Phenomenology of γZ interference structure functions

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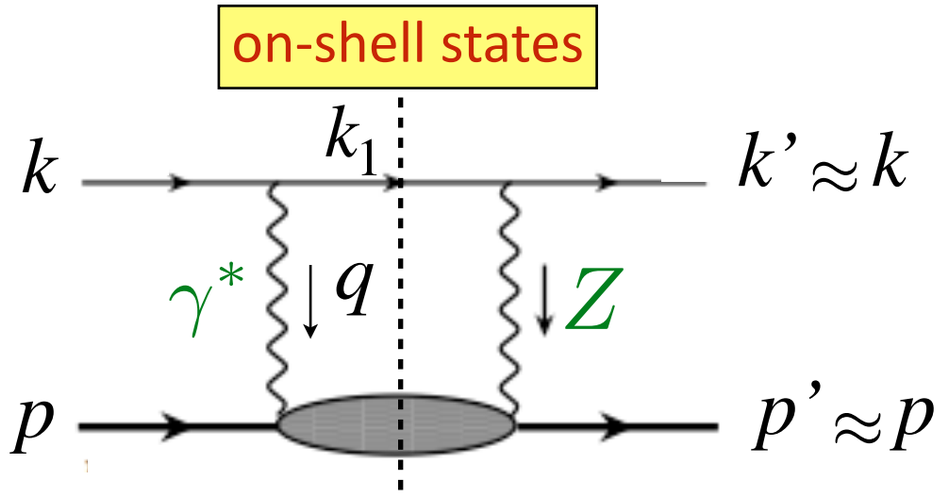
Workshop on "Precision Physics with MOLLER"
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†**Collaborators:** Kyle Shiells (UM), Wally Melnitchouk (JLab), Sienna English (UM), Nathan Hall, Anthony Thomas and Ross Young (Adelaide)

“Modern” approach to box diagrams

- PVES at low Q^2 and large E (Qweak)
- GH formulated box corrections in terms of forward angle dispersion relations using γZ structure functions
- Data-driven theoretical analysis in terms of structure functions

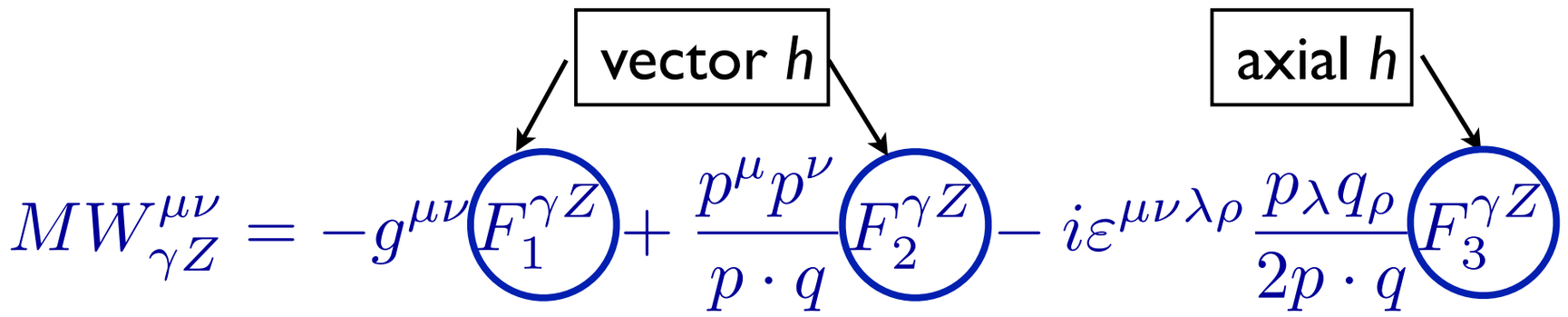
*Gorchtein, Horowitz, PRL **102** (2009) 091806*



Forward scattering limit: $|f\rangle \approx |i\rangle$

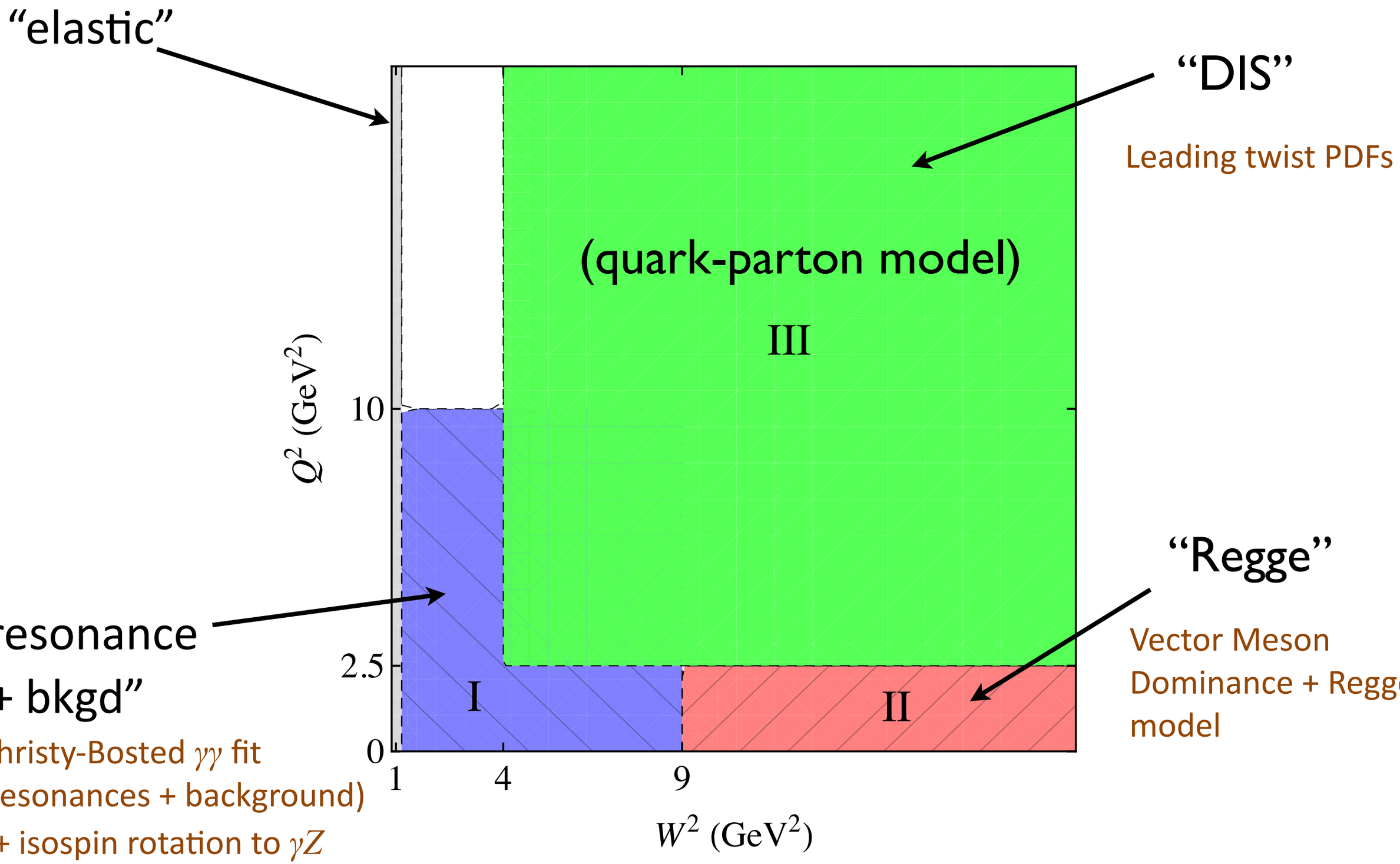
$$\mathcal{I}m\langle i|\mathcal{M}^{\gamma Z}|i\rangle \sim \int d^3\,k_1\ \frac{L_{\mu\nu}W^{\mu\nu}}{q^2(q^2-M_Z^2)}$$

hadronic tensor:



Structure function map: 3 distinct regions

Need structure functions for $A_e \times V_h$ ($F_{1,2}^{\gamma Z}$) and $V_e \times A_h$ ($F_3^{\gamma Z}$)



Vector h correction: $A_e \times V_h$

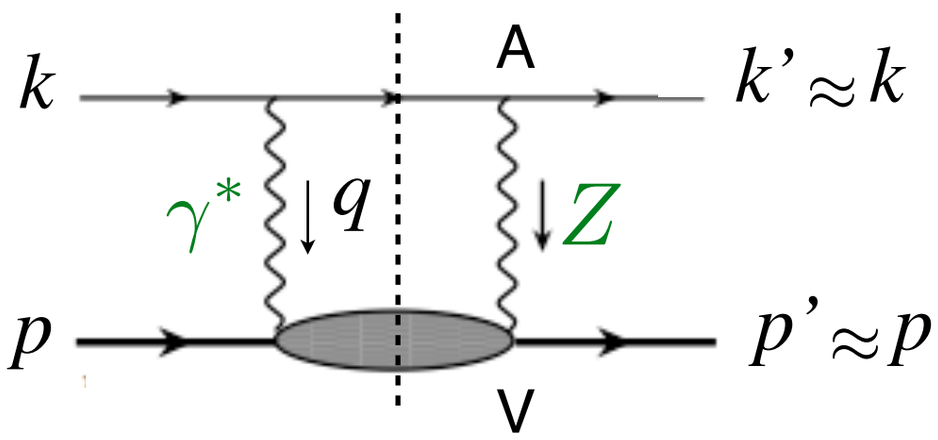
- Vector h correction $\Box^V_{\gamma Z}$ vanishes at $E = 0$ (atomic PV limit), but PVES is at finite energy (*e.g.* Qweak $E \sim 1$ GeV)

⟶ Use forward limit dispersion relation

$$\Re e\, \Box^V_{\gamma Z}(E) = \frac{2E}{\pi} \int_0^\infty dE' \, \frac{1}{E'^2 - E^2} \, \Im m\, \Box^V_{\gamma Z}(E')$$

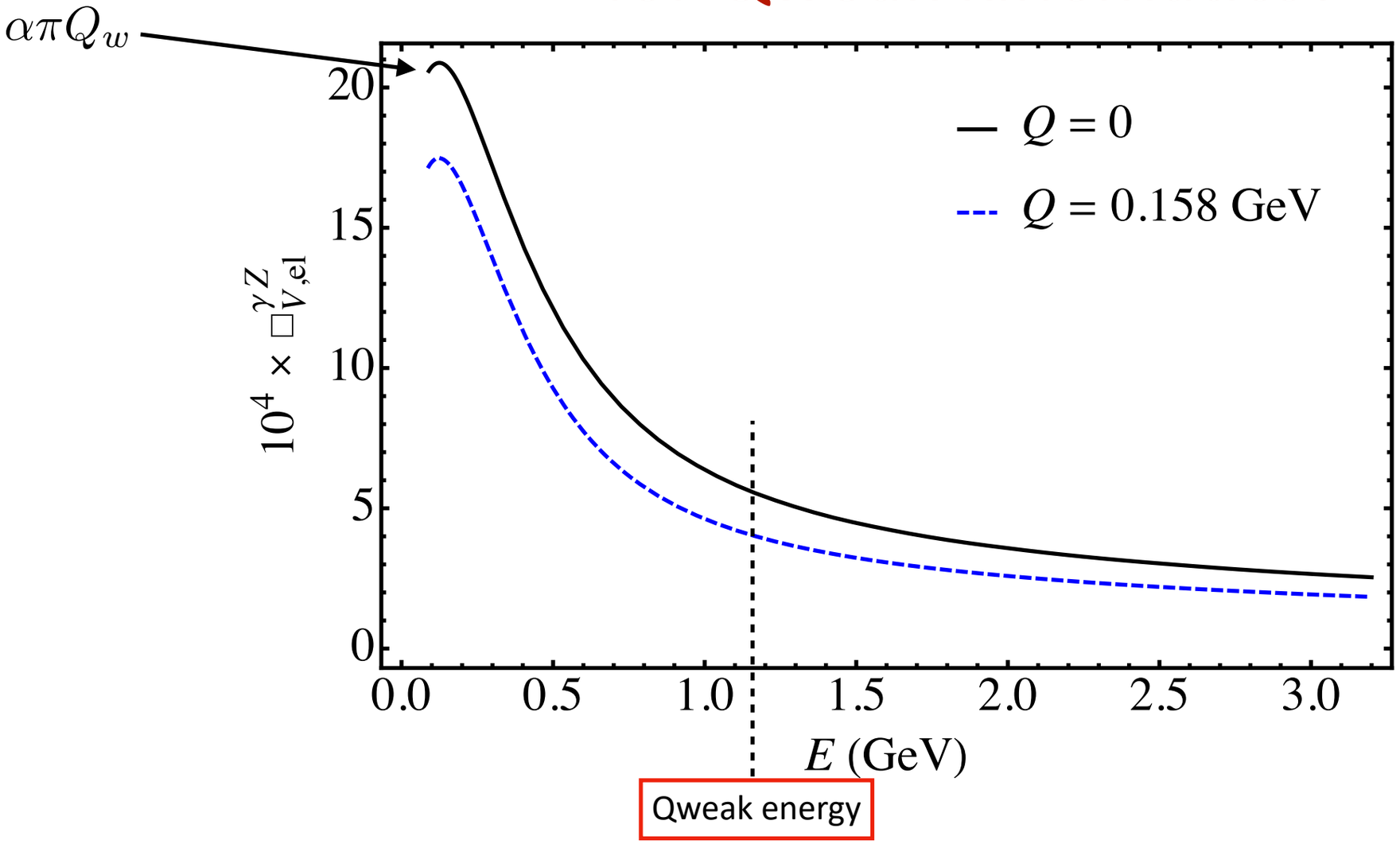
⟶ imaginary part given by

$$\Im m\, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\rm max}} \frac{dQ^2}{1 + Q^2/M_Z^2}$$



$$\times \left(F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\,(Q^2_{\rm max} - Q^2)}{Q^2 (W^2 - M^2 + Q^2)} \right)$$

How good is the forward angle approximation for Qweak kinematics?



- Ratio is

  0.825 ± 0.005

- Independent of E

- Compare forward angle ($Q = 0$) γZ box for **elastic** (nucleon) intermediate state with exact result at Qweak kinematics ($Q^2 = 0.026\ \mathrm{GeV}^2$)
- Note that for the elastic part $\Box^{\gamma Z}_{V,\mathrm{el}}(0) \neq 0$
- Discarded in atomic PV because it corresponds to Coulomb correction to Z exchange (already accounted for in electron bound state wavefunction)

Main issues for $\Box^V_{\gamma Z}$

- Relatively little known about $F_i^{\gamma Z}$ interference structure functions below HERA measurements, with $Q^2 \geq 1500\ \mathrm{GeV}^2$

- Fit $F_i^{\gamma\gamma}$ over all kinematics in Q^2 and W, then "rotate" to $F_i^{\gamma Z}$ using available theoretical/phenomenological constraints

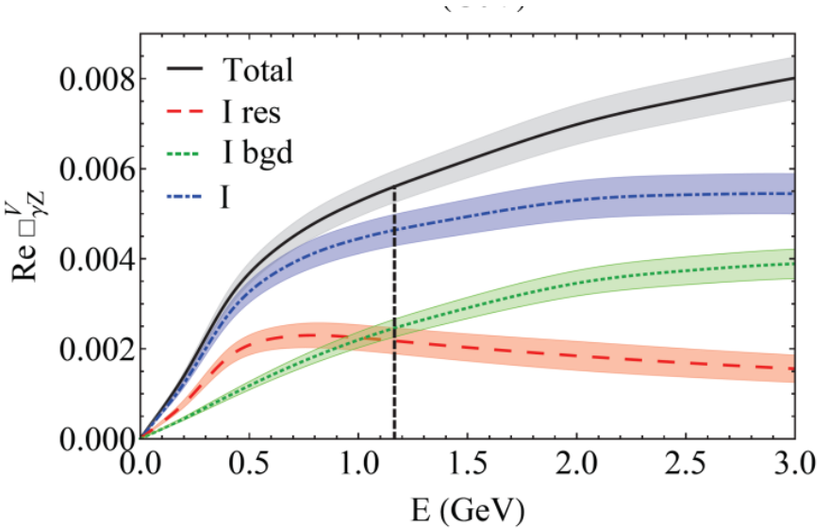
  ⟶ *e.g.* isospin symmetry

$$\langle N^*|J_Z^\mu|p\rangle = (1 - 4\sin^2\theta_W)\langle N^*|J_\gamma^\mu|p\rangle - \langle N^*|J_\gamma^\mu|n\rangle$$

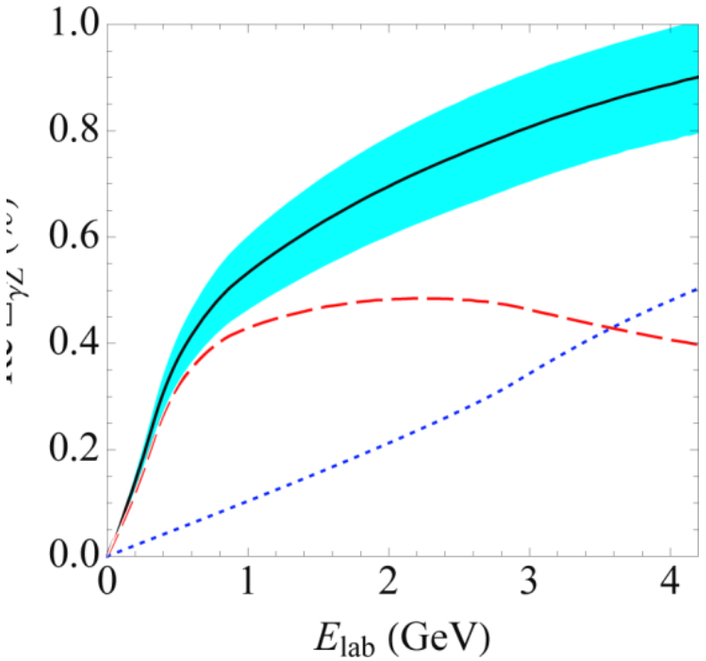
- AJM model (approach): require structure functions to match across different kinematic regions (constrains free parameters)

Three groups doing independent analyses

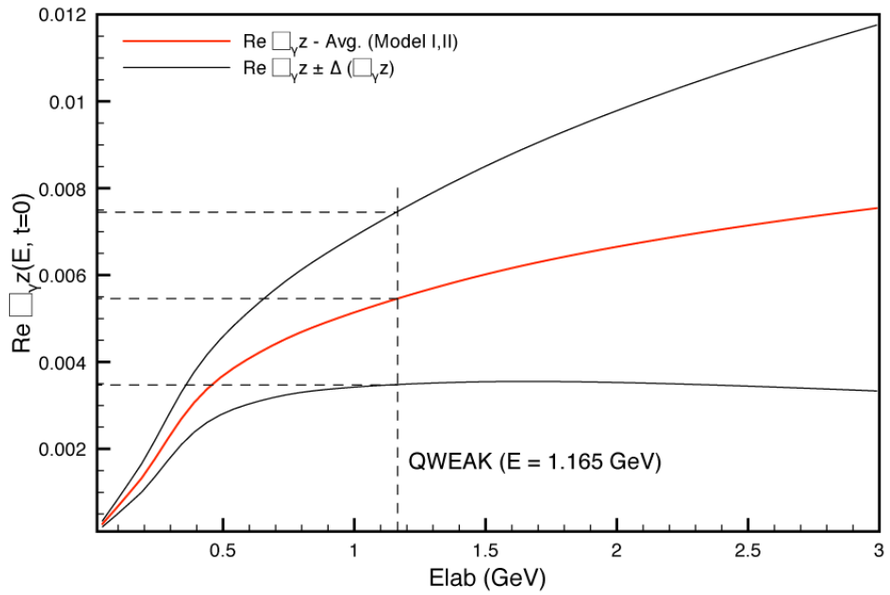
Hall *et al.*
PRD 88, 013011 (2013)



Carlson and Rislow
PRD 83, 113007 (2011)



Gorchtein *et al.*
PRC 84, 015502 (2011)



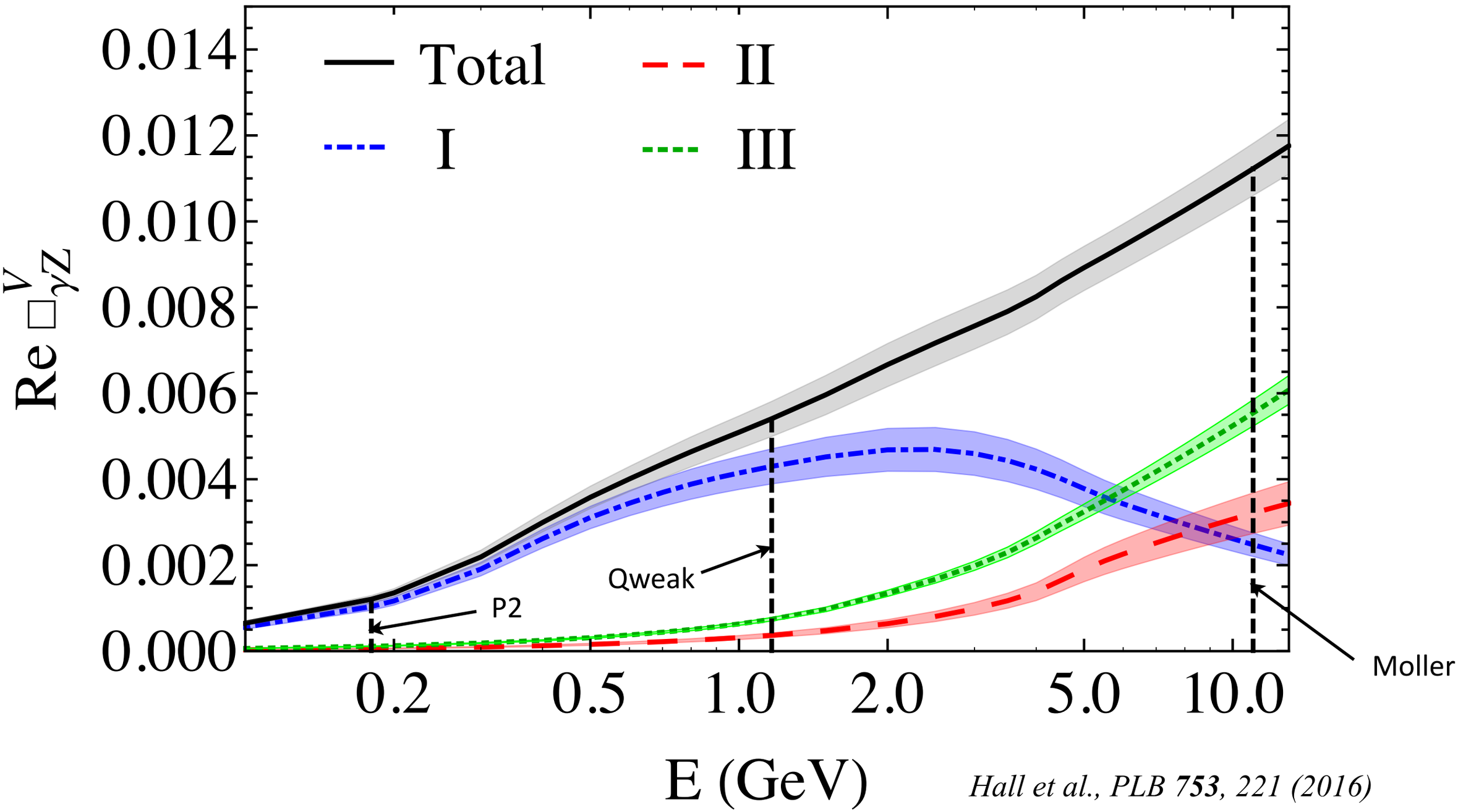
Qweak energy: $\mathrm{Re}\Box^V_{\gamma Z}(E = 1.165\,\mathrm{GeV})$ **8% correction!**

$(5.6 \pm 0.36) \times 10^{-3}$ $(5.7 \pm 0.9) \times 10^{-3}$ $(5.4 \pm 2.0) \times 10^{-3}$

- Mainly different treatments of low Q^2, low W region background contributions
- Agree on overall magnitude of 8% correction, but disagree on errors and details

Contribution from different regions to $\Box^V_{\gamma Z}$

(relative to weak charge of 0.071)

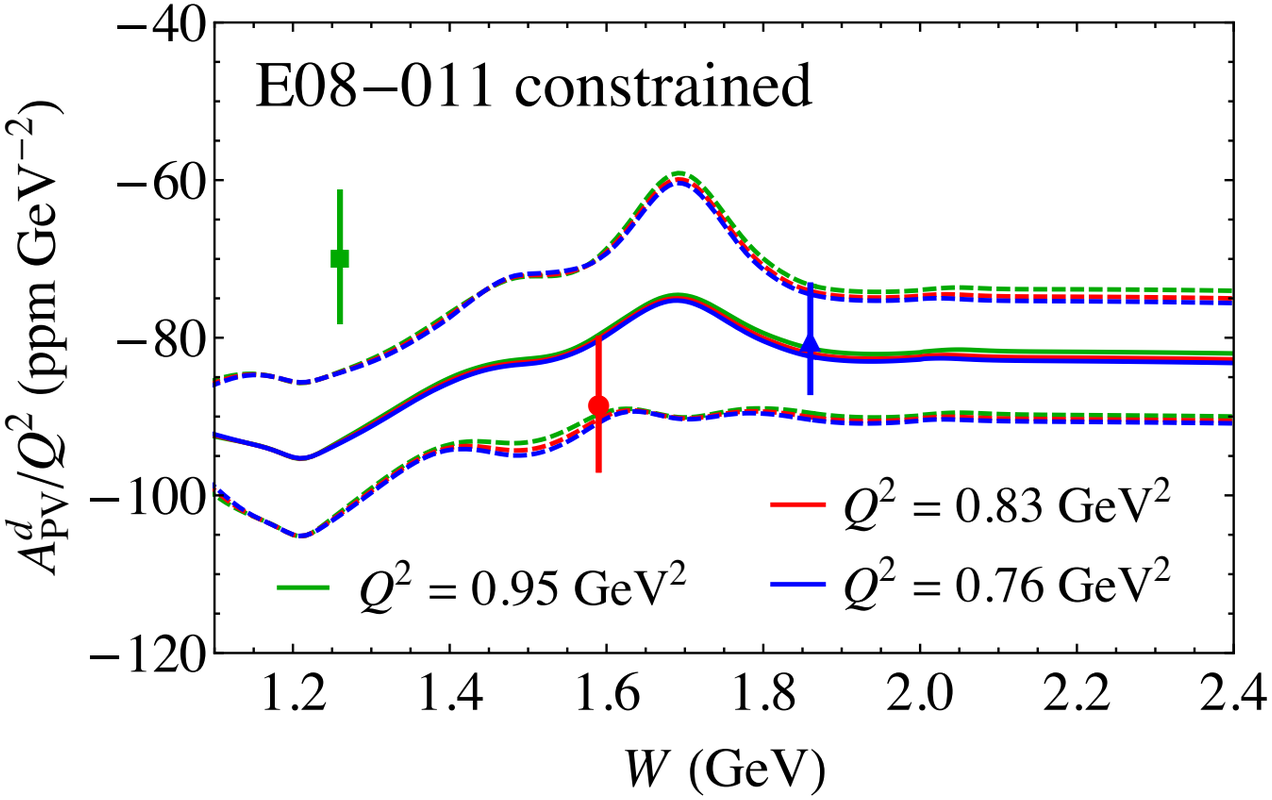


*Hall et al., PLB **753**, 221 (2016)*

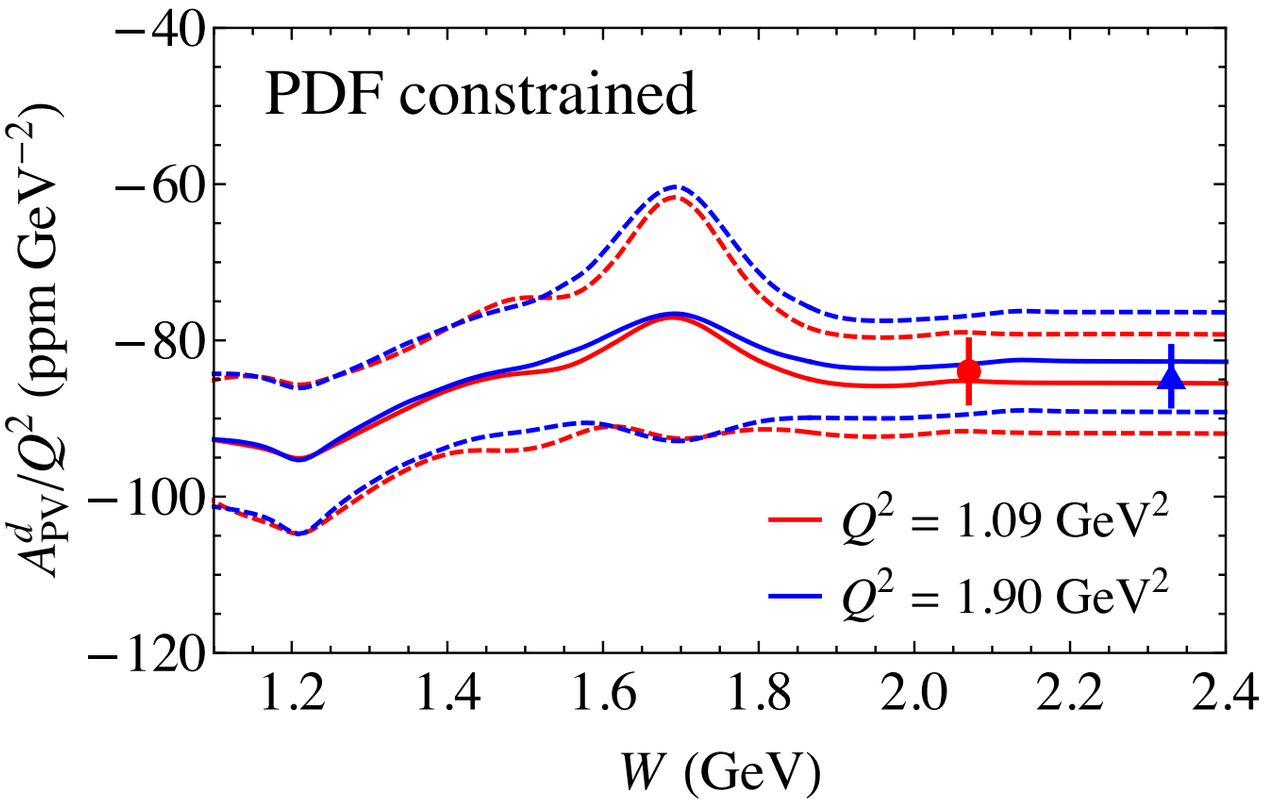
Constraints from PV inelastic asymmetries

Parity-violating Deep Inelastic Scattering (PVDIS) asymmetry allows a direct measurement of the γZ structure functions

$$A_{\rm PV} = g_A^e \left(\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\right) \frac{xy^2 F_1^{\gamma Z} + (1-y)F_2^{\gamma Z} + \frac{g_V^e}{g_A^e}(y - y^2/2)xF_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y)F_2^{\gamma\gamma}}$$

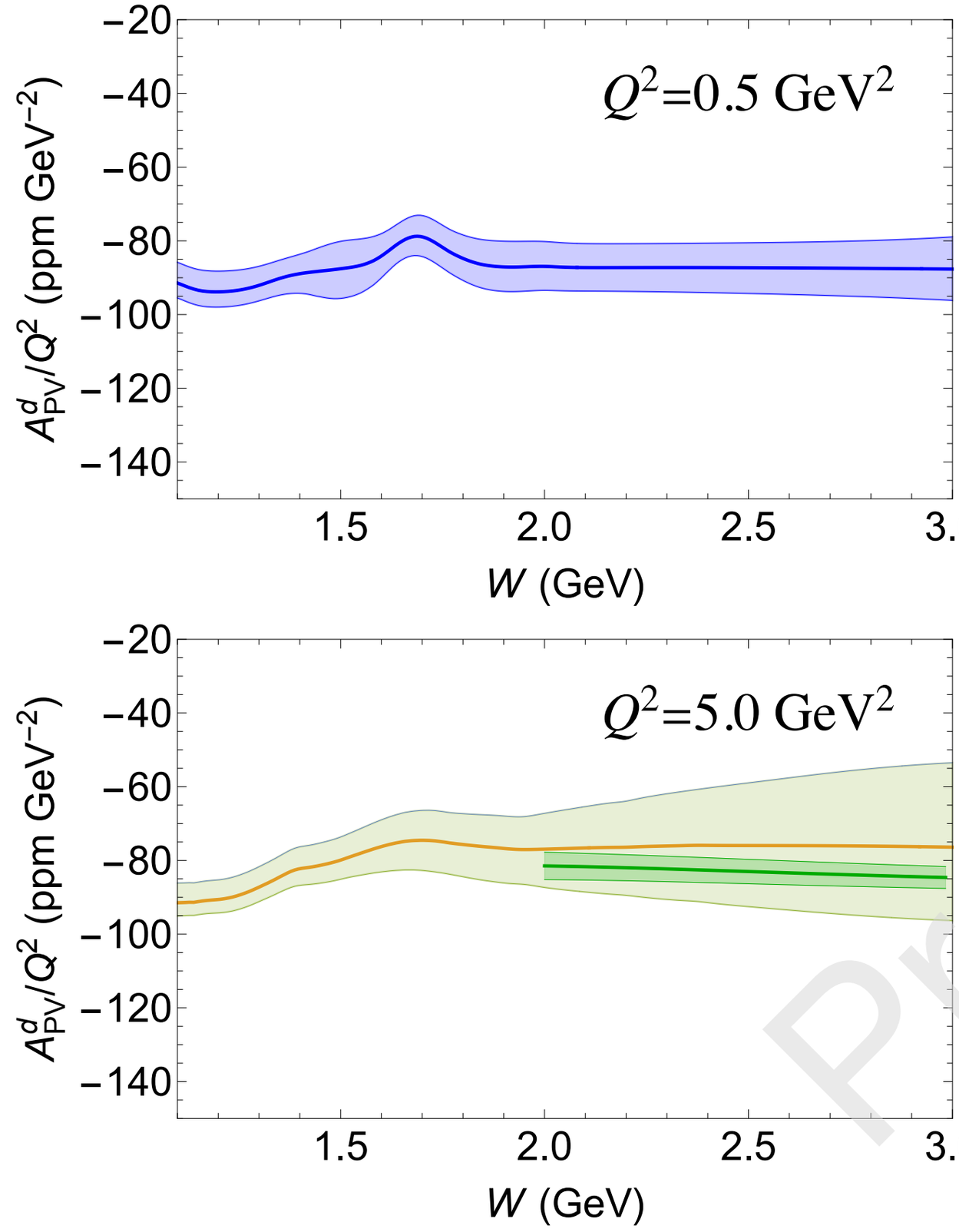


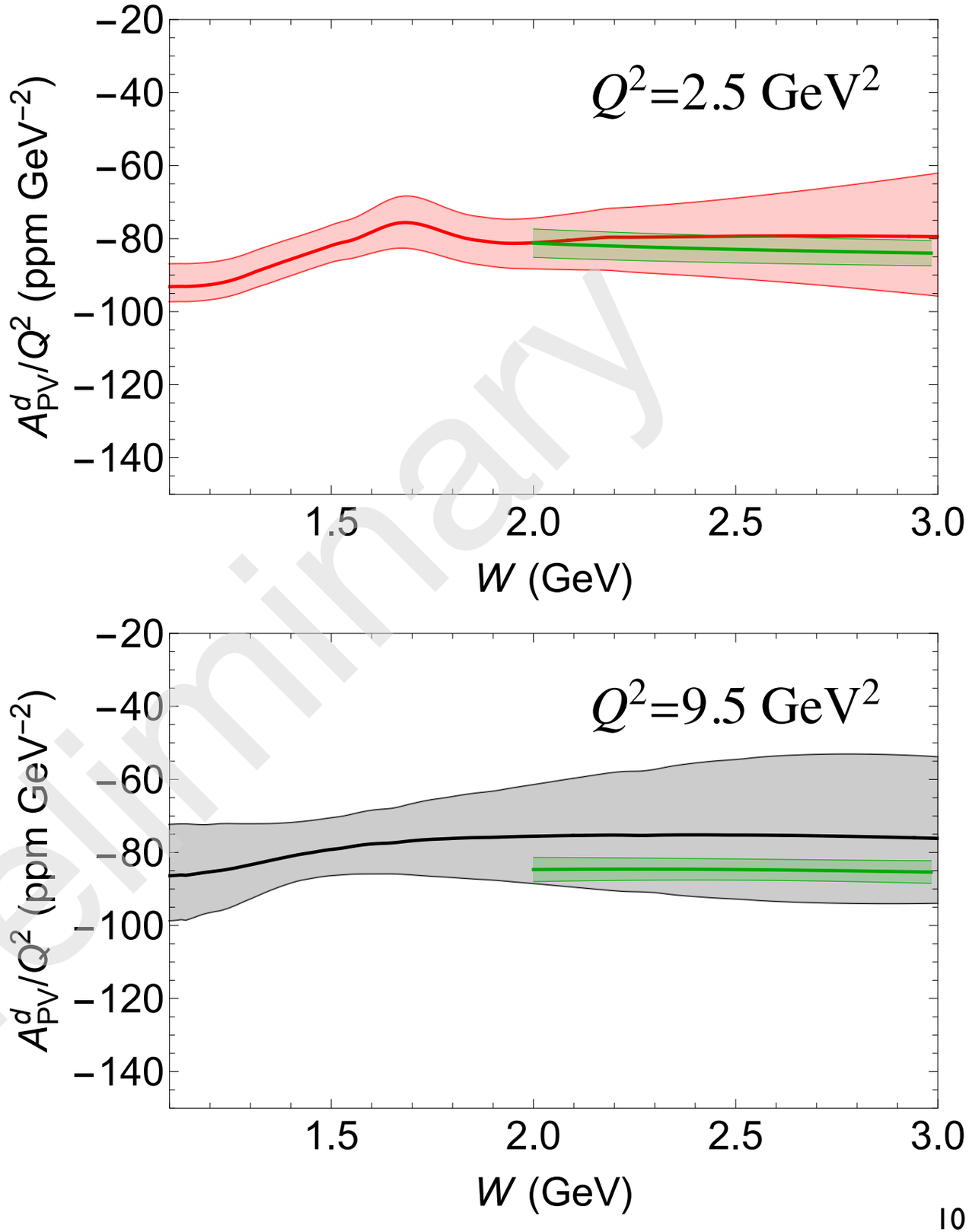
*Wang et al. PRL **111**, 082501 (2013)*



E08-011: *Wang et al. Nature **506**, 67 (2014)*

AJM model A_{PV}(d) for selected SOLID kinematics (E=11 GeV)





Preliminary

Improvements underway (also relevant for γW box)

- Use most recent PDF datasets (*e.g.* JAM24)
- All three groups use Christy-Bosted (CB) parametrization of resonance region (pre 2010).
  - This is a parametrization of *N** states to fit **inclusive** cross section data
  - It is **not** based on transition form factors and branching ratios of produced resonant states, for which exclusive meson production is suited.
    - Suggests that other more recent parametrizations should be explored (*e.g.* CLAS)
- Incorporate recent improvements to $F_3(W^2, Q^2)$ structure function using neutrino data for Regge region
- Lattice QCD calculations

Axial h correction: $V_e \times A_h$

- Relevant for PVES, atomic parity-violation, and β decay (via γW boxes) at low energy

$$\Box_A^{\gamma Z}(E=0) \sim g_V^e \int_0^\infty dQ^2 \, \frac{\alpha(Q^2)}{Q^2(1+Q^2/M_Z^2)} \int_0^{x_{\rm max}} dx \; F_{3p}^{\gamma Z}(x,Q^2) \, \frac{1+2r}{(1+r)^2},$$

$$x = \frac{Q^2}{W^2 - M^2 + Q^2}, \qquad r = \sqrt{1 + 4M^2x^2/Q^2}.$$ Blunden *et al.* PRL **107**, 181861 (2011)

In DIS region integrand can be expressed in terms of moments of structure functions by letting $x_{\rm max} \to 1$.

$$M_3^{\gamma Z(n)} = \int_0^1 dx \, x^{n-1} F_3^{\gamma Z}(x,Q^2)$$

Allows for systematic small corrections to a model-independent leading term

$\underline{n=1}$ $\qquad M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$

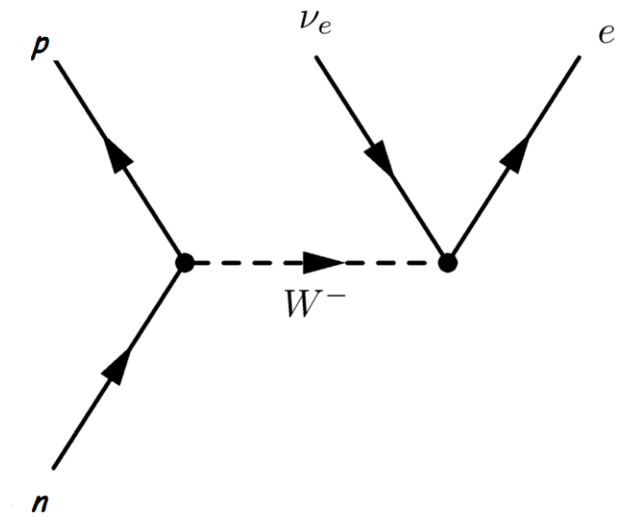
$\longrightarrow$ γZ analog of Gross-Llewellyn Smith sum rule for F_3^W

$\longrightarrow$ connection with 1980's result of Marciano & Sirlin

Huge increase in theoretical work due to application to …

Superallowed 0^+ to 0^+ β decays

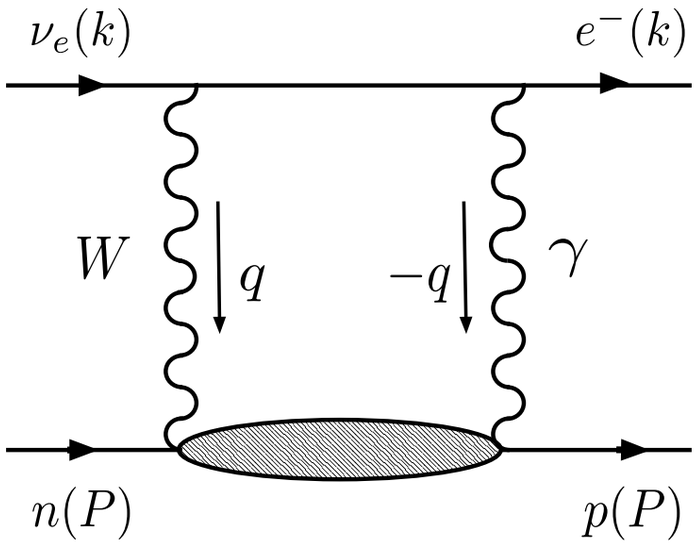
$$|V_{ud}|^2 = \frac{0.97148(20)}{1+\Delta_R^V},$$



CKM unitarity requires $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

Inner radiative correction term (nucleus-independent)

$$\Delta_R^V = \frac{\alpha}{2\pi}\left[3\ln\frac{M_Z}{M} - \ln\frac{M_W}{M_Z}\right] + 2\,\Box_A^{\gamma W},$$



- γW box is largest source of hadronic uncertainty
- Dispersive approach turned out to be a much harder problem than γZ

Seng *et al.* (2018, 2019) found the correct dispersion representation of $\Box_A^{\gamma W}$ using isospin symmetry. Seng, Gorchtein, & Ramsey-Musolf, PRD **100**, 013001 (2019)

Analogous to the contribution to the weak axial charge. At zero energy

$$\Box_A^{\gamma W} = \frac{\alpha}{2\pi}\int_0^\infty dQ^2\,\frac{1}{Q^2(1+Q^2/M_W^2)}\int_0^{x_{\rm max}} dx\;F_3^{(0)}(x,Q^2)\,\frac{1+2r}{(1+r)^2}.$$

In general: $F_3^{(0)} = F_{3p}^{\gamma Z} - F_{3n}^{\gamma Z};\quad F_3^W \approx F_{3p}^{\gamma Z} + F_{3n}^{\gamma Z}$

In DIS region: $F_3^{(0)} = (e_u+e_d)\,(u_v-d_v)\,;\quad F_3^W \approx (e_u-e_d)\,(u_v+d_v)$

Sum rules for leading moments (GLS)

$$\int_0^1 dx\,F_{3\,(\rm DIS)}^{(0)}(x,Q^2) = \frac{1}{3}\left(1-\frac{\alpha_s(Q^2)}{\pi}+\mathcal{O}(\alpha_s^2)\right),$$
$$\int_0^1 dx\,F_{3\,(\rm DIS)}^{W}(x,Q^2) = 3\left(1-\frac{\alpha_s(Q^2)}{\pi}+\mathcal{O}(\alpha_s^2)\right).$$

Use sum rules plus data on F_3^W from $\nu p + \bar{\nu} p$

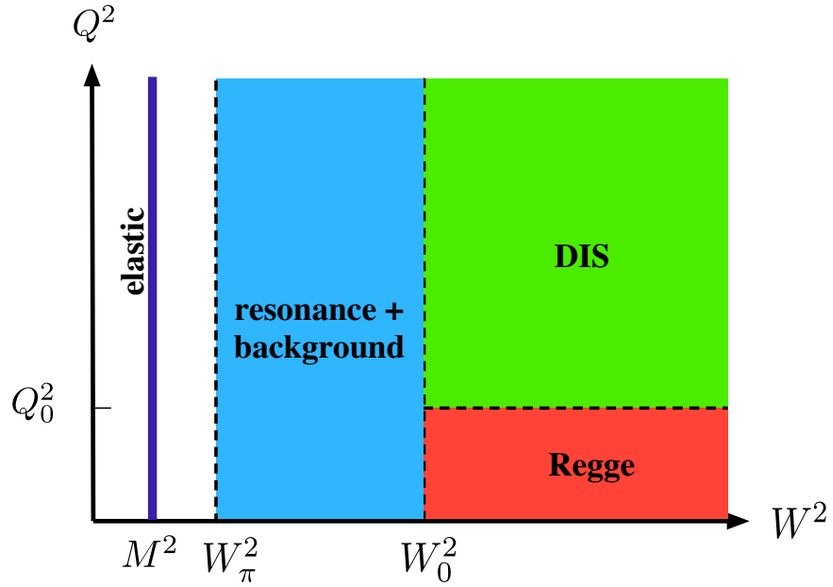
$\longrightarrow$ Found a shift in $\Box_A^{\gamma W}$ from $3.52(11)\times10^{-3}$ to $3.79(10)\times10^{-3}$.

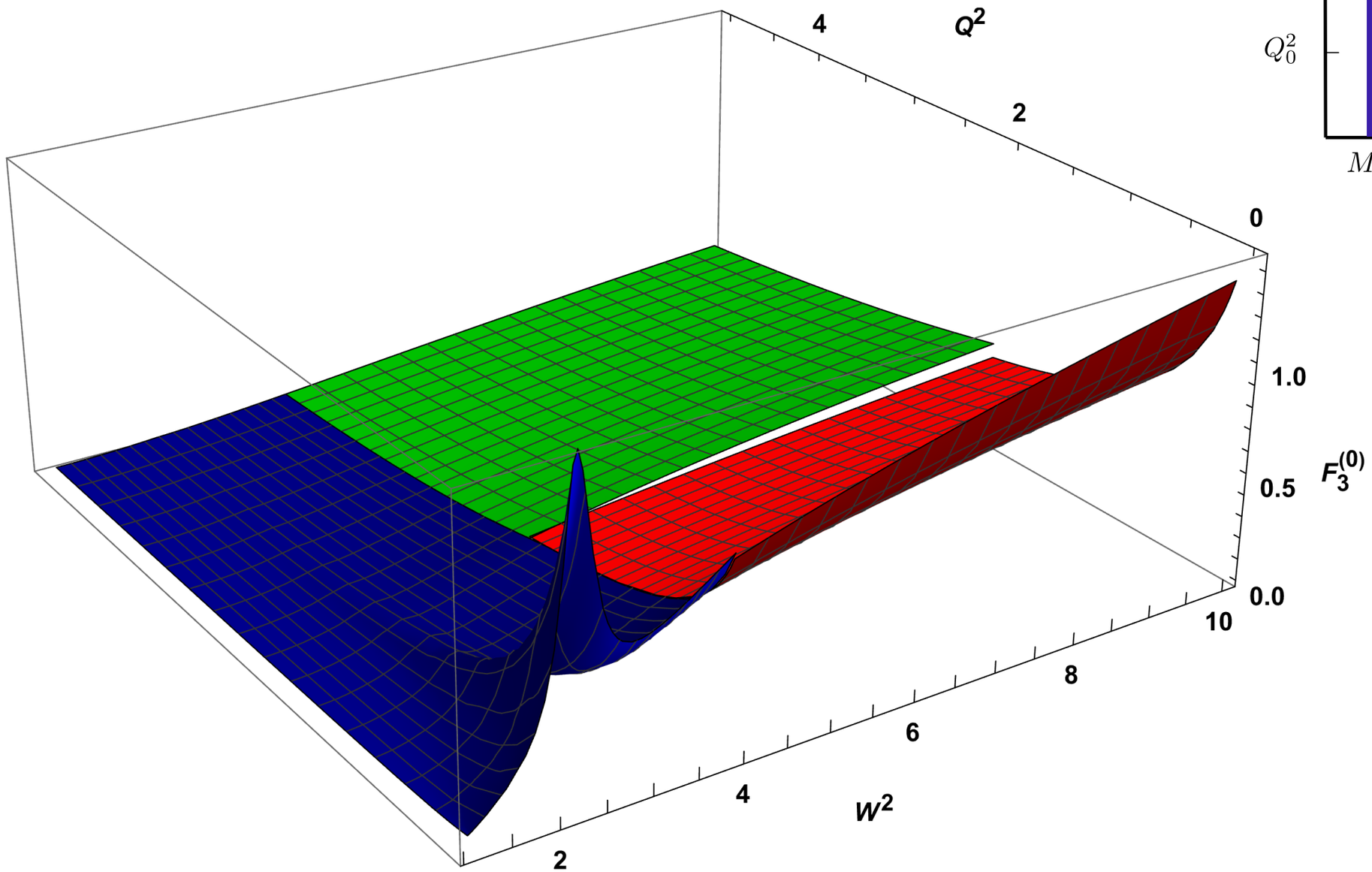
Effect is to move further away from unitarity in CKM matrix.

Alternative to using sum rules

Shiells *et al.* (2021): calculation based on constructing $F_3^{(0)}(W^2, Q^2)$ and imposing continuity across kinematic boundaries (AJM approach)

Shiells, PGB & Melnitchouk, PRD **104**, 033101 (2021)





Regge contribution has largest uncertainty

At low Q^2 and high W^2, the strong interaction becomes nonperturbative.

Model of Capella *et al.* from ν-N scattering fits:

$$F_{3\,(\mathrm{Reg})}^{(0)}(x,Q^2) = A^{p-n}\, x^{-\alpha_R}(1-x)^c \left(\frac{Q^2}{Q^2+\Lambda_R^2}\right)^{\alpha_R}$$

The true Q^2 dependence of this structure function is not well-determined by theory.

Match this function to the well-known value in the DIS region around $Q^2 = 2\ \mathrm{GeV}^2$ AND constrain it from available data on F_3^W from $\nu p + \bar{\nu} p$

$$A^{p-n} = A^{p+n}\ \frac{1}{9}$$

OR

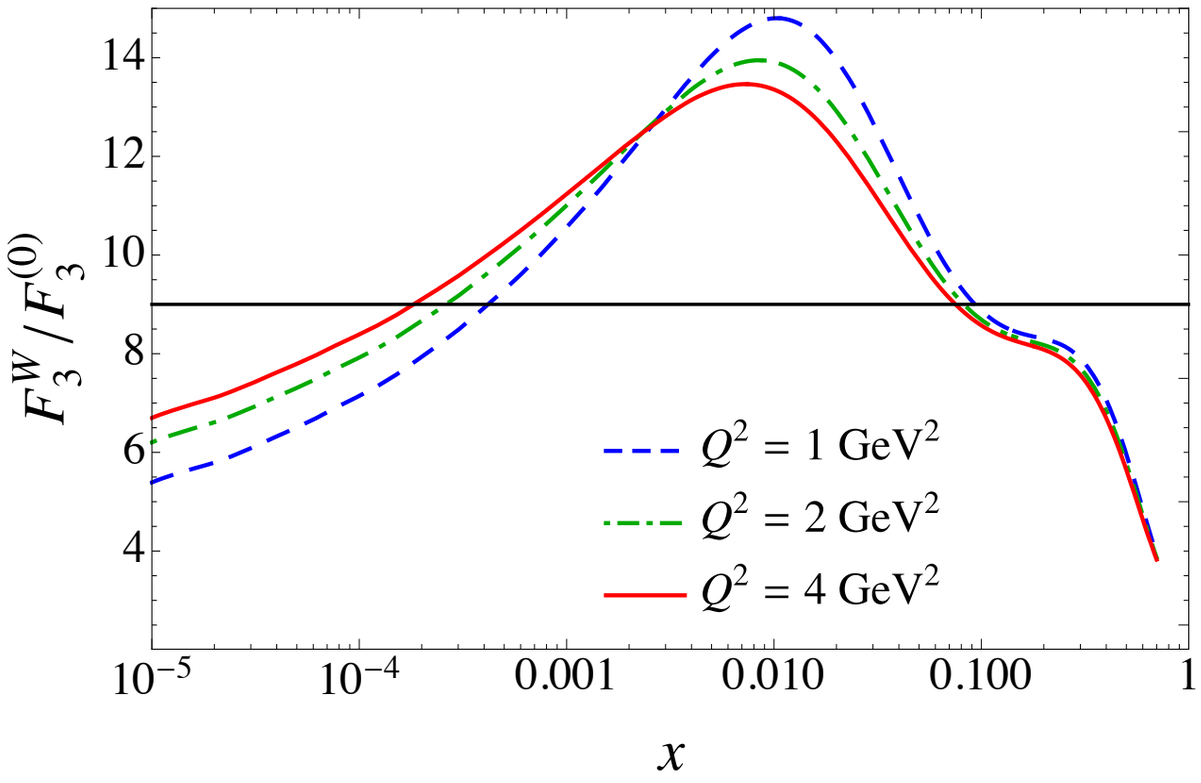
$$A^{p-n} = A^{p+n}\ \left.\frac{F_{3\,(\mathrm{Reg})}^{(0)}}{F_{3\,(\mathrm{Reg})}^{W}}\right|_{Q^2=Q_0^2}$$

Sum-rule constrained
(independent of x)

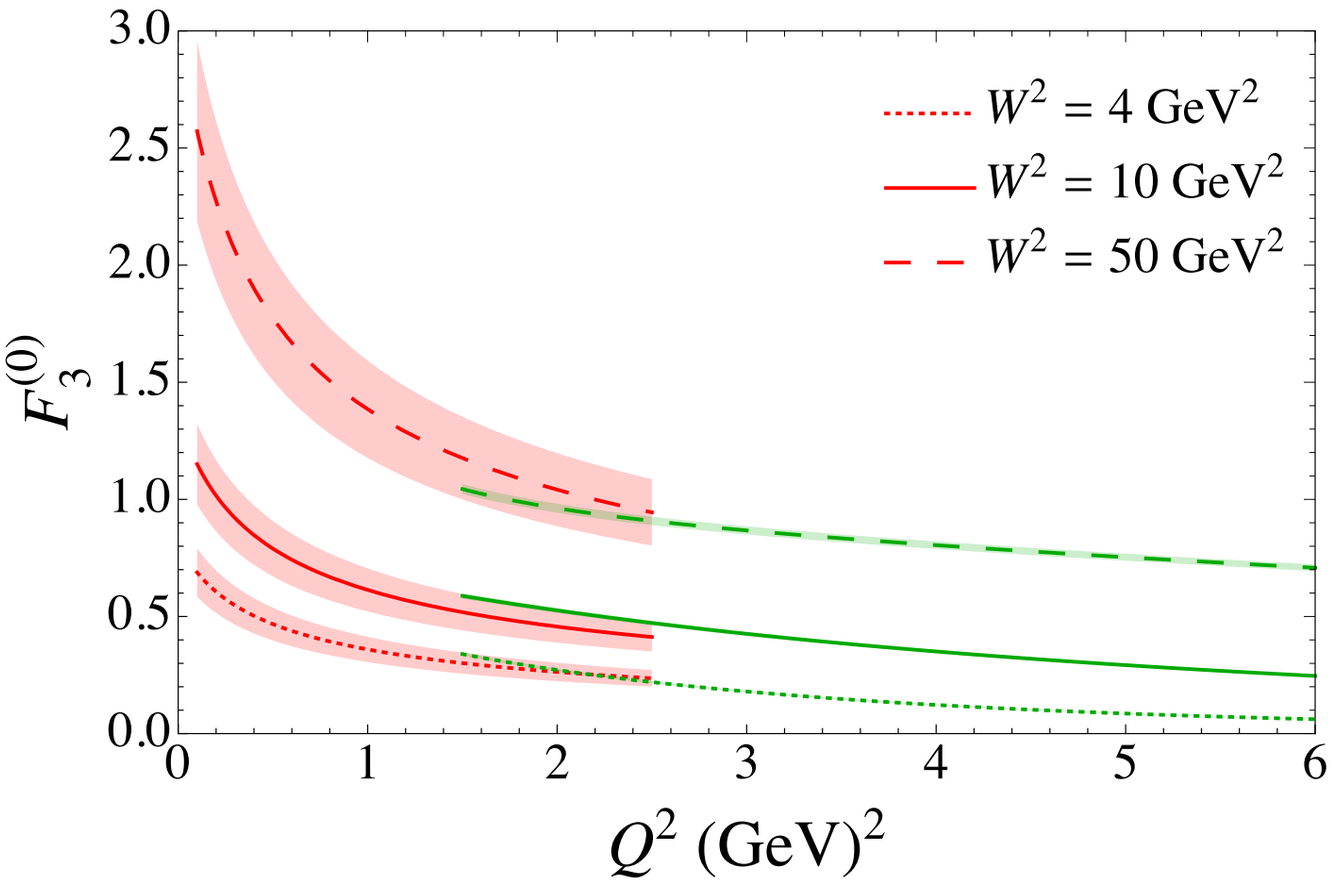
PDF constrained
(x dependent)

Regge contribution has largest uncertainty

Is the ratio 9 as a function of x?



Matching Regge to DIS at low Q^2



Elastic contribution:

$$F_{3\,(\mathrm{el})}^{(0)}(Q^2) = -\left[G_M^p(Q^2) + G_M^n(Q^2)\right] G_A(Q^2)\, Q^2 \delta(W^2 - M^2)$$

- Isoscalar magnetic form factor from recent fits (including TPE)
- Axial form factors from νN scattering (major source of uncertainty)

Resonance contributions: (isoscalar, so very small)

- Scale Christy-Bosted background to match at boundary

DIS contribution: (use pQCD and factorization)

- convolution at NLO

$$F_{3,\mathrm{DIS}}^{(0)}(x, Q^2) = \int_x^1 \frac{dz}{z} C_3^{(1)}(z) \times \frac{1}{3}\left(u_v(x/z, Q^2) - d_v(x/z, Q^2)\right)$$

- small effect over LO

$\Box_A^{\gamma W}$ $(\times 10^{-3})$	SBM[a]	SGRM[b]	Lattice QCD[c]
elastic	1.05(4)	1.06(6)	⋮
resonance	0.04(1)	—	⋮
Regge + $(Q^2 < 2)$ bgd	0.52(7)	0.56(8)	1.49(7)
DIS + $(Q^2 > 2)$ bgd	2.28(3)[d]	2.17(0)	2.16(6)
Total	3.89(9)	3.79(10)	3.65(7)

Agree within uncertainties

[a] Shiells *et al.*, PRD**104**, 033003 (2021)

[b] Seng *et al.*, PRD**100**, 013001 (2019)

[c] Ma *et al.*, PRL**132**, 191901 (2024)

[d] 2.19(3) without a 4% enhancement from running of $\alpha_{\rm em}$.

Summary

- Dispersion approach based on structure function data is a significant improvement over old methods
- Recent progress in lattice QCD calculations of non-perturbative part
- PDF region provides constraints on model-dependence of isospin rotation
- Direct comparison with PV inelastic data in resonance and DIS regions strongly constrains $\Box_V^{\gamma Z}$
- Regge and background contributions give largest uncertainties in $\Box_A^{\gamma Z}$