Beam Normal SSA in the Resonance Region

(meaning here the Delta(1232) region)

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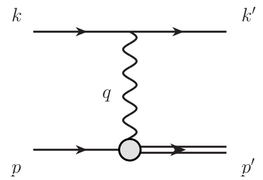
BNSSA for Resonances

- BNSSA = Beam Normal Single Spin Asymmetry
- Beam = incoming electron
- Normal= incoming electron polarized perpendicular to scattering plane
- Single Spin = other spins not measured, or summed over
- Not forbidden by parity or time reversal invariance
- (Constrast: BLSSA forbidden by parity)

$$B_n = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

Arrow is spin up or down for incoming electron

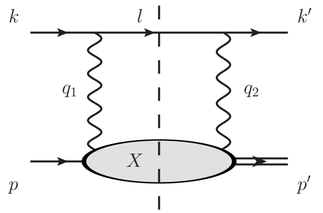
BNSSA



- BUT, calculation in lowest order, or one photon exchange, gives BNSSA = 0.

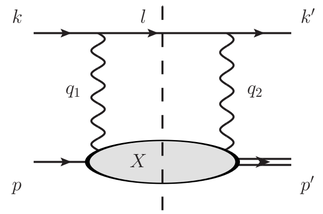
- To get a non-zero BNSSA need a second interaction, involving the observed particle.

- E.g., if only electron observed in final state, need another photon exchange.



- For resonances, like the Delta(1232), decays occur and allow observing pion (or nucleon) which can have strong interaction with another final state particle and allow a non-zero BNSSA (will come back to this late).

BNSSA w/ only electron observed



- Two photon exchange
- Using time reversal invariance, rotation invariance, and unitarity prove

$$B_n \simeq \frac{2\,\mathrm{Im}\left[\left(T_{1\gamma}\right)^*_{fi}\left(\mathrm{Abs}\,T_{2\gamma}\right)_{fi}\right] - \left|\left(\mathrm{Abs}\,T_{2\gamma}\right)_{fi}\right|^2}{\sum\limits_{\mathrm{spins}} |T_{1\gamma}|^2}$$

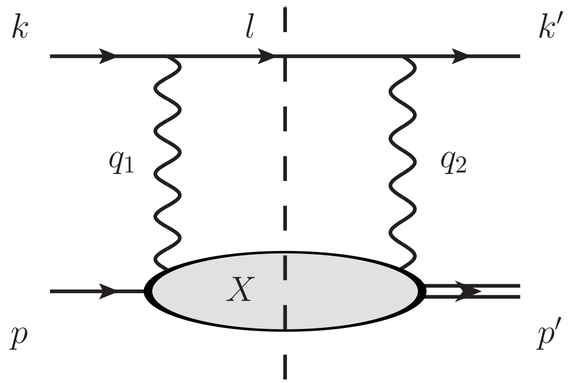
- Usually drop second term in numerator, because it is higher order in α
- Left with interference term between 1γ and 2γ exchange

Definition and comments

- $T_{fi} = \langle f \,|\, T \,|\, i \rangle$ = matrix element of T-matrix, a.k.a., scattering amplitude

- Absorptive part,

$$\left(\text{Abs}\, T_{2\gamma} \right)_{fi} \equiv \sum_n T^*_{nf} T_{ni}$$



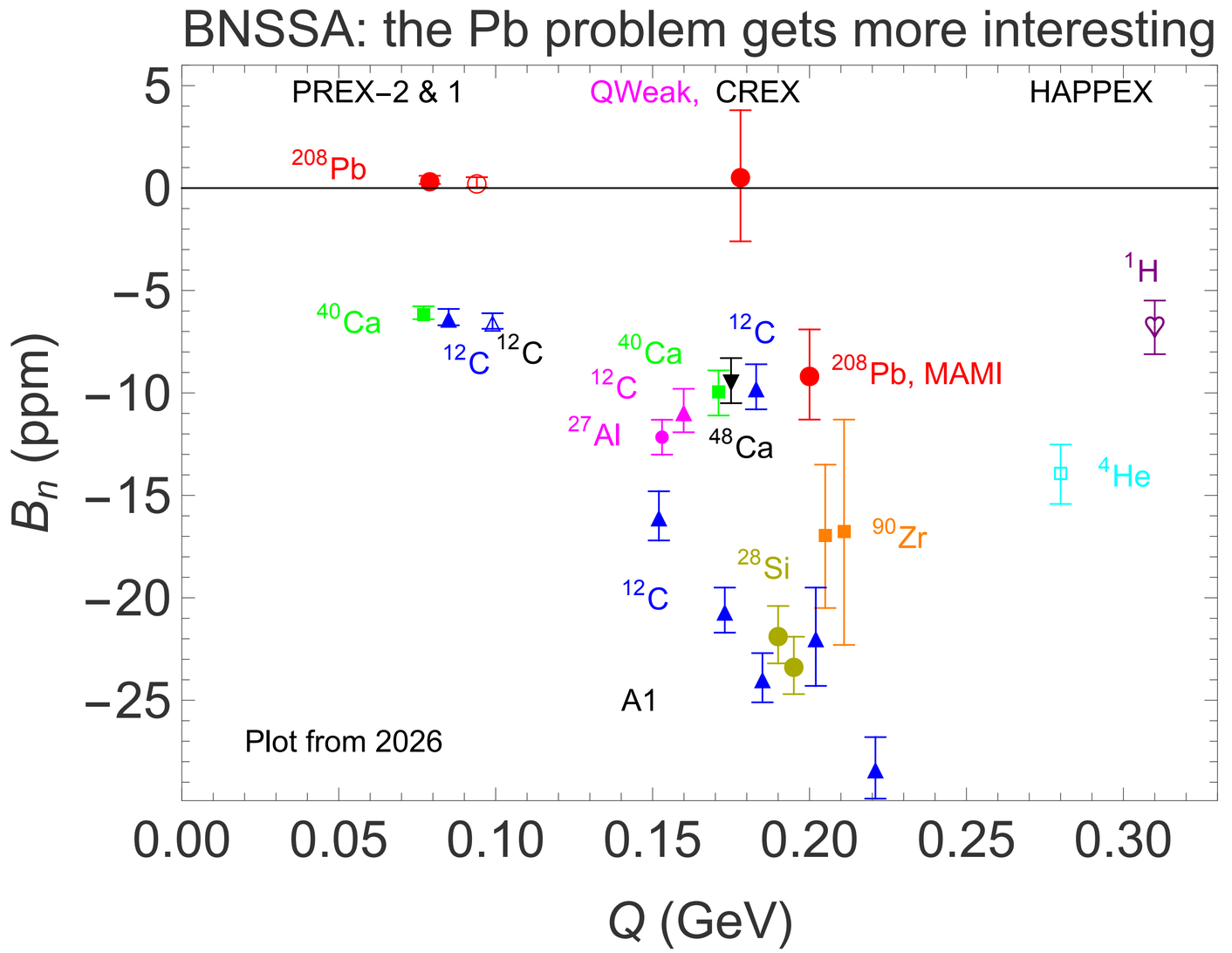
- In Abs(…) the intermediate states are real states, i.e., the particles in the Feynman diagram for it are on shell.

- So that if "*X*" is a nucleon or resonance, the form factors and such at the vertices are the same as in 1γ exchange scattering, and are measured quantities, at least in some cases. ∴ easier than general loop diagram.

More commentary

- That $|\text{Abs}|^2$ is small compared to $T \circ$ Abs pretty clear if expansion parameter is α as it is for single nucleon target. Not so clear if the target is a heavy nucleus, where expansion is in $Z\alpha$. ($Z = 82$ for Pb.)
- $\exists$ data on BNSSA for various nuclei. See plot. Note especially Pb.
- Will not pursue here. See alternative from Tereshchuk and Afanasev.

BNSSA for nuclei



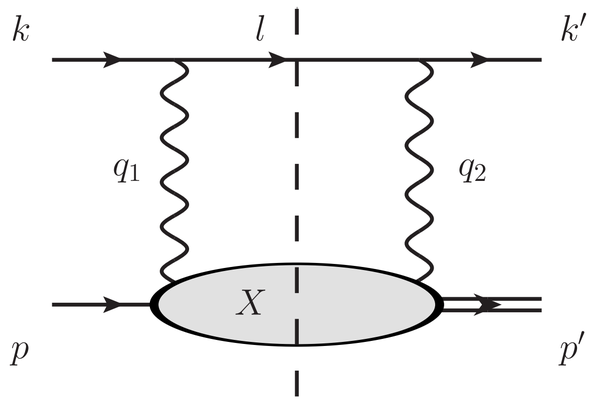
Interest in BNSSA and 2γ process

- 2γ processes became interesting in years 2000ff when polarization transfer measurements of proton form factor G_E that differed from result of Rosenbluth analysis of unpolarized scattering

- Culprit became Rosenbluth and radiative corrections, specifically 2γ terms. In unpolarized *e*-*p* scattering the G_M terms dominated G_E at high Q^2, so small corrections to G_M strongly affected the extracted G_E

(Continued)

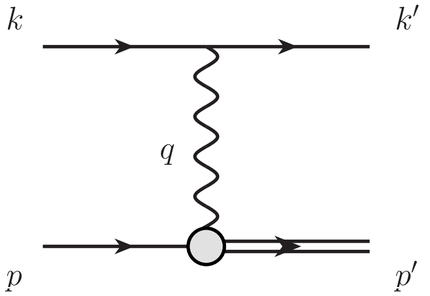
- More specific to resonance production, there is little data on diagonal resonance metrics, e.g., for $\Delta \to \Delta$, what are the magnetic moments, charge radii, EM form factors, EM polarizabilities? To measure them, one first has to make the resonance, and then let it interact with a photon. Hard! But 2γ processes do it naturally.



- In general, 1γ exchange is bigger. But BNSSA only happens when there is 2γ exchange, and so isolates it and allows special resonance studies.

Hiatus: $1\gamma \quad p \to \Delta^+$ production

- Useful for denominator of $B_n \simeq 2\,\mathrm{Im}\left[\left(T_{1\gamma}\right)^*_{fi}\left(\mathrm{Abs}\,T_{2\gamma}\right)_{fi}\right] \Big/ \sum_{\mathrm{spins}} |\,T_{1\gamma}|^2$.



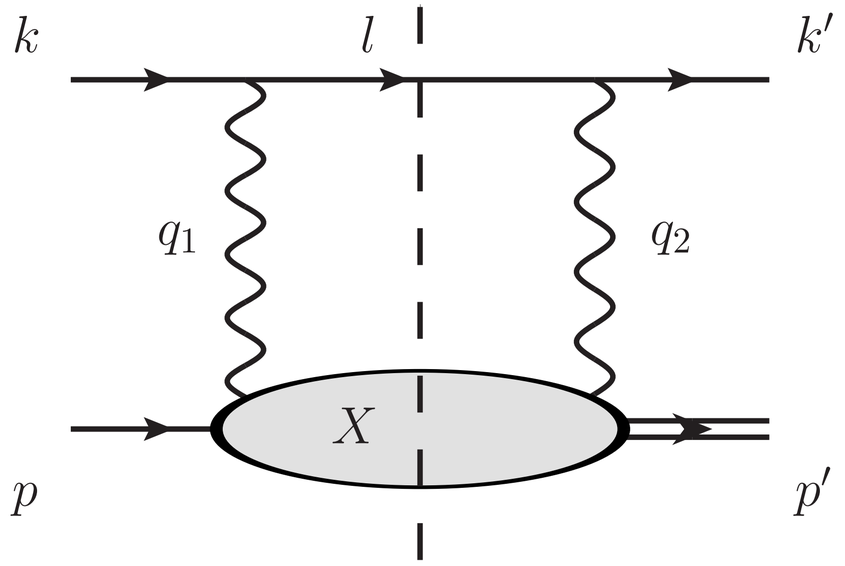
- Results to be given using Jones-Scadron ampltudes $G_E^*, G_M^*,$ and G_C^* .

- $\sum_{\mathrm{spins}} |\,T_{1\gamma}|^2 \equiv \frac{e^4}{Q^2} D_{1\gamma}(s, Q^2)$ and

- $D_{1\gamma}(s, Q^2) = \frac{2\left[(M_\Delta - M_N)^2 + Q^2\right](M_\Delta + M_N)^2}{(1-\varepsilon)\,M_N^2} \times \left[G_M^{*2} + 3G_E^{*2} + \varepsilon \frac{Q^2}{M_\Delta^2} G_C^{*2}\right]$

ϵ = usual

2γ exchange

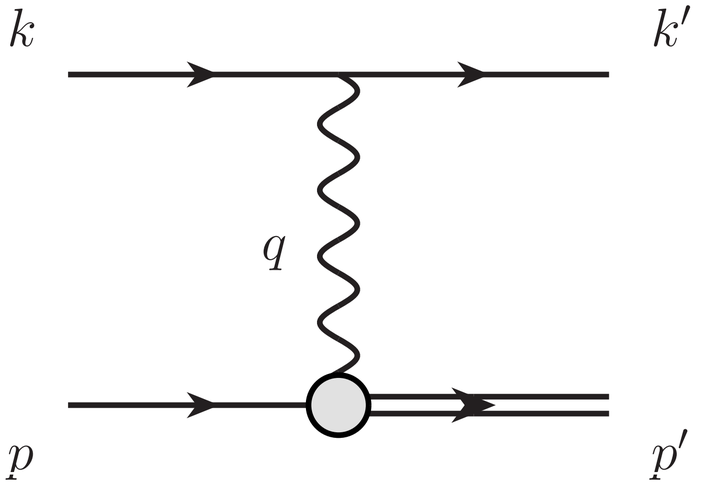


- Given diagram and meaning,

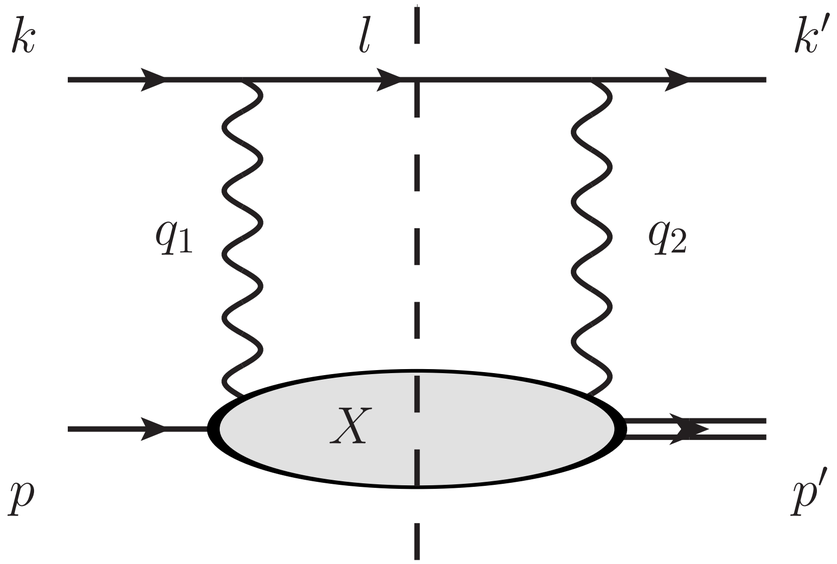
$$\text{Abs}\, T_{2\gamma} = \int \frac{d^3\vec{l}}{(2\pi)^3 2E_l} \bar{u}(k', s'_e)\gamma_\mu(\gamma \cdot l + m_e)\gamma_\nu\, u(k, s_e) \times \frac{e^4}{Q_1^2 Q_2^2} \cdot W^{\mu\nu}(p', \lambda'; p, \lambda)$$

- and

$$W^{\mu\nu}(p', \lambda'; p, \lambda) = \sum_X (2\pi)^4\, \delta^4(p + q_1 - p_X) \times \langle \Delta(p', \lambda') \,|\, J^{\dagger\mu}(0) \,|\, X \rangle \langle\, X \,|\, J^\nu(0) \,|\, N(p, \lambda) \rangle$$



B_n



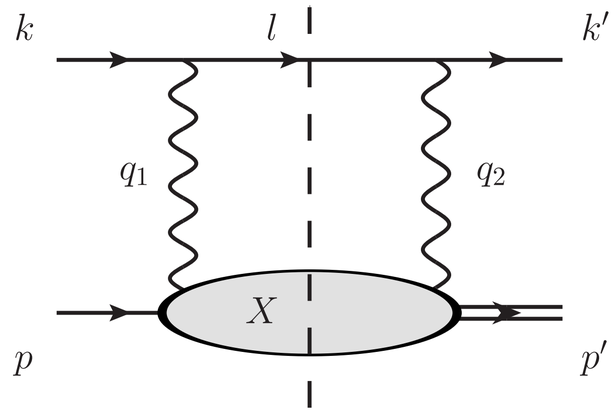
- Combine terms on previous side with 1γ terms, and change d^3l for other variables, and get

$$B_n = -\frac{e^2}{D_{1\gamma}(s,Q^2)}\frac{1}{(2\pi)^3}\int dW^2\left(\frac{s-W^2}{8\,s}\right)\times\int d\Omega_1\frac{1}{Q_1^2\,Q_2^2}\,\mathrm{Im}\left(L_{\kappa\mu\nu}\,H^{\kappa\mu\nu}\right)$$

Various terms

- $L_{\kappa\mu\nu} = \mathrm{Tr}\left\{\gamma_\kappa(\gamma\cdot k' + m_e)\gamma_\mu(\gamma\cdot l + m_e)\gamma_\nu\gamma_5\,\gamma\cdot\xi\,(\gamma\cdot k + m_e)\right\}$
- ξ is vector representing polarization up. (Unit vector in *y*-direction.)
- Point to note: $L_{\kappa\mu\nu} \propto m_e$. B_n will be small!
- $H^{\kappa\mu\nu} = \left[N \to \Delta \text{ stuff}\right]^\kappa \times W^{\mu\nu}(p', \lambda'; p, \lambda)$

Narrow intermediate states X



- I.e., X are just some resonances
- $H^{\kappa\mu\nu} \equiv (2\pi)\delta(W^2 - M_X^2)\tilde{H}^{\kappa\mu\nu}$
- $$B_n = -\frac{1}{(2\pi)^2}\frac{e^2}{D_{1\gamma}(s,Q^2)}\left(\frac{s-M_X^2}{8\,s}\right)\theta(s-M_X^2)\times\int \mathrm{d}Q_1^2\mathrm{d}Q_2^2\frac{J^{-1}(Q_1,Q_2)}{Q_1^2\,Q_2^2}\mathrm{Im}\left(L_{\kappa\mu\nu}\,\tilde{H}^{\kappa\mu\nu}\right)$$
- J is just a Jacobian that lets us write $d\Omega_1$ as $dQ_1^2\,dQ_2^2$.
- The $W^{\mu\nu}$ in $H^{\kappa\mu\nu}$ is now just a product of terms for $N \to X$ and $X \to \Delta$ at the lower two vertices of the Feynman diagram.

Break

- From a 1937 article in the Reviews of Modern Physics by E. Condon: “Readers who are not interested in the details of quantum-mechanical calculations will find great pleasure in skipping the rest of this section”.
- Would you have liked to see this quote earlier?
- Could we get away with such a statement in a modern paper?

Calculate

- "product of terms for $N \to X$ and $X \to \Delta$": we are now prepared to get numerical results.
- First case: Intermediate baryon is nucleon.
- Need $N \to N$ and $N \to \Delta$. Both are (fairly) well known.
- Second case: Intermediate baryon is Delta.
- Need $N \to \Delta$ and $\Delta \to \Delta$. First is known. Second is not.

What to do for $\Delta \to \Delta$?

- Basic answer: use an existing lattice calculation.
- EM current matrix element for $\Delta \to \Delta$ has four form factors, with names and values at $Q^2 = 0$ denoted by

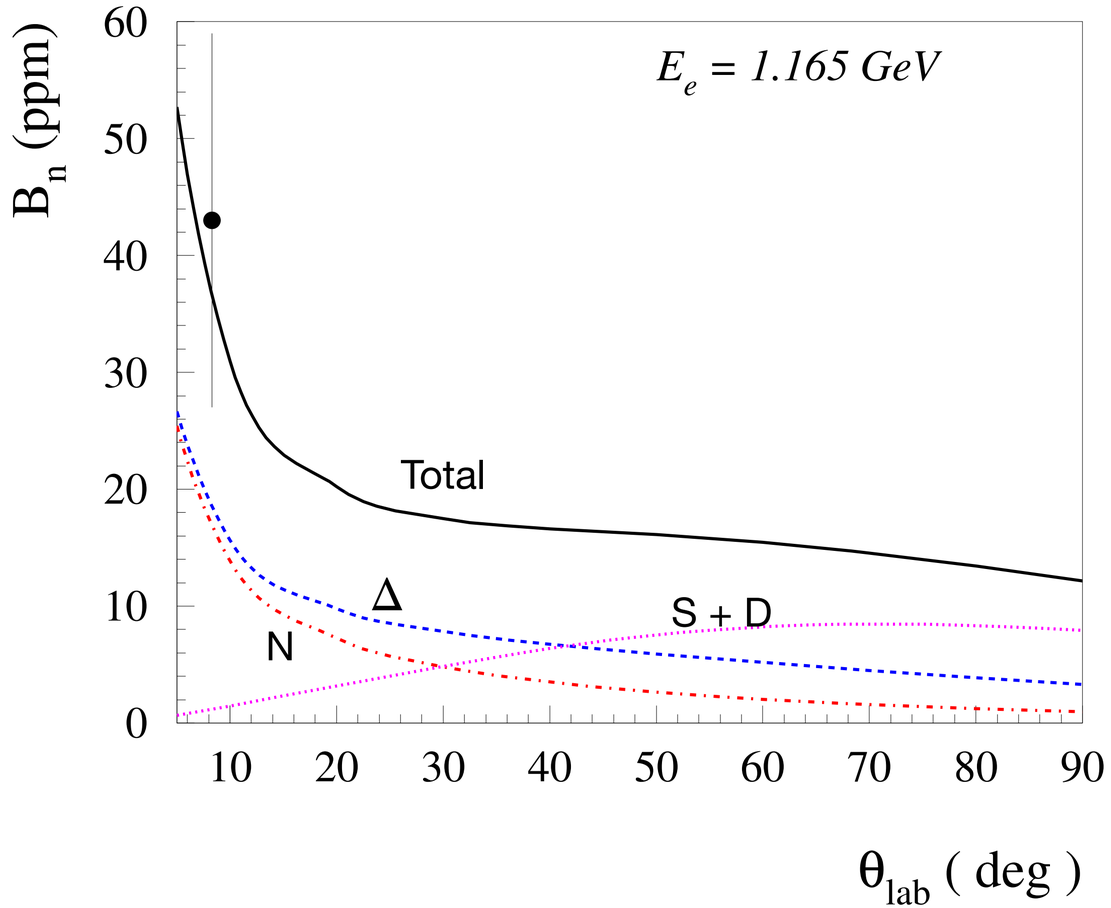
$$e_\Delta = G_{E0}^\Delta(0), \quad \mu_\Delta = \frac{e}{2M_\Delta} G_{M1}^\Delta(0), \quad \mathcal{Q}_\Delta = \frac{e}{M_\Delta^2} G_{E2}^\Delta(0), \quad \mathcal{O}_\Delta = \frac{e}{2M_\Delta^3} G_{M3}^\Delta(0)$$

- Left hand sides are charge, magnetic moment, electric quadruple moment, and magnetic octuple moment.
- Also got some information on Q^2 dependence from the lattice.

Also included S_{11} and D_{13}

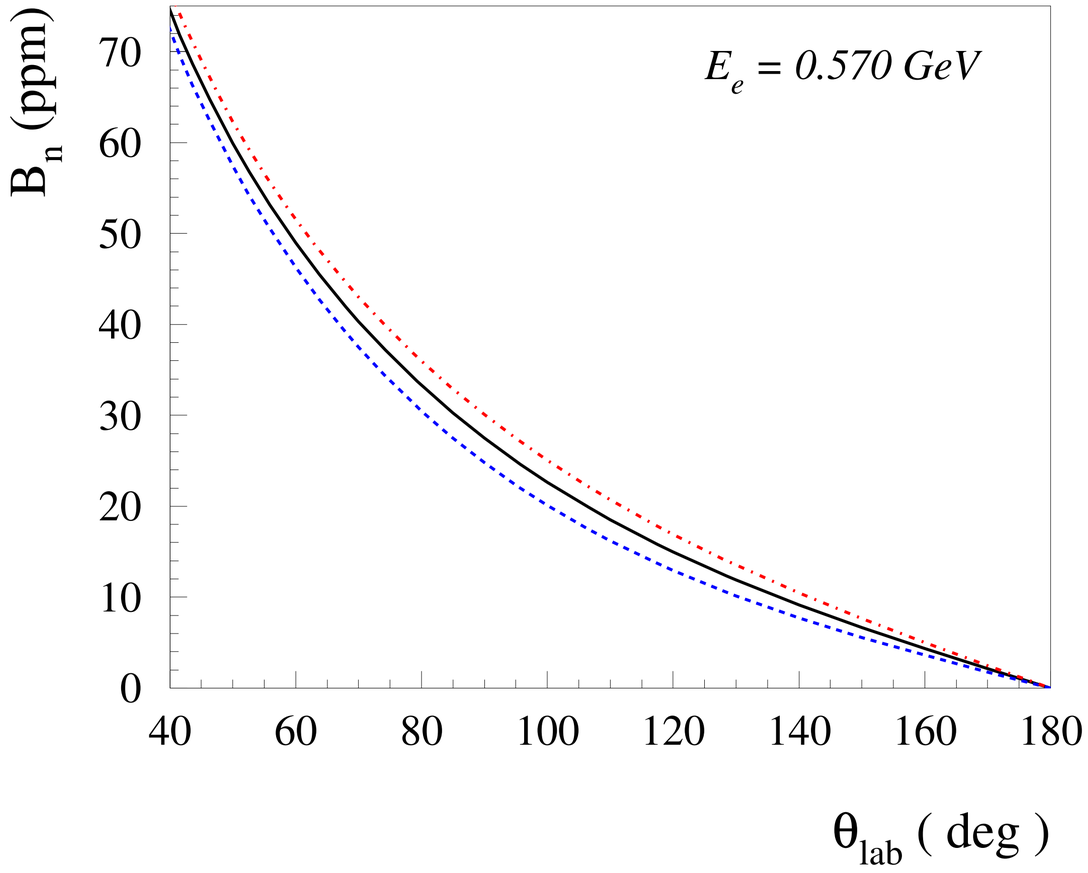
- With masses, $S_{11}(1535)$ and $D_{13}(1520)$.
- Obtained EM current matrix elements by writing constituent quark model wave functions for these states, and obtaining parameters by comparing to similar calculations for the nucleon and Delta.
- Basics in F. Close's 1979 book, "An Introduction to Quarks and Partons".

A general B_n result



- Data point from QWeak spin-off, Nuruzzaman arXiv:1510.00449
- Contributing resonances must have mass
  $W < \sqrt{s} = \sqrt{M_N^2 + 2M_N E_{\text{beam}}}$,
  here 1.75 GeV.
- Can go to lower energy to eliminate higher resonances

A sensitivity result



- Three curves, for values $\mu_\Delta = \{1.5, 3.0, 4.5\}\, e/(2M_\Delta)$, bottom to top.

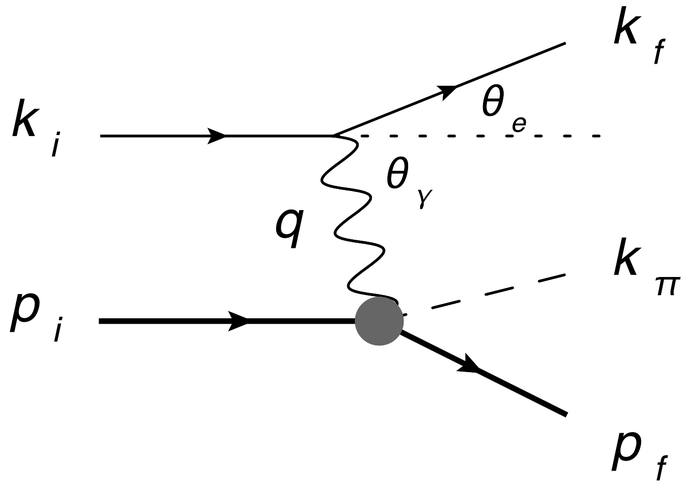
- ($W_{\text{max}} = 1.4$ GeV)

If other particles observed

- So far: even in inelastic scattering, if only the electron is observed and weak interactions are not considered, BNSSAs are forbidden in one-photon exchange.

- However, if the reaction is $ep \rightarrow e\pi N$, one may observe the pion (say), by itself or along with the electron. The pion experiences strong final state interactions, and these interactions give different phases to the amplitudes for the two electron spins, and lead to nonzero beam normal spin asymmetries.

Case where both e and π seen



- Ancient knowledge: can write cross section as photon flux factor times $\gamma^* p$ CM cross section,

$$\frac{d\sigma}{d^3k_f\, d\Omega_\pi^{\rm CM}} = E_f^2 \Gamma \frac{d\sigma_{\rm v}}{d\Omega_\pi^{\rm CM}}$$

- Formerly with no electron polarization. If electron polarized ⊥ to it momentum, at angle ϕ_{Se} to the electron scattering plane, ∃ extra terms,

$$\frac{d\sigma_{\rm v}}{d\Omega_\pi^{\rm CM}} = \frac{d\sigma_{\rm v}^{\rm unpol}}{d\Omega_\pi^{\rm CM}} + \sin\phi_{Se} \frac{d\sigma_{\rm v}^{n}}{d\Omega_\pi^{\rm CM}} + \cos\phi_{Se} \frac{d\sigma_{\rm v}^{s}}{d\Omega_\pi^{\rm CM}}$$

BNSSA

- From this $B_n = \dfrac{d\sigma_{\rm v}^n/d\Omega_\pi^{\rm CM}}{d\sigma_{\rm v}^{\rm unpol}/d\Omega_\pi^{\rm CM}}$

- Can calculate in terms of standard multipole amplitudes for pion electroproduction, which are available in tabulations like MAID,

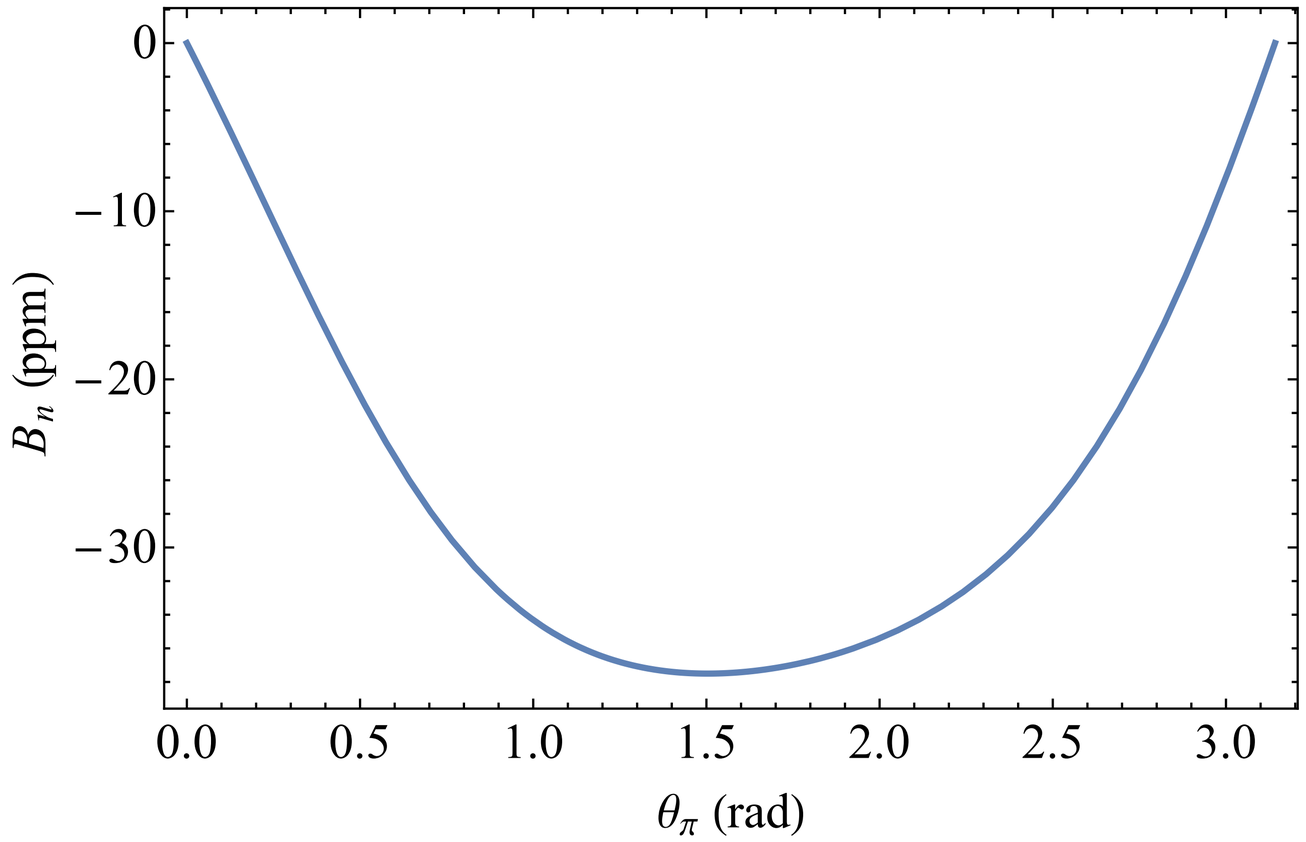
$$\frac{d\sigma_{\rm v}^n}{d\Omega_\pi^{\rm CM}} = \frac{2m_e}{Q}(1-\epsilon)D'\sin\theta_\pi^{\rm CM}\cos\phi_\pi^{\rm CM}$$

- $D' = D'_0 + D'_1\cos\theta_\pi^{\rm CM}$;

$$D'_0 = -\frac{|\vec{k}_\pi^{\rm CM}|}{k_\gamma^{\rm CM}}\frac{Q}{\omega_\gamma}\,{\rm Im}\left\{L_{0+}^*(3E_{1+}-M_{1+}+M_{1-}) - (2L_{1+}-L_{1-})^*E_{0+}\right\}$$

D'_1 also known

One plot for e and π seen



- For
  $E_e = 1.155$ GeV,$\theta_e = 7.9°$,
  $W = 1.232$ GeV, and $\phi_\pi = 0°$

Ending

- BNSSA arises from interference of one-photon and imaginary part of two-photon amplitudes
- Intermediate states on mass shell, so can access $\Delta \to \Delta$ and $N^* \to \Delta$
- Showed currently possible theory results for JLab and Mainz energies
- Did get close to the one available data point
- Can also get non-zero BNSSA in one-photon exchange if other (strongly interacting) particles are observed

Beyond the end

Slide 2

- First line W&M Green
  - First subline W&M Gold alternate
    - Second subline W&M Gold