Status of Electroweak Precision Calculations for MOLLER

Juhun Kwak

University of Pittsburgh

11.05.26.

based primarily on

1912.08220: fermionic NNLO with $\hat{s}(m_Z)$

2202.11976: $\hat{s}(m_Z) - \hat{s}(0)$ and finite-Q^2

2610.12287: bosonic NNLO with $\hat{s}(0)$

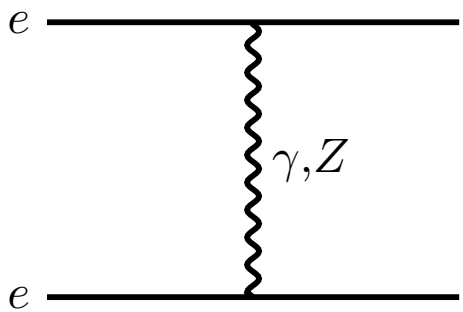




Parity-Violating Møller Scattering

Parity violating asymmetry in low-energy Møller scattering is given by

$$A_{\mathrm{LR}} = \frac{d\sigma_{\mathrm{L}} - d\sigma_{\mathrm{R}}}{d\sigma_{\mathrm{L}} + d\sigma_{\mathrm{R}}} = \frac{G_F Q^2}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4} \underbrace{(1 - 4\sin^2\theta_W + \delta Q_W^e)}_{Q_W^e}$$



where $y = Q^2/s = -t/s$, $Q^2 \ll m_Z^2$.

$$A_{\mathrm{LR}}^{\mathrm{LO}} \propto \left(e_L \, Z \, e_L \times e \, \gamma \, e^* \right)^2 - \left(e_R \, Z \, e_R \times e \, \gamma \, e^* \right)^2 \propto 1 - 4\sin^2\theta_W$$

$$\delta Q_W^e = \sum_{n_l} \sum_{n_f=1}^{n_l} \delta Q_{W(n_l,n_f)}^e$$

n_l : number of total loops

n_f : number of fermion loops

Target precision $\lesssim 0.1\delta Q_{W,\mathrm{exp}}^e \approx 1.1 \times 10^{-4} \approx 0.2\% Q_{W(0,0)}^e$.

QED Corrections

$$A_{\mathrm{LR}} = \frac{d\sigma_{\mathrm{L}} - d\sigma_{\mathrm{R}}}{d\sigma_{\mathrm{L}} + d\sigma_{\mathrm{R}}} = \frac{|\mathcal{M}_{LL}|^2 + |\mathcal{M}_{LR}|^2 - |\mathcal{M}_{RL}|^2 - |\mathcal{M}_{RR}|^2}{|\mathcal{M}_{LL}|^2 + |\mathcal{M}_{LR}|^2 + |\mathcal{M}_{RL}|^2 + |\mathcal{M}_{RR}|^2}$$



- Pure QED corrections completely cancels in the numerator, but enters in the asymmetry through the denominator.
- Fully differential prediction requires careful treatment of real radiation.
- NNLO calculation ($\supset$ NLO + 2γ) available thanks to McMule (Monte Carlo for MUon and other LEptons).
- See Banerjee et al. PRD '22 for concrete analysis for PRad II.

Renormalization Scale and $\delta Q^e_{W(1,1)}$

$\sin^2\theta_W$ is $\overline{\text{MS}}$ renormalized. Depending on the scale choice,

$\sin^2\theta_W(m_Z)$

- practical for computation
- useful for EFT matching
- $\delta Q^e_{W(1,1)} \supset \ln m_f^2/m_Z^2$ is a loop effect

$\sin^2\theta_W(0)$

- better choice for phenomenology
- $\delta Q^e_{W(1,1)}$ is absorbed in the running of $\sin^2\theta_W$

Once could also choose different expansion parameters:

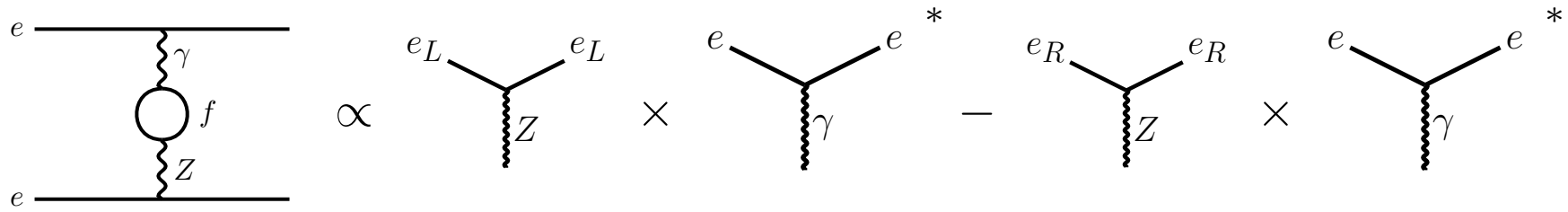
Expansion in α

- straightforward to compute

Expansion in G_μ

- translated from α-scheme
- partial cancellation of common effects in NC and CC

Renormalization Scale and $\delta Q^e_{W(1,1)}$



$$\propto 1 - 4\sin^2\theta_W$$

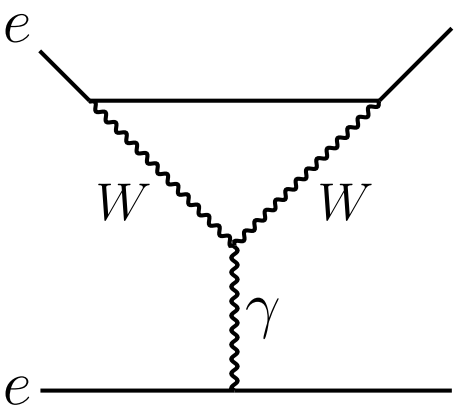
- With $\mu = m_Z$, $\delta Q^e_{W(1,1)} \sim -40\% Q^e_{W(0,0)}$ due to $\ln\left(m_f^2/m_Z^2\right)$.
- With $\mu = 0$, $\delta Q^e_{W(1,1)} \lesssim -1\% Q^e_{W(0,0)}$.

Technically, $\mu = Q^2$. The finite momentum effect is small:

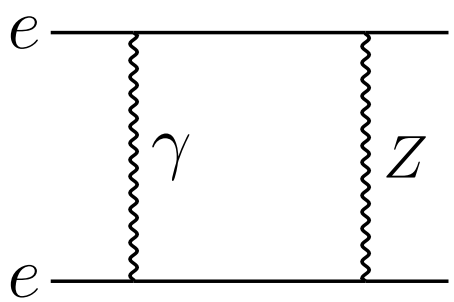
$$F_2(y = 1/2, Q^2) = -4\cos^2\theta_W \sin^2\theta_W \left[\Pi_{\gamma Z}(-Q^2) - \Pi_{\gamma Z}(0)\right] \sim 10^{-5}$$

Czarnecki & Marciano PRD '96

Remaining NLO



- Not suppressed by $1 - 4\sin^2\theta_W$ due to W-boson-fermion coupling
- No large logs appears either

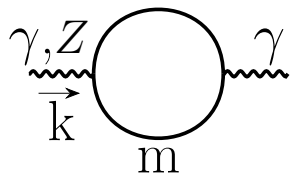


- Suppressed by $1 - 4\sin^2\theta_W$
- Enhanced by $\ln\left(Q^2/m_Z^2\right) \sim \mathcal{O}(10)$

These classes of contribution could in principle reach $\sim 10\%$ of the leading order, but fortuitous cancellation yields $\delta Q^e_{W(1,0)} \sim 1.7\% Q^e_{W(0,0)}$.

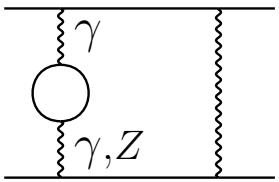
Loop Calculation Techniques - Dispersion Method

QFT optical theorem allows for a relation between the imaginary part of a self-energy and the matrix element of pair producing particles in the loop:



$$= \underbrace{\frac{N_c g_1 g_2}{12\pi}\left(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{m^2}\right)}_{c_\varepsilon} + \frac{k^2}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi_T(\sigma)}{\sigma - k^2 - i0}\frac{d\sigma}{\sigma}$$

where $\mathrm{Im}\Pi_T(\sigma) = \dfrac{N_c g_1 g_2}{12\pi}\left(1 + \dfrac{2m^2}{\sigma}\right)\sqrt{1 - \dfrac{4m^2}{\sigma}}\,\Theta(\sigma - 4m^2)$.



$$\sim \int \frac{d^D k}{i\pi^{D/2}} \frac{N(q)}{\Pi_j^5((q+k_j)^2 - m_j^2)}\left[c_\varepsilon - \frac{q^2}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi_T(\sigma)}{q^2 - \sigma + i0}\frac{d\sigma}{\sigma}\right]$$

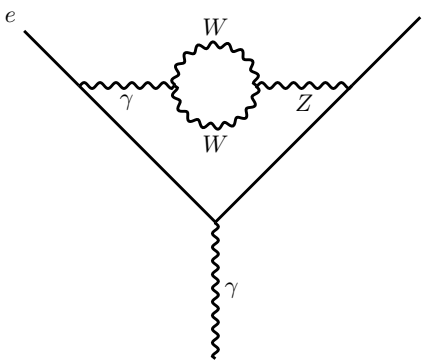
which is a one-loop integral multiplied by some coefficients; semi-analytic.

see also Bauberger et al. Nucl. Phys. '95, Awramik et al. JHEP '06, Aleksejevs '18

Loop Calculation Techniques - Method of Regions

1. Split loop momenta into regions according to the scales of the problem
2. Expand the integrands in series with corresponding scaling
3. Integrates all regions with $[0, \infty)$ as bounds
4. Add contributions from all regions; unphysical terms cancel.

Beneke & Smirnov Nucl. Phys. '98, Smirnov '01, Pak & Smirnov '10,...



$$\sim \frac{1}{(q_1^2 - m_e^2)((q_1 - p_a)^2 - m_\gamma^2)((q_1 - p_a)^2 - m_Z^2)} \times \frac{1}{((q_1 - p_a)^2 - m_e^2)(q_2^2 - m_W^2)((q_2 - q_1 - p_a)^2 - m_W^2)}$$

where $\{p_a, p_b\} \ll m_Z$ are (comb. of) external momenta

Hard-Hard region:

$\{q_1, q_2\} \sim m_Z \gg \{p_a, p_b\}$

Hard-Soft region:

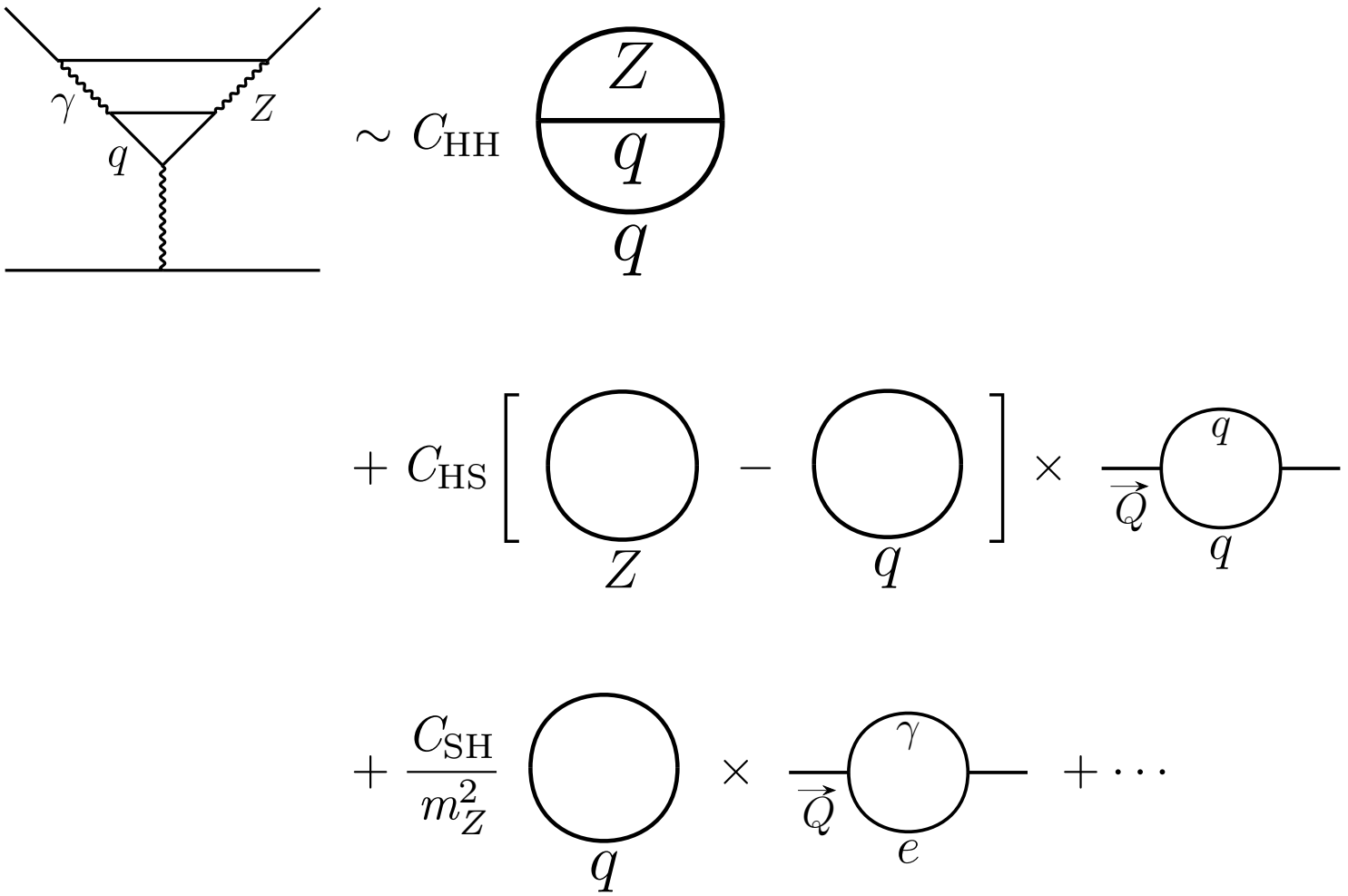
~~$q_1 \sim m_Z \gg q_2 \sim \{p_a, p_b\}$~~

$q_2 \sim m_Z \gg q_1 \sim \{p_a, p_b\}$

Soft-Soft region:

~~$\{q_1, q_2\} \sim \{p_a, p_b\} \ll m_Z$~~

Loop Calculation Techniques - Method of Regions



gives two-loop vacuum integrals and one-loop integrals; fully analytic!

Fermionic NNLO - Threshold Masses

The hadronic effects in quark masses are parametrized with threshold masses:

1. Obtain $\Delta_s\hat{\alpha}$ from $e^+e^- \to$ hadron data
2. Use $SU(3)$ flavor symmetry breaking to separate flavor and determine the t'Hooft scale where the quark of interest decouples

Erler & Ramsey-Musolf PRD '05

- Up to α_s^4 included in RGE
- lattice results are used to confirm and improve predictions from data, handle singlet contribution and OZI-violating effects, etc.

Erler & Ferro-Hernández JHEP '18 and refs therein

$$\bar{m}_b = 3.990 \text{ GeV} \qquad \bar{m}_c = 1.185 \text{ GeV}$$
$$\bar{m}_s = 342^{+48}_{-53} \text{ MeV} \qquad \bar{m}_{u,d} = 246^{+54}_{-57} \text{ MeV}$$

This gives $\sim 10^{-5}$ uncertainty for the fermionic NNLO results.
Even with $m_{q-0.5m_q}^{+1.0m_q}$ for light quark, the uncertainty is $\lesssim 6 \times 10^{-5}$.

Work on-going for results with data-driven method.

$\hat{s}^2(m_Z) - \hat{s}^2(0)$ Translation

1912.02280 computes fermionic NNLO without QCD effects.
For a phenomenological numeric, one should rewrite

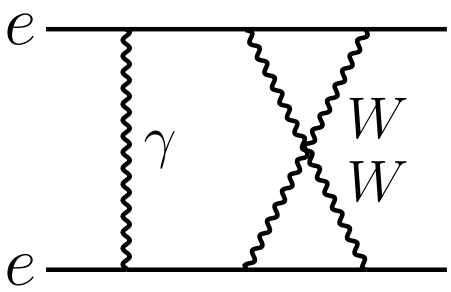
$$\hat{s}^2(m_Z) = \hat{s}^2(0) - [\hat{s}^2(0) - \hat{s}^2(m_Z)]$$

where $\hat{s}^2(\mu) = \sin^2\hat{\theta}_W(\mu)$. The shift $\hat{s}^2(m_Z) - \hat{s}^2(0)$ is computed by relating to the two-loop running of $\hat{\alpha}$.

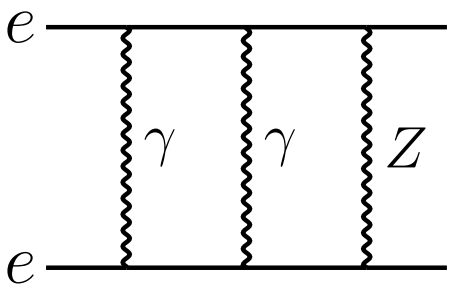
- Uncertainties from data, flavor separation, and pQCD combined gives $\sim 2 \times 10^{-5}$ for $\hat{s}^2(0)$ or equivalently, $\sim 8 \times 10^{-5}$ for Q^e_W.
- Effects from finite-Q^2 at two-loop is $\sim 10^{-6}$.
- No bosonic NNLO was included. This is also true for the input for $\delta Q^e_{W(2,0)}$, but the uncertainty from this is negligible.

Bosonic NNLO

2610.12287 focuses on two classes of contributions (rep. diagrams below):



- Not suppressed by $1 - 4\sin^2\theta_W$
- Enhanced by $\ln\left(Q^2/m_W^2\right) \sim \mathcal{O}(10)$



- Suppressed by $1 - 4\sin^2\theta_W$
- Enhanced by $\ln^2(Q^2/m_Z^2) \sim \mathcal{O}(100)$
- Only retain the log squared term, extracted from the HH region of MoR

Fully analytic results (anc file `Moller_BB.res` on arXiv) thanks to MoR.

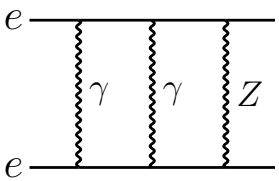
Comparison with Literature

- Czarnecki performed a full EW NNLO calculations (arXiv:2610.40247)
- Expansion in s/m_Z^2 and m_e^2/s
- Bosonic NNLO in good agreement except for terms $\propto (1 - 4\sin^2\theta_W)$. As contributions to Q_W^e in units of 10^{-3} $(y = 0.4)$:

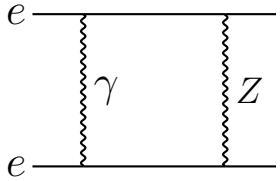
	2610.12287	2610.40247
$(L = \ln Q^2/m_Z^2)^2$	0.174	0.087
remainder ($\propto L$ or constant)	not calculated estimate ~ 0.044	0.062
$(1 - 4\sin^2\theta_W)$ sum	0.174 ± 0.044	0.149
$\delta Q^e_{W(2,0)}$	-0.134	-0.164

Uncertainties from Missing EW Calculations

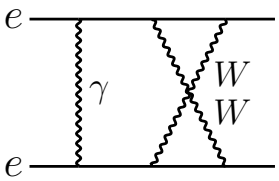
2610.12287's error estimation scheme ($L = \ln\left(Q^2/m_Z^2\right)$):



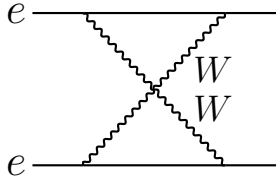
$$\sim -\left(\frac{3}{2}+\frac{1}{3}\right)\frac{\alpha}{\pi}L$$



$$\approx 0.151\times 10^{-3} \qquad [0.174\times 10^{-3}?]$$

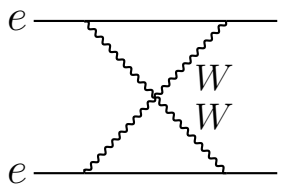


$$\sim -\left(\frac{3}{2}+\frac{1}{3}\right)\frac{\alpha}{\pi}L$$



$$\approx -0.328\times 10^{-3} \qquad [-0.270\times 10^{-3}]$$

two-loop remainder $\sim -6.9\dfrac{\alpha}{\pi}$

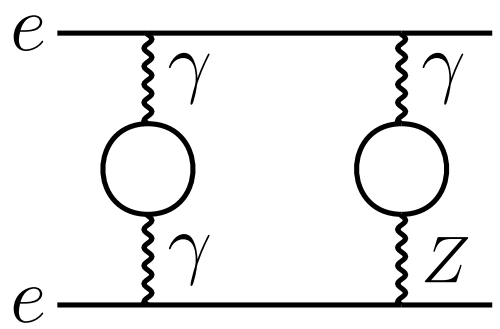


$$\approx -0.092\times 10^{-3} \qquad [-0.038\times 10^{-3}]$$

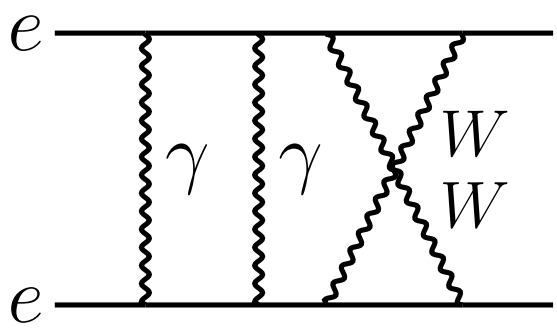
This is a demonstration using two-loop: the numbers following $\approx$ is the estimate; [the numbers in brackets] are actual calculations. In other words, these are not parts of the remaining uncertainty.

Uncertainty from Missing EW Calculations

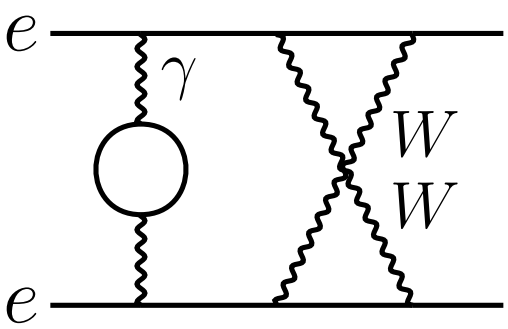
Estimates for the largest remaining contributions (**PRELIMINARY**):



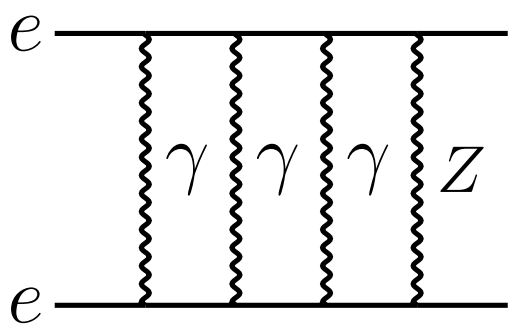
$\approx 1.5\times10^{-5}$



$\approx 2\times10^{-5}$



$\approx 1.5\times10^{-4}$



$\approx 10^{-5}$

Adding all uncertainties in quadrature gives $\sim 1.9 \times 10^{-4}$.
This is probably conservative; refinement coming soon.

Scheme Dependence

In different schemes, $10^3\,Q_W^e$ are ($\delta\,Q_{W,\text{exp}}^e \approx 1.1$):

	$\sin^2\theta_W(0)$	$\sin^2\theta_W(m_Z)$	
	2202.11976 & 2610.12287	2610.40247 (data | lattice)	
(0,0)	45.560	75.12	
(1,1)	0.341	−33.34	− 33.64
(1,0)	0.774	3.01	
(2,2)	0.070	−0.26	− 0.27
(2,1)	−1.146	1.39	1.39
(2,0)	−0.134	−0.45	
Remainder	$0.029_{Q^2\neq 0} \pm 0.19$	$(-0.08 + 0.02)_{\text{QCD}} \pm 0.04$ ± 0.07	± 0.02
Total	45.494 ± 0.19	45.46 ± 0.08	45.17 ± 0.04

Note that the input values are slightly different.

Summary and Outlook

- Theoretical uncertainty for MOLLER is under control.
- Work on-going to resolve the 1/2 discrepancy in the $(1 - 4\sin^2\theta_W)L^2$ terms between 2610.12287 and 2610.40247.

Complementary Works/Potential Improvement:

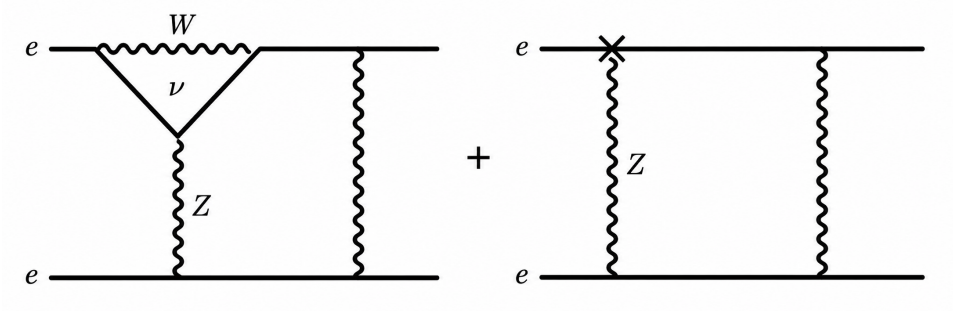
- Data-driven approach for fermion loops coming soon. Lattice QCD?
- Refinement on uncertainty from missing calculations also coming soon.
- Full EFT approach to rigorously resum all logs (McMule).

γ_5 and Dimensional Regularization

γ_5 is ill-defined in D-dimension. We are forced to give up one of

- Anticommutativity $\{\gamma_5, \gamma_\mu\} = 0$
- Trace Identity (cyclicity)
- Gauge Invariance

In our case, some bubble and vertex subloop in a box diagrams requires care:

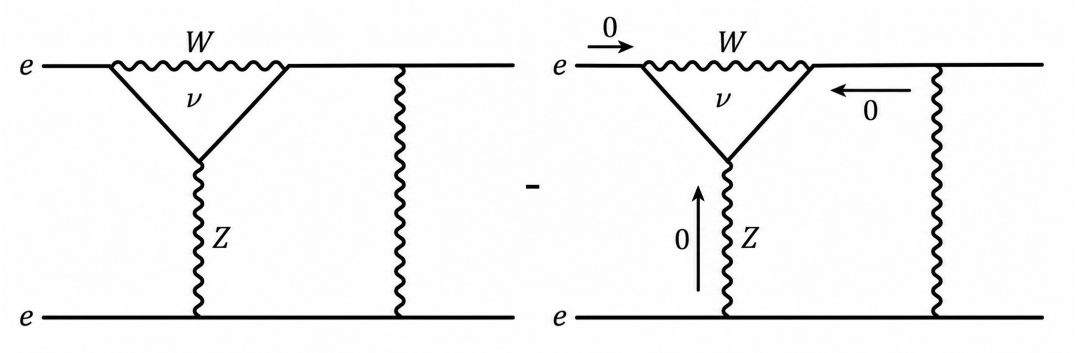


Typical approaches include

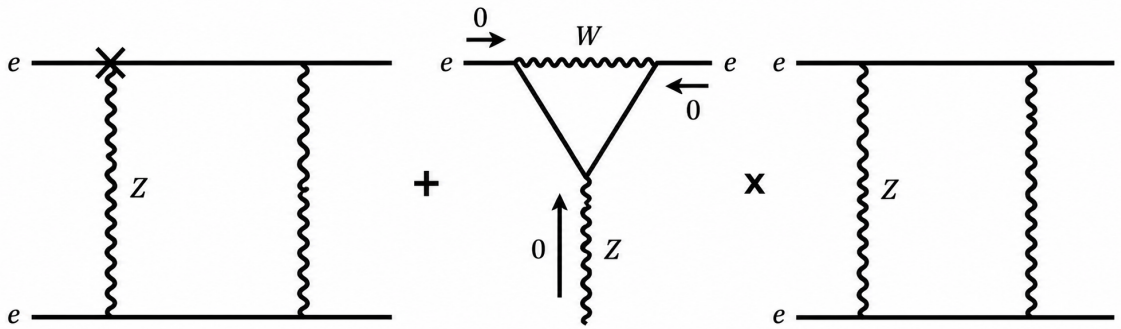
- NDR — Chanowitz, Furman, and Hinchliffe Nuc. Phys. B '79
- BMHV — Breitenlohner & Maison '77, 't Hooft & Veltman '72

γ_5 with "Momentum Subtraction"

We want to calculate the first column. To do this, we first calculate in 4D



which is now finite thanks to the subtraction term. Then calculate again in 4D



which is also finite. The sum of the two lines gives the desired result since the second column adds to zero. Note that we have not used dim. reg.
This also works for diagrams with IR divergence that is regulated by m_γ.