Projections for the Running Weak Mixing Angle

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Precision Physics with MOLLER Workshop
Jefferson Lab
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HIM HELMHOLTZ Helmholtz-Institut Mainz



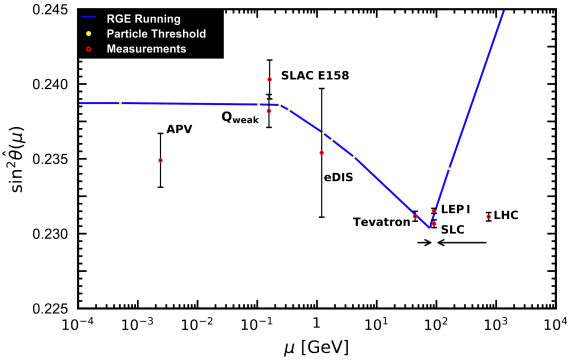
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Running of the weak mixing angle

$$\sin^2\theta_W(M_Z) \xrightarrow{\text{Standard Model evolution}} \sin^2\theta_W(0)$$

- How to perform the running from the Z pole to low energies?
- What are the main contributions to its uncertainty?
- How to handle the non-perturbative running?

The weak mixing angle $\sin^2\theta_W$



[PDG]

- Beyond tree level, the value depends on the definition used.
- On-shell definition:

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

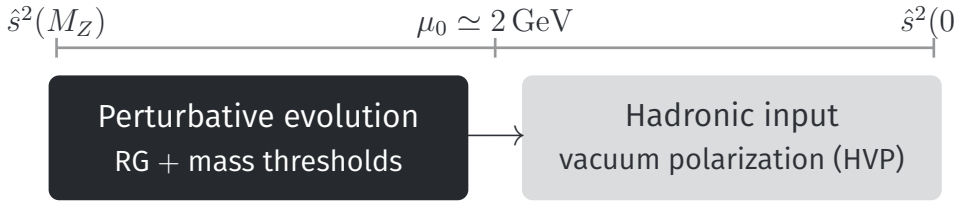
- Running parameter in $\overline{\text{MS}}$:

$$\hat{s}^2(\mu) \equiv \sin^2\hat{\theta}_W(\mu) = \frac{\hat{g}'^2(\mu)}{\hat{g}^2(\mu) + \hat{g}'^2(\mu)}$$

- We want to evolve $\hat{s}^2(\mu)$ from the Z pole to low energies.

Running from the Z pole to low energies

$$\hat{s}^2(0) \equiv \hat{\kappa}(0)\,\hat{s}^2(M_Z)$$



- Hadronic vacuum polarization (HVP) contributes to photon propagation, $\Pi^{(\gamma,\gamma)}$, and photon–Z mixing, $\Pi^{(Z,\gamma)}$.
- Weak-angle running needs separate ud, s, and disconnected contributions because the two currents weight flavors differently.

[Erler, Ramsey-Musolf, hep-ph/0409169] [Erler, Ferro-Hernández, 1712.09146]

HVP IN THREE PRECISION OBSERVABLES

Subtracted hadronic vacuum polarization:

$$\bar{\Pi}^{(A,B)}(-Q^2) \equiv \Pi^{(A,B)}(-Q^2) - \Pi^{(A,B)}(0), \qquad A, B = \gamma, Z$$

Electromagnetic current: $\bar{\Pi}^{(\gamma,\gamma)}$

$$\Delta\alpha_{\rm had}(-Q^2) = 4\pi\alpha\,\bar{\Pi}^{(\gamma,\gamma)}(-Q^2)$$

$$a_\mu^{\rm HVP,LO} = \int_0^\infty dQ^2\, w_\mu(Q^2)\bar{\Pi}^{(\gamma,\gamma)}(-Q^2)$$

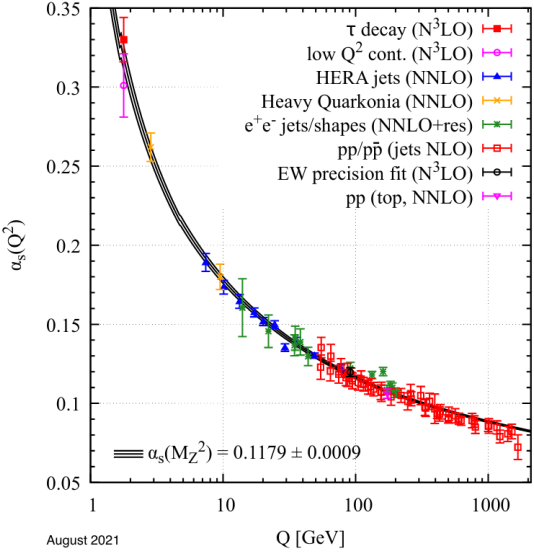
Photon–Z mixing: $\bar{\Pi}^{(Z,\gamma)}$

$$\delta_{\rm had}\sin^2\theta_W(-Q^2) \simeq -4\pi\alpha\,\bar{\Pi}^{(Z,\gamma)}(-Q^2)$$

Additive hadronic change relative to $Q^2 = 0$, at leading order.

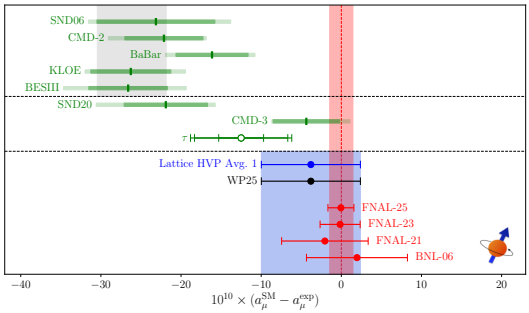
Same flavor-resolved QCD information; different currents and momentum weights.

Non-perturbative information



- QCD is a strongly coupled theory in the hadronic regime at $Q \sim 300\,\mathrm{MeV}$.
- Perturbative expansion fails $\lesssim 1\,\mathrm{GeV}$.
- Two routes to obtain non-perturbative HVP:
  - From experimental data: The R-ratio.
  - Ab-initio theory: Lattice QCD.

Non-perturbative information

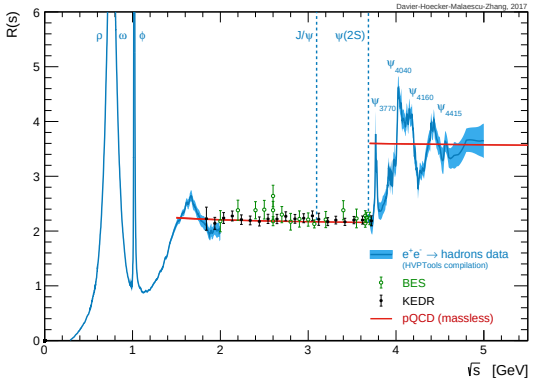


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  - Ab-initio theory: Lattice QCD.
- Disagreement in predicting $a_\mu^{\mathrm{HVP,LO}}$ $\rightarrow$ Standard Model prediction now based on lattice QCD for $a_\mu^{\mathrm{HVP,LO}}$.

[Muon $g-2$ Theory Initiative, 2505.21476]

The R-ratio approach

R-ratio: $\quad R(s) = \dfrac{\sigma^0(e^+e^- \to \text{hadrons}(+\gamma))}{\sigma_{\text{pt}}}\,, \qquad \sigma_{\text{pt}} = \dfrac{4\pi\alpha^2}{3s}$



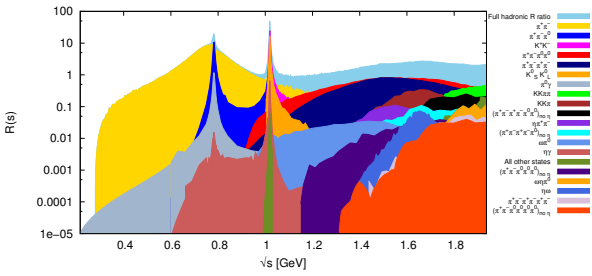
- Data-driven evaluation from

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi}\,\mathcal{P}\int_{4m_\pi^2}^{\infty} ds\,\frac{R(s)}{s(M_Z^2 - s)}$$

[Davier et al., 1706.09436] [Keshavarzi et al., 1802.06229]

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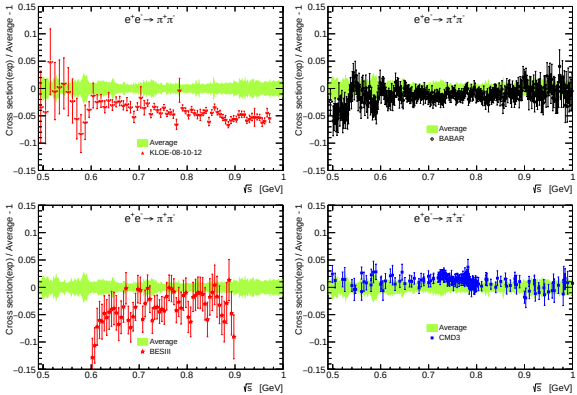
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- At low energies, $R(s)$ assembled from exclusive channels.
- For $\sin^2\theta_W$, a model-dependent flavor separation is necessary.

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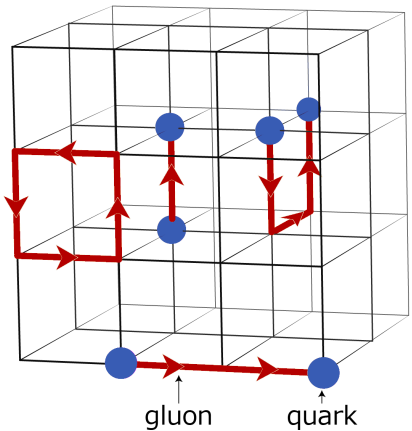
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- At low energies, $R(s)$ assembled from exclusive channels.
- For $\sin^2\theta_W$, a model-dependent flavor separation is necessary.
- Several σ discrepancies in the dominant $e^+e^- \to \pi^+\pi^-$ channel cannot be explained.

See also BaBar 2026 [BABAR, 2611.04596]

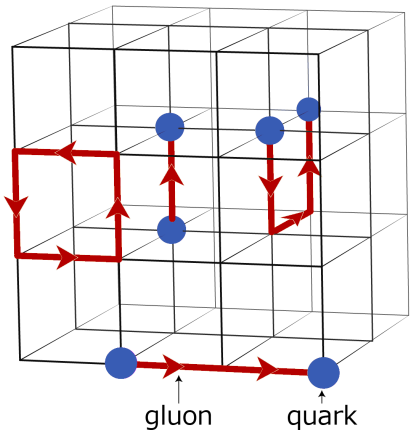
[Malaescu]

Lattice QCD



- Ab-initio approach to obtain the vacuum polarization functions: lattice QCD

Lattice QCD



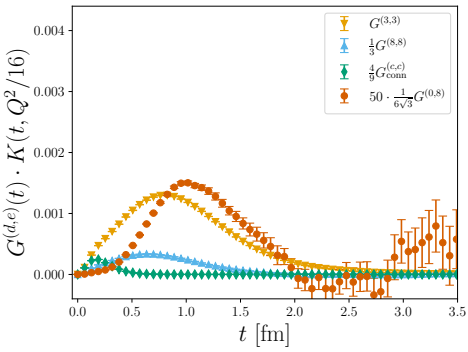
- Ab-initio approach to obtain the vacuum polarization functions: lattice QCD
- Formulate the theory
  - on a finite grid $\rightarrow$ regulator Λ_{UV}.
  - in finite volume $\rightarrow \Lambda_{\mathrm{IR}}$.
  - in Euclidean space-time
  - as a Boltzmann distribution
- Compute expectation values $\langle \mathrm{O} \rangle$ by sampling the QCD path integral with Markov Chain Monte Carlo methods.

The QCD Lagrange density

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=1}^{N_f} \bar{\psi}_f (\not{D} + m_f)\psi_f + \tfrac{1}{4}\mathrm{tr} F_{\mu\nu}F^{\mu\nu}$$

- Contains $N_f + 1$ **bare** parameters (gauge coupling and N_f quark masses)
- Renormalize the theory from hadronic input, e.g., m_Ω, m_π, m_K, m_{D_s}, m_{B_s}.
  $\rightarrow$ All other observables are **predictions**.
- Freedom of choice on how to discretize $\mathcal{L}_{\mathrm{QCD}}$: Wilson, twisted mass, staggered, domain wall, overlap, ...
- *Ab-initio* predictions for QCD after lifting the cutoffs:
  - Λ_{IR}: Infinite-volume limit.
  - Λ_{UV}: Continuum limit.

Vacuum polarization from lattice QCD



Obtain vacuum polarization function from

$$\bar{\Pi}^{(\alpha,\beta)}(-Q^2) = \int_0^\infty dt\, G^{(\alpha,\beta)}(t)\, \hat{K}(t, Q^2).$$

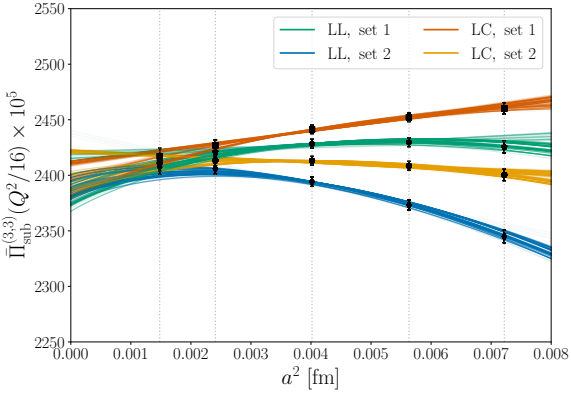
$G^{(\alpha,\beta)}(t)$: Euclidean current-current correlation function.

$\hat{K}(t, Q^2)$: Known analytic kernel function.

- Flavor separation straightforward on the lattice.
- Signal-to-noise problem in some channels.

[Bernecker and Meyer, 1107.4388] [Conigli , SK et al., 2511.01623]

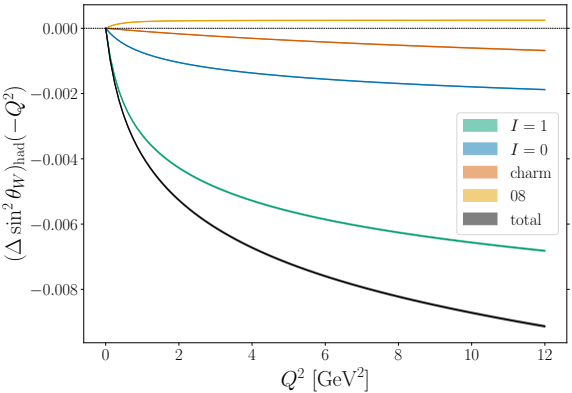
Vacuum polarization from lattice QCD



- Lift the cutoffs: **Continuum limit** and correction for finite-volume effects.

[Conigli, SK et al., 2607.03370] [Conigli, SK et al., 2511.01623]

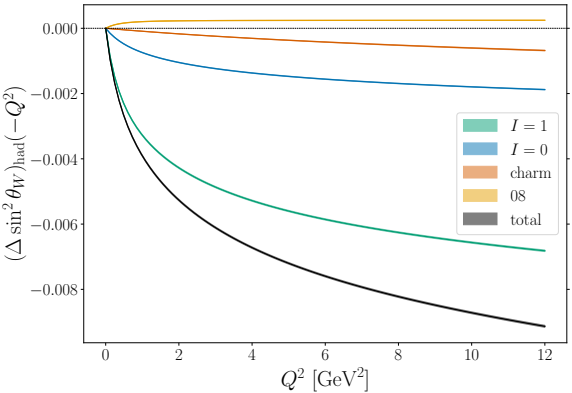
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- Lift the cutoffs: **Continuum limit** and correction for finite-volume effects.
- Obtain spacelike running of isovector, isoscalar, charm and singlet-octet (08) contributions.

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Vacuum polarization from lattice QCD

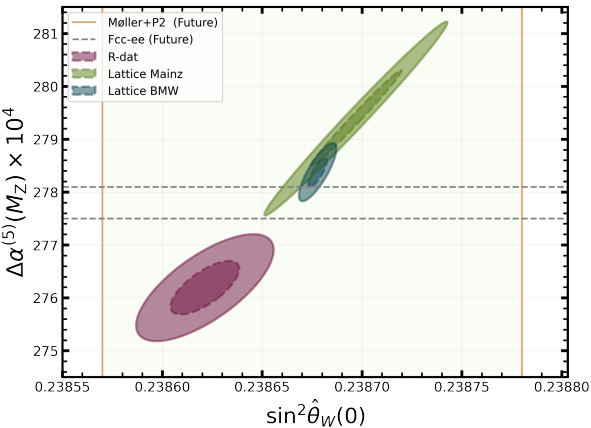


- Lift the cutoffs: **Continuum limit** and correction for finite-volume effects.
- Obtain spacelike running of isovector, isoscalar, charm and singlet-octet (08) contributions.

- Lattice QCD supplies the flavor-resolved HVP at a matching scale.
- Conversion to $\overline{\rm MS}$, followed by RG evolution and threshold matching, gives $\hat{\kappa}(0)$.

[Conigli, SK et al., 2607.03370] [Conigli, SK et al., 2511.01623]

Theory-driven evolution of the weak mixing angle



- In [Erler, SK et al., 2406.16691]: Combination of perturbative running with
  - Experimental R-ratio [Davier et al., 1908.00921]
  - Lattice QCD from Mainz [Cè et al., 2203.08676]
  - Lattice QCD from BMW [Borsanyi et al., 1711.04980]
- Well-known discrepancy from a_μ^{HVP} reappears.

Uncertainties from running well below MOLLER precision!

How much has the running uncertainty improved?

- Results could be updated using the new lattice QCD evaluation of $\bar{\Pi}$ in [Conigli, SK et al., 2607.03370] [Conigli, SK et al., 2511.01623].

Contribution to $\delta\hat{s}^2(0)$	Mainz 2022	possible update
	in units of 10^{-5}, at fixed $\hat{s}^2(M_Z)$	
Hadronic lattice QCD input	2.3	≈ 0.9
$\alpha_s(M_Z)$	0.5	0.4
Condensates	0.2	0.2
Quark masses and perturbative remainder	0.2	0.2
Combined running uncertainty	≈ 2.4	≈ 1.0

- The strong coupling constant α_s is the next-largest input in this budget – the most precise evaluation being from lattice QCD [Dalla Brida et al., 2501.06633].

Implications for MOLLER

Indicative uncertainties expressed as precision on $\hat{s}(0)$:

$$\underbrace{\sim 1 \times 10^{-5}}_{\text{running}} \ll \underbrace{\sim 13 \times 10^{-5}}_{\text{current direct } Z\text{-pole input}} < \underbrace{\sim 30 \times 10^{-5}}_{\text{MOLLER goal}}$$

- Precision of the **running** is already much better than needed for MOLLER and P2.
- Lattice QCD provides precise information for the non-perturbative running including flavor separation.
- Data-driven evaluations currently not consistent due to discrepancies in experimental results for $e^+e^- \to \pi^+\pi^-$.
- Improvements of the lattice QCD analysis and several independent results are foreseeable given the precision targets for a_μ^{HVP} and $\alpha(M_Z)$ (in the FCC era).