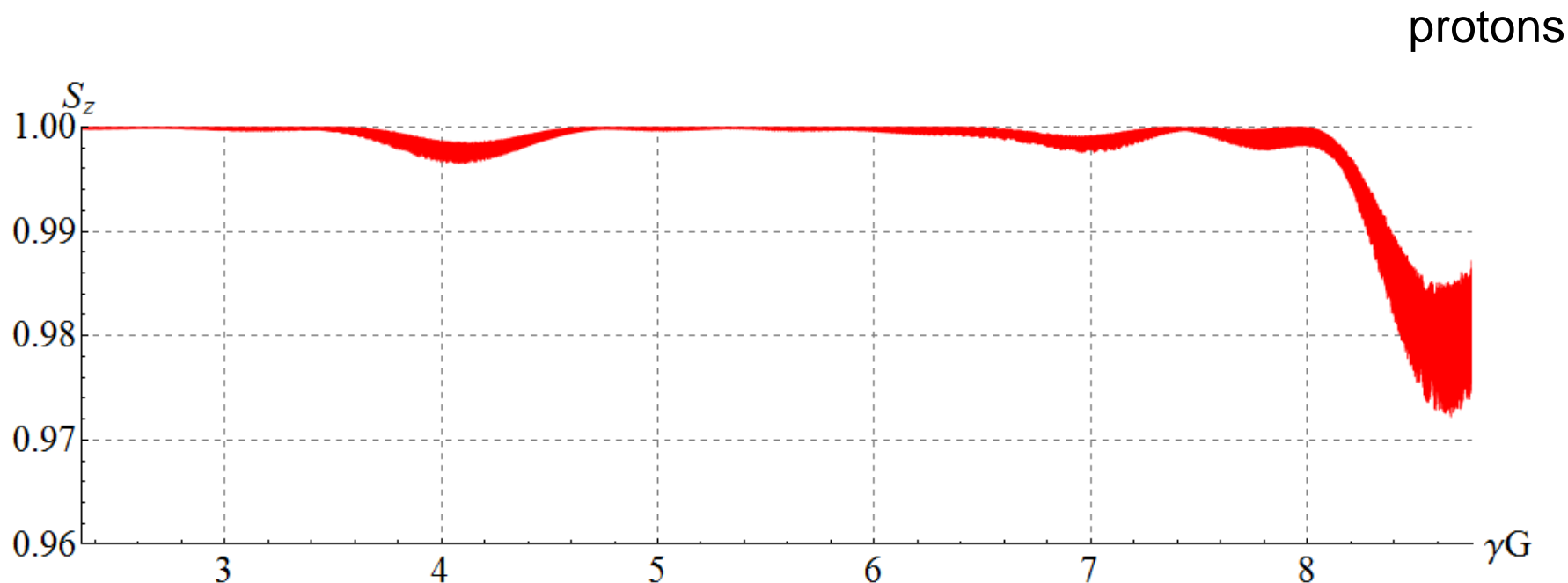


- Precession rate gives the value of the coherent part of the spin resonance strength



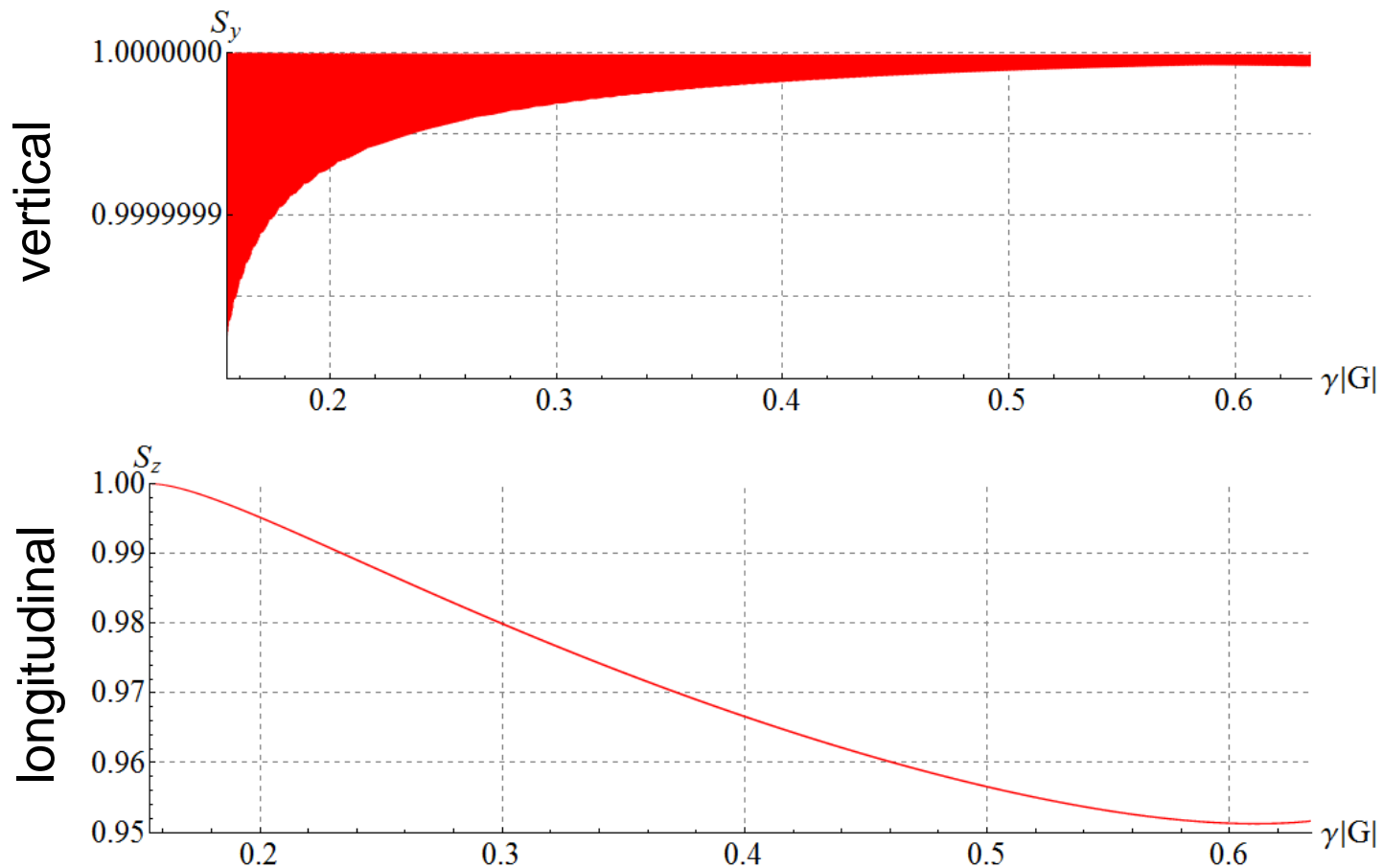
# Stabilization of Proton Spin

- Figure-8 booster with transverse quadrupole misalignments
- 0.1 Tm (maximum) spin stabilizing solenoid
- On-momentum particle with 1 cm offsets in x and y



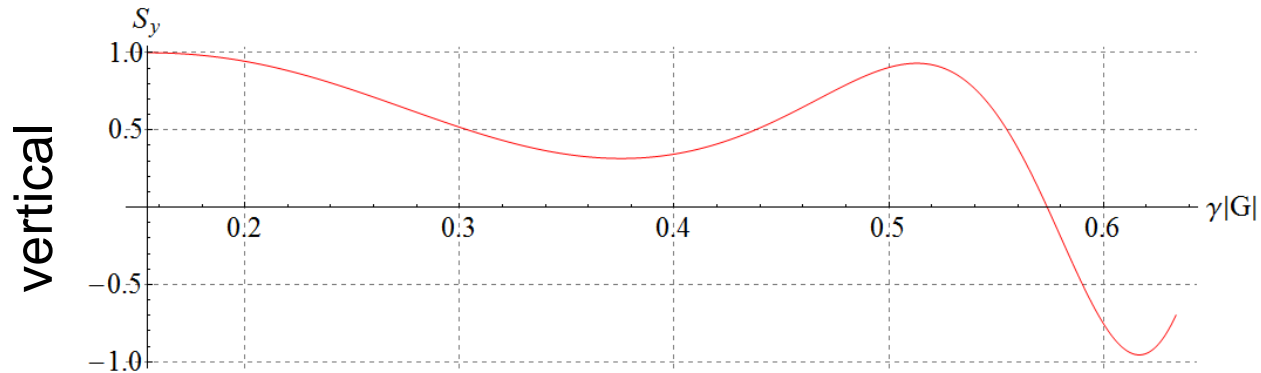
# Incoherent Part of Resonance Strength

- Ideal figure-8 booster, on-momentum particles with 1 cm offsets in x and y
- **Stable polarization direction vertical as expected!**
- Incoherent part of the resonance strength  $< 10^{-8}$



# Coherent Part of Resonance Strength

- Same transverse quadrupole misalignments as for protons
- On-momentum particle launched along CO, coherent strength part  $<10^{-6}$



deuterons

- 0.1 Tm solenoid, on-momentum particle with 1 cm offsets in x and y

