Determination of the Polarization Observables C_x, C_z, and P_y for the Quasi-Free Mechanism in $\vec{\gamma} d \to K^+ \vec{\Lambda} n$

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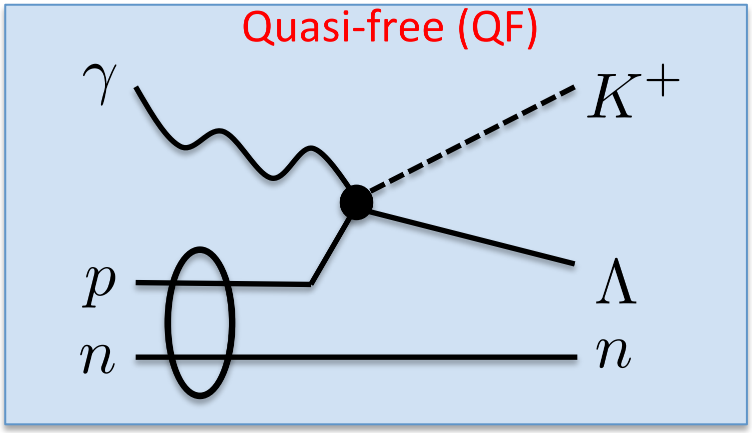
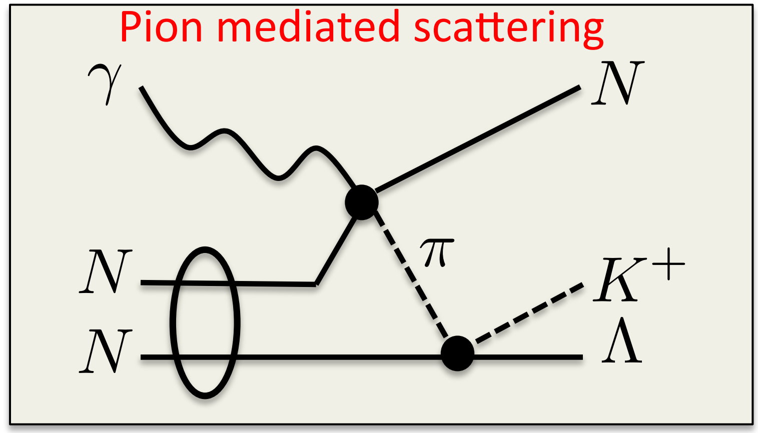
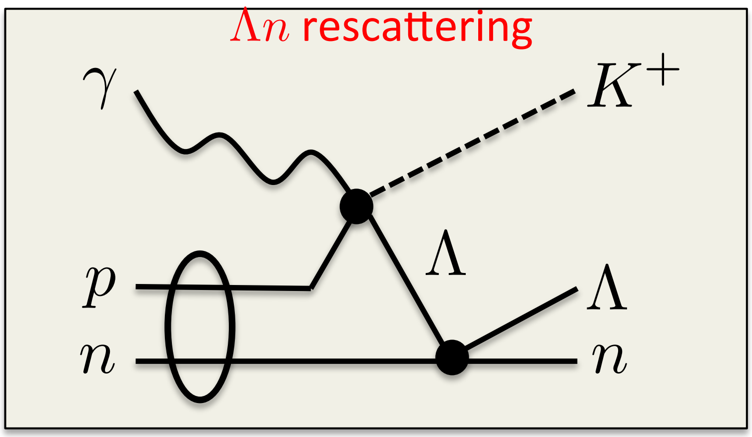
WHERE DISCOVERIES BEGIN

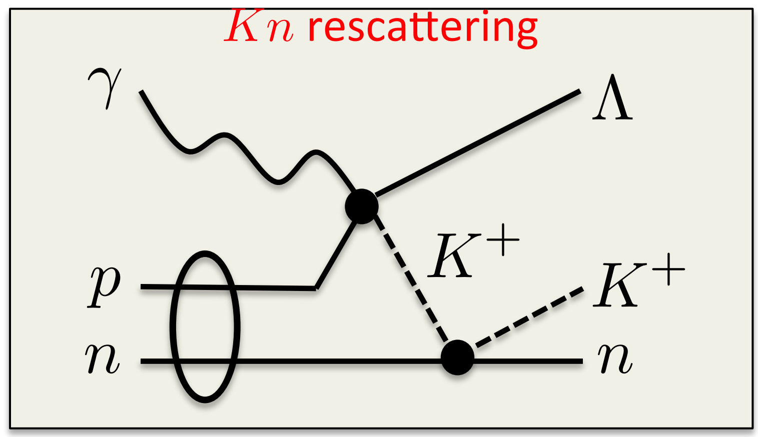
clas

Main Mechanisms of $\overrightarrow{\gamma} d \to K^+ \overrightarrow{\Lambda} n$







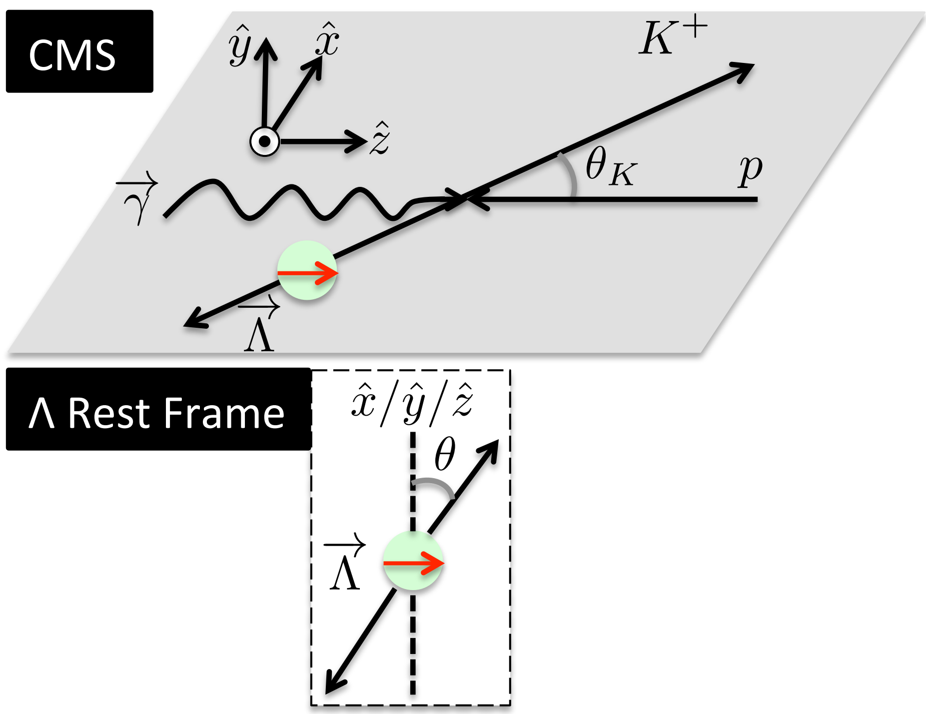


Experimental Observables

Helicity-dependent polarized differential cross section for hyperon photoproduction off the nucleon.

$$\frac{d\sigma^{\pm}}{d\Omega} = \frac{d\sigma}{d\Omega}_{|unpol} (1 \pm \alpha P_{circ} C_x \cos\theta_x \pm \alpha P_{circ} C_z \cos\theta_z + \alpha P_y \cos\theta_y)$$

Λ self-analyzing power: $\alpha = 0.642 \pm 0.013$



$$\begin{cases} \hat{z} = \hat{p}_\gamma \\ \hat{y} = \frac{\hat{p}_\gamma \times \hat{p}_K}{|\hat{p}_\gamma \times \hat{p}_K|} \\ \hat{x} = \hat{y} \times \hat{z} \end{cases}$$

The g13a Experiment

- Circularly polarized photon beam
- E_e = 2 GeV; 2.65 GeV
- Electron beam polarization: up to 85%
- Photon beam polarization: [27%, 80%]
- Target: LD_2, unpolarized, 40 cm long, upstream of CLAS center

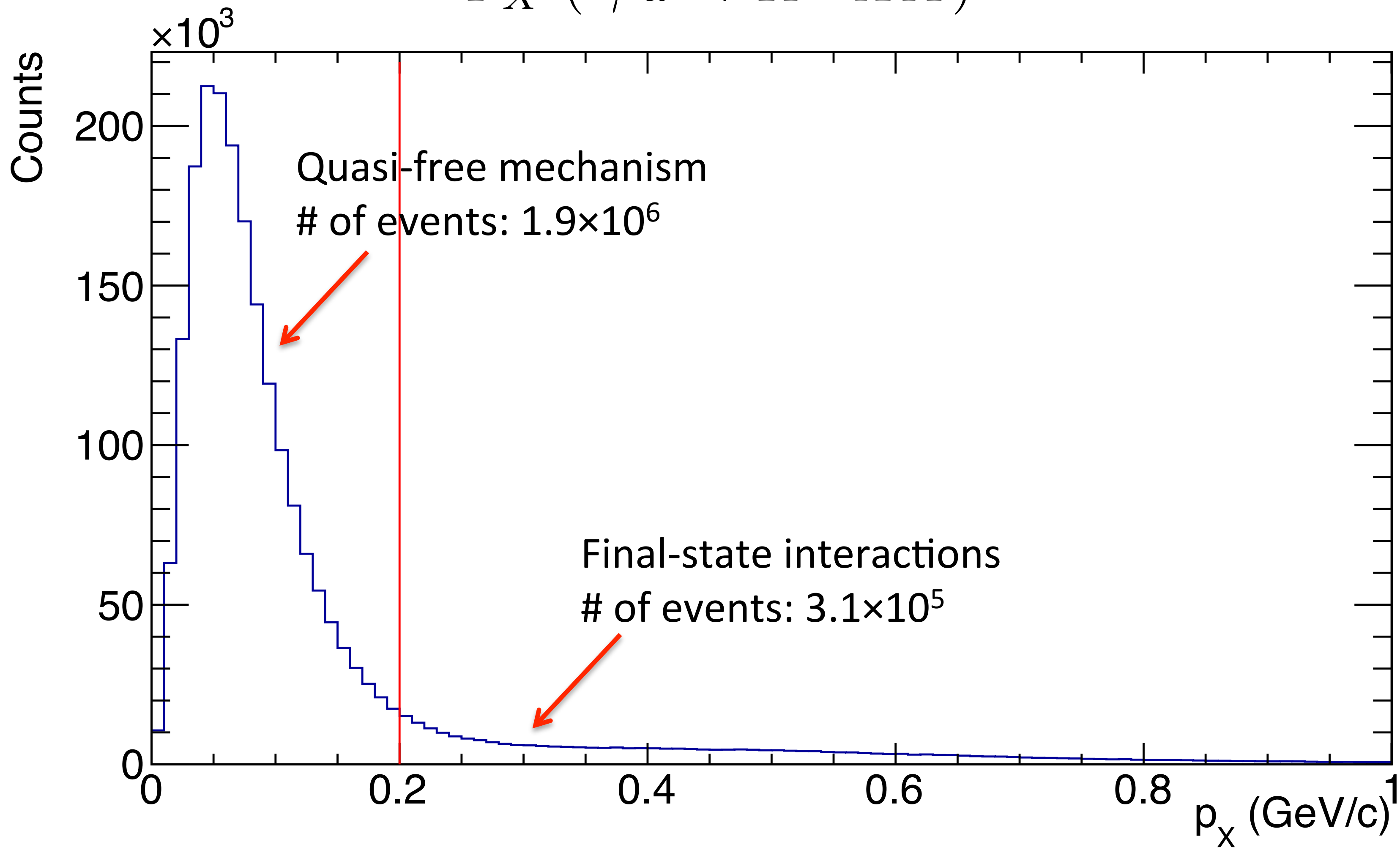
$$\vec{\gamma}\, d \rightarrow \boxed{K^+}\, \vec{\Lambda}\, n$$

$$\Lambda \rightarrow \boxed{p\pi^-}$$

Detected in CLAS

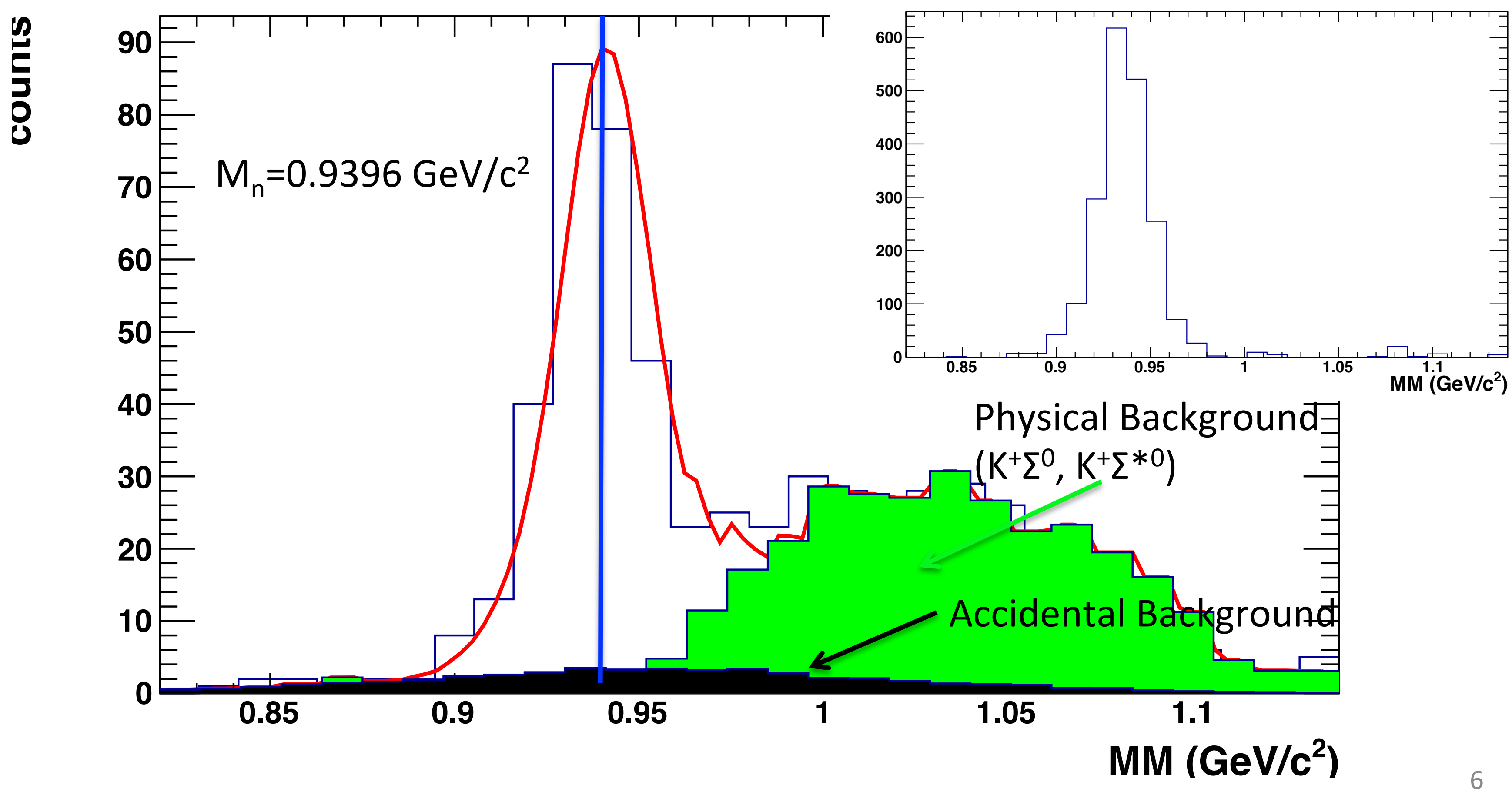
Selection of Quasi-Free Events

$$P_X\ (\overrightarrow{\gamma} d \rightarrow K^+ \overrightarrow{\Lambda} X)$$



Background Subtraction

$$MM = \sqrt{(\widetilde{p}_{\gamma} + \widetilde{p}_{d} - \widetilde{p}_{K^+} - \widetilde{p}_{p} - \widetilde{p}_{\pi^-})^2}$$



Observable-Extraction Methods

- One-dimensional fit:

$$Asym = \frac{Y^+ - Y^-}{Y^+ + Y^-} = \frac{\int\int \frac{d\sigma^+}{d\Omega} d(cos\theta_y) d(\cos\theta_{z/x}) - \int\int \frac{d\sigma^-}{d\Omega} d(cos\theta_y) d(\cos\theta_{z/x})}{\int\int \frac{d\sigma^+}{d\Omega} d(cos\theta_y) d(\cos\theta_{z/x}) + \int\int \frac{d\sigma^-}{d\Omega} d(cos\theta_y) d(\cos\theta_{z/x})} = \alpha P_{circ} C_{x/z} \cos\theta_{x/z}$$

- Two-dimensional fit:

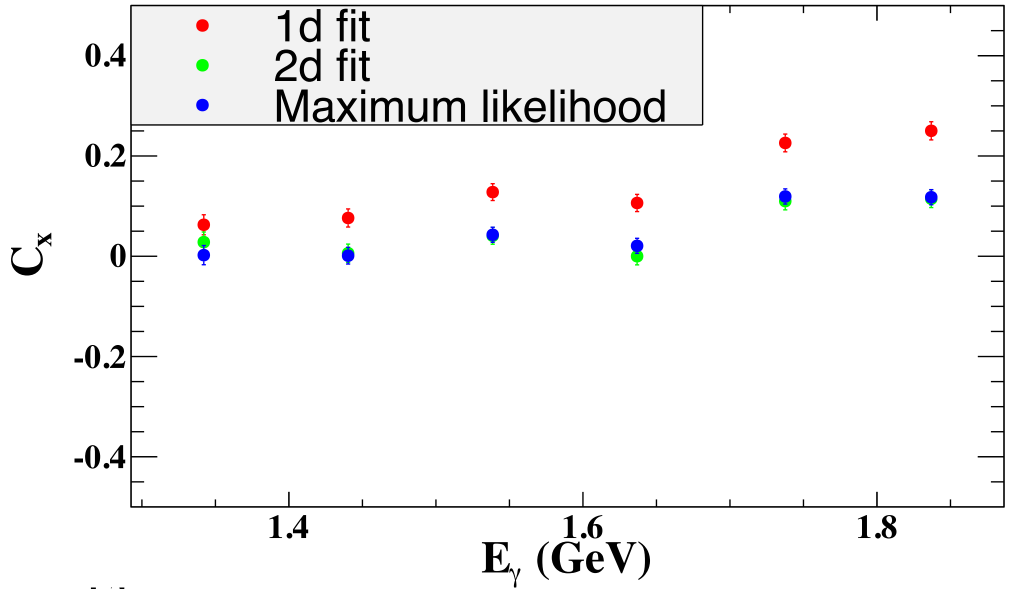
$$Asym = \frac{Y^+ - Y^-}{Y^+ + Y^-} = \frac{\int \frac{d\sigma^+}{d\Omega} d(cos\theta_y)) - \int \frac{d\sigma^-}{d\Omega} d(cos\theta_y)}{\int \frac{d\sigma^+}{d\Omega} d(cos\theta_y)) + \int \frac{d\sigma^-}{d\Omega} d(cos\theta_y)} = \alpha P_{circ} C_x \cos\theta_x + \alpha P_{circ} C_z \cos\theta_z$$

- Maximum likelihood Method:

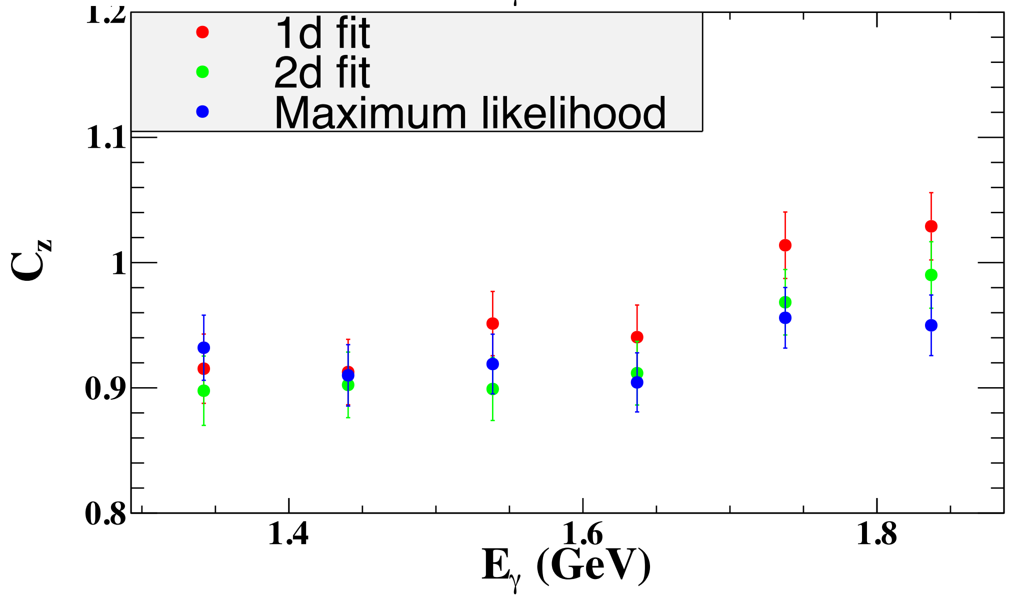
$$PDF = \frac{d\sigma}{d\Omega}_{|unpol} (1 \pm \alpha P_{circ} C_x \cos\theta_x \pm \alpha P_{circ} C_z \cos\theta_z + \alpha P_y \cos\theta_y)$$

Results for C_x and C_z from Different Methods

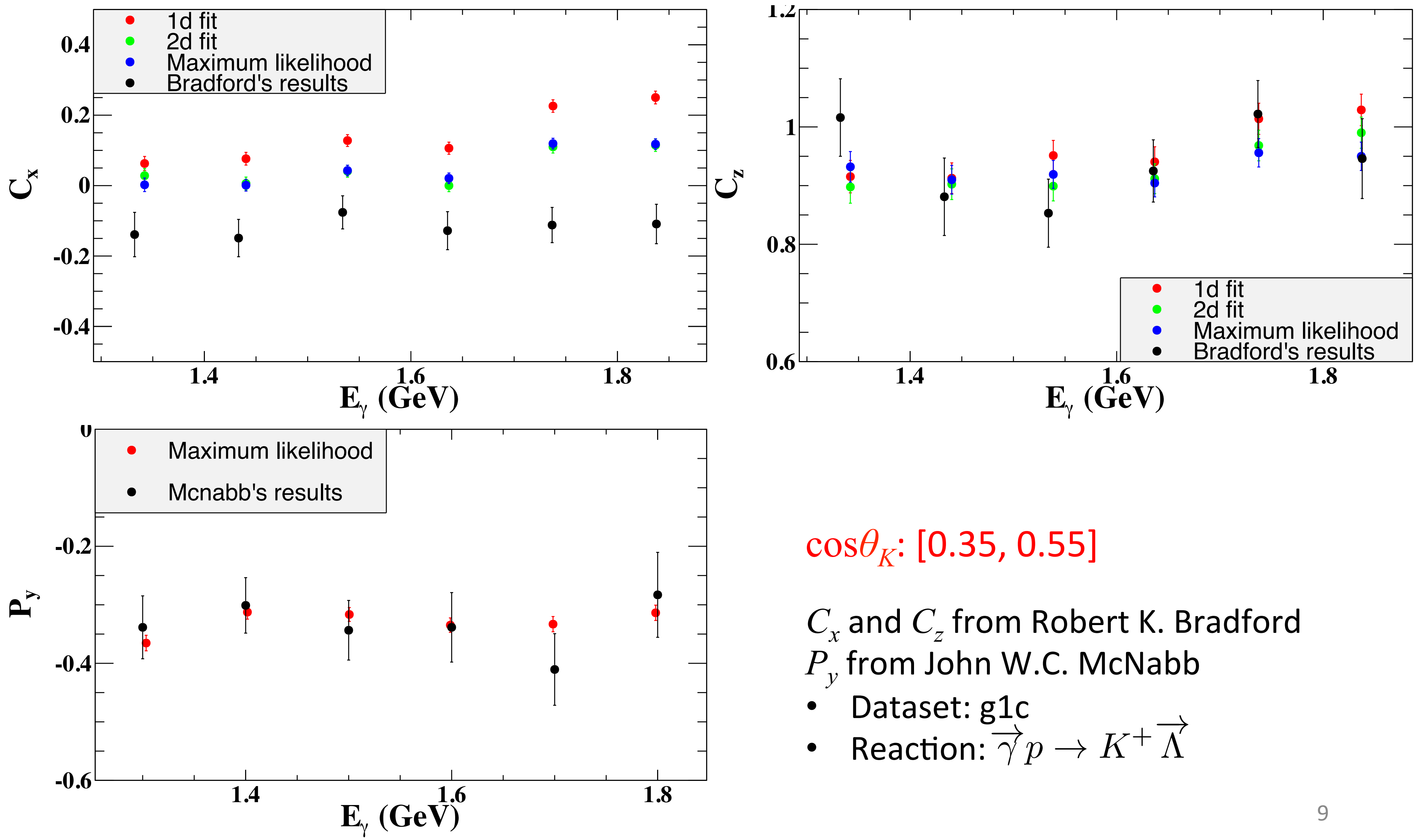
$\cos\theta_K$: [0.35, 0.55]



$\cos\theta_K$: [0.35, 0.55]



Comparison With g1c Results



$\cos\theta_K$: [0.35, 0.55]

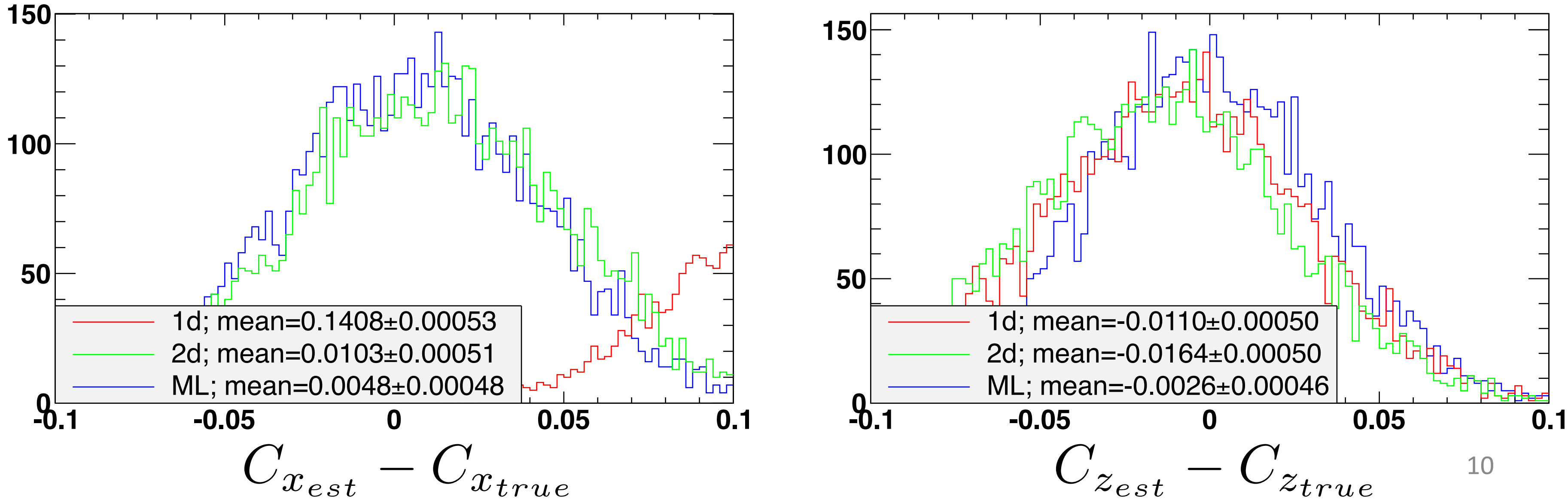
C_x and C_z from Robert K. Bradford
P_y from John W.C. McNabb

- Dataset: g1c
- Reaction: $\overrightarrow{\gamma} p \to K^+ \overrightarrow{\Lambda}$

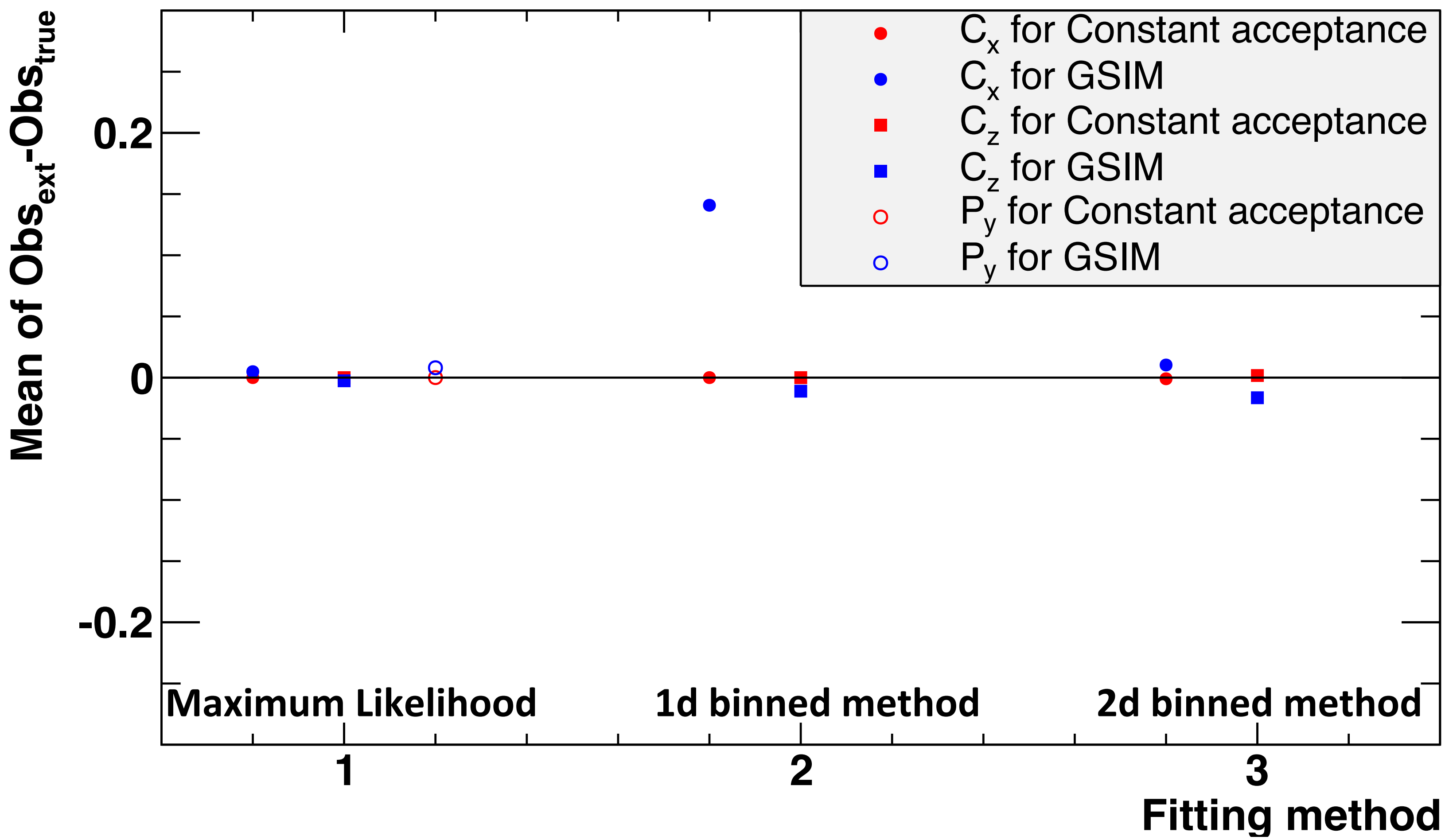
Simulation Study to Understand Different Methods

A study was used to evaluate potential bias of the maximum likelihood method and the binned methods.

- 6000 different experiments, with 10^6 events in each experiment, were generated according to the differential polarized cross section with realistic values of C_x, C_z, and P_y for $\vec{\gamma} p \rightarrow K^+ \vec{\Lambda}$.
- Generated data were processed through GSIM and gpp.
- After raw data were skimmed, the observables were extracted using the maximum likelihood method and the binned methods.

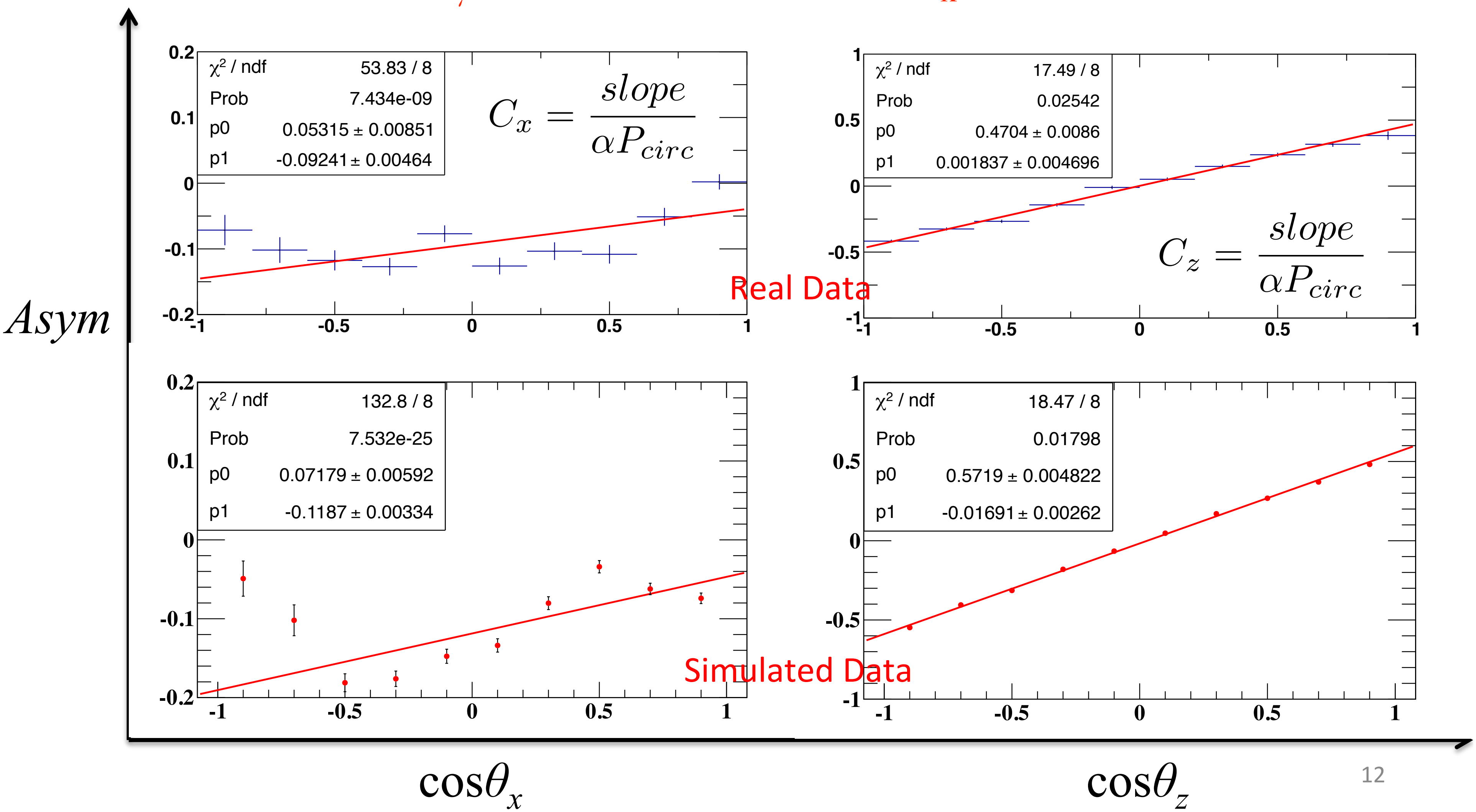


Simulation Study to Compare Different Methods



Examples of 1D Fit

E_γ: [1.5875, 1.6875] GeV and $\cos\theta_K$: [0.35, 0.55]



Why is the Bias Small for C_z from 1D Fit?

In the spherical coordinate system:

$$\begin{cases} \cos\theta_x = \sin\theta\cos\phi \\ \cos\theta_y = \sin\theta\sin\phi \\ \cos\theta_z = \cos\theta \end{cases}$$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

θ_x, θ_y, and θ_z are not independent.

Event yield: $Y^{\pm}(\theta,\phi) = N_\gamma^{\pm} N_T \sigma^{\pm}(\theta,\phi) A(\theta,\phi)$

Integral over ϕ: $Y^{\pm}(\theta) = c(A(\theta) \pm \alpha P_{circ} C_x \sin\theta A_x(\theta) \pm \alpha P_{circ} C_z \cos\theta A(\theta) + \alpha P_y \sin\theta A_y(\theta))$

$$A(\theta) = \int_0^{2\pi} A(\theta,\phi)\,\mathrm{d}\phi;\ \ A_x(\theta) = \int_0^{2\pi} A(\theta,\phi)\cos\phi\,\mathrm{d}\phi;\ \ A_y(\theta) = \int_0^{2\pi} A(\theta,\phi)\sin\phi\,\mathrm{d}\phi$$

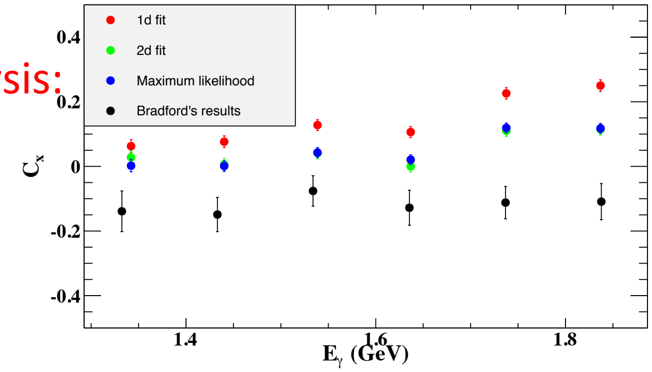
$$A_x(\theta) = \int_0^{2\pi} A(\theta,\phi)\cos\phi\,\mathrm{d}\phi < \int_0^{2\pi} A(\theta,\phi)|\cos\phi|\,\mathrm{d}\phi < |\cos\phi|_{max}\int_0^{2\pi} A(\theta,\phi)\,\mathrm{d}\phi = \int_0^{2\pi} A(\theta,\phi)\,\mathrm{d}\phi = A(\theta)$$

Asymmetry: $Asym = \dfrac{Y^+ - Y^-}{Y^+ + Y^-} = \dfrac{\alpha P_{circ} C_x \sin\theta A_x(\theta) + \alpha P_{circ} C_z \cos\theta A(\theta)}{A(\theta) + \alpha P_y \sin\theta A_y(\theta)}$

Generally, $|C_x| << |C_z|, |P_y| < |C_z|$

Therefore, $Asym \approx \alpha P_{circ} C_z \cos\theta_z$

Why is the Bias Large for C_x from 1D Fit?



Spherical coordinate system for the convenience of C_x analysis:

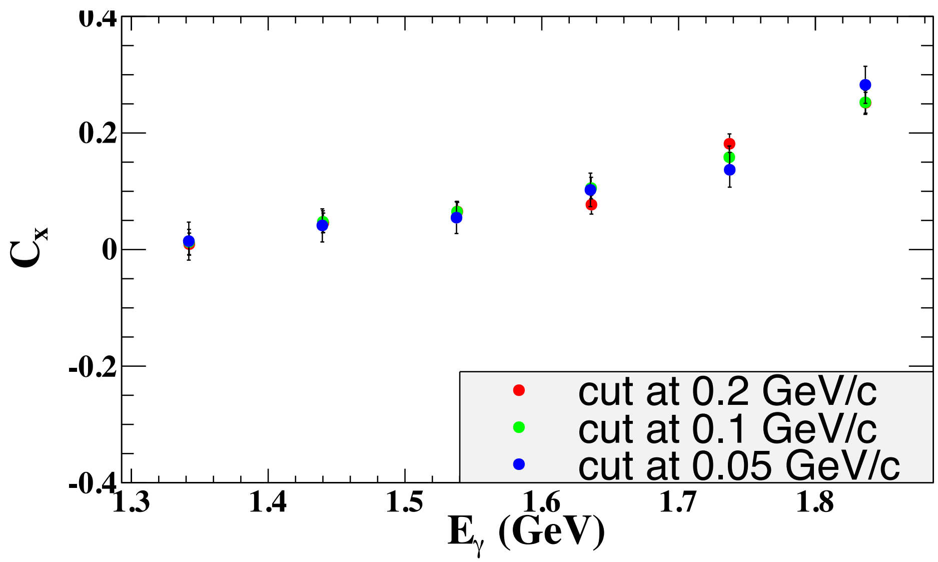
$$\begin{cases} \cos\theta_x = \cos\theta \\ \cos\theta_y = \sin\theta\cos\phi \\ \cos\theta_z = \sin\theta\sin\phi \end{cases}$$

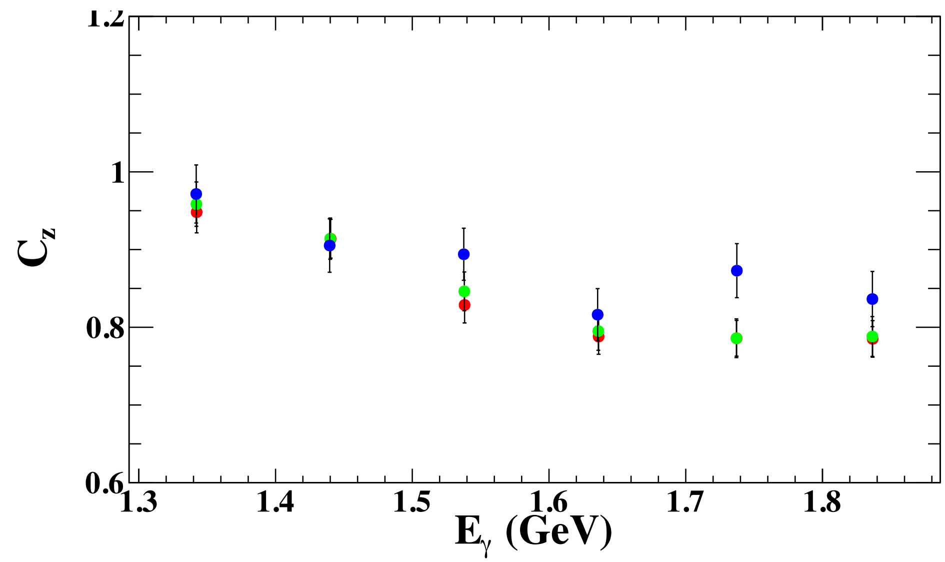
Asymmetry: $$Asym = \frac{Y^+ - Y^-}{Y^+ + Y^-} = \frac{\alpha P_{circ} C_x \cos\theta A(\theta) + \alpha P_{circ} C_z \sin\theta A_z(\theta)}{A(\theta) + \alpha P_y \sin\theta A_y(\theta)}$$

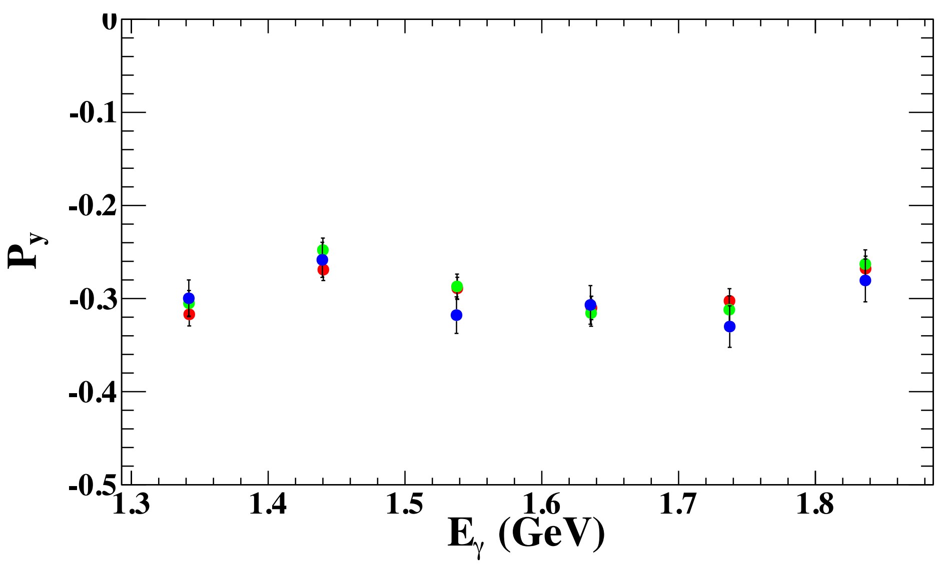
In general, C_x is small relative to C_z and P_y, so C_z and P_y terms do not cancel. Therefore, the asymmetry for C_x is not a linear function of $\cos\theta_x$.

- The effect of acceptance cannot be ignored in 1D fit, especially for C_x.
- The situation with P_y is somewhat in-between C_x and C_z if it's extracted by 1D fit.
- 2D fitting can reduce the effect of the acceptance to some extent.

Effect of Missing Momentum Cut



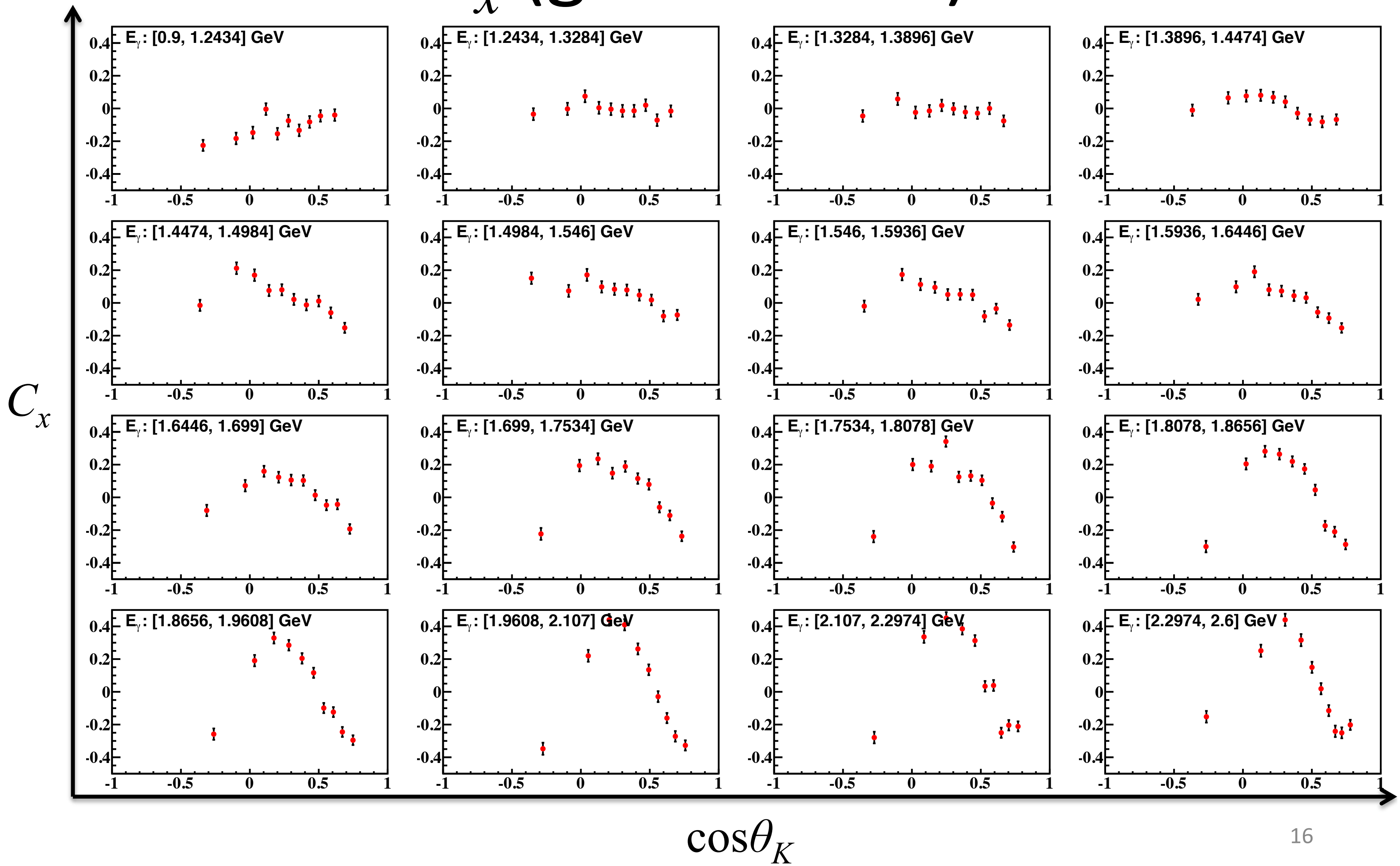




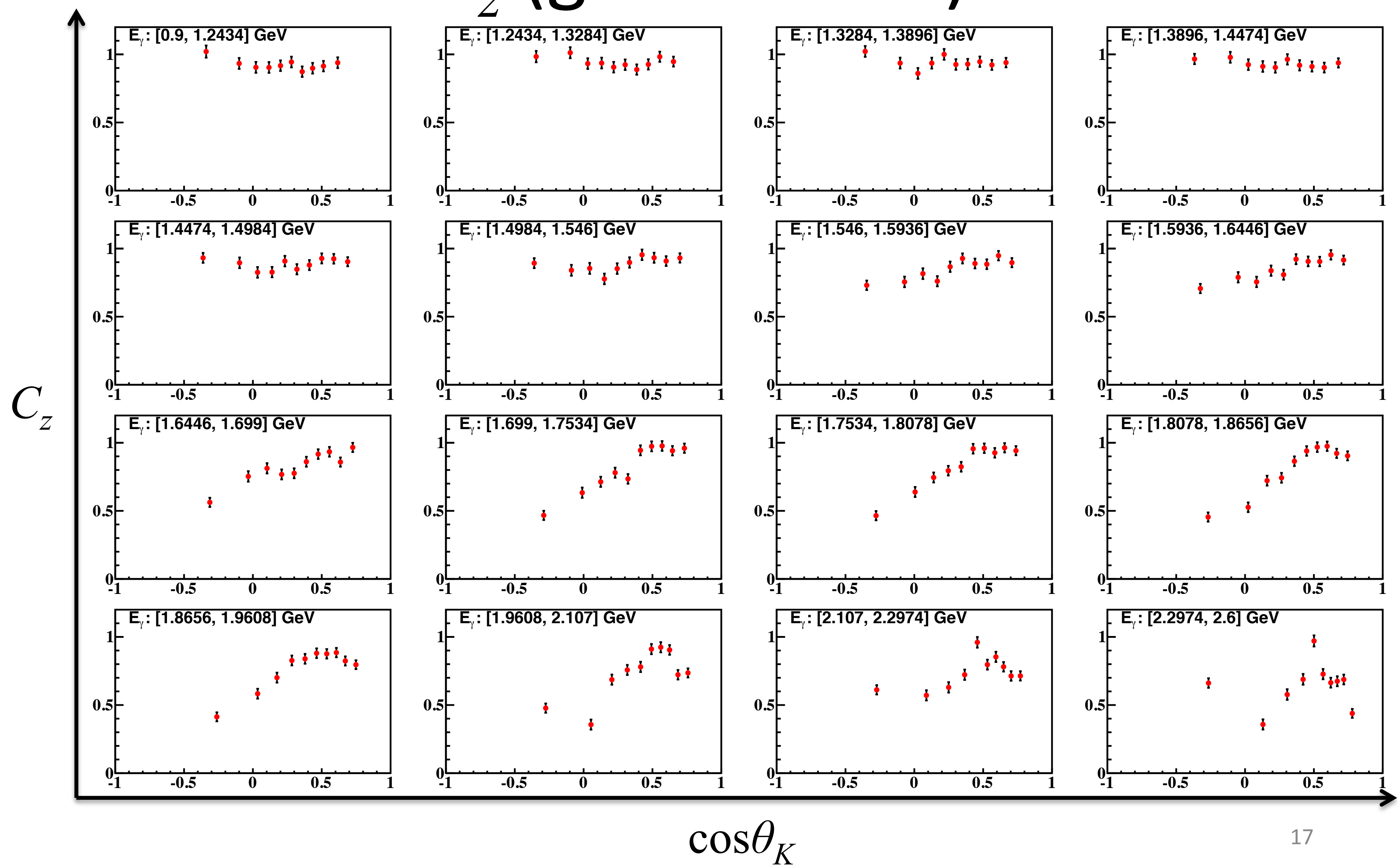
$\cos\theta_K$: [0.15, 0.35]

The missing momentum was cut at points 0.2, 0.1 and 0.05 GeV/c.

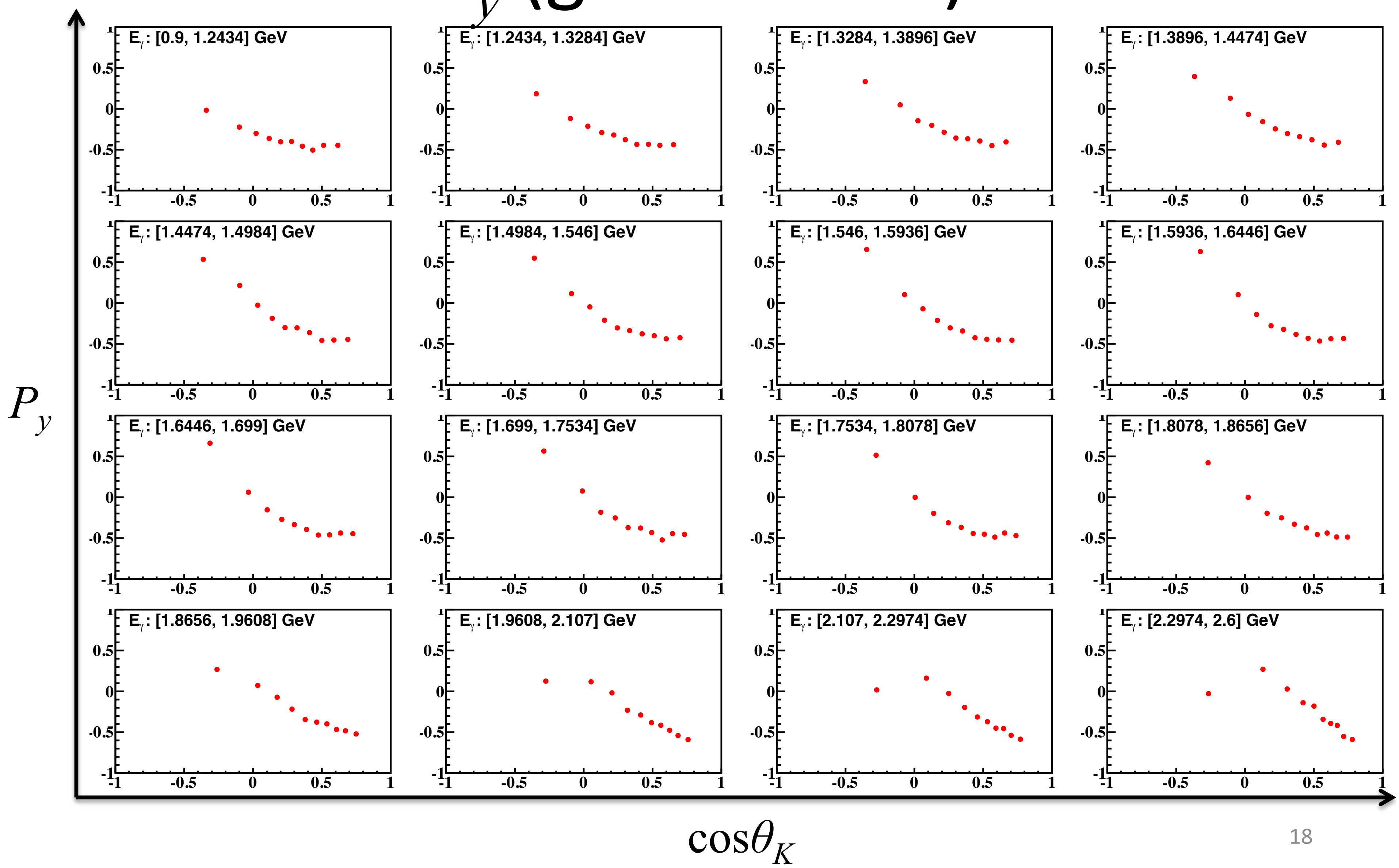
C_x (g13 Results)



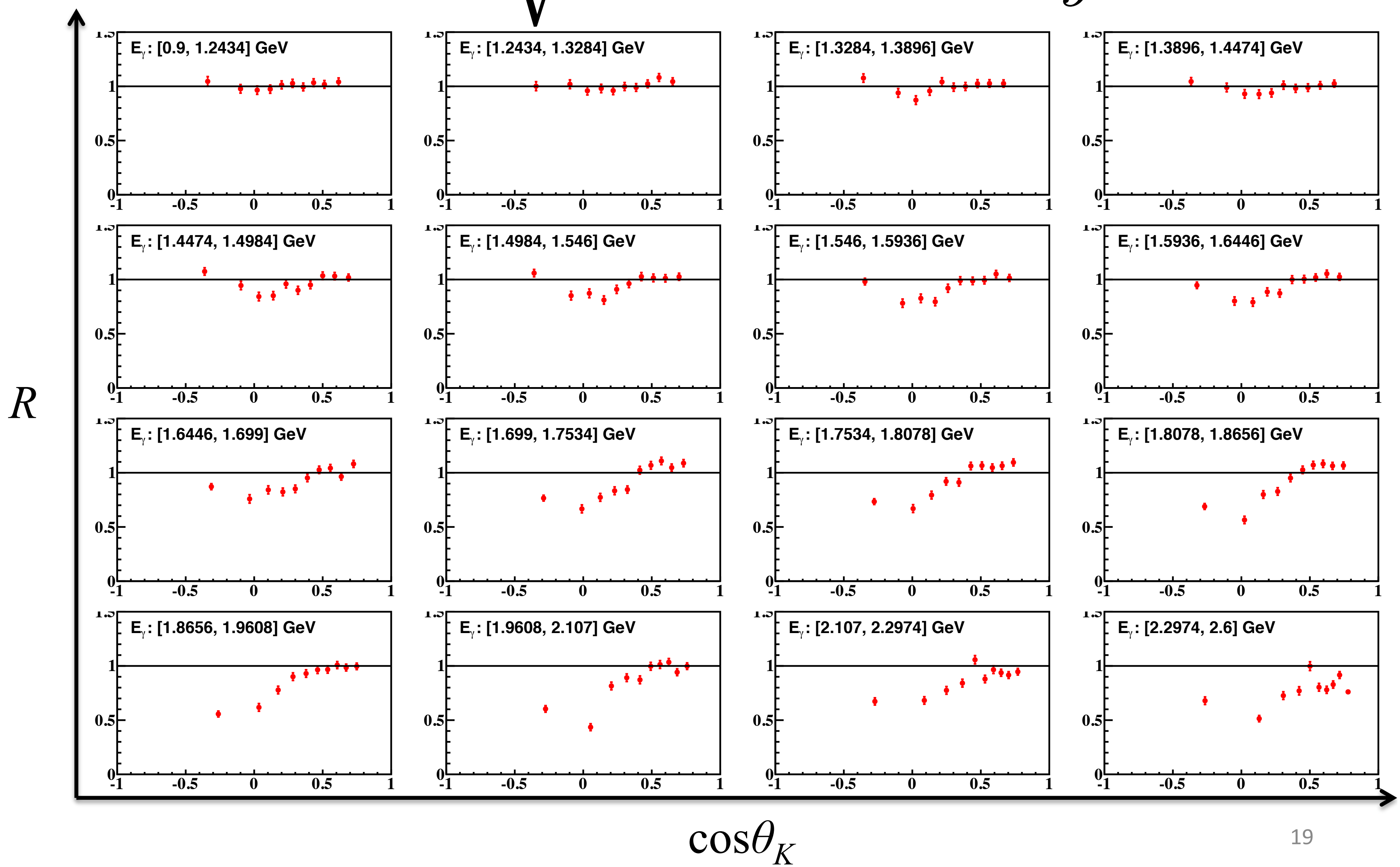
C_z (g13 Results)



P_y (g13 Results)



$$R = \sqrt{C_x^2 + C_z^2 + P_y^2}$$



Summary and Outlook

- Comprehensive results for C_x, C_z, and P_y in the kinematic bins of E_γ (0.9 – 2.6 GeV) and $\cos\theta_K$ (-1, 1) have been obtained for $K^+\Lambda$ photoproduction off the bound proton.
- Fermi-motion does not seem to significantly influence the polarization observables C_x, C_z, and P_y.
- Systematic uncertainties to be estimated.
- Currently the C_x discrepancy g1c vs g13 is not understood.
- To analyze free proton data from g13.
- To publish results eventually.

Backup Slides

Why Study $K\Lambda$ Channel?

- The study of nucleon resonance excitation plays an important role in building a comprehensive picture of the strong interaction.
- The theoretical work on quark models in the intermediate energy range predicts a rich resonance spectrum.
- Besides have been observed in $\pi N \rightarrow \pi N$ scattering experiments, those "missing" resonances may couple strongly to other channels, such as $K\Lambda$ and $K\Sigma$ channels.

Interests of QF Study in $\overrightarrow{\gamma} d \rightarrow K^+ \overrightarrow{\Lambda} n$

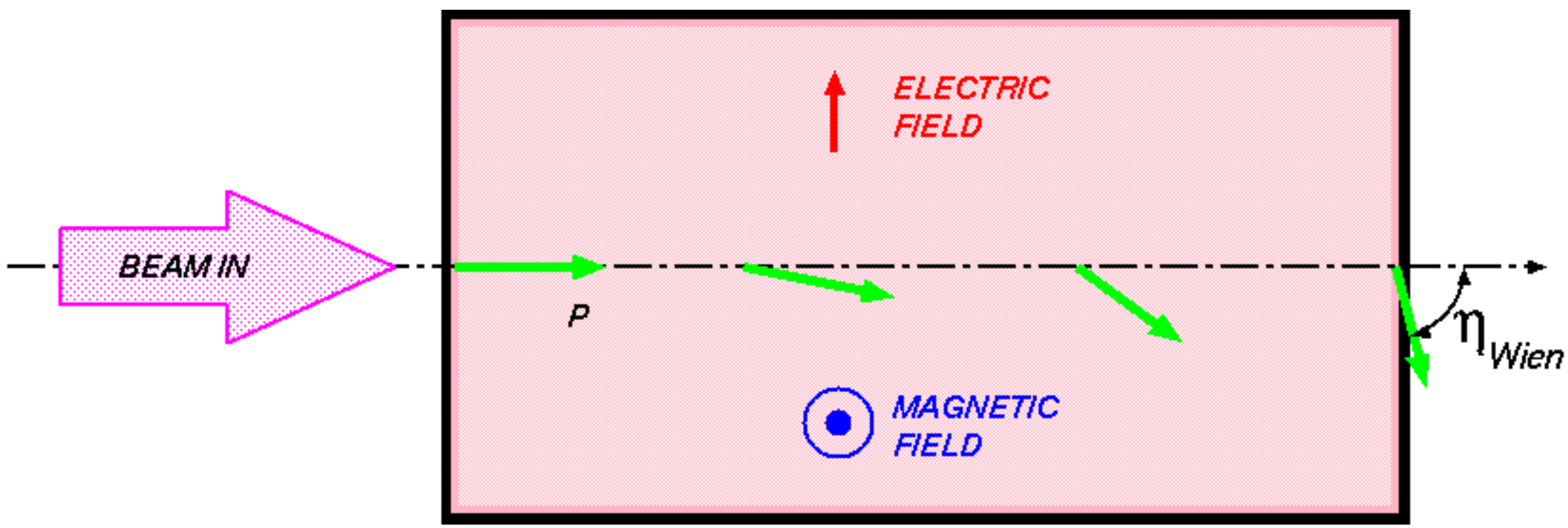
- Many experimental results for $\overrightarrow{\gamma} p \rightarrow K^+ \overrightarrow{\Lambda}$ have been published, from total cross section to polarization observables.

- This study is to extract polarization observables for the quasi-free mechanism in $\overrightarrow{\gamma} d \rightarrow K^+ \overrightarrow{\Lambda} n$, where the target proton is not free, but bounded in deuterium.

- The comparison of results between reactions with free and bounded proton target can be used to test how great influence of the fermi-motion in the deuterium system.

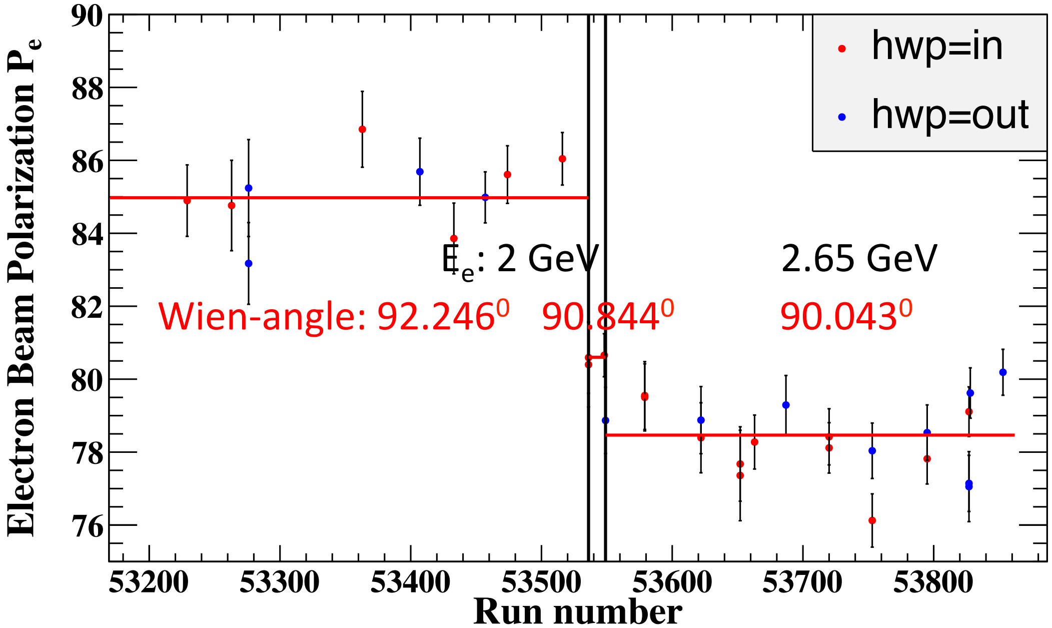
Wien Angle

- The electron moves in a straight line but the spin precesses around the magnetic field. The rotated angle is called Wien angle which determines the degree of beam polarization



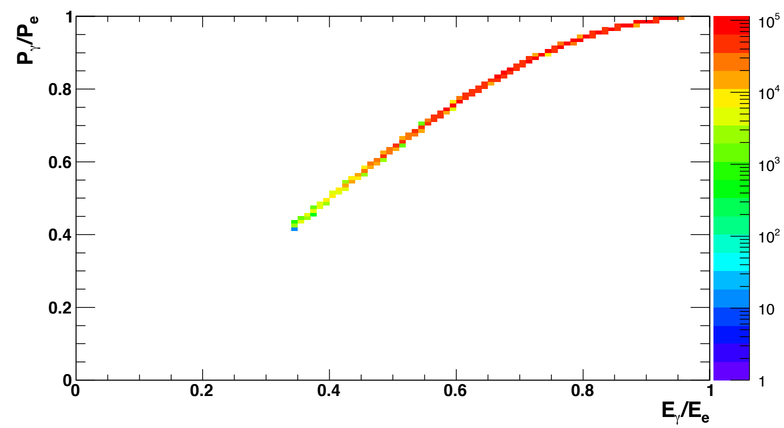
Photon Polarization

The electron polarization for some special runs were measured by the Mϕller polarimeter.



The polarization of the photon beam was calculated using the Maximon and Olson relation

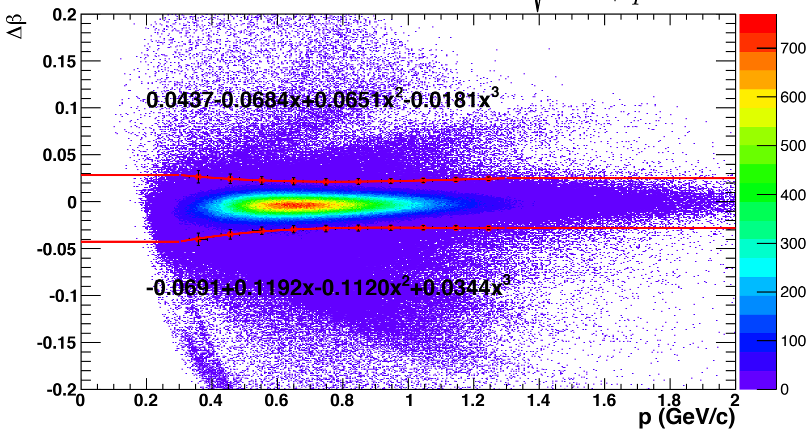
$$P_{cir} = \frac{E_\gamma(E + \frac{1}{3}E')P_e}{E^2 + E'^2 - \frac{2}{3}EE'}$$



Reaction Yields

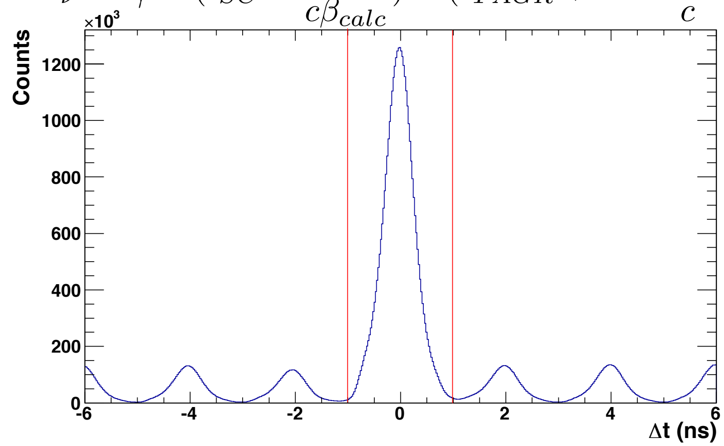
Particle Identification

$$\Delta\beta = \beta_{meas} - \beta_{calc} = \beta_{meas} - \sqrt{\frac{p^2}{m^2+p^2}}$$

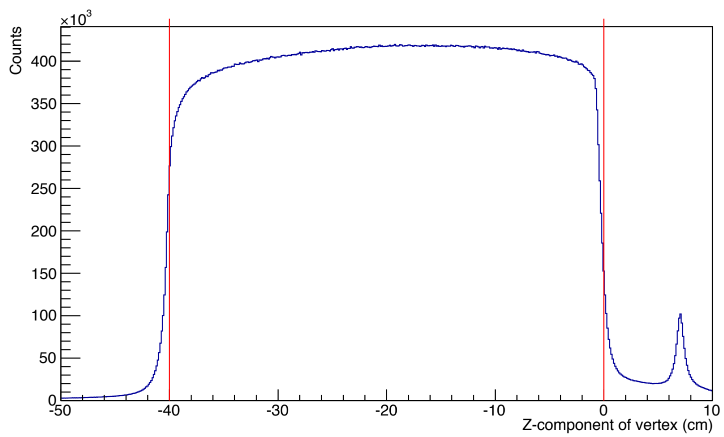


Photon Selection

$$\Delta t = t_\nu - t_\gamma = (t_{SC} - \frac{d_{SC}}{c\beta_{calc}}) - (t_{TAGR} + \frac{z+20\ (cm)}{c})$$

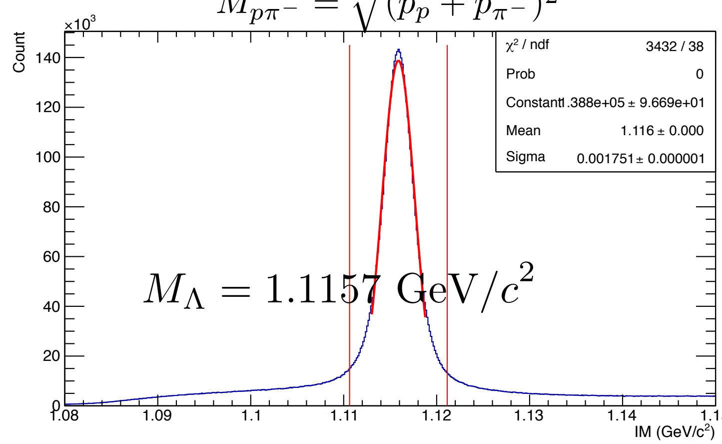


Z-Vertex Cut



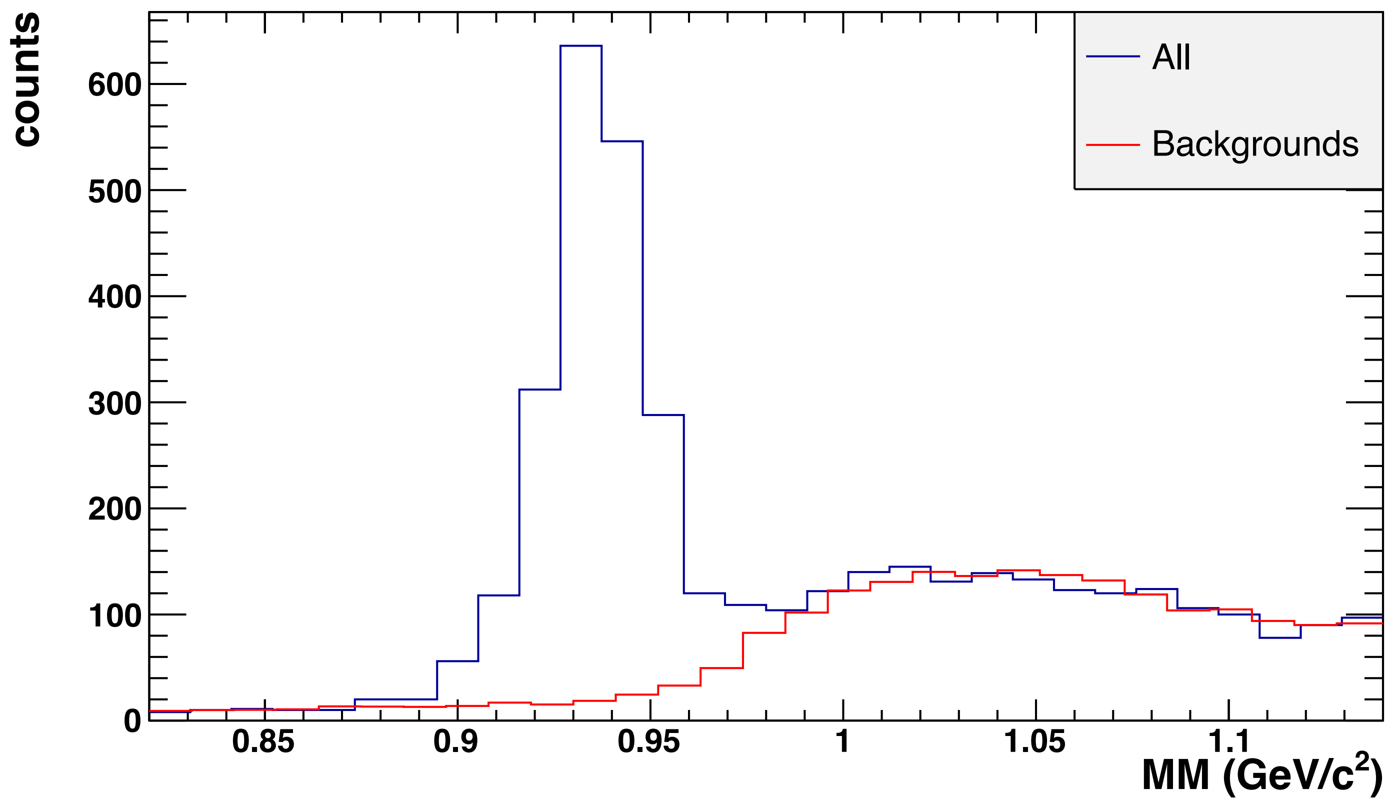
Invariant-Mass Cut

$$M_{p\pi^-} = \sqrt{(\widetilde{p}_p + \widetilde{p}_{\pi^-})^2}$$



Method 2 of Observable-Extraction: Weight for Each Event

Weight = (All – Backgrounds) / All



Method 2 of Observable-Extraction: The Maximum Likelihood Method

Non-normalized Probability Density Function (PDF) defined from the polarized differential cross section:

$$PDF = \frac{d\sigma}{d\Omega}_{|unpol}(1 \pm \alpha P_{circ} C_x \cos\theta_x \pm \alpha P_{circ} C_z \cos\theta_z + \alpha P_y \cos\theta_y)$$

Total likelihood is the product of the likelihoods for all individual events:

$$logL = b + \sum_{i=1}^{n^+} log[(1 + \alpha P^i_{circ} C_x \cos\theta^i_x + \alpha P^i_{circ} C_z \cos\theta^i_z + \alpha P_y \cos\theta^i_y) w^i]$$

$$\sum_{j=1}^{n^-} log[(1 - \alpha P^j_{circ} C_x \cos\theta^j_x - \alpha P^j_{circ} C_z \cos\theta^j_z + \alpha P_y \cos\theta^j_y) w^j]$$

Next side will introduce how to set weight w^i and w^j for each event.

Why the Maximum Likelihood Method Can Ignore Acceptance?

Non-normalized PDF with consideration of acceptance:

$$PDF = A(\theta,\phi)\sigma^{\pm}(\theta,\phi;C_x,C_z,P_y)w$$

Total likelihood: $$logL(C_x,C_z,P_y) = \sum_{i=1}^{n^+} logA(\theta_i,\phi_i) + \sum_{j=1}^{n^-} logA(\theta_j,\phi_j)$$

$$+ \sum_{i=1}^{n^+} log[\sigma^+(\theta_i,\phi_i;C_x,C_z,P_y)w_i] + \sum_{j=1}^{n^-} log[\sigma^-(\theta_j,\phi_j;C_x,C_z,P_y)w_j]$$

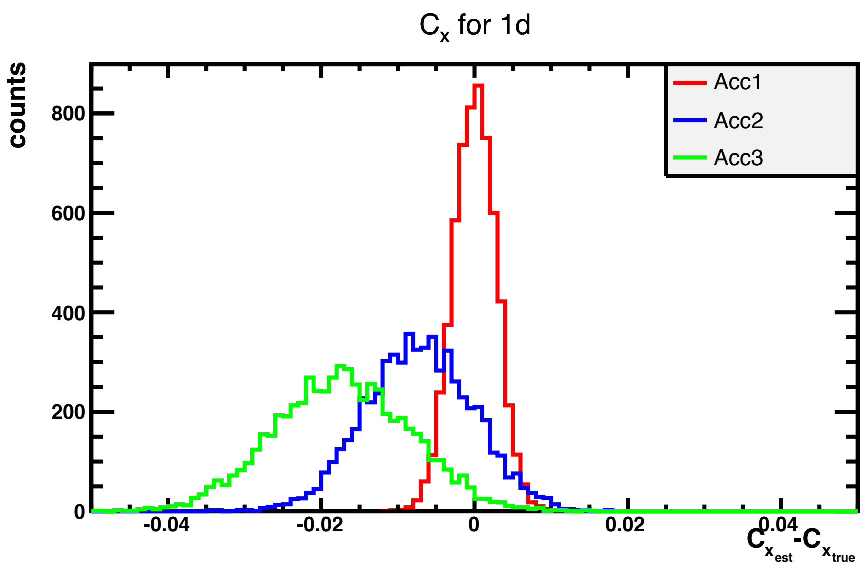
Equation array to obtain C_x, C_z, and P_y:

$$\begin{cases} \frac{\partial logL(C_x,C_z,P_y)}{\partial C_x} = \frac{\partial \sum_{i=1}^{n^+} log[\sigma^+(\theta_i,\phi_i;C_x,C_z,P_y)w_i]}{\partial C_x} + \frac{\partial \sum_{j=1}^{n^-} log[\sigma^-(\theta_j,\phi_j;C_x,C_z,P_y)w_j]}{\partial C_x} = 0 \\ \frac{\partial logL(C_x,C_z,P_y)}{\partial C_z} = \frac{\partial \sum_{i=1}^{n^+} log[\sigma^+(\theta_i,\phi_i;C_x,C_z,P_y)w_i]}{\partial C_z} + \frac{\partial \sum_{j=1}^{n^-} log[\sigma^-(\theta_j,\phi_j;C_x,C_z,P_y)w_j]}{\partial C_z} = 0 \\ \frac{\partial logL(C_x,C_z,P_y)}{\partial P_y} = \frac{\partial \sum_{i=1}^{n^+} log[\sigma^+(\theta_i,\phi_i;C_x,C_z,P_y)w_i]}{\partial P_y} + \frac{\partial \sum_{j=1}^{n^-} log[\sigma^-(\theta_j,\phi_j;C_x,C_z,P_y)w_j]}{\partial P_y} = 0 \end{cases}$$

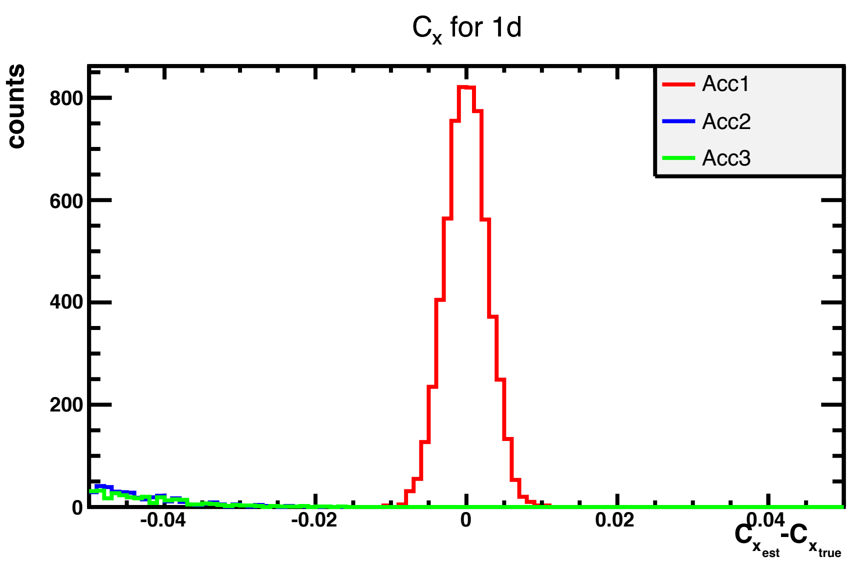
Acceptance is cancelled because it’s independent of polarization observables.

Comparison of 2-Track and 3-Track Topology

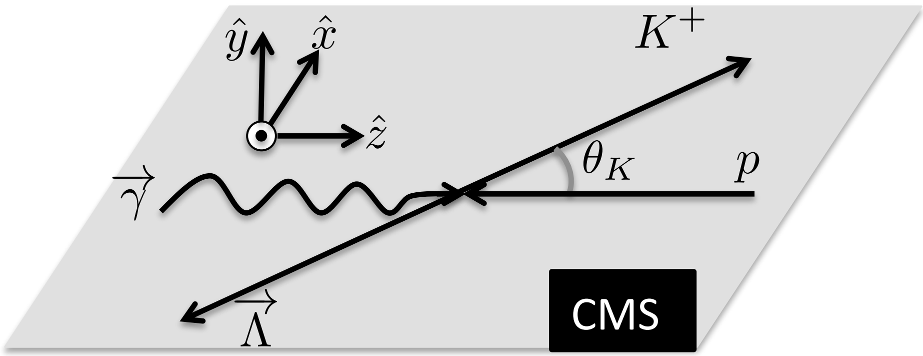
2-Track



3-Track

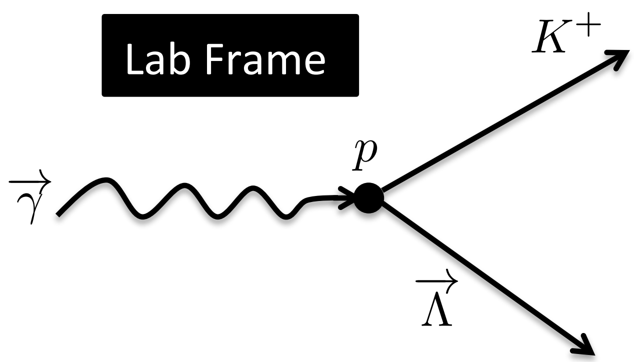


Axis Conventions



Axis Convention 1:

$$\begin{cases} \hat{z} = \hat{p}_\gamma \\ \hat{y} = \frac{\hat{p}_\gamma \times \hat{p}_K}{|\hat{p}_\gamma \times \hat{p}_K|} \\ \hat{x} = \hat{y} \times \hat{z} \end{cases}$$



Axis Convention 2:

$$\begin{cases} \hat{z} = \hat{p}_\gamma \\ \hat{y} = \frac{\hat{p}_\gamma \times \hat{p}_K}{|\hat{p}_\gamma \times \hat{p}_K|} \\ \hat{x} = \hat{y} \times \hat{z} \end{cases}$$

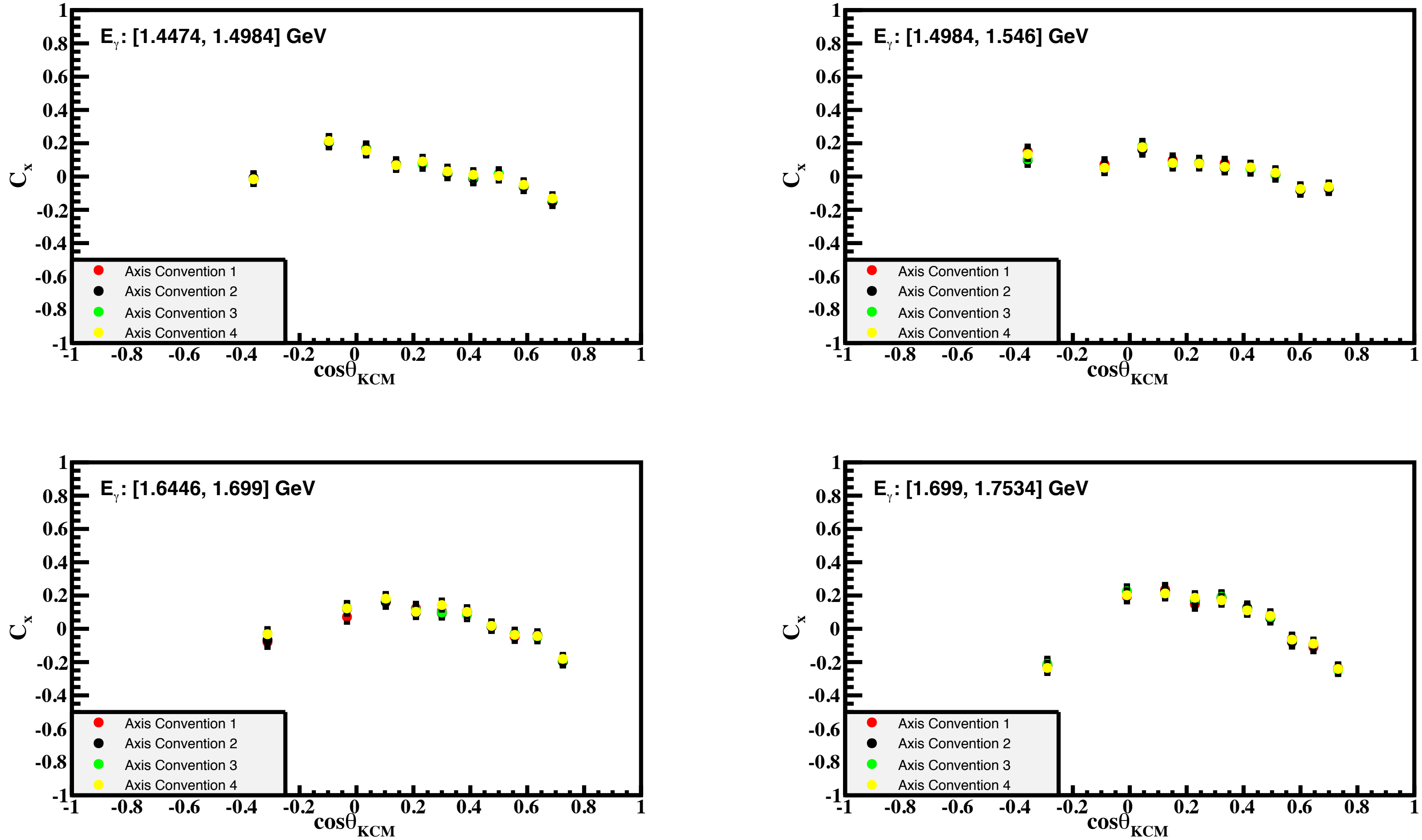
Axis Convention 3:

$$\begin{cases} \hat{z} = \hat{p}_\gamma \\ \hat{y} = \frac{\hat{p}_\Lambda \times \hat{p}_K}{|\hat{p}_\Lambda \times \hat{p}_K|} \\ \hat{x} = \hat{y} \times \hat{z} \end{cases}$$

Axis Convention 4:

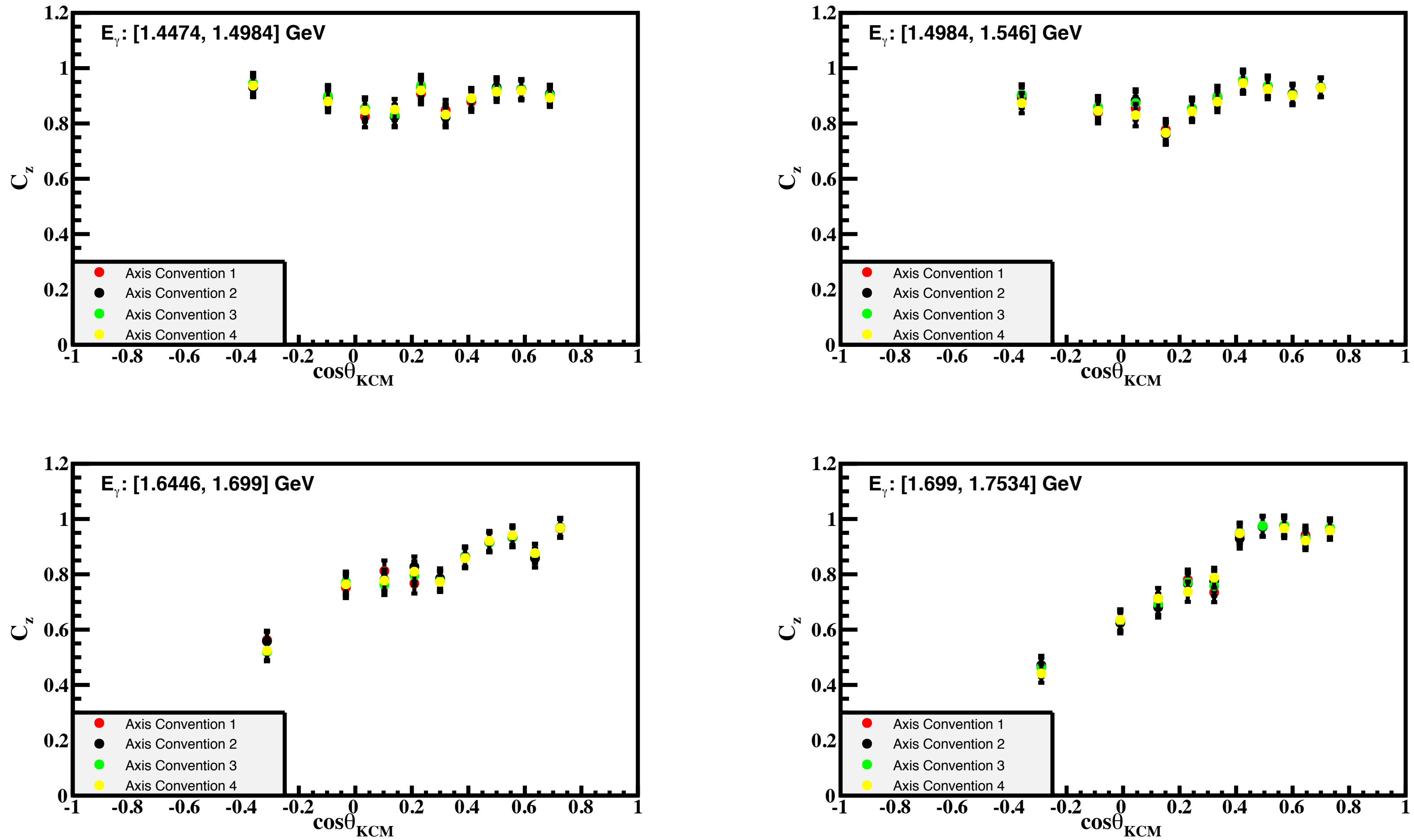
$$\begin{cases} \hat{z} = \frac{\hat{p}_\Lambda + \hat{p}_K}{|\hat{p}_\Lambda + \hat{p}_K|} \\ \hat{y} = \frac{\hat{p}_\Lambda \times \hat{p}_K}{|\hat{p}_\Lambda \times \hat{p}_K|} \\ \hat{x} = \hat{y} \times \hat{z} \end{cases}$$

C_x



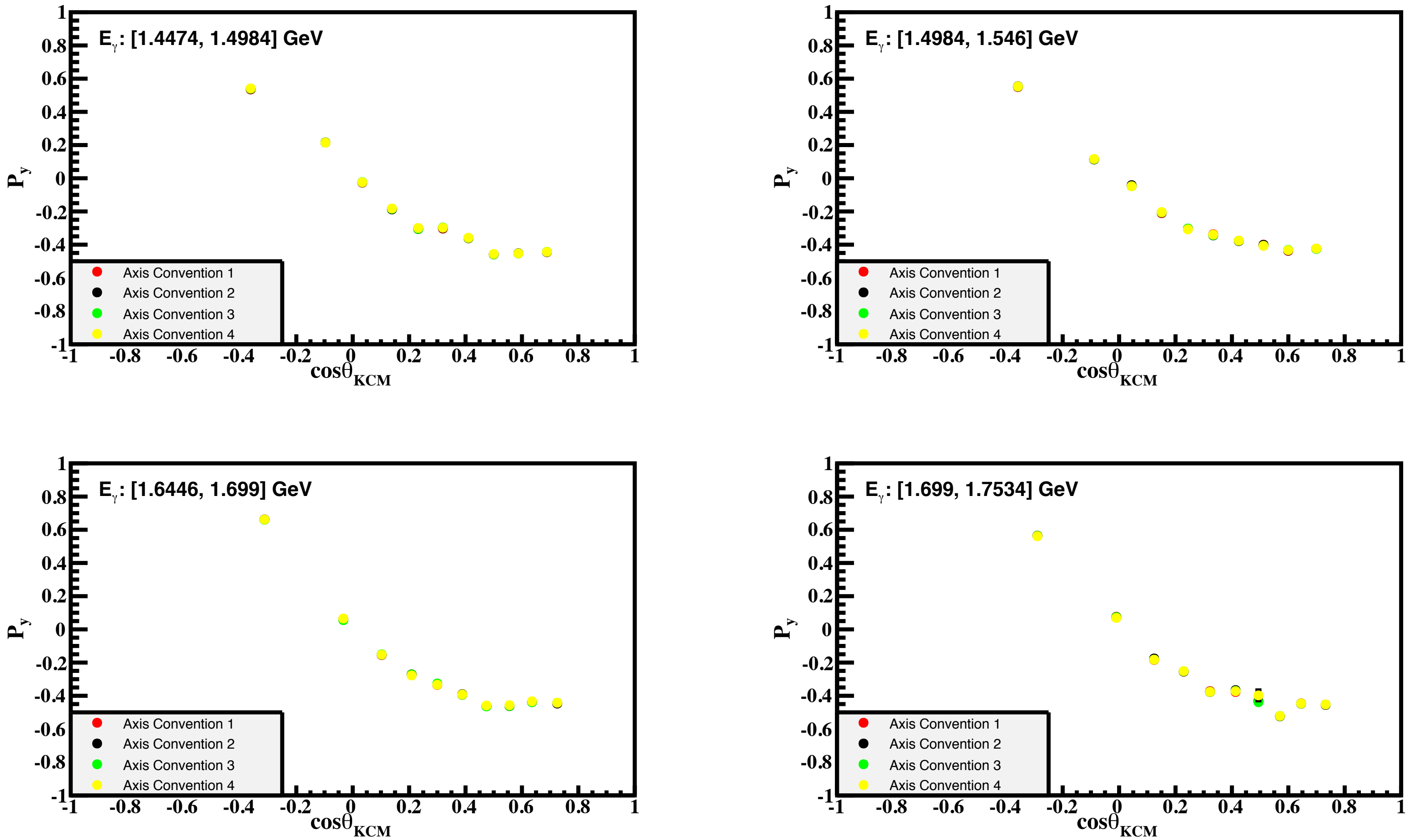
Eγ: [1.4474, 1.4984] GeV
Eγ: [1.4984, 1.546] GeV
Eγ: [1.6446, 1.699] GeV
Eγ: [1.699, 1.7534] GeV
Axis Convention 1
Axis Convention 2
Axis Convention 3
Axis Convention 4
C_x
cosθ_KCM

C_z



E$_\gamma$: [1.4474, 1.4984] GeV
E$_\gamma$: [1.4984, 1.546] GeV
E$_\gamma$: [1.6446, 1.699] GeV
E$_\gamma$: [1.699, 1.7534] GeV
C_z
$\cos\theta_{KCM}$
Axis Convention 1
Axis Convention 2
Axis Convention 3
Axis Convention 4

P_y

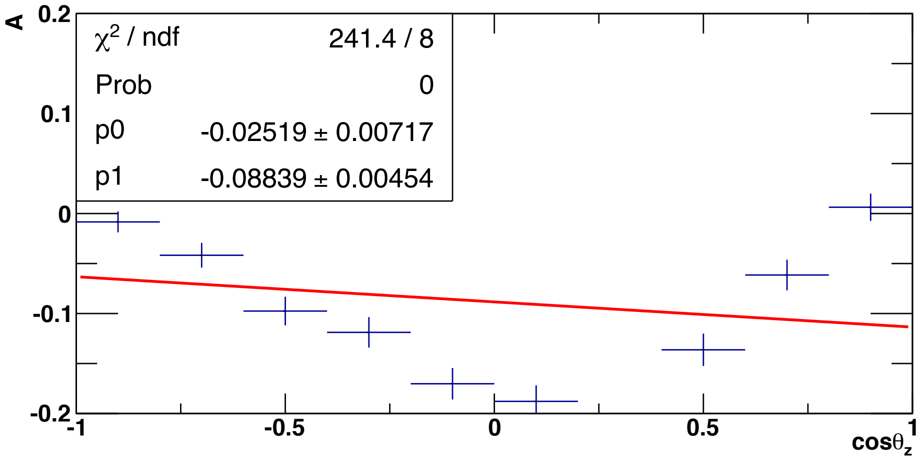


Eγ: [1.4474, 1.4984] GeV
Eγ: [1.4984, 1.546] GeV
Eγ: [1.6446, 1.699] GeV
Eγ: [1.699, 1.7534] GeV
Axis Convention 1
Axis Convention 2
Axis Convention 3
Axis Convention 4
Py
cosθKCM
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

C_y

E_γ: [1.5875, 1.6875] GeV and $\cos\theta_K$: [0.35, 0.55]

Real data



Simulated data