On the RF resonant polarimetry

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Abstract

I will discuss the basics of the dynamic theory and main constraints of the resonance RF polarimetry. The Hamiltonian and basic equations of the dynamical system: polarized beam in a ring + spin-resonance SRF cavity will be presented. Importance of taking into account of the charged particle orbit modulation by the spin precession in the ring (*neo Stern-Gerlach effect*) will be demonstrated. This phenomenon offers a frequent increase of the strength of polarized beam interaction with resonance cavity. Possible measures of reduction of the spin tune spread will be discussed, as well.

Outline

- Spin Hamiltonian
- Spin in synchrotron
- Spin to orbit mode (SOM) in synchrotron
- Spin mode – cavity (SMC) interaction
- Tuning
- Noise (has been discussed in the original paper)
- Summary

Spin in the external fields

- Thomas – BMT equation:

$$\dot{\boldsymbol{S}} = \boldsymbol{W} \times \boldsymbol{S}\,; \qquad \boldsymbol{W} \equiv -\frac{e}{m}\{(\frac{1}{\gamma} + G)\boldsymbol{B}_{\perp} + \frac{1+G}{\gamma}\boldsymbol{B}_{\parallel} + \left(G + \frac{1}{\gamma+1}\right)\boldsymbol{E} \times \boldsymbol{v}\}$$

- Spin in magnetic field of a synchrotron
- Spin in RF fields
- *Forced spin motion* $\boldsymbol{n}(\boldsymbol{p}, \boldsymbol{r}, t)$
- Spin *adiabatic invariant* $S_n = \boldsymbol{Sn}(\boldsymbol{p}, \boldsymbol{r}, t)$
- Spin *free precession around* $\boldsymbol{n}$: $\dot{\Psi} = \nu(I_{orb})$; and *tune spread*
- Spin ***response function*** in focusing field of a synchrotron
- Spin resonances, etc. …

…We know how the fields affect the spin…

But how the spin affects the orbit and the RF field ?

How to describe this?

Spin Hamiltonian

…Nothing is better than the ***Hamiltonian***. ..
How to get the Hamiltonian?

Very simple – in two steps!

Step one
BMT Hamiltonian for spin: $H_{BMT} = \boldsymbol{WS}$

Step two:
Total Hamiltonian linear in spin :

$$H = \frac{1}{2}\sum_{\lambda}(\mathcal{P}^2 + \omega^2 Q^2)_{\lambda} + \sum_{\boldsymbol{j}}\{\sqrt{m^2 + (\boldsymbol{P} - e\boldsymbol{A})^{\boldsymbol{2}}} + \boldsymbol{WS}\}_{\boldsymbol{j}}$$

- $\boldsymbol{A} = \boldsymbol{A}_{ext} + \boldsymbol{A}_c;$ $\boldsymbol{A}_c = Q(t)\boldsymbol{e}(\boldsymbol{r});$
- $\boldsymbol{E}_c = -\mathcal{P}(t)\boldsymbol{e}(\boldsymbol{r});$ $\boldsymbol{B}_c = Q(t)\boldsymbol{b} \equiv Q(t)\boldsymbol{\nabla} \times \boldsymbol{e}(\boldsymbol{r});$
- $\boldsymbol{W} = \boldsymbol{W}_{ext} + \boldsymbol{W}_c$

Once it cannot be simpler, it is unique and must be true…

Spin to orbit mode (SOM) in a synchrotron

Spin force: $\dot{P}_\alpha = -\boldsymbol{v}\nabla_\alpha \boldsymbol{A} - \nabla_\alpha \boldsymbol{WS}$; $\dot{\boldsymbol{r}} = \frac{\partial H}{\partial \boldsymbol{P}} = \boldsymbol{v} + \frac{\partial}{\partial \boldsymbol{P}}(\boldsymbol{WS})$; $\boldsymbol{v} \equiv \frac{\boldsymbol{P}-e\boldsymbol{A}}{\sqrt{m^2+(\boldsymbol{P}-e\boldsymbol{A})^2}}$

$$\boldsymbol{W}_{ext} = \boldsymbol{W}_0 + \boldsymbol{w}; \qquad \boldsymbol{W}_0 \Longrightarrow GB_{y0}$$

$$w_x \Longrightarrow G\frac{\partial B_x}{\partial y}y\,; \qquad w_y = G\frac{\partial B_y}{\partial x}x\,; \qquad w_\parallel \Longrightarrow \frac{1+G}{\gamma}\frac{\partial B_{y0}}{\partial s}y + \frac{G}{\gamma}B_{y0}\frac{dy}{dz}$$

$$\dot{P}_y = ev\frac{\partial B_x}{\partial y}y - \frac{\partial \boldsymbol{w}}{\partial y}\boldsymbol{S}\,; \qquad \dot{y} = \frac{P_y}{\gamma m} + \frac{\partial \boldsymbol{w}}{\partial P_y}\boldsymbol{S}$$

$$\gamma m\ddot{y} - ev\frac{\partial B_y}{\partial x}y = \frac{e}{m}\left[G\frac{\partial B_y}{\partial x}S_x + \frac{1+G}{\gamma}\frac{\partial B_y}{\partial z}S_v + \frac{G}{\gamma}\frac{d}{dz}(B_{y0}S_v)\right] \equiv F_{sp}$$

$$y'' + ny = \frac{F_{sp}}{pv} \Longrightarrow \approx -n\frac{G}{mv}S_x\,; \qquad y = y_b + y(\boldsymbol{S}_\perp)$$

$$y(\boldsymbol{S}_\perp) = \frac{G}{mv}\hat{f}\boldsymbol{S}_\perp$$

Spin mode – SRF cavity interaction

- RF field general equations:

$$\dot{\mathcal{P}} = -\frac{\partial H}{\partial Q} = -\omega^2 Q + e\sum_j \boldsymbol{ve} + \frac{e}{m}\sum_j \{[\left(\frac{1}{\gamma} + G\right)\boldsymbol{b}_\perp + \frac{1+G}{\gamma}\boldsymbol{b}_\parallel]\boldsymbol{S}\}_j\,;$$

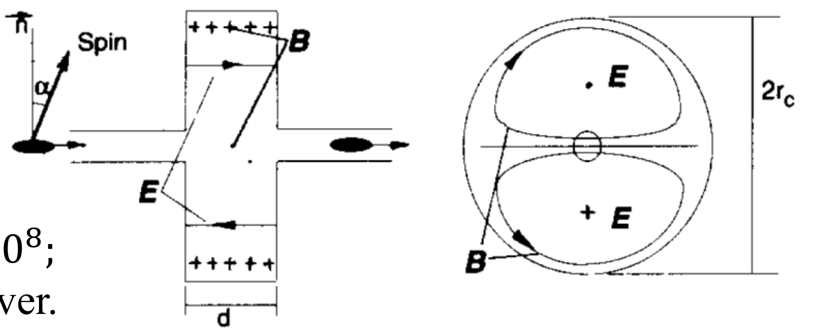
$$\dot{Q} = \frac{\partial H}{\partial \mathcal{P}} = \mathcal{P} + \frac{e}{m}\sum_j \{\left(G + \frac{1}{\gamma+1}\right)(\boldsymbol{e}\times\boldsymbol{v})\boldsymbol{S}\}_j$$

- After reduction to the main terms:

$$\ddot{Q} + 2\Lambda\dot{Q} + \omega^2 Q = \Longrightarrow \approx e\sum_j \{\boldsymbol{ve} + \frac{G}{m}\boldsymbol{b}_\perp\boldsymbol{S}\}_j \Longrightarrow \approx e\sum_j \{v\frac{\partial e_\parallel}{\partial y}(y_b + y_{sp}) + \frac{G}{m}\boldsymbol{b}_\perp\boldsymbol{S}\}_j$$

- Final equation for spin's RF field in a resonant cavity:

$$(\ddot{Q} + 2\Lambda\dot{Q} + \omega^2 Q)_{sp} = N\frac{eG}{m} < b_x\cdot > (S_x + \hat{f}S_\perp)_{coh}$$



Assume: $\omega = 2\pi \times 10^9$ Hz ; $\Delta\nu_{sp} = 10^{-4}$

Then best inquired passive SRF cavity quality is about 10^8; this might be not needed at an adequately sensitive receiver.

Summary

RF arrangement for non-touching beam spin monitoring involves the following basic elements:

- The spin-resonance (SRF) cavity (***spin driver***) to cause spin precession around a vertical or horizontal (rotating) axis, similarly to spin motion in NMR;
- The spin driver can be used for a significant reduction of the spin tune spread
- A passive superconducting resonator (single or a few in a row) to be excited by the precessing coherent spin by mean of
- Spin mode interaction with resonance cavity can be frequently enhanced at use of the spin – orbit force through the focusing quadrupole lattice of athe synchrotron caused by the spin precession;
- Monitoring RF circuit (a loop, filter, amplifier, scope) to determine the spin-related high frequency voltage accumulated in the resonator.

Not the end of the story

but

Thank you for your attention!