

Mechanical Structure of Nucleons and Nuclei

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Jefferson Lab

CENTER for NUCLEAR FEMTOGRAPHY

Pressure in the proton

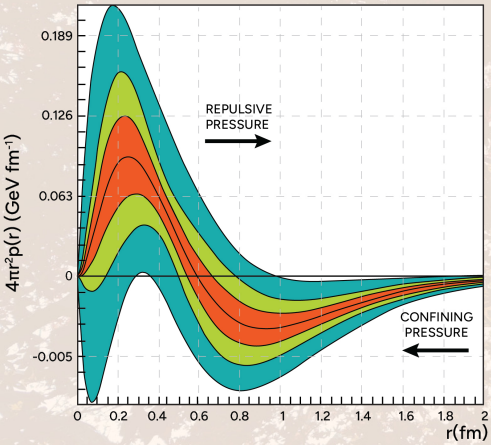


Figure: Burkert, Elouadrhiri & Girod, Nature (2018)

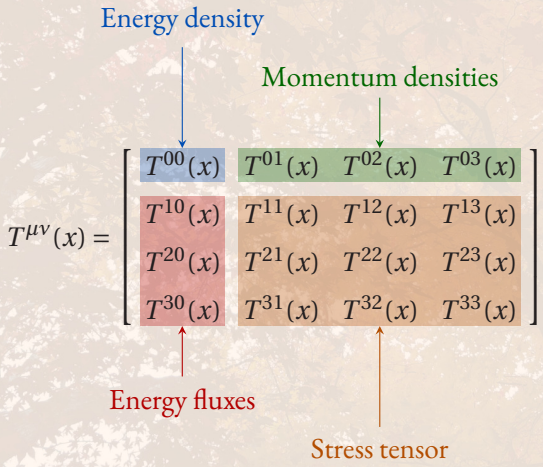
- Pressure in the proton has become a hot topic.
- Detailed theoretical studies:
  - Polyakov, Phys Lett B (2003)
  - Polyakov & Schweitzer, Int J Mod Phys A (2018)
  - Lorcé, Moutarde & Trawinski, Eur Phys J C (2019)
- Empirical extractions happening at JLab!
  - Burkert, Elouadrhiri & Girod, Nature (2018)
  - Duran &al., Nature (2023)
  - Chatagnon *et al.*, Phys Rev C (2026)
- Popular science articles
  - Science Daily (2018)
  - Quanta magazine (2024)

CLAS measurements


The Energy-Momentum Tensor

The energy-momentum tensor

- The energy-momentum tensor describes **density** and **flow** of energy & momentum.
- Also known as the stress-energy tensor.

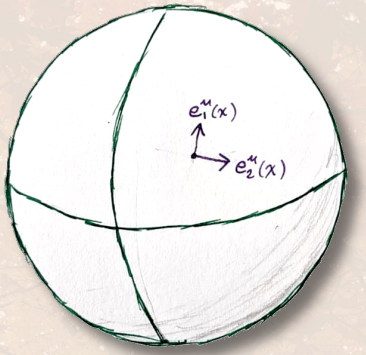


Coupling matter to spacetime

- The energy-momentum tensor (EMT) couples matter to spacetime.
- Does so through **vierbein**:

$$\eta^{ab} e_a{}^{\mu}(x) e_b{}^{\nu}(x) = g^{\mu\nu}(x)$$

$$\underbrace{g_{\mu\nu}(x)}_{\text{spacetime metric}} e_a{}^{\mu}(x) e_b{}^{\nu}(x) = \underbrace{\eta_{ab}}_{\text{Minkowski metric}}$$



  - Local Lorentz frame
  - *Roughly*: square root of the metric
  - **Necessary to couple gravity to spinors**

- EMT defined through functional derivative:

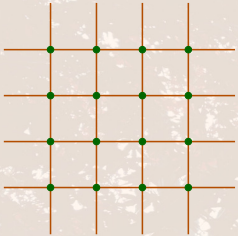
$$T^{\mu}{}_{\nu}(x) = \frac{e_a{}^{\mu}(x)}{\sqrt{-g(x)}} \frac{\delta S}{\delta e_a{}^{\nu}(x)}$$

  - EMT describes how matter responds to spacetime deformation

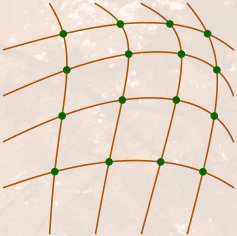
A. Einstein, Sitzungsberichte der Preussischen Akademie der Wissenschaften (1928) 217
L. Rosenfeld, Mémoires de l'Academie royale de Belgique 18 (1938)
E. Cartan, *The Theory of Spinors*

Response to spacetime deformations

- Deforming spacetime transforms the vierbein:



$$x \mapsto x - \xi(x)$$



$$e_a{}^\nu(x) \mapsto e_a{}^\nu(x) - \big(\partial_\mu \xi^\nu(x)\big) e_a{}^\mu(x)$$

- In turn transforms the action:

$$\delta S_{\text{QCD}} = -\int \mathrm{d}^4 x \, \overbrace{T^{\mu\nu}(x)}^{\text{Four-dimensional stress tensor}} \underbrace{\partial_\mu \xi_\nu(x)}_{\text{Four-dimensional strain-torsion tensor}}$$

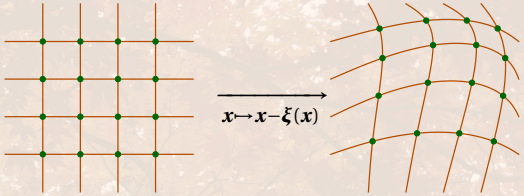
  - Curiously similar form to the stress-strain rule

Lorcé, Metz, Pasquini & Rodini, JHEP 11 (2021) 121
AF & Lorcé, *in preparation* (2026)

- 3×3 sub-matrix of the energy-momentum tensor.

$$\begin{bmatrix} T^{00}(x) & T^{01}(x) & T^{02}(x) & T^{03}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix}$$

Stress tensor



- Work needed to deform a system:

$$W[x \mapsto x - \xi(x)] = -\int \mathrm{d}^3x \overbrace{T^{ij}(x)}^{\text{stress}} \Big(\overbrace{\tfrac{1}{2}\big(\partial_i \xi_j(x) + \partial_j \xi_i(x)\big)}^{\text{strain}} + \overbrace{\tfrac{1}{2}\big(\partial_i \xi_j(x) - \partial_j \xi_i(x)\big)}^{\text{torsion}} \Big)$$

  - **Stress**: internal forces resisting *spatial* deformation

Lorcé, Metz, Pasquini & Rodini, JHEP 11 (2021) 121
AF & Lorcé, *in preparation* (2026)

- Continuity equation (integral form):

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\left[\int_V \mathrm{d}^3x\, T^{0j}(\boldsymbol{x},t)\right]}_{\text{Momentum in region}} = -\underbrace{\oint_{\partial V} \mathrm{d}S\, \hat{n}_i T^{ij}(\boldsymbol{x},t)}_{\text{Flux out of region}}$$

$$T^{\mu\nu}(x) = \begin{bmatrix} T^{00}(x) & T^{01}(x) & T^{02}(x) & T^{03}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix}$$

- Stresses are **momentum flux densities**
  - Include **particle fluxes** & **forces**



Particle flux



Force

- **Kinetic pressure is pressure**
  - It's the microscopic origin of gas & fluid pressure
  - Photon flux is the origin of radiation pressure

Bat photo: Froschauer Ann, USFWS (pixnio)
Sisyphus image: Miikka Holkeri (website)


EMT Form Factors

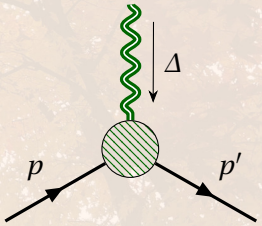
- Energy-momentum tensor parametrized using **EMT form factors (EMT-FFs)**

$$\langle \boldsymbol{p}' | \hat{T}^{\mu\nu}_{q,g}(0) | \boldsymbol{p} \rangle = 2P^\mu P^\nu A_{q,g}(\Delta^2) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{2} D_{q,g}(\Delta^2) + 2m^2 g^{\mu\nu} \bar{c}_{q,g}(\Delta^2)$$

  - Also known as **gravitational form factors**
  - EMT is source of gravitation, but we're not doing gravitational physics

- General form from Lorentz covariance
- EMT-FFs encode hadron structure

$$P^\mu = \frac{1}{2}(p + p')^\mu$$
$$\Delta^\mu = (p' - p)^\mu$$



Kobzarev & Okun, Sov Phys JETP 16 (1963) 1343
H. Pagels, Phys Rev 144 (1966) 1250
X. Ji, PRL78 (1997) 610

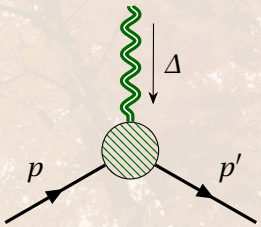
EMT form factors: spin-zero

- Energy-momentum tensor parametrized using **EMT form factors**

$$\langle \boldsymbol{p}'|\hat{T}^{\mu\nu}_{q,g}(0)|\boldsymbol{p}\rangle = \overbrace{2P^\mu P^\nu A_{q,g}(\Delta^2)}^{\text{barycentric motion \& dispersion}} + \underbrace{\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{2} D_{q,g}(\Delta^2) + 2m^2 g^{\mu\nu}\bar{c}_{q,g}(\Delta^2)}_{\text{internal pressure \& forces}}$$

- General form from Lorentz covariance
- **EMT-FFs encode hadron structure**

$$P^\mu = \frac{1}{2}(p+p')^\mu$$
$$\Delta^\mu = (p'-p)^\mu$$



Kobzarev & Okun, Sov Phys JETP 16 (1963) 1343
H. Pagels, Phys Rev 144 (1966) 1250
X. Ji, PRL78 (1997) 610

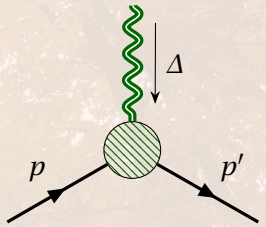
- Spin-half has **additional EMT form factors** related to angular momentum

$$\langle \boldsymbol{p}', s' | \hat{T}^{\mu\nu}_{q,g}(0) | \boldsymbol{p}, s \rangle = \bar{u}(\boldsymbol{p}', s') \Bigg\{ \overbrace{\frac{P^\mu P^\nu}{m} A_{q,g}(\Delta^2)}^{\text{motion \& dispersion}} + \overbrace{\frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{4m} D_{q,g}(\Delta^2) + m \bar{c}_{q,g}(\Delta^2)}^{\text{pressure \& forces}} + \underbrace{\frac{iP^{\{\mu}\sigma^{\nu\}\Delta}}{2m} J_{q,g}(\Delta^2)}_{\text{angular momentum}} \underbrace{- \frac{iP^{[\mu}\sigma^{\nu]\Delta}}{2m} S_{q,g}(\Delta^2)}_{\text{fermion spin}} \Bigg\} u(\boldsymbol{p}, s)$$

- **New symmetric part** related to total angular momentum
- **Antisymmetric part** related to elementary fermion spin

$$P^\mu = \frac{1}{2}(p + p')^\mu$$

$$\Delta^\mu = (p' - p)^\mu$$



Leader & Lorcé, Phys Rept 541 (2014) 163

- Momentum sum rule:

$$\sum_{q,g} A_c(0) = 1$$

- Angular momentum sum rule (Ji sum rule):

$$\sum_{q,g} J_c(0) = \frac{1}{2}$$

- Local momentum conservation:

$$\sum_{q,g} \bar{c}_c(\Delta^2) = 0$$

  - **Violated for open sub-systems**
  - Momentum can be exchanged between quarks & gluons
  - $\bar{c}_c(\Delta^2)$ **quantifies forces on** c

X. Ji, Phys Rev Lett 78 (1997) 610
Polyakov & Son, JHEP 09 (2018) 156

Relationship to GPDs

- EMT-FFs can be found from **generalized parton distributions** (GPDs):

$$\int_{-1}^{1} \mathrm{d}x\, x H_{q,g}(x,\xi,t) = A_{q,g}(t) + \xi^2 D_{q,g}(t)$$

$$\int_{-1}^{1} \mathrm{d}x\, x E_{q,g}(x,\xi,t) = 2J_{q,g}(t) - A_{q,g}(t) - \xi^2 D_{q,g}(t)$$

  - Using Ji's convention for gluon GPDs
  - Relationships from **polynomiality**—consequence of Lorentz covariance

- Measuring x-dependent GPDs can give the EMT-FFs
  - Double DVCS, SDHEPs, etc.

- Near-threshold quarkonium production gives direct access to EMT-FFs

Geyer, Robaschik, Bordag & Hořejši, Z Phys C 26 (1985) 591 (GPDs)
Ji, J Phys G 24 (1998) 1181 (polynomiality)
K. Mamo & I. Zahed, Phys Rev D 101 (2020) 086003 (vector quarkonium production)

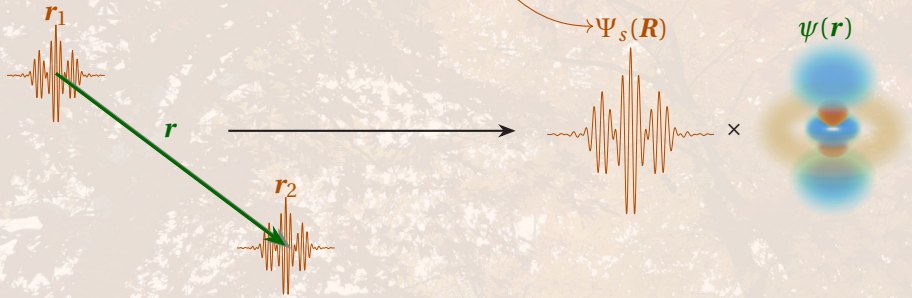

Spatial Stress Distributions

- Momentum transfer → densities via Fourier transforms:

EMT form factors appear here

$$\langle \Psi | \hat{T}^{ij}(\boldsymbol{x}, t) | \Psi \rangle = \sum_{s,s'} \int \mathrm{d}^3 R \int \frac{\mathrm{d}^3 \Delta}{(2\pi)^3} \Psi^*_{s'}(\boldsymbol{R}, t) \langle p', s' | \hat{T}^{ij}(0) | p, s \rangle \Psi_s(\boldsymbol{R}, t)\, \mathrm{e}^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{x}-\boldsymbol{R})} \Big|_{2i\boldsymbol{P} \to \overleftrightarrow{\boldsymbol{\nabla}}}$$

- This depends on the proton's wave packet $\Psi_s(\boldsymbol{R}, t)$



- Need a bit more effort to distill out ***internal* densities**

Li *et al.*, Phys Lett B 838 (2023) 137676
AF & Miller, Phys Rev D 108 (2023) 034008

Dynamic and internal stress

$$\langle\Psi|\hat{T}^{ij}(\boldsymbol{x},t)|\Psi\rangle = \sum_{s,s'}\int \mathrm{d}^3R\Big\{\overbrace{-\Psi^*_{s'}(\boldsymbol{R},t)\frac{\overleftrightarrow{\nabla}^i\overleftrightarrow{\nabla}^j}{4M^2}\Psi_s(\boldsymbol{R},t)}^{\text{dynamic pressures}}\underbrace{\mathfrak{a}_{s's}(\boldsymbol{x}-\boldsymbol{R})}_{\text{internal mass density}} - \overbrace{\frac{i\Psi^*_{s'}(\boldsymbol{R},t)\overleftrightarrow{\nabla}^i\Psi_s(\boldsymbol{R},t)}{2M}}^{\text{barycentric flux}}\underbrace{\mathfrak{p}^j_{s's}(\boldsymbol{x}-\boldsymbol{R})}_{\text{internal momentum density}}$$

$$\overbrace{-\frac{i\Psi^*_{s'}(\boldsymbol{R},t)\overleftrightarrow{\nabla}^j\Psi_s(\boldsymbol{R},t)}{2M}}^{\text{barycentric flux}}\underbrace{\mathfrak{f}^i_{s's}(\boldsymbol{x}-\boldsymbol{R})}_{\text{internal mass flux}} + \overbrace{\Psi^*_{s'}(\boldsymbol{R},t)\Psi_s(\boldsymbol{R},t)}^{\text{barycentric probability}}\underbrace{\mathfrak{t}^{ij}_{s's}(\boldsymbol{x}-\boldsymbol{R})}_{\text{internal stresses}}\Big\}$$



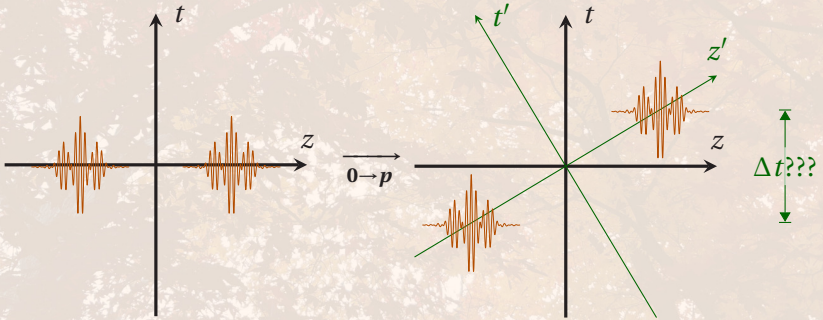
Image: dynamic pressure from wind

- **Internal densities** smeared out by **barycentric distributions**
- **Dynamic pressures** due to barycentric motion & wave packet dispersion
- **Internal stress** due to internal motions *and* forces
  - Smeared by barycentric probability
- **Strictly non-relativistic** breakdown
  - Afforded by Galilei symmetry (absoluteness of simultaneity / fixed t)

AF, Phys Rev D 112 (2025) 034037
Cosyn, AF & Sosa, Phys Rev C 113 (2026) 055208

Relativity of simultaneity

- Momentum-space wave packet contains boosted versions of composite system
  - Boosts mix up planes of simultaneity
  - Internal constituents get boosted to **different times**



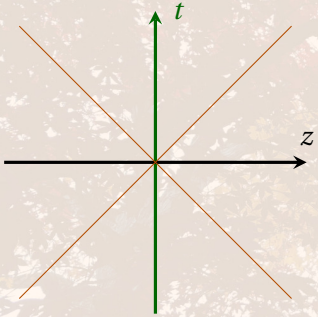
- Cannot decompose structure to overall wave packet ⊗ internal structure at fixed t
- **Light front coordinates** fix this by defining a **new, boost-invariant time variable**!

A. Einstein, Annalen Phys 17 (1905) 891

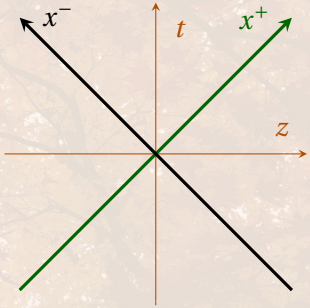
Light front coordinates are a different foliation of spacetime.

- Involves redefining *equal-time surfaces*.

$$x^{\pm} = \frac{1}{\sqrt{2}}\left(t \pm z\right) \qquad \boldsymbol{x}_{\perp} = (x, y) \qquad x^{+} = \frac{1}{\sqrt{2}}\left(t + z\right) = \text{time}$$



Minkowski coordinates



Light front coordinates

P.A.M. Dirac, Rev Mod Phys 21 (1949) 392
Brodsky, Pauli & Pinsky, Phys Rept 301 (1998) 299
AF, *A friendly introduction to the light front* (lecture series)

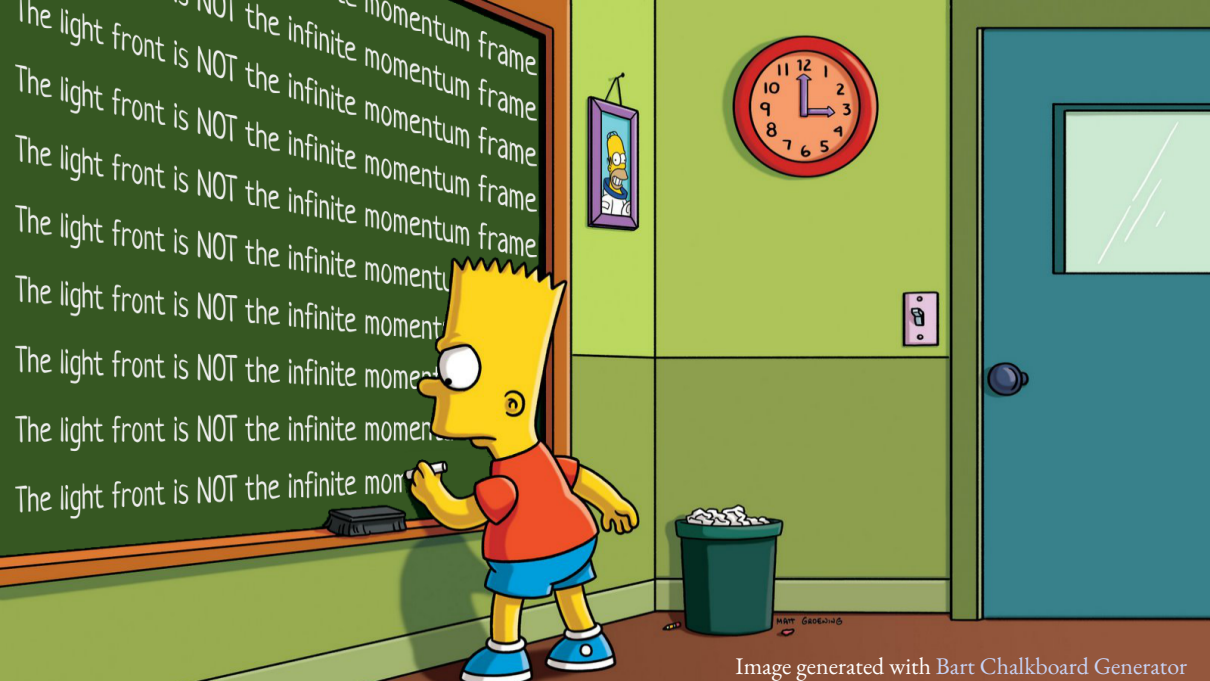


Image generated with Bart Chalkboard Generator

Equal-"time" surfaces are just a convention

- The *round-trip* speed of light is invariant.
- The one-way speed of light being c is a *definition*—not an empirical fact.
  - Pointed out in Einstein's original paper.
- Redefining "time" means changing this.
  - Light front coordinates do exactly this!

B durch einen in B befindlichen Beobachter möglich. Es ist aber ohne weitere Festsetzung nicht möglich, ein Ereignis in A mit einem Ereignis in B zeitlich zu vergleichen; wir haben bisher nur eine „A-Zeit" und eine „B-Zeit", aber keine für A und B gemeinsame „Zeit" definiert. Die letztere Zeit kann nun definiert werden, indem man *durch Definition* festsetzt, daß die „Zeit", welche das Licht braucht, um von A nach B zu gelangen, gleich ist der „Zeit", welche es braucht, um von B nach A zu gelangen. Es gehe nämlich ein Lichtstrahl zur „A-Zeit" t_A von A nach B ab, werde zur „B-Zeit" t_B in B gegen A zu reflektiert und gelange zur „A-Zeit" t'_A nach A zurück. Die beiden Uhren laufen definitionsgemäß synchron, wenn

$$t_B - t_A = t'_A - t_B.$$

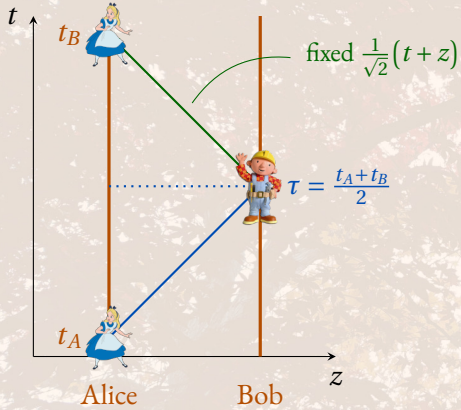
A. Einstein, Annalen Phys 17 (1905) 891

Veritasium, "Why No One Has Measured The Speed of Light" (didactic overview / YouTube)

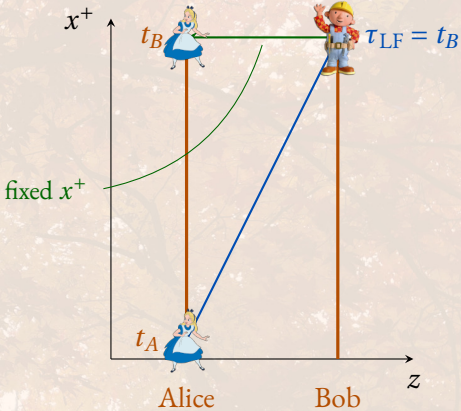
Anderson, Stedman & Vetharaniam, Phys. Rept. 295 (1998) 93 (technical review)

Synchronization conventions

Einstein synchronization

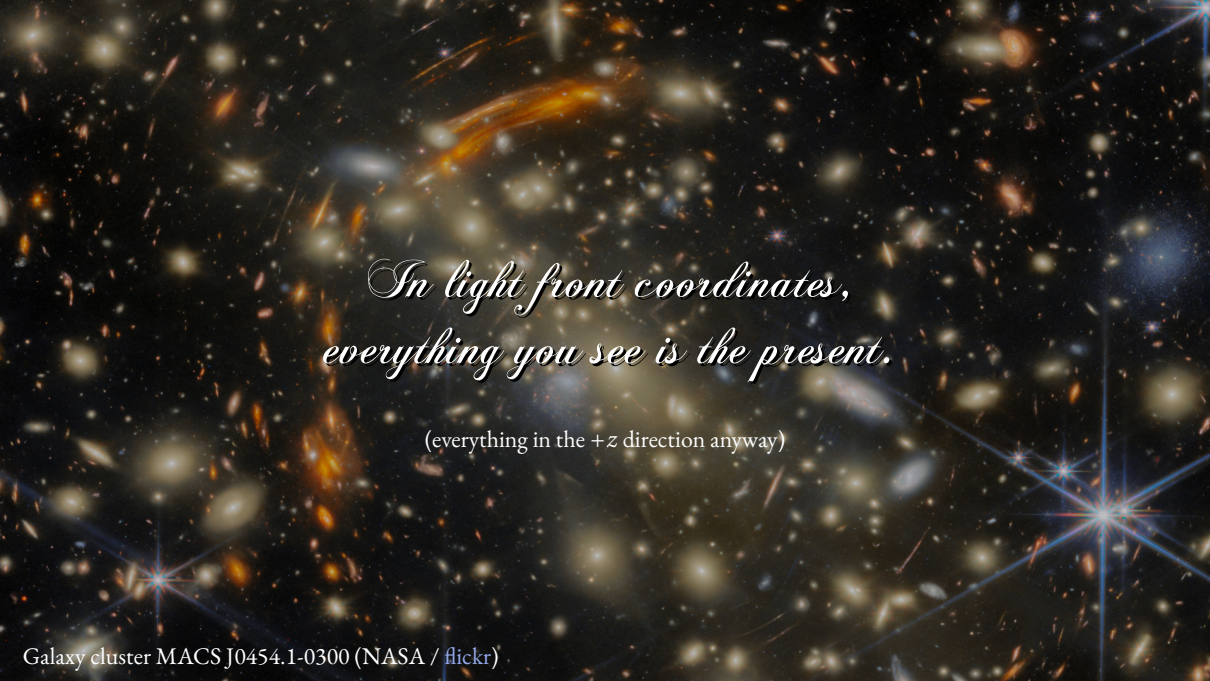


Light front synchronization

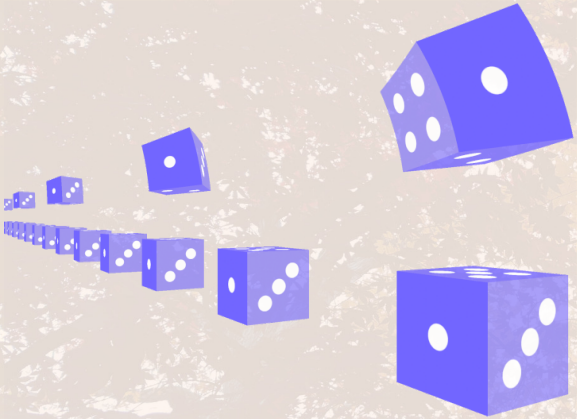


- **Einstein synchronization** defined to be isotropic.
- **Light front synchronization** defines hyperplanes with fixed $x^+ = \frac{1}{\sqrt{2}}(t+z)$ to be "simultaneous."
  - Light travels instantaneously in $-z$ direction *by definition.*

A. Einstein, Annalen Phys 17 (1905) 891
AF & Miller, Phys Rev D 107 (2023) 074036



Galaxy cluster MACS J0454.1-0300 (NASA / flickr)



Dice images by Ute Kraus,
https://www.spacetimetravel.org/

- Lorentz-boosted objects *appear rotated*.
  - **Terrell rotation**
  - Optical effect: contraction + delay

- **Light front transverse boost**
  undoes Terrell rotation:

$$B_x^{(\mathrm{LF})} = \frac{1}{\sqrt{2}}\left(K_x + J_y\right)$$

  - Standard boost + counter-rotation
  - Leaves x^+ (time) invariant
  - Part of the **Galilei subgroup**

J. Terrell, Phys Rev 116 (1959) 1041
M. Burkardt, Int J Mod Phys A 18 (2003) 173

- Poincaré group has a $(2+1)$D **Galilei subgroup**.
  - x^+ is time and $\boldsymbol{x}_\perp$ is space under this subgroup.
  - $P^+ = \frac{1}{\sqrt{2}}\left(E_{\boldsymbol{p}} + p_z\right)$ is the central charge.
  - x^+ and P^+ are invariant under this subgroup!
- Light front time gives **fully relativistic** 2D picture that looks a lot like non-relativistic physics.



$$\frac{\mathrm{d}\boldsymbol{P}_\perp}{\mathrm{d}x^+} = P^+ \frac{\mathrm{d}^2\boldsymbol{x}_\perp}{\mathrm{d}x^{+2}}$$

$$H = H_{\text{rest}} + \frac{\boldsymbol{P}_\perp^2}{2P^+}$$

$$\boldsymbol{v}_\perp = \frac{\boldsymbol{P}_\perp}{P^+}$$

G. Pinski, J Math Phys 9 (1986) 1927
M. Burkardt, Int J Mod Phys A 18 (2003) 173

$$\int_{\mathbb{R}} \mathrm{d}x^- \langle\Psi|\hat{T}^{ij}(x)|\Psi\rangle = \sum_{s,s'} \int \mathrm{d}^2 R_\perp \Bigg\{ \overbrace{-\Psi^*_{s'} \frac{\overleftrightarrow{\nabla}^i \overleftrightarrow{\nabla}^j}{4(P^+)^2} \Psi_s}^{\text{dynamic pressures}} \underbrace{\mathfrak{a}_{s's}(\boldsymbol{x}_\perp - \boldsymbol{R}_\perp)}_{\text{internal } P^+ \text{ density}} - \overbrace{\frac{i\Psi^*_{s'} \overleftrightarrow{\nabla}^i \Psi_s}{2P^+}}^{\text{barycentric flux}} \underbrace{\mathfrak{p}^j_{s's}(\boldsymbol{x}_\perp - \boldsymbol{R}_\perp)}_{\text{internal momentum density}}$$

$$- \overbrace{\frac{i\Psi^*_{s'} \overleftrightarrow{\nabla}^j \Psi_s}{2P^+}}^{\text{barycentric flux}} \underbrace{\mathfrak{f}^i_{s's}(\boldsymbol{x}_\perp - \boldsymbol{R}_\perp)}_{\text{internal } P^+ \text{ flux}} + \overbrace{\Psi^*_{s'} \Psi_s}^{\text{barycentric probability}} \underbrace{\mathfrak{t}^{ij}_{s's}(\boldsymbol{x}_\perp - \boldsymbol{R}_\perp)}_{\text{internal stresses}} \Bigg\}$$



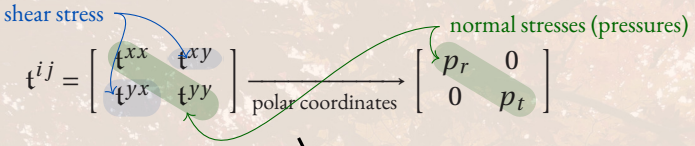
Image: dynamic pressure from wind

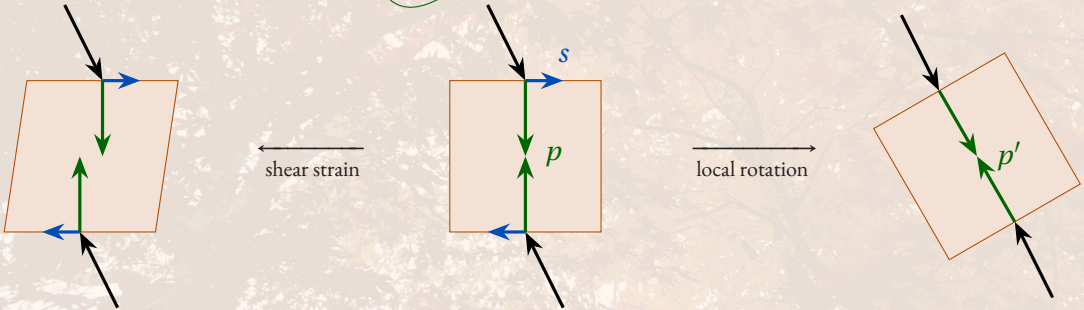
- Echoes non-relativistic breakdown—light front Galilei symmetry!
- EMT-FFs $D_q(\boldsymbol{\Delta}_\perp^2)$ and $\bar{c}_q(\boldsymbol{\Delta}_\perp^2)$ in stress tensor.

$$\mathfrak{t}^{ij}_q(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \left(\frac{\Delta^i_\perp \Delta^j_\perp - \delta^{ij}\boldsymbol{\Delta}^2_\perp}{4P^+} D_q(\boldsymbol{\Delta}^2_\perp) - \frac{m^2}{P^+}\delta^{ij}\bar{c}_q(\boldsymbol{\Delta}^2_\perp) \right) \mathrm{e}^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}$$

Lorcé, Moutarde & Trawinski, Eur Phys J C 79 (2019) 89
AF & Miller, Phys Rev D 103 (2021) 094023

Principal stresses on the light front





- **Normal stresses** push (compression) or pull (tension) on material element
- **Shear stresses** cause medium to shear
- Shear stresses eliminated by diagonalization—get **principal stresses**

C.S. Jog, *Continuum Mechanics*
Polyakov & Schweitzer, Int J Mod Phys A (2018)

Principal stresses in proton

$$\sum_{q,g} \mathrm{t}^{ij}_{q,g}(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \left(\frac{\Delta^i_\perp \Delta^j_\perp - \delta^{ij}\boldsymbol{\Delta}^2_\perp}{4P^+} \right) D(\boldsymbol{\Delta}^2_\perp)\, \mathrm{e}^{-i\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp}$$

Combined tripole form

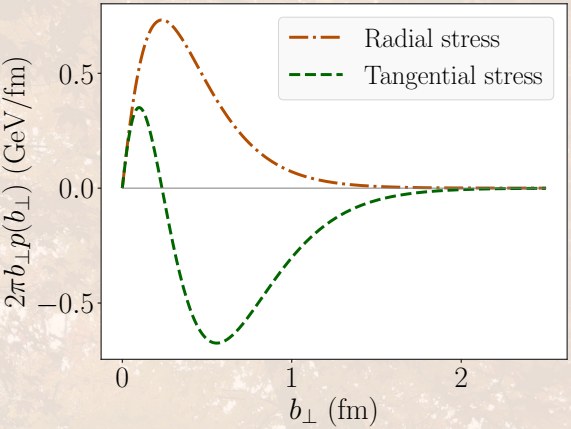
$$D(t) = \frac{D_0}{(1 - t/\Lambda^2)^3}$$

$$D_0 = D_0^{(q)} + D_0^{(g)} = -3.46$$

$$\Lambda = 1.12 \text{ GeV}$$

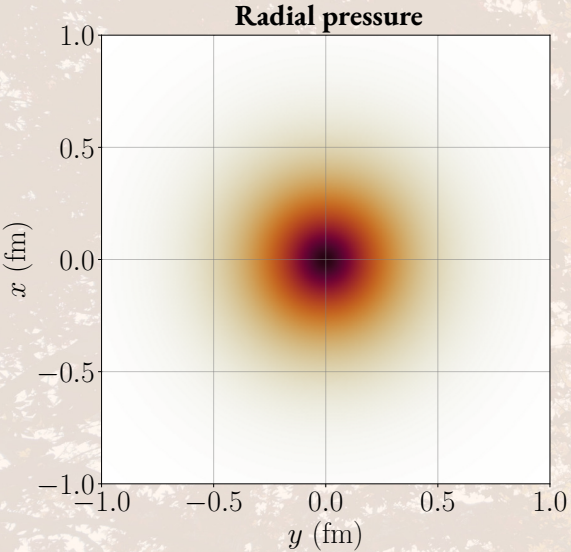
Burkert, Elouadrhiri & Girod, Nature (2018) (quark)

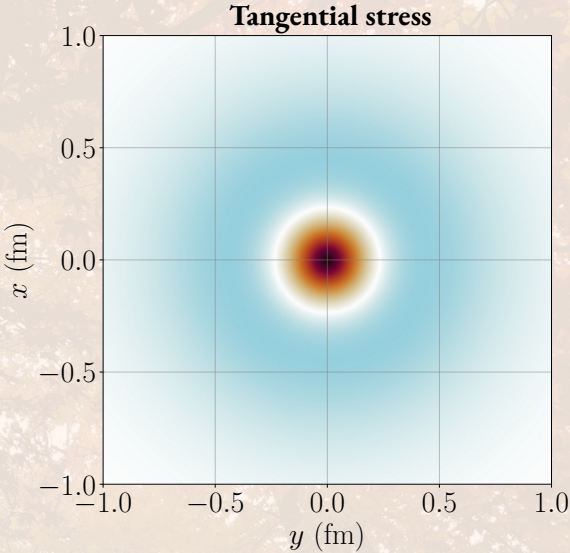
Duran &al., Nature (2023) (gluon)



- **Radial stress** is strictly positive in the proton—**compression**
- **Tangential stress** is negative at periphery—**tension**

2D principal stress distributions





Proton is like a liquid drop with a *very* fuzzy, diffuse boundary

Polyakov, Phys Lett B (2003)


Mechanical Interpretation

Positive or negative stress?

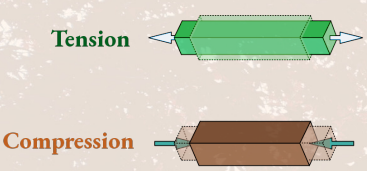


Image: MikeRun (Wikimedia), very modified

Hanging monkeys

- **Negative stress** / negative "pressure"
- **Tension** / pulling / stretching
- Highest monkey under most tension

$$\nabla_i T^{ij} = -\rho g \hat{z}^j = f^j_{\text{grav}}$$

Monkey image by Angie Freese (Instagram)

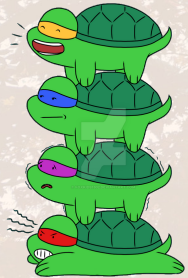


Stacked Turtles

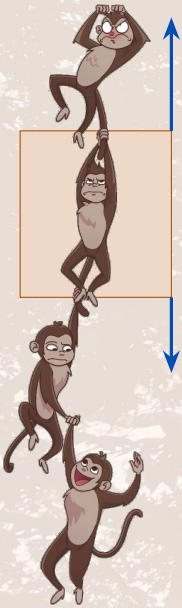
- **Positive stress** / positive "pressure"
- **Compression** / pushing / squishing
- More compression at bottom of pile

$$\nabla_i T^{ij} = -\rho g \hat{z}^j = f^j_{\text{grav}}$$

Turtle image by FoxKids1302 (DeviantArt)



Cauchy momentum equation



- **Cauchy momentum equation** for an *open, static* system:

$$f^j_{\text{net}}(\boldsymbol{x}, t) = f^j_{\text{external}}(\boldsymbol{x}, t) - \nabla_i T^{ij}(\boldsymbol{x}, t) = 0$$

  - Balance between **external forces** & **internal forces**
  - Can infer external forces from stress tensor!

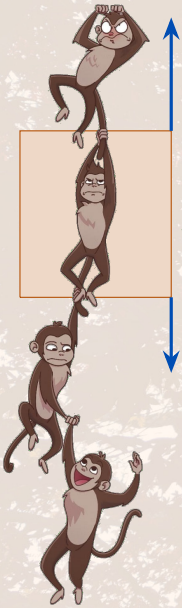
- Works for expectation values in quantum systems!

$$\langle f^j_{\text{external}}(\boldsymbol{x}, t)\rangle = \nabla_i \langle T^{ij}(\boldsymbol{x}, t)\rangle$$

  - Schrödinger, Ann. Phys. 387 (1927) 265
  - Nielsen & Martin, PRB 32 (1985) 3780
  - Polyakov & Son, JHEP 09 (2018) 156

- Stress tensor in hydrogen atom returns Coulomb force law!
  - AF, Phys Rev D 111 (2025) 034047

Monkey image by Angie Freese (Instagram)



$F_{\text{up}} = +3Mg$

$F_g = -Mg$

$F_{\text{down}} = -2Mg$

- Internal stress tensor:

$$\mathrm{t}_c^{ij}(\boldsymbol{b}) = \int \frac{\mathrm{d}^n \Delta}{(2\pi)^n} \left(\frac{\Delta^i \Delta^j - \delta^{ij} \boldsymbol{\Delta}^2}{4P^+} D_c(\boldsymbol{\Delta}^2) \quad -\frac{M^2}{P^+} \delta^{ij} \bar{c}_c(\boldsymbol{\Delta}^2) \right) \mathrm{e}^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}}$$

  - $n = 2$ for light front, $n = 3$ for non-relativistic

- $\bar{c}(\boldsymbol{\Delta}^2)$ quantifies force imbalance
  - Hence provides access to net external force

- $D(\boldsymbol{\Delta}^2)$ quantifies balancing forces from all sides
  - Provides ambient pressures
  - Similar to deep sea pressure, stress in bridge, etc.
  - All moneys have the same $\bar{c}$—differ only in D
  - **This is physical and can be felt**

Monkey image by Angie Freese (Instagram)

Example: $\bar{c}$ in non-relativistic systems

- Particle $\bar{c}(\Delta^2)$ for two-body non-relativsitic state:

$$\bar{c}_n(\Delta^2) = \frac{1}{2\chi_n\overline{\chi}_n M^2 \Delta}\int_0^\infty \mathrm{d}r\left(u'(r)u''(r) - u(r)u'''(r)\right) j_1\left(\overline{\chi}_n \Delta r\right)$$

  - Auxiliary quantities:

$$\chi_n = \frac{m_n}{M}$$

Mass fraction

$$\overline{\chi}_n = 1 - \chi_n$$

Mass fraction complement

- Using Schödinger equation and integration by parts:

$$\bar{c}_n(\Delta^2) = \frac{1}{M\Delta}\int_0^\infty \mathrm{d}r\, u^2(r)\left(-V'(r)\right) j_1\left(\overline{\chi}_n \Delta r\right)$$

  - **Can be inverted to find the force law using $\bar{c}(\Delta^2)$!**

$$-V'(r) = \frac{1}{\overline{\chi}_n^3 |\psi(r)|^2}\frac{\mathrm{d}}{\mathrm{d}b}\left[\int_0^\infty \mathrm{d}\Delta\, \Delta^2 \bar{c}_n(\Delta^2)\, j_0(\Delta b)\right]\Big|_{b=\overline{\chi}_n r}$$

  - Reproduces Coulomb force (hydrogen atom), central part of nuclear force (deuteron)

AF, Phys Rev D 111 (2025) 034047
Cosyn, AF & Sosa Phys Rev C 113 (2026) 055208
AF, 2609.33151

Need for the other form factors

- Need $\bar{c}_q(t)$ for full pressure

$$\mathfrak{t}_q^{ij}(\boldsymbol{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\left(\frac{\Delta_\perp^i\Delta_\perp^j - \delta_\perp^{ij}\boldsymbol{\Delta}_\perp^2}{4P^+}D_q(\boldsymbol{\Delta}_\perp^2) - \frac{m^2}{P^+}\delta_\perp^{ij}\bar{c}_q(\boldsymbol{\Delta}_\perp^2)\right)\mathrm{e}^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}$$

- $\bar{c}_q(t)$ also gives force distribution

$$\langle f_c^j(\boldsymbol{b}_\perp)\rangle = -m\nabla_j\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\bar{c}_c(\boldsymbol{\Delta}_\perp^2)\,\mathrm{e}^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}$$

  - Makes $\bar{c}_q(t)$ the most exciting form factor! (in my opinion...)

- Unclear how to measure except in **D2 scheme**:

$$\bar{c}_q(t) = -\bar{c}_g(t) = \frac{1}{4}\Big(\underbrace{F_\sigma(t)}_{\text{sigma term form factor}} - A_q(t)\Big) - \frac{t}{16m^2}\Big(2J_q(t) - A_q(t) - 3D_q(t)\Big)$$

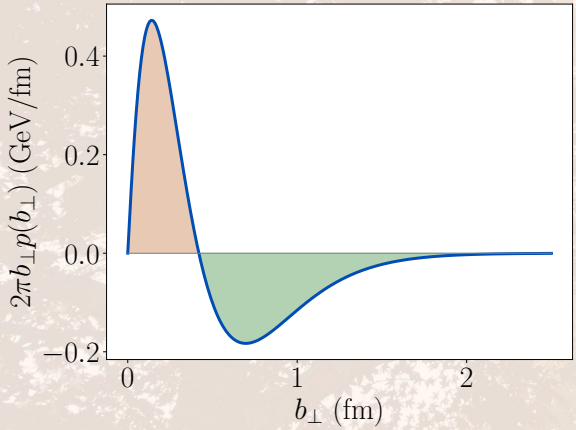
  - Need *all* EMT-FFs—**need x-dependence of GPDs**

Polyakov & Son, JHEP 09 (2018) 156
Lorcé, Moutarde & Trawinski, Eur Phys J C 79 (2019) 89
Metz, Rodini & Pasquini, Phy .Rev D 102 (2020) 114042

Isotropic pressure—average of principal stresses:

$$p(\boldsymbol{b}_\perp) = \frac{1}{2}\Big(p_r(\boldsymbol{b}_\perp) + p_t(\boldsymbol{b}_\perp)\Big) = -\frac{1}{4P^+}\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\boldsymbol{\Delta}_\perp^2 D(\boldsymbol{\Delta}_\perp^2)\,\mathrm{e}^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}$$

- Often just called "pressure" (e.g., in BEG paper)



von Laue condition

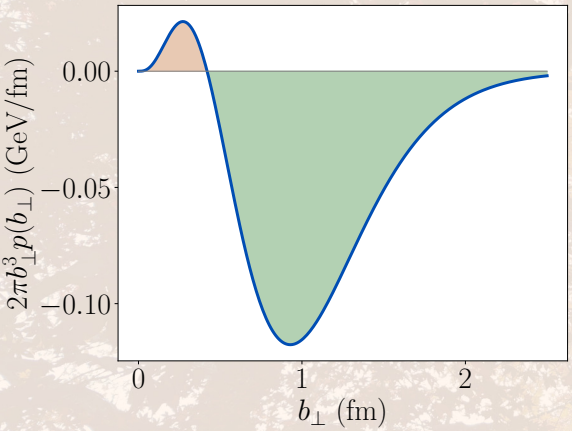
$$\int \mathrm{d}^2 b_\perp\, p(\boldsymbol{b}_\perp) = 0$$

- Satisfied for every hadron
- Can be shown from:

$$\langle \boldsymbol{p}|T^{ij}(0)|\boldsymbol{p}\rangle = 0$$

von Laue, Annalen Phys 340 (1911) 524

Polyakov & Schweitzer, Int J Mod Phys A 33 (2018) 1830025



Polyakov's conjecture

$$D(0) = P^+ \int \mathrm{d}^2 b_\perp \, b_\perp^2 \, p(\boldsymbol{b}_\perp) < 0$$

- Seems to be true for hadrons
- Seems intuitive:
  - Compression against collapse
  - Tension against disintegration
- **Is false for some systems!**

Perevalova, Polyakov & Schweitzer, PRD 94 (2016) 054024

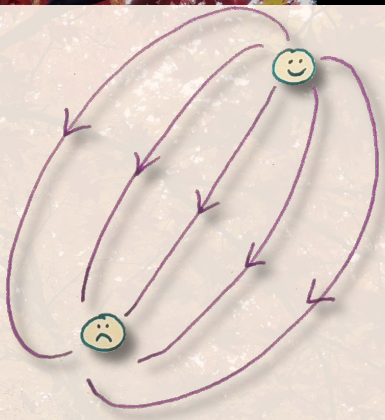
Polyakov & Schweitzer, Int J Mod Phys A 33 (2018) 1830025

Example: static abelian fields

- Two-body non-relativistic case again
- Assume **static abelian fields**
  - **Static**: time-independent
  - **Abelian**: no self-interactions
- Per-configuration stress depends on field spin:

$$T^{ij}_{\text{static}} = (-1)^s \left\{ (\nabla_i \phi)(\nabla_j \phi) - \frac{1}{2}\delta^{ij}\left((\boldsymbol{\nabla}\phi)^2 + \mu^2\phi^2\right) \right\}$$

  - Inherited from sign of Lagrangian

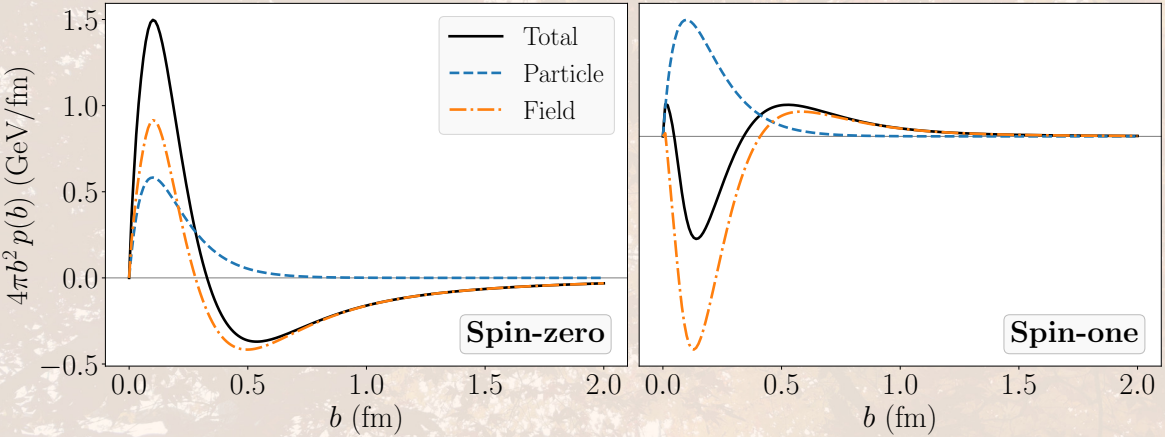


- Recipe for form factors:

$$\langle \boldsymbol{p}' | \hat{T}^{ij}_{\phi}(0) | \boldsymbol{p} \rangle = \underbrace{\int \mathrm{d}^3 r \, |\psi(\boldsymbol{r})|^2 \overbrace{\int \mathrm{d}^3 b \, \mathrm{e}^{i\boldsymbol{\Delta}\cdot\boldsymbol{b}} T^{ij}_{\phi}\left(0; \boldsymbol{b} - \eta_1 \overline{\chi}_1 \boldsymbol{r}, \boldsymbol{b} - \eta_2 \overline{\chi}_2 \boldsymbol{r}\right)}^{\text{Fourier transform of classical field configuration}}}_{\text{Ensemble average over particle configurations}}$$

AF, 2609.33151

Isotropic pressure distribution



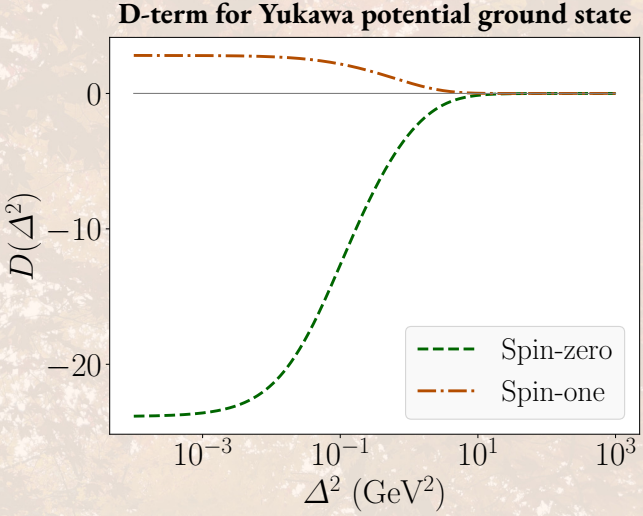
$V(r) = -\dfrac{\alpha\, e^{-\mu r}}{r}$ $\quad \alpha = 1$

$m_1 = m_2 = 1$ GeV $\quad \mu = 0.1$ GeV

- Exact same potential & wave function
- Spin of force field radically changes pressure

AF, 2609.33151

- Sign of D-term from field spin
  - Unrelated to stability
- **Spin-odd: postive**
  - Includes electromagnetic systems
  - Hydrogen atom has $D(0) > 0$
- **Spin-even: negative**
- Hadrons break this pattern—**why?**
  - Gluons are spin one...
  - Something to learn about QCD!



AF, 2609.33151


Wrapping Up

Summary & Outlook

- EMT & its form factors give mechanical stresses
  - These are the same thing as momentum flux densities
- Divergence of stress tensor gives forces
  - This makes $\bar{c}_q(\Delta^2)$ the most exciting form factor!
- Light front framework gives relativistic spatial stress distributions
- We don't really understand why the D-term is negative for hadrons
  - It's not a stability condition
  - It breaks the pattern in abelian theories
  - Something interesting to learn about QCD here?
- We need *all* the EMT form factors for pressure and force densities

Thank you for your time!

Generative AI was **not** used in making these slides.