

Workshop on the Physics Program with High Luminosity Facilities



Longitudinal Photons in the Factorization of Hard Scattering Processes L/T Separation R-SIDIS (Theory)

Oct. 06 2026

Leonard Gamberg

With Alessandro Bacchetta, Matteo Cerutti, Richard Whitehill

<https://arxiv.org/abs/2610.06628>

Preamble Motivation

Owing to kinematic reach & dynamical mechanisms of TMD observables in SIDIS, extraction of cross sections & mapping onto the nucleon's partonic structure TMDs & collinear PDFs/FFs continue to present significant theory & pheno. challenges (see talk Nobuo Sato)

A quantitatively reliable description requires among other things

- Consistent control of power corrections
- QED radiative effects (talk of Igor Akushevich)
- Vector meson background
- Matching of small & large transverse momentum regions within QCD Factorization
- We consider impact of *longitudinal structure* function on extraction TMDs in SIDIS
 - $\longrightarrow$ entails subleading power TMD factorization
 - $\longrightarrow$ entails matching small & large- $P_{h\perp}$ ($\approx zq_T$) regimes within QCD factorization

Preamble Motivation

- Underexplored ingredient role of longitudinally polarized virtual photons
unpolarized SIDIS “SF” $F_{UU,L}$ impact on SIDIS multiplicities TMD region $q_T \ll Q$

$$\begin{aligned} M^h(Q^2, x, z, P_{h\perp}) &= \frac{F_{UU,T}}{F_T + \epsilon F_L} \longrightarrow \frac{F_{UU,T} + \epsilon F_{UU,L}}{F_T + \epsilon F_L} \\ &= \frac{F_{UU,T}}{F_T + \epsilon F_L} (1 + \epsilon R_{SIDIS}) \end{aligned}$$

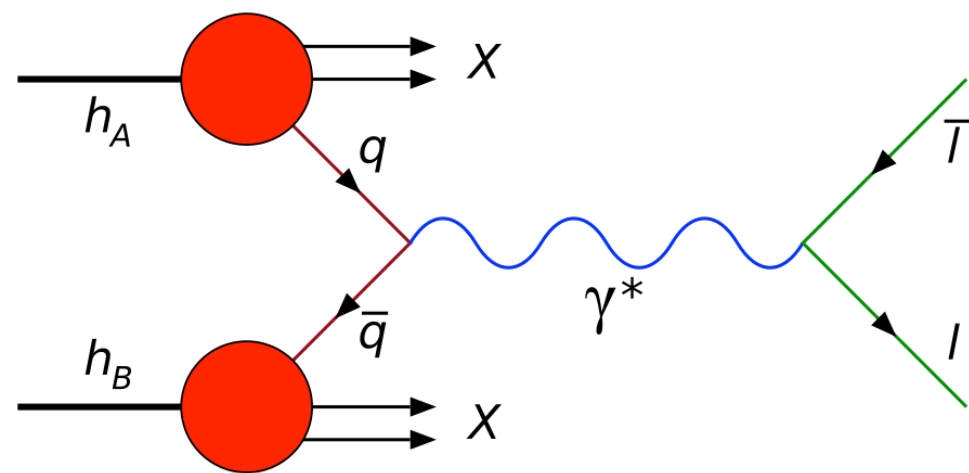
given the challenge describe unpolarized SIDIS & DY cross section ... frameworks,
e.g. MAPTMD24 TMD fit, SIDIS normalization factor Bacchetta et al., JHEP (2024)

- Could directly impact phenomenological extractions of unpolarized TMDs

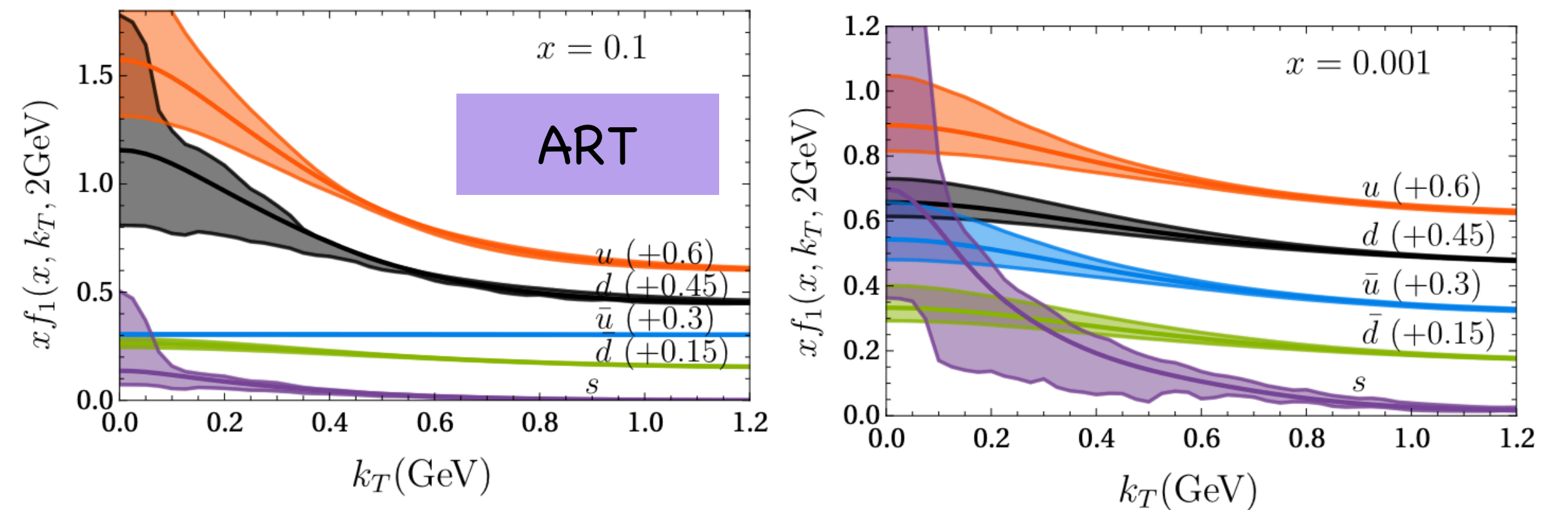
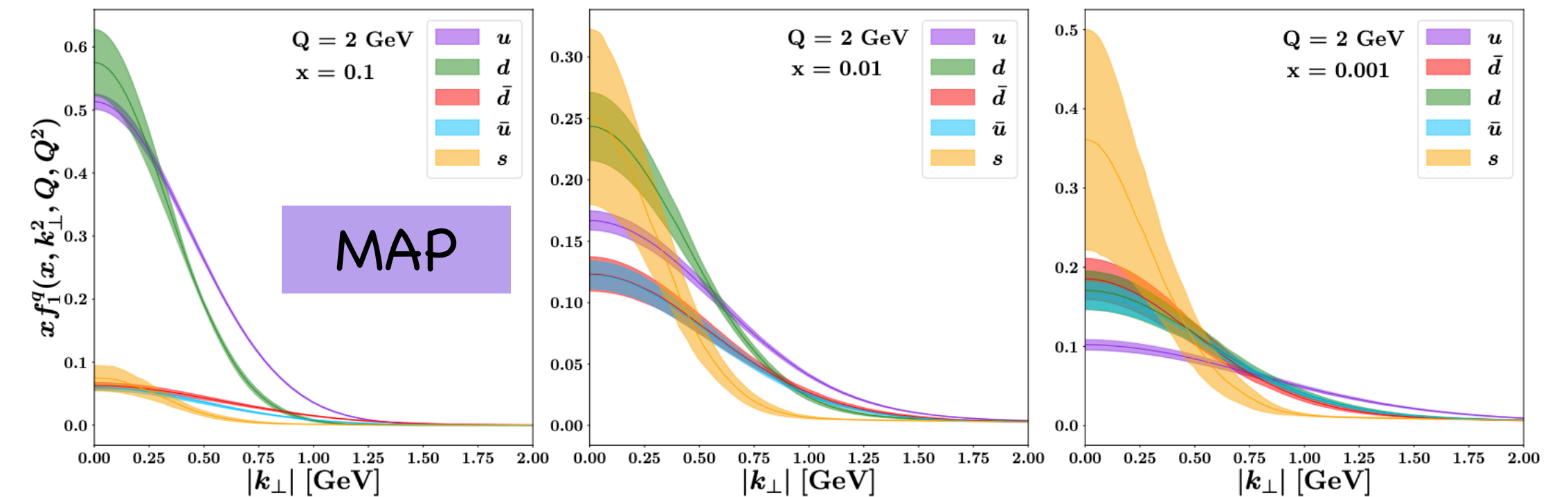
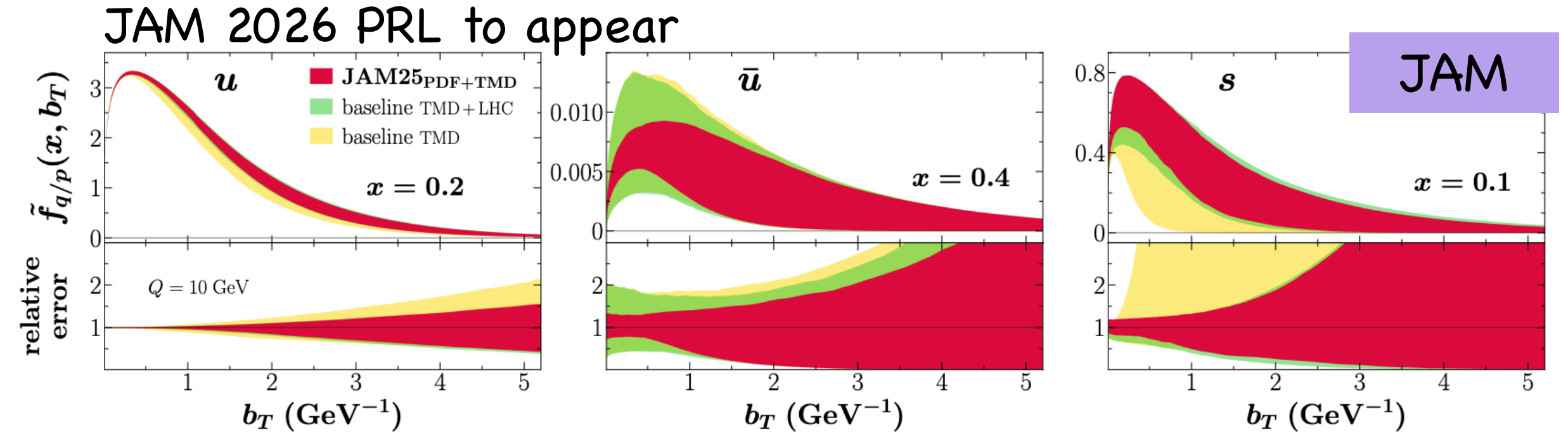
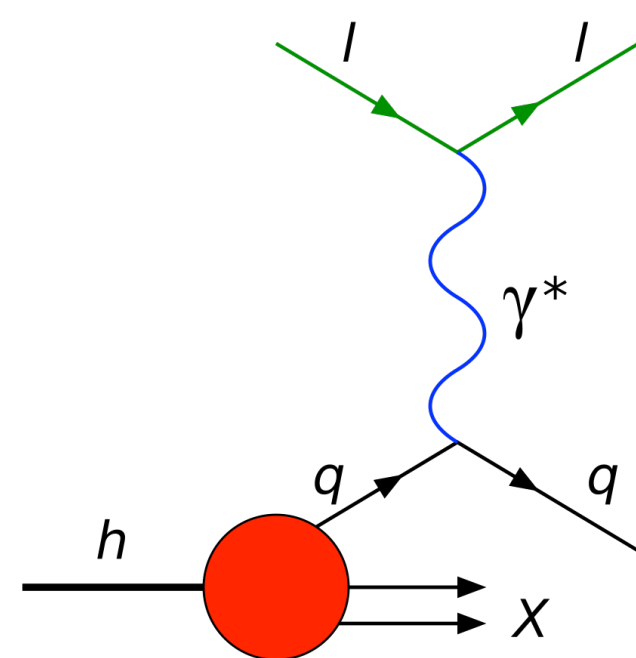
Impact on TMD extractions ?

- 1) Universality* and factorization
- 2) Global Analysis
- 3) Lattice (talk of Xiangdong Ji)

Drell-Yan



SIDIS



Preamble Motivation

- Also related & underexplored contribution $F_{UU,L}$ @ $q_T \gg Q$

$$M^h(Q^2, x, z, P_{h\perp}) = \frac{F_{UU,T}}{F_T + \epsilon F_L} (1 + \epsilon R_{SIDIS})$$

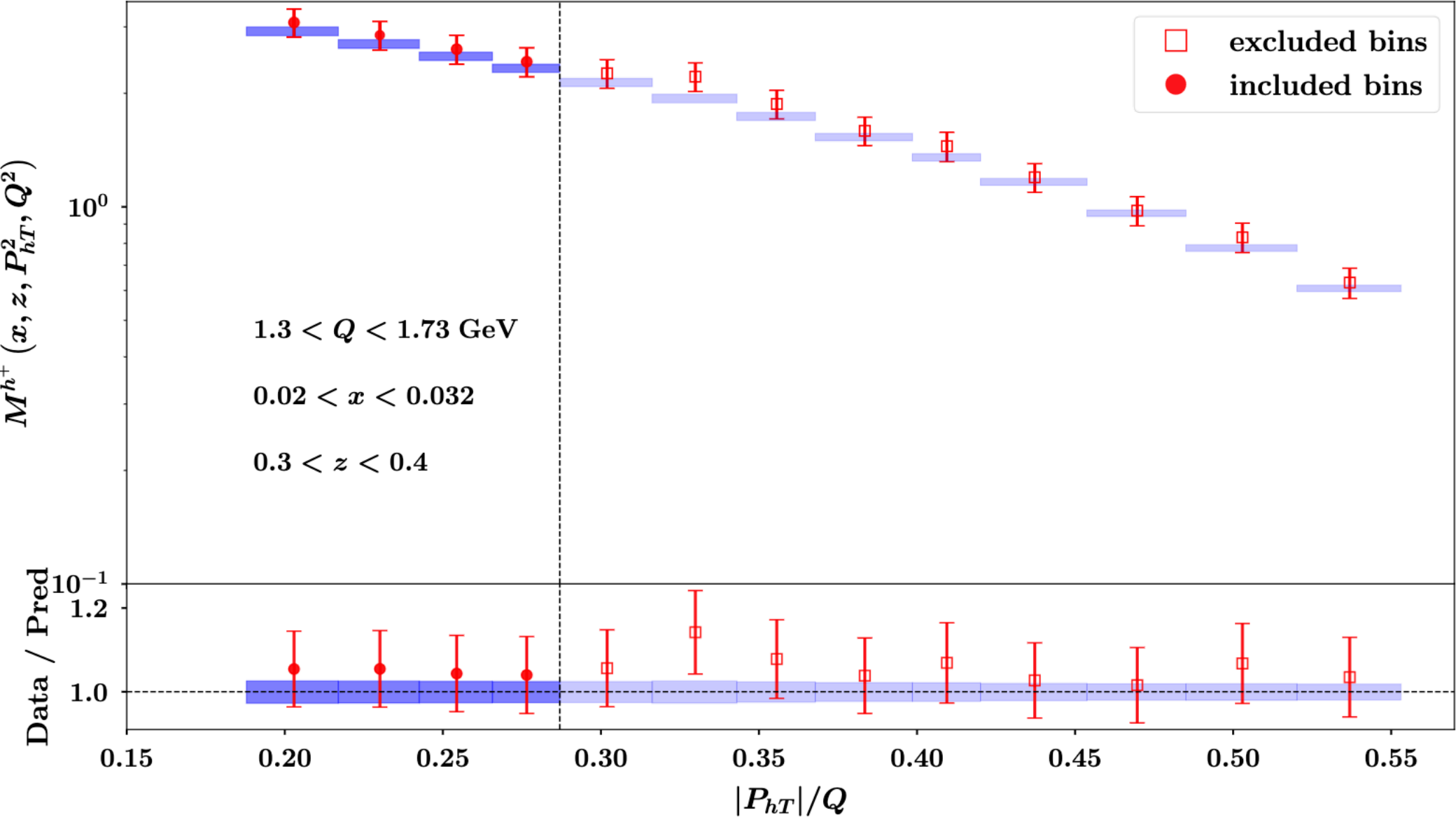
- In this study consider the $F_{UU,L}$ in a consistent factorization framework spanning the entire transverse-momentum range *panning available q_T range*
- Lever arm to probe “TMD contribution” of $\gamma_{L/T}^*$ contributions

Preamble Motivation

Multiplicity: SIDIS cross section normalized to DIS

$$M^h(Q^2, x, z, P_{h\perp}) \equiv \frac{dx dQ^2}{d^2\sigma^{\text{DIS}}(Q^2, x)} \frac{d^4\sigma^h(Q^2, x, z, P_{h\perp})}{dx dQ^2 dz dP_{h\perp}}$$

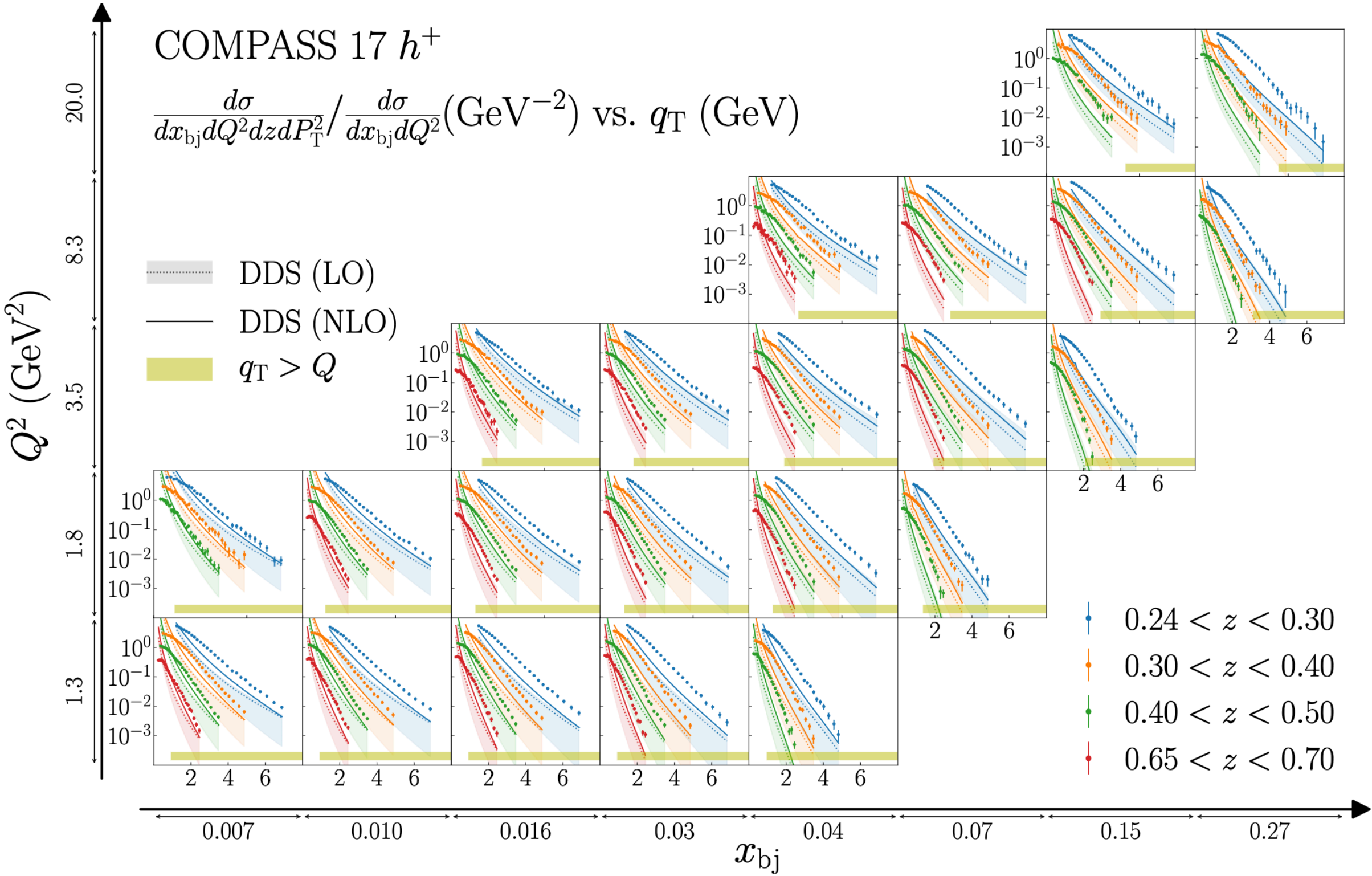
MAP Bacchetta et. al., arXiv:2206.07598v2 [hep-ph] (2022)



W term describe “large” P_{hT} data surprisingly well beyond “TMD region”? MAPTMD22

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evolution	FF evolution
NLL	0	1	2	LO	LO
N ² LL	1	2	3	NLO	NLO
N ³ LL ⁻	2	3	4	NNLO	NLO
N ³ LL	2	3	4	NNLO	NNLO

Gonzalez-Hernandez et. al., arXiv:1808.04396v2 [hep-ph] (2018)



FO theory does not describe “large” q_T data so well

Take away: Interplay of large & small P_{hT}

Preamble Motivation

Necessitates a unified treatment of

- low- $P_{h\perp} = zq_T$ region governed by TMD factorization $M \ll q_T \sim Q$
- high- $P_{h\perp} = zq_T$ region governed collinear factorization $M \sim q_T \ll Q$
controlled implementation small & large q_T momentum matching

$$\sigma_{\text{SIDIS}} \propto \left| \text{Diagram 1} \right|^2 \approx \left| \text{Diagram 2} \right|^2 \otimes \left| \text{Diagram 3} \right|^2 \otimes \left| \text{Diagram 4} \right|^2$$

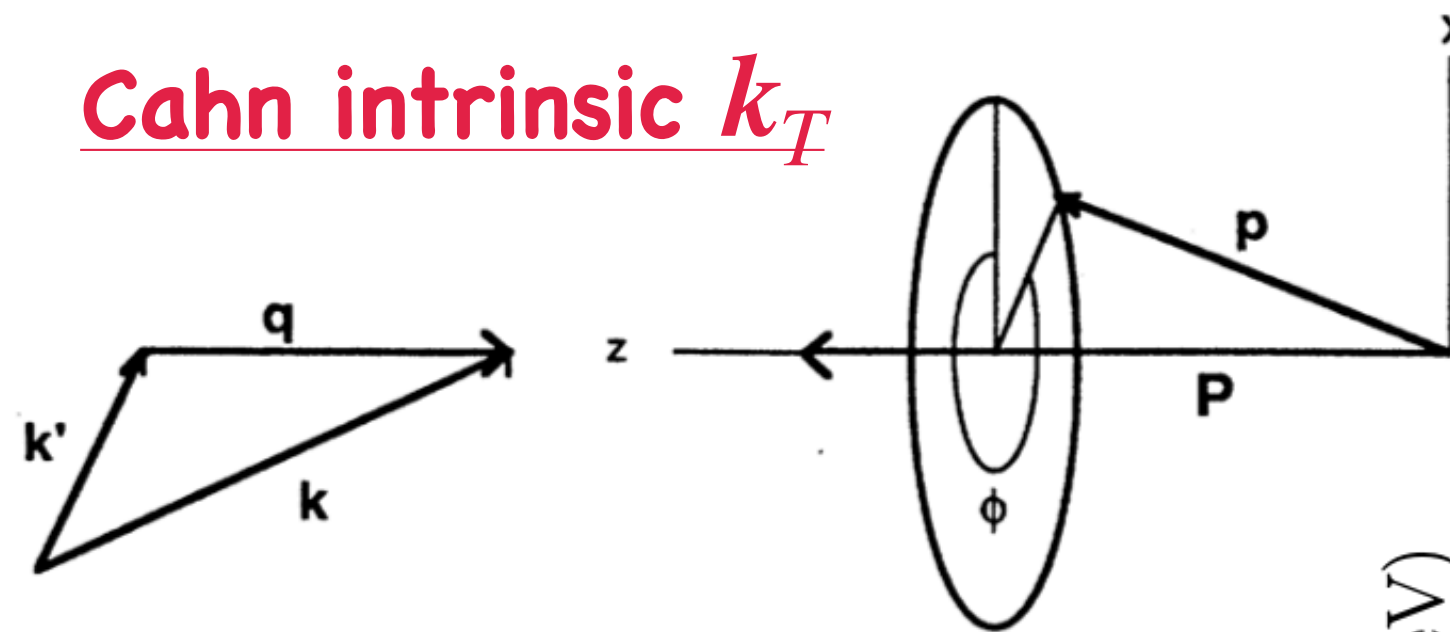
- “ $W + Y$ ” matching in the language of CSS
(nb so far only at leading power (LP), & at NLP e.g. Cahn etc ...)

1. $\implies$ Matching “low” (TMD) to “high” (collinear) P_T (or q_T) spectrum
2. Discussion: power counting & $\exists$ a resummation formalism generalized CSS?
3. Challenge of Factorization at LP NLP NNLP in the hard scale Q

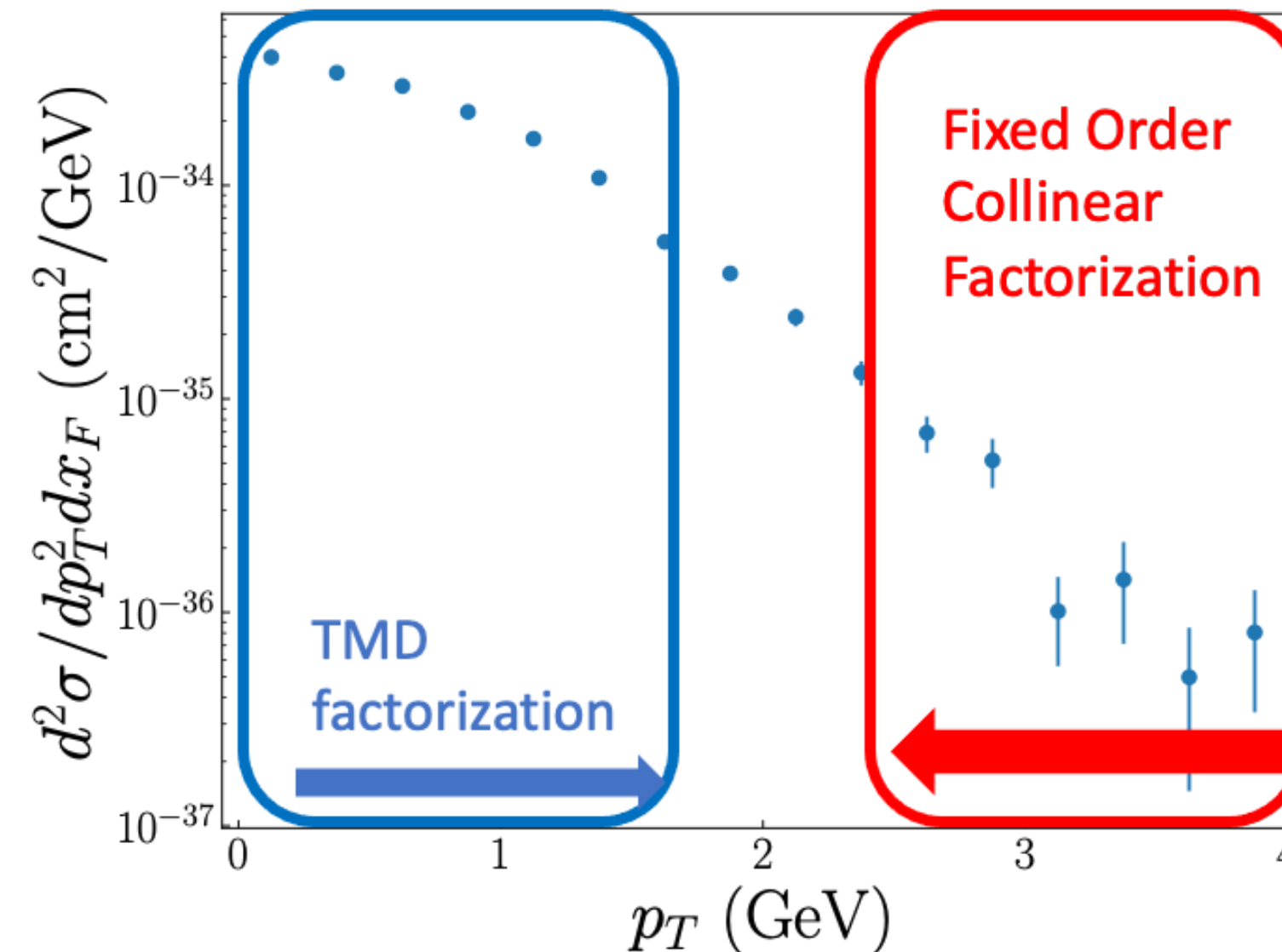
Two mechanisms? *Matching...*

Factorization & Matching collinear to TMD unpolarized/angle independent Collins Soper Stermann NPB 1985

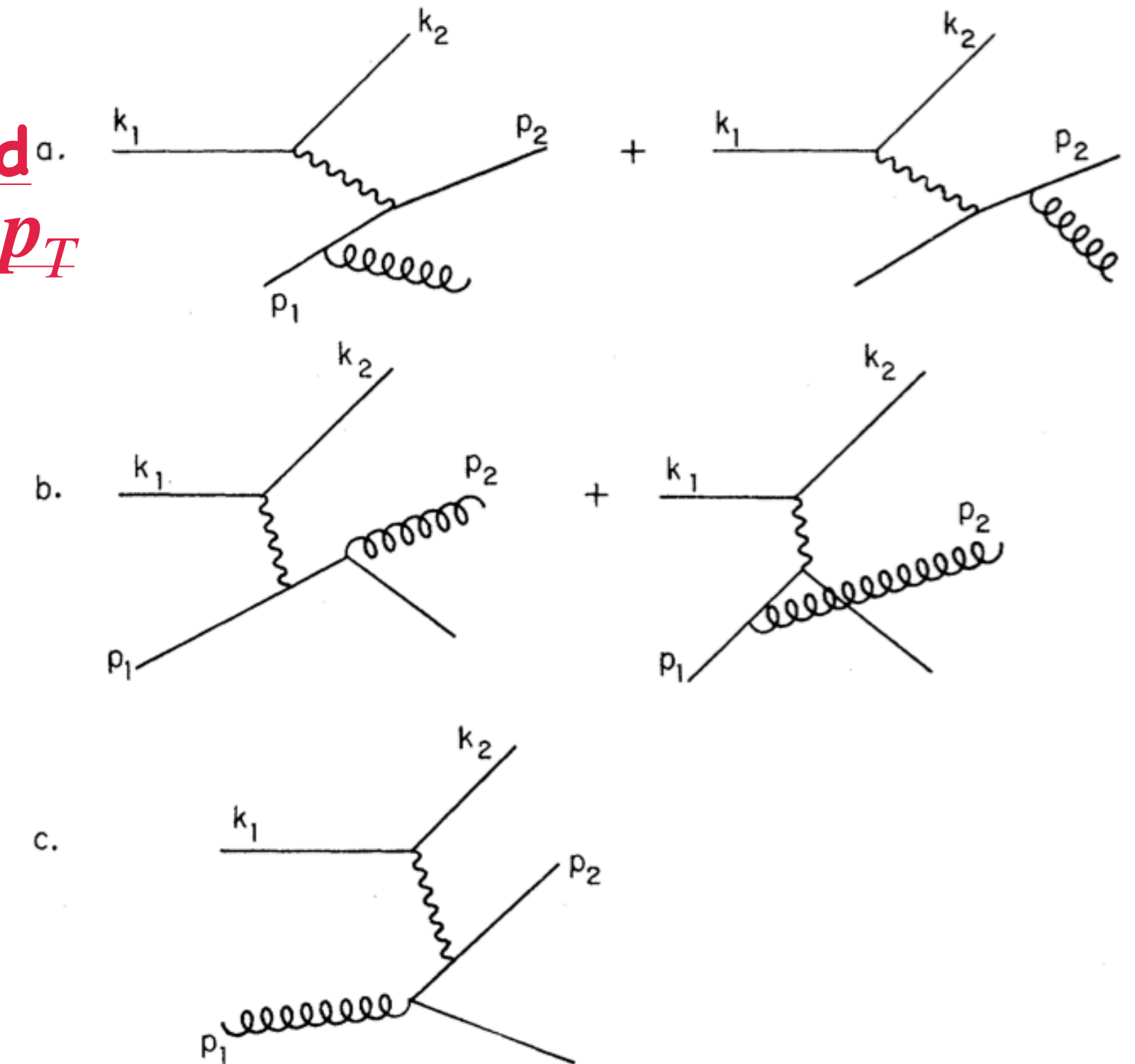
Cahn intrinsic k_T



- “TMD” region
($p_T \sim k_T$) $\sim q_T \ll Q$



Georgi & Politzer hard gluon bremsstrahlung p_T



- “Collinear” region

$$\Lambda_{qcd} \ll q_T \sim Q$$

Comprehensive study of matching the hi & low $q_T = P_{h\perp} / z$ in the AY (overlap) region in SIDIS attention given to azimuthal and polarization dependence Bacchetta, Boer, Diehl, Mulders JHEP (2008)

SIDIS basics unpolarized

$$\ell P \longrightarrow \gamma_{T/L}^* \longrightarrow \ell' h X$$

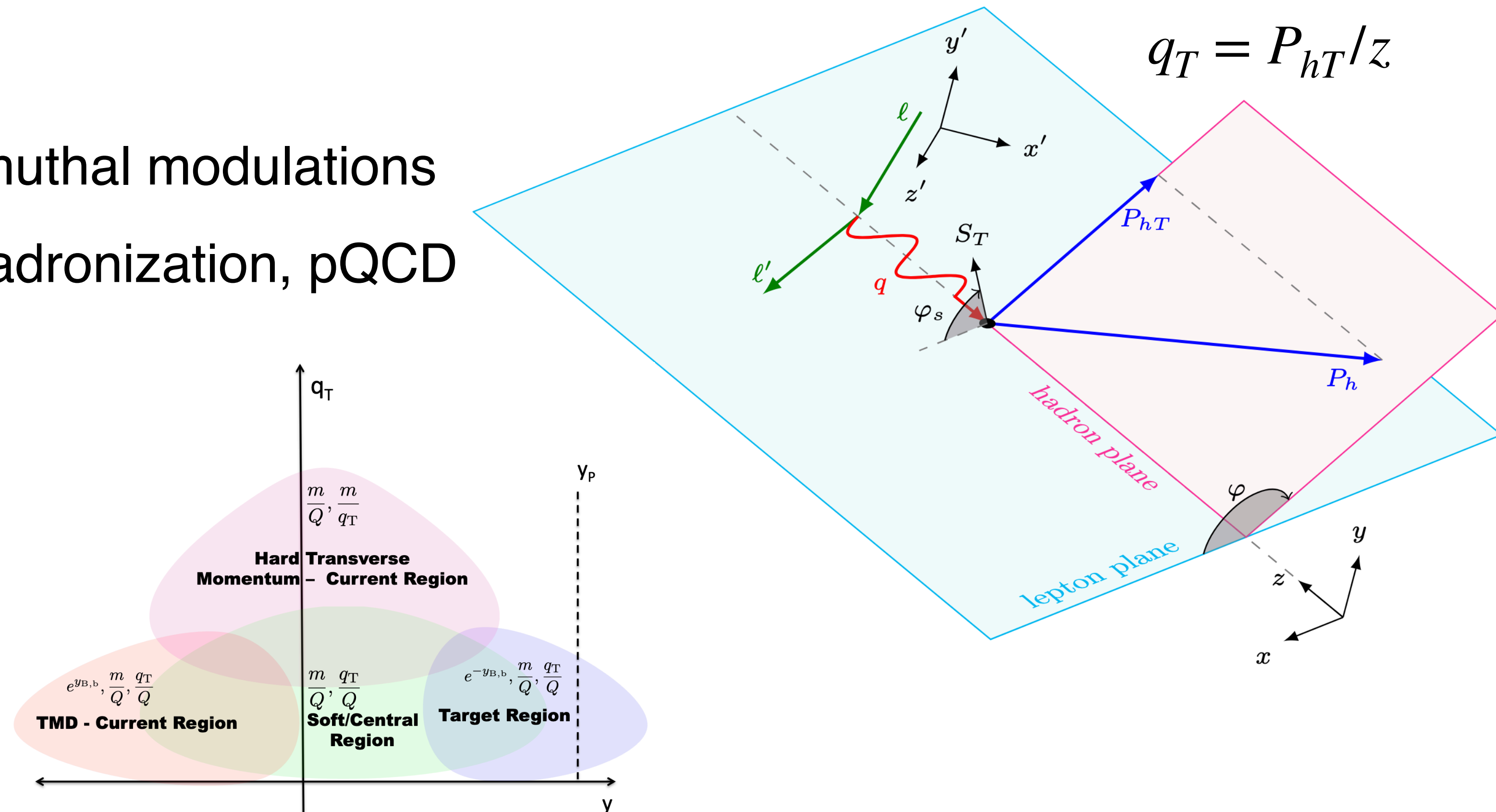
$$\frac{d\sigma}{dx dy dz dq_T^2 d\varphi} = \frac{\alpha^2 y}{Q^2(1-\varepsilon)} \left[F_{UU,T} + \varepsilon F_{UU,L} - \sqrt{2\varepsilon(1+\varepsilon)} F_{UU}^{\cos\varphi} \cos\varphi + \varepsilon F_{UU}^{\cos 2\varphi} \cos 2\varphi \right]$$

Observation of final-state hadron*

- Unique correlations encoded in azimuthal modulations
- Bridge between hadron structure, hadronization, pQCD

*produced via current fragmentation

Boglione, Collins, LG, J.-O. Hernandez
Rogers, Sato, Phys.Lett.B 766 (2017)

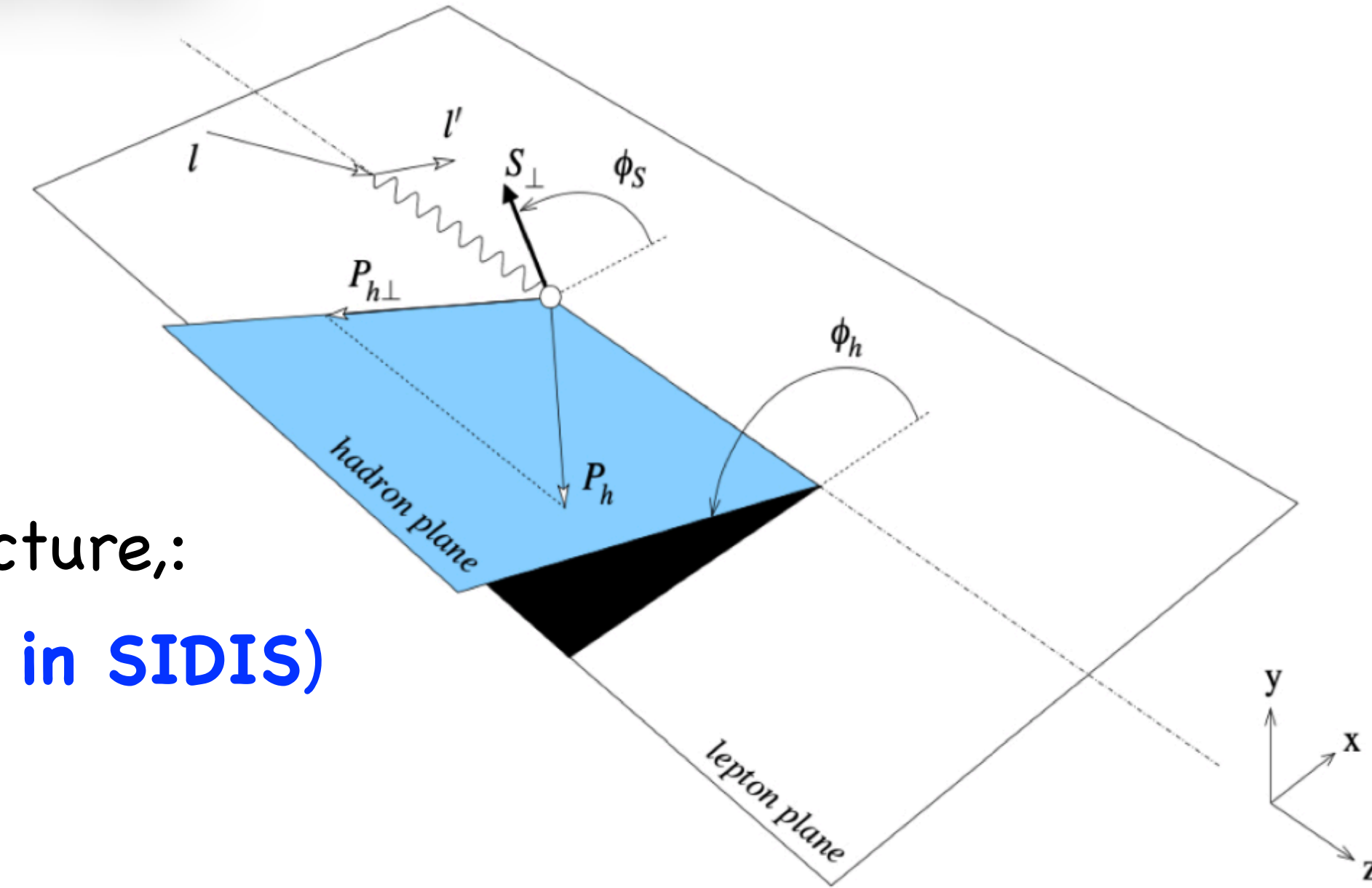


$F_{UU,L}$ NNLP structure function

- $F_{UU,L}$ & $R_{SIDIS} = \frac{\sigma_L}{\sigma_T} \sim \frac{F_{UU,L}}{F_{UU,T}}$
- Age old topic: interesting opportunity to study hadron structure,;
Ratio of longitudinal and transverse $\gamma_{T/L}^*$ cross section (now in SIDIS)
classic power suppressed $\sim (1/Q)^2$ in DIS ...
- Suggested extension beyond DIS: Feynman 1972 "Photon Hadron Interactions"
& Ravndal PLB 1973, Cahn 1989 PRD 1989:

$$R = \frac{\sigma_L}{\sigma_T} = \frac{4 (m^2 + \langle p_{\perp}^2 \rangle)}{Q^2} \stackrel{?}{\Rightarrow} \frac{F_L}{F_T} \stackrel{?}{\sim} \frac{F_{UU,L}}{F_{UU,T}}$$

$\langle p_{\perp}^2 \rangle$ intrinsic parton transverse momentum



R_{SIDIS} What do we know

$$F_{UU,L} \text{ \& } R_{SIDIS} \equiv \frac{F_{UU,L}}{F_{UU,T}} \stackrel{?}{\sim} \frac{\sigma_L}{\sigma_T} = R_{DIS}$$

- Often assumed $R_{SIDIS} \approx R_{DIS}$ however the latter independent of $(x, z, P_{h\perp}, Q^2)$
- Few measurements of $F_{UU,L}(x, z, P_{h\perp})$ – JLab Hall C E12-06-104: Nucleon R SIDIS
- In TMD pheno @ low transverse momentum often assumed negligible

20th century interpretation collinear DIS physics

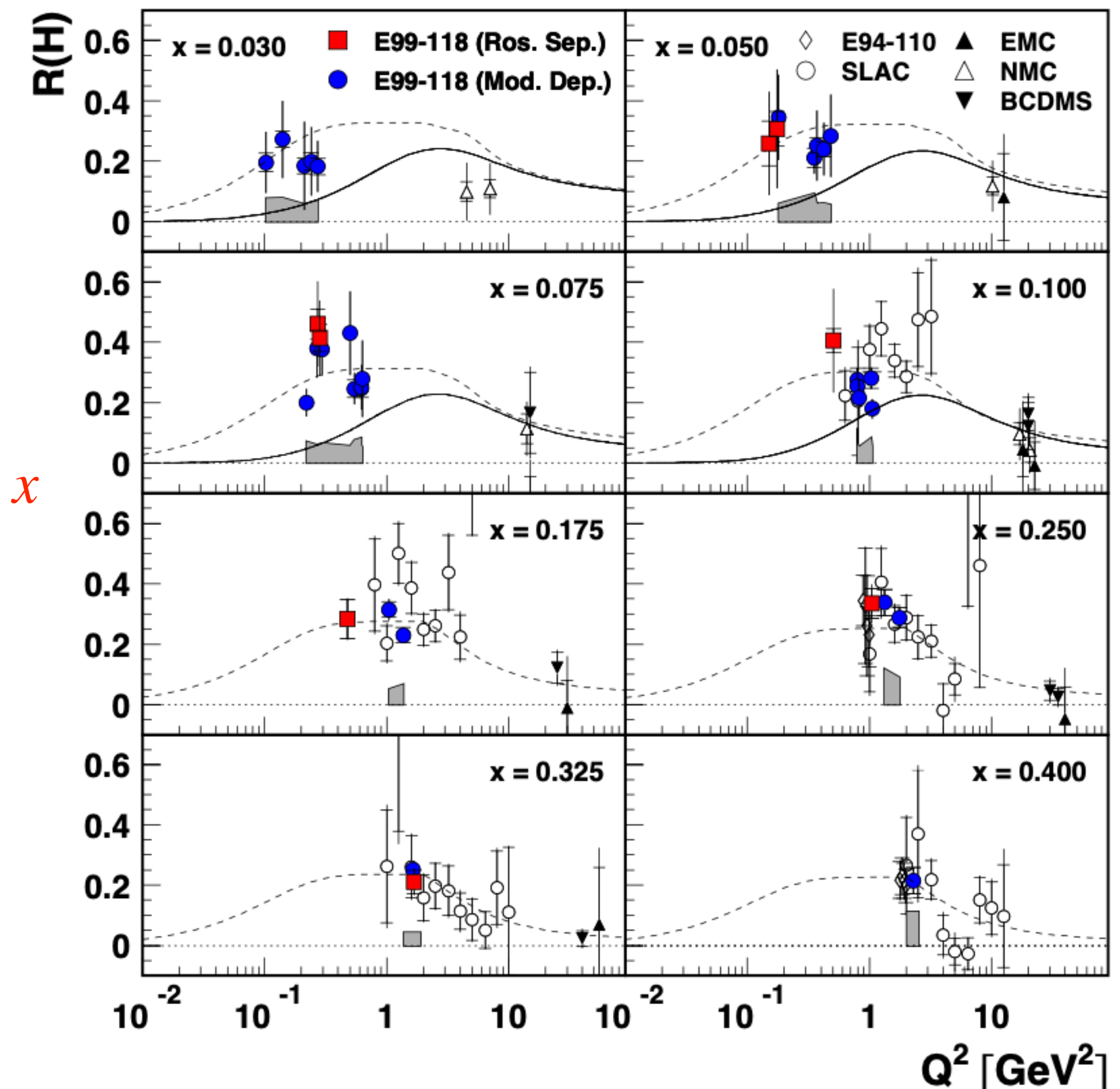
Recall inclusive cross section: σ_T and σ_L
 or structure functions F_L (F_2 & F_1) & F_T (F_1) ,
 via absorption of longitudinal vs. transverse photons

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_L}{F_T} = \frac{1}{2xF_1} \left\{ F_2 \left(1 + \frac{4x^2M^2}{Q^2} \right) - 2xF_1 \right\}$$

- Zero in **parton model** scaling limit $\lim_{Q^2 \rightarrow \infty} R \approx \frac{4x^2M^2}{Q^2} \rightarrow 0$,
- Suppressed but **leading twist** in PQCD $\{F_2 - 2xF_1\} \sim \frac{\alpha_s(Q)}{2\pi} C_2(F) x$

Comparison of values of $R(x, Q^2)$ for hydrogen from
 JLab exp. (E99-118) to results of other exps.

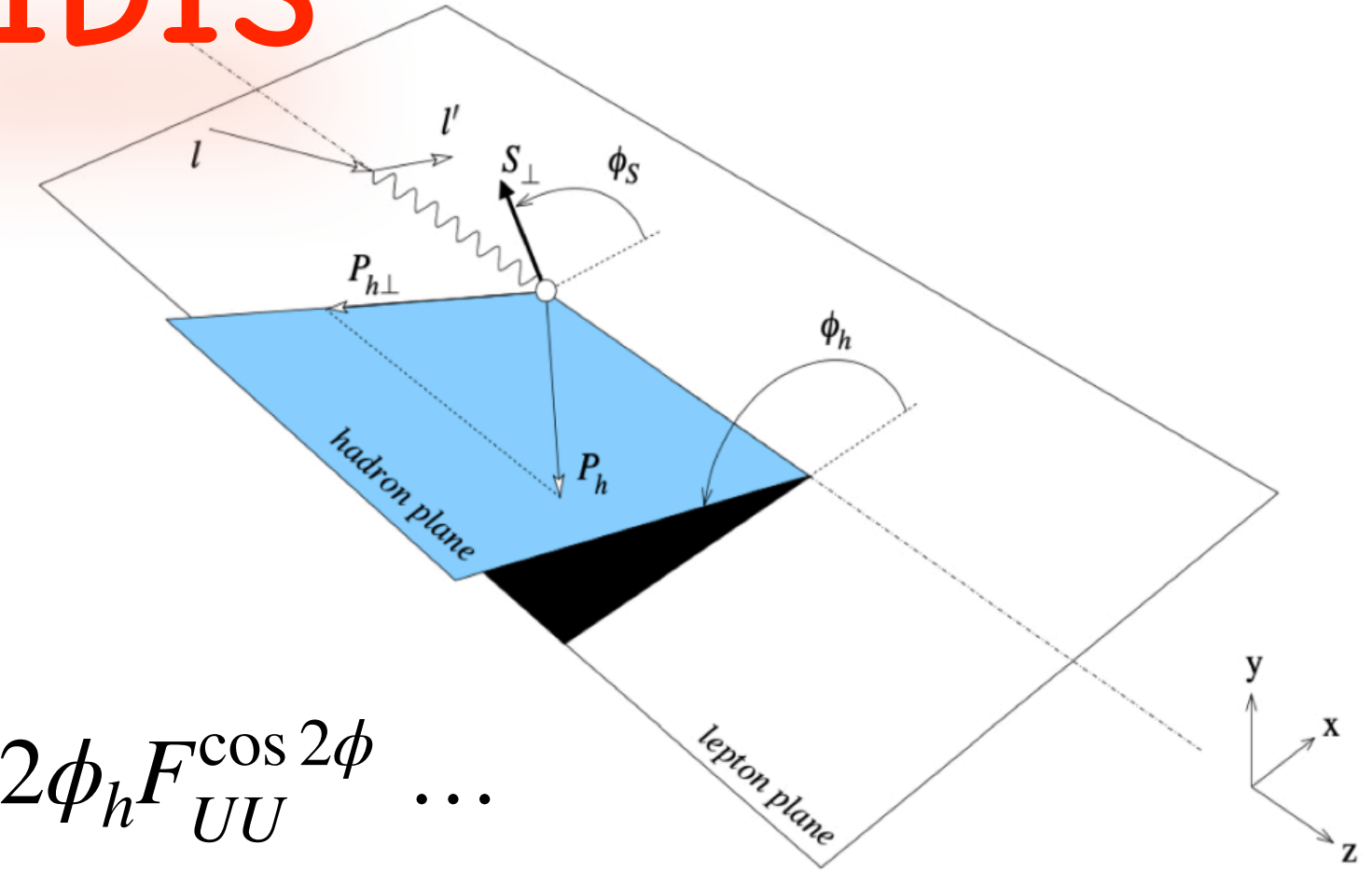
JLab exp. (E99-118) Prl 2007



21st century interpretation: from DIS to SIDIS

$$R = \frac{\sigma_L}{\sigma_T} \longrightarrow \frac{F_{UU,L}}{F_{UU,T}}$$

$$\frac{d\sigma}{dx dy dz d\phi_h dP_{h\perp}} = \frac{\alpha^2}{x Q^2} \frac{y}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{UU}^{\cos \phi} + \varepsilon \cos 2\phi_h F_{UU}^{\cos 2\phi} \dots \right.$$



- Does $\exists$ TMD observable for $F_{UU,L}$?
- Tree level TMD factorization Mulders et al. NPB 1996 Bacchetta et al. JHEP 2007

$$F_{UU,T} = \mathbb{C} [f_1 D_1] ,$$

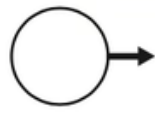
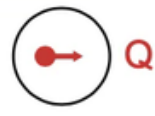
$$F_{UU,L} = \mathbb{C} [. ? .]$$

$$\mathcal{C}[wfD] = \sum_a x e_a^2 \int d^2 \mathbf{p}_T d^2 \mathbf{k}_T \delta^{(2)}(\mathbf{p}_T - \mathbf{k}_T + \mathbf{q}_T) w(\mathbf{p}_T, \mathbf{k}_T) f^a(x, p_T^2) D^a(z, k_T^2)$$

Reminder: recall Quark correlator LP, NLP

n.b. "tree level" Mulders Tangerman 1996 Bacchetta 2007

$$\Phi(x, p_T) = \frac{1}{2} \left\{ f_1 \not{n} + f_{1T}^\perp \frac{S_{Ti} \varepsilon^{ij} p_{Tj}}{M} \not{n} + \dots \right\} + \frac{M}{2P^+} \left\{ e + \dots + f^\perp \frac{\not{p}_T}{M} \dots \right\}$$

Leading Quark TMDPDFs  Nucleon Spin  Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{Unpolarized}$		$h_1^\perp = \text{Boer-Mulders}$
	L		$g_1 = \text{Helicity}$	$h_{1L}^\perp = \text{Worm-gear}$
	T	$f_{1T}^\perp = \text{Sivers}$	$g_{1T}^\perp = \text{Worm-gear}$	$h_1 = \text{Transversity}$ $h_{1T}^\perp = \text{Pretzelosity}$

Subleading Quark TMDPDFs

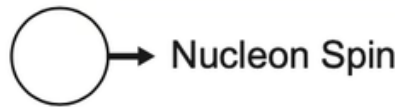
		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f^\perp, g^\perp$	e, h
	L	$f_L^\perp, g_L^\perp$	e_L, h_L
	T	$f_T, f_T^\perp, g_T, g_T^\perp$	$e_T, e_T^\perp, h_T, h_T^\perp$

Reminder: recall Quark correlator LP, NLP

n.b. “tree level” Mulders Tangerman 1996 Bacchetta 2007

$$\Phi(x, p_T) = \frac{1}{2} \left\{ f_1 \not{n} + f_{1T}^\perp \frac{S_{Ti} \varepsilon^{ij} p_{Tj}}{M} \not{n} + \dots \right\} + \frac{M}{2P^+} \left\{ e + \dots + f^\perp \frac{\not{p}_T}{M} \dots \right\} + \frac{M^2}{(2P^+)^2} \left\{ \dots \quad ?? \quad \dots \right\}$$

Leading Quark TMDPDFs



		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{Unpolarized}$		$h_1^\perp = \text{Boer-Mulders}$
	L		$g_1 = \text{Helicity}$	$h_{1L}^\perp = \text{Worm-gear}$
	T	$f_{1T}^\perp = \text{Sivers}$	$g_{1T}^\perp = \text{Worm-gear}$	$h_1 = \text{Transversity}$ $h_{1T}^\perp = \text{Pretzelosity}$

Subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f^\perp, g^\perp$	e, h
	L	$f_L^\perp, g_L^\perp$	e_L, h_L
	T	$f_T, f_T^\perp, g_T, g_T^\perp$	$e_T, e_T^\perp, h_T, h_T^\perp$

Sub-subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	???	
	L		
	T		

Context TMD Correlator at tree level NNLP "twist 4"

Goeke, Metz, Schlegel PLB 2005

NNLP:

$$\Phi^{[\gamma^-]} = \frac{M^2}{(P^+)^2} \left[f_3(x, \vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{3T}^\perp(x, \vec{k}_T^2) \right]$$

- Correlator at tree level @ "twist" 4
- previously of academic interest
- factorization relatively unexplored

Recall *LP* & *NLP*:

$$\Phi^{[\gamma^+]} = f_1(x, \vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{1T}^\perp(x, \vec{k}_T^2),$$

$$\Phi^{[\gamma^i]} = \frac{M}{P^+} \left[\frac{k_T^i}{M} \left(f^\perp(x, \vec{k}_T^2) - \frac{\varepsilon_T^{jk} k_{Tj} S_{Tk}}{M} f_T^{\perp'}(x, \vec{k}_T^2) \right) + \dots \right]$$

Sub-subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f_3 = \odot$ Unpolarized	
	L		
	T	$f_{3T}^\perp = \uparrow \odot - \downarrow \odot$ Sivers-type	

Leading Quark TMDPDFs  Nucleon Spin  Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \odot$ Unpolarized		$h_1^\perp = \uparrow \odot - \downarrow \odot$ Boer-Mulders
	L		$g_1 = \rightarrow \odot - \leftarrow \odot$ Helicity	$h_{1L}^\perp = \rightarrow \odot - \leftarrow \odot$ Worm-gear
	T	$f_{1T}^\perp = \uparrow \odot - \downarrow \odot$ Sivers	$g_{1T}^\perp = \rightarrow \odot - \leftarrow \odot$ Worm-gear	$h_1 = \uparrow \odot - \downarrow \odot$ Transversity $h_{1T}^\perp = \rightarrow \odot - \leftarrow \odot$ Pretzelosity

Subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f^\perp, g^\perp$	e, h
	L	$f_L^\perp, g_L^\perp$	e_L, h_L
	T	$f_T, f_T^\perp, g_T, g_T^\perp$	$e_T, e_T^\perp, h_T, h_T^\perp$

Context TMD Correlator at tree level NNLP “twist 4”

Goeke, Metz, Schlegel PLB 2005

NNLP:

$$\Phi^{[\gamma^-]} = \frac{M^2}{(P^+)^2} \left[f_3(x, \vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{3T}^\perp(x, \vec{k}_T^2) \right]$$

- Correlator at tree level @ “twist” 4
- previously of academic interest
- factorization relatively unexplored

Sub-subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f_3 = \odot$ Unpolarized	
	L		
	T	$f_{3T}^\perp = \uparrow \odot - \downarrow \odot$ Sivers-type	

Twist-4 contributions SIDIS.

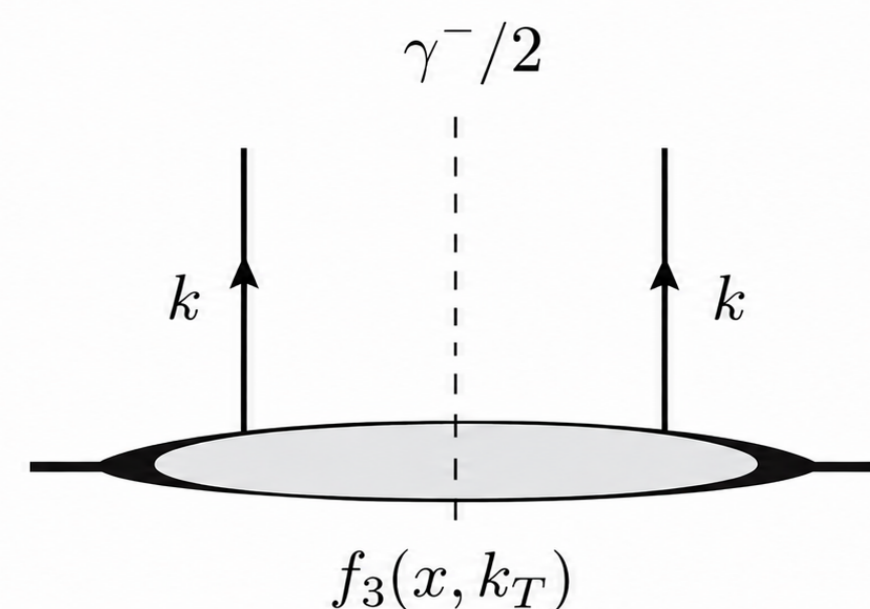
Need multiparton correlations also ...

Wei et al., PRD 95, 074017 (2017)

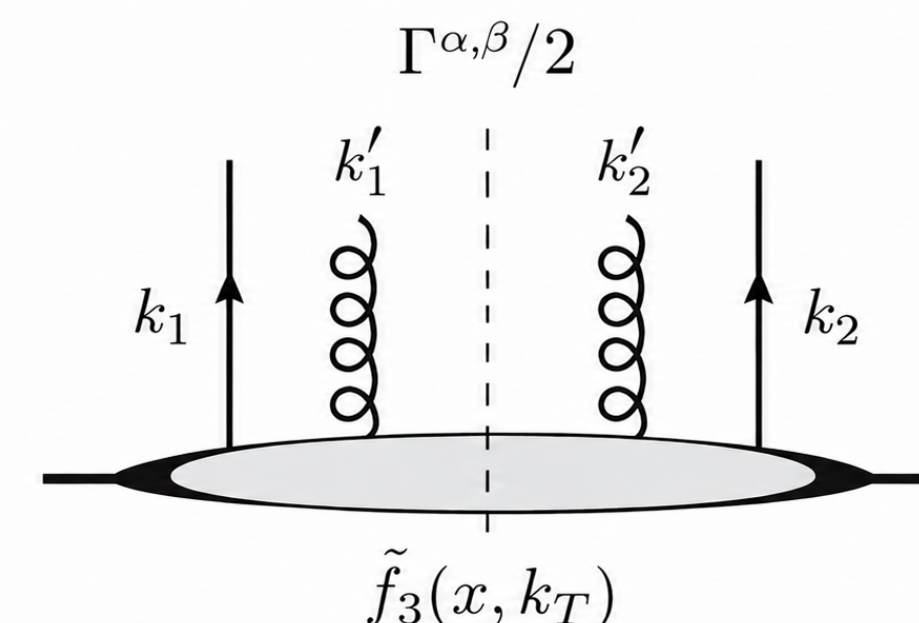
Qiu & Sterman, NPB 353, 105 (1991)

Qiu & Sterman, NPB 353, 137 (1991)

Intrinsic twist-4 quark-quark TMD (f_3)



Dynamical twist-4 quark-gluon-gluon-quark TMD ($\tilde{f}_3$)



Context TMD Correlator at tree level NNLP “twist 4”

NNLP: NNLP: some discussion in Bacchetta et al.
Matches and mismatches JHEP 2008

$$F_{UU,L} = \frac{M^2}{Q^2} \mathbb{C} \left[w(p_T) f_1 D_1 + \dots \right] = \frac{M^2}{Q^2} \mathbb{C} \left[\frac{4p_T^2}{M^2} f_1 D_1 + \dots \right]$$

- Correlator at tree level @ “twist” 4
- previously of academic interest
- factorization ~~relatively unexplored~~

R_{SIDIS} NLP TMDs ?

R_{sidis} prelim MAP coll. Sizable up to 20%

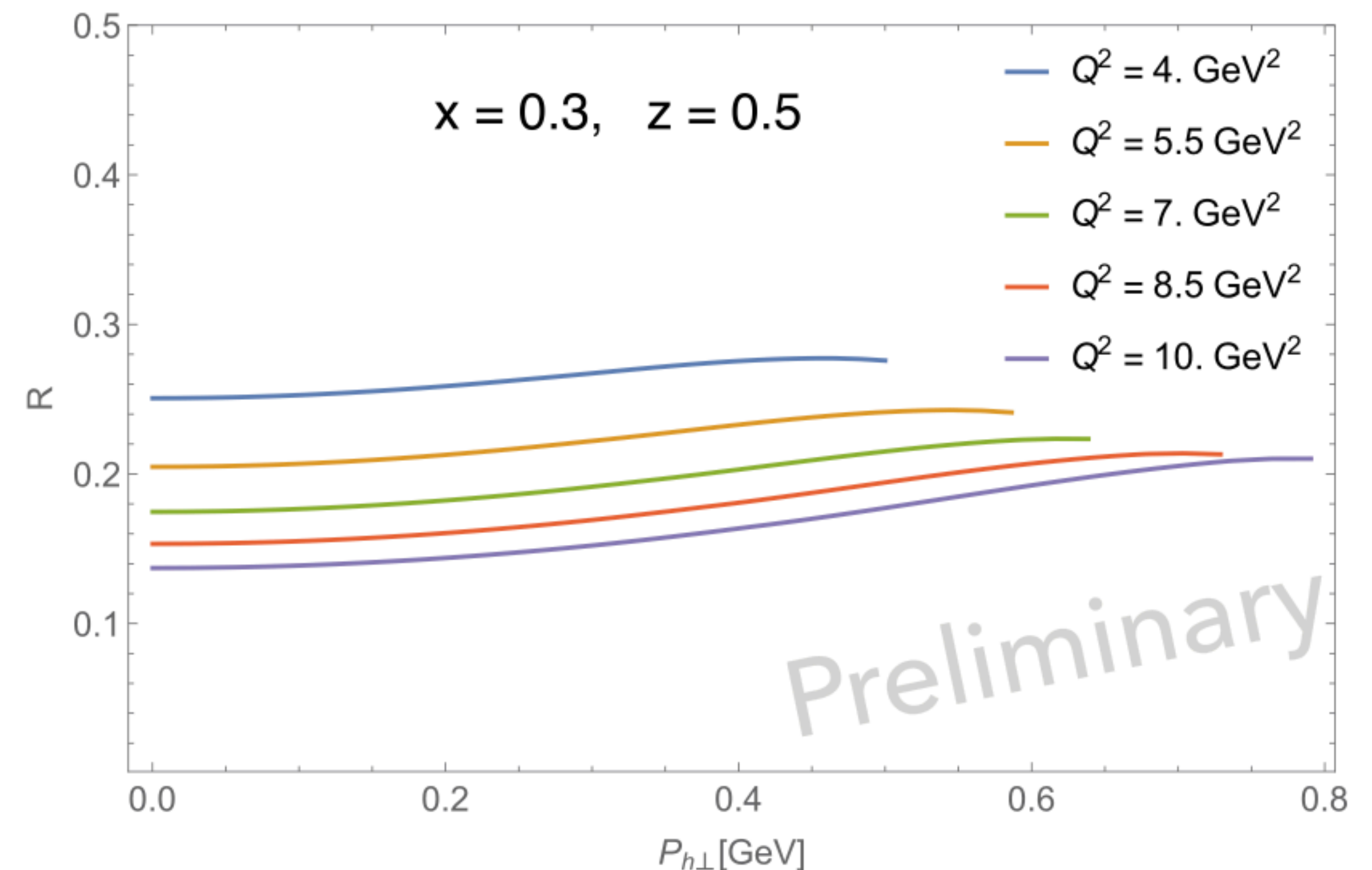
$$\frac{d\sigma}{dx dy dz d\phi_S d\phi_h dP_{h\perp}} = \frac{\alpha^2}{xQ^2} \frac{y}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} \right.$$

$$F_{UU,L} = \frac{M^2}{Q^2} \mathbb{C} \left[\frac{4p_T^2}{M^2} f_1 D_1 \right] \quad R = \frac{F_{UU,L}}{F_{UU,T}}$$

ε ratio of longitudinal and transverse photon flux

- Findings demonstrate $F_{UU,L}$ can't be ignored
- substantial & essential for an accurate interpretation of $F_{UU,T}$
- which is associated with LP TMDs
- SIDIS normalization in TMD factorization ?

Bacchetta & Cerruti MAP Eur. Phys. J. A (2024) 60:173



Estimate of R_{SIDIS} versus hadron transverse momentum $P_{h\perp}$ @ fixed x and z and for values Q^2 , compatible with JLab22 kinematics, using MAP22 TMD analysis

As stated we consider also q_T $F_{UU,L}(x, z, q_T, Q)$

Take “granular” look: both @ small & large $q_T \approx P_{h\perp}/z$ structure functions

- $F_{UU,T}(x, z, q_T, Q)$

- $F_{UU,L}(x, z, q_T, Q)$

Key point for SIDIS & global analysis (GA) and whether the low & high q_T physics necessary for GA ?

- Explore thru factorization low & high q_T “matching”

- i) consider power counting: necessary conditions for match
 - ii) direct calculation: sufficient conditions for match

e.g. Allows extension of simultaneous fits of colliner pdfs & TMDs JAM PRL 2026 to appear
Barry, Prokudin, Anderson, Cocuzza, LG, Melnitchouk, Moffat, Pitonyak, Qiu, Sato, Vladimirov, Whitehill

$$\tilde{f}(x, b_T; \mu, \zeta) = [C \otimes f](x, b_T; \mu_0, \zeta_0) \times e^{S(b_T; \mu, \mu_0, \zeta, \zeta_0)} f_{NP}(x, b_T)$$

Explore thru factorization low & high q_T “matching”

i) consider power counting: necessary conditions for match

ii) direct calculation: sufficient conditions for match

Power counting in regions

Bacchetta et al. JHEP 2008

Power counting $F_{XY,Z} \longrightarrow F_{UU,L} \text{ \& } F_{UU,T} \dots$

- Low TMD factorization

$$M \sim q_T \ll Q$$

$$F_{UU,T} \sim \mathbb{C} [f_1 D_1]$$

Leading power

$$F_{UU,L} \sim \frac{M^2}{Q^2} \mathbb{C} \left[\frac{p_T^2}{M^2} f_1 D_1 \right]$$

Sub-sub leading

- High collinear factorization

$$M \ll q_T \sim Q$$

$$F_{UU,T} = \alpha_s \mathbb{F} [f_1 D_1]$$

Leading power

$$F_{UU,L} = \alpha_s \mathbb{F} [f_1 D_1]$$

Leading power

Power counting in “intermediate region”

Power counting $F_{UU,L}$

- Low TMD factorization $\longrightarrow M \ll q_T \ll Q \longleftarrow$ • High collinear factorization

$$F_{UU,L}^{M \ll q_T \ll Q} \sim \alpha_s \frac{q_T^2}{Q^2} \frac{M^2}{q_T^2} \mathcal{F}[f_1 D_1]$$

$$F_{UU,L}^{M \ll q_T \ll Q} \sim \alpha_s \frac{M^2}{q_T^2} \frac{q_T^2}{Q^2} \mathcal{F}[f_1 D_1]$$

There could be a Match

$$F_{UU,L}^{M \ll q_T \ll Q} \sim \frac{\alpha_s}{Q^2} \mathcal{F}[f_1 D_1]$$

Explore thru factorization low & high q_T “matching”

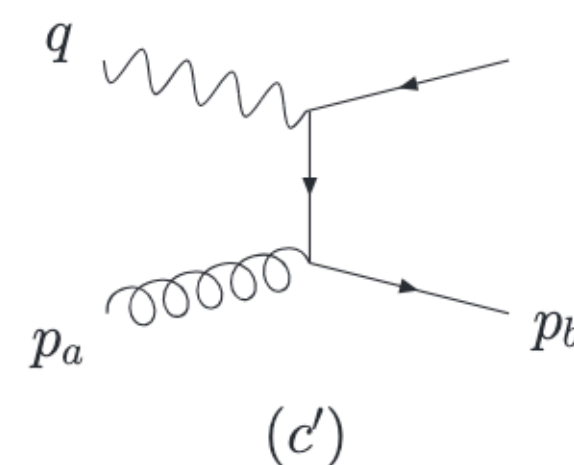
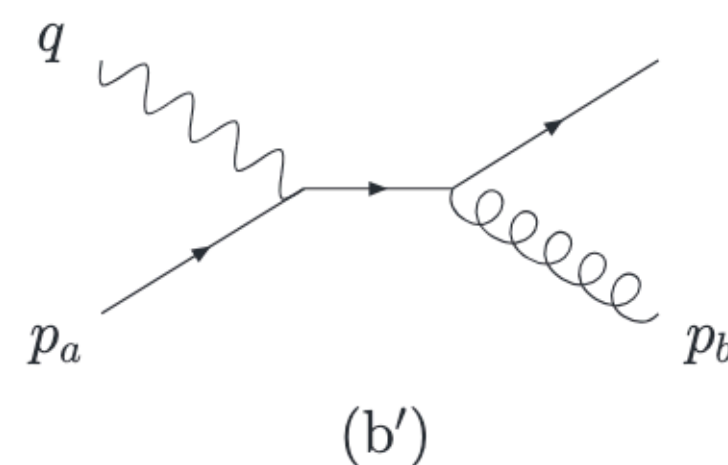
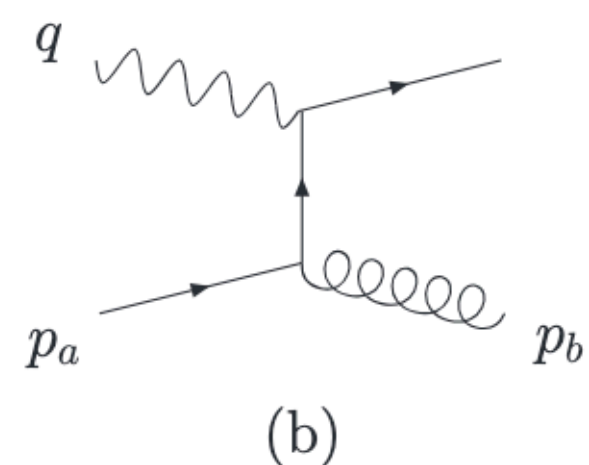
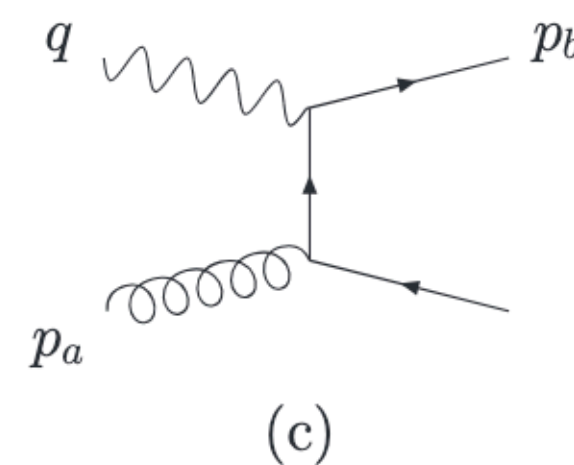
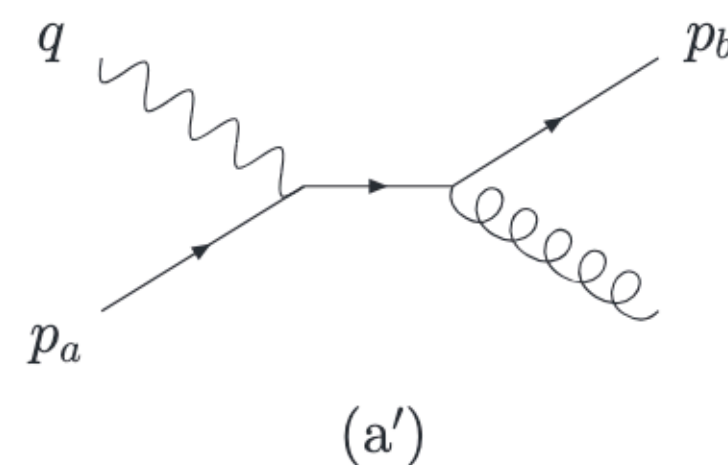
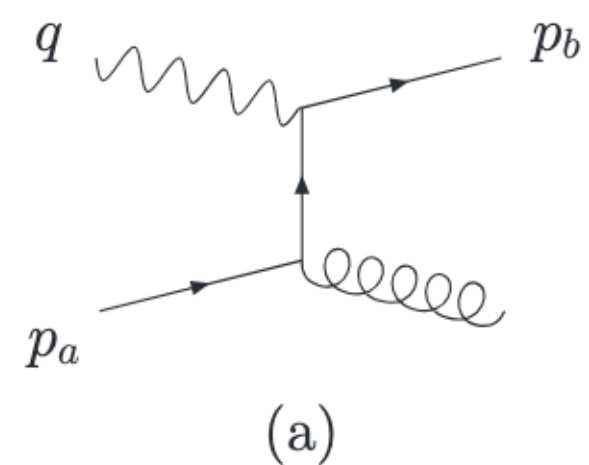
i) consider power counting: necessary conditions for match

ii) direct calculation: sufficient conditions for **match**

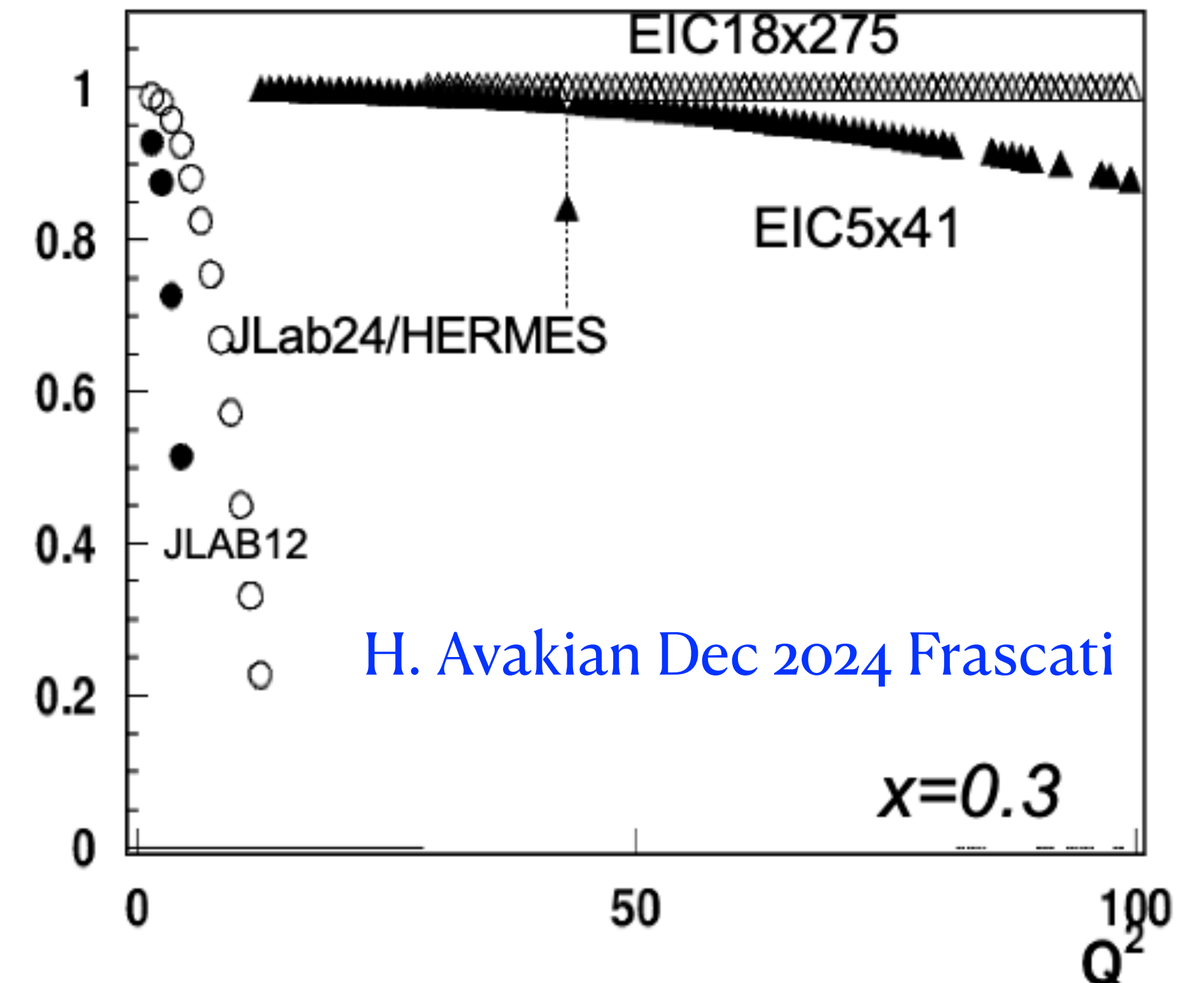
Consider $F_{UU,L}$ & R_{SIDIS} @ large $P_{h\perp}$

- @ large $P_{h\perp}$, $F_{UU,L} \sim F_{UU}^{\cos 2\phi_h}$ — see Bacchetta et al. JHEP 2008 “Matches & Mis-matches”:
so in principle hard gluon radiation — “collinear P_T factorization applies CSS 1985
Catani et al. 1997-2015, Nadolsky, Vogelsang Koike NPB 2005 ... many others

$$\frac{d\sigma}{dx dy dz d\phi_h dP_{h\perp}} = \frac{\alpha^2}{xQ^2} \frac{y}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \dots \right.$$



ε



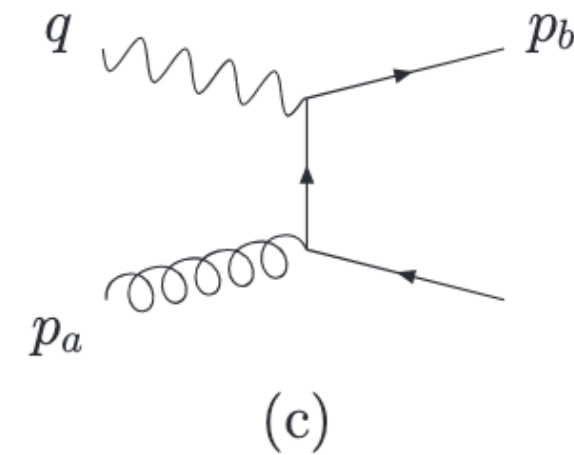
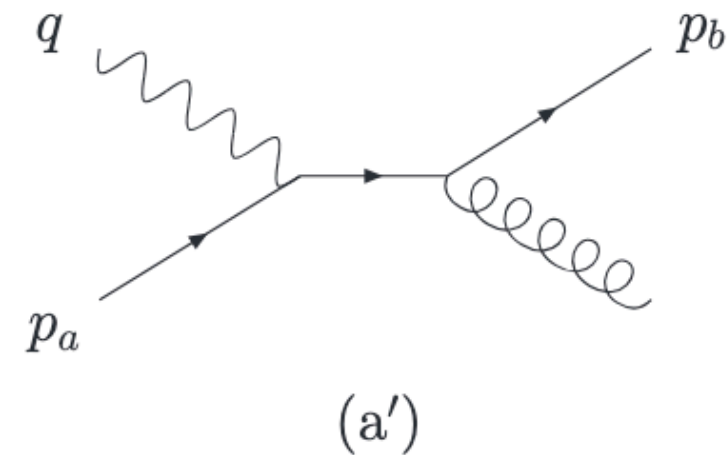
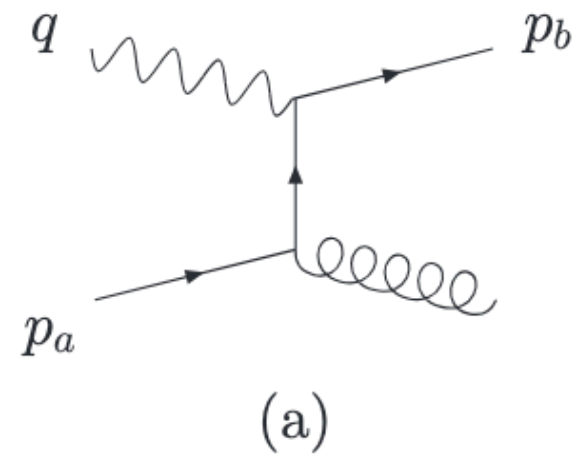
H. Avakian Dec 2024 Frascati

Large $q_T \sim P_{h\perp}/z$ well established factorization **Leading POWER** $F_{UU,T}$

$$M \ll q_T \sim Q$$

i.e.

$$F_{UU,T/L} = \frac{1}{Q^2} \frac{\alpha_s}{2\pi^2 z} \int_{\xi_{\min}}^1 \frac{d\xi}{\xi} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} \delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \\ \times \sum_{ij} C_{UU,T/L}^{(ij)}(\hat{x}, Q^2, \hat{z}, q_T) f_{i/N}(\xi, Q^2) D_{h/j}(\zeta, Q^2),$$

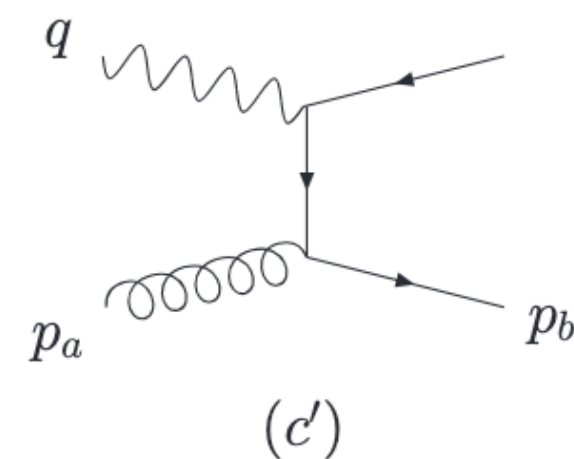
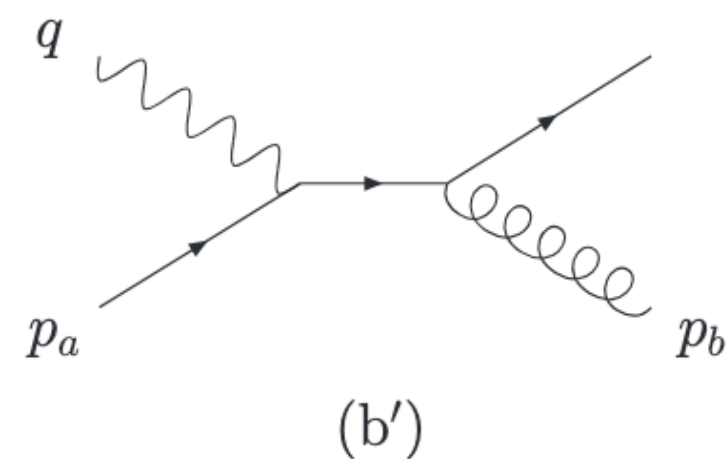
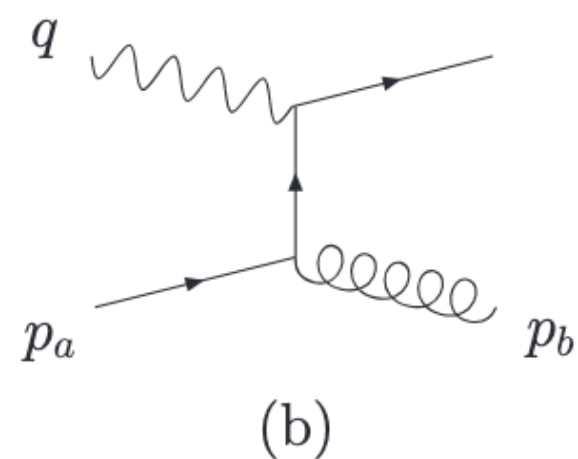


$$\bullet \gamma^* q \rightarrow qq$$

$$C_{UU,T}^{(qq)} = 2C_F e_q^2 \left\{ 2\hat{x}\hat{z} + \left[2 + \frac{(1-\hat{x})^2 + (1-\hat{z})^2}{\hat{x}\hat{z}} \right] \frac{Q^2}{q_T^2} \right\}$$

$$\bullet \gamma^* q \rightarrow gq$$

$$C_{UU,T}^{(qg)} = 2C_F e_q^2 (1-\hat{z}) \left\{ 2\hat{x} + \frac{1 + (\hat{x} - \hat{z})^2}{\hat{x}\hat{z}^2} \frac{Q^2}{q_T^2} \right\}$$



$$\bullet \gamma^* g \rightarrow q\bar{q}$$

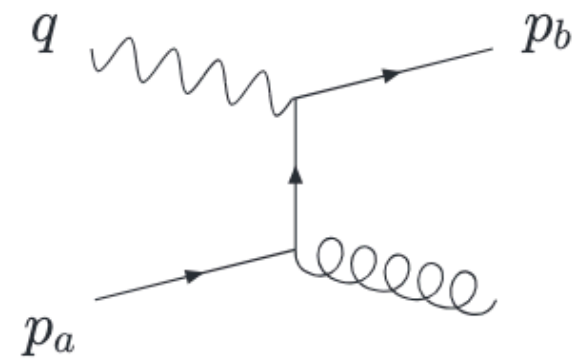
$$C_{UU,T}^{(gq)} = 2T_R e_q^2 (1-\hat{x}) \left\{ 4\hat{x} + \frac{1 - 2\hat{x}(1-\hat{x}) - 2\hat{z}(1-\hat{z})}{\hat{x}\hat{z}^2} \frac{Q^2}{q_T^2} \right\}$$

Large $q_T \sim P_{h\perp}/z$ well established factorization **Leading POWER** $F_{UU,L}$

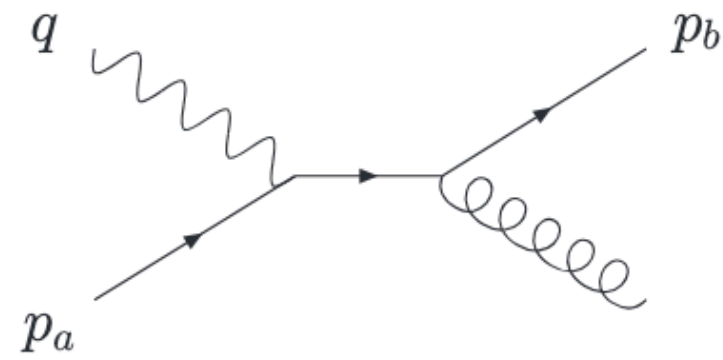
$$M \ll q_T \sim Q$$

i.e.

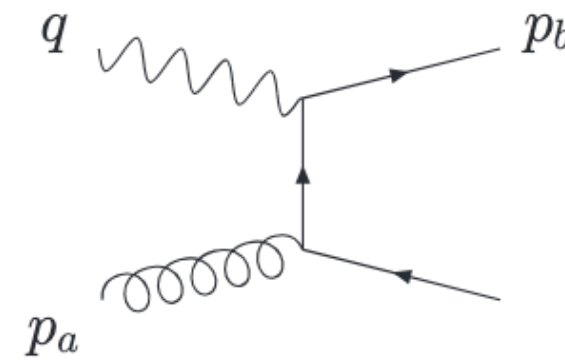
$$F_{UU,T/L} = \frac{1}{Q^2} \frac{\alpha_s}{2\pi^2 z} \int_{\xi_{\min}}^1 \frac{d\xi}{\xi} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} \delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \\ \times \sum_{ij} C_{UU,T/L}^{(ij)}(\hat{x}, Q^2, \hat{z}, q_T) f_{i/N}(\xi, Q^2) D_{h/j}(\zeta, Q^2),$$



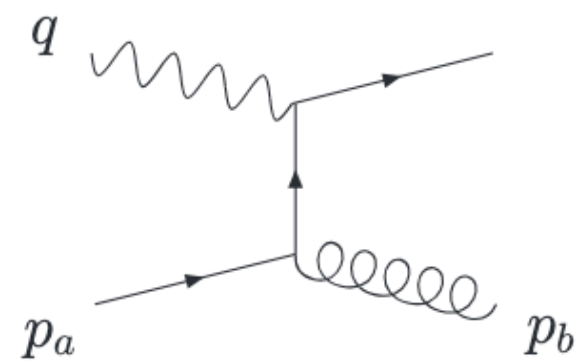
(a)



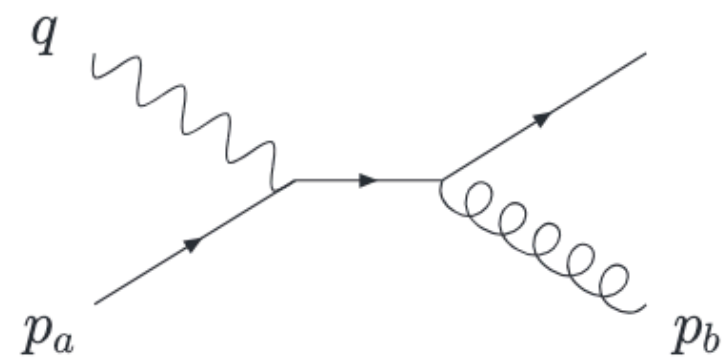
(a')



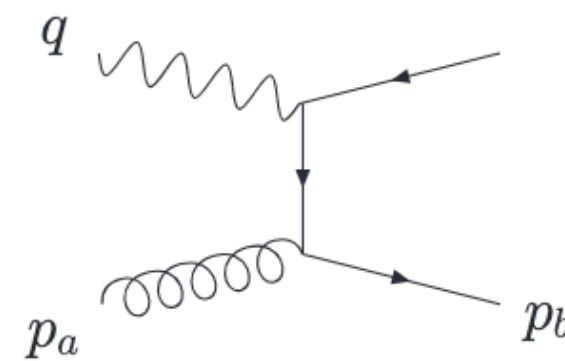
(c)



(b)



(b')



(c')

$$\bullet \gamma^* q \rightarrow qg$$

$$C_{UU,L}^{(qq)} = 8C_F e_q^2 \hat{x} \hat{z}$$

$$\bullet \gamma^* q \rightarrow gq$$

$$C_{UU,L}^{(qg)} = 8C_F e_q^2 \hat{x} (1 - \hat{z})$$

$$\bullet \gamma^* g \rightarrow q\bar{q}$$

$$C_{UU,L}^{(gq)} = 16T_R e_q^2 \hat{x} (1 - \hat{x})$$

Power behavior $F_{UU,L}$ & $F_{UU,T}$ in **AY** region, $M \ll q_T \ll Q$

$M \ll q_T \sim Q$

$$F_{UU,T/L} = \frac{1}{Q^2} \frac{\alpha_s}{2\pi^2 z} \int_{\xi_{\min}}^1 \frac{d\xi}{\xi} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} \delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \\ \times \sum_{ij} C_{UU,T/L}^{(ij)}(\hat{x}, Q^2, \hat{z}, q_T) f_{i/N}(\xi, Q^2) D_{h/j}(\zeta, Q^2),$$

$M \ll q_T \ll Q$

Asymptotic region

$$F_{UU,T} = \frac{1}{q_T^2} \frac{\alpha_s}{2\pi^2 z^2} \sum_a x e_a^2 \left[f_1^a(x) D_1^a(z) L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x) (D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq})(z) \right. \\ \left. + (P_{qq} \otimes f_1^a + P_{qg} \otimes f_1^g)(x) D_1^a(z) \right]$$

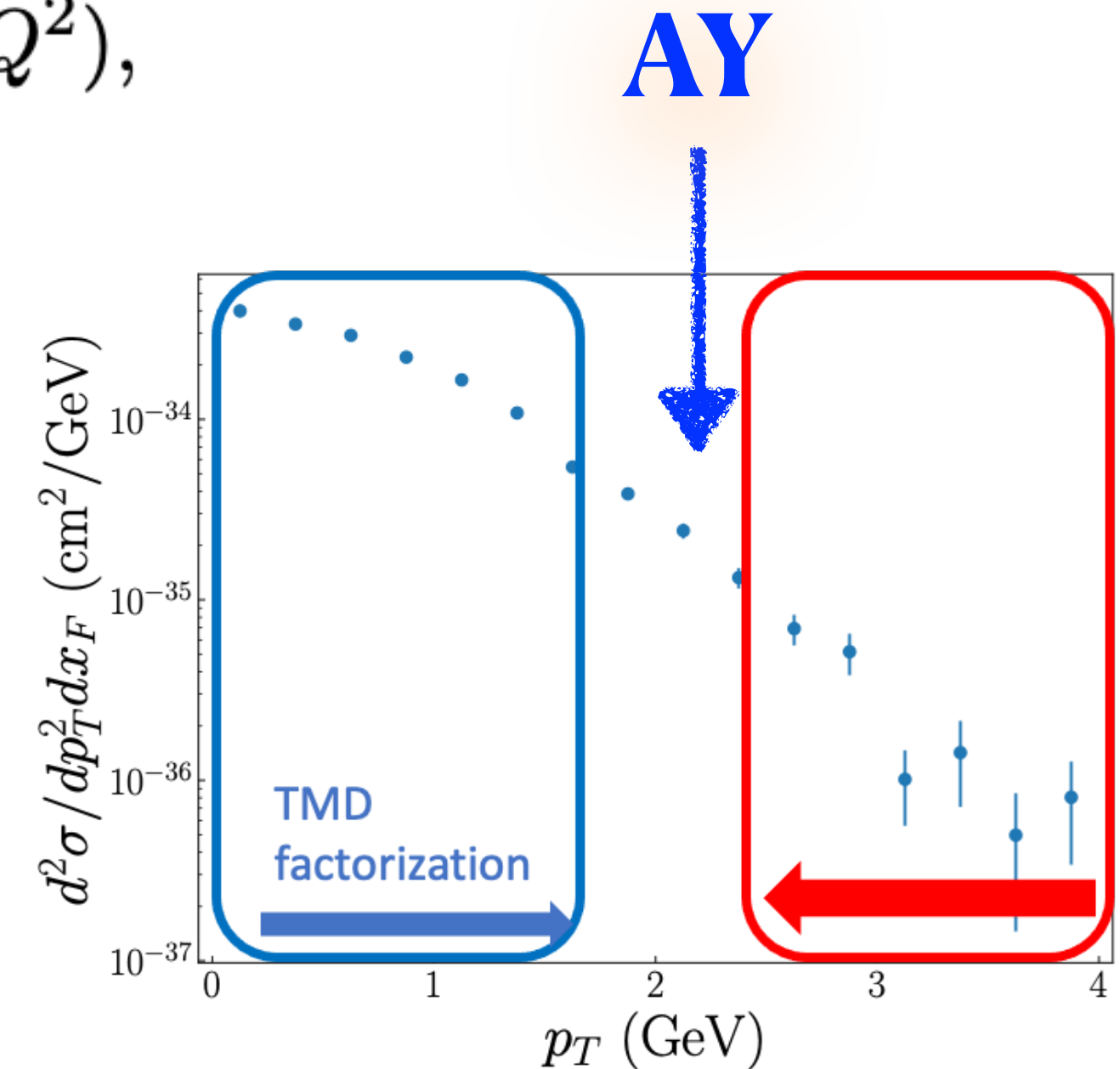
$$F_{UU,L} = \frac{\mathbb{1}}{Q^2} \frac{\alpha_s}{2\pi^2 z^2} \sum_a x e_a^2 \left[f_1^a(x) D_1^a(z) L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x) (D_1^a \otimes P_{qq}'' + D_1^g \otimes P_{gq}'')(z) \right. \\ \left. + (P_{qq}'' \otimes f_1^a + P_{qg}'' \otimes f_1^g)(x) D_1^a(z) \right]$$

!!

$$F_{UU,L}^{M \ll q_T \ll Q} \sim \frac{\alpha_s}{Q^2} \mathcal{F}[f_1 D_1]$$

• A match can be $F_{UU,L}$

• can $F_{UU,L}$ be resummed? See e.g. Berger Qiu Zhang, PRD D65 (2002)



$$L\left(\frac{Q^2}{q_T^2}\right) \equiv C_F \left[2 \ln\left(\frac{Q^2}{q_T^2}\right) - 3 \right]$$

Phenomenological Estimate of subleading power TMD dynamics in unpolarized sector

<https://arxiv.org/abs/2610.06628>

$$F_{UU,L} \quad \& \quad R_{\text{SIDIS}} = F_{UU,L} / F_{UU,T}$$

Phenomenological Estimate of subleading power TMD dynamics in unpolarized sector **SETUP**

- Our determination R_{SIDIS} based on use TMD PDF and TMD FF grids from the MAP22 extraction which are used to compute the W -term contribution to both $F_{UU,T}$ and $F_{UU,L}$
- Use the FO calculation at $\mathcal{O}(\alpha_S)$ for both structure functions, and we use the MMHT2014 and DSS14 sets at NLO for PDFs and FFs, respectively
- To achieve consistent matching for the W and FO terms we use TMD grids at NNLL accuracy
- To achieve consistent matching for the W and FO terms we phenomenological matching scheme
- Focus here on differential multiplicities in transverse momentum, and study the size of R_{SIDIS} in the kinematic region covered by COMPASS data

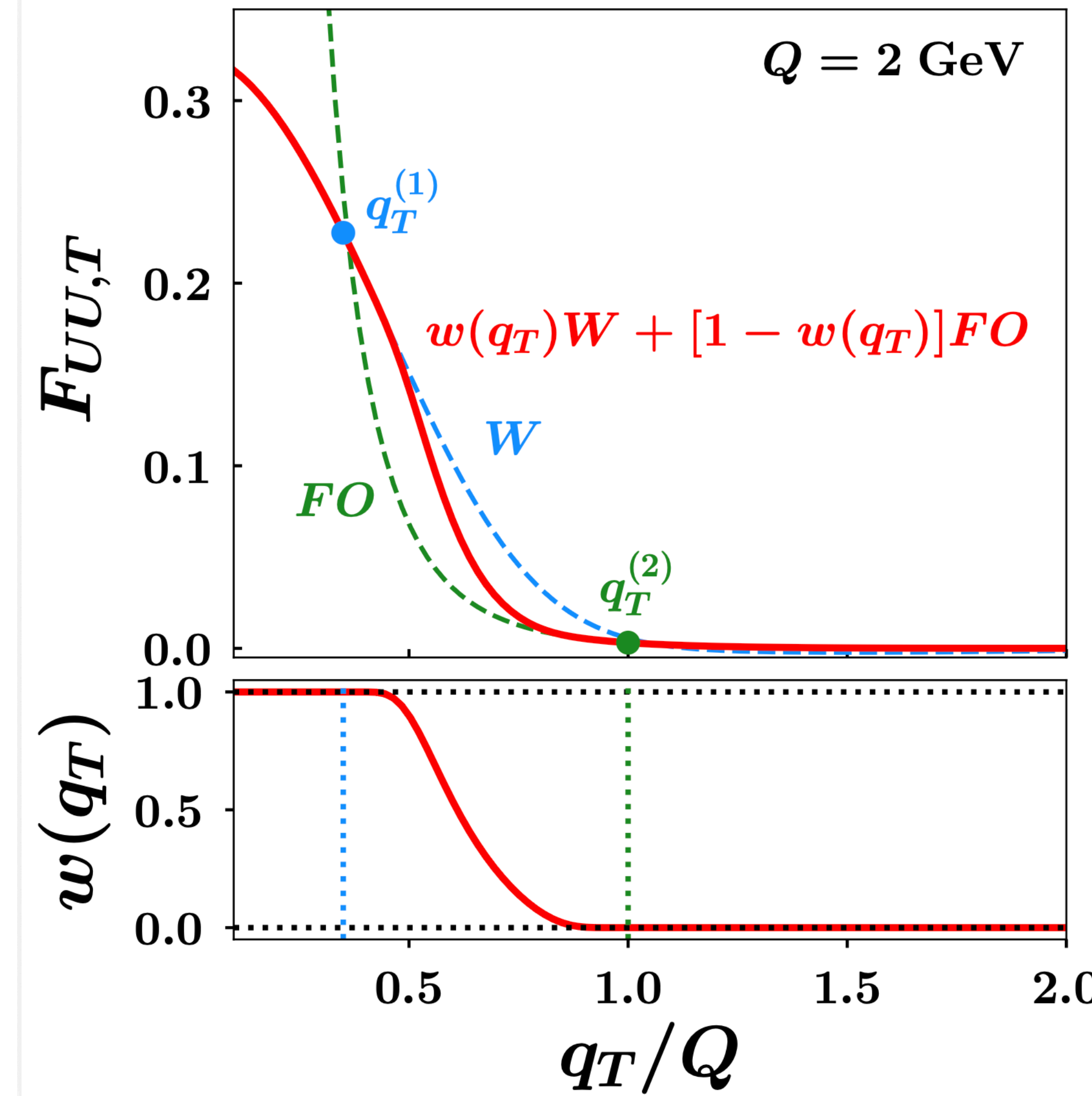
Building our model: matching

Since $W + Y$ not yet feasible phenomenologically

- use transition function to stitch together TMD & FO
- use intersection points between W & FO to separate scales
- ensure analyticity point-by-point

$$F_{UU,T} = w(q_T) F_{UU,T}^{[M \sim q_T \ll Q]} + (1 - w(q_T)) F_{UU,T}^{[M \ll q_T \sim Q]}$$

$$F_{UU,T}^{[M \sim q_T \ll Q]} = \mathbb{C}[f_1 D_1], \quad F_{UU,T}^{[M \ll q_T \sim Q]} = \text{Fixed order}$$



$w(q_T)$ is-transition function

Transverse momentum dependence

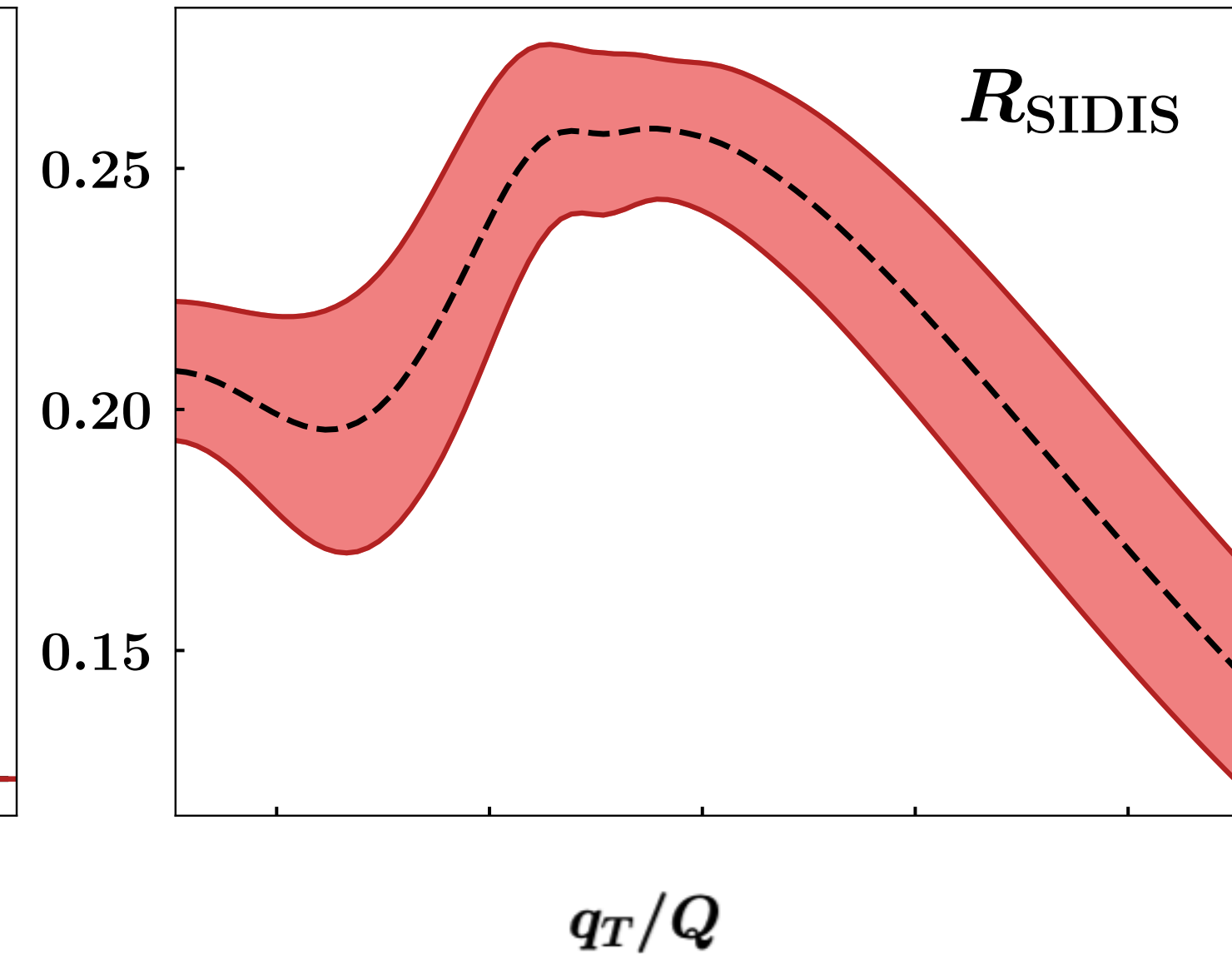
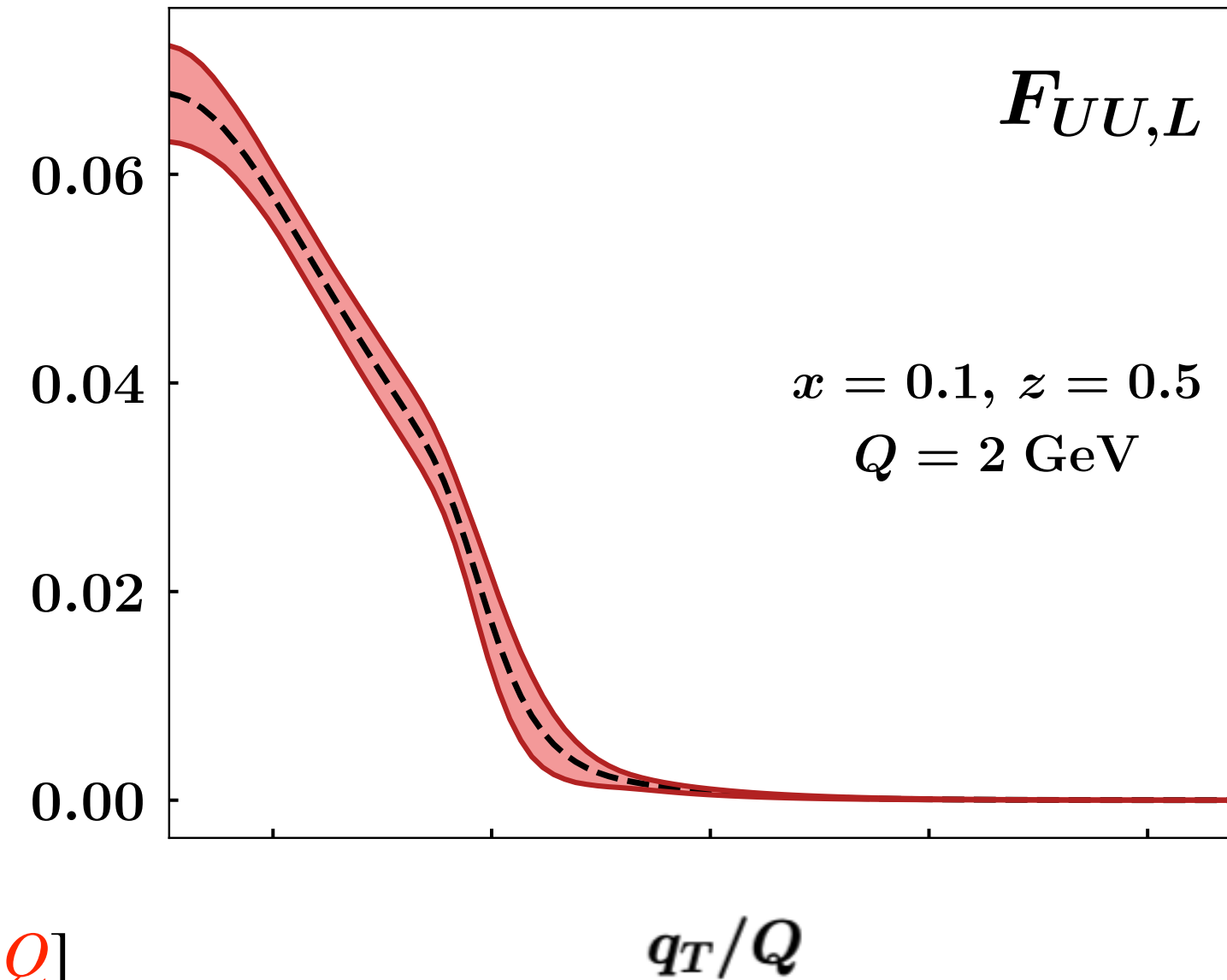
$$R_{SIDIS} = \frac{F_{UU,L}}{F_{UU,T}}$$

→ weight for TMD contribution

$$w(p_T) = \frac{4p_T^2}{1 + 4p_T^2/Q^2} \rightarrow \frac{4p_T^2}{Q^2} \text{ when } p_T \sim 0$$

$$F_{UU,L} = w(q_T) F_{UU,L}^{[M \sim q_T \ll Q]} + (1 - w(q_T)) F_{UU,L}^{[M \ll q_T \sim Q]}$$

$$F_{UU,L}^{[M \sim q_T \ll Q]} = \frac{M^2}{Q^2} \mathbb{C} [w(p_T) f_1 D_1], \quad F_{UU,L}^{[M \ll q_T \sim Q]} = \text{Fixed order}$$



Uncertainty quantification

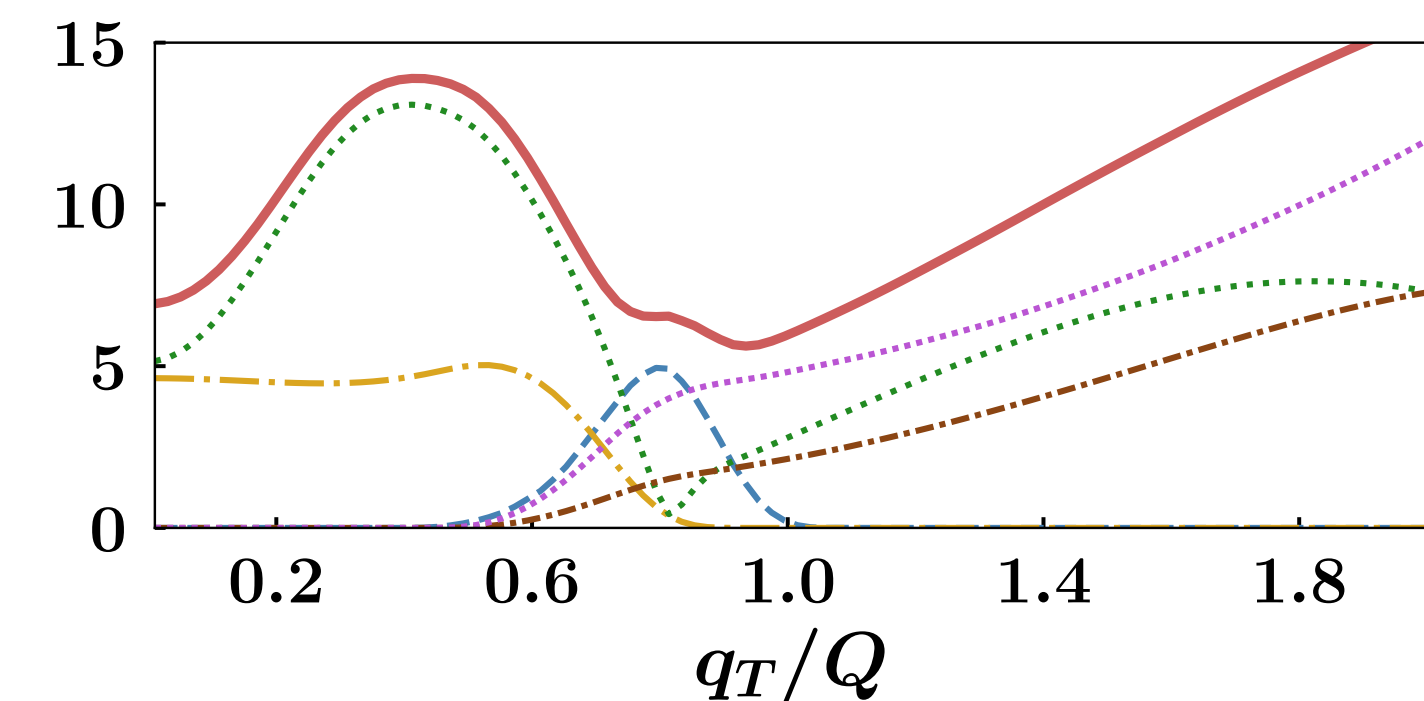
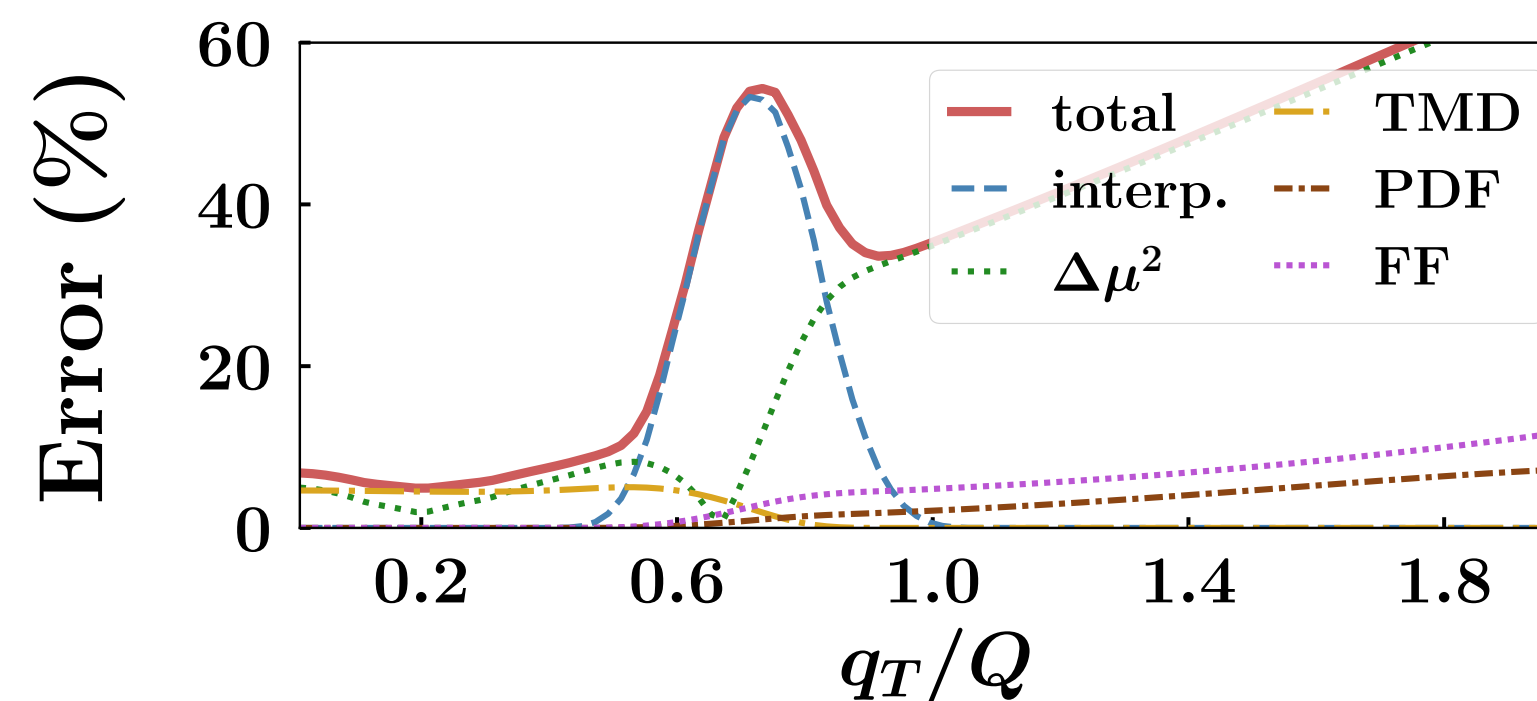
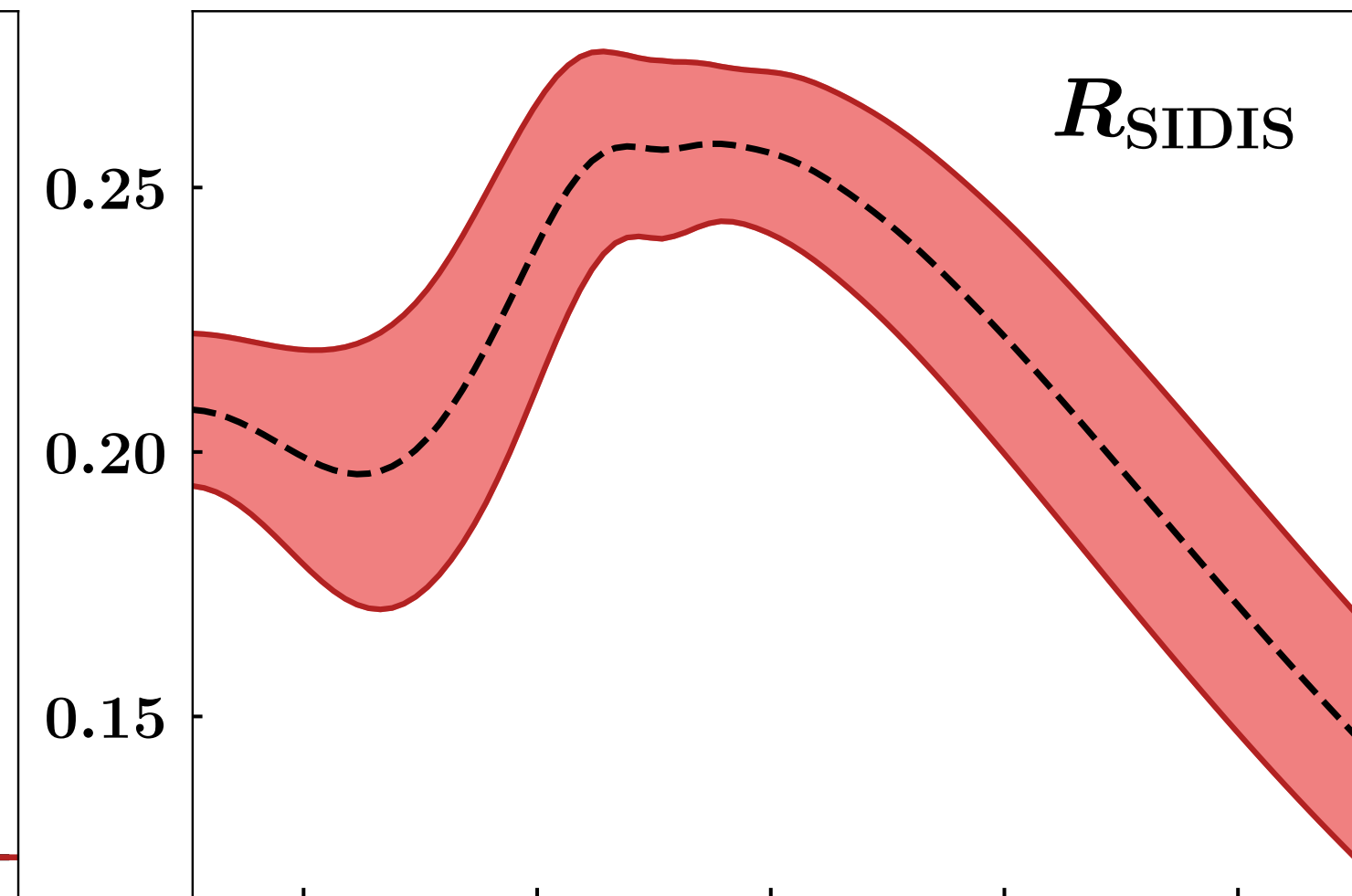
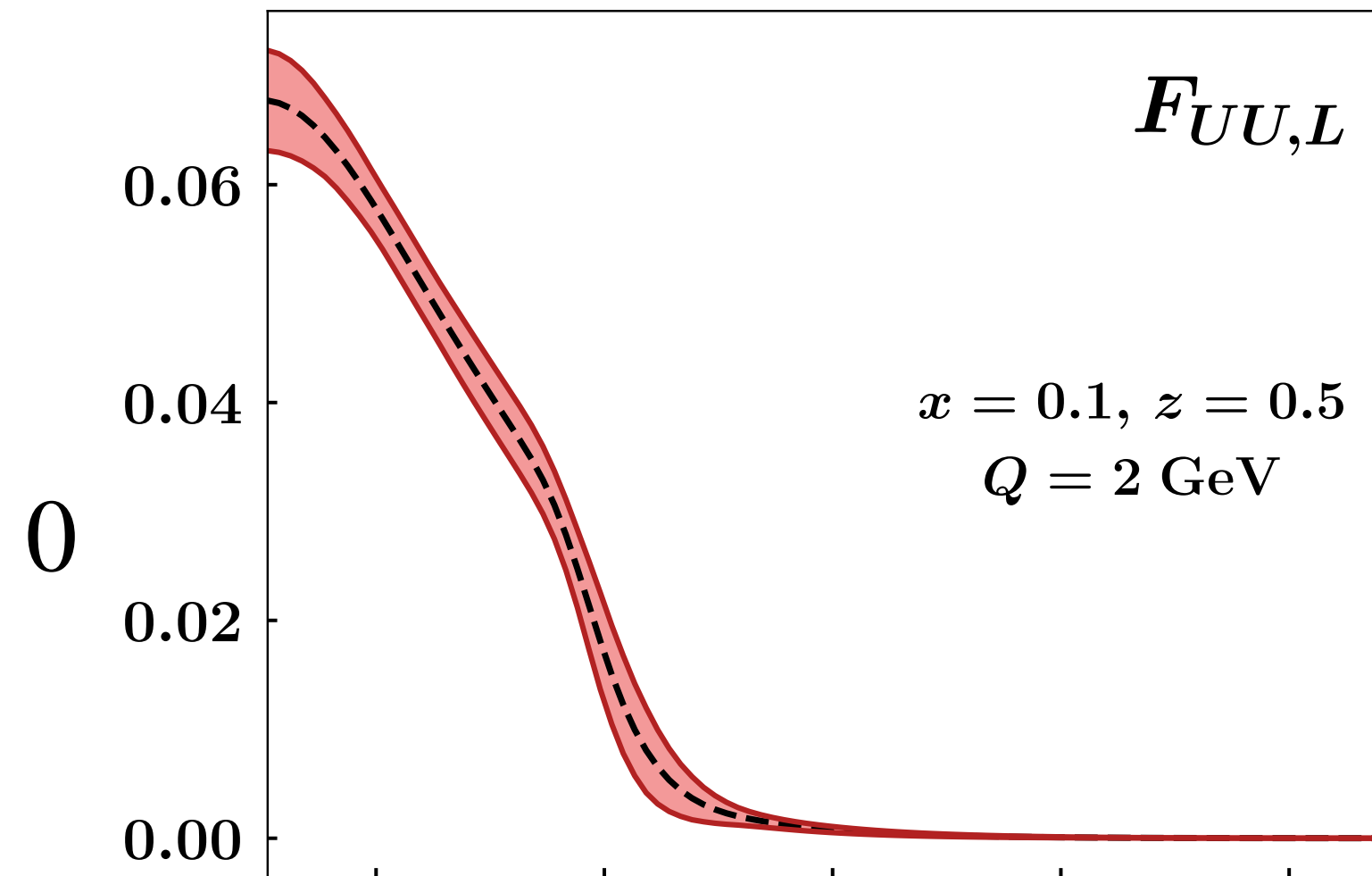
Use same interpolation for $F_{UU,T/L}$

→

$$w(p_T) = \frac{4p_T^2}{1 + 4p_T^2 / Q^2} \rightarrow \frac{4p_T^2}{Q^2} \text{ when } p_T \sim 0$$

→ 4 types of uncertainty:

- (1) interpolation between W & FO
- (2) Scale variation
- (3) TMD replicas
- (4) Collinear hessian sets



Pheno impacts/opportunities

R_{SIDIS} predictions at Compass kinematics

→ full error budget

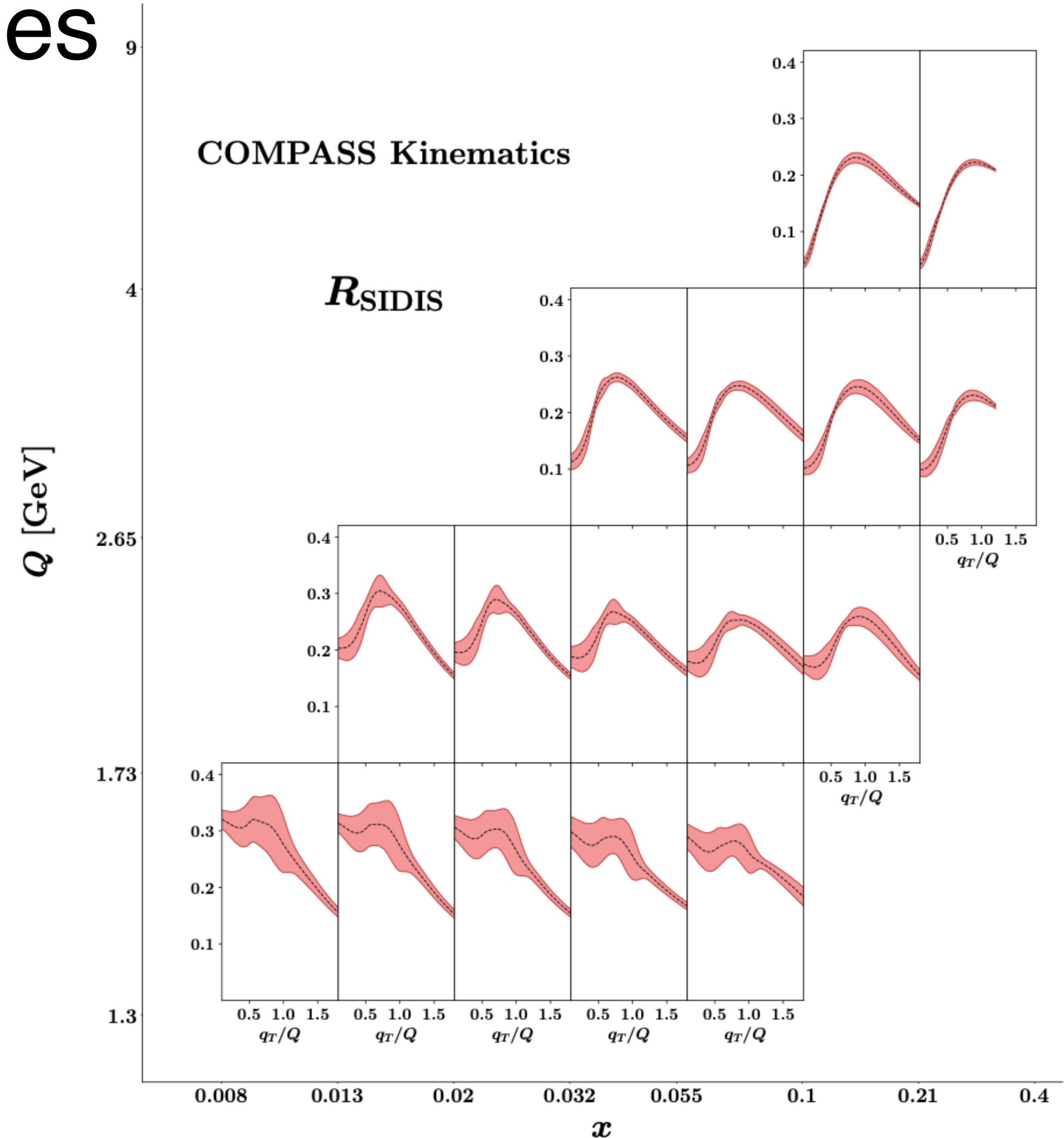
→ trends with increasing Q

(1) Magnitude of $R_{\text{SIDIS}}(q_T \rightarrow 0)$ decreases

(2) $R_{\text{SIDIS}}(q_T \rightarrow 0) \sim (q_T/Q)^2$

(3) Uncertainty decreases

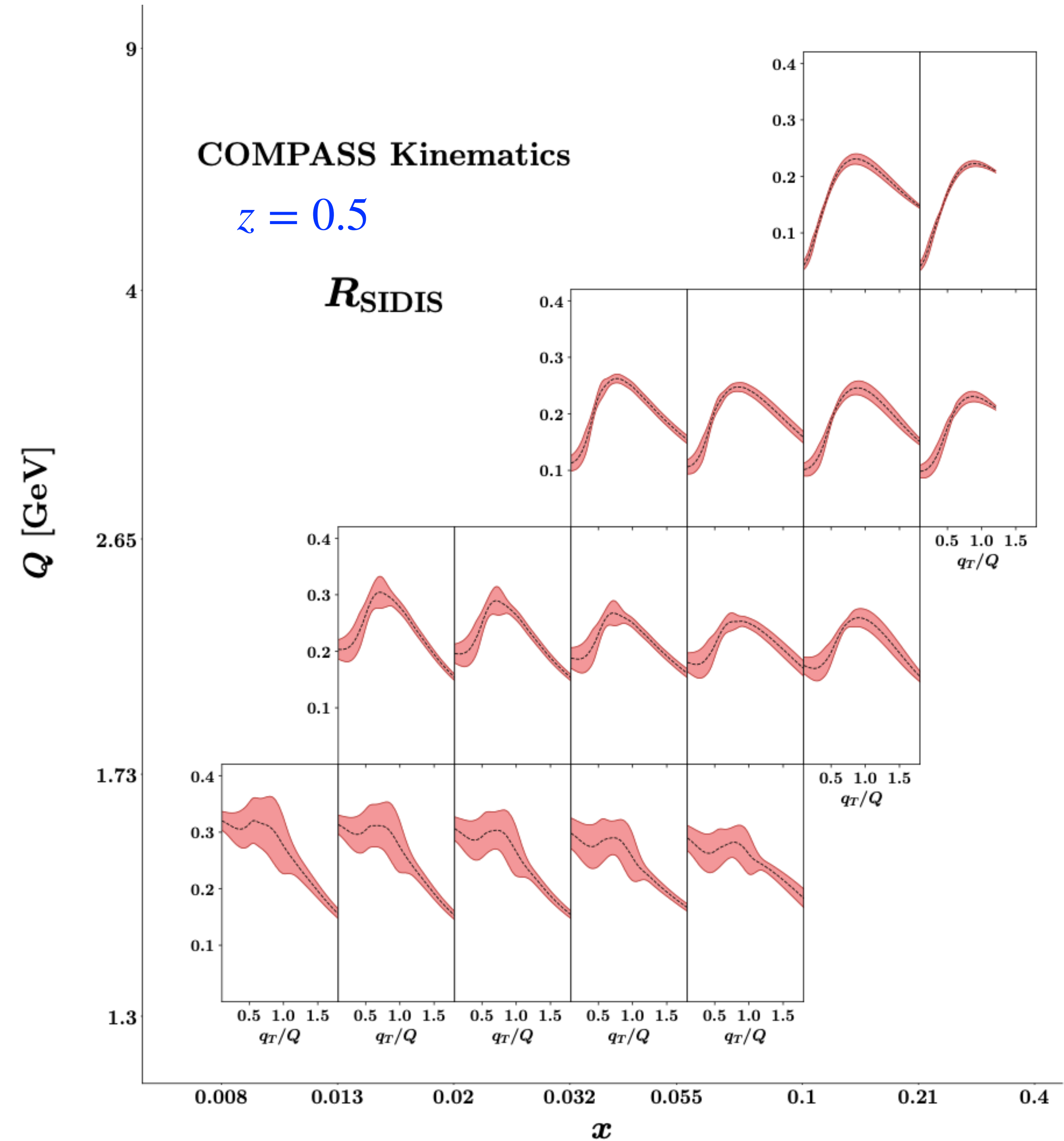
How do theory & experimental uncertainties compare?



Impact on Multiplicities

$$M^h(Q^2, x, z, P_{h\perp}) = \frac{F_{UU,T}}{F_T + \epsilon F_L} \left(1 + \epsilon R_{SIDIS} \right)$$

- Both DIS and SIDIS cross sections given by the sum of the T and L structure functions
- In current SIDIS pheno studies contribution of $F_{UU,L}$ in the numerator neglected
- Inclusion ?? reduce normalization mismatch observed in the description of the hadron multiplicities at high theoretical accuracy Bacchetta et al. MAP-JHEP 2022
- Inclusion ?? more robust results in studies of angular modulations, spin asymmetries, where the denominator measured observables is represented by unpolarized cross section



Probe for sub-leading physics

Truncated moments of R_{SIDIS} from large P_{hT} ($\equiv P_{h\perp}$)

$$F_{UU,T/L} = \frac{1}{Q^2} \frac{\alpha_s}{2\pi^2 z} \int_{\xi_{\min}}^1 \frac{d\xi}{\xi} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} \delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \\ \times \sum_{ij} C_{UU,T/L}^{(ij)}(\hat{x}, Q^2, \hat{z}, q_T) f_{i/N}(\xi, Q^2) D_{h/j}(\zeta, Q^2),$$

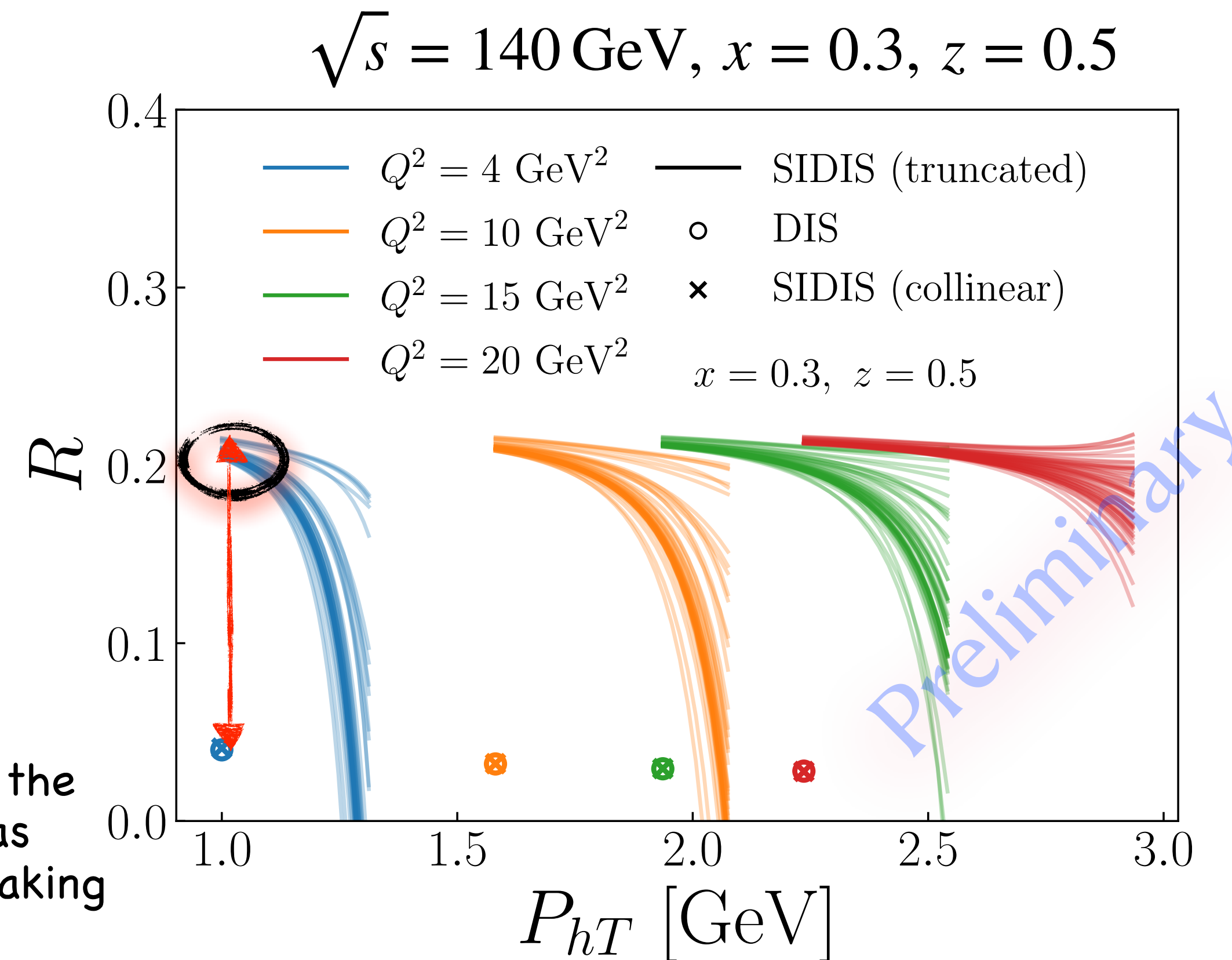
$$R_{SIDIS}^{\text{truncated}} = \int_{P_{hT}} d^2 \vec{P}'_{hT} F_{UU,L}^{(FO)} / \int_{P_{hT}} d^2 \vec{P}'_{hT} F_{UU,T}^{(FO)} \quad \text{vs.}$$

$$R_{DIS} = F_L / F_T$$

Comments:

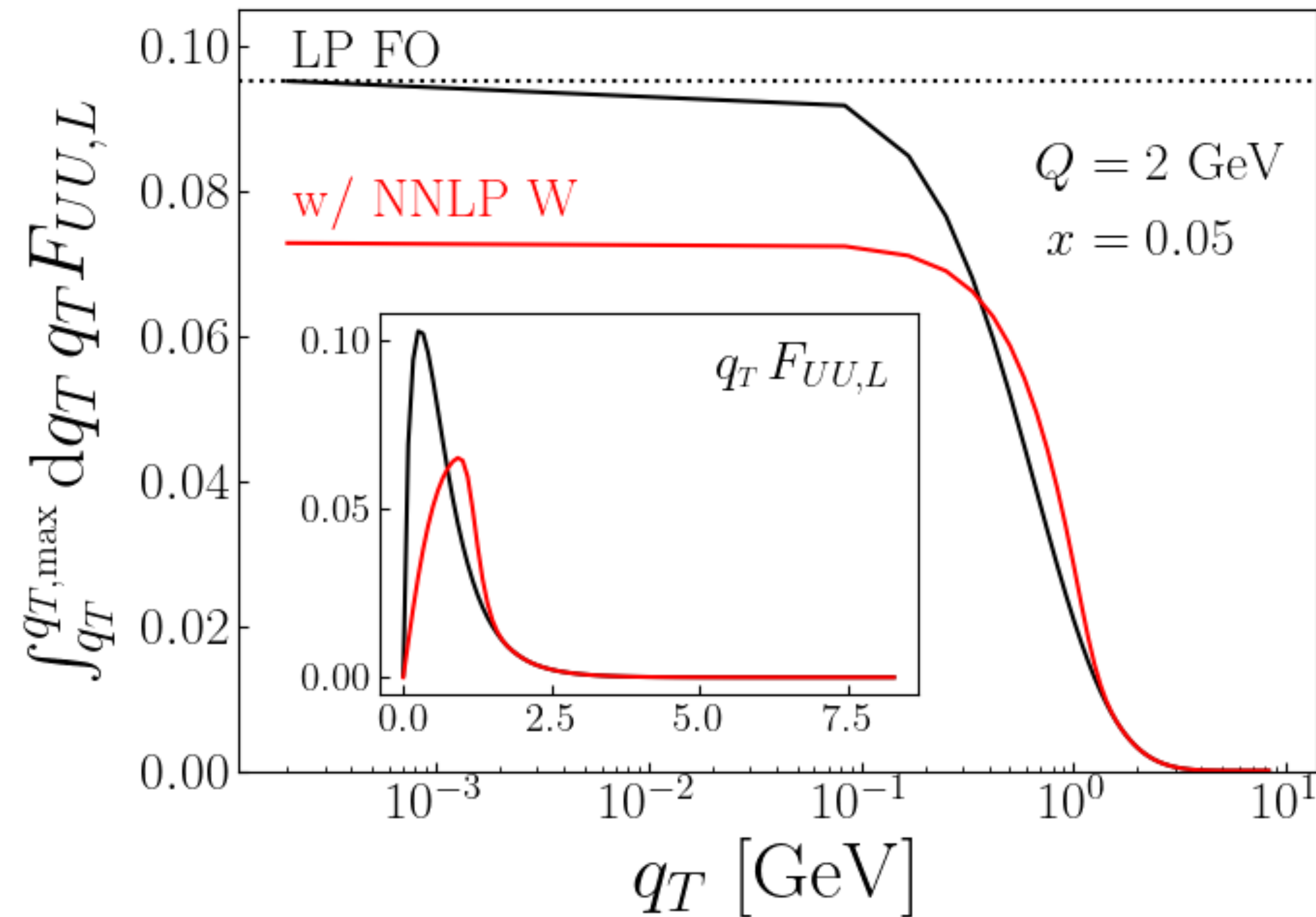
- Truncated moment is sig. larger than $P_{h\perp}$ integrated SIDIS—indication of
- power corrections TMD contribution ?
- What role does sub-leading $F_{UU,L}^{(W)}$ play?

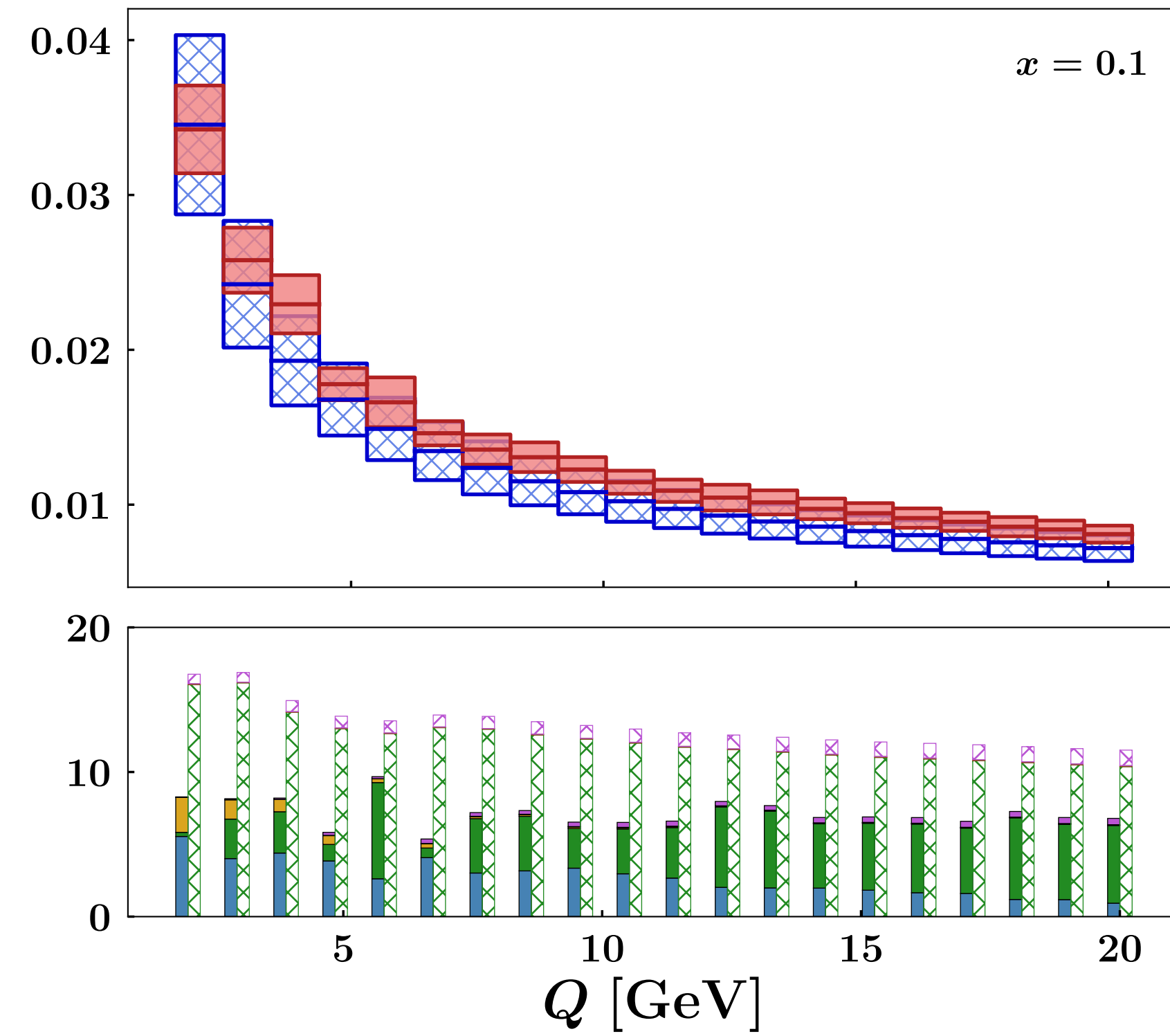
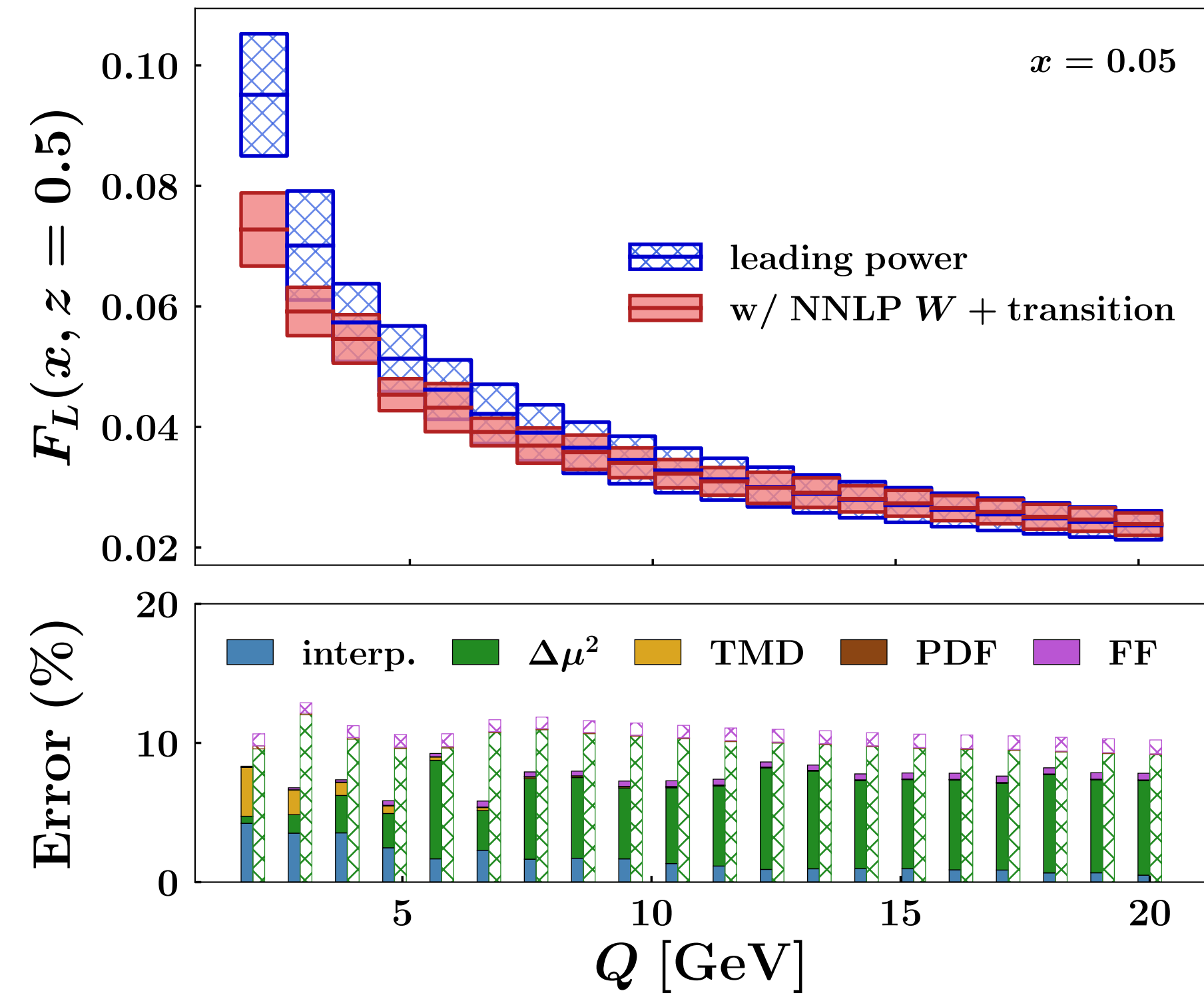
Nb: Bands generated by computing the observable on subset of JAM replicas (from recent W+charm analysis) & taking the mean $\pm$ standard deviation



Probe for sub-leading physics

$$\int_{P_{h\perp\min}^2/z^2}^{q_{T\max}^2} F(x, z, Q^2, P_{h\perp})$$





The integrated $F_{UU,L}$ function over q_T as a function of Q at $x = 0.1$ & $z = 0.5$ and $x = 0.01$ & $z = 0.5$

Red band: integrated structure function obtained from our model

Blue band corresponds to the integral the fixed-order calculation.

Lower plot we show the breakdown and the size of the different sources of uncertainties.

Sub-leading contributions in W term $q_T \ll Q$

Kinematic power corrections for TMD factorization theorem of semi-inclusive deep-inelastic scattering

Sara Piloñeta^a and Alexey Vladimirov^a

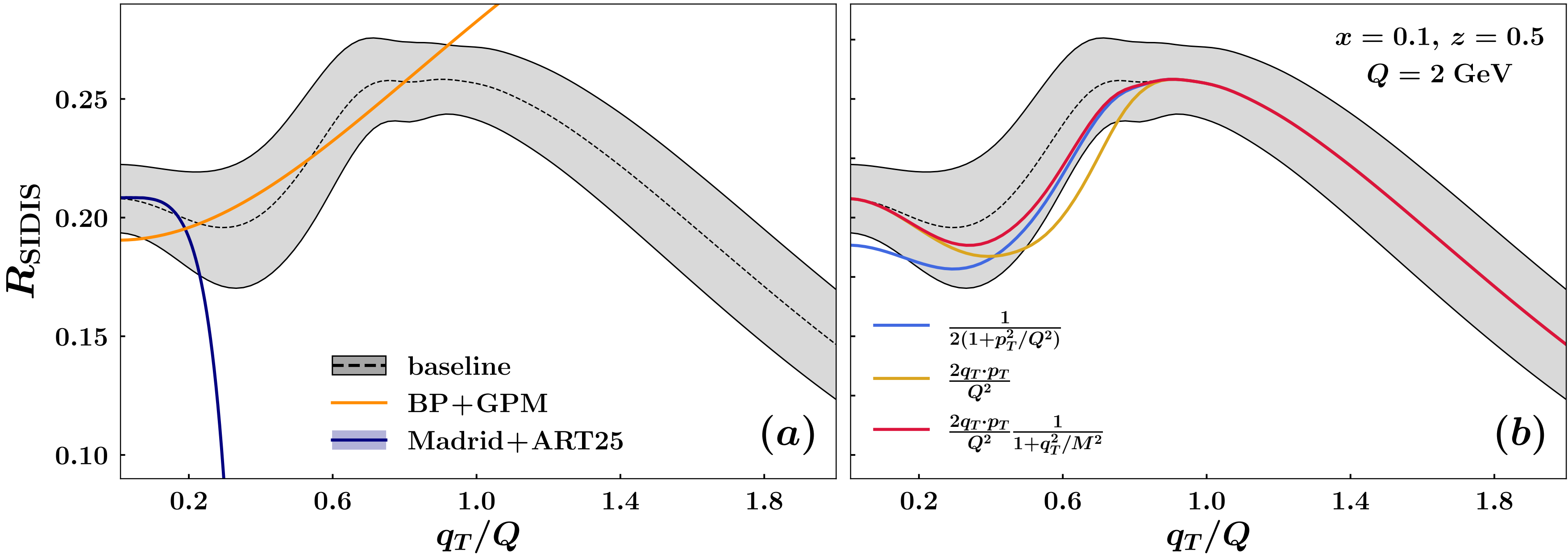
JHEP 2026

Next-to-next-to-leading power corrections to unpolarized Semi-Inclusive Deep Inelastic Scattering

Ian Balitsky^{a,c} Alexei Prokudin^{b,c}

JHEP 2026

<https://arxiv.org/abs/2610.06628>



Summary

Longitudinal Structure function $F_{UU,L}$

- indispensable to probe transverse proton structure SIDIS multiplicities ?
- lots of challenges in formal/phenomenological description of q_T spectrum
- Progress in sub-leading TMD physics
- pheno estimates with uncertainties of $F_{UU,L}$ and R_{SIDIS} considering interplay of W & FO
- Future — impacts on multiplicities/collinear SIDIS
- Anticipate H1 Lumi measurements of R_{SIDIS} to better constrain the longitudinal contribution
Hall C, Solid LITE

Pheno cont.

$F_{UU,L}$ predictions @ Compass kinematics

