

Measuring Electron and Proton Electrical Dipole Moments

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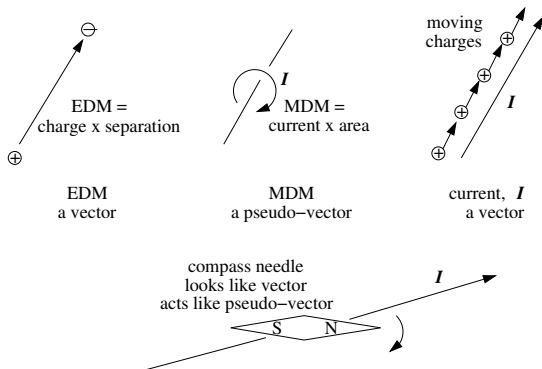
BNL “AGS Analogue” Ring as EDM Prototype

Proposed Resonant Polarimetry Test Using COSY Deuteron Beam

Most of this talk is extracted from the following papers:

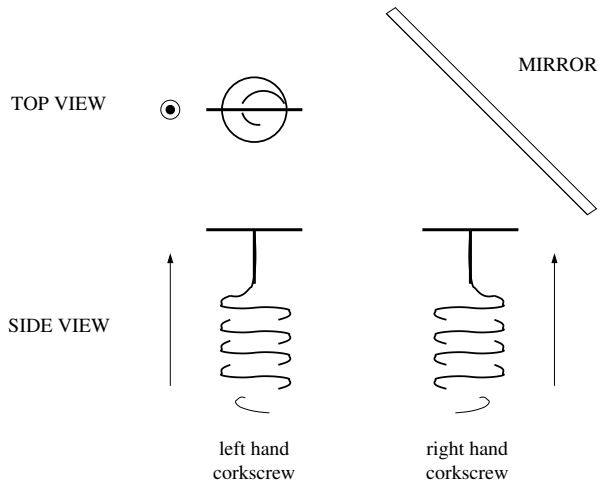
- ▶ R. and J. Talman, *Symplectic orbit and spin tracking code for all-electric storage rings*, Phys. Rev. ST Accel Beams **18**, ZD10091, 2015
- ▶ R. and J. Talman, *Electric dipole moment planning with a resurrected BNL Alternating Gradient Synchrotron electron analog ring*, Phys. Rev. ST Accel Beams **18**, ZD10092, 2015
- ▶ R. Talman, *Frequency domain storage ring method for Electric Dipole Moment measurement*, arXiv:1508.04366 [physics.acc-ph], 18 Aug 2015
- ▶ R. Talman and J. Talman, *Octupole focusing relativistic self-magnetometer electric storage ring “bottle”*, arXiv:1512.00884 [physics.acc-ph], 2 Dec, 2015

4 Force Field Symmetries: Vectors and Pseudovectors



- ▶ An electric dipole moment (EDM) points from plus charge toward minus charge—the “orientation” of a true **vector**.
- ▶ The axis of a magnetic dipole moment (MDM) is perpendicular to a current loop, whose direction gives a different “orientation”. The MDM is a **pseudo-vector**.
- ▶ Oersted effect, Ampère interpretation : how does the compass needle know which way to turn?

5 Difference Between Vector and Pseudovector



6 Capsule history of force field symmetries

- ▶ Newton: Gravitational field, (inverse square law) central force. Reflection symmetry (P) is automatic.
- ▶ Coulomb: By analogy, electric force is the same (i.e. central)
- ▶ Ampere: How can compass needle near a current figure out which way to turn? Magnetic field is **pseudo-vector**. A **right hand rule** is somehow built into E&M and into the compass needle.
- ▶ The upshot: By introducing **pseudo-vector** magnetic field, E&M respects reflection symmetry (but compound objects need not exhibit the symmetry—e.g. compass needle).

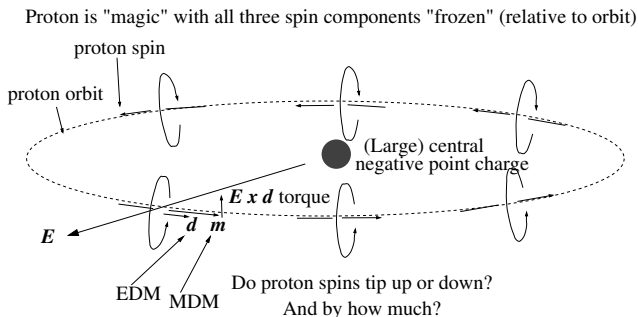
Lee, Yang, etc: A particle with spin (**pseudo-vector**), say “down”, can decay more up than down (**vector**);

- ▶ viewed in a mirror, this statement is reversed.
- ▶ i.e. weak decay force violates reflection symmetry (P).

Fitch, Cronin, etc: protons, etc. have both MDM and EDM. This violates both parity (P) and time reversal (T).

- ▶ Current task: How to exploit the implied symmetry violation to measure the EDM of proton, electron, etc?

7 EDM Sensitive Configuration; Frozen Spin Proton Orbiting Point Charge



Two issues:

- ▶ Can the tipping angle be measurably large for plausibly large EDM, such as 10^{-30} e-cm? With modern, frequency domain, technology, yes
- ▶ Can the symmetry be adequately preserved when the idealized configuration above is approximated in the laboratory? This is the main issue

8 My opinions:

- ▶ Every effort has to be made to insure symmetric (especially up-down) construction and positioning of all storage ring elements, and of injected beams.
- ▶ Nevertheless, irreducible apparatus uncertainties will *always* cause initial condition asymmetries to be large compared to the maximum possible further asymmetry caused by EDM's during runs of realistic duration.
- ▶ However, phase-locked particle trap technology will be able to measure precession *deviation* ascribable only to EDM's to significantly high accuracy during runs of realistic duration.
- ▶ Eventual precision for protons: 10^{-29} e-cm with carbon polarimetry, 10^{-30} e-cm if resonant polarimetry works.
- ▶ Eventual precision for electrons: 10^{-30} e-cm if resonant polarimetry succeeds; not competitive with molecular physics if resonant polarimetry fails.

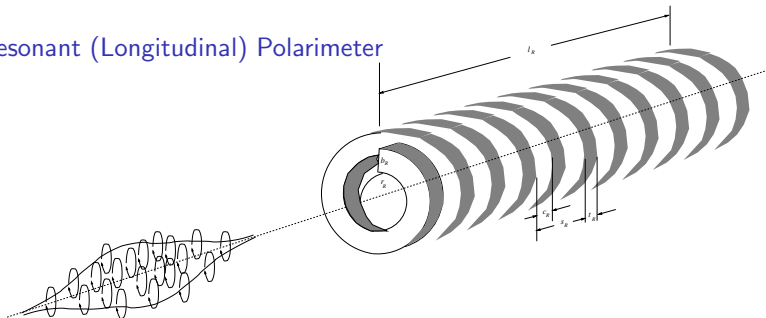
9 Why Measure EDM?

- ▶ Violations of parity (P) and time reversal (T) in the standard model are insufficient to account for excess of particles over anti-particles in the present day universe.
- ▶ Any non-zero EDM of electron or proton would represent a violation of both P and T, and therefore also CP.

Comments:

- ▶ Beam direction reversal is possible in all-electric storage ring, with all parameters except injection direction held fixed. This is crucial for reducing systematic errors.
- ▶ “Frozen spin” operation in all-electric storage ring is only possible with electrons or protons—by chance their anomalous magnetic moment values allow it. The “magic” kinetic energies are 14.5 MeV for e, 235 MeV for p.

10 Resonant (Longitudinal) Polarimeter



- ▶ Longitudinally polarized beam approaching a superconducting helical resonator.
- ▶ Beam polarization is due to the more or less parallel alignment of the individual particle spins, indicated here as tiny current loops.
- ▶ The helix is the inner conductor of a helical transmission line, open at both ends. The cylindrical outer conductor is not shown.
- ▶ High Q, (transverse) polarimetry was first proposed by Derbenev in 1993. But it has not yet been successfully demonstrated.

11 Resonant (Longitudinal) Polarimeter Response

- ▶ The Faraday's law E.M.F. induced in the resonator has one sign on input and the opposite sign on output.
- ▶ At high enough resonator frequency these inputs no longer cancel.
- ▶ The key parameters are particle speed v_p and (transmission line) wave speed v_r .
- ▶ The lowest frequency standing wave for a line of length l_r , open at both ends, has $\lambda_r = 2l_r$;

$$B_z(z, t) \approx B_0 \sin \frac{\pi z}{l_r} \sin \frac{\pi v_r t}{l_r}, \quad 0 < z < l_r. \quad (1)$$

- ▶ The (Stern-Gerlach) force on a dipole moment \mathbf{m} is given by

$$\mathbf{F} = \nabla(\mathbf{B} \cdot \mathbf{m}). \quad (2)$$

- ▶ The force on a magnetic dipole on the axis of the resonator is

$$F_z(z, t) = m_z \frac{\partial B_z}{\partial z} = \frac{\pi m_z B_0}{l_r} \cos \frac{\pi z}{l_r} \sin \frac{\pi v_r t}{l_r}. \quad (3)$$

- 12 At position $z = v_p t$ a magnetic dipole traveling at velocity v_p is subject to force

$$F_z(z) = \frac{\pi m_z B_0}{l_r} \cos \frac{\pi z}{l_r} \sin \frac{\pi(v_r/v_p)z}{l_r}. \quad (4)$$

Integrating over the resonator length, the work done on the particle, as it passes through the resonator, is

$$\Delta U(v_r/v_p) = m_z B_0 \left[\frac{\pi}{l_r} \int_{z=0}^{l_r} \cos \frac{\pi z}{l_r} \sin \frac{\pi(v_r/v_p)z}{l_r} dz \right]. \quad (5)$$

- See plot.

13 Energy lost in resonator

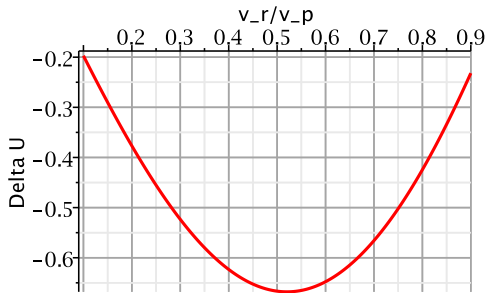


Figure: Plot of energy lost in resonator $\Delta U(v_r/v_p)$ as given by the bracketed expression in Eq. (5).

- ▶ For $v_r = 0.51 v_p$, the energy transfer from particle to resonator is maximized.
- ▶ **With particle speed twice wave speed, during half cycle of resonator, B_z reverses phase as particle proceeds from entry to exit.**

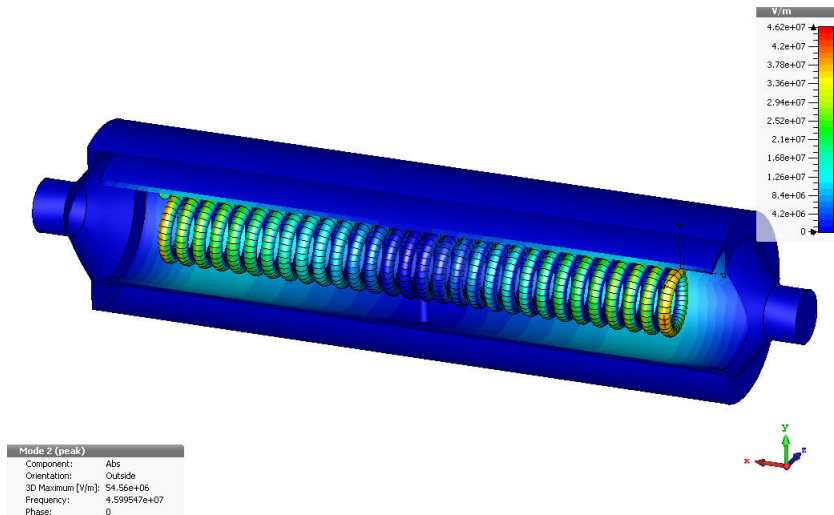
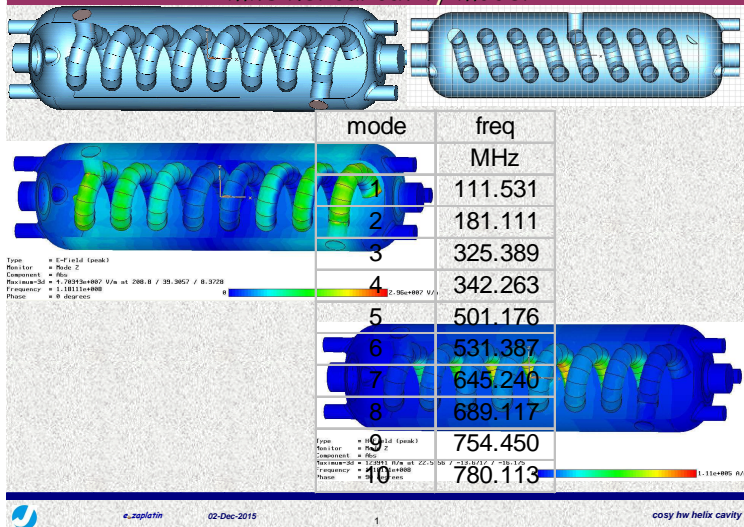


Figure: Evgeny Zaplatin (Jülich) CAD drawing for resonant polarimeter roughly matched for particle speed $\beta_p \approx 0.2$, wave speed $\beta_r \approx 0.1$.

mws helical cavity model



16 EDM Measurement Ring

(passive) resonant polarimeters

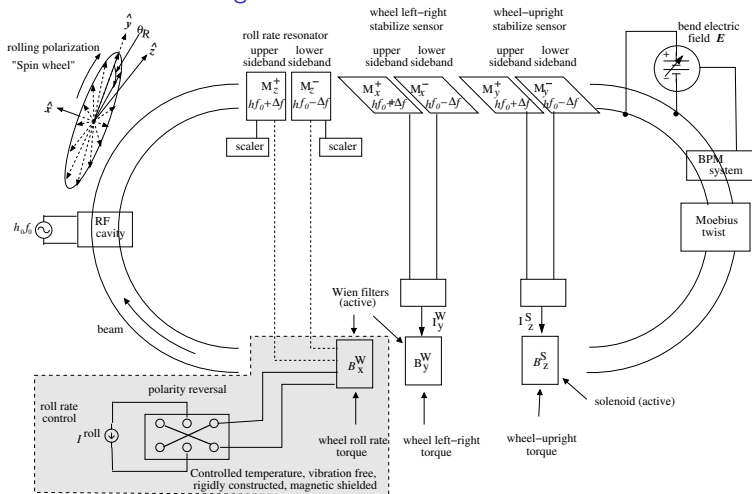


Figure: Cartoon of the EDM ring and its spin control. The Koop polarization “spin wheel” in the upper left corner “rolls” along the ring, always upright, and aligned with the orbit. The boxes at the bottom apply torques to the magnetic moments without altering the design orbit.

- ▶ Because of the rolling polarization the resonator excitation appears as upper and lower sidebands of the revolution frequency (and its harmonics).
- ▶ Elements with superscript “W” are Wien filters; superscript “S” indicates solenoid.
- ▶ **The frequency domain EDM signal is the difference between forward and backward spin wheel rotation frequencies, when the B_x^W Wien filter polarity is reversed.**
- ▶ EDM measurement accuracy (as contrasted with precision) is limited by the reversal accuracy occurring in the shaded region.
- ▶ Precision is governed by scaler precision. This is a benefit obtained by moving the EDM sensitivity from polarimeter intensity to polarimeter frequency response.

18 Spin wheel stabilizing torques

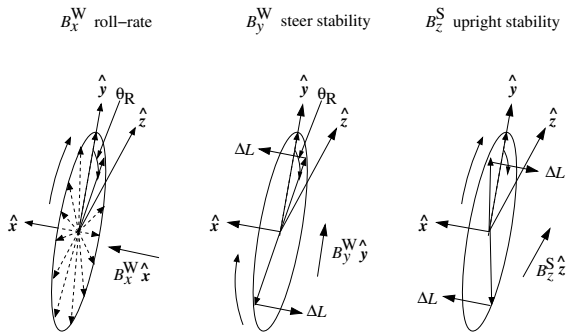


Figure: Roll-plane stabilizers: Wien filter $B_x^W \hat{x}$ adjusts the “wheel” roll rate, Wien filter $B_y^W \hat{y}$ steers the wheel left-right, Solenoid $B_z^S \hat{z}$ keeps the wheel upright.

- ▶ A Wien filter does not affect the particle orbit (because the crossed electric and magnetic forces cancel) but it acts on the particle magnetic moment (because there is a non-zero magnetic field in the particle's rest frame).

- ▶ A Wien torque

$$\hat{\mathbf{x}} \times (\hat{\mathbf{y}}, \hat{\mathbf{z}}) S = (\hat{\mathbf{z}}, -\hat{\mathbf{y}}) S$$

changes the roll-rate.

- ▶ A Wien torque

$$\hat{\mathbf{y}} \times (\hat{\mathbf{z}}, \hat{\mathbf{x}}) S = (\hat{\mathbf{x}}, -\hat{\mathbf{z}}) S$$

steers the wheel left-right.

- ▶ (Without affecting the orbit) a solenoid torque

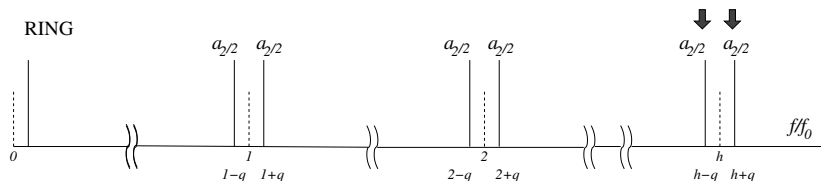
$$\hat{\mathbf{z}} \times (\hat{\mathbf{x}}, \hat{\mathbf{y}}) S = (\hat{\mathbf{y}}, -\hat{\mathbf{x}}) S$$

can keep the wheel upright.

20 Why “Rolling Polarization”? and Why the EDM Signal Survives

- ▶ Polarized “spin wheel” was proposed by Koop for different (but important) reason—to cancel $\Delta\gamma$ spin decoherence.
- ▶ Here the primary purpose of the rolling polarization is to shift the resonator response frequency *away from* harmonic of revolution frequency. (Aside: for polarimeter test with non-frozen spin this comes for free.)
- ▶ This is essential to protect the polarization response from being overwhelmed by direct response to beam charge or beam current.
- ▶ Since the EDM torque is always in the plane of the wheel its effect is to alter the roll rate.
- ▶ Reversing the roll direction (with beam direction fixed) does not change the EDM contribution to the roll.
- ▶ **The difference between forward and backward roll-rates measures the EDM (as a frequency difference).**

21 Rolling Magnetization Frequency Spectra



- ▶ Frequency spectra of the beam polarization drive signal to the resonant polarimeter.
- ▶ The operative polarimetry sideband lines are indicated by dark arrows.
- ▶ The resonator Q -value has to be high enough to resolve the side-bands.

22 Polarimeter room temperature bench test

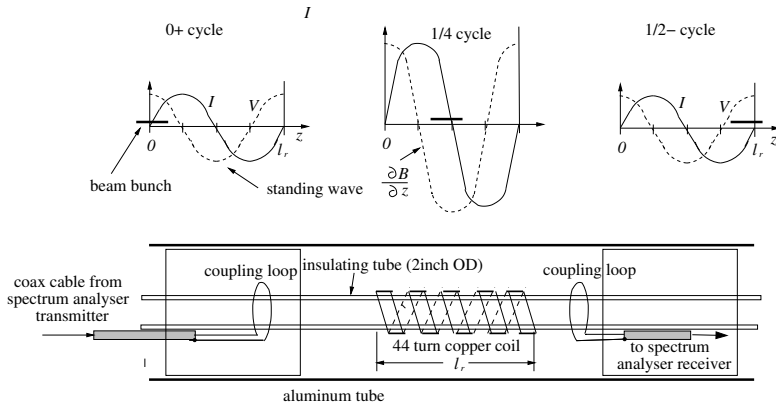


Figure: . Room temperature bench test set-up of prototype resonant polarimeter, with results shown in next figure. The coil length is $l_r=11$ inches. Beam magnetization is emulated by the spectrum analyser tracking generator (transmitter).

- ▶ Resonator excitation is detected by a single turn loop connected to the spectrum analyser receiver.
- ▶ This would be an appropriate pick-up in the true polarimetry application though, like the resonator, the preamplifier would have to be at cryogenic temperature to maximize the signal to noise ratio.
- ▶ The figures above the apparatus are intended to complete the analogy to a situation in which the transmitter is replaced by the passage of a beam bunch.
- ▶ The particle and wave speeds are arranged to maximize the energy transfer from beam to resonator.

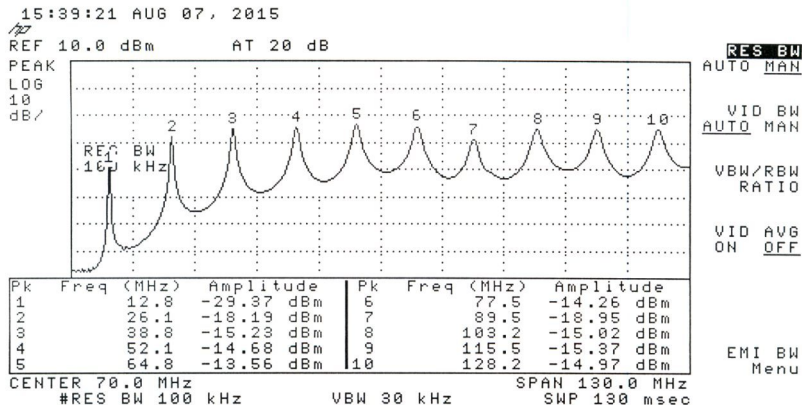
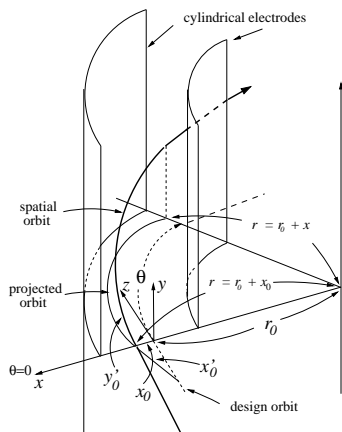


Figure: Frequency spectrum observed using the bench test shown in previous figure. Ten normal modes of the helical transmission line are visible.

25 No-Vertical-Focusing, Purely-Cylindrical Deflection



- ▶ No focusing (except geometric) pure-cylindrical field is ideal for long spin coherence time. But particles escape vertically.
- ▶ Curing this with octupole (not quadrupole) focusing enhances **self-magnetometry suppression of radial magnetic field** (the leading systematic EDM error source).

26 Self-Magnetometer Octupole-Focusing-Only Relativistic “Bottle”

- ▶ In a *purely electric* ring clockwise (CW) and counter-clockwise (CCW) orbits would be identical, irrespective of ring positioning and powering errors.
- ▶ This symmetry is guaranteed by time reversal invariance (T).
- ▶ Any average radial magnetic field error $\langle \Delta B_r \rangle$ causes both a systematic EDM error and a vertical orbit shift between CW and CCW beams.
- ▶ Canceling the shift cancels the EDM systematic error.
- ▶ Self-magnetometry measures this shift, enabling its cancellation.
- ▶ The accuracy of radial magnetic field suppression:
 $\langle \Delta B_r \rangle \approx \pm 3 \times 10^{-16}$ Tesla.
- ▶ Small enough to reduce the systematic error in the proton EDM measurement into a range where realistically small deviations from standard model predictions can be measured.

27 CW/CCW vertical orbit shift due to $\langle B_r \rangle$

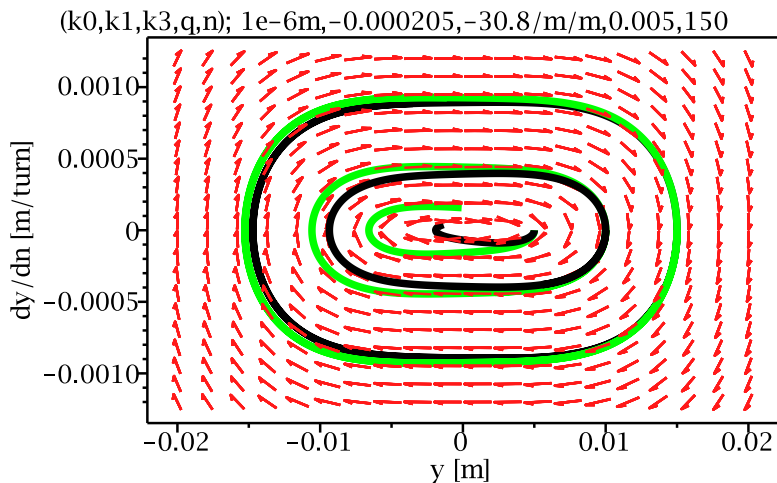


Figure: CW/CCW vertical shift in relativistic self-magnetometer electric storage ring “bottle” with only octupole focusing. Green orbits are CW, black CCW.

28 Tunes in Octupole-Focusing-Only Bottle

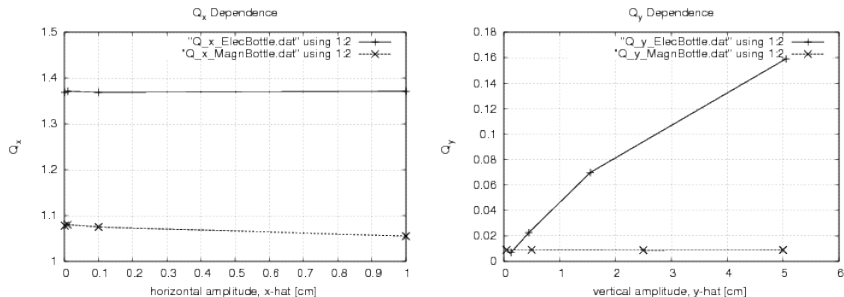


Figure: Vertical Tunes in electric, octupole-only focusing ring and conventional quadrupole-focusing storage rings: Q_x on the left, Q_y on the right. The strikingly novel feature is the near-linear dependence of Q_y on vertical amplitude in the octupole-only electric case.

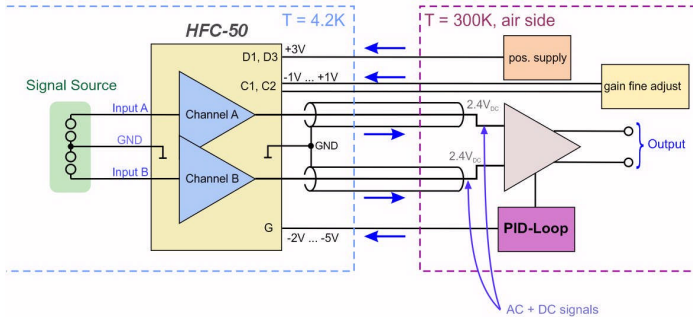


Figure: Commercial, cryogenic, low noise, high gain, dual pre-amp for transmission line signal extraction from cryogenic to room temperature environments.

30 Wien Filter

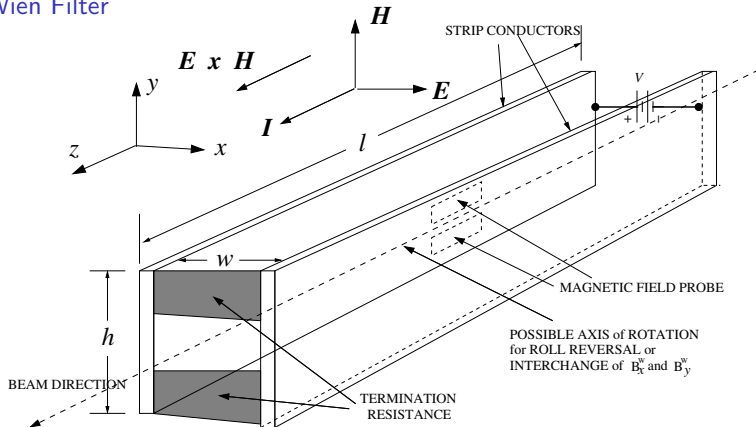


Figure: Stripline Wien filter dimensions. With electromagnetic power and beam traveling in the same direction, the electric and magnetic forces tend to cancel. Termination resistance R is adjusted for exact cancelation. For rolling polarization reversal, the Wien filter current to be reversed will be a conveniently low value, such as 5 A.

31 Wien reversal current bridge monitor

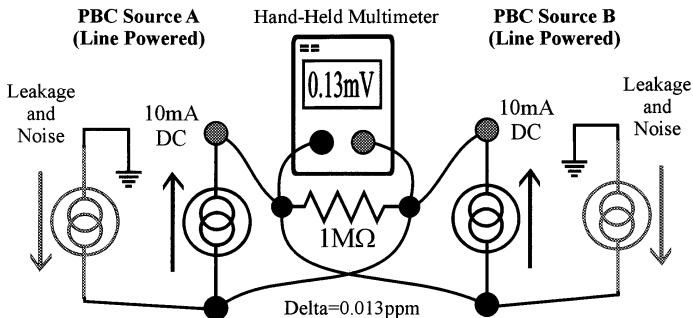


Figure: Current bridge used for high precision current monitoring. Copied from CERN, PBC reference. One current is the active current, the other a highly stable reference current. Even hand-held, 1 part in 10^8 precision is obtained. Wien current reversal precision will be monitored every run by recording the potentiometer voltage with Wien current in one arm and standard current in the other.

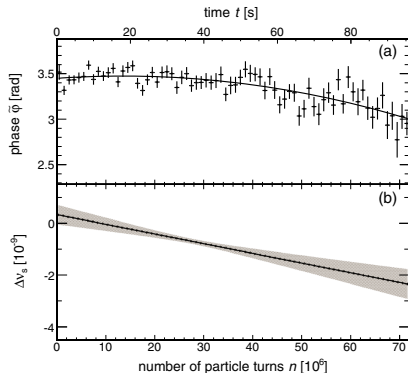


FIG. 3 (color online). (a) Phase $\tilde{\varphi}$ as a function of turn number n for all 72 turn intervals of a single measurement cycle for $|\nu_s^{\text{fix}}| = 0.160\,975\,407$, together with a parabolic fit. (b) Deviation $\Delta\nu_s$ of the spin tune from ν_s^{fix} as a function of turn number in the cycle. At $t \approx 38$ s, the interpolated spin tune amounts to $|\nu_s| = (16\,097\,540\,628.3 \pm 9.7) \times 10^{-11}$. The error band shows the statistical error obtained from the parabolic fit, shown in panel (a).

33 Achievable Precision (i.e. not including systematic error) say for electron

EDM in units of (nominal value) 10^{-29} e-cm = \tilde{d}

$2 \times \text{EDM}(\text{nominal})/\text{MDM}$ precession rate ratio:

$$2\eta^{(e)} = 0.92 \times 10^{-15} \approx 10^{-15}$$

duration of each one of a pair of runs = T_{run}

smallest detectable fraction of a cycle = $\eta_{\text{fringe}} = 0.001$

N_{FF} = EDM induced fractional fringe shift per pair of runs

$$= \frac{(2\eta^{(e)})\tilde{d}}{\eta_{\text{fringe}}} h_r f_0 T_{\text{run}} \quad \left(\begin{array}{l} \text{e.g.} \\ \approx \end{array} \tilde{d} \frac{10^{-15} \cdot 10 \cdot 10^7 \cdot 10^3}{10^{-3}} = 0.1\tilde{d} \right),$$

Assumed roll rate reversal error : $\pm \eta^{\text{rev.}} \stackrel{\text{e.g.}}{=} 10^{-10}$

$\sigma_{FF}^{\text{rev.}}$ = roll reversal error measured in fractional fringes

$$= \pm \frac{f^{\text{roll}} \eta^{\text{rev.}} T_{\text{run}}}{\eta_{\text{fringe}}} \quad \left(\begin{array}{l} \text{e.g.} \\ \approx \end{array} \pm \frac{10^2 \cdot 10^{-10} \cdot 10^3}{10^{-3}} = 10^{-2} \right).$$

particle	$ d_{\text{elec}} $ current upper limit e-cm	excess fractional cycles per pair of 1000 s runs	error after 10^4 pairs of runs e-cm	roll reversal error e-cm
neutron	3×10^{-26}			
proton	8×10^{-25}	$\pm 8 \times 10^3$	$\pm 10^{-30}$	$\pm 10^{-30}$
electron	10^{-28}	± 1	$\pm 10^{-30}$	$\pm 10^{-30}$

34 Resonator Test Considerations

Resonator cannot be built until test configuration is chosen !

Longitudinal or transverse?

- ▶ I favor longitudinal—but both are needed eventually.

Electron:

- ▶ Three orders of magnitude larger MDM.
- ▶ Linac or circular? Probably circular—linac requires difficult rolling-polarization development, and polarization signal is weaker than circular.
- ▶ (Non-frozen) longitudinal spin component oscillates at frequency (spin tune) favorably far from revolution harmonics.

Proton or deuteron?:

- ▶ Deuteron signal is weak.
- ▶ Same (non-frozen, adjustable) spin tune advantage
- ▶ But availability of simultaneous carbon scattering polarimetry at COSY would be enormously helpful for initial resonant polarimetry commissioning.

Conclusion: Probably circular, polarized electron beam is simplest, especially if

- ▶ polarization is high,
- ▶ sufficiently long spin coherence time.
- ▶ can be rotated into horizontal plane,
- ▶ there is experience with resonant detection,
- ▶ co-existing polarimetry exists.

35 Time Permitting

36 Long Term EDM Program

- ▶ Design and build resonant polarimeter and circuitry
- ▶ Confirm (longitudinal, helical) resonant polarimetry using polarized electron or proton beam, or (currently most promising) the same polarized deuteron beam at COSY, Jülich, shown in earlier slide.
- ▶ Confirm (transverse, e.g. TE101) resonant polarimetry using polarized electron or proton beam
- ▶ Build 5 m diameter, 14.5 MeV electron ring (e.g. at Wilson Lab, J-Lab, Bonn, etc.)
- ▶ Measure electron EDM
- ▶ Attack electron EDM systematic errors
- ▶ Meanwhile, same program as above for 235 MeV protons in 40 m radius, all electric ring (e.g. at BNL, FNAL, COSY, etc.)

37 Conclusions Regarding EDM Measurement

- ▶ Successful application of the method depends on two not yet established experimental methods: resonant polarimetry (promising theoretically) and “rolling polarization trap” operation—meaning stable, phase-locked, rolling polarization operation—(promising experimentally, at COSY).
- ▶ It is thermal noise in the resonant polarimeter that limits EDM measurement *precision*.
- ▶ A successful single beam fill will include at least one forward/backward reversal of the roll (not beam) direction, with roll frequency precisely measured both before and after.
- ▶ One (of many) successful single beam EDM measurements will consist of four data sets, roll forward and backward, with EDM effect on (spin wheel vertical) and off (spin wheel horizontal “background” measurement).

- ▶ Almost all AC magnetic field effects cancel. Only pure DC (or rather less than 0.01 Hz) ΔB_r error field gives a spurious EDM signal.
- ▶ Only CW/CCW beam direction reversal (with suppressed relative vertical displacement) can reduce this systematic error. Preferable, but not absolutely necessary, for the beams to be present simultaneously.
- ▶ Alternate runs with beam direction reversed EDM systematic error due to residual radial magnetic field.
- ▶ Expressed as EDM upper limit, measurement *precision* of 10^{-30} e-cm after year-long running, for either electrons and protons, can be expected.

39 BNL “AGS Analogue” Ring as EDM Prototype

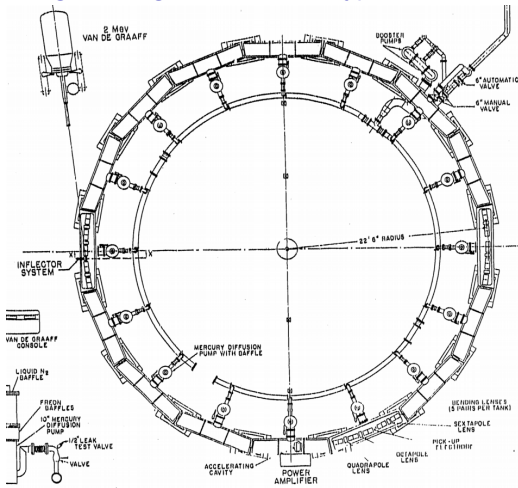


Figure: The 1955 AGS-Analogue lattice as reverse engineered from available documentation—mainly the 1953 BNL-AEC proposal letter. Except for insufficient straight section length, and the 10 MeV rather than 14.5 MeV energy, this ring could be used to measure the electron EDM.

Field strength (magnetic type)	
at injection	10.5 gauss
at 10 MeV	74 gauss
Field strength (electrostatic type)	
at injection	3 kV/cm
at 10 MeV	22 kV/cm
Rise time	.01 sec
Phase transition energy	2.8 MeV
Frequency (final)	7 mc
Frequency change	5% %
Volts/turn	150 V
RF power	about 1 kw
No. of betatron wavelengths	about 6.2
aperture	1 X 1 in.
Betatron amplitude for 10^{-3} rad. error	0.07 in.
Maximum stable amplitude, synchrotron osc.	-0.16 in.
Radial spacing of betatron resonances	about 0.4 in.
Vacuum requirement	about 10^{-6} mm Hg

Total power requirements will be small and available with existing installations. The test shack seems to be a suitable location since the ring will be erected inside a thin magnetic shield which can be thermally insulated and heated economically.

We estimate the cost to be approximately \$600,000, distributed as shown in the following table:

<u>Model</u>	<u>Direct</u>	<u>Overhead</u>	<u>Total</u>	<u>Inflate to 2015</u>
Staff S. & W.	\$135,000	\$ 65,000	\$200,000	\$M 1.76
Van de Graaff	70,000	-	70,000	0.62
Other E. & S.	130,000	-	130,000	1.14
Shops	<u>135,000</u>	<u>65,000</u>	<u>200,000</u>	<u>1.76</u>
	\$470,000	\$130,000	\$600,000	\$M 5.27

41 Resonant Polarimetry Test Using COSY Deuteron Beam

Table: An earlier slide showed spin phase stability to better than 2π for about 100 s time duration, long enough for EDM measurement.

experiment			TEST	TEST	TEST(pessimistic)	TEST(optimistic)
parameter	symbol	unit	proton	deuteron	deuteron	deuteron
bunches/ring	N_b		116	1	10	10
beam			COSY	COSY	COSY	COSY
conductor			SC	SC	SC	SC
RF solenoid			✓	✓	✓	✓
magnetization freq.	F_{roll}	MHz	0.12	0.12	0.12	0.12
free or driven?			free	free	free	free
ring frequency	f_0	MHz	9.804	0.75		
magnetic moment	μ	eV/T	0.88e-7	0.2703e-8		
β	β		0.6	0.434		
resonator frequency	f_r	MHz	$116f_0 \pm 0.12$	$110f_0 \pm 0.12$		
resonator radius	r_r	cm	1.80	2		
resonator length	l_r	m	2	0.920		
temperature	T	° K	4	10	10	4
phase velocity/c	β_r		0.408	0.22		
quality factor	$Q_{res.}$		1e6	1e6	1e6	1e7
response time	$Q_{res.}/f_r$	s	0.0088	0.012		0.12
beam current	I	A	0.001	0.0001	0.0001	0.001
particles	N_e		6.4e9	0.832e8	0.832e8	0.832e9
particles/bunch	N_e/N_b		5.5e7	0.832e8	0.832e7	0.832e8
magnetic field	H_r	Henry	1.15e-8	0.90e-9		0.90e-7
resonator current	I_r	A	2.6e-9	0.90e-10		0.90e-8
magnetic induction	B_r	T	1.4e-14	1.13e-15		1.13e-13
max. resonator energy	U_r	J	3.8e-25	1.19e-27		1.19e-23
noise energy	$\overline{U_m}$	J	2.8e-23	6.9e-23		
S/N(ampl.)	$\sqrt{U_r/\overline{U_m}}$	$s^{-1/2}$	0.117	0.0042	0.0042	0.66
S/N(synch.)	$(S/N)\sqrt{f_0}$		phase-lock?	phase-lock?	phase-lock?	phase-lock?
S/N(lock-in)	$(S/N)\sqrt{f_0}$		$1248\sqrt{T[s]}$	$37.7\sqrt{T[s]}$	$37.7\sqrt{T[s]}$	$5960\sqrt{T[s]}$
S/N(2pol-coh.)	$(S/N)\sqrt{f_0}$		$>> 1248\sqrt{T[s]}$	$>> 37.7\sqrt{T[s]}$	$>> 37.7\sqrt{T[s]}$	$>> 5960\sqrt{T[s]}$