Factorized Approach to Radiative Corrections

Precision Physics with MOLLER Experiment

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1. Introduction

2. Joint QCD+QED Factorization for DIS

3. Møller Scattering in the Factorized Approach

4. Summary

Radiative Corrections in Precision Lepton Scattering

- Precision measurements requires radiative corrections to be included in the analysis.
- For a longitudinally polarized electron beam,

$$A_{\mathrm{PV}} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}.$$

- Radiation changes rates and kinematics. Bin migration need not cancel in A_{PV}.
- Weak effects and real/virtual QED corrections must use the same measurement definition.

→ Can we systematically separate **universal electron radiation** from the **hard scattering**?

Fixed Order, Effective Theory and Factorization

Method	Main task	Role in the factorized approach
Fixed-order calculation	Real and virtual corrections	Calculation of the hard part
Low-energy effective theory	Integrating out heavy particles and resumming large logarithms	Modification of the hard part
Collinear factorization	Separation of universal long-distance collinear radiation from short-distance hard scattering	Resummed collinear logarithms and calculable hard scattering

$$\underbrace{d\sigma \sim F_{\text{in}} \otimes H \otimes D_{\text{out}}}_{\text{schematic factorized structure}}$$

- Fixed-order calculations already include real radiation and infrared cancellation.

→ **Factorization** combines fixed order with resummation without double counting.

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QED Radiation in Lepton-Hadron Scattering

- Hard lepton–hadron scattering produces both QCD and QED radiation.
- Standard **collinear QCD factorization** separates universal long-distance dynamics into parton distribution functions (PDFs), while the short-distance partonic scattering is perturbatively calculable:

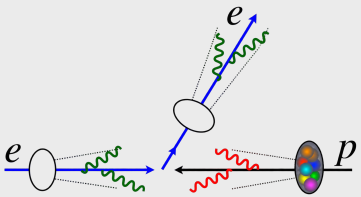
$$\mathrm{d}\sigma = \sum_i f_{i/h} \otimes \mathrm{d}\hat{\sigma}_{\ell i}.$$

- Factorization makes QCD corrections systematically improvable: fixed-order hard coefficients describe resolved emissions, while PDF evolution resums universal collinear logarithms.
- Incoming and outgoing leptons also radiate photons. For electron beams, the interval $m_e \to Q$ generates sizeable $\alpha \ln\left(Q^2/m_e^2\right)$ terms and crosses the hadronic regime, so it also contains nonperturbative long-distance physics. **E.g.,** $e \to e + \gamma(\to q\bar{q}) \to \cdots$

→ This motivates extending the established collinear-factorization framework to include collision-induced QED radiation.

Inclusive ep Deep Inelastic Scattering

- Due to QED radiation from the incoming/outgoing leptons, the momentum transfer q is not fixed. Hadron is probed by the virtual photon with momentum $\hat{q}$ instead of q.



measured	$q \to \hat{q}$	**experienced**
from	$Q^2 := -q^2 \to \hat{Q}^2 := -\hat{q}^2$	**by**
leptons	$x_B = \dfrac{Q^2}{2P \cdot q} \to \hat{x}_B := \dfrac{\hat{Q}^2}{2P \cdot \hat{q}}$	**proton**

- Measurement with fixed Q^2 and x_B could cover a kinematic range of $\hat{Q}^2$ and $\hat{x}_B$ even with the approximation of one-photon exchange.

$$x_B \to \hat{x}_B \in [x_B, 1], \quad Q^2 \to \hat{Q}^2 \in \left[\frac{Q^2(1-y)}{1-x_B y}, \frac{Q^2}{1-y+x_B y}\right], \quad y := \frac{P \cdot q}{P \cdot l}$$

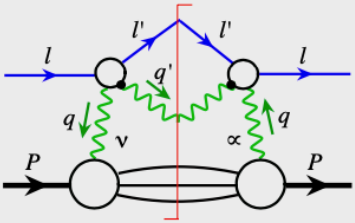
→ $\hat{Q}^2$ can be smaller than Q^2 and even **no longer a hard scale**.

Radiative Corrections

- Traditionally, a simple RC factor is applied to correct the measured cross section to the Born level.

$$\sigma_{\text{measured}} = \text{RC} \otimes \sigma_{\text{Born}}$$

- However, such RC approaches could suffer from perturbative **pinched singularity** from QED radiation when $q^2 \sim 0$. l' is balanced by unobserved radiative photon(s).



→ inclusive analogue of the **Bethe-Heitler process**.

$$E' \frac{d\sigma^{\text{RC}}_{eh\to eX}}{d^3\ell'} \propto \int d^4q \left[L^{(1)}_{\mu\nu}(\ell, \ell', q) \frac{1}{q^2 - i\epsilon} \times W_h^{\mu\nu}(q, P) \frac{1}{q^2 + i\epsilon} \right] \to \infty$$

- Keeping the exchanged photon as a **hard probe** requires cuts or parameters on radiated-photon phase space; these are collision-energy dependent.

Joint QCD+QED Factorization for DIS

Factorization Formula for $e(\ell)p(P) \to e(\ell') + X$ at leading power

$$E'\frac{\mathrm{d}\sigma}{\mathrm{d}^3\ell'} \approx \frac{1}{2s}\sum_{ija}\int_{\zeta_{\min}}^1 \frac{\mathrm{d}\zeta}{\zeta^2}\int_{\xi_{\min}}^1 \frac{\mathrm{d}\xi}{\xi} D_{e/j}\left(\zeta,\mu^2\right) f_{i/e}\left(\xi,\mu^2\right) \int_{x_{\min}}^1 \frac{\mathrm{d}x}{x} f_{a/p}(x,\mu^2)\, \hat{H}_{ia\to jX}\left(\xi\ell, xP, \frac{\ell'}{\zeta}, \mu^2\right)$$

→ **Predictive power:** universal LDFs/PDFs/FFs, perturbative evolution, calculable hard parts.

- $f_{j/p}$: **universal** parton distribution function (PDF);
- $f_{i/e}/D_{e/j}$: **universal** lepton distribution/fragmentation functions (LDFs/LFFs) resum both QCD, QED and mixed collinear logarithms;
- $\hat{H}_{ia\to jX}$: **perturbative calculable** hard scattering coefficient

$$\hat{H}_{ia\to jX}\left(\xi\ell, xP, \ell'/\zeta, \mu^2\right) = \sum_{m,n}\alpha^m\alpha_s^n \hat{H}^{(m,n)}_{ia\to jX}\left(\xi\ell, xP, \ell'/\zeta, \mu^2\right)$$

- The cross section without QED radiation can be recovered by setting

$$D_{e/j}\left(\zeta,\mu^2\right) = \delta(1-\zeta), \quad f_{i/e}\left(\xi,\mu^2\right) = \delta(1-\xi), \quad m = 0 \text{ for } \sum_{m,n}$$

Lepton Distribution Function

- Joint factorization leads to mixed evolution of LDFs.

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ \hdashline f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix} = \overbrace{\begin{pmatrix} P_{ee}^{(1,0)} & P_{e\bar{e}}^{(2,0)} & P_{e\gamma}^{(1,0)} & P_{eq}^{(2,0)} & P_{e\bar{q}}^{(2,0)} & P_{eg}^{(2,1)} \\ P_{\bar{e}e}^{(2,0)} & P_{\bar{e}\bar{e}}^{(1,0)} & P_{\bar{e}\gamma}^{(1,0)} & P_{\bar{e}q}^{(2,0)} & P_{\bar{e}\bar{q}}^{(2,0)} & P_{\bar{e}g}^{(2,1)} \\ P_{\gamma e}^{(1,0)} & P_{\gamma\bar{e}}^{(1,0)} & P_{\gamma\gamma}^{(1,0)} & P_{\gamma q}^{(1,0)} & P_{\gamma\bar{q}}^{(1,0)} & P_{\gamma g}^{(1,1)} \\ \hdashline P_{qe}^{(2,0)} & P_{q\bar{e}}^{(2,0)} & P_{q\gamma}^{(1,0)} & P_{qq}^{(0,1)} & P_{q\bar{q}}^{(0,2)} & P_{qg}^{(0,1)} \\ P_{\bar{q}e}^{(2,0)} & P_{\bar{q}\bar{e}}^{(2,0)} & P_{\bar{q}\gamma}^{(1,0)} & P_{\bar{q}q}^{(0,2)} & P_{\bar{q}\bar{q}}^{(0,1)} & P_{\bar{q}g}^{(0,1)} \\ P_{ge}^{(2,1)} & P_{g\bar{e}}^{(2,1)} & P_{g\gamma}^{(1,1)} & P_{gq}^{(0,1)} & P_{g\bar{q}}^{(0,1)} & P_{gg}^{(0,1)} \end{pmatrix}}^{\text{QED}\to\text{QED}\ \ \text{QCD}\to\text{QCD}} \otimes \begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ \hdashline f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix}.$$

- Evolution kernels $P_{ij}^{(m,n)}$ are perturbatively calculable in both QED and QCD.

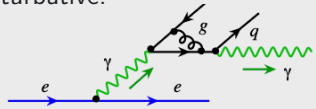
$$P_{ij}(\xi,\mu^2) = \sum_{n,m=0}^{\infty}\left(\frac{\alpha_{\rm em}(\mu^2)}{2\pi}\right)^n\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^m \hat{P}_{ij}^{(n,m)}(\xi) =: \sum_{n,m=0}^{\infty} P_{ij}^{(n,m)}(\xi,\mu^2),$$

Input LDFs as Nonperturbative Boundary

- Perturbative QCD+QED evolution determines the **scale dependence**, but not the distributions at the starting scale:

$$\{f_{i/e}(\xi,\mu_0^2)\} \xrightarrow{\text{QCD+QED evolution}} \{f_{i/e}(\xi,\mu^2)\} \longrightarrow \text{predictions.}$$

- **Choose $\mu_0 = m_c$:** below this scale, the QCD part of $\gamma \to q\bar{q}$ is not reliably perturbative.



- **Parameterize the boundary:**

$$f_{i/e}(\xi,\mu_0^2) = \frac{N_i \xi^{\alpha_i}(1-\xi)^{\beta_i}}{B(1+\alpha_i, 1+\beta_i)},$$

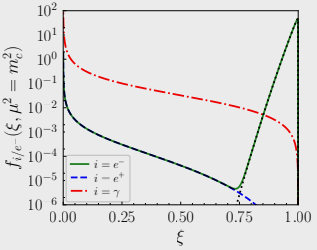
with electron-number and momentum sum rules.

→ Existing measurements generally have traditional radiative corrections pre-applied, so they cannot directly constrain these universal inputs without reanalysis.

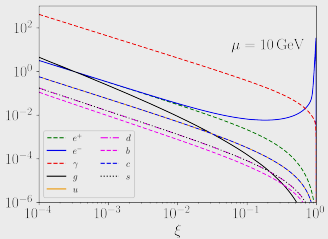
Default Input Model

Qiu, Watanabe, 2607.07664

- At $\mu_0 = m_c = 1.3\,\mathrm{GeV}$:
  - match valence-e Mellin moments to NLO QED;
  - obtain γ and sea leptons by QED evolution from m_e to m_c;
  - set $q, \bar{q}, g = 0$ and **generate them through evolution** above μ_0.
- The last choice gives a conservative **lower-limit model** for the electron's hadronic content.



input at μ_0



evolved to $\mu = 10$ GeV

→ **Long-term strategy:** fit the common input LDFs to data without pre-applied radiative corrections, then evolve them to any process and hard scale.

Modeling Assumptions for the Phenomenological Analysis

- **Test input-shape dependence:** compare the baseline LF1 with two alternatives at $\mu_0 = m_c$,

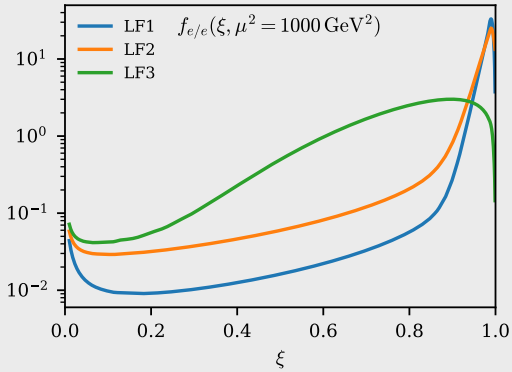
  LF2: $(\alpha_e, \beta_e) = (0.125, 50)$,

  LF3: $(\alpha_e, \beta_e) = (0.5, 5)$.

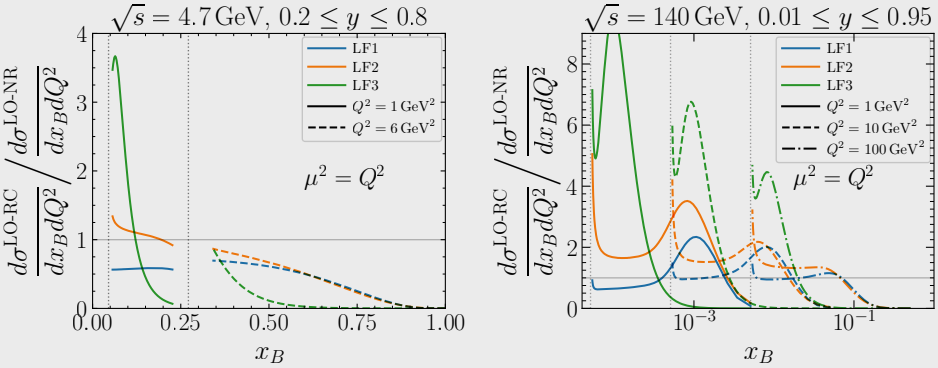
- **Recall**

$$f_{i/e}(\xi, \mu_0^2) = \frac{N_i \xi^{\alpha_i} (1-\xi)^{\beta_i}}{B(1+\alpha_i, 1+\beta_i)},$$

- Use the same model shape for the outgoing LFF and incoming LDF.



Resummed Collinear Contribution



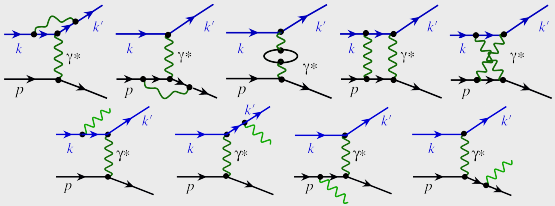
- Tree level hard part with model LDFs and LFFs at $\mu_0 = m_c$, evolved to $\mu = Q$.
- The effect of QED radiation is significant & sensitive to the shape of LDFs and LFFs.

Partonic Hard Functions beyond LO

- The hard function $\hat{H}_{ia\to jX}$ is perturbatively calculable and IR/CO safe.
- Joint factorization can naturally go **beyond the one-photon exchange approximation**.
- NLO QED corrections to hard scattering coefficient: ***Phys.Rev.D 112 (2025) 5, 056007***

$$\hat{H}^{(1)}_{eq\to eX} = \underbrace{\hat{\sigma}^{(1)}}_{\text{FO}} \underbrace{-D^{(1)}_{e/e}\otimes\hat{H}^{(0)}_{eg\to eX} - f^{(1)}_{e/e}\otimes\hat{H}^{(0)}_{eq\to eX} - f^{(1)}_{q/p}\otimes\hat{H}^{(0)}_{eq\to eX} - f^{(1)}_{\gamma/p}\otimes\hat{H}^{(0)}_{e\gamma\to eX}}_{\text{subtraction of collinear divergences}}$$

Hadron's parton distributions are not pure QCD.



- two photon exchange: IR divergent;
- virtual + real emission: IR safe;
- CO divergence: factorized into PDFs/LDFs/LFFs.

Hard Functions at NLO QED

- e_q^* helps to isolate contributions from different diagrams and cancellation of divergences.

$$\widehat{H}^{(1)}_{eq\to eX}(\hat{s},\hat{t},\hat{u}) = \frac{4e_q^2\alpha^3}{\pi(1-\hat{v})^2\,\hat{s}}\left\{e_l^2\,\frac{2(1+\hat{v}^2)}{9\,\hat{v}}\left(3\ln\left[\frac{(1-\hat{v})\hat{s}}{\mu^2}\right]-5\right)\delta(1-\hat{w})\right.$$

$$+ e_q\left(a_1\delta(1-\hat{w}) + a_2\frac{1}{[1-\hat{w}]_+} + a_4\right)$$

$$+ e_q^2\left(b_1\delta(1-\hat{w}) + b_2\frac{1}{[1-\hat{w}]_+} + b_3\left[\frac{\ln(1-\hat{w})}{1-\hat{w}}\right]_+ + b_4\right)$$

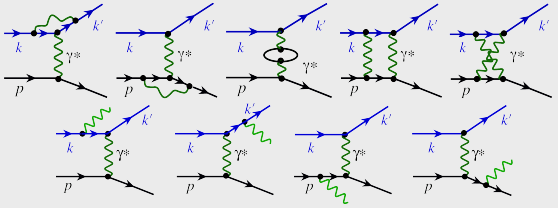
$$\left. + c_1\delta(1-\hat{w}) + c_2\frac{1}{[1-\hat{w}]_+} + c_3\left[\frac{\ln(1-\hat{w})}{1-\hat{w}}\right]_+ + c_4\right\}$$

$e_l^2 := \sum_f N_f^C e_f^2$, vacuum polarization

two-photon exchange, UV & CO safe

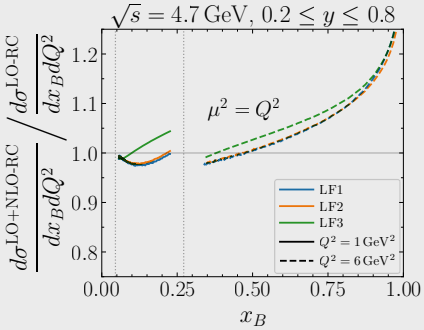
radiation from q, **same as NLO QCD**

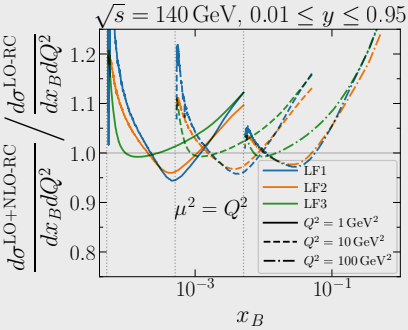
RC from electron, need $f_{\gamma/p}$



- a_*, b_*, c_*: analytic functions of $\hat{v}, \hat{w}$
- $\hat{t} =: -(1-\hat{v})\hat{s}$
- $\hat{u} =: -\hat{v}\hat{w}\hat{s}$.

Numerical Impact of the NLO QED Hard Part





$$R_H = d\sigma^{\text{LO+NLO-RC}}/d\sigma^{\text{LO-RC}} \quad \text{at fixed } x_B, Q^2, \text{ with } \mu^2 = Q^2.$$

- Identical evolved LDFs/LFFs and proton PDFs isolate the **NLO hard contribution**.

→ Hard corrections complement collinear evolution and depend on kinematics.

Cammarota, Qiu, Watanabe, Zhang, Phys. Rev. D 112 (2025) 056007

Parity Violating Deep Inelastic Scattering

Parity Violating Lepton-Spin Asymmetry

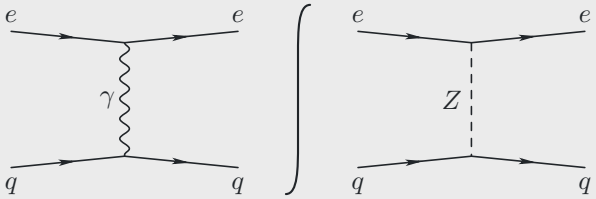
$$A_{\mathrm{PVE}} := \frac{\sigma_{e(\lambda=1)p\to eX} - \sigma_{e(\lambda=-1)p\to eX}}{\sigma_{e(\lambda=1)p\to eX} + \sigma_{e(\lambda=-1)p\to eX}} =: \frac{\Delta\sigma_{ep\to eX}}{\sigma_{ep\to eX}(\mathrm{calculated})}$$

- Previous framework is hard to extend to full EW&QCD factorization directly.
- For SoLID, Z/W is too heavy to radiate. EW&QCD factorization might be applicable at heavy gauge boson limit.

$$E_{l'}\frac{\mathrm{d}\sigma_{lP\to l'X}}{\mathrm{d}^3 l'} \approx \int_{\zeta_{\min}}^{1}\frac{\mathrm{d}\zeta}{\zeta^2}D_{e/e}\left(\zeta,\mu^2\right)\int_{\xi_{\min}}^{1}\mathrm{d}\xi f_{e(\lambda_k)/e(\lambda_l)}\left(\xi,\mu^2\right)\left(E_{k'}\frac{\mathrm{d}\hat{\sigma}_{kP\to k'X}}{\mathrm{d}^3 k'}\right)_{k=\xi l,k'=l'/\zeta}$$

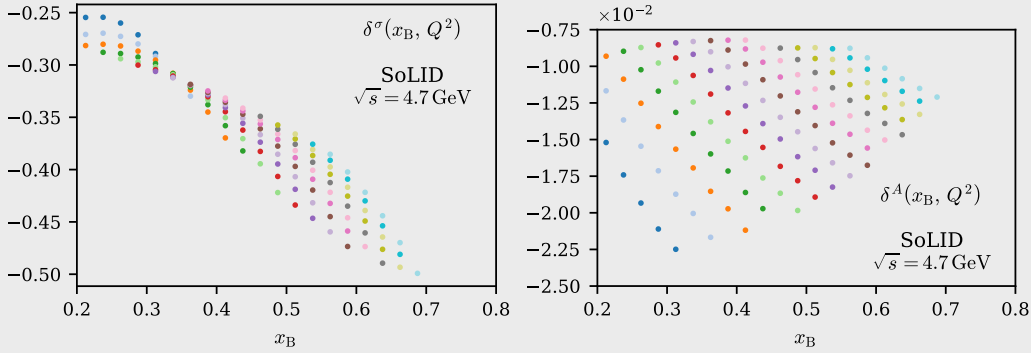
One-Vector-Boson Exchange

- Only interference between γ and Z exchange contributes to A_{PVE} at LO.



$$\frac{\mathrm{d}\Delta\hat{\sigma}}{\mathrm{d}\hat{y}} = \frac{2\pi\alpha^2 e_q}{m_Z^2 \sin^2\theta_{\mathsf{W}} \cos^2\theta_{\mathsf{W}} \hat{y}} \left\{ e_q \sin^2\theta_{\mathsf{W}} \left[1 + (1-\hat{y})^2\right] + I_3^q \left[2\sin^2\theta_{\mathsf{W}} \left(1 - (1-\hat{y})^2\right) - 1\right] \right\}$$

Asymmetry with Default LDF/LFF



- few-percent asymmetry shifts

$$\sigma = \sigma^{\mathrm{NR}}(1+\delta^\sigma), \quad A_e = A_e^{\mathrm{NR}}(1+\delta^A)$$

→ Asymmetries are more insensitive to the effects of QED radiation compared to cross section.

- We assume helicity-independent LDFs, which can be further tested in factorization approach.

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A Factorized View of the Møller Cross Section

- Asymmetry A_{PV} at Møller with polarized electron beam and unpolarized target:

$$A_{\mathrm{PV}} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \frac{\Delta\sigma}{\sigma},$$

- Both numerator(polarized) and denominator(unpolarized) require radiative corrections.
- Collinear factorization can be applied to both numerator and denominator.

$$e^- e^- \to e^- + X, \qquad d(\Delta)\sigma^{\mathrm{resum}} \sim \sum_{abc} f_{a/e} \otimes f_{b/e} \otimes H_{ab\to cX} \otimes D_{e/c}.$$

- **Two incoming LDFs, one observed-electron LFF**: the recoil electron and unobserved radiation are inclusive; f, D: universal collinear radiation and evolution. H: finite hard scattering after subtraction.
- Møller (11 GeV, Born estimate): $p_T \sim 40$–50 MeV $\gg m_e$, $L_e = \ln\left(p_T^2/m_e^2\right) \simeq 9$. Collinear logs are enhanced; their higher-order impact should be quantified at the required precision.
- For wider energy range, the factorization framework can be matched to fixed-order calculations

$$d\sigma = d\sigma^{\mathrm{resum}} + d\sigma^{\mathrm{FO}} - d\sigma^{\mathrm{asy}}$$

Unpolarized Scattering: Factorization Verified at NLO

- **Factorization:** $e^-e^- \to e^-(p) + X$

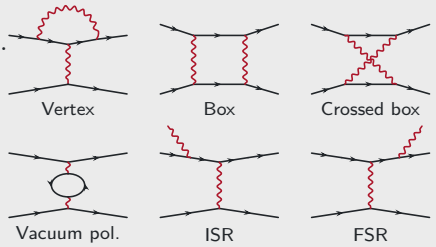
$$E_p \frac{d\sigma}{d^3p} = \frac{1}{2S} \sum_{abc} \int \frac{dz}{z^2} D_{e/c} \int \frac{dx_1}{x_1} f_{a/e} \int \frac{dx_2}{x_2} f_{b/e}\, \widehat{H}_{ab\to cX}.$$

Leading power; two LDFs and one LFF.

- **LO: full t–u interference**

$$\frac{d\hat{\sigma}^{\rm LO}_{ee\to eX}}{dv} = \frac{4\pi\alpha^2(v^2 - v + 1)^2}{s(1-v)^2 v^2}, \qquad v = -\frac{u}{s}.$$

NLO: representative topologies



Representative diagrams; exchange channels included.

- **NLO matching: five collinear subtraction terms**

$$\widehat{H}^{(0,3)}_{ee} = \mathcal{H}^{(0,3)}_{ee} - D^{(1)}_{e/e} \otimes_z \widehat{H}^{(0,2)}_{ee} - f^{(1)}_{e/e} \otimes_2 \widehat{H}^{(0,2)}_{ee} - f^{(1)}_{\gamma/e} \otimes_2 \widehat{H}^{(0,2)}_{e\gamma} - f^{(1)}_{e/e} \otimes_1 \widehat{H}^{(0,2)}_{ee} - f^{(1)}_{\gamma/e} \otimes_1 \widehat{H}^{(0,2)}_{\gamma e},$$

$$\widehat{H}^{(0,3)}_{eq} = \mathcal{H}^{(0,3)}_{eq} - D^{(1)}_{e/e} \otimes_z \widehat{H}^{(0,2)}_{eg} - f^{(1)}_{q/p} \otimes_2 \widehat{H}^{(0,2)}_{eq} - f^{(1)}_{\gamma/p} \otimes_2 \widehat{H}^{(0,2)}_{e\gamma} - f^{(1)}_{e/e} \otimes_1 \widehat{H}^{(0,2)}_{eq}$$ **for DIS**

Result: IR/CO singularities cancel after subtraction $\rightsquigarrow$ finite NLO hard part.

Fixed-Order Calculations and Our Next Step

Fixed-order calculations

- **Unpolarized denominator:** fully differential NNLO QED in McMule, including real photons and vacuum polarization. *Banerjee et al. (2022)*

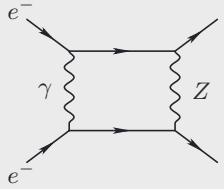
  QED radiation in the transverse asymmetry: *Dixon & Schreiber (2004)*
- **Polarized numerator and asymmetry:** two-loop virtual EW corrections in the low-energy expansion; LEFT combines NLO EW, LL evolution and NNLO QED.

  Petriello (2003); Aleksejevs et al. (2010); Du et al. (2021); Erler et al. (2022); Kollatzsch et al. (2025); Chen et al. (2026); Czarnecki (2026)

What can the factorized approach add?

- **Evolve LDFs and LFFs** to organize higher-order electron-mass logarithms beyond fixed order.
- **Match to fixed order** and subtract the overlap, retaining finite hard and EW contributions.

γZ Box: Large Logarithms and EFT Resummation



Representative box; crossed graphs implied.

Widely separated scales: $s, |t|, |u| \ll M_Z^2$

$$\delta\mathcal{M}_{\gamma Z} \supset \mathcal{M}_Z^{(0)}\, \mathcal{O}\left(\frac{\alpha}{\pi} \ln \frac{M_Z^2}{s}\right).$$

Schematic, IR-subtracted logarithmic contribution.

- For Møller, $\sqrt{s} \simeq 0.106\,\text{GeV}$: $\ln\left(M_Z^2/s\right) \simeq 13.5$.
- Higher-order towers $[\alpha \ln\left(M_Z^2/s\right)]^n$ require resummation.

LEFT: match at $\mu_h \sim M_Z$, evolve to a low scale

$$\mathcal{L}_{\text{LEFT}} \supset \sum_i C_i(\mu)\mathcal{O}_i, \qquad \mu\frac{d\boldsymbol{C}}{d\mu} = \boldsymbol{\gamma}^T \boldsymbol{C}, \qquad \boldsymbol{C}(\mu_s) = U(\mu_s, \mu_h)\boldsymbol{C}(\mu_h).$$

- RG evolution resums the logs; include threshold matching and low-scale matrix elements.

Kollatzsch et al., JHEP 09 (2025) 196, Sec. 2 and App. B

1. Introduction

2. Joint QCD+QED Factorization for DIS

3. Møller Scattering in the Factorized Approach

4. Summary

Summary and Outlook

- Precision measurements require systematic theoretical predictions of radiative effects.
- Factorization provides such a framework, separating universal radiation from hard scattering.
- This approach has been applied to lepton–nucleon scattering, with collinear evolution, NLO hard parts and numerical studies.
- It can be extended to Møller scattering and complement existing fixed-order calculations.

→ We will perform NLO calculations before Møller takes the data.

