# Coupled transient thermal and electromagnetic finite element analysis for the simulation of quench effects in superconducting magnets

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Coupled nonlinear transient thermal, electromagnetic and circuit models have been developed for the simulation of quench in superconducting magnets. Finite elements methods are used for the thermal and electromagnetic fields simulations. The simulations are closely coupled because the heat sources for the thermal model are determined by the electromagnetic and circuit models. The strongly anisotropic thermal conductivity of typical superconducting coil structures and the highly non-linear material characteristics require a matching anisotropic mesh and adaptive time integration procedures.

## 1. Introduction

Superconducting magnets probably quench because micron movements of the wire cause local heating that raises the superconductor above its critical temperature. This resistive, normal zone will propagate through a low temperature superconducting coil if the conductor is not cryogenically stable. Thermal conduction causes the quench to spread and ohmic heating in the normal zone provides an expanding heat source. Once the total resistance of the normal zone is appreciable, or when a protection circuit switches in, the coil current begins to decay. The current decay causes a changing magnetic field that generates rate dependent losses from eddy currents in the twisted multi-filamentary superconductor; this heat source exists even in the superconducting zone of the coil. The rise in temperature in the windings and the internal voltages developed during this quench process are a critical issue for magnet safety, in addition the eddy currents induced in support structures during a quench may result in large Lorentz forces that can cause damage.

Approximate adiabatic models have been used to achieve good results for the time profile of the current decay [1]. More accurate methods based on transient thermal finite element simulations have also been used to obtain temperature and voltage distributions [2]. To implement a complete solution to quench modelling, including rate dependent losses and eddy currents in the structure, a non-linear thermal simulation has been closely coupled to a non-linear transient electromagnetic simulation, including electric circuits.

## 2. Physical model

Superconducting magnets typically consist of several coils each formed from a few turns (tens) of insulated cable or many turns (thousands) of insulated wire. The coils are often vacuum impregnated with filled resin or wax to create a solid self supporting structure. They may be wound onto metal formers that can be part of the protection system; eddy currents in the formers will generate losses during a quench and may heat the surface of a coil above its critical temperature.

Although it would be possible to exactly model the details of the wire, insulation and filler in a coil with a few turns, it is impractical for a magnet with thousands of turns. Macroscopic models for the average specific heat can be easily derived from the volume fractions of the specific heat of the materials used in the coil. However at low temperatures the average macroscopic anisotropic thermal conductivity cannot be derived from simple models of the structure, heat is conducted by phonons and these are reflected by material boundary interfaces. Measurements are the only reliable source of macroscopic thermal conductivity data for the composite coil structure.

The coils forming a magnet are often connected in series to a power supply or to a superconducting switch used for persistent mode operation. Protection circuits are used to control the behaviour of the magnet during a quench, these consist of resistors or diodes connected in parallel to each coil. They provide an alternative current path and dump energy outside the coil. The quenching may also be detected and active protection methods applied, such as discharging current into heaters that are attached to the coils in order to raise their temperature and make them resistive.

## 3. Algorithm and methods

The equation defining a non-linear transient temperature fields is

$$\rho C(T)\frac{dT}{dt} - \nabla.\kappa(T)\nabla T = Q \qquad (1)$$

where $T$ is the temperature, $\rho$ the mass density, $C$ the specific heat, $\kappa$ the thermal conductivity tensor and Q the volume heat source. The weak form of the transient thermal equation after applying the Galerkin method is shown in equation 2.

$$\int_{\Omega} W\rho C(T)\frac{dT}{dt}d\Omega + \int_{\Omega}\nabla W.\kappa(T)\nabla T\,d\Omega$$
$$= \int_{\Omega} Q\,d\Omega - \int_{\Gamma} W\kappa(T)\nabla T \cdot d\Gamma \qquad (2)$$

In the context of quench simulations, the interesting feature of equation 2 is the heat source $Q$ and the strongly non-linear thermal conductivity, specific heat and boundary condition terms that can be used represent heat transfer to helium liquid or gas.

The low frequency limit of Maxwell's equation describing transient electromagnetic fields is

$$\nabla \times \frac{1}{\mu}\nabla \times \vec{A} + \sigma\left(\frac{d\vec{A}}{dt} + \nabla V\right) = \vec{J} \qquad (3)$$

where $\vec{A}$ is the magnetic vector potential, $V$ an electric scalar potential that may be conveniently used, $\mu$ is the magnetic permeability tensor, $\sigma$ the electrical conductivity tensor and $\vec{J}$ is a prescribed current density. Accurate, fast calculation of the flux density in the superconducting coils is essential for this application so that the non-linear terms in the coupled thermal equation can be reliably predicted. An algorithm including the coils in the finite element mesh model was therefore constructed. In the macroscopic model of the coil, the prescribed current density $\vec{J}$ in equation 3 must be related to the current $I$ flowing in the turns of the coil. Equation 4 shows the relationship between the coil current $I_p$ in coil $p$ and the required current density $\vec{J}$, in terms of the vector turns density $\vec{N}_p$.

$$\vec{N}_p I_p = \vec{J} \qquad (4)$$

An edge variable finite element method [3] was used to solve the electromagnetic field equation. The coil turns density is known from the winding configuration, it can be converted to a turns density vector using the winding configuration and this turns density vector must then be placed in the correct function space for the edge vector elements.

The electromagnetic field formulation has been coupled to a circuit model that represents the protection circuit. This was achieved using loop equations to model the protection circuits. A typical, idealised circuit loop equation for a loop containing coil $p$ is shown in equation

5, where $\Omega_p$ is the volume of the coil, $\vec{E}_j$ is an edge shape function in the coil volume, $R$ is a resistance in the loop and $V$ is a voltage in the loop.

$$\iiint_{\Omega_p}\vec{N}_p.\vec{E}_j\frac{d\vec{A}_j}{dt}d\Omega_p + I_p R = V \qquad (5)$$

In this application the resistance $R$ may be composed of terms from external protection circuit resistors or diodes and terms from the developing normal zones in the coil.

*Coupling*

When the superconducting wire at some position in a coil rises above its critical field and temperature it becomes resistive. The resistance $R$ in the loop equations therefore includes a term $R_p$ evaluated from a volume integral over the resistive parts of all coils as shown in equation 6.

$$R_p = \int_{\Omega_p}\frac{|N_p|}{a}\left(\frac{1}{f_{Cu}\sigma_{cu}(B,T) + f_{SC}\sigma_{SC}(B,T)}\right)d\Omega \qquad (6)$$

Equation 6 includes $a$ the cross sectional area of the superconducting wire, the fraction of each material $f$ and the material conductivity which may be a function of the flux density and temperature. In practise the implementation allows any number of materials to be included in a wire and their properties may be functions of other variables if required.

The heat source density at any resistive point in a coil has a contribution from ohmic heating as shown in equation 7.

$$Q_{ohmic} = \frac{|N_p|}{a}\left(\frac{1}{f_{Cu}\sigma_{cu}(B,T) + f_{SC}\sigma_{SC}(B,T)}\right)I_p^2 \qquad (7)$$

Losses are also present in superconductors whenever there is a time dependent flux density. A number of different mechanisms are required to describe this loss, however the most significant component during the quench process is related to eddy currents flowing between superconducting filaments through the resistive copper matrix. Approximate methods of analysis have been used to derive the form of this loss [4] and these results have been compared with measurements. An accepted expression for this "rate dependent" loss is shown in equation 8, where $\lambda$ is the volume fraction of superconductor in the winding and $d$ is the diameter of the superconducting filaments.

$$Q_{rate} = \frac{2}{3\pi}\lambda J_c(T,B)d\frac{dB}{dt} \qquad (8)$$

The software implementation was designed so that any form of heat source could be supplied, dependent on the flux density, its time derivative, the temperature and the current in the wire.

### 4. Implementation

The coupled equations described above were implemented in the OPERA-3d program using components of its ELEKTRA and TEMPO modules. Facilities to model any functional material property were available in the program using its internal algebraic interpreter. These facilities were extended by adding interpolation in multidimensional tabulated data to support the quench requirements. This extendable feature allows properties such as electrical conductivity to be made a function of any physical variables that may be required, in the example shown above this includes flux density and temperature.

OPERA-3d provides a range of standard coil descriptions and the field from these coils is evaluated by Biot-Savart integration. Facilities to automatically convert these coils to meshed volumes with known winding orientation were added, including options to generate anisotropic mesh sizes linked to the winding orientation. This feature was added to overcome the normal limitation of automatic tetrahedral meshing that produces an isotropic mesh size distribution.

The transient thermal and electromagnetic field simulations were implemented using an adaptive Galerkin time stepping method. Newton Raphson non-linear equation solution methods were included, however the highly non-linear material properties caused a strong interaction between the adaptive time stepping and non-linear equation solution. The effect appeared as the adaptive time step attempted to increase, this caused larger temperature changes in a time step and hence the non-linear thermal equations became increasingly expensive to solve. The non-linear thermal equations produce an asymmetric Jacobian matrix, another reason for the cost of the non-linear solution being excessive. The most efficient solution was obtained by using adaptive time stepping based on a comparison of two half steps with a full step, updating the material characteristics after each half time step.

### 5. Results

Analysis was performed on the PT55 Target Magnet. The magnet was designed and manufactured at the Rutherford Appleton Laboratory [6]. It consisted of 4 coils each independently protected by diode rings placed in parallel with the coils, clamping the terminal voltage of the each coil to a maximum of 2 Volts. The anisotropic thermal conductivity of the coil was measured. The magnet produced a central field of 2.5T and was wound with a wire that had a 2:1 copper to superconductor ratio and 10 micron diameter superconducting filaments. The coil & former geometry is shown in Fig. 1.
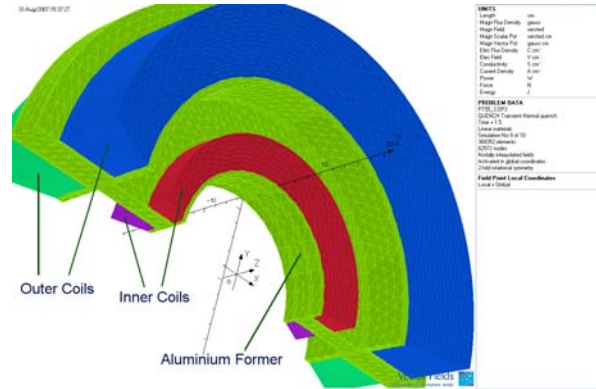


*Fig. 1. PT55 Target Magnet Coil and former arrangement.*

When this magnet was designed the available quench analysis tools were not capable of including rate dependent losses. During the design phase the losses were estimated from adiabatic quench calculations and it was clear that these losses would cause temperature rises that raised large volumes of the coils above their critical temperature. Quench times were estimated for the coils and the adiabatic model was used to evaluate the safety of the coil system.

A first analysis of the PT55 polarised target magnet using the new QUENCH Solver was presented in [5]. This initial analysis included the coupling of circuit equations characterising the 4 coils with the transient thermal equations to predict the propagation of the quench, but ignored the presence of coil formers and hence the effect of former-induced eddy currents on the quench velocities. The analysis did, however, include the effect of rate dependent losses: without rate dependent losses a quench should not spread to other coils. However, as confirmed by measurements, the results clearly showed that all coils became resistive.

The present results advance the analysis one step further, introducing not only rate dependent loss but also the coupling of all three components of the QUENCH solver, namely circuit equations, transient thermal and transient electromagnetic equations in order to accurately predict the effect of the coil former induced eddy currents. Fig. 2 shows the prediction of eddy currents in the coil former at one particular instant while Fig. 3 shows the temperature distribution in the coils at time=0.1s and 1 sec, illustrating the spread of the quench over the highly anisotropic conductor volume. The quench spreads to other coils as a result of rate dependent losses in the coils as well as heat conduction through the aluminium former. However, it is always possible that a random movement in the coils forced by the rapid change in the stress distribution could have affected the real magnet.   The calculated and measured current decay in the PT55 magnet following the quench is shown in Fig. 4.
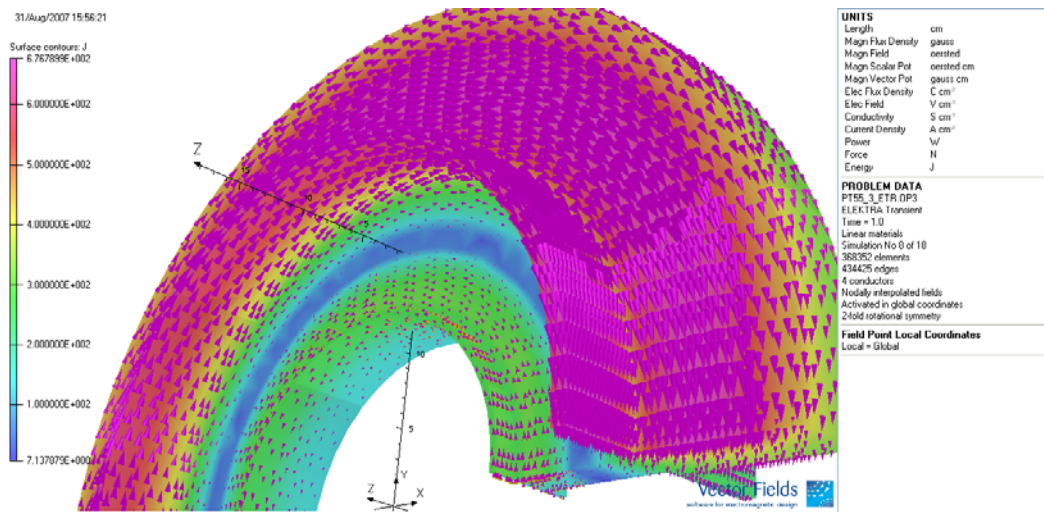
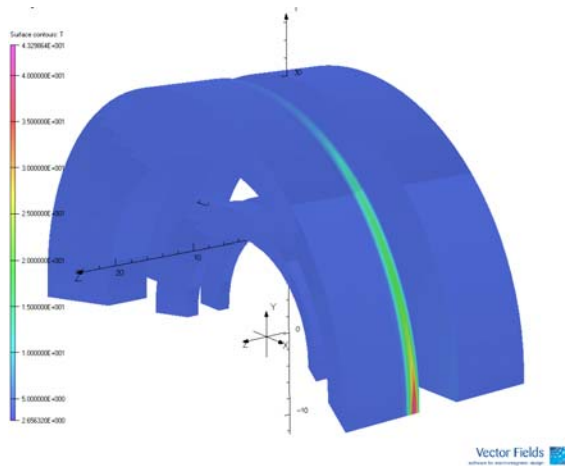*Fig. 2. Eddy currents in the Aluminium Former (Time=1 sec).*
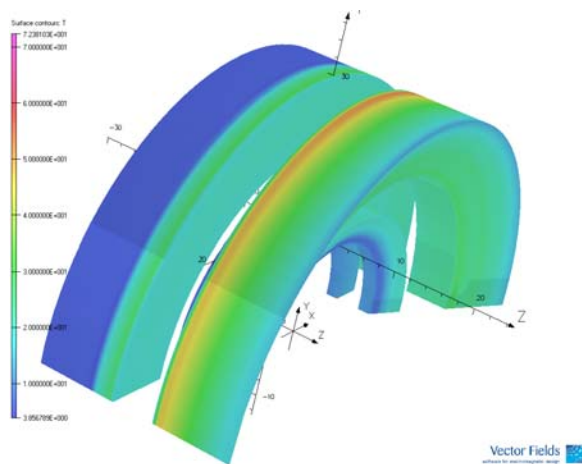


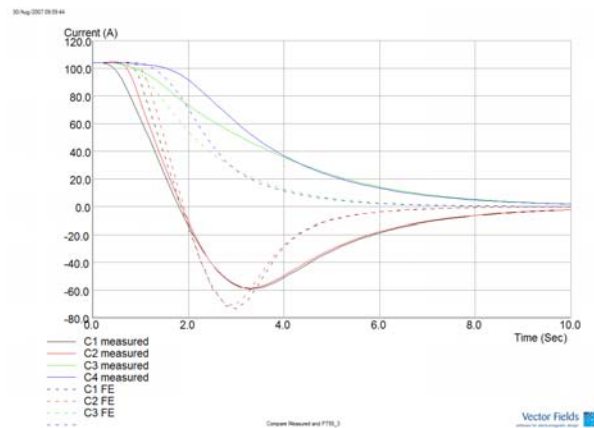*Fig. 3. (a) Temperature at Time=0.1 sec.*



*Fig. 4. Calculated and measured decay of the current in the PT55 magnet coils.*

## 6. Conclusions

Strongly coupled transient finite element thermal, electromagnetic and circuit simulations have been implemented to simulate quench in superconducting magnets. Efficient solutions to the non-linear equations were achieved by using material property updates after each partial step in an adaptive time integration procedure. Results have been compared with measurement for a multi-coil magnet system, the simulations demonstrate how *dB/dt* losses in the coils as well as the heat conduction through the aluminium former determine the quench behaviour of the coils. The agreement with measurement is very satisfactory.

## References

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*Fig. 3. (b) Temperature at Time=1 sec.*

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