

Precision Physics with MOLLER Workshop

RESONANCE ESTIMATES FOR SINGLE SPIN ASYMMETRIES IN ELECTRON-NUCLEON SCATTERING

BARBARA PASQUINI

Università degli Studi di Pavia & INFN Pavia

UNIVERSITÀ DI PAVIA

INFN Istituto Nazionale di Fisica Nucleare

Outline

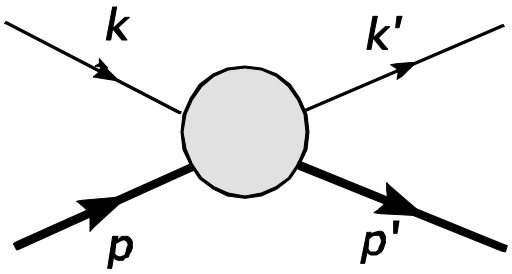
- Formalism to describe elastic electron-nucleon scattering beyond the one-photon exchange approximation

- Normal Spin Asymmetries in elastic electron-nucleon scattering as clean observables for studying two-photon effects

- Estimates of resonance contribution to NSAs in comparison with available measurements

BP, Vanderhaeghen, hep-ph/0405303

Elastic e-N scattering: general amplitude



$$K \equiv \frac{1}{2}(k + k')$$

$$P \equiv \frac{1}{2}(p + p')$$

$$q \equiv k - k' = p' - p$$

Kinematical invariants :

$$Q^2 \equiv -q^2$$

$$\nu \equiv K \cdot P$$

Lorentz + Parity + Time reversal invariance ⟶ 6 structures

$\bar{u}(k')\,u(k) \cdot \bar{u}(p')\,u(p)$	$\bar{u}(k')\,Pu(k) \cdot \bar{u}(p')\,Ku(p)$
$\bar{u}(k')\,u(k) \cdot \bar{u}(p')\,K\,u(p)$	$\bar{u}(k')\,Pu(k) \cdot \bar{u}(p')\,u(p)$
$\bar{u}(k')\,\gamma_5\,u(k) \cdot \bar{u}(p')\,\gamma_5\,u(p)$	$\bar{u}(k')\,\gamma_5\,u(k) \cdot \bar{u}(p')\,\gamma_5\,K\,u(p)$

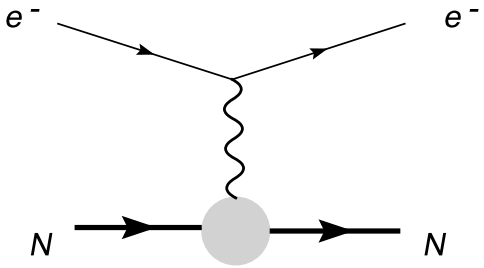
e- helicity flip | e- helicity conserving

$$T = T_{\text{flip}} + T_{\text{non-flip}}$$

$\tilde{F}_4, \tilde{F}_5, \tilde{F}_6$ | $\tilde{G}_M, \tilde{G}_E, \tilde{F}_3$

6 invariant amplitudes functions of Q^2, ν

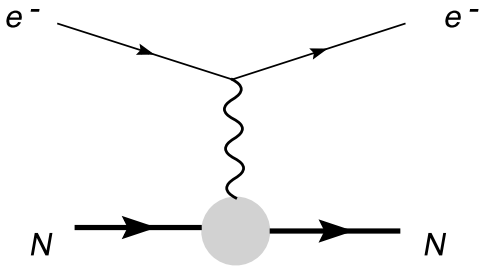
Born approximation



$$\tilde{G}_M^{\text{Born}}(Q^2,\nu) = G_M(Q^2) \qquad \tilde{G}_E^{\text{Born}}(Q^2,\nu) = G_E(Q^2)$$

$$\tilde{F}_3^{\text{Born}} = \tilde{F}_5^{\text{Born}} = \tilde{F}_5^{\text{Born}} = \tilde{F}_6^{\text{Born}} = 0$$

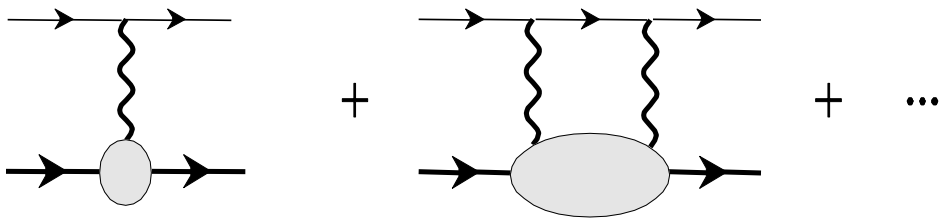
➡ Born approximation



$$\tilde{G}_M^{\text{Born}}(Q^2,\nu) = G_M(Q^2) \qquad \tilde{G}_E^{\text{Born}}(Q^2,\nu) = G_E(Q^2)$$

$$\tilde{F}_3^{\text{Born}} = \tilde{F}_5^{\text{Born}} = \tilde{F}_5^{\text{Born}} = \tilde{F}_6^{\text{Born}} = 0$$

➡ beyond the Born approximation



$$\tilde{G}_M(Q^2,\nu) = G_M(Q^2) + \delta G_M(\nu,Q^2) \qquad \tilde{G}_E(Q^2,\nu) = G_E(Q^2) + \delta G_E(\nu,Q^2)$$

$$\tilde{F}_3, \tilde{F}_4, \tilde{F}_5, \tilde{F}_6 \neq 0$$

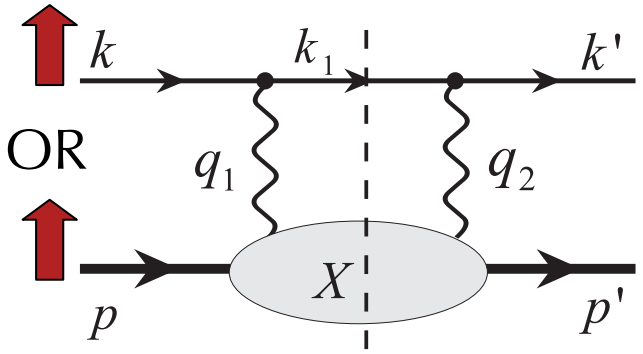
Normal Spin Asymmetries in elastic eN scattering

➡ directly proportional to the Imaginary part of 2-photon exchange amplitudes

$$\mathrm{NSA} = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

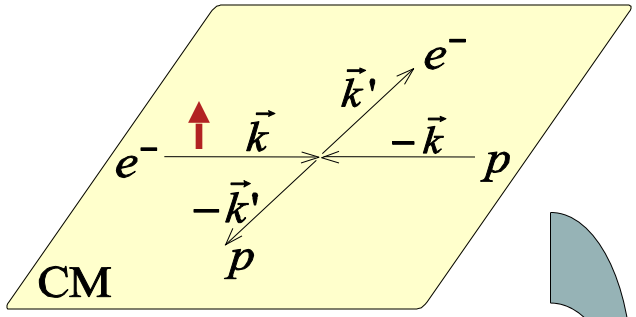
spin of beam OR target

NORMAL to scattering plane



on-shell intermediate state

Normal beam spin asymmetry



$\uparrow \equiv (\hat{k} \times \hat{k}')$

$T_{fi} \equiv T_{\uparrow}(\vec{k}, \vec{k}')$

$|T_{fi}|^2 \sim \sigma_{\uparrow}$

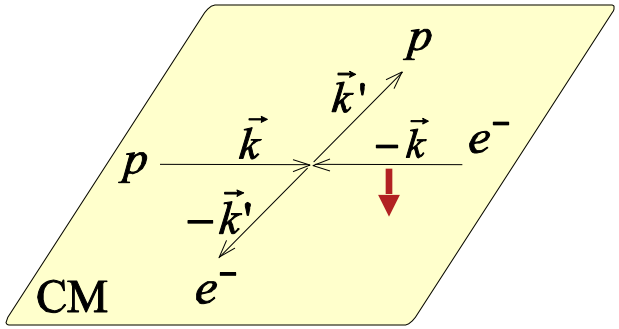
time reversed states

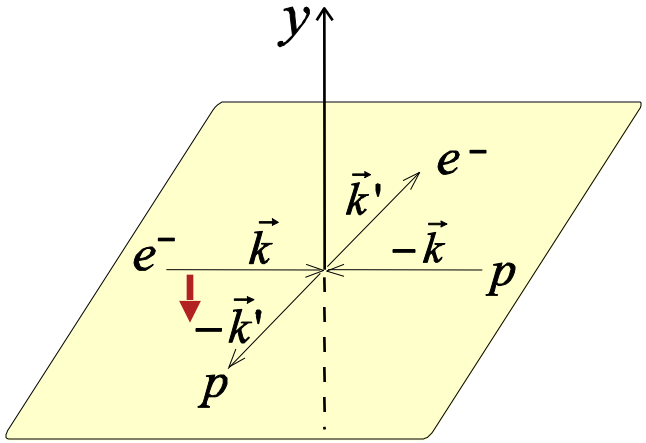
$i \to \tilde{i}$

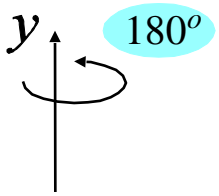
$f \to \tilde{f}$

momenta and spins reversed

$T_{\tilde{f}\tilde{i}} \equiv T_{\downarrow}(-\vec{k}, -\vec{k}')$







rotation of 180^o
around axis $\perp$ to plane

$T_{\downarrow}(-\vec{k}, -\vec{k}') = \eta\, T_{\downarrow}(\vec{k}, \vec{k}')$

phase

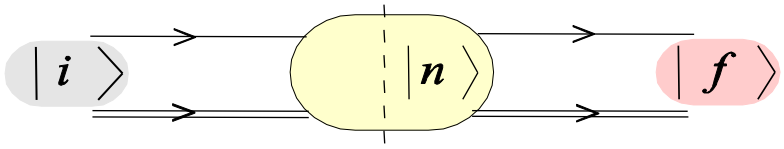
$|T_{\tilde{f}\tilde{i}}|^2 = |T_{\downarrow}(\vec{k}, \vec{k}')|^2 \sim \sigma_{\downarrow}$

[De Rujula, Kaplan, de Rafael, NPB35, 365 (1971)]

$$\mathrm{NSA} = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow} = \frac{|T_{fi}|^2 - |T_{\tilde{f}\tilde{i}}|^2}{|T_{fi}|^2 + |T_{\tilde{f}\tilde{i}}|^2}$$

- Unitarity $\qquad SS^\dagger = S^\dagger S = I \;\text{ with }\; S_{fi} = \delta_{fi} - iT_{fi}$

$$\Longrightarrow \quad i(T_{fi} - T_{fi}^\dagger) \equiv \mathrm{Abs}T_{fi} = \sum_n T_{fn}^\dagger T_{ni}$$



On-shell

- Time-reversal invariance $\quad |T_{fi}|^2 \equiv |T_{\tilde{i}\tilde{f}}|^2$

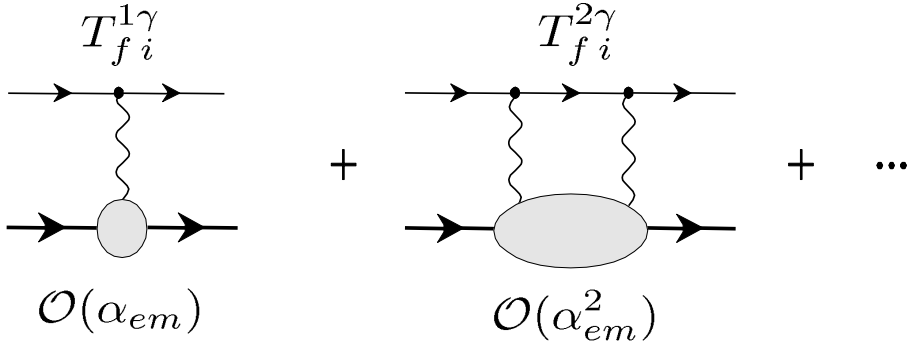
$$\begin{cases} |\mathrm{Abs}\, T_{f\,i}|^2 \;=\; |T_{f\,i}|^2 \;+\; |T_{\tilde{f}\,\tilde{i}}|^2 \;-\; 2\,\mathrm{Re}(T_{f\,i}\,T_{i\,f}) \\ \\ 2\,\mathrm{Im}\,(T_{f\,i}^*\mathrm{Abs}\,T_{f\,i}) \;=\; 2\,\mathrm{Re}\,(|T_{f\,i}|^2 \;-\; T_{f\,i}^* T_{i\,f}^*) \end{cases}$$

$$2\,\mathrm{Im}\,(T_{f\,i}^*\mathrm{Abs}\,T_{f\,i}) \;-\; |\mathrm{Abs}\,T_{f\,i}|^2 \;=\; |T_{f\,i}|^2 \;-\; |T_{\tilde{f}\,\tilde{i}}|^2 \;\sim\; \mathrm{NSA}$$

$$2\,\mathrm{Im}\,(T^*_{f\,i}\mathrm{Abs}\,T_{f\,i}) - |\mathrm{Abs}\,T_{f\,i}|^2 = |T_{f\,i}|^2 - |T_{\tilde{f}\,\tilde{i}}|^2 \sim \mathrm{NSA}$$

- Perturbation theory in α_{em}

$$T_{f\,i} =$$



→ to $\mathcal{O}(\alpha^2_{\mathrm{em}})$: $|T^{1\gamma}_{fi}|^2 - |T^{1\gamma}_{\tilde{i}\tilde{f}}|^2 = 0$

1γ exchange gives no contribution to spin asymmetries

→ to $\mathcal{O}(\alpha^3_{\mathrm{em}})$: $|T_{fi}|^2 - |T_{\tilde{i}\tilde{f}}|^2 = 2\,\mathrm{Im}\,(T^{*\,1\gamma}_{fi}\mathrm{Abs}T^{2\gamma}_{fi})$

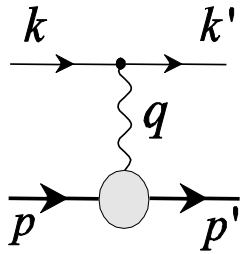
spin asymmetries arise from interference of

1γ exchange and absorptive part of 2γ exchange

➡ to $\mathcal{O}(\alpha_{\rm em})$:

$$\text{NSA} = \frac{\text{Im}(T_{fi}^{*\,1\gamma}\,\text{Abs}T_{fi}^{2\gamma})}{\sum_{\text{spin}} |\,T_{fi}^{1\gamma}\,|^2}$$

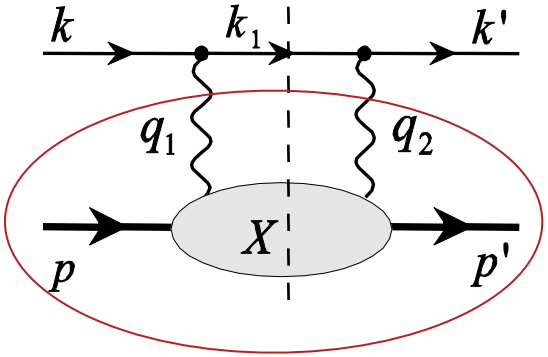
➡ 1γ exchange



$$\sum_{\text{spin}} |\,T_{fi}^{1\gamma}\,|^2 = \frac{e^4}{Q^4} D(s, Q^2)$$

functions of elastic nucleon form factors

➡ 2γ exchange

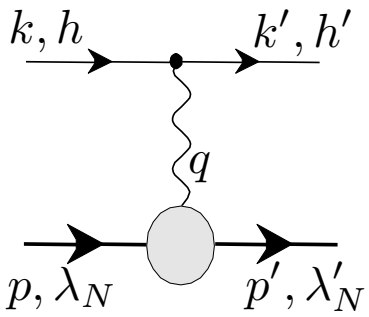


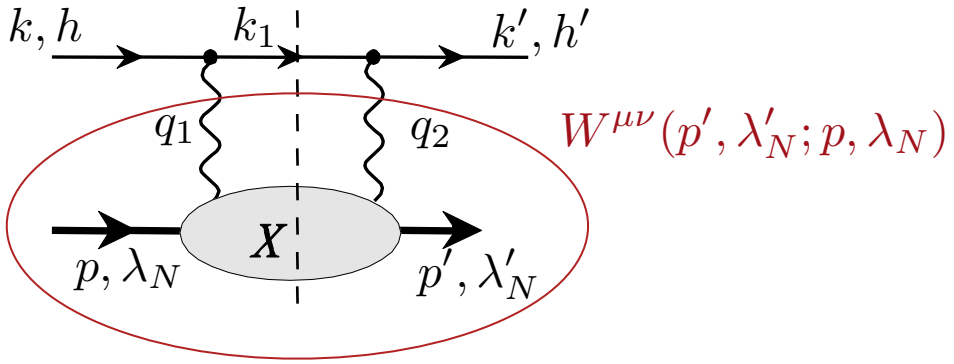
absorptive part of doubly virtual Compton scattering

on-shell intermediate state ($M_x^2 = W^2$)

$$\begin{aligned} \text{Abs}\,T^{2\gamma} &= e^4 \int \frac{d^3\vec{k}_1}{(2\pi)^3 2E_{k_1}} \bar{u}(k', h')\gamma_\mu(\gamma \cdot k_1 + m_e)\gamma_\nu u(k, h) \\ &\times \frac{1}{Q_1^2 Q_2^2} W^{\mu\nu}(p', \lambda_N'; p, \lambda_N) \end{aligned}$$

$$\mathrm{NSA} = \frac{\mathrm{Im}(T_{fi}^{*\,1\gamma}\,\mathrm{Abs}T_{fi}^{2\gamma})}{\sum_{\mathrm{spin}} |\,T_{fi}^{1\gamma}\,|^2}$$





$$\mathrm{NSA} = -\frac{1}{(2\pi)^3}\frac{e^2Q^2}{D(s,Q^2)}\int_{M^2}^{(\sqrt{s}-m_e)^2} dW^2\,\frac{|\vec{k}_1|}{4\sqrt{s}}\int d\Omega_{k_1}\frac{1}{Q_1^2\,Q_2^2}\,\mathrm{Im}\,\{L_{\alpha\mu\nu}\,H^{\alpha\mu\nu}\}$$

leptonic tensor $L_{\alpha\mu\nu}(h, h')$

hadronic tensor: $H^{\alpha\mu\nu}(\lambda_N, \lambda'_N) = W^{\mu\nu} \cdot \left[\bar{u}(p', \lambda'_N)\,\left(G_M\gamma^\alpha - F_2\,P^\alpha/M\right)\,u(p, \lambda_N)\right]^*$

Beam normal spin asymm.

$h \parallel (\hat{k} \times \hat{k}')$

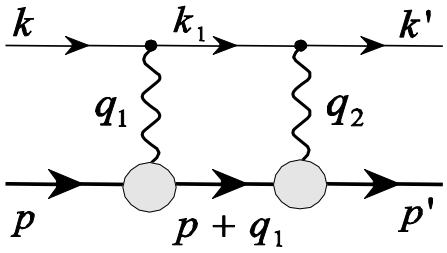
Target normal spin asymm.

$\lambda_N \parallel (\hat{k} \times \hat{k}')$

and sum over spins unpolarized particles

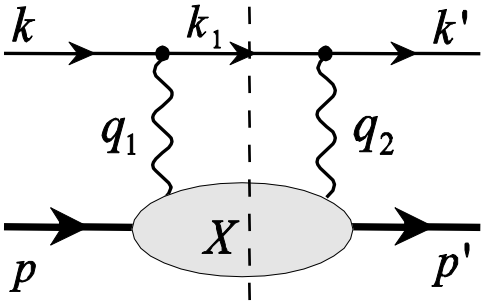
Models for the hadronic tensor

- Elastic contribution



on-shell nucleon intermediate states

- Inelastic contribution



X= π N

resonant and non-resonant π N intermediate states ($W_{\max} = 2$ GeV)
calculated with MAID

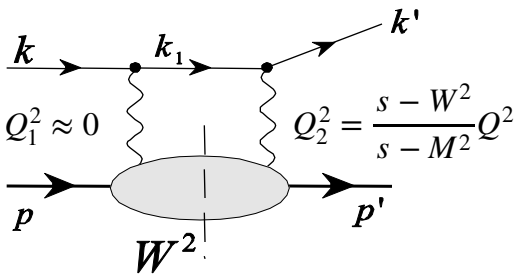
[Drechsel, Hanstein, Kamalov, Tiator, NPA645 (1999); EPJA34 (2007)]

Kinematical limits

$Q_1^2 \approx 0,\ Q_2^2 \neq 0$

$\downarrow$

$\vec{k}_1 \parallel \vec{k}$

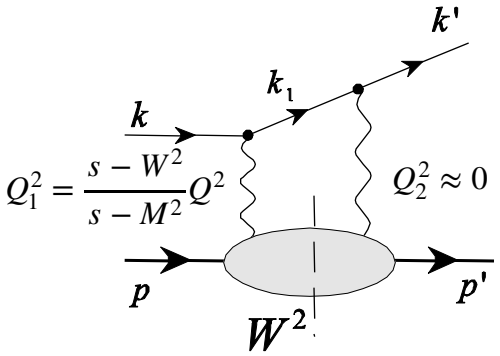


Quasi - VCS

$Q_1^2 \neq 0,\ Q_2^2 \approx 0$

$\downarrow$

$\vec{k}_1 \parallel \vec{k}'$

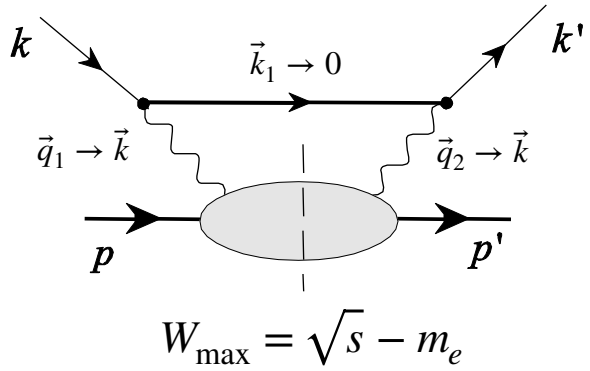


Quasi - VCS

$Q_1^2 \approx 0,\ Q_2^2 \approx 0$

$\downarrow$

$|\vec{k}_1| = 0,\ W_{\max} = \sqrt{s} - m_e$



Quasi - RCS

Order of magnitude for NSA

- Beam Normal Spin Asymmetry

$$L^{\nu\mu\alpha} \sim \mathcal{O}(m_e)$$

$$\text{BNSA} \sim \alpha_{em} \cdot \frac{m_e}{E_e} \sim 10^{-5} - 10^{-6}$$

- Target Normal Spin Asymmetry

$$L^{\nu\mu\alpha} \sim \mathcal{O}(1)$$

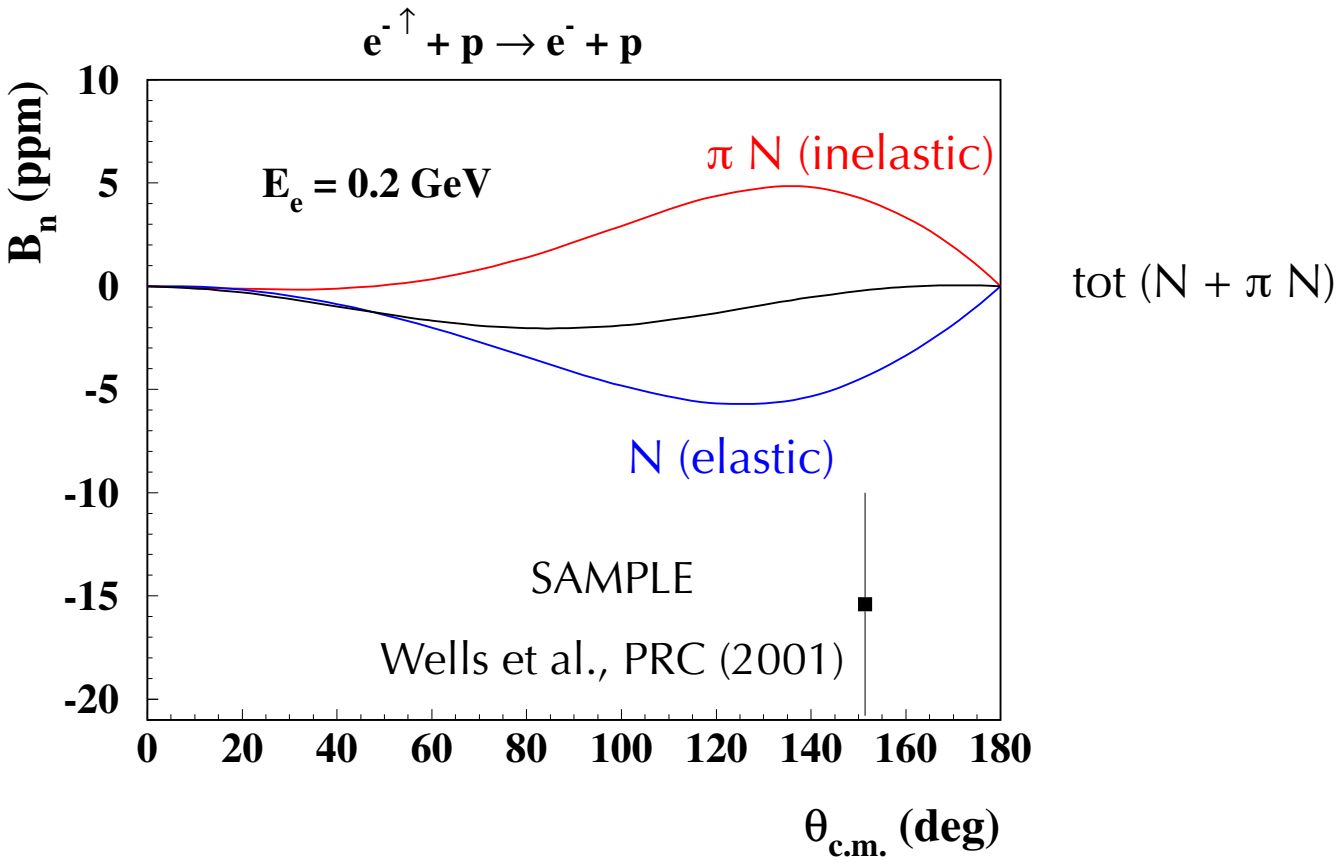
$$\text{TNSA} \sim \alpha_{em} \sim 10^{-2}$$

Measurements of TBSA

Experiment	E_{lab} (GeV)	θ_{lab} (°)	Q^2 (GeV^2)	Experimental B_n (ppm)
SAMPLE (2001)	0.2	146.1	0.1	-15.4 ± 5.4
A4 (2005)	0.855	35.0	0.230	$-8.52 \pm 2.31 \pm 0.87$
	0.569	35.0	0.106	$-8.59 \pm 0.89 \pm 0.75$
G0 (2007)	3.031	7.5	0.15	$-4.06 \pm 0.99 \pm 0.63$
	3.031	9.6	0.25	$-4.28 \pm 1.87 \pm 0.98$
G0 (2011)	0.362	108.0	0.22	-176.5 ± 9.4
	0.687	108.0	0.63	-21 ± 24
HAPPEX (2012)	3.026	6.0	0.099	-6.80 ± 1.54
A4 (2017)	0.315	145.0	0.22	$-94.83 \pm 6.02 \pm 4.07$
	0.420	145.0	0.350	$-99.55 \pm 6.73 \pm 4.63$
A4 (2020)	0.315	34.1	0.032	$-2.22 \pm 0.40 \pm 0.43$
	0.42	34.1	0.057	$-6.88 \pm 0.53 \pm 0.42$
	0.510	34.1	0.082	$-9.32 \pm 0.63 \pm 0.62$
	0.855	34.1	0.218	$-7.46 \pm 1.22 \pm 1.55$
	1.508	34.1	0.613	$-0.06 \pm 2.89 \pm 1.90$
Q_{weak} (2020)	1.149	7.9	0.0248	$-5.194 \pm 0.067 \pm 0.082$

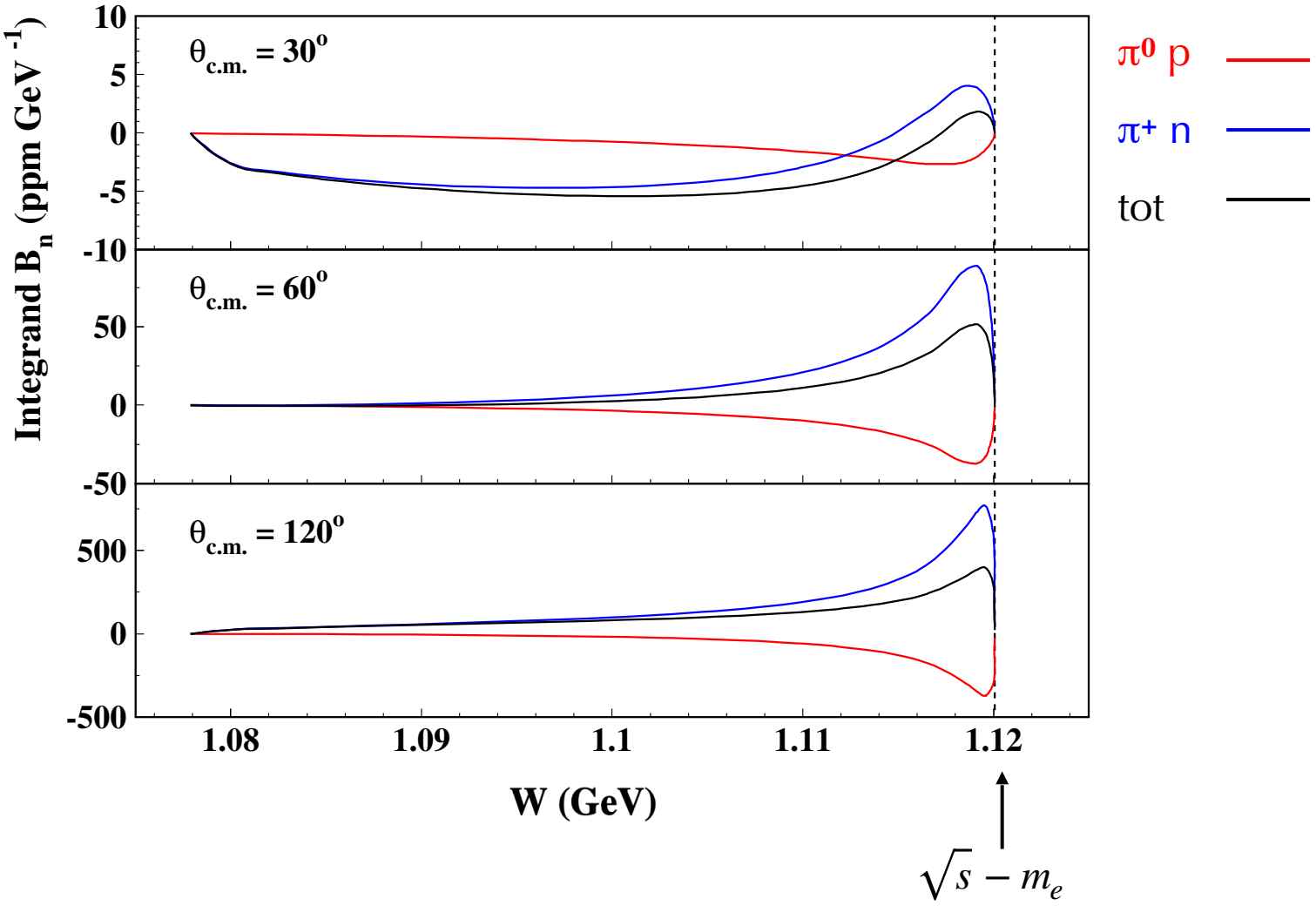
Beam normal spin asymmetry

$E_e = 0.2$ GeV

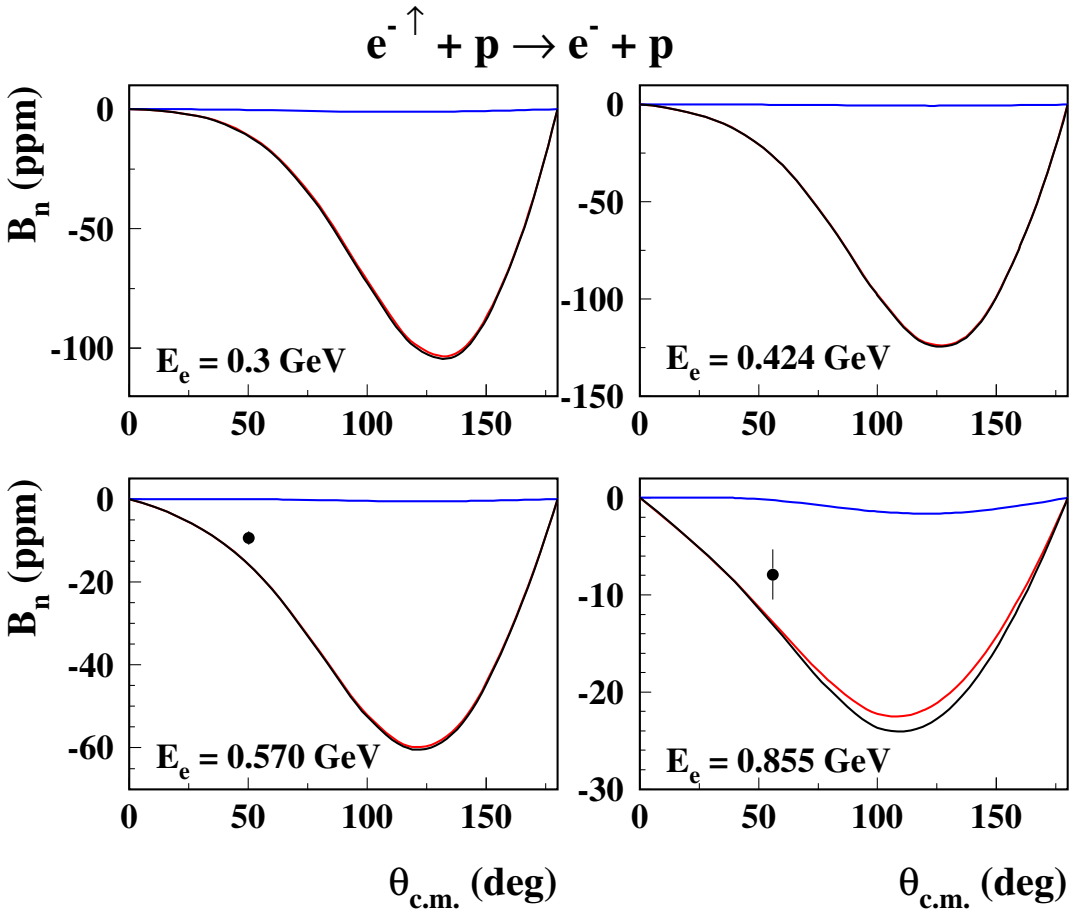


Integrand: beam normal spin asymmetry

$E_e = 0.2$ GeV



Beam normal spin asymmetry



N (elastic)

π N (inelastic)

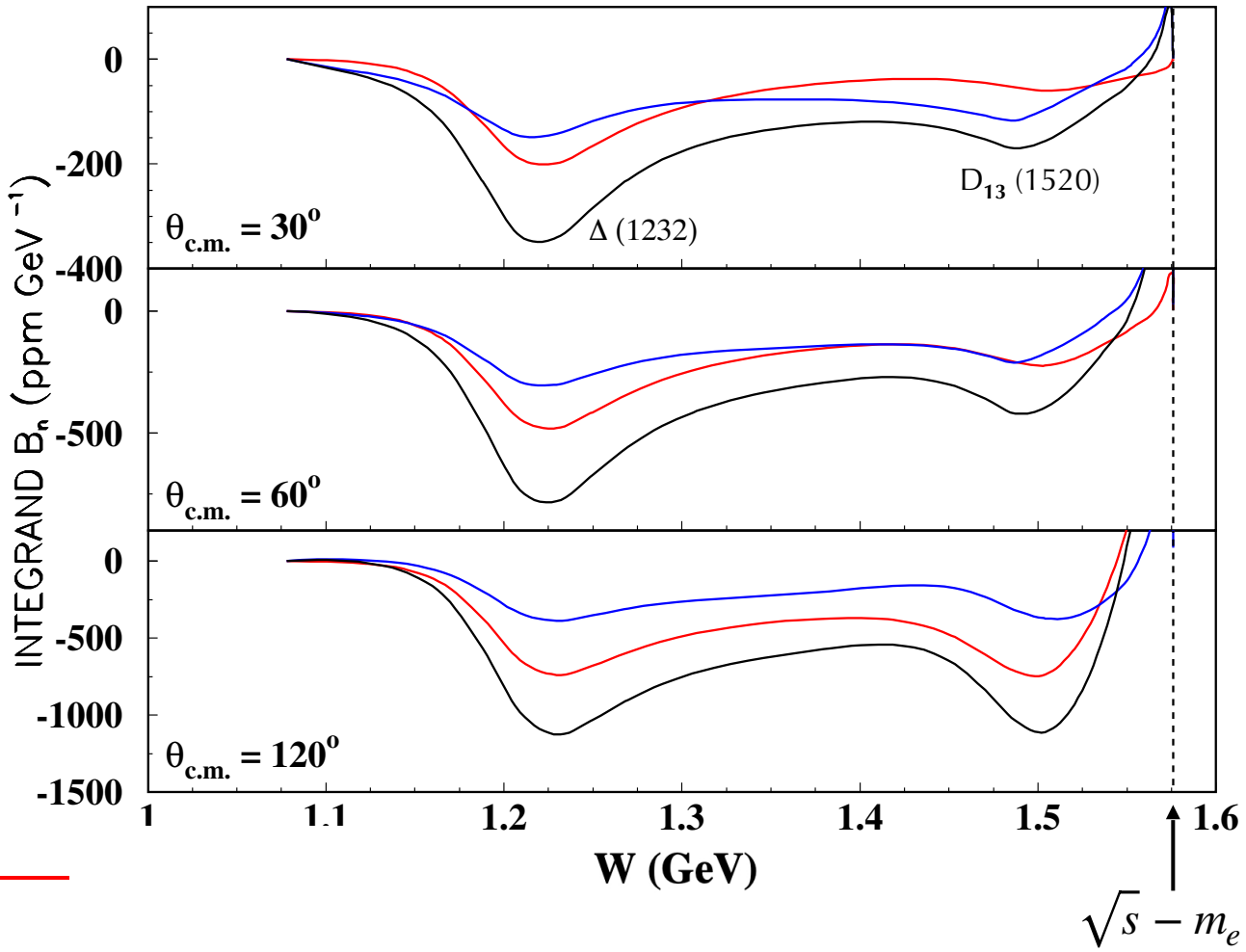
total (N + π N)

MAMI data

A4 , PRL 94, 082001(2005)

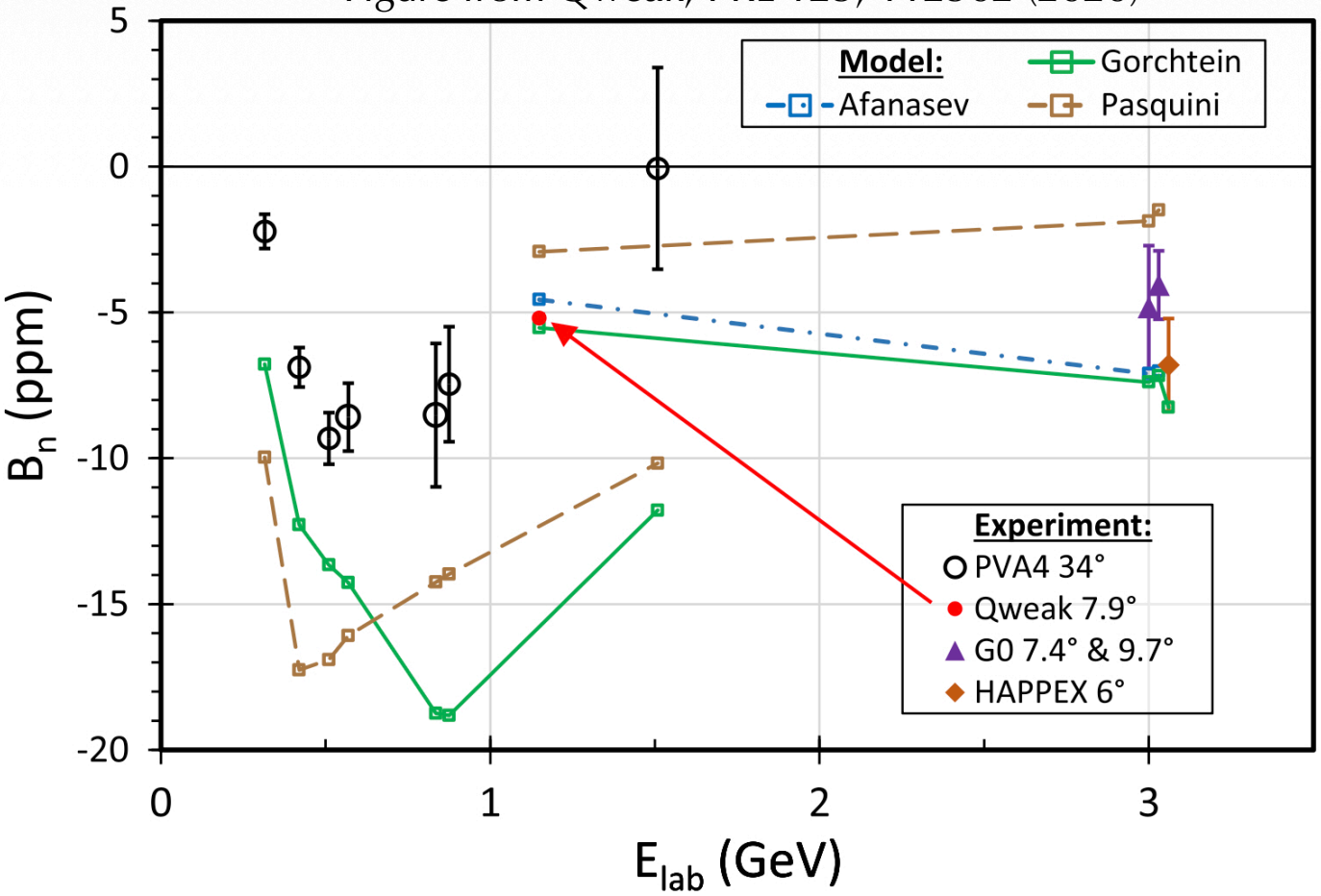
Integrand: beam normal spin asymmetry

$E_e = 0.855$ GeV



Beam normal spin asymmetry at forward angles

Figure from Qweak, PRL 125, 112502 (2020)



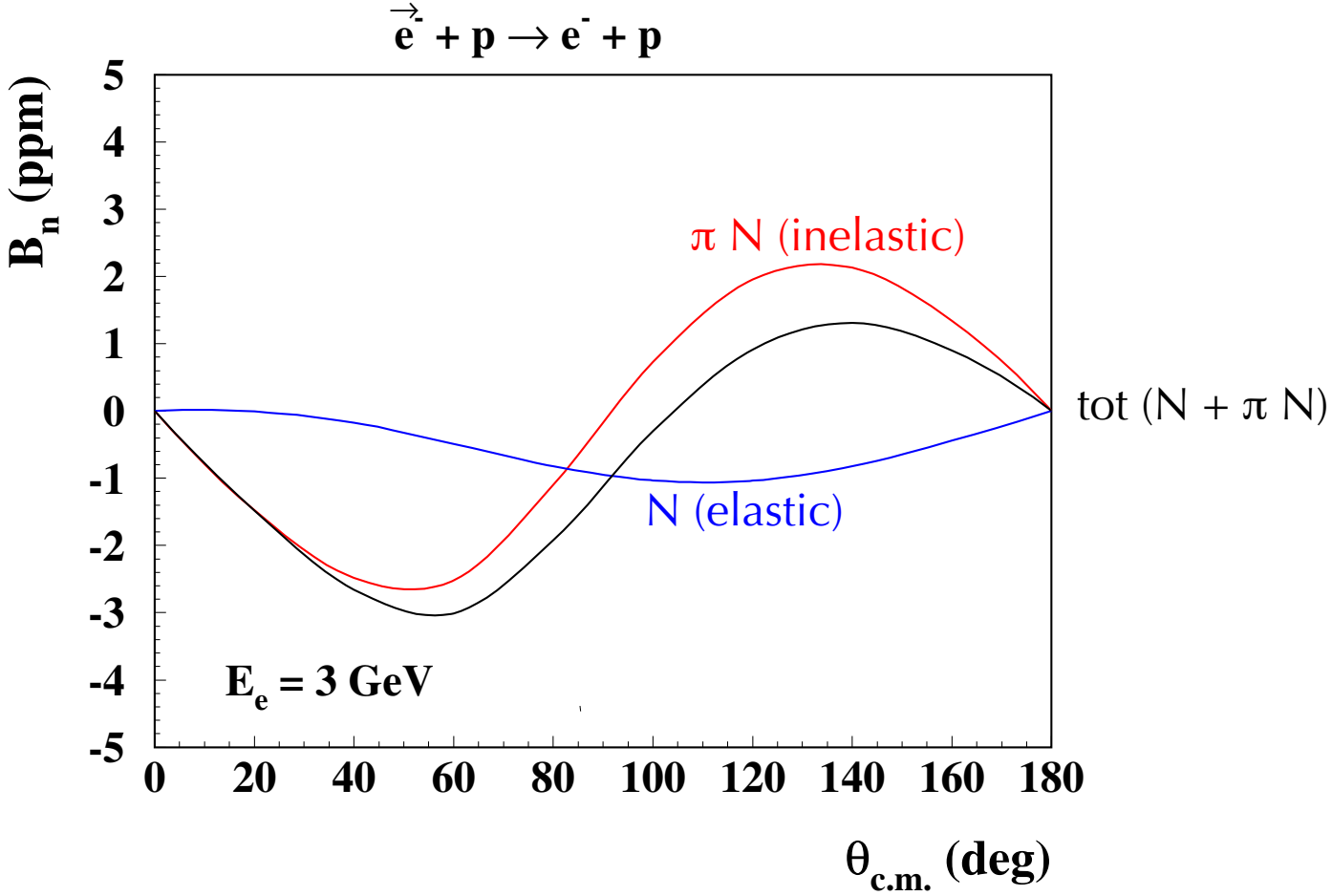
Afanasev, Merenkov, PLB **599**, 48 (2004) ----- M. Gorchtein, PLB **644**, 322 (2007)

Forward-hadronic tensor calculated from total photo-absorption cross section with a phenomenological Ansatz for Q^2dependence

Ahmed, Blunden, Melnitchouk, PRC**108**, 055202 (2023)
similar results: overshoot the SAMPLE and A4, good agreement with data at $E_e > 1$GeV)

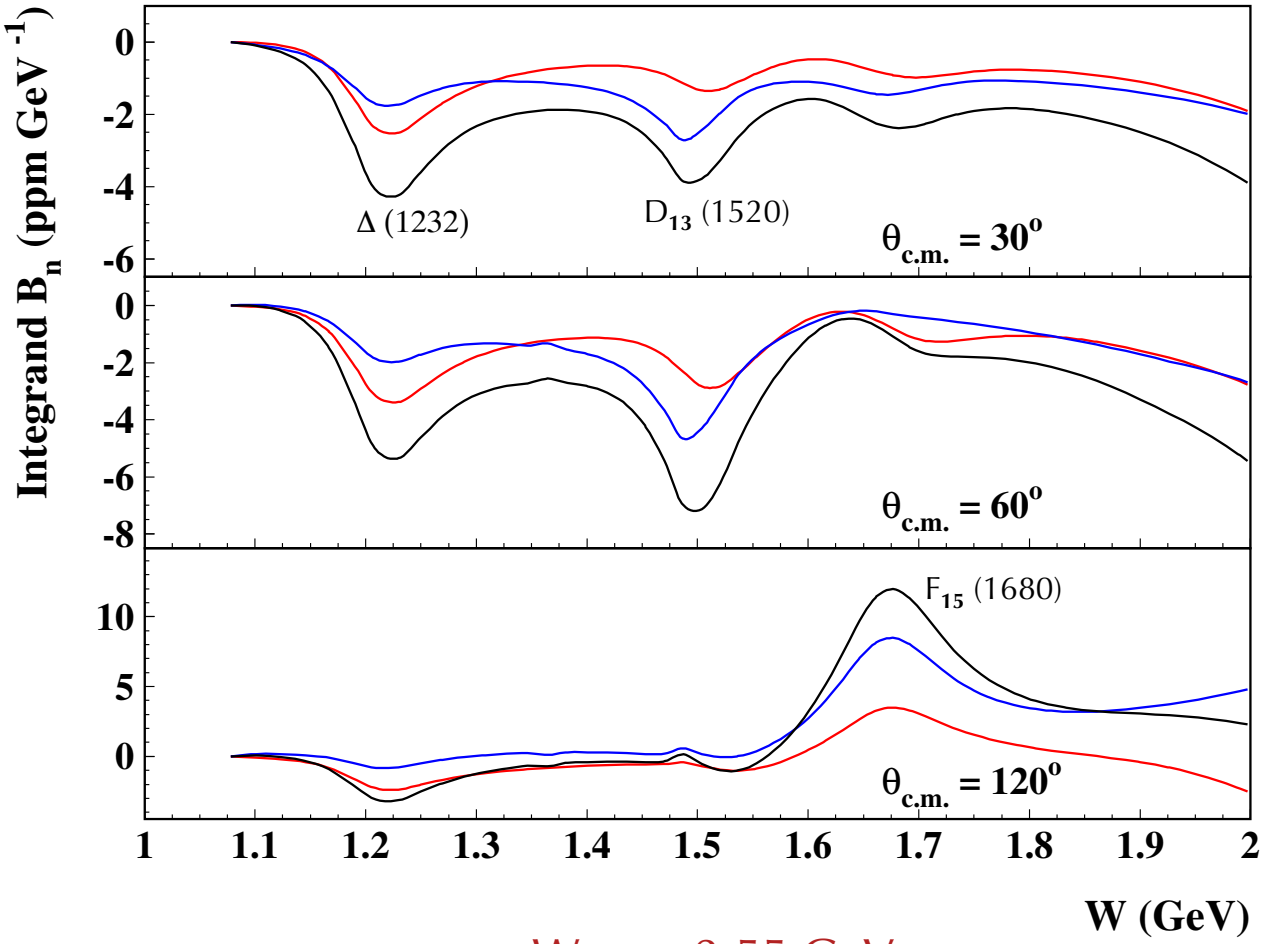
Beam normal spin asymmetry

$E_e = 3$ GeV



Integrand: beam normal spin asymmetry

E_e = 3 GeV



π^0 p

π^+ n

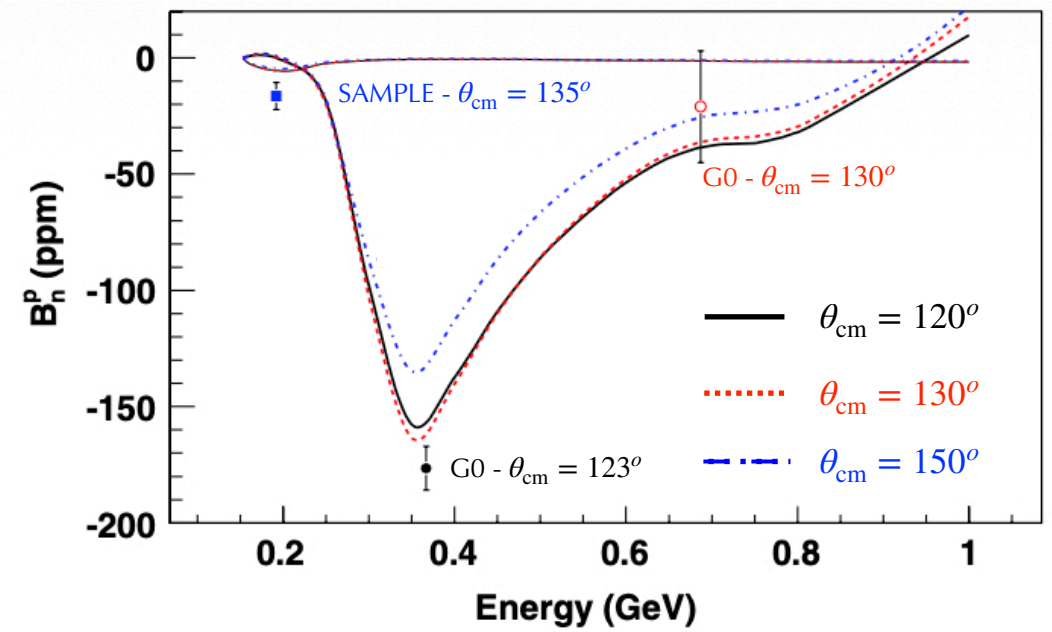
tot

W_{max} = 2.55 GeV,

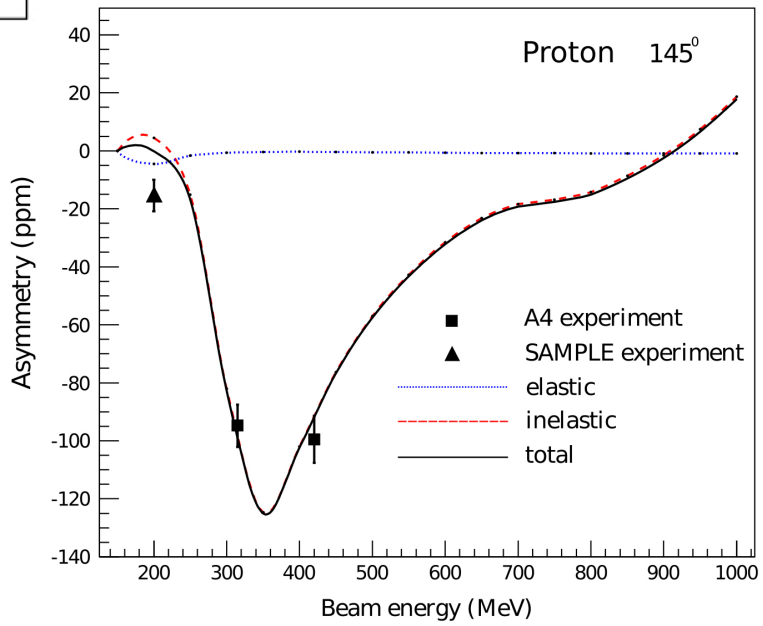
but with MAID we can integrate only up to W = 2 GeV

Beam normal spin asymmetry at backward angles

G0 Coll., PRL 107, 022501 (2011)

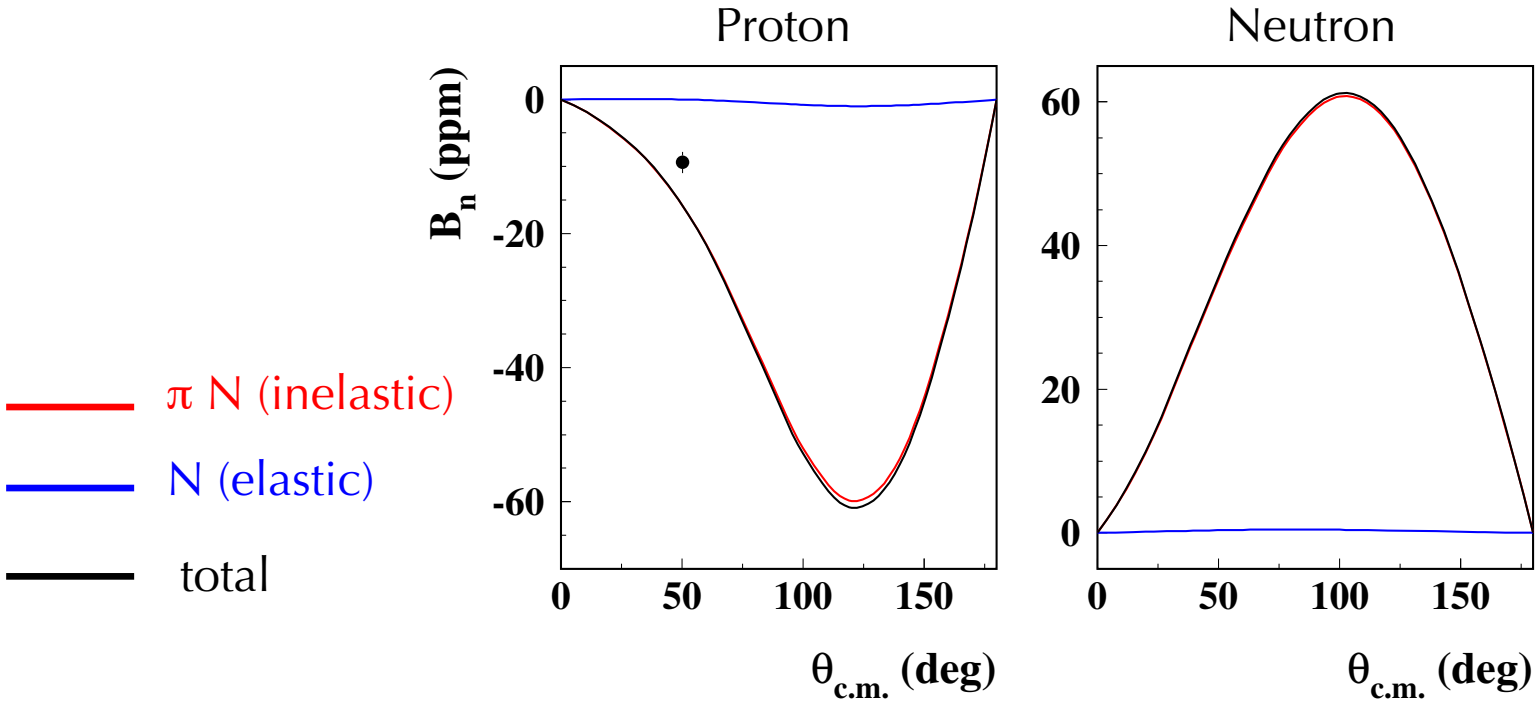


MAMI-A4, PRL 119, 012501 (2017)



Beam normal spin asymmetry

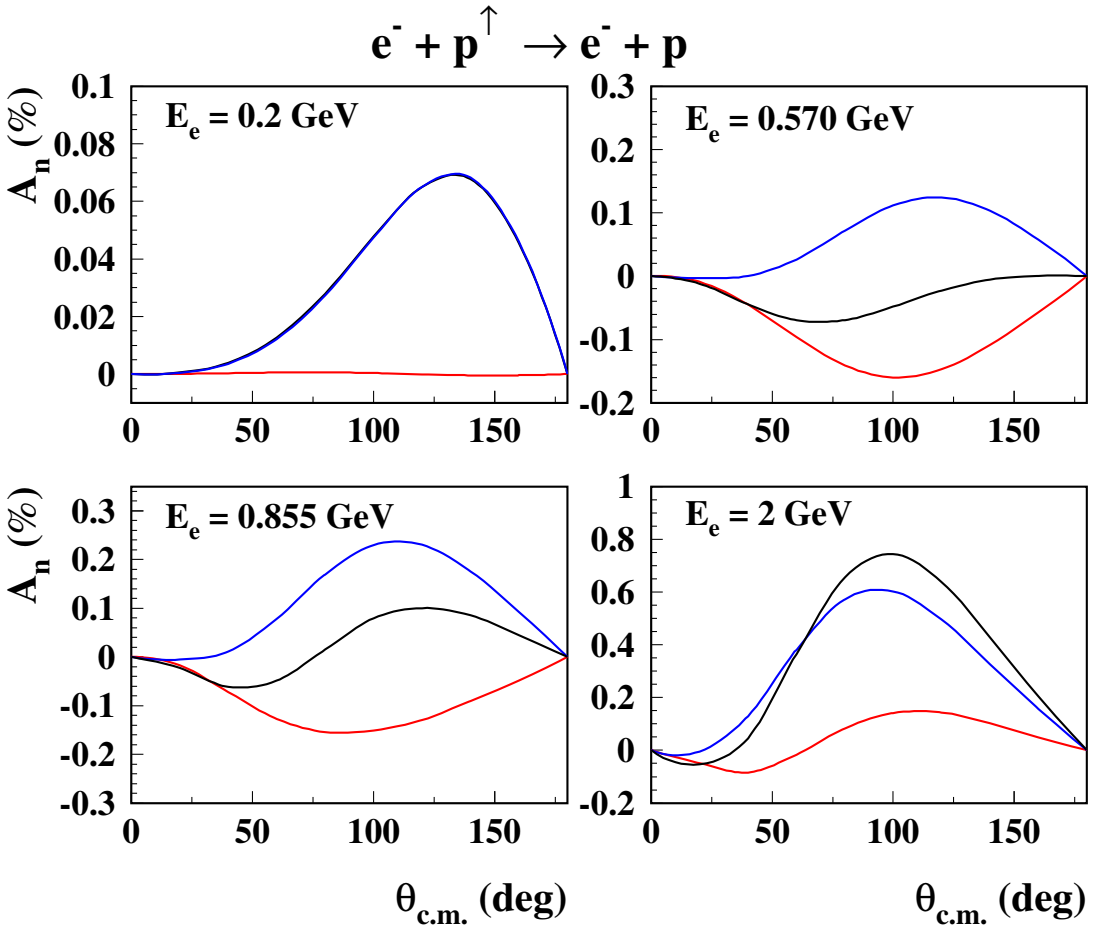
$E_e = 0.570$ GeV



$$B_n = \frac{2\,m_e}{Q}\sqrt{2\,\varepsilon\,(1-\varepsilon)}\sqrt{1+\frac{1}{\tau}}\left(G_M^2 + \frac{\varepsilon}{\tau}\,G_E^2\right)^{-1}$$

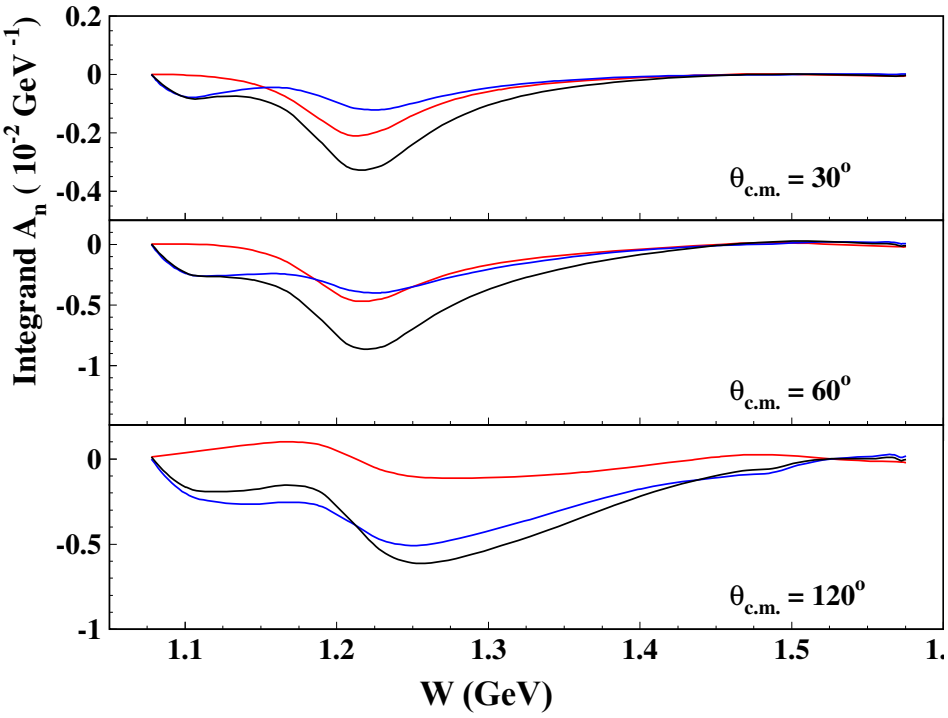
$$\times \quad \left\{-\tau\,G_M\,\mathrm{Im}\left(\tilde{F}_3 + \frac{1}{1+\tau}\,\frac{\nu}{M^2}\,\tilde{F}_5\right) - G_E\,\mathrm{Im}\left(\tilde{F}_4 + \frac{1}{1+\tau}\,\frac{\nu}{M^2}\,\tilde{F}_5\right)\right\}$$

Target normal spin asymmetry

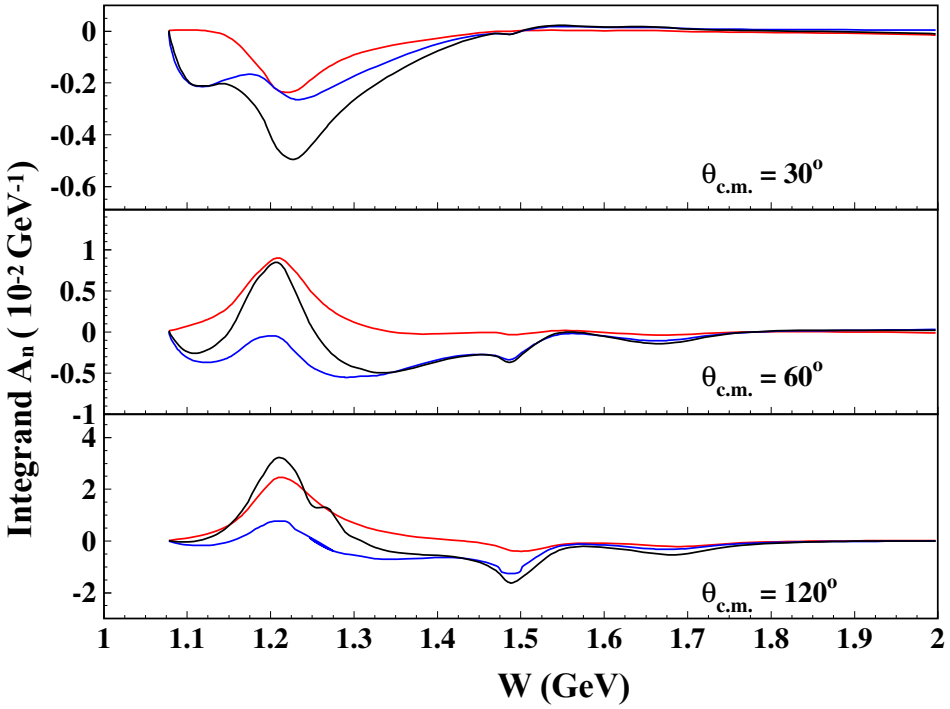


Integrand: target normal spin asymmetry

E_e = 0.855 GeV



E_e = 2 GeV



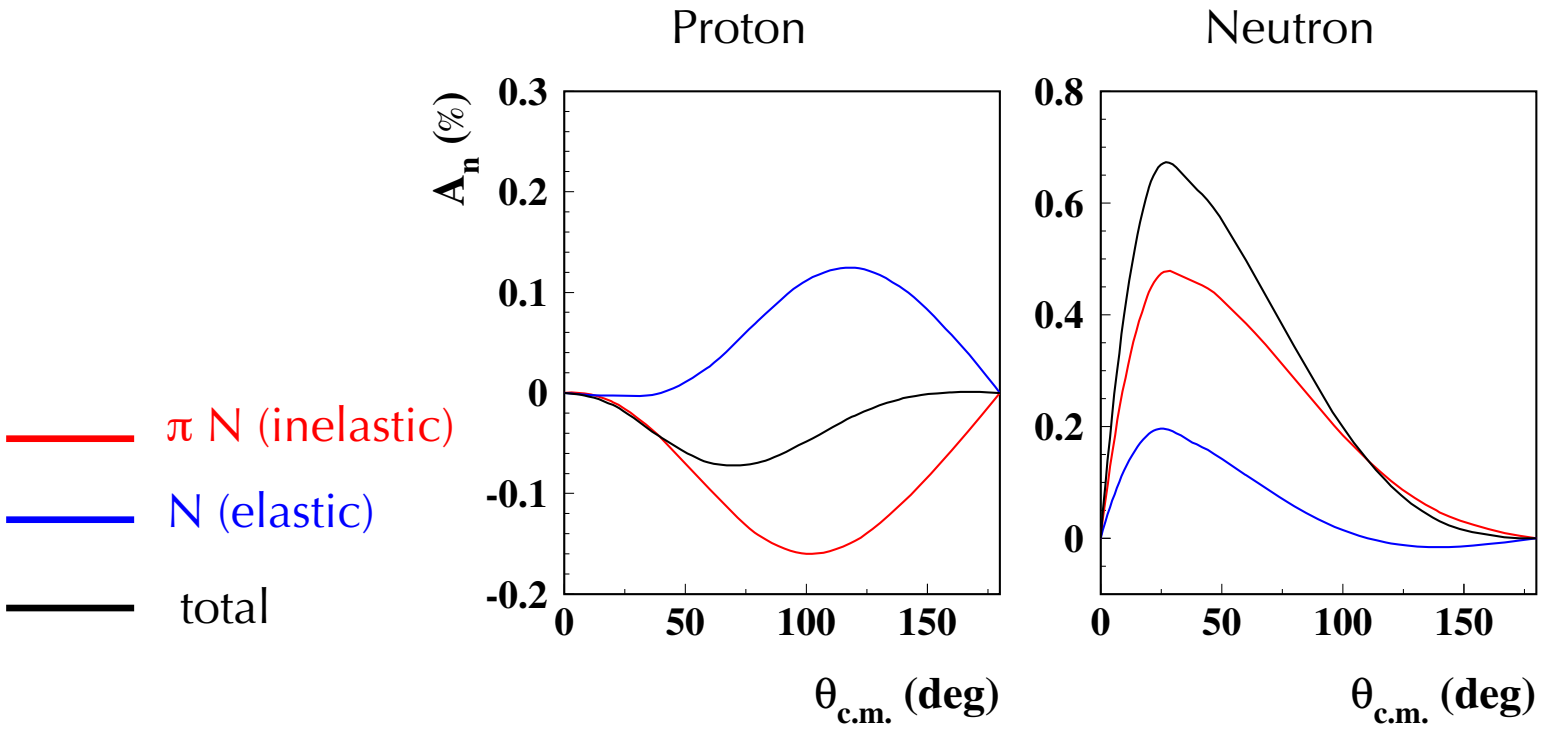
π^0 p

π^+ n

tot

Target normal spin asymmetry

$E_e = 0.570$ GeV



$$A_n = \sqrt{\frac{2\,\varepsilon\,(1+\varepsilon)}{\tau}}\,\left(G_M^2 + \frac{\varepsilon}{\tau}\,G_E^2\right)^{-1}$$

$$\times \quad \left\{ - G_M\,\mathrm{Im}\left(\delta\tilde{G}_E + \frac{\nu}{M^2}\tilde{F}_3\right) + G_E\,\mathrm{Im}\left(\delta\tilde{G}_M + \left(\frac{2\varepsilon}{1+\varepsilon}\right)\frac{\nu}{M^2}\tilde{F}_3\right)\right\}$$

Conclusions

- Beam Normal Spin Asymmetry measured across $0.2 \leq E_e \leq 3$ GeV, providing key insights into both resonant and non resonant contributions

- Calculation of two-photon exchange via full doubly-virtual Compton scattering tensor, using as input phenomenological pion electroproduction amplitudes (MAID, $W_{\max} = 2$ GeV)

- Forward angles ($E_e \leq 1$ GeV): available model calculations systematically overestimate asymmetry compared to data

- Backward angles ($E_e \leq 1$ GeV): good overall agreement between resonance-estimate with MAID and data

- Higher energies: contributions beyond the one-pion channel become essential and MAID calculation misses the quasi-RCS peak region

- Target Normal Spin Asymmetry: Unlike beam normal SSA, the quasi-RCS near-singularity is absent, suppressing inelastic contributions relative to elastic ones

- Proton ($E_e \leq 1$ GeV): cancellation between elastic and inelastic contributions yields small values ($A_n \sim 0.1\,\%$)

- Neutron ($E_e \leq 1$ GeV): no cancellation; larger asymmetry expected ($A_n \sim 1\,\%$)