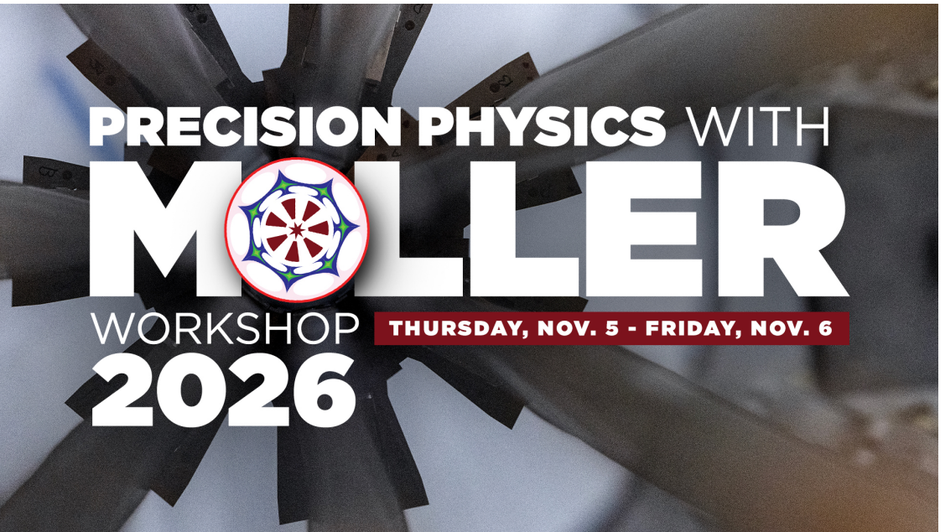
MOLLER Physics Extraction and Running of $\sin^2\theta_W$

Yury Kolomensky
UC Berkeley/LBNL

November 6, 2026



What does MOLLER Measure ?

- Two left-right asymmetries: longitudinal (electroweak) and transverse (QED, confirms absolute normalization)
- Experimental measurement: $A_{raw} = \dfrac{\text{Rate}_R - \text{Rate}_L}{\text{Rate}_R + \text{Rate}_L}$
- Convert to a physics asymmetry (c.f. E158 PRL, Ciprian's talk):

$$A_{\text{phys}} = \frac{1}{P_b \epsilon} \frac{A_{\text{raw}} - \Sigma_i \Delta A_i}{1 - \Sigma_i f_i}.$$

  - f: dilution factors
  - ε: linearity corrections
  - ΔA: background asymmetries
  - in MOLLER, a deconvolution method (a bit more advanced than E158)

Standard Model Prediction

- E158 prescription (order O(α)):

$$A_{\mathrm{PV}} = -\mathcal{A}(Q^2, y)\rho^{(e;e)}[1 - 4\sin^2\theta_{\mathrm{W}}^{\mathrm{eff}}(Q) + \Delta].$$

Interesting stuff

"Analyzing power":
kinematics, QED radiative corrections

Low-energy value of
NC/CC couplings

EW boxes etc

Standard Model Prediction

- Analyzing power

$$\mathcal{A}\,(Q^2, y) = \frac{G_F Q^2}{\sqrt{2}\pi\alpha(Q)} \frac{1-y}{1+y^4+(1-y)^4} \mathcal{F}_{\mathrm{QED}}$$

Kinematics (from simulations, calibration)

QED corrections
(internal brem, etc)

Standard Model Prediction

- Another way to rewrite this

$$A_{\mathrm{PV}} = -\mathcal{A}(Q^2, y)\rho^{(e;e)}[1 - 4\sin^2\theta_{\mathrm{W}}^{\mathrm{eff}}(Q) + \Delta].$$

$$Q_W^e(Q^2)$$

Low-energy coupling of Z to electron

Includes Z propagators, so integrates over energy scales. E158 treated scale dependence as an uncertainty, which is of O(α^2).

- Does this hold for O(α^2) ? Scale uncertainty should be O(α^3), small enough.
- Use this to extract experimental value of $Q_W^2(Q^2)$ and $\sin^2\theta_W^{\mathrm{eff}}$

Standard Model Prediction

- Yet another way to rewrite this

$$A_{\mathrm{PV}} = -\mathcal{A}(Q^2, y)\rho^{(e;e)}[1 - 4\sin^2\theta_{\mathrm{W}}^{\mathrm{eff}}(Q) + \Delta].$$

$$=$$

$$\hat{Q}_W^e(\sin^2\hat{\theta}_W(M_Z), M_Z, \alpha, \ldots)$$

- Computed in a particular renormalization scheme. So can pick a scheme (MSbar), integrate over SM nuisance parameters, invert the relationship to extract experimental value of $\sin^2\hat{\theta}_W(M_Z)$
- What would the theory community prefer ?

Transverse Asymmetries

- Experimental measurement: $A_{raw} = \dfrac{\text{Rate}_\uparrow - \text{Rate}_\downarrow}{\text{Rate}_\uparrow + \text{Rate}_\downarrow}$
- Convert to a physics asymmetry (c.f. Ciprian's talk):

$$A_{\text{phys}} = \frac{1}{P_b \epsilon} \frac{A_{\text{raw}} - \Sigma_i \Delta A_i}{1 - \Sigma_i f_i}.$$

  - Deconvolution method can extract A_{phys} for ee, ep elastic, ep inelastic
- Extract vector analyzing power

$$A_T = \frac{\alpha m}{\sqrt{s}} \sin\theta \sin\phi \times f(t,u) = \alpha_T \sin\phi$$

Transverse Asymmetries

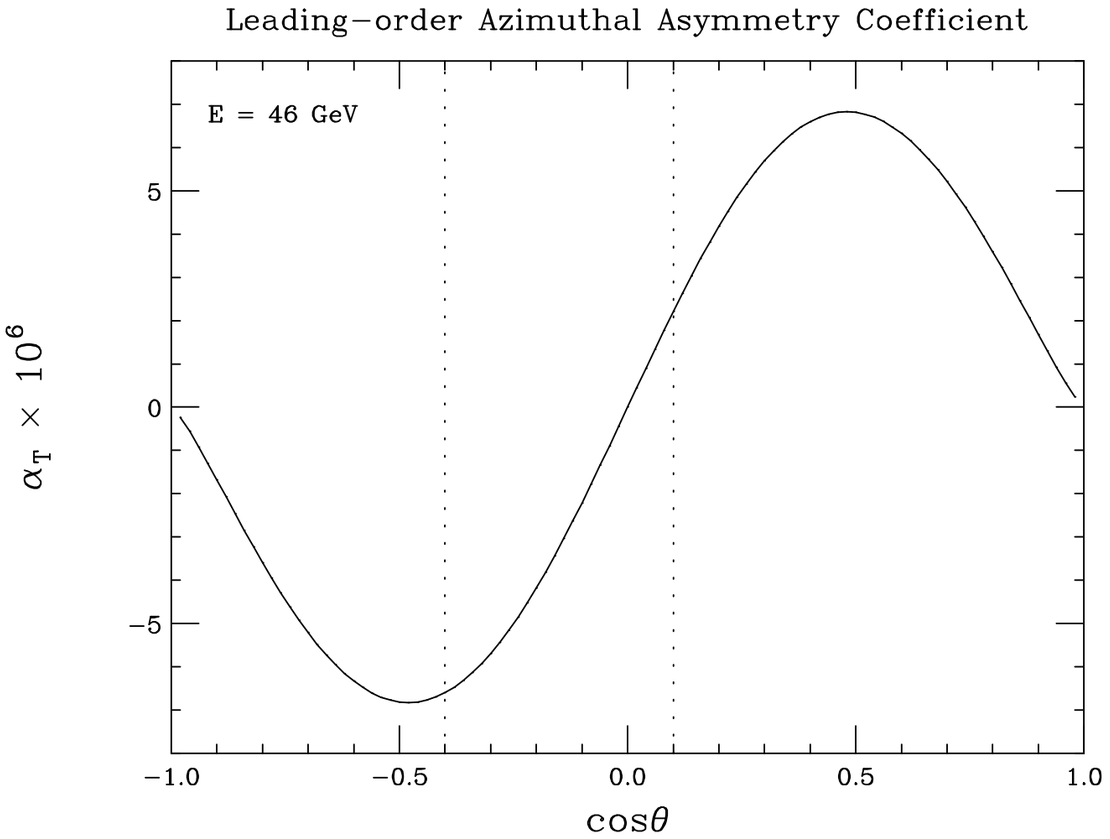


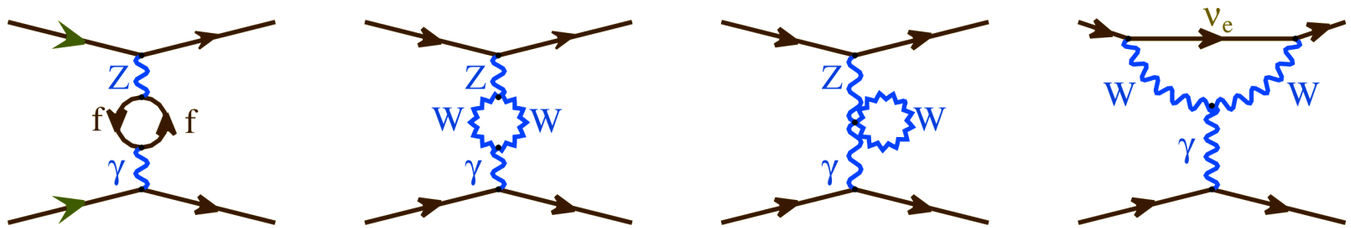
FIG. 2: The azimuthal asymmetry coefficient at leading order, α_T^{LO}, as a function of $\cos\theta$, for $E = 46$ GeV, $\sqrt{s} \approx 217$ MeV. The vertical dotted lines indicate the approximate acceptance of E158 for leading-order kinematics.

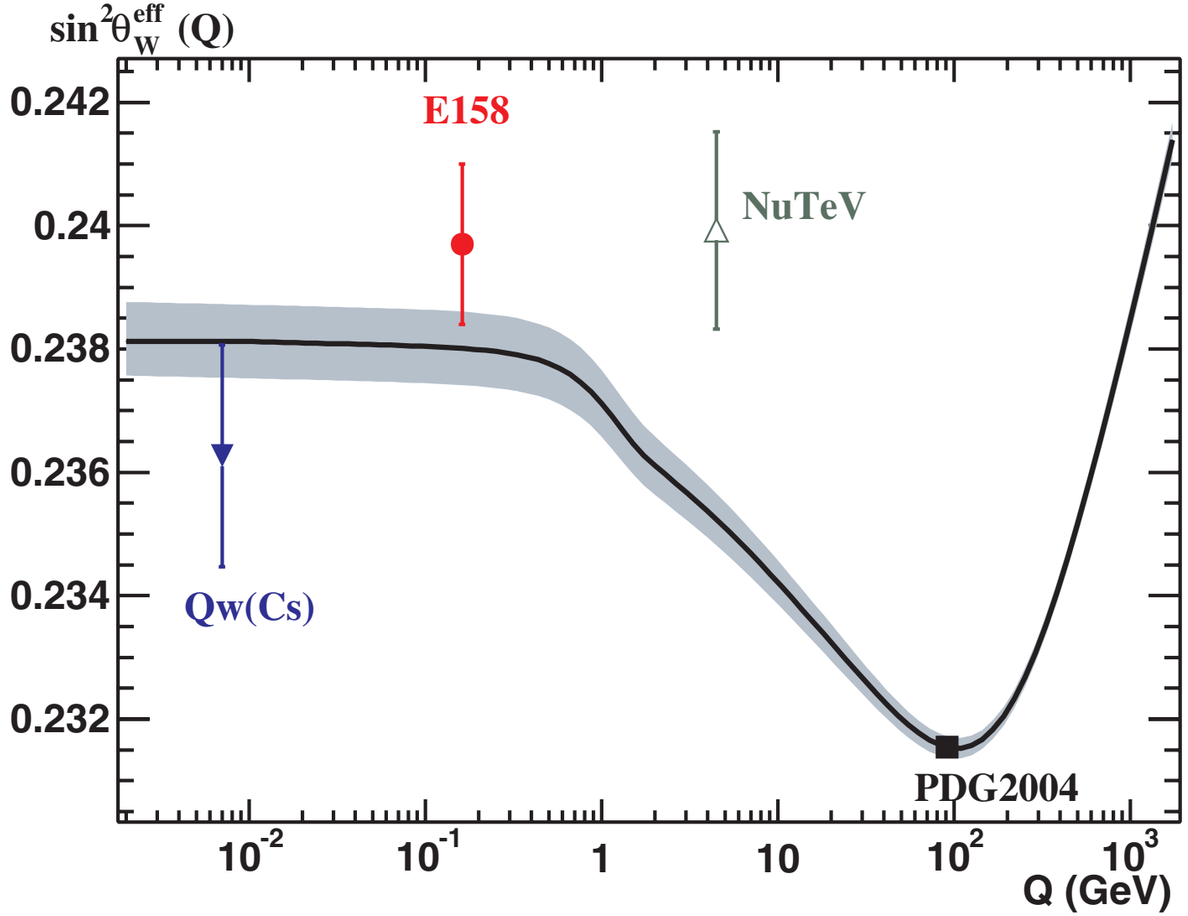
Dixon and Schreiber, PRD **69**, 113001 (2004), PRD **71**, 059903 (2005) (erratum)

What we need from theory

- SM prediction for A_{PV} and A_T, ideally to order $O(\alpha^2)$, on the timescale of the final publications
  - $O(\alpha)$ calculations for both A_{PV} and A_T done circa 2004; $O(\alpha^2)$ for A_{PV} is partially done
- Which means
  - Running of $\sin^2\theta_W$ (c.f. S Kuberski's talk, this talk)
  - Electroweak or EM corrections (boxes, vertices) to order $O(\alpha^2)$
  - Internal brem and corresponding counter-terms to order $O(\alpha^2)$
  - Kinematic distributions, i.e. double-differential cross sections $\frac{d^2\sigma}{dE'd\Omega}$
  - Ideally kinematic distributions for the backgrounds
  - The rest (i.e. external radiation, backgrounds) are handled in Geant4

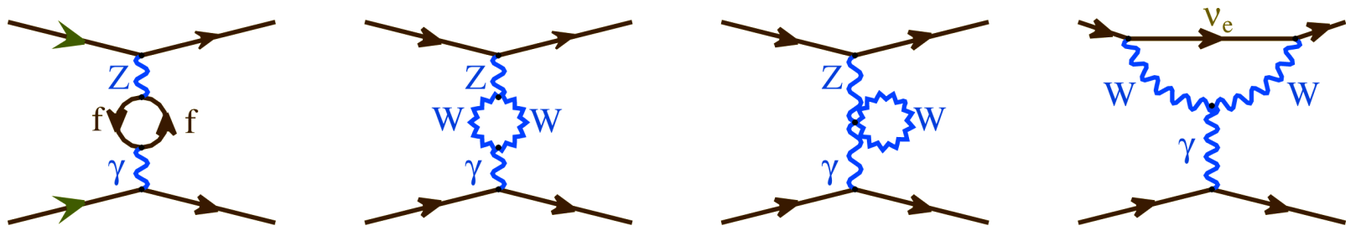
Weak Mixing Angle at Low Energies

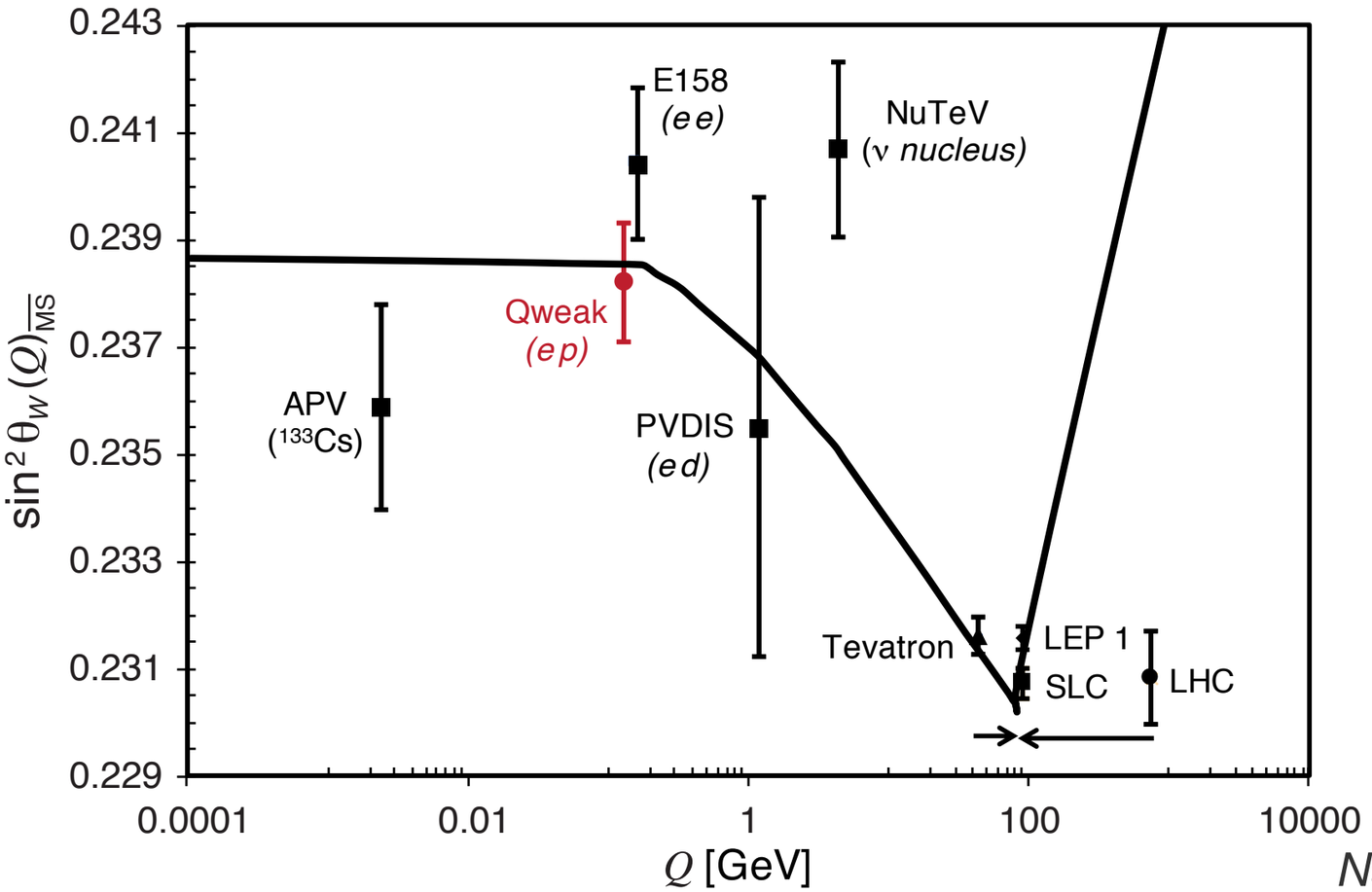




PRL **95,** 081601 (2005)

Weak Mixing Angle at Low Energies





Nature 557 (2018) 7704, 207

Electroweak Radiative Corrections

- Few points to note:
  - We have promised that the radiative corrections would be computed to "two loops" ($O(\alpha^2)$) before MOLLER takes data. Work on this is ongoing.

    ☞ Some recent progress

Hadronic effects in Møller scattering at NNLO

Jens Erler (Mainz U., Inst. Kernphys.), Rodolfo Ferro-Hernández (Mainz U., Inst. Kernphys.), Ayres Freitas (Mainz U., Inst. Kernphys.) (Feb 24, 2022)

Published in: *JHEP* 08 (2022) 183 • e-Print: 2202.11976 [hep-ph]

Bosonic Contribution to
Parity-Violating Møller Scattering at NNLO

Lisong Chen,[a] Jens Erler,[a] Ayres Freitas,[b] Juhun Kwak[b]

[a]PRISMA^{++} *Cluster of Excellence and Institute for Nuclear Physics, Johannes Gutenberg-University, 55099 Mainz, Germany*

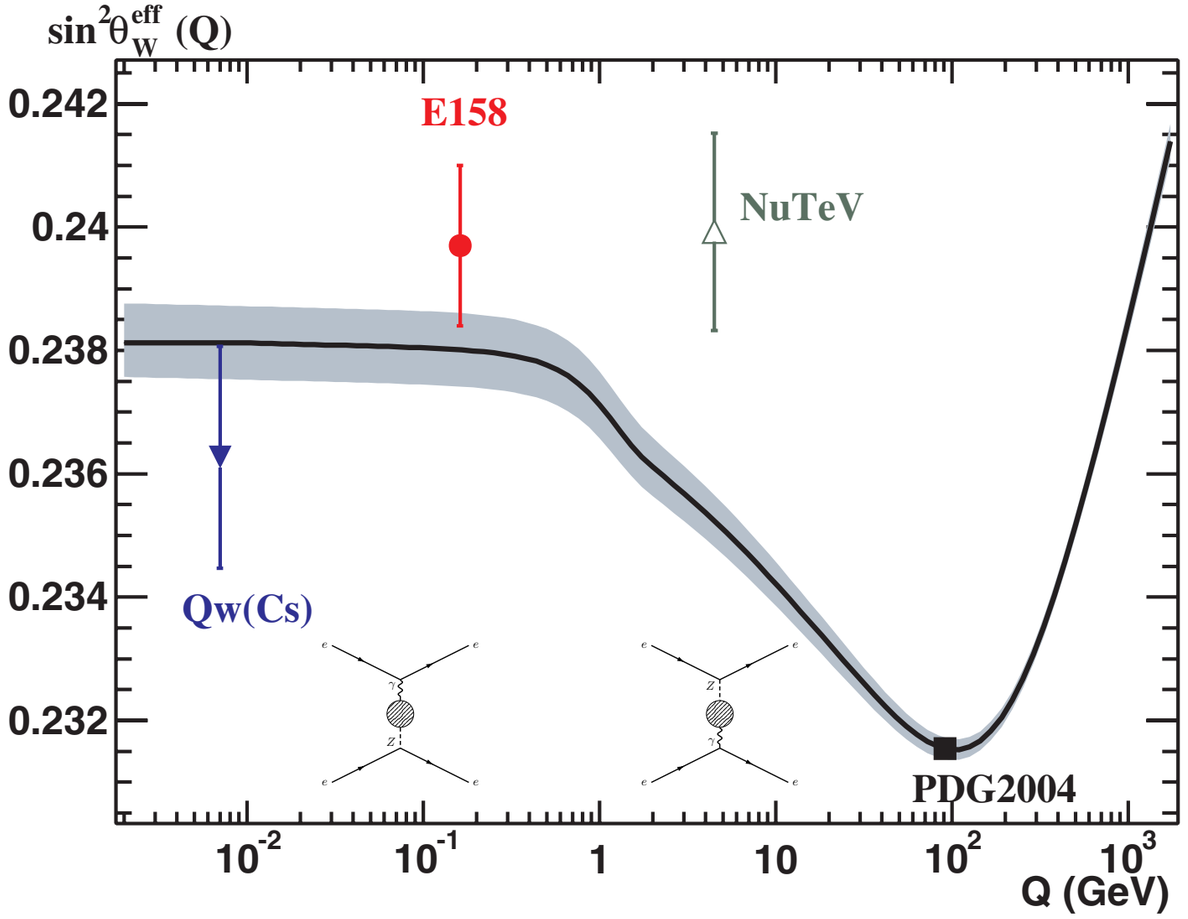
[b]*Pittsburgh Particle-physics Astro-physics & Cosmology Center (PITT-PACC), Department of Physics & Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA*

arXiv:2610.12287

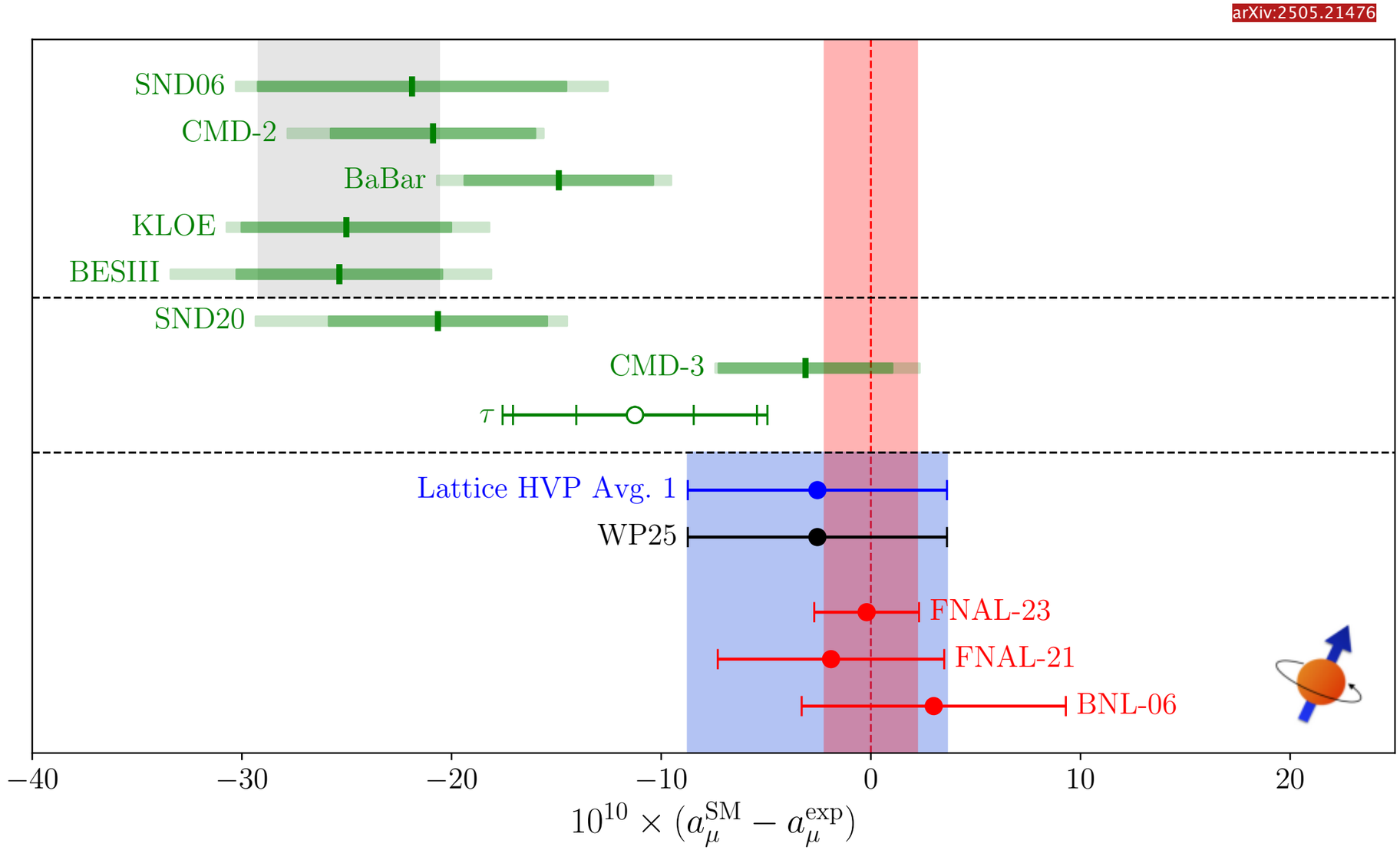
  - But uncertainties on the Standard Model prediction are not necessarily dominated by $O(\alpha^2)$ effects
  - Dominant uncertainty, historically, has been hadronic vacuum polarization contribution to γ-Z mixing

Hadronic Vacuum Polarization

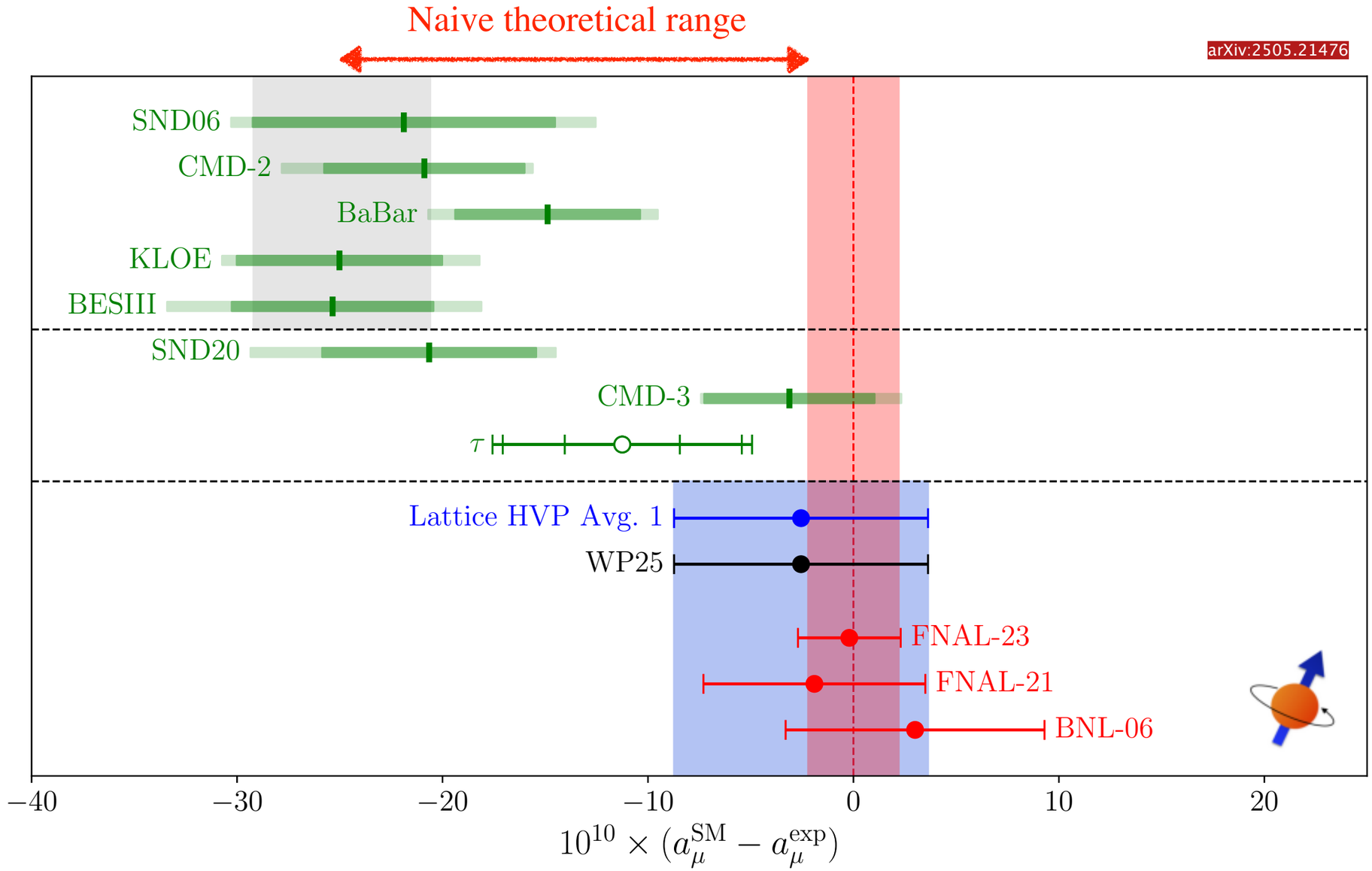
- The value of $\sin^2\theta_W^{eff}$ used by E158 was computed based on the estimate of hadronic vacuum polarization contributions circa 1970s
- Many things have changed since then:
  - ✓ Much more precise data !
  - ✓ RGE analysis of the running of electroweak constants
  - ✓ Precision electroweak fits
  - ✓ g-2 saga
- When we set out to update it, no updated calculation of the HVP was available in print
- We wanted to pay careful attention to the experimental uncertainties and experimental discrepancies
  - Radamovich, Nazymko, Kolomensky, in prep



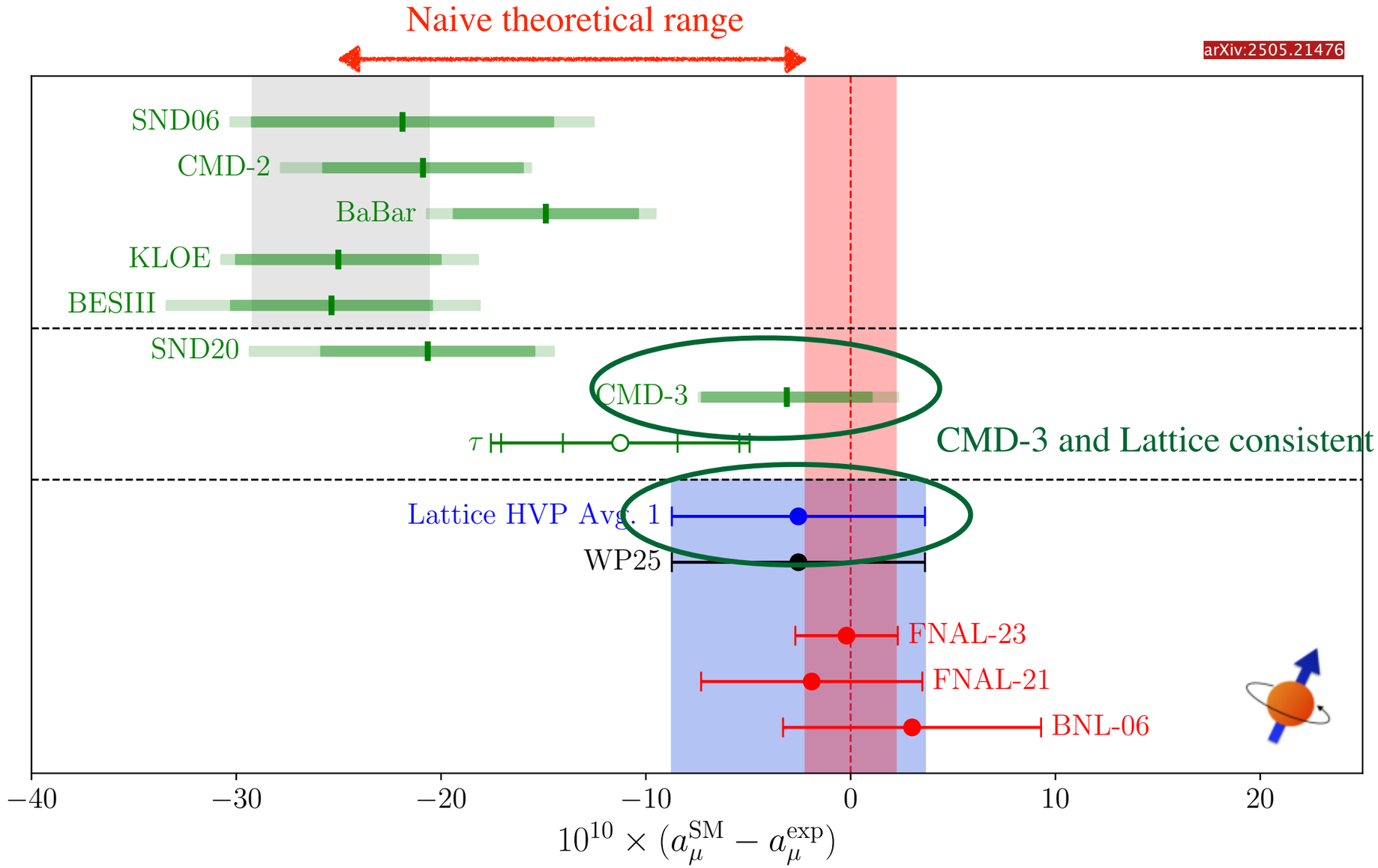
g-2 saga


arXiv:2505.21476
SND06
CMD-2
BaBar
KLOE
BESIII
SND20
CMD-3
τ
Lattice HVP Avg. 1
WP25
FNAL-23
FNAL-21
BNL-06
−40
−30
−20
−10
0
10
20
$10^{10} \times (a_\mu^{\mathrm{SM}} - a_\mu^{\mathrm{exp}})$

g-2 saga


Naive theoretical range
arXiv:2505.21476
SND06
CMD-2
BaBar
KLOE
BESIII
SND20
CMD-3
τ
Lattice HVP Avg. 1
WP25
FNAL-23
FNAL-21
BNL-06
−40
−30
−20
−10
0
10
20
$10^{10} \times (a_\mu^{\mathrm{SM}} - a_\mu^{\mathrm{exp}})$

g-2 saga


Naive theoretical range
arXiv:2505.21476
SND06
CMD-2
BaBar
KLOE
BESIII
SND20
CMD-3
τ
CMD-3 and Lattice consistent
Lattice HVP Avg. 1
WP25
FNAL-23
FNAL-21
BNL-06
−40
−30
−20
−10
0
10
20
$10^{10} \times (a_\mu^{\mathrm{SM}} - a_\mu^{\mathrm{exp}})$

Key Concept

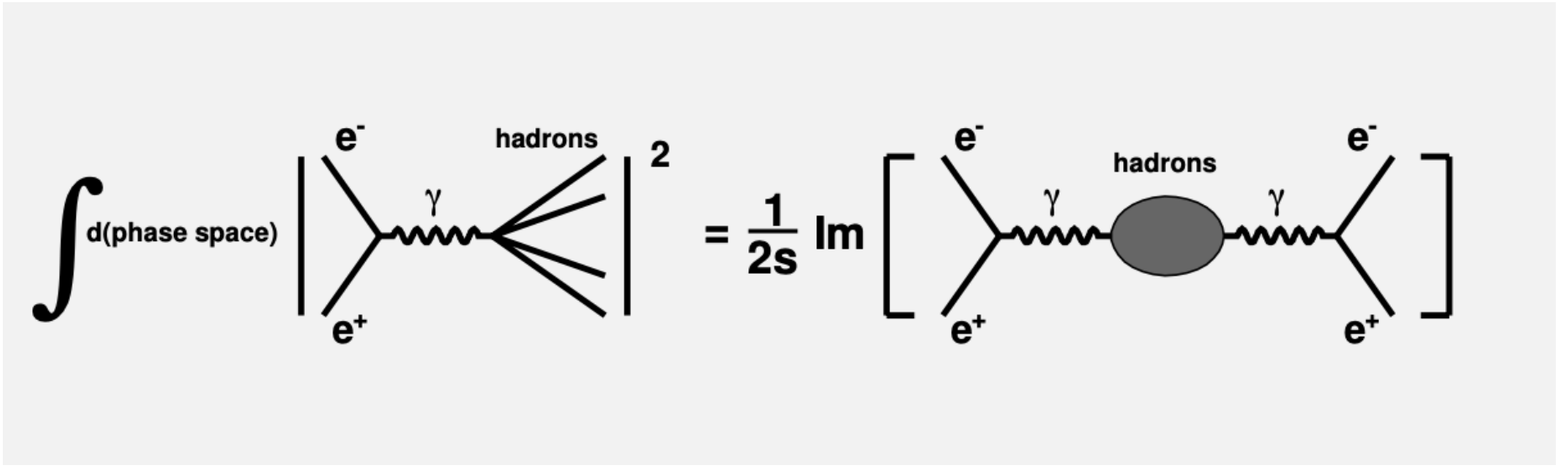


Figure 1: Optical theorem example.

- Since HVP cannot be computed perturbatively, use cross section data for $e^+e^- \rightarrow$ hadrons to integrate numerically
  - Similar approach has been used to estimate HVP contributions to g-2 (until switch to lattice)
  - Several detailed analyses for g-2 and $\Delta\alpha_{had}$ available and a plethora of datasets
  - So can test machinery on g-2 and $\Delta\alpha_{had}$, then apply to $\sin^2\theta_W^{eff}$

Hadronic Contributions to Running of $\sin^2\theta_W$

Traditionally express: $\sin^2\theta_W^{eff}(Q) = \kappa(Q)\sin^2\theta_W(M_Z)_{\overline{MS}}$

$$\kappa(0) = 1 - \frac{\alpha}{2\pi\hat{s}^2}\left(\frac{1}{3}\sum_{\mathrm{f},\ \mu_\mathrm{f}<M_Z}(T_{3f}Q_f - 2\hat{s}^2Q_f^2)\ln\frac{\mu_f^2}{M_Z^2} + \left(\frac{7}{2}\hat{c}^2 + \frac{1}{12}\right)\ln\hat{c}^2 + \left(\frac{7}{9} - \frac{\hat{s}^2}{3}\right)\right)$$

Replace sum over quarks with an integral over the data

$$\frac{1}{3}\sum_{f=\mathrm{quarks}}(T_{3f}Q_f - 2\hat{s}_Z^2Q_f^2)\ln\frac{m_f^2}{M_Z^2} \equiv -8\pi^2\Pi_{\mathrm{had}}^{\gamma Z}(0).$$

Weak Mixing Angle at Low Energies

- Three ways to compute the evolution of the weak mixing angle:
  - Czarnecki/Marciano: explicit calculation of the gamma-Z mixing by integrating over the $e^+e^-\rightarrow$hadrons cross sections. Requires separation into the flavor final states

$$\frac{1}{3}\sum_{f=\text{quarks}}(T_{3f}Q_f - 2\hat{s}_Z^2 Q_f^2)\ln\frac{m_f^2}{M_Z^2} \equiv -8\pi^2\Pi_{\text{had}}^{\gamma Z}(0). \qquad \Pi_{\text{had}}^{\gamma Z}(0) = \left(\frac{1}{2} - \hat{s}_Z^2\right)\Pi_{I=1}(0) - \frac{\hat{s}_Z^2}{9}\Pi_{I=0}(0) + \frac{1}{3}\left(\frac{1}{4} - \frac{\hat{s}_Z^2}{3}\right)\Pi_{s\bar{s}}(0) + \ldots,$$

  - Jegerlehner: flavor separation, calculation of EW couplings:

$$\sin^2\theta_{\text{eff}}(-M_Z^2) = \left(\frac{1-\Delta\alpha_2(-M_Z^2)}{1-\Delta\alpha(-M_Z^2)} + \ldots\right)\sin^2\theta_{\text{eff}}(0)$$

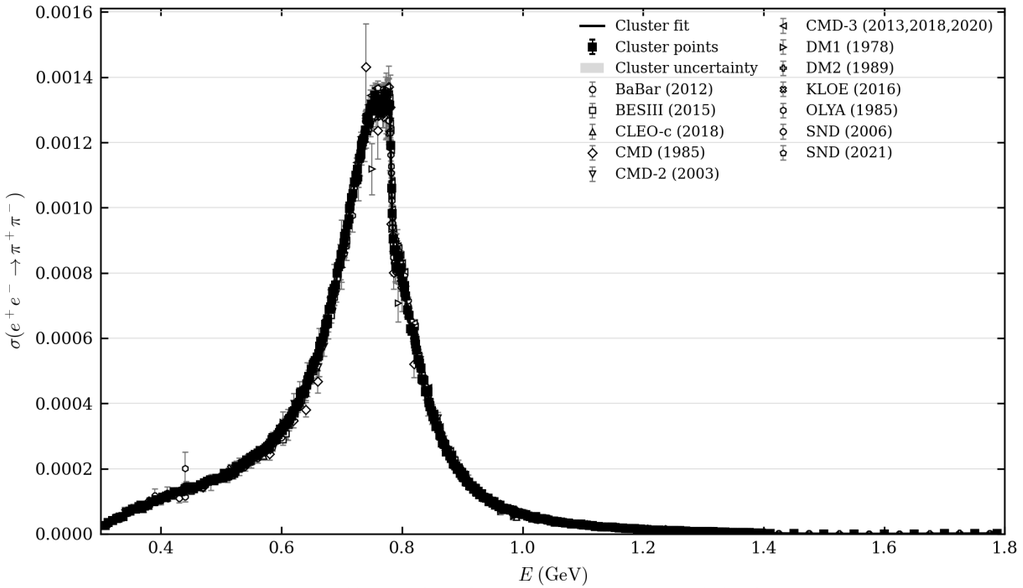
  - Erler et al.: renormalization group equation evolution

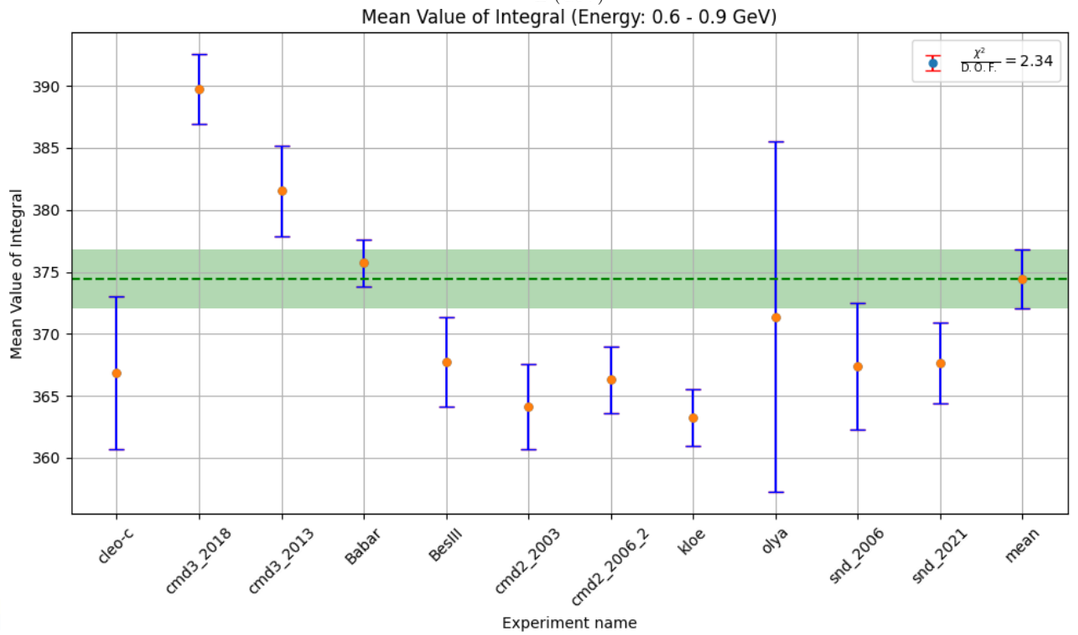
$$\hat{s}^2(\mu) = \hat{s}^2(\mu_0)\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \lambda_1\left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\right] + \frac{\hat{\alpha}(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu)\right].$$

☞ Experimental data enters implicitly by fitting for the effective quark masses and in the singlet contribution $\sigma(\mu)$

Example: $\pi^+\pi^-$ Channel

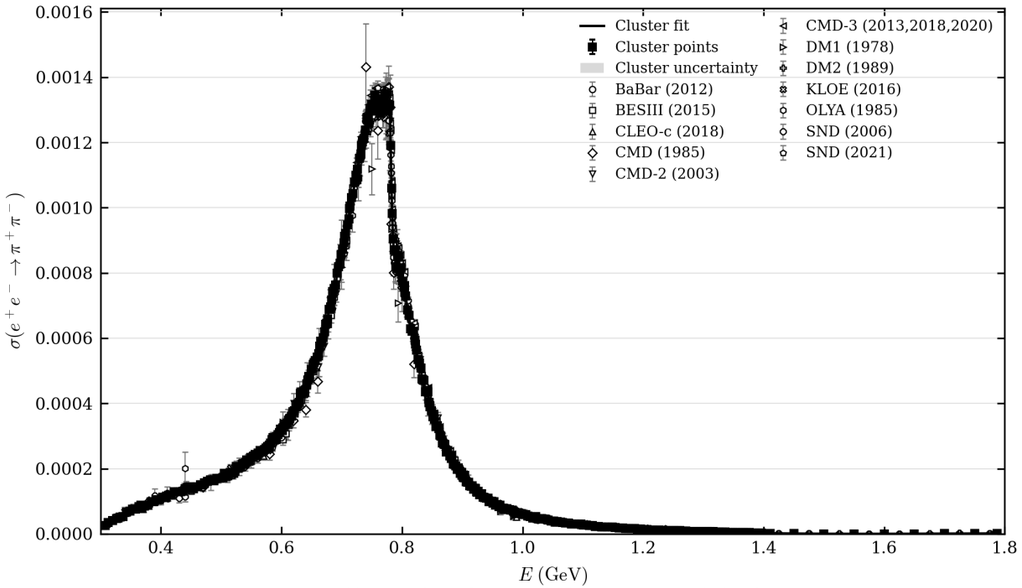
- Main source of problems for g-2
- Clear discrepancies between experiments
  - CMD3 vs KLOE produce the largest range
  - g-2 Theory Initiative has recently decided to ignore all data and instead rely on the lattice calculation (which agrees with CMD3)
    - ☞ For our analysis, we use the "PDG recipe": add uncorrelated systematic uncertainties to experiments until χ2/df=1, then compute weighted average
    - ☞ But also compute the variation if only one experiment is used, e.g. CMD3

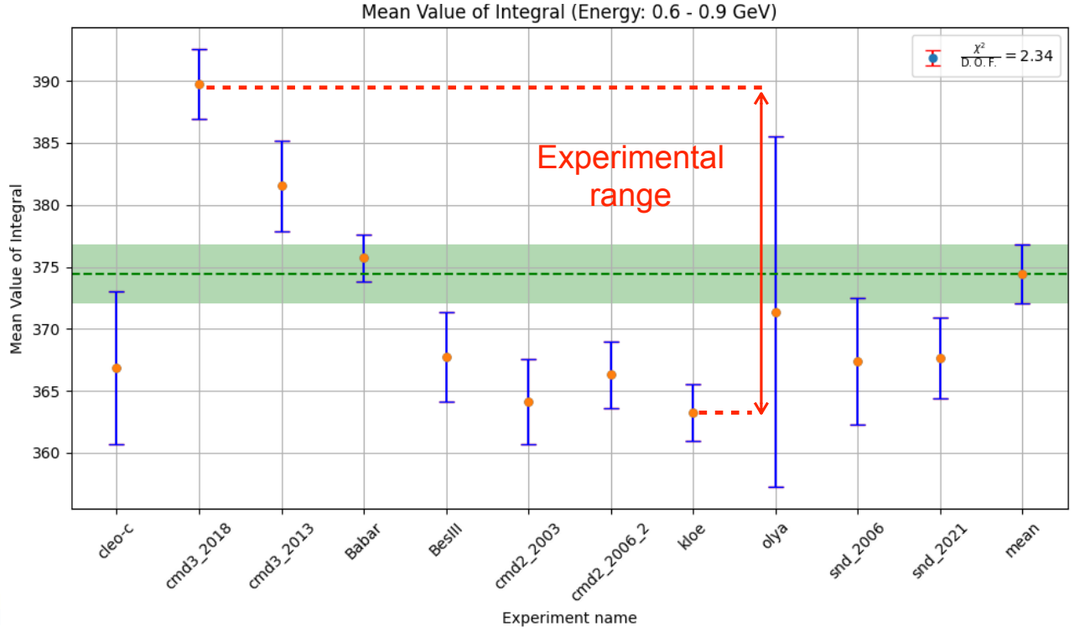




Example: $\pi^+\pi^-$ Channel

- Main source of problems for g-2
- Clear discrepancies between experiments
  - CMD3 vs KLOE produce the largest range
  - g-2 Theory Initiative has recently decided to ignore all data and instead rely on the lattice calculation (which agrees with CMD3)
    - ☞ For our analysis, we use the "PDG recipe": add uncorrelated systematic uncertainties to experiments until χ2/df=1, then compute weighted average
    - ☞ But also compute the variation if only one experiment is used, e.g. CMD3





Cross-check: $\Delta\alpha_{\mathrm{2had}}$

Channel	$\Delta\alpha_{\mathrm{2had}} \cdot 10^{-4}$	**Comment**
$\pi^+\pi^-$ (-2.178% ω)	$25.045 \pm 0.03 \pm 0.11$	Subtracted ω
$2\pi^+2\pi^-$	$2.58 \pm 0.00 \pm 0.06$	
$\pi^+\pi^-2\pi^0$	$3.25 \pm 0.02 \pm 0.13$	
$3\pi^+3\pi^-$	$0.03 \pm 0.00 \pm 0.00$	
$2\pi^+2\pi^-2\pi^0$	$0.24 \pm 0.02 \pm 0.04$	
$\omega\pi^0$	$0.73 \pm 0.02 \pm 0.01$	
Sum (light) (multiplied by 0.5)	$15.94 \pm 0.34 \pm 0.19$	–
K^+K^-	$1.24 \pm 0.02 \pm 0.04$	clear $s\bar{s}$
K_SK_L	$1.27 \pm 0.07 \pm 0.10$	
ϕ	$0.03 \pm 0.00 \pm 0.00$	
$K\bar{K}\,\pi$	$0.19 \pm 0.01 \pm 0.06$	
$K\bar{K}\,2\pi$	$0.22 \pm 0.02 \pm 0.03$	
Sum (multiplied by 0.75) ($s\bar{s}$)	$2.21 \pm 0.08 \pm 0.12$	–
$\eta\pi^+\pi^-$, ...	$0.33 \pm 0.01 \pm 0.02$ (combined)	
ψ, J/ψ ...	4.21 ± 0.10	BW integrals
$\Upsilon(1S$–$6S)$	0.67 ± 0.03	
Inclusive + pQCD tail	105.51 ± 0.36	
Final Summation	$128.12 \pm 0.11_{\mathrm{stat}} \pm 0.23_{\mathrm{syst}} \pm 0.38_{\mathrm{incl}} \pm 0.04_{\eta} \pm 0.64_{25\%}$	*(to get $\Delta\alpha_{\mathrm{2had}}$)*

Can compare to other existing calculations, e.g. Erler, Jegerlehner, etc. Systematic uncertainties include flavor separation uncertainties, e.g. η/η' mixing, ssbar contributions

Final answer

- Individual flavor contributions:
  - $\Pi_{I=1}(0) = 0.164 \pm 0.00078$
  - $\Pi_{I=0}(0) = 0.181 \pm 0.00456$
  - $\Pi_{s\bar{s}}(0) = 0.301 \pm 0.00160$
  - $\Pi_{c}(0) = 0.216 \pm 0.004$
  - $\Pi_{b}(0) = 0.1561 \pm 0.004$

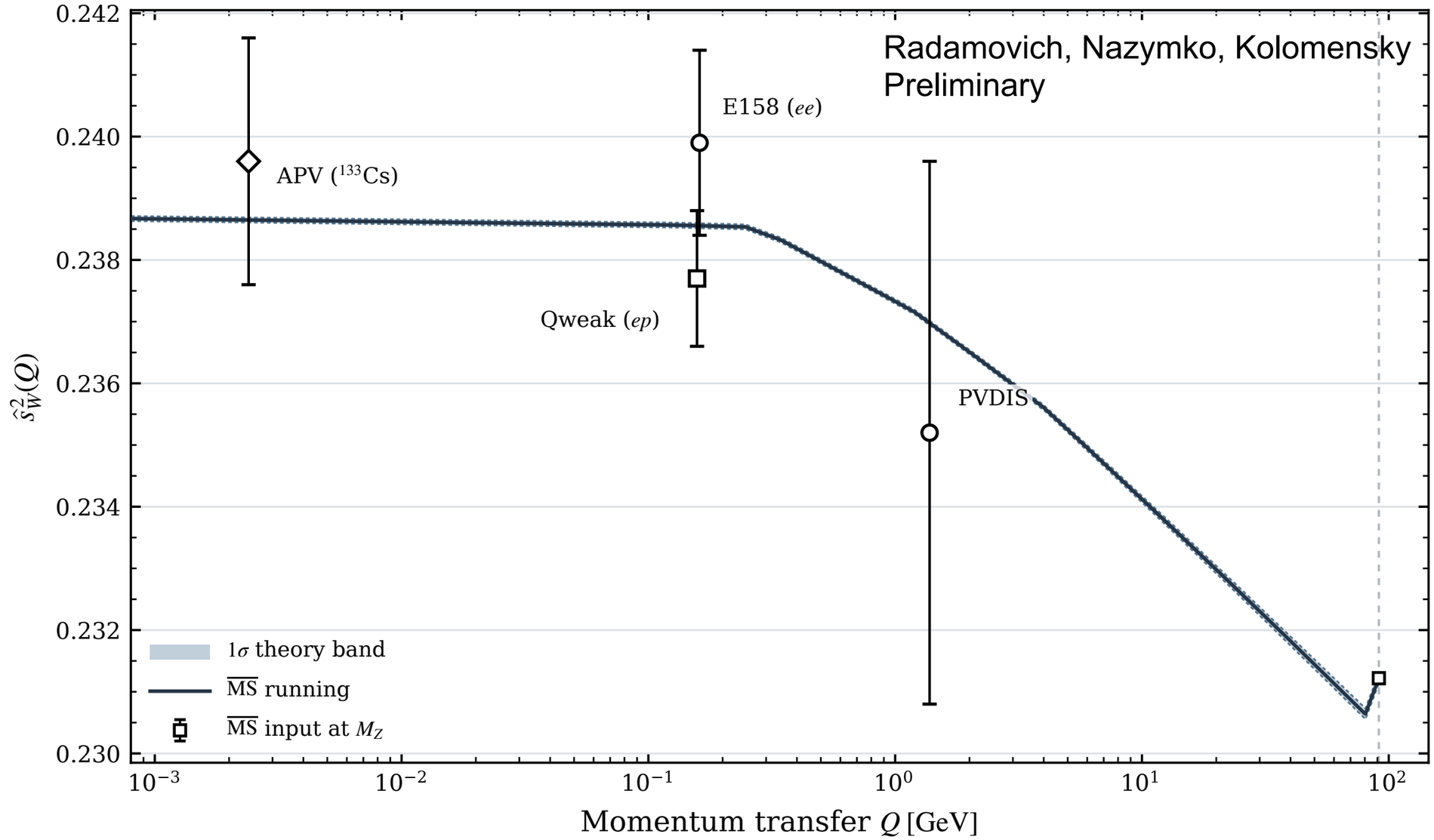
$$\Pi^{\gamma Z}_{\text{had}}(0) = \left(\frac{1}{2} - \hat{s}^2_Z\right)\Pi_{I=1}(0) - \frac{\hat{s}^2_Z}{9}\Pi_{I=0}(0) + \frac{1}{3}\left(\frac{1}{4} - \frac{\hat{s}^2_Z}{3}\right)\Pi_{s\bar{s}}(0) + \ldots,$$

- Results in

$$\hat{\kappa}(0) = 1.03225 \pm 0.00017$$

$$\sin\hat{\theta}_W(0) = 0.23868 \pm 0.00004\,(\text{HVP}) \pm 0.00008\,(\text{SM})$$

Running of $\sin^2\theta_W$ (MSbar scheme)



Comparison with other calculations

Source	$\sin^2\theta_W(0)$	Comment
This work (2025)	0.23868 ± 0.00004	Data-driven HVP evaluation
Czarnecki–Marciano (1996) [12]	0.23867 ± 0.00016	One-loop electroweak analysis
Jens Erler, Rodolfo Ferro-Hernandez and Simon Kuberski (2024) [25]	0.23869 ± 0.00002	RGE evolution with lattice-QCD input
Erler, Ferro–Hernández & Freitas (2022) [10]	0.23857 ± 0.00005	NNLO electroweak prediction
Jegerlehner (2017) [15]	0.23865 ± 0.00016	Adler–function method

Table VIII. Comparison of determinations of the low-energy $\overline{\text{MS}}$ correction factor $\widehat{\kappa}(0)$. The last column displays the shifted and rescaled quantity $10^4[\widehat{\kappa}(0) - 1]$.

Input or determination	$\widehat{\kappa}(0)$	$10^4[\widehat{\kappa}(0) - 1]$
This work, full $e^+e^- \to$ hadrons input	1.03225 ± 0.00017	322.5 ± 1.7
This work, CMD-3-based $\pi^+\pi^-$ input	1.03250 ± 0.00017	325.0 ± 1.7
LQCD input [21]	1.03234 ± 0.00010	323.4 ± 1.0

Comparison with other calculations

Most recent:

Theory Driven Evolution of the Weak Mixing Angle

Jens Erler[1], Rodolfo Ferro-Hernández[1] and Simon Kuberski[2]
[1]PRISMA$^+$ *Cluster of Excellence and Institute for Nuclear Physics, Johannes Gutenberg-University, 55099 Mainz, Germany*
[2]*Department of Theoretical Physics, CERN, 1211 Geneva 23, Switzerland*
(Dated: June 25, 2024)

parameter	result	correlations		
$\hat{\kappa}(0)-1$	$(323.4\pm1.0)\times10^{-4}$	1.0	0.98	0.9
$\Delta\alpha^{(5)}(M_Z)$	$(279.4\pm0.9)\times10^{-4}$	0.98	1.0	0.8
$a_\mu^{\rm hvp}$	$(72.0\pm1.6)\times10^{-9}$	0.8	0.8	1.0
$\hat{\kappa}(0)-1$	$(320.1\pm0.8)\times10^{-4}$	1.0	0.7	0.5
$\Delta\alpha^{(5)}(M_Z)$	$(276.2\pm0.5)\times10^{-4}$	0.7	1.0	0.8
$a_\mu^{\rm hvp}$	$(69.4\pm0.4)\times10^{-9}$	0.5	0.8	1.0

TABLE III: Values, errors, and correlations for the running of $\sin^2\hat{\theta}_W$, of the hadronic contribution to $\hat{\alpha}(M_Z)$, and to a_μ when the input is provided by LQCD [35] (upper panel) and by e^+e^- data [17] (lower panel).

Our result

$$\hat{\kappa}(0)_{CMD-3}-1=(325.0\pm1.7)\times10^{-4}$$

$$\hat{\kappa}(0)_{e+e-}-1=(322.5\pm1.7)\times10^{-4}$$

Variation vs Experimental Values and Lattice

- Can replace average data in $\pi^+\pi^-$ channel with CMD-3 only

$$\sin^2\hat{\theta}_W(0) = 0.23868 \pm 0.00004$$

$$\sin^2\hat{\theta}_W(0)_{\text{CMD-3}} = 0.23874 \pm 0.00004$$

☞ Results in a shift up by 6×10^{-5}, compared to error on $\sin^2\theta_W(M_Z)$ of 8×10^{-5}, and MOLLER expected uncertainty of 26×10^{-5}

☞ Similar shift down if we used e.g. only KLOE data

- Conservative proposal (Bayesian priors)

☞ $\sin^2\theta_W(0) = 0.23868 \pm 0.00008$ (EW, gauss) ± 0.00006 (theory, flat) for prediction

☞ Or $\sin^2\theta_W(0) = 0.23868 \pm 0.00009$ (SM theory, 68% CL)

- Comments from theory community welcome

Many Thanks

- To Daniil Radamivich and Daria Nazymko for slogging through many theory papers and performing a tour de force analysis





  - Paper in draft form, waiting on my final edits
- Oleg Zenin (IHEP) for providing access to readable data
- Fred Jegerlehner, Jens Erler, Bill Marciano for much inspiration

Backup: Exclusive Data

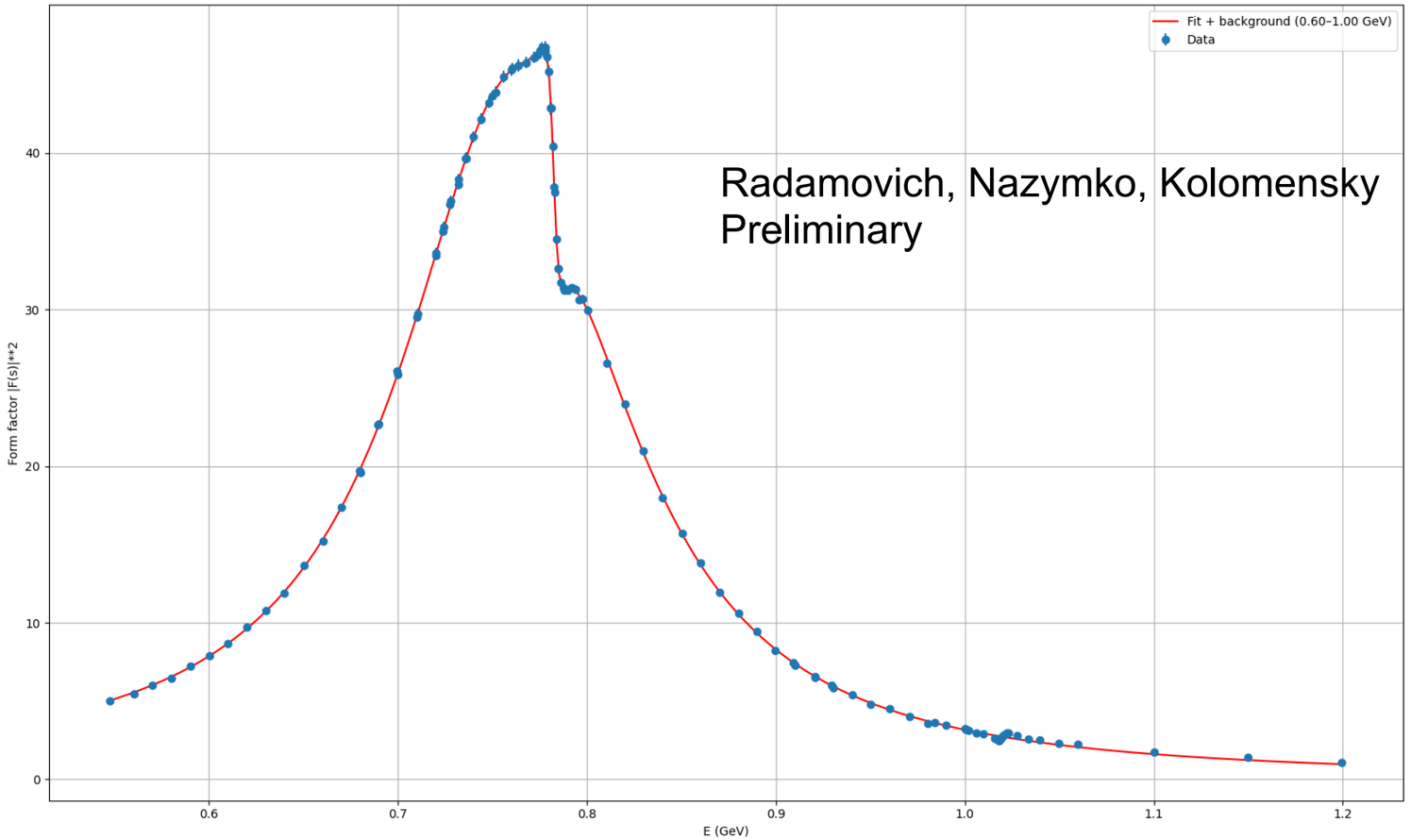


Figure 2: Fit to the $\pi^+\pi^-$ form factor squared, $|F_\pi(s)|^2$, in the ρ–ω interference region. The fit (red line) accurately reproduces the experimental data (blue points), enabling a robust estimate of the ρ contribution for flavor-separation purposes.

Backup: Exclusive Data

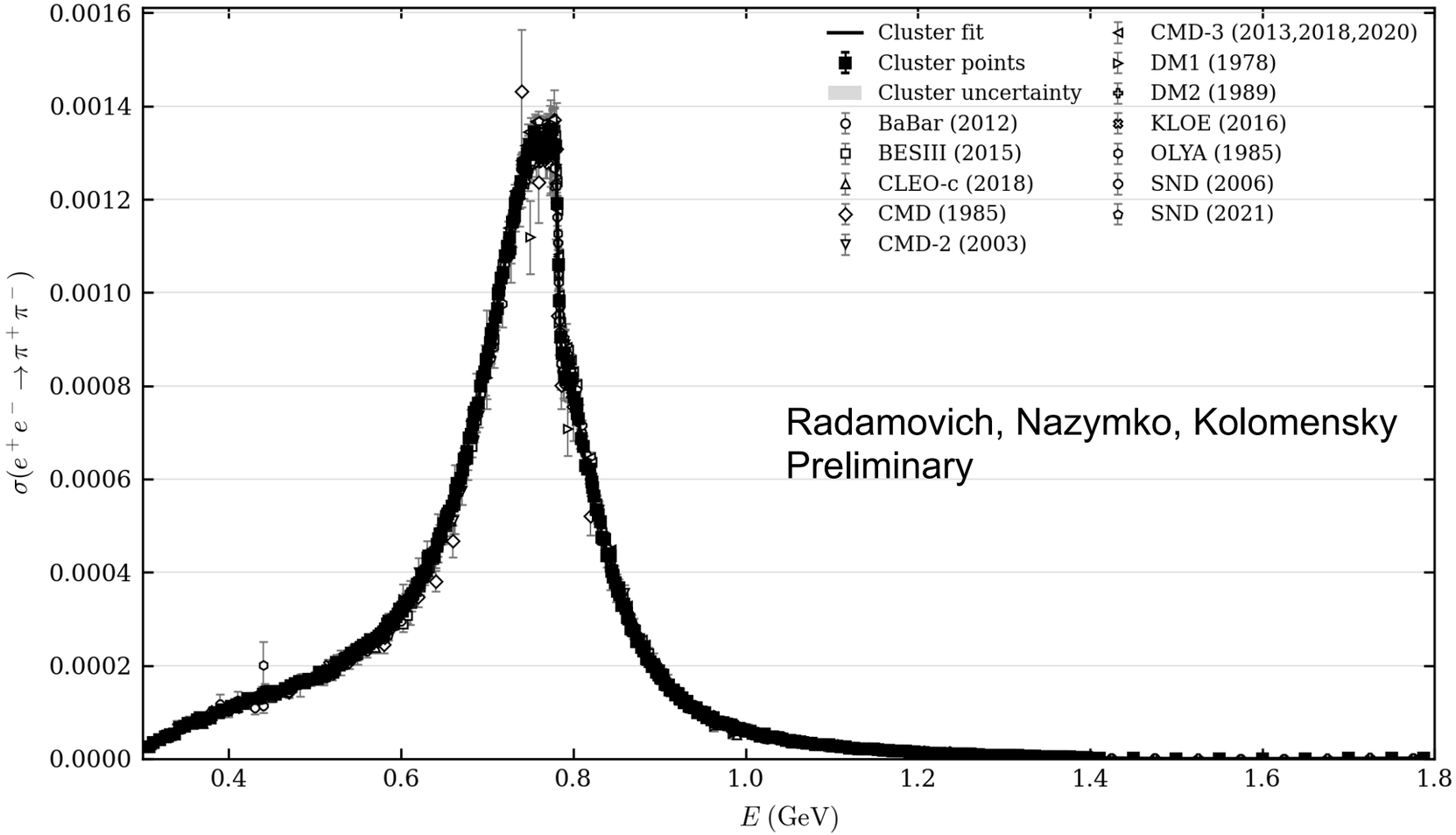


Figure 3: The cm of our data clusterization (red points with error band) for the $\pi^+\pi^-$ cross section, shown against the various sets of experimental data (scattered points) used in the fit.

Backup: Exclusive Data

Channel	$\Pi_{I=1}(0)$			$\Pi_{I=0}(0)$			$\Pi_{s\bar{s}}(0)$		
	Central	**Stat.**	**Syst.**	**Central**	**Stat.**	**Syst.**	**Central**	**Stat.**	**Syst.**
$\pi^+\pi^-$	4.45811	0.00269	0.03289	—	—	—	—	—	—
$2\pi^+2\pi^-$	0.38979	0.00096	0.05120	—	—	—	—	—	—
$\pi^+\pi^-2\pi^0$	0.52282	0.00185	0.03289	—	—	—	—	—	—
$3\pi^+3\pi^-$	0.00490	0.00003	0.00004	—	—	—	—	—	—
$2\pi^+2\pi^-2\pi^0$	0.06391	0.00101	0.00953	—	—	—	—	—	—
$\omega\pi^0$	0.02490	0.00030	0.00057	—	—	—	—	—	—
$\pi^+\pi^-\pi^0$	—	—	—	0.67341	0.00194	0.05600	—	—	—
$2\pi^+2\pi^-\pi^0$	—	—	—	0.03803	0.00087	0.00571	—	—	—
$\pi^+\pi^-3\pi^0$	—	—	—	0.03357	0.00127	0.00229	—	—	—
7π	—	—	—	0.00092	0.00017	0.00046	—	—	—
K^+K^-	—	—	—	—	—	—	0.42666	0.00074	0.00268
K_SK_L	—	—	—	—	—	—	0.23107	0.00077	0.00187
$\bar{K}K\pi^0$	—	—	—	—	—	—	0.06185	0.00197	0.00264
$\bar{K}K2\pi$	—	—	—	—	—	—	0.03021	0.00085	0.00168
$\bar{K}K3\pi$	—	—	—	—	—	—	0.00114	0.00020	0.00038
Channels with η contributions (split by flavor)									
$\eta\,\omega$	—	—	—	0.00778	0.00055	0.00036	0.00232	0.00017	0.00011
$\eta\,2\pi$	0.03511	0.00048	0.00045	—	—	—	0.01049	0.00014	0.00013
$\eta\,\pi^+\pi^-\pi^0$	—	—	—	0.02537	0.00077	0.00027	0.00758	0.00023	0.00008
$\eta\,\omega\,\pi^0$	0.00161	0.00202	0.00102	—	—	—	0.00048	0.00052	0.00030
$\eta\,\phi$	—	—	—	—	—	—	0.01010	0.00072	0.00047
Total	**5.50005**	**0.00357**	**0.06983**	**0.78208**	**0.00213**	**0.05629**	**0.78190**	**0.00712**	**0.00855**

Table V: Channel-by-channel contributions to the isovector, isoscalar, and strange components of $\Pi^{\gamma Z}_{\rm had}(0)$. An en-dash (—) indicates no contribution from a channel to that component.