

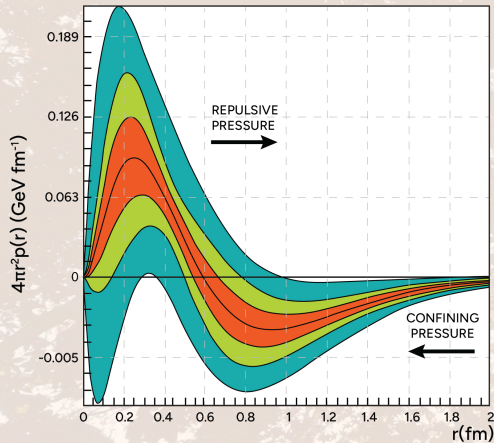
Mechanical Structure of the Nucleon

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October 7, 2026

 Jefferson Lab

 CENTER for
NUCLEAR FEMTOGRAPHY



- 🍁 Pressure in the proton has become a hot topic.
- 🍁 Detailed theoretical studies:
 - 🍃 Polyakov, *Phys Lett B* (2003)
 - 🍃 Polyakov & Schweitzer, *Int J Mod Phys A* (2018)
 - 🍃 Lorcé, Moutarde & Trawinski, *Eur Phys J C* (2019)
- 🍁 Empirical extractions happening at JLab!
 - 🍃 Burkert, Elouadrhiri & Girod, *Nature* (2018)
 - 🍃 Duran & al., *Nature* (2023)
 - 🍃 Chatagnon *et al.*, *Phys Rev C* (2026)
- 🍁 Popular science articles
 - 🍃 Science Daily (2018)
 - 🍃 Quanta magazine (2024)

Figure: Burkert, Elouadrhiri & Girod, *Nature* (2018)

1 The Energy-Momentum Tensor

- What is the EMT?
- What is the hadronic stress tensor?

2 EMT Form Factors

- How do we formally describe hadronic stresses?

3 Spatial Stress Distributions

- How do EMT-FFs entail stress distributions?
- How do we interpret these distributions?

A photograph of a brick building with autumn trees in the foreground and a clear blue sky. The text "The Energy-Momentum Tensor" is overlaid in a stylized font.

The Energy-Momentum Tensor

- ✳ The energy-momentum tensor describes **density** and **flow** of energy & momentum.
- ✳ Also known as the stress-energy tensor.

Energy density

Momentum densities

$$T^{\mu\nu}(x) = \begin{bmatrix} T^{00}(x) & T^{01}(x) & T^{02}(x) & T^{03}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix}$$

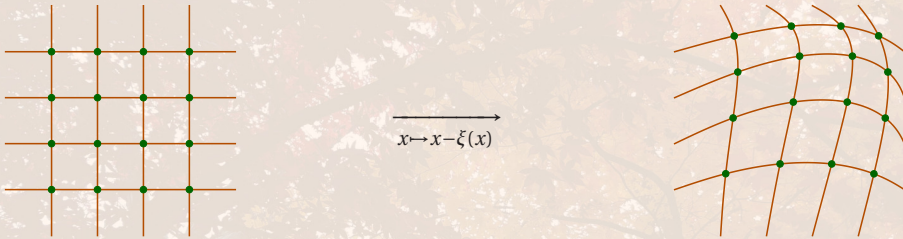
Energy fluxes

Stress tensor

The diagram illustrates the components of the energy-momentum tensor $T^{\mu\nu}(x)$. The tensor is represented as a 4x4 matrix. The components are color-coded and labeled with arrows:

- Energy density:** $T^{00}(x)$ (blue box, blue arrow from the label).
- Momentum densities:** $T^{01}(x)$, $T^{02}(x)$, and $T^{03}(x)$ (green boxes, green arrow from the label).
- Energy fluxes:** $T^{10}(x)$, $T^{20}(x)$, and $T^{30}(x)$ (red boxes, red arrow from the label).
- Stress tensor:** $T^{11}(x)$, $T^{12}(x)$, $T^{13}(x)$, $T^{21}(x)$, $T^{22}(x)$, $T^{23}(x)$, $T^{31}(x)$, $T^{32}(x)$, and $T^{33}(x)$ (orange boxes, orange arrow from the label).

- Conserved current from *local* spacetime translations (**Noether's second theorem**):



- **Noether's theorems:** symmetries imply conservation laws
- *Local* translation: move spacetime differently everywhere

- The **energy-momentum tensor (EMT)** is the associated conserved quantity

$$\delta_{\xi} S_{\text{QCD}} = - \int d^4x \underbrace{T_{\text{QCD}}^{\mu\nu}(x)}_{\text{Energy-Momentum Tensor}} \partial_{\mu} \xi_{\nu}(x) = 0$$

$$\Sigma_q \left\{ \frac{i}{2} \bar{q}(x) \gamma^{\mu} \overleftrightarrow{D}^{\nu} q(x) \right\} - 2 \text{Tr} [G^{\mu\lambda} G^{\nu}_{\lambda}] + \frac{1}{2} g^{\mu\nu} \text{Tr} [G^{\lambda\sigma} G_{\lambda\sigma}]$$

Felsager, *Geometry, Particles, and Fields*

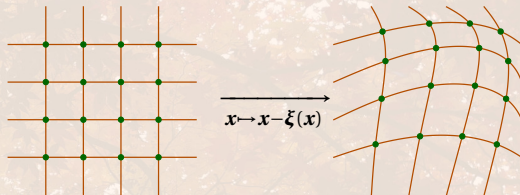
Montesinos & Flores, *Rev Mex Fis* 52 (2006) 29

AF, *Phys Rev D* 113 (2026) 016011

🍁 3×3 sub-matrix of the energy-momentum tensor.

$$\begin{bmatrix} T^{00}(x) & T^{01}(x) & T^{02}(x) & T^{03}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix}$$

↑
Stress tensor



🍁 Work needed to deform a system:

$$W[x \mapsto x - \xi(x)] = - \int d^3x \overbrace{T_{\text{QCD}}^{ij}(x)}^{\text{stress tensor}} \left(\overbrace{\frac{1}{2}(\partial_i \xi_j(x) + \partial_j \xi_i(x))}^{\text{strain}} + \overbrace{\frac{1}{2}(\partial_i \xi_j(x) - \partial_j \xi_i(x))}^{\text{torsion}} \right)$$

🍂 **Stress:** internal forces resisting *spatial* deformation

🍂 Deformations are a virtual pressure gauge

- Continuity equation (integral form):

$$\frac{d}{dt} \underbrace{\left[\int_V d^3x T^{0j}(\mathbf{x}, t) \right]}_{\text{Momentum in region}} = - \underbrace{\oint_{\partial V} dS \hat{n}_i T^{ij}(\mathbf{x}, t)}_{\text{Flux out of region}}$$

$$T^{\mu\nu}(x) = \begin{bmatrix} T^{00}(x) & T^{01}(x) & T^{02}(x) & T^{03}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix}$$

- Stresses are **momentum flux densities**

Include **particle fluxes & forces**



Particle flux

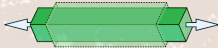


Force

- Kinetic pressure is pressure**

- It's the microscopic origin of gas & fluid pressure
- Photon flux is the origin of radiation pressure

Tension



Compression

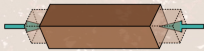


Image: MikeRun ([Wikimedia](#)), very modified

Hanging monkeys

- 🍁 **Negative stress** / negative “pressure”
- 🍁 **Tension** / pulling / stretching
- 🍁 Highest monkey under most tension

$$\nabla_i T^{ij} = -\rho g \hat{z}^j = f_{\text{grav}}^j$$

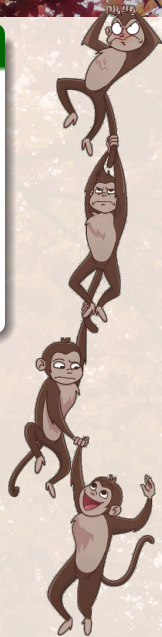
Monkey image by Angie Freese ([Instagram](#))

Stacked Turtles

- 🍁 **Positive stress** / positive “pressure”
- 🍁 **Compression** / pushing / squishing
- 🍁 More compression at bottom of pile

$$\nabla_i T^{ij} = -\rho g \hat{z}^j = f_{\text{grav}}^j$$

Turtle image by FoxKids1302 ([DeviantArt](#))



🍁 Cauchy momentum equation for an *open, static* system:

$$f_{\text{net}}^j(\mathbf{x}, t) = f_{\text{external}}^j(\mathbf{x}, t) - \nabla_i T^{ij}(\mathbf{x}, t) = 0$$

- Balance between **external forces** & **internal forces**
- Can infer external forces from stress tensor!

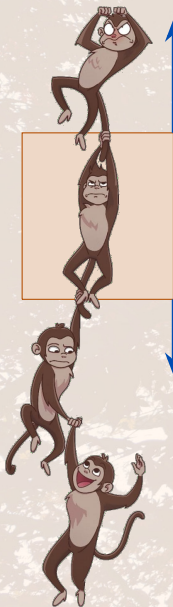
🍁 Works for expectation values in quantum systems!

$$\langle f_{\text{external}}^j(\mathbf{x}, t) \rangle = \nabla_i \langle T^{ij}(\mathbf{x}, t) \rangle$$

- Schrödinger, *Ann. Phys.* 387 (1927) 265
- Nielsen & Martin, *PRB* 32 (1985) 3780
- Polyakov & Son, *JHEP* 09 (2018) 156

🍁 Stress tensor in atom returns Coulomb force law!

- AF, *Phys Rev D* 111 (2025) 034047
- Could map out strong nuclear forces via stress tensor!



EMT Form Factors

- Energy-momentum tensor parametrized using **EMT form factors (EMT-FFs)**

$$\langle \mathbf{p}' | \hat{T}_{q,g}^{\mu\nu}(0) | \mathbf{p} \rangle = 2P^\mu P^\nu A_{q,g}(\Delta^2) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{2} D_{q,g}(\Delta^2) + 2m^2 g^{\mu\nu} \bar{c}_{q,g}(\Delta^2)$$

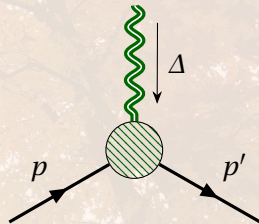
- Also known as **gravitational form factors**
- EMT is source of gravitation, but we're not doing gravitational physics

- General form from Lorentz covariance

- EMT-FFs encode hadron structure

$$P^\mu = \frac{1}{2}(p + p')^\mu$$

$$\Delta^\mu = (p' - p)^\mu$$



Kobzarev & Okun, *Sov Phys JETP* 16 (1963) 1343

H. Pagels, *Phys Rev* 144 (1966) 1250

X. Ji, *PRL* 78 (1997) 610

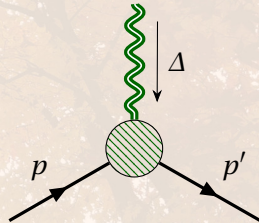
- Energy-momentum tensor parametrized using **EMT form factors**

$$\langle \mathbf{p}' | \hat{T}_{q,g}^{\mu\nu}(0) | \mathbf{p} \rangle = \underbrace{2P^\mu P^\nu A_{q,g}(\Delta^2)}_{\text{barycentric motion \& dispersion}} + \underbrace{\frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{2} D_{q,g}(\Delta^2) + 2m^2 g^{\mu\nu} \bar{c}_{q,g}(\Delta^2)}_{\text{internal pressure \& forces}}$$

- General form from Lorentz covariance
- EMT-FFs encode hadron structure**

$$P^\mu = \frac{1}{2}(p + p')^\mu$$

$$\Delta^\mu = (p' - p)^\mu$$



Kobzarev & Okun, [Sov Phys JETP 16 \(1963\) 1343](#)

H. Pagels, [Phys Rev 144 \(1966\) 1250](#)

X. Ji, [PRL78 \(1997\) 610](#)

- 🍁 Spin-half has **additional EMT form factors** related to angular momentum

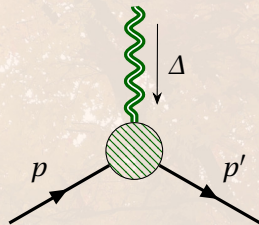
$$\langle \mathbf{p}', s' | \hat{T}_{q,g}^{\mu\nu}(0) | \mathbf{p}, s \rangle = \bar{u}(\mathbf{p}', s') \left\{ \overbrace{\frac{p^\mu p^\nu}{m} A_{q,g}(\Delta^2)}^{\text{motion \& dispersion}} + \overbrace{\frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} D_{q,g}(\Delta^2) + m \bar{c}_{q,g}(\Delta^2)}^{\text{pressure \& forces}} \right. \\ \left. + \underbrace{\frac{i P^{[\mu} \sigma^{\nu]} \Delta}{2m} J_{q,g}(\Delta^2)}_{\text{angular momentum}} - \underbrace{\frac{i P^{[\mu} \sigma^{\nu]} \Delta}{2m} S_{q,g}(\Delta^2)}_{\text{fermion spin}} \right\} u(\mathbf{p}, s)$$

🍁 **New symmetric part** related to total angular momentum

🍁 **Antisymmetric part** related to elementary fermion spin

$$P^\mu = \frac{1}{2}(p + p')^\mu$$

$$\Delta^\mu = (p' - p)^\mu$$



🍁 Momentum sum rule:

$$\sum_{q,g} A_c(0) = 1$$

🍁 Angular momentum sum rule (Ji sum rule):

$$\sum_{q,g} J_c(0) = \frac{1}{2}$$

🍁 Local momentum conservation:

$$\sum_{q,g} \bar{c}_c(\Delta^2) = 0$$

🍂 **Violated for open sub-systems**

🍂 Non-zero $\bar{c}_c(\Delta^2)$ **quantifies forces**—Cauchy momentum equation!

$$\langle f_c^j(\mathbf{b}) \rangle = -m \nabla_j \int \frac{d^3 \Delta}{(2\pi)^3} \bar{c}_c(\Delta^2) e^{-i\Delta \cdot \mathbf{b}}$$

🍂 Called **non-conserved form factor** — quantifies local conservation violation

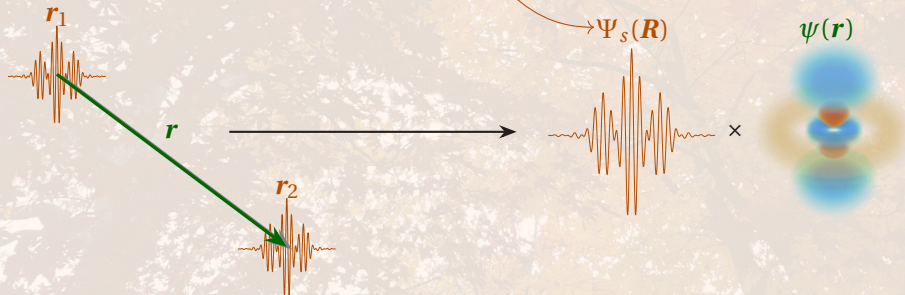
A photograph of a brick building with autumn trees in the foreground and a clear blue sky. The text "Spatial Stress Distributions" is overlaid in a white, cursive font with a red shadow effect.

Spatial Stress Distributions

- 🍁 Momentum transfer $\rightarrow$ densities via Fourier transforms:

$$\langle \Psi | \hat{T}^{ij}(\mathbf{x}, t) | \Psi \rangle = \sum_{s,s'} \int d^3R \int \frac{d^3\Delta}{(2\pi)^3} \underbrace{\Psi_{s'}^*(\mathbf{R}, t) \langle p', s' | \hat{T}^{ij}(0) | p, s \rangle \Psi_s(\mathbf{R}, t)}_{\text{EMT form factors appear here}} e^{-i\Delta \cdot (\mathbf{x} - \mathbf{R})} \Big|_{2i\mathbf{P} \rightarrow \overleftrightarrow{\nabla}}$$

- 🍁 This depends on the proton's wave packet $\Psi_s(\mathbf{R}, t)$



- 🍁 Need a bit more effort to distill out **internal densities**

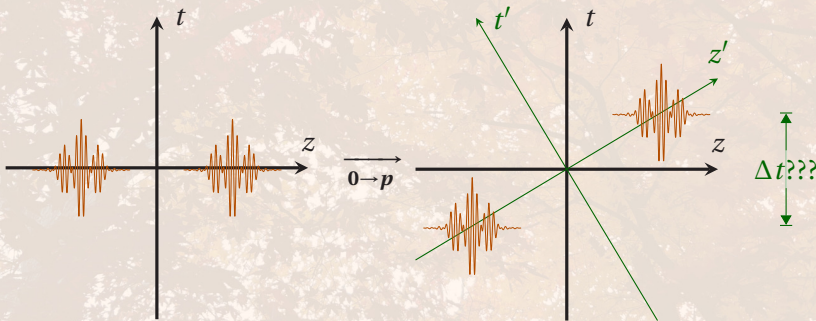
$$\begin{aligned}
 \langle \Psi | \hat{T}^{ij}(\mathbf{x}, t) | \Psi \rangle = \sum_{s,s'} \int d^3 R \left\{ \right. & \overbrace{\left(-\Psi_{s'}^*(\mathbf{R}, t) \frac{\overleftrightarrow{\nabla}^i \overleftrightarrow{\nabla}^j}{4M^2} \Psi_s(\mathbf{R}, t) \right)}^{\text{dynamic pressures}} \underbrace{a_{s's}(\mathbf{x} - \mathbf{R})}_{\text{internal mass density}} \\
 & - \overbrace{\left(\frac{i\Psi_{s'}^*(\mathbf{R}, t) \overleftrightarrow{\nabla}^i \Psi_s(\mathbf{R}, t)}{2M} \right)}^{\text{barycentric flux}} \underbrace{p_{s's}^j(\mathbf{x} - \mathbf{R})}_{\text{internal momentum density}} \\
 & - \overbrace{\left(\frac{i\Psi_{s'}^*(\mathbf{R}, t) \overleftrightarrow{\nabla}^j \Psi_s(\mathbf{R}, t)}{2M} \right)}^{\text{barycentric flux}} \underbrace{f_{s's}^i(\mathbf{x} - \mathbf{R})}_{\text{internal mass flux}} \\
 & \left. + \overbrace{\left(\Psi_{s'}^*(\mathbf{R}, t) \Psi_s(\mathbf{R}, t) \right)}^{\text{barycentric probability}} \underbrace{t_{s's}^{ij}(\mathbf{x} - \mathbf{R})}_{\text{internal stresses}} \right\}
 \end{aligned}$$



Image: dynamic pressure from wind

- 🍁 **Internal densities** smeared out by **barycentric distributions**
- 🍁 **Dynamic pressures** due to barycentric motion & wave packet dispersion
- 🍁 **Internal stress** due to internal motions *and* forces
 - 👉 Smeared by barycentric probability
- 🍁 **Strictly non-relativistic** breakdown
 - 👉 Afforded by Galilei symmetry (absoluteness of simultaneity)

- 🍁 Momentum-space wave packet contains boosted versions of composite system
 - 👉 Boosts mix up planes of simultaneity
 - 👉 Internal constituents get boosted to **different times**



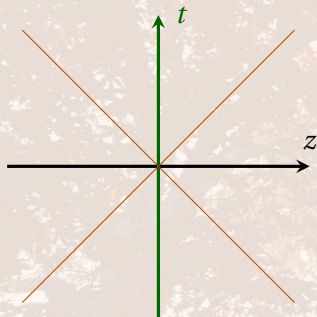
- 🍁 Cannot decompose structure to overall wave packet $\otimes$ internal structure at fixed t
- 🍁 **Light front coordinates** fix this by defining a **new, boost-invariant time variable!**

🍁 **Light front coordinates** are a different foliation of spacetime.

👉 Involves redefining *equal-time surfaces*.

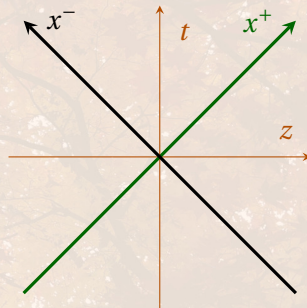
$$x^{\pm} = \frac{1}{\sqrt{2}}(t \pm z)$$

$$\mathbf{x}_{\perp} = (x, y)$$



Minkowski coordinates

$$x^{+} = \frac{1}{\sqrt{2}}(t + z) = \text{time}$$

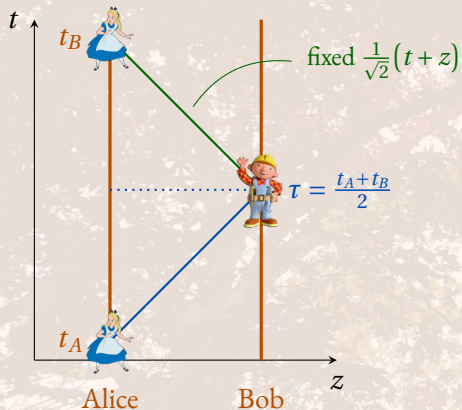


Light front coordinates

P.A.M. Dirac, [Rev Mod Phys 21 \(1949\) 392](#)

Brodsky, Pauli & Pinsky, [Phys Rept 301 \(1998\) 299](#)

AF, [A friendly introduction to the light front](#) (lecture series)



🍁 Fixed t surfaces require coordination

- Distant clocks must be synchronized
- Must wait for light to arrive
- Reconstruct after the fact
- **Ruined by boosts**

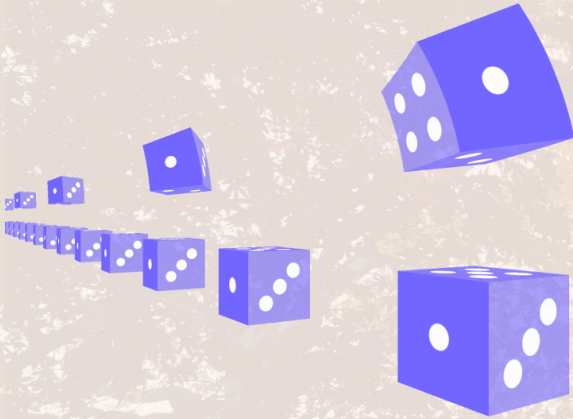
🍁 Fixed x^+ surfaces require looking

- Look in the $+z$ direction
- That's a fixed x^+ surface
- **Invariant under boosts!**
- Allow relativistic densities

A. Einstein, [Annalen Phys 17 \(1905\) 891](#)

Brodsky, Pauli & Pinsky, [Phys Rept 301 \(1998\) 299](#)

AF & Miller, [Phys Rev D 107 \(2023\) 074036](#)



🍁 Lorentz-boosted objects *appear rotated*.

🍂 **Terrell rotation**

🍂 Optical effect: contraction + delay

🍁 **Light front transverse boost**
undoes Terrell rotation:

$$B_x^{(\text{LF})} = \frac{1}{\sqrt{2}}(K_x + J_y)$$

🍂 Standard boost + counter-rotation

🍂 Leaves x^+ (time) invariant

🍂 Part of the **Galilei subgroup**

Dice images by Ute Kraus,

<https://www.spacetime travel.org/>

J. Terrell, *Phys Rev* 116 (1959) 1041

M. Burkardt, *Int J Mod Phys A* 18 (2003) 173

- 🍁 Poincaré group has a (2 + 1)D **Galilei subgroup**.
 - 🍁 x^+ is time and $\mathbf{x}_\perp$ is space under this subgroup.
 - 🍁 $P^+ = E_{\mathbf{p}} + p_z$ is the central charge.
 - 🍁 x^+ and P^+ are invariant under this subgroup!
- 🍁 Light front time gives **fully relativistic** 2D picture that looks a lot like non-relativistic physics.



$$\frac{d\mathbf{P}_\perp}{dx^+} = P^+ \frac{d^2 \mathbf{x}_\perp}{dx^{+2}}$$

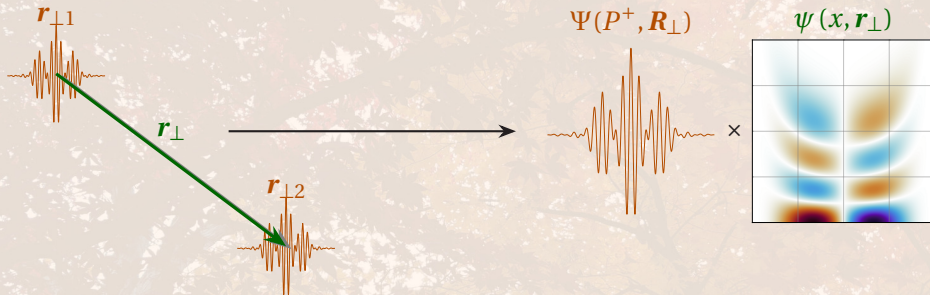
$$H = H_{\text{rest}} + \frac{\mathbf{P}_\perp^2}{2P^+}$$

$$\mathbf{v}_\perp = \frac{\mathbf{P}_\perp}{P^+}$$

G. Pinski, [J Math Phys 9 \(1986\) 1927](#)

M. Burkardt, [Int J Mod Phys A 18 \(2003\) 173](#)

- Wave packet separation works for *transverse* spatial coordinates



- Works thanks to the Galilei subgroup
- Apparently stuck with 2D spatial densities
 - Momentum fraction x gives third dimension
 - x dependence visible in **impact parameter parton distributions**—beyond scope of this talk

Brodsky, Pauli & Pinsky, [Phys Rept 301 \(1998\) 299](#)

M. Burkardt, [Int J Mod Phys A 18 \(2003\) 173](#)

Vary, Honkanen, Li, Maris & Brodsky, [Phys Rev C 81 \(2010\) 035205](#)

$$\int_{\mathbb{R}} dx^- \langle \Psi | \hat{T}^{ij}(x) | \Psi \rangle = \sum_{s,s'} \int d^2 R_{\perp} \left\{ \underbrace{-\Psi_{s'}^* \frac{\overleftrightarrow{\nabla}^i \overleftrightarrow{\nabla}^j}{4(P^+)^2} \Psi_s}_{\text{dynamic pressures}} \underbrace{a_{s's}(\mathbf{x}_{\perp} - \mathbf{R}_{\perp})}_{\text{internal } P^+ \text{ density}} - \underbrace{\frac{i\Psi_{s'}^* \overleftrightarrow{\nabla}^i \Psi_s}{2P^+}}_{\text{barycentric flux}} \underbrace{p_{s's}^j(\mathbf{x}_{\perp} - \mathbf{R}_{\perp})}_{\text{internal momentum density}} \right. \\ \left. - \frac{i\Psi_{s'}^* \overleftrightarrow{\nabla}^j \Psi_s}{2P^+} \underbrace{f_{s's}^i(\mathbf{x}_{\perp} - \mathbf{R}_{\perp})}_{\text{barycentric flux}} + \underbrace{\Psi_{s'}^* \Psi_s}_{\text{barycentric probability}} \underbrace{t_{s's}^{ij}(\mathbf{x}_{\perp} - \mathbf{R}_{\perp})}_{\text{internal stresses}} \right\}$$



Image: dynamic pressure from wind

- 🍁 Echoes non-relativistic breakdown—light front Galilei symmetry!
- 🍁 EMT-FFs $D_q(\Delta_{\perp}^2)$ and $\bar{c}_q(\Delta_{\perp}^2)$ in stress tensor.

$$t_q^{ij}(\mathbf{b}_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} \left(\frac{\Delta_{\perp}^i \Delta_{\perp}^j - \delta^{ij} \Delta_{\perp}^2}{4P^+} D_q(\Delta_{\perp}^2) - \frac{m^2}{P^+} \delta^{ij} \bar{c}_q(\Delta_{\perp}^2) \right) e^{-i\Delta_{\perp} \cdot \mathbf{b}_{\perp}}$$

- 🍁 Start with a closed system (sum quarks & gluons—no $\bar{c}$):

$$t_{\text{LF}}^{ij}(\mathbf{b}) = \int \frac{d^3 \Delta_{\perp}}{(2\pi)^2} \frac{\Delta^i \Delta^j - \delta^{ij} \Delta^2}{4P^+} D(\Delta_{\perp}^2) e^{-i\Delta_{\perp} \cdot \mathbf{b}_{\perp}} = \frac{-\nabla_i \nabla_j + \delta_{ij} \nabla^2}{4P^+} \underbrace{\int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} D(\Delta_{\perp}^2) e^{-i\Delta_{\perp} \cdot \mathbf{b}_{\perp}}}_{\equiv \mathcal{D}(r)}$$

Polyakov pressure potential

- 🍁 Simple tripole model:

$$D(\Delta_{\perp}^2) = \frac{D_0}{(1 + \Delta_{\perp}^2 / \Lambda^2)^3} \quad \Rightarrow \quad \mathcal{D}(b_{\perp}) = \frac{D_0 \Lambda^4 b_{\perp}^2}{16\pi} \underbrace{K_2(\Lambda b_{\perp})}_{\text{modified Bessel function}}$$

- 🍁 Tripole from perturbative QCD; D_0 & Λ from experiment or lattice

Combined tripole form

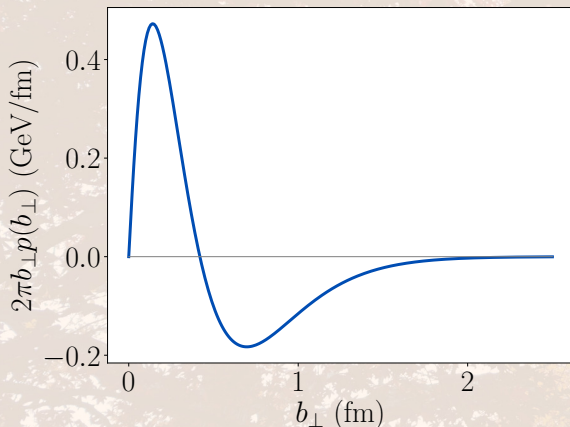
$$D_0 = D_0^{(q)} + D_0^{(g)} = -3.46$$

$$\Lambda = 1.12 \text{ GeV}$$

Polyakov & Schweitzer, [Int J Mod Phys A \(2018\)](#)
 Lorcé, Moutarde & Trawinski, [Eur Phys J C \(2019\)](#)
 AF & Miller, [Phys Rev D 103 \(2021\) 094023](#)

Burkert, Elouadrhiri & Girod, [Nature \(2018\)](#)
 Duran & al., [Nature \(2023\)](#)

$$t_{\text{LF}}^{ij}(\mathbf{b}_{\perp}) = \frac{D_0 \Lambda^5 b_{\perp}}{128\pi P^+} \left\{ \left(\Lambda b_{\perp} K_0(\Lambda b_{\perp}) - 2K_1(\Lambda b_{\perp}) \right) \underbrace{\delta^{ij}}_{\text{isotropic pressure}} - 2\Lambda b_{\perp} K_0(\Lambda b_{\perp}) \underbrace{\left(\hat{b}^i \hat{b}^j - \frac{1}{2} \delta^{ij} \right)}_{\text{shear stresses}} \right\}$$



Combined tripole form

$$p(r) = \frac{D_0 \Lambda^5 b_{\perp}}{128\pi P^+} \left(\Lambda b_{\perp} K_0(\Lambda b_{\perp}) - 2K_1(\Lambda b_{\perp}) \right)$$

$$D_0 = D_0^{(q)} + D_0^{(g)} = -3.46$$

$$\Lambda = 1.12 \text{ GeV}$$

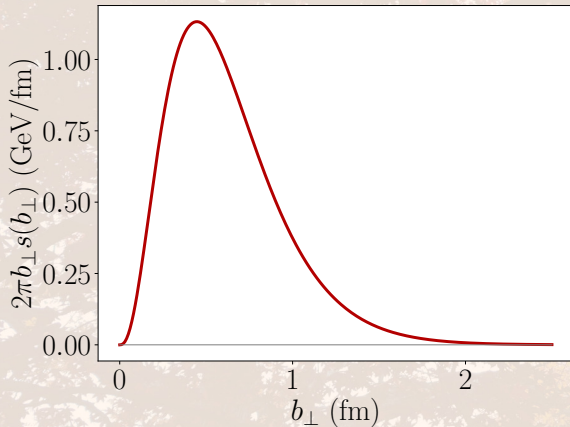
Burkert, Elouadrhiri & Girod, [Nature \(2018\)](#)

Duran & al., [Nature \(2023\)](#)

🍁 **Isotropic pressure:** average stress over all directions

👉 Often just called “pressure” in the literature

$$t_{\text{LF}}^{ij}(\mathbf{b}_{\perp}) = \frac{D_0 \Lambda^5 b_{\perp}}{128\pi P^+} \left\{ \left(\Lambda b_{\perp} K_0(\Lambda b_{\perp}) - 2K_1(\Lambda b_{\perp}) \right) \overbrace{\delta^{ij}}^{\text{isotropic pressure}} - 2\Lambda b_{\perp} K_0(\Lambda b_{\perp}) \overbrace{\left(\hat{b}^i \hat{b}^j - \frac{1}{2} \delta^{ij} \right)}^{\text{shear stresses}} \right\}$$



Combined tripole form

$$s(r) = -\frac{D_0 \Lambda^6 b_{\perp}^2}{64\pi P^+} K_0(\Lambda b_{\perp})$$

$$D_0 = D_0^{(q)} + D_0^{(g)} = -3.46$$

$$\Lambda = 1.12 \text{ GeV}$$

Burkert, Elouadrhiri & Girod, [Nature \(2018\)](#)

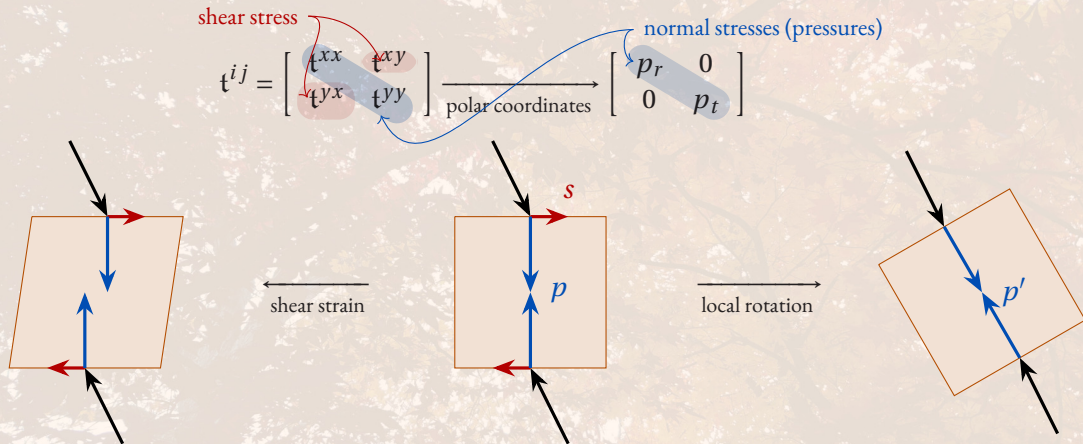
Duran & al., [Nature \(2023\)](#)



Shear stress: anisotropy in stress tensor



Describes how material elements are sheared by the stresses



- 🍁 **Normal stresses** push (compression) or pull (tension) on material element
- 🍁 **Shear stresses** cause medium to shear
- 🍁 Shear stresses eliminated by diagonalization—get **principal stresses**

$$t_{\text{LF}}^{ij}(\mathbf{b}_{\perp}) = \frac{D_0 \Lambda^5 b_{\perp}}{64\pi P^+} \left\{ -K_1(\Lambda b_{\perp}) \overbrace{\hat{b}^i \hat{b}^j}^{\text{radial pressure}} + \left(\Lambda b_{\perp} K_0(\Lambda b_{\perp}) - K_1(\Lambda b_{\perp}) \right) \overbrace{\left(\delta^{ij} - \hat{b}^i \hat{b}^j \right)}^{\text{tangential stresses}} \right\}$$

Combined tripole form

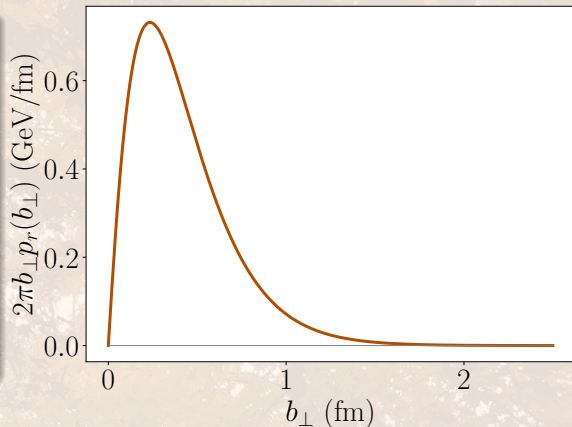
$$p_r(r) = -\frac{D_0 \Lambda^5 b_{\perp}}{64\pi P^+} K_1(\Lambda b_{\perp})$$

$$D_0 = D_0^{(q)} + D_0^{(g)} = -3.46$$

$$\Lambda = 1.12 \text{ GeV}$$

Burkert, Elouadrhiri & Girod, [Nature \(2018\)](#)

Duran & al., [Nature \(2023\)](#)



Radial pressure is strictly positive in the proton—but not in all systems

$$t_{\text{LF}}^{ij}(\mathbf{b}_{\perp}) = \frac{D_0 \Lambda^5 b_{\perp}}{64\pi P^+} \left\{ -K_1(\Lambda b_{\perp}) \overbrace{\hat{b}^i \hat{b}^j}^{\text{radial pressure}} + \left(\Lambda b_{\perp} K_0(\Lambda b_{\perp}) - K_1(\Lambda b_{\perp}) \right) \overbrace{\left(\delta^{ij} - \hat{b}^i \hat{b}^j \right)}^{\text{tangential stresses}} \right\}$$

Combined tripole form

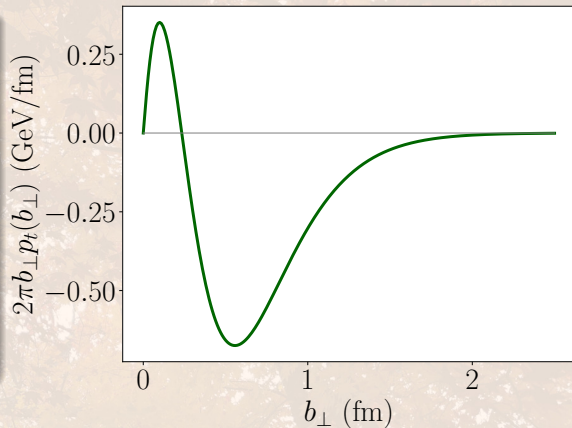
$$p_t(r) = \frac{D_0 \Lambda^5 b_{\perp}}{64\pi P^+} \left(\Lambda b_{\perp} K_0(\Lambda b_{\perp}) - K_1(\Lambda b_{\perp}) \right)$$

$$D_0 = D_0^{(q)} + D_0^{(g)} = -3.46$$

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Burkert, Elouadrhiri & Girod, [Nature \(2018\)](#)

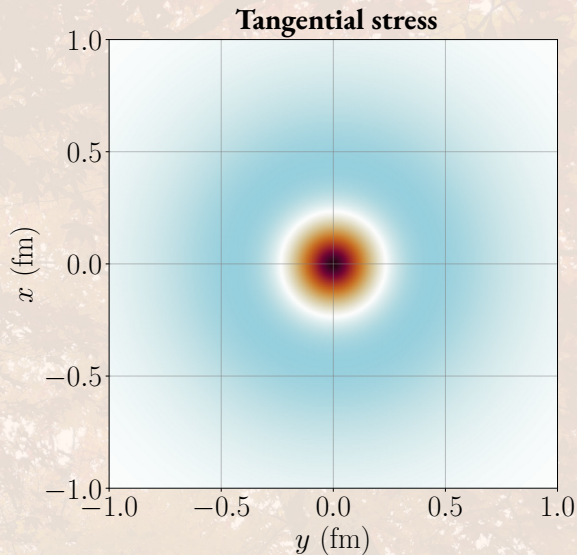
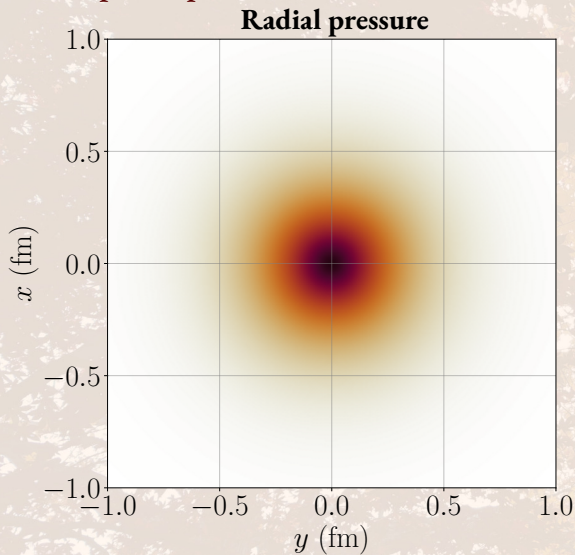
Duran & al., [Nature \(2023\)](#)



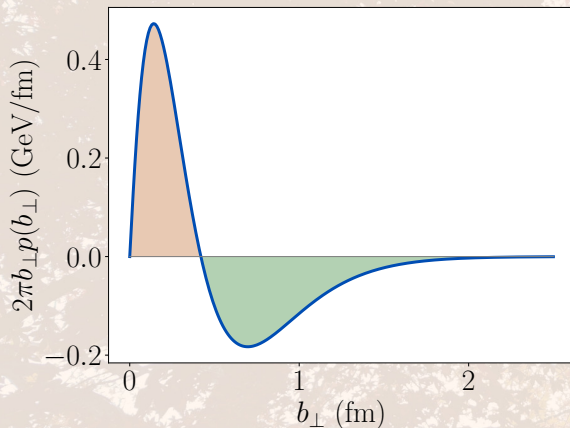
Diffuse **tangential tension**



Proton is like a liquid drop with a *very* fuzzy, diffuse boundary



Proton is like a liquid drop with a *very* fuzzy, diffuse boundary



von Laue condition

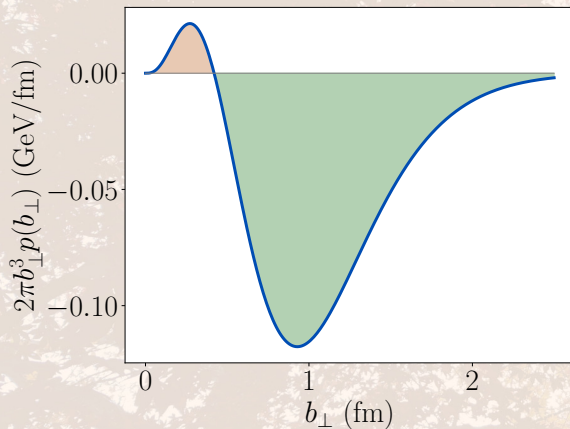
$$\int d^2 b_{\perp} p(\mathbf{b}_{\perp}) = 0$$

- 🍁 Satisfied for every hadron
- 🍁 Can be shown from:

$$\langle \mathbf{p} | T^{ij}(0) | \mathbf{p} \rangle = 0$$

von Laue, *Annalen Phys* 340 (1911) 524

Polyakov & Schweitzer, *Int J Mod Phys A* 33 (2018) 1830025



Polyakov's conjecture

$$D(0) = P^+ \int d^2 b_\perp b_\perp^2 p(\mathbf{b}_\perp) < 0$$

- 🍁 Seems to be true for hadrons
- 🍁 Seems intuitive:
 - 🍂 Compression against collapse
 - 🍂 Tension against disintegration
- 🍁 **Is false for some systems!**

Perevalova, Polyakov & Schweitzer, [PRD 94 \(2016\) 054024](#)

Polyakov & Schweitzer, [Int J Mod Phys A 33 \(2018\) 1830025](#)

- Sign of D-term correlated with spin of force-mediating field

- Potential & wave function underdetermine D-term!

- Spin-odd: positive**

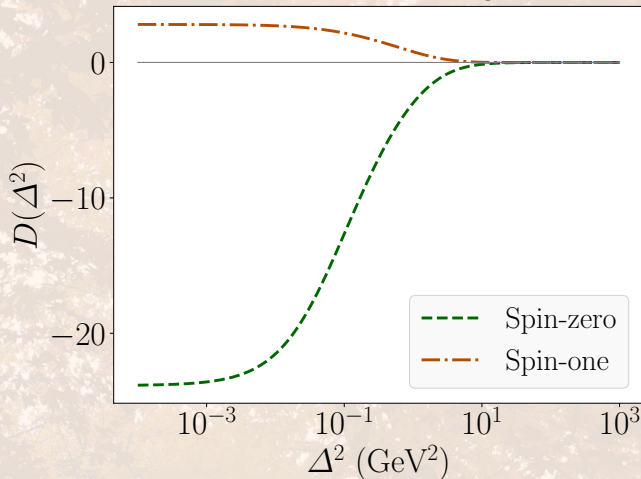
- Includes electromagnetic systems
 - Hydrogen atom has $D(0) > 0$

- Spin-even: negative**

- Hadrons break this pattern—**why?**

- Gluons are spin one...
 - Something to learn about QCD!

D-term for Yukawa potential ground state



A photograph of a brick building with autumn trees in the foreground and a clear blue sky. The text "Wrapping Up" is overlaid in a white, cursive font with a red shadow effect.

Wrapping Up

- 🍁 Stresses are momentum flux densities
 - 🍂 How a system responds to deformations
 - 🍂 Positive/negative stress related to compression/tension
- 🍁 Divergence of stress tensor gives forces between sub-components.
 - 🍂 This makes $\bar{c}_q(\Delta^2)$ the most exciting form factor.
 - 🍂 **Can map out color force distributions!**
- 🍁 Light front densities give relativistically exact 2D stress distributions
 - 🍂 3D Breit frame densities are a non-relativistic approximation
- 🍁 We don't really understand why the D-term is negative for hadrons
 - 🍂 Something interesting to learn about QCD here?

Thank you for your time!