# QED Radiative Corrections in SIDIS: the Best Approximation and Radiative Azimuthal Mixing in Spin Asymmetries

Igor Akushevich
Duke University, Durham, NC USA



*in collaboration with*
*Haiyan Gao, Sasha Ilyichev,*
*Vlad Khachatryan, Youjie Lin, Tianbo Liu,*
*and W. Melnitchouk*

e-mail: igor.akushevich@duke.edu

# Introduction and Outline

- SIDIS is a primary tool for extracting multidimensional nucleon structure, including TMDs and fragmentation functions.
- The precision achieved at JLab and expected at the EIC requires radiative-correction calculations beyond the traditional level of accuracy.
- This motivates a reexamination of SIDIS radiative corrections, their theoretical foundations, higher-order contributions, and associated systematic uncertainties.

**Outline**

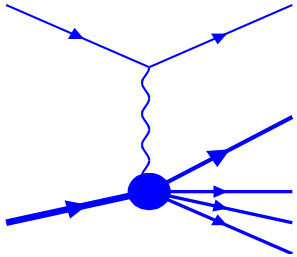
- Traditional approaches to SIDIS radiative corrections
- A hybrid framework: combining the strengths of traditional and factorized approaches and constructing the best approaximation for RC in SIDIS
- Radiative azimuthal mixing and its phenomenological implications
- Open theoretical questions and future directions

Part II:

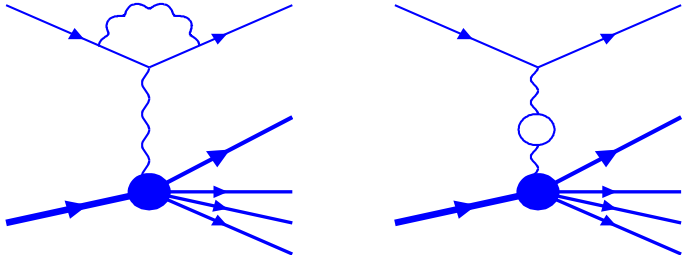
# Traditional Approaches to SIDIS Radiative Corrections

# Contribution to RC in SIDIS

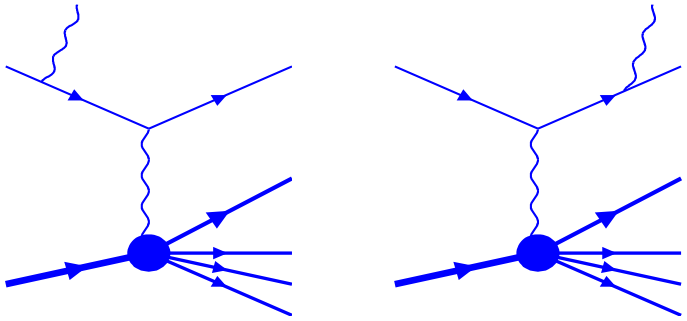
The Born cross section



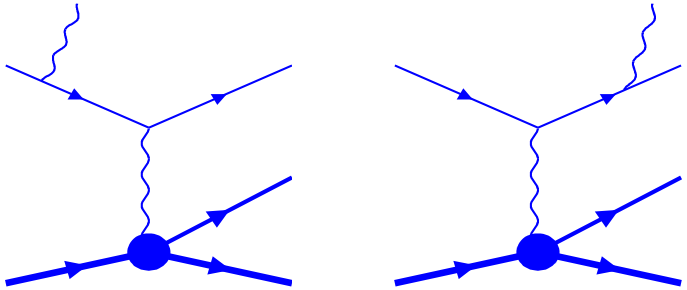
Loop diagrams



Emission of a radiated photon (semi-inclusive processes)



Emission of a radiated photon (exclusive processes)



# Current Status of Our Calculations

- RC to unpolarized SIDIS cross section, angular structure
  - Akushevich I, Shumeiko N, and Soroko A (1999) Radiative effects in the processes of hadron electroproduction. *European Physical Journal*, C10(4), pp.681-687.
- Radiative tail from exclusive peak
  - Akushevich I, Ilyichev A, and Osipenko M (2009) Lowest order QED radiative corrections to five-fold differential cross section of hadron leptoproduction. *Physics Letters* B672(1), pp.35-44.
- General calculation of RC for polarized particles
  - Akushevich I and Ilyichev A (2019) Lowest order QED radiative effects in polarized SIDIS. *Physical Review* D100(3), p.033005.
- Leading Log and the best approximation
  - Akushevich I, Srednyak S, and Ilyichev A (2024) Exact and leading order radiative effects in semi-inclusive deep inelastic scattering. *Physical Review* D109(7), p.076028.
- Radiative tail from exclusive peak
  - Akushevich I, Avakian H, Ilyichev A, Srednyak S (2023) Complete lowest order radiative corrections in semi-inclusive scattering of polarized particles. *The European Physical Journal* A59(10), 246.

# Exact Formulae for RC in SIDIS

The total RC is calculated exactly in our paper (PR D100 (2019) 033005):

$$\sigma_{RC} = \frac{\alpha}{\pi}\exp(\delta_{inf})\left(\delta_{VR} + \delta^{l}_{vac} + \delta^{h}_{vac}\right)\sigma_B + \sigma^{F}_{R} + \sigma_{AMM} + \sigma^{ex}_{R}$$

Here $\delta_{VR}$ and $\delta^{l,h}_{vac}$ come from the radiation of soft photons and the effects of vacuum polarization, the correction $\delta_{VR}$ is infrared-free sum of factorized parts of real and virtual photon radiation, and $\sigma^{F}_{R}$ and $\sigma^{ex}_{R}$ are contributions from the process of emission of an additional real photon:

$$\begin{aligned}
\sigma_B &= \frac{\alpha^2 S_x^2}{4Q^4\sqrt{\lambda_Y}S}\sum_{i=1}^{9}\theta_i^B\mathcal{H}_i \\
\sigma^{F}_{R} &= -\frac{\alpha^3 S S_x^2}{32\pi^2\lambda_S\lambda_Y}\int\limits_{\tau_{\min}}^{\tau_{\max}}d\tau\int\limits_{0}^{2\pi}d\phi_k\int\limits_{0}^{R_{\max}}dR\sum_{i=1}^{9}\left[\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\theta_{ij}\frac{R^{j-2}}{\tilde{Q}^4} - \frac{\theta_{i1}\mathcal{H}_i}{RQ^4}\right], \\
\sigma^{ex}_{R} &= -\frac{\alpha^3 S S_x^2}{2^8\pi^5\lambda_S\lambda_Y}\int\limits_{\tau_{\min}}^{\tau_{\max}}d\tau\int\limits_{0}^{2\pi}d\phi_k\sum_{i=1}^{9}\sum_{j=1}^{k_i}\frac{\tilde{\mathcal{H}}^{ex}_i\theta_{ij}R_{ex}^{j-2}}{(1+\tau-\mu)\tilde{Q}^4}.
\end{aligned}$$

Explicit expressions for the functions $\theta_{ij}$ are given in Appendix B of our paper.

# Leading, Next-to-Leading, and Exact Contributions to RC

By “exactly” calculated RC we understand the estimation of the lowest order RC contribution with any predetermined accuracy. The structure of the dependence on the electron mass in RC cross section:

$$\sigma_{RC} = A \log \frac{Q^2}{m^2} + B + O(m^2/Q^2)$$

where $A$ and $B$ do not depend on the electron mass $m$.

- If only $A$ is kept, this is the leading log approximation.
- If both contributions are kept (i.e., contained $A$ and $B$), this is the calculation with the next-to-leading accuracy, practically equivalent to exact calculation.

Three approaches to extract the leading Log contribution for the SIDIS cross section (i.e., to calculate $A$)

- use our exact formulae, collect all terms that result in leading log after integration over photon angles, combine them into the final expression
- extract the poles that correspond to radiation collinear to initial and final electron, integrate over angles, and find the factorizing form traditional for leading log calculations.
- use the method of the electron structure functions.

# Leading Log: Extraction from Exact Formulae

The exact expression for $\sigma_R^F$ is:

$$\sigma_R^F = -\frac{\alpha^3 S S_x^2}{32\pi^2 \lambda_S \lambda_Y} \int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \int_0^{R_{\max}} dR \sum_{i=1}^{9} \left[ \sum_{j=1}^{k_i} \tilde{\mathcal{H}}_i \theta_{ij} \frac{R^{j-2}}{\tilde{Q}^4} - \frac{\theta_{i1} \mathcal{H}_i}{R Q^4} \right],$$

Analysis of the integrand and tracing the origin of the leading log allows us to extract the leading log terms from the exact formulae:

$$\int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \sum_{j=1}^{k_i} \theta_{ij}^s R^{j-2} = -4\pi l_m \frac{\sqrt{\lambda_Y}}{S} \frac{1+z_1^2}{z_1(1-z_1)} \theta_i^{Bs}$$

$$\int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \sum_{j=1}^{k_i} \theta_{ij}^p R^{j-2} = -4\pi l_m \frac{\sqrt{\lambda_Y}}{X} \frac{1+z_2^2}{1-z_2} \theta_i^{Bp}$$

# Leading Log: Exclusive Radiative Tail

Exact contribution calculated inour 2019 paper is:

$$\sigma_R^{ex} = -\frac{\alpha^3 S S_x^2}{2^8\pi^5\lambda_S\lambda_Y}\int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \sum_{i=1}^{9}\sum_{j=1}^{k_i} \frac{\tilde{\mathcal{H}}_i^{ex}\theta_{ij}R_{ex}^{j-2}}{(1+\tau-\mu)\tilde{Q}^4}.$$

where $R_{ex} = \frac{p_x^2-m_u^2}{1+\tau-\mu}$. The Born cross section of exclusive process is:

$$\sigma_{ex}^B = \frac{\alpha^2 S_x}{32\pi^3 Q^4 S\sqrt{\lambda_Y}}\sum_{i=1}^{9}\mathcal{H}_i^{ex}\theta_i^B$$

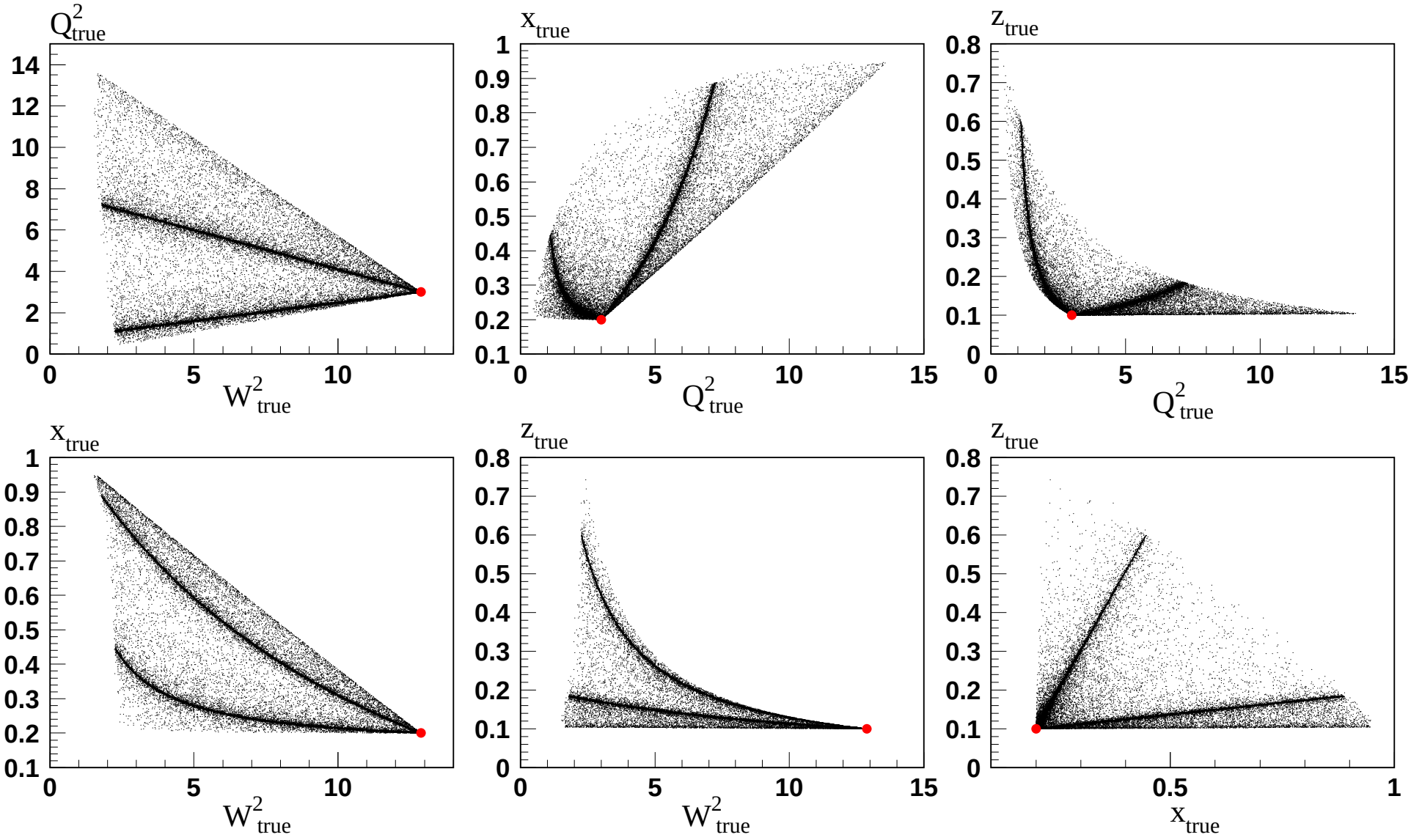
Therefore we have

$$\sigma_{sex}^1 = \frac{\alpha}{2\pi} l_m \sqrt{\frac{\lambda_Y^{se}}{\lambda_Y}}\frac{S_x^2}{S' S_x^{se}}\frac{1+z_{1ex}^2}{1-z_{1ex}}\sigma_{sex}^B,$$

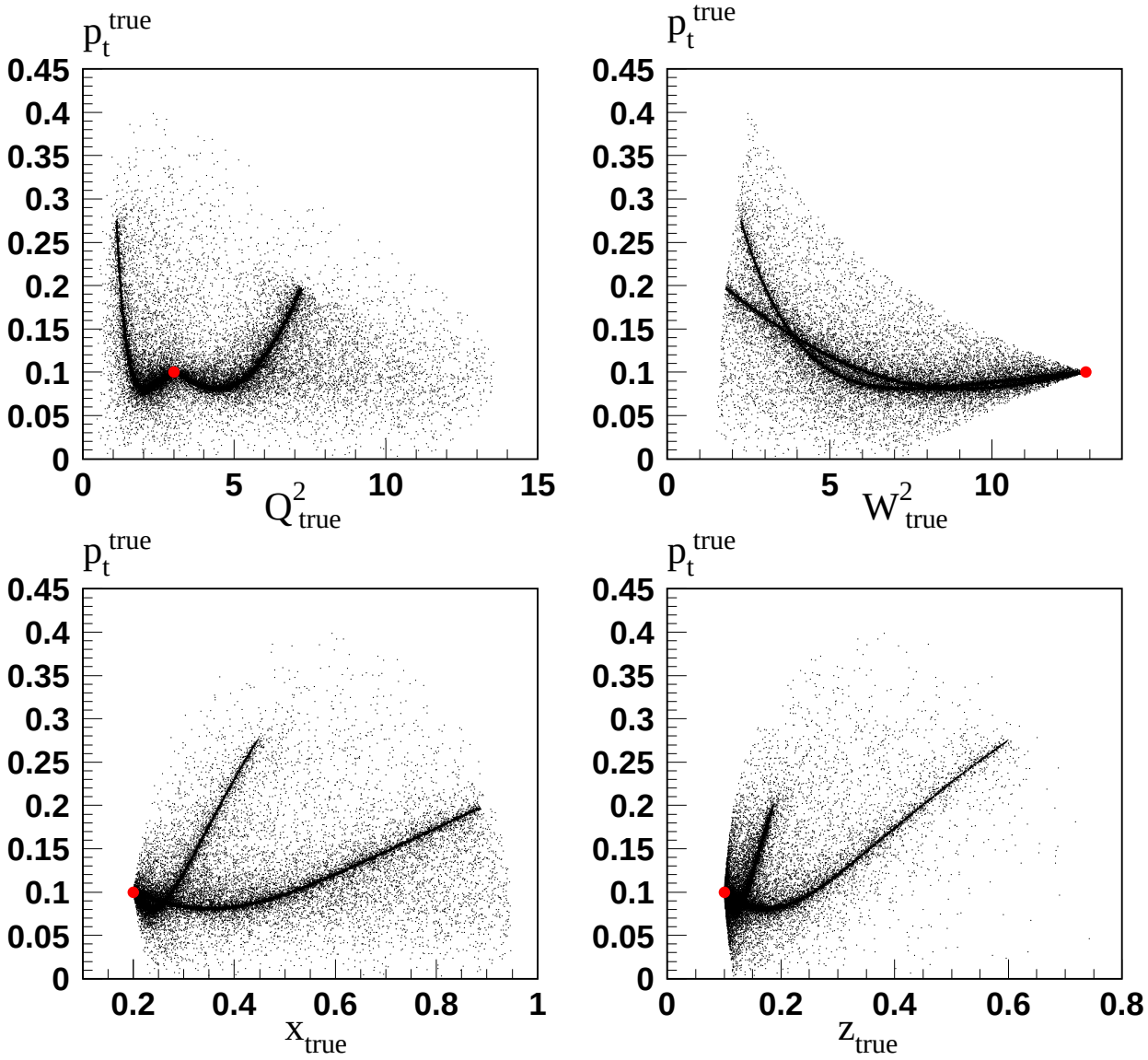
$$\sigma_{pex}^1 = \frac{\alpha}{2\pi} l_m \sqrt{\frac{\lambda_Y^{pe}}{\lambda_Y}}\frac{S_x^2}{X' S_x^{pe}}\frac{1+z_{2ex}^2}{1-z_{2ex}}\sigma_{pex}^B,$$

where $z_{1ex} = 1-(p_x^2-m_u^2)/S'$ and $z_{2ex} = (1+(p_x^2-m_u^2)/X')^{-1}$.

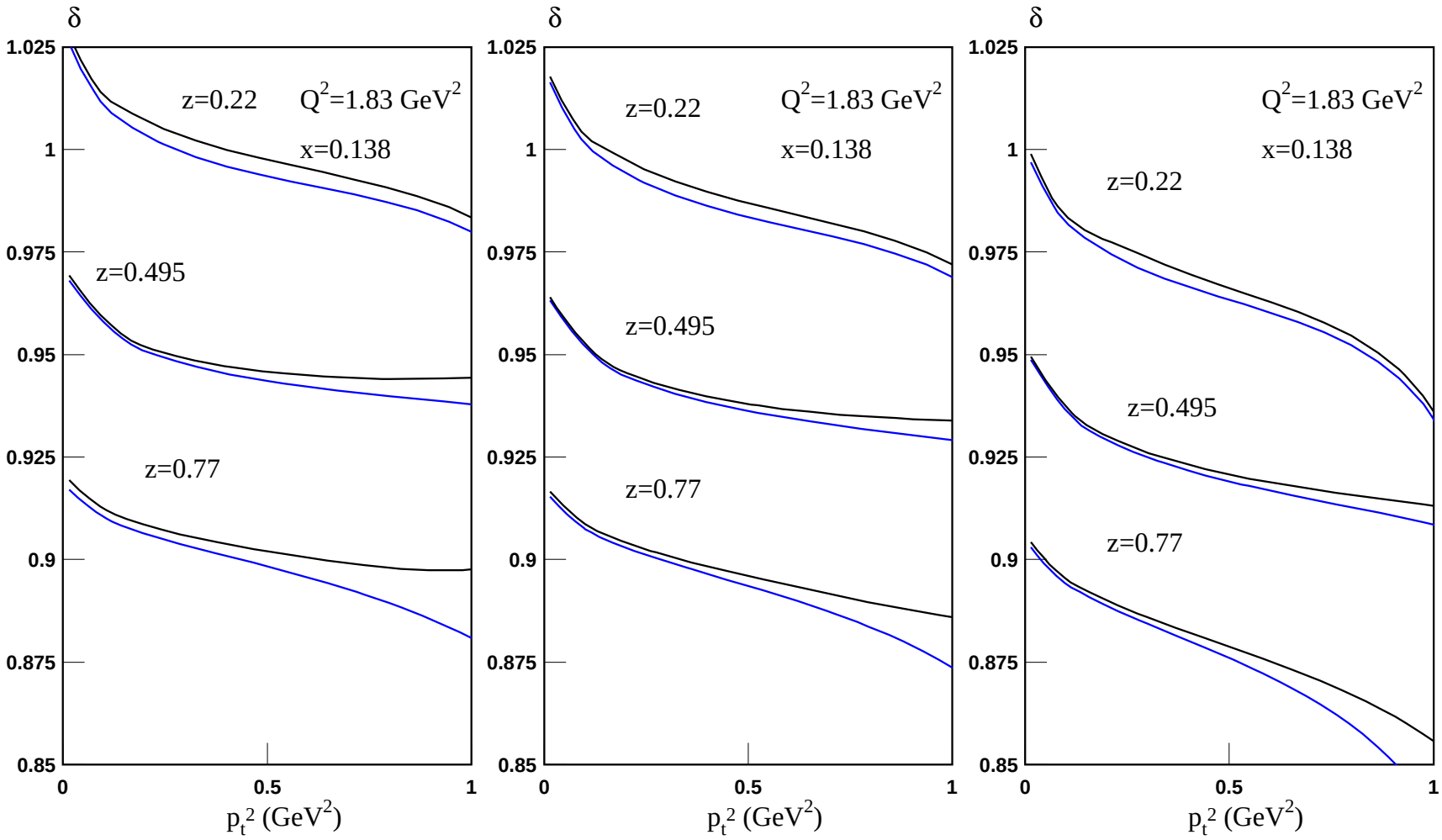
# Kinematic Regions for SIDIS SFs (x,Q2,W2,z)



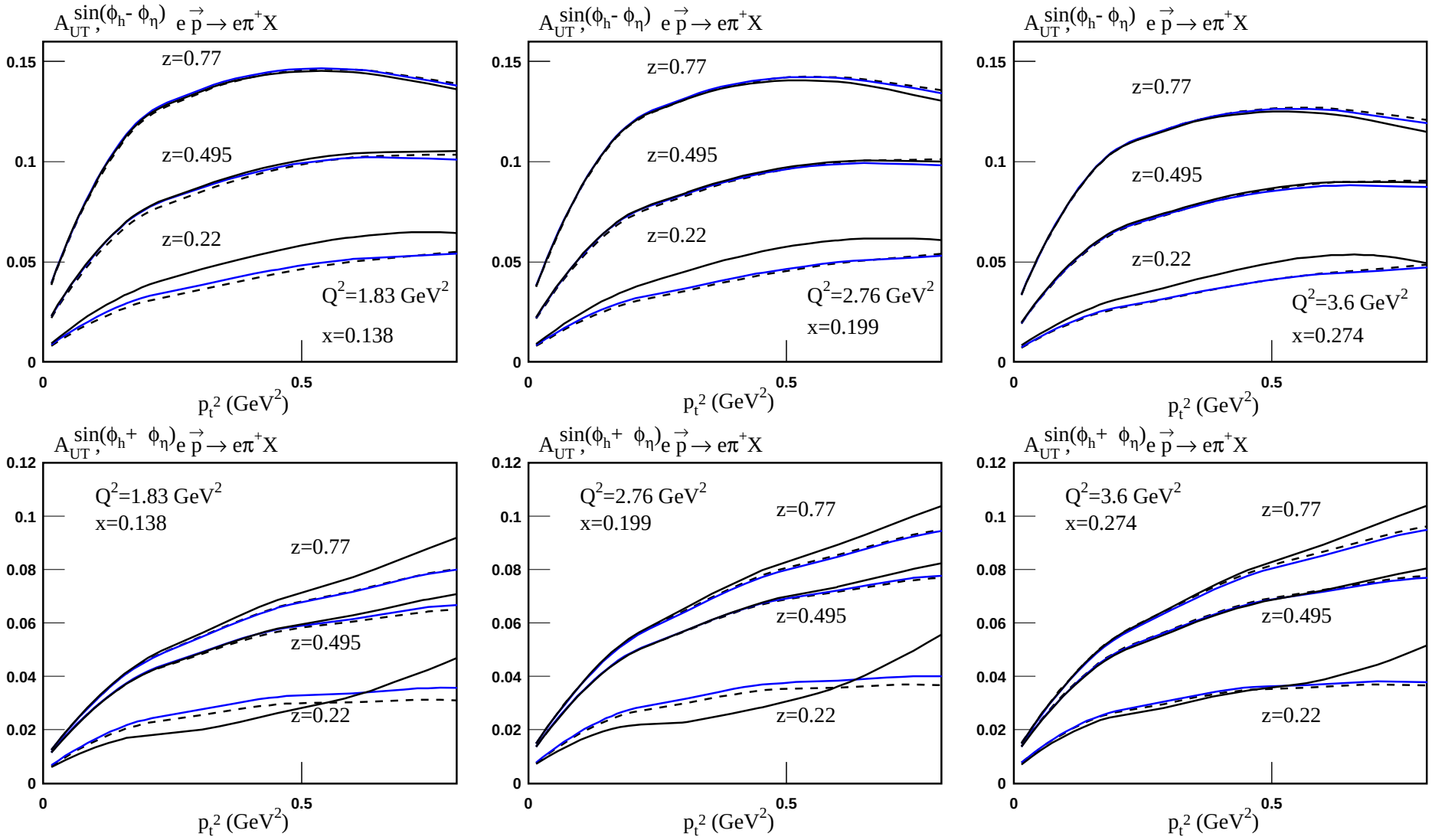
# Kinematic Region for SIDIS SFs (pt)



# RC to Unpolarized Cross Section



# RC to Collins and Sivers Asymmetries



Part III:

# A Hybrid Framework: Combining the Strengths of Traditional and Factorized Approaches

# Hybrid Approach: Motivation and Goal

- Precision SIDIS measurements require accurate treatment of QED radiative effects.
- Two conceptually different frameworks are currently available:
  - Traditional approach: exact $\mathcal{O}(\alpha)$ radiative corrections + ESF resummation.
  - Factorized approach: LDF/LFF factorization with perturbative evolution.
- Both frameworks aim to describe the same physical cross section.
- The relation between their predictions is not completely understood.
- Goal: understand similarities, differences, and domains of applicability.

**Main questions:**

- Formulate the traditional and factorized computational schemes.
- Identify the universal LL contributions in both approaches.
- Compare the resulting LL approximations analytically.
- Compare LL+NL contributions at lowest and higher orders numerically.
- Determine the theoretical origin of the observed discrepancies.

*I. Akushevich, H. Gao, A. Ilyichev, S. Jia, V. Khachatryan, Y. Lin, T. Liu, and W. Melnitchouk, "QED radiative effects in semi-inclusive deep-inelastic scattering: traditional and factorized approaches," arXiv:2608.20294 [hep-ph] (2026).*

# Traditional and Factorized Approaches

Traditional approach $\sigma_{\rm RC} = \sigma_B + \sigma_V + \sigma_R + \sigma_{\rm ERT}$

- Exact treatment of real and virtual corrections.
- Explicit phase-space integration.
- Infrared cancellation performed before final integration.
- Electron structure functions for higher-order RC in LL.
- Exclusive radiative tail treated separately.

Factorized approach $\sigma = D_i \otimes H \otimes D_f$

- Universal lepton distribution and fragmentation functions.
- Convolution of LDFs/LFFs with the hard scattering cross section.
- QED and QCD radiation treated within the same framework.
- Mellin-space evolution and all-order resummation of collinear logarithms.
- Natural implementation of higher-order corrections.

The two approaches are based on different factorization and resummation strategies. Their relation at leading and nonleading orders is the subject of the present comparison.

# Factorized Approach

$$\frac{\mathrm{d}^6\sigma^{\mathrm{obs}}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi\,\mathrm{d}z\,\mathrm{d}\phi_h\,\mathrm{d}p_t^2} = \int_{\xi_{\mathsf{min}}}^{1} \mathrm{d}\xi\, f_{e/e}(\xi,Q^2) \int_{\zeta_{\mathsf{min}}(\xi)}^{1} \mathrm{d}\zeta\, D_{e/e}(\zeta,Q^2)\, \widehat{J}\, \frac{\mathrm{d}^6\hat{\sigma}\left(\alpha(Q^2)\right)}{\mathrm{d}\hat{x}\,\mathrm{d}\hat{y}\,\mathrm{d}\hat{\phi}\,\mathrm{d}\hat{z}\,\mathrm{d}\hat{\phi}_h\,\mathrm{d}\hat{p}_t^2}\,,$$

In factorized framework, collinear QED radiation produces large logarithms order by order in perturbation theory. Their resummation is achieved through ESF obeying QED DGLAP evolution equations.

$$\frac{\mathrm{d}f_{i/e}(\xi,Q^2)}{\mathrm{d}\log Q^2} = \frac{\alpha(Q^2)}{2\pi} \sum_j P_{ij}(\xi) \otimes_\xi f_{j/e}(\xi,Q^2),$$

$$\frac{\mathrm{d}D_{e/i}(\zeta,Q^2)}{\mathrm{d}\log Q^2} = \frac{\alpha(Q^2)}{2\pi} \sum_j D_{e/j}(\zeta,Q^2) \otimes_\zeta P_{ji}(\zeta),$$

where $i,j = e,\bar{e},\gamma$, and the symbol $\otimes_x$ denotes the Mellin convolution,

$$\frac{\mathrm{d}^6\sigma^{\mathrm{obs}}_{\mathcal{O}(\alpha),\mathrm{LL+NL}}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi\,\mathrm{d}z\,\mathrm{d}\phi_h\,\mathrm{d}p_t^2} = \frac{\alpha(m^2)}{2\pi}\left[\frac{4}{3}\log\frac{Q^2}{m^2}\,\sigma_B + \int_{\xi_{\mathsf{min}}}^{1}\mathrm{d}\xi\left[\frac{1+\xi^2}{1-\xi}\left(\log\frac{Q^2}{(1-\xi)^2m^2}-1\right)\right]_+ \widehat{J}\,\hat{\sigma}_B\Big|_{\zeta=1}\right.$$
$$\left. + \int_{\zeta_{\mathsf{min}}(1)}^{1}\mathrm{d}\zeta\left[\frac{1+\zeta^2}{1-\zeta}\left(\log\frac{Q^2}{(1-\zeta)^2m^2}-1\right)\right]_+ \widehat{J}\,\hat{\sigma}_B\Big|_{\xi=1}\right].$$

# Analytical and Numerical Comparison

- Analytical comparison showed coincidence of the Born cross section and the lowest order RC. Specifically, the exact coincidence was found for:
  - the structure of the cross sections and vacuum polarization corrections,
  - expressions for shifted variables that are argument of the Born cross section in the integrands,
  - expressions for Jacobian transformation, lower integration limits, and splitting functions.
- Numeric comparison was performed for kinematical conditions of JLab, HERMES, and EIC
  - Excellent agreement at Born and LL level was found for both unpolarized cross section and spin asymmetries.
  - Differences become visible when finite nonleading contributions are included.
  - For unpolarized $\pi^+$ electroproduction cross sections, the discrepancies reach approximately

$$2\% \quad (\mathrm{JLab}), \qquad 7\% \quad (\mathrm{HERMES}), \qquad 5\% \quad (\mathrm{EIC}).$$

  - Similar effects are observed for Collins and Sivers asymmetries.
- Discrepancies arise from finite NLL terms with dominant term: $P(z)\ln(1-z)$

# Physical Interpretation

In the factorized approach, the transverse-momentum integration is performed independently of the kinematic constraints of a particular process (i.e., from $0$ to $\infty$) and generates $\log(1-z)$

$$\int_0^\infty \frac{dk_T^2}{k_T^2+(1-z)^2m^2} \;\to\; \ln\frac{\mu_R^2}{(1-z)^2m^2} = \ln\frac{\mu_R^2}{m^2} - 2\ln(1-z).$$

In an explicit radiative-correction calculation, the integration is performed with physical phase-space boundaries, $k_{T,\max}^2(z) = Q^2(1-z)^2/4z$,

$$\int_0^{k_{T,\max}^2(z)} \frac{dk_T^2}{k_T^2+(1-z)^2m^2} = \ln\left(1+\frac{Q^2}{4zm^2}\right),$$

- Universal factorization requires transverse-momentum integrations to be defined independently of process-specific kinematic boundaries, implying an extension of the upper integration limit to $k_T^2 = \infty$.
- The resulting discrepancy reflects the incompatibility between exact kinematics and the universality principle of operator-based factorization.

# The Best Approximation of RC to SIDIS

Construction of the best approximation in which:

- the order $O(\alpha)$ corrections are calculated exactly,
- the exclusive RCs are calculated separately,
- the higher order RCs are computed using leading- and next-to-leading-log contributions,
- radiation from hadrons and quarks is incorporated along with radiation from leptons.

This requires a redefinition of the non-radiated cross section and careful attention to avoid all forms of double counting. For example, if only LL terms are accounted for

$$\sigma_B + \sigma_{RC} = \sigma_B + A \log \frac{Q^2}{m^2} + B + O(m^2/Q^2) = A \log \frac{Q^2}{m^2} + \sigma_{hard}$$

where $\sigma_{hard}$ is our new non-radiated cross section. The best approximation is:

$$\sigma_{best}^{in} = \frac{S_x^2}{p_l} \int\limits_{z_{1i}}^{1} dz_1 D(z_1, Q^2) \int\limits_{\hat{z}_{2i}}^{1} \frac{dz_2}{z_2^2} D(z_2, Q^2) r^2 \left( \frac{z_1}{z_2} Q^2 \right) \frac{\hat{p}_l \sigma_{hard}(\hat{S}, \hat{Q}^2, \hat{x}, \hat{z}, \hat{p}_t, \cos\hat{\phi}_h)}{(z_1 S - X/z_2)^2},$$

$$r(Q^2) = \sum_{i=0}^{\infty} \left( \frac{\alpha}{2\pi} \delta_{\rm vac}^l(Q^2) \right)^i = \left[ 1 - \frac{\alpha}{2\pi} \delta_{\rm vac}^l(Q^2) \right]^{-1},$$

Similarly the formulae for the exclusive radiative tail can be updated.

Part IV:

# Radiative Azimuthal Mixing

# and

# its Phenomenological Implications

# Radiative Azimuthal Mixing

- At Born level, azimuthal weights project out individual structure functions.
- Example:

$$\langle \sin(\phi_h - \phi_S) \rangle = \int d\phi_h \, d\phi_S \, \sin(\phi_h - \phi_S) \, d\sigma_B \propto F_{UT}^{\sin(\phi_h - \phi_S)}$$

- Photon emission changes the azimuthal structure of the cross section.
- Orthogonality of the Born harmonics is lost.
- The weighted radiative cross section receives contributions from all 18 SIDIS structure functions.

$$\langle \sin(\phi_h - \phi_S) \rangle_R = \int d\phi_h \, d\phi_S \, \sin(\phi_h - \phi_S) \, d\sigma_R = \sum_{i=1}^{18} M_i \, F_i$$

- This phenomenon is called **radiative azimuthal mixing**.

Radiative azimuthal mixing is the loss of the one-to-one correspondence between SIDIS asymmetries and structure functions due to QED radiation.

This effect can modify extracted spin asymmetries at the several-percent level and therefore contributes to precision-systematics studies.

# Sivers and Collins asymmetries

The general definitions and the expressions at the Born level are:

- Sivers asymmetry

$$A_{UT}^{\sin(\phi_h-\phi_\eta)} = \frac{2\int_0^{2\pi} d\phi_\eta \int_0^{2\pi} d\phi_h \sin(\phi_h-\phi_\eta)\sigma^{UT}}{\int_0^{2\pi} d\phi_\eta \int_0^{2\pi} d\phi_h \sigma^{UT}}, \qquad A_{UT}^{\sin(\phi_h-\phi_\eta)} = \frac{F_{UT,T}^{\sin(\phi_h-\phi_\eta)}}{F_{UU,T}}$$

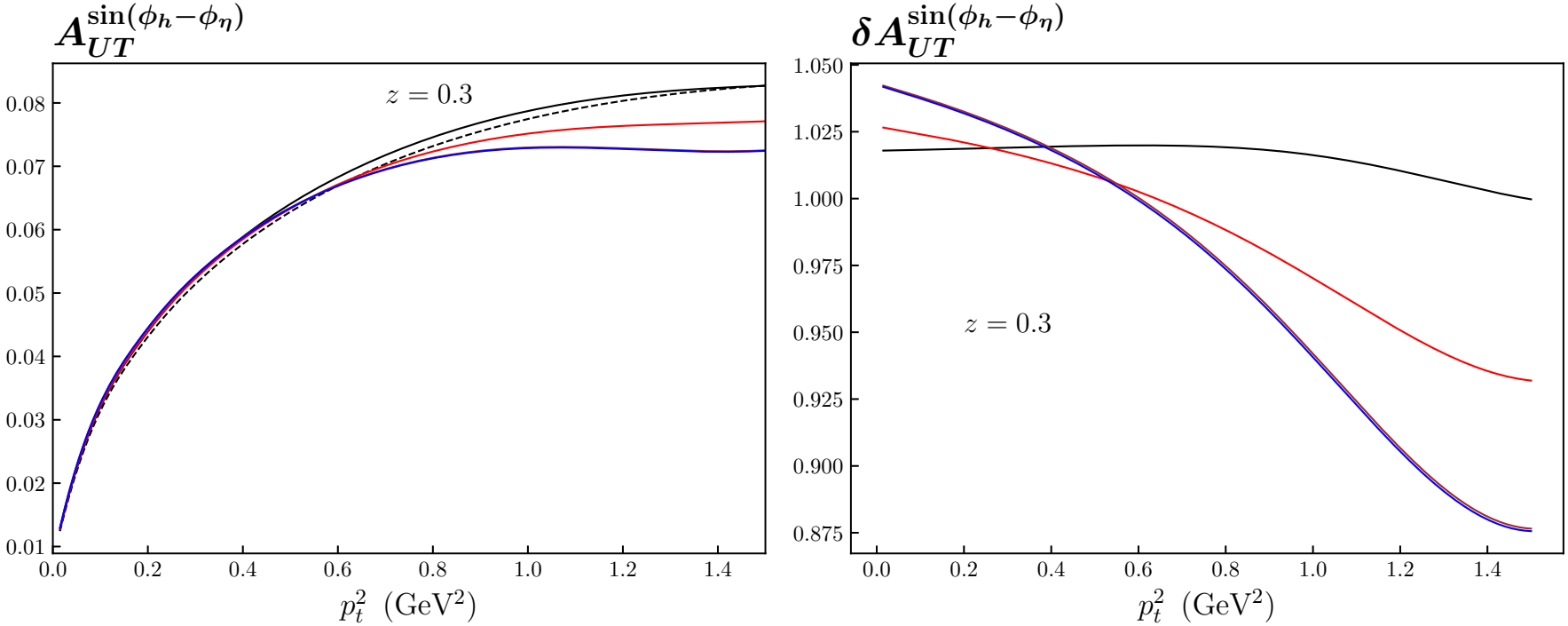
- Collins asymmetry

$$A_{UT}^{\sin(\phi_h+\phi_\eta)} = \frac{2\int_0^{2\pi} d\phi_\eta \int_0^{2\pi} d\phi_h \sin(\phi_h+\phi_\eta)\sigma^{UT}}{\varepsilon\int_0^{2\pi} d\phi_\eta \int_0^{2\pi} d\phi_h \sigma^{UT}}, \qquad A_{UT}^{\sin(\phi_h+\phi_\eta)} = \frac{F_{UT,T}^{\sin(\phi_h+\phi_\eta)}}{F_{UU,T}}$$

# Sivers and Collins asymmetries

We added step by step the different contributions in order to see the quantities of the effect

- Basic contributions $F_{UU,T}$ and $F_{UT,T}^{\sin(\phi_h-\phi_\eta)}$ ($F_{UT}^{\sin(\phi_h+\phi_\eta)}$)
- Unpolarized contributions $F_{UU}^{\cos\phi_h}$ and $F_{UU}^{\cos 2\phi_h}$
- Transverse contributions $F_{UT}^{\sin(\phi_\eta)}$, $F_{UT}^{\sin(3\phi_h-\phi_\eta)}$, $F_{UT}^{\sin(2\phi_h-\phi_\eta)}$
- Longitudinal contributions $F_{UL}^{\sin(\phi_h)}$ $F_{UL}^{\sin(2\phi_h)}$.



# Born cross section

$$
\begin{aligned}
\frac{d^6\sigma_B}{dx\,dy\,d\phi\,dz\,d\phi_h\,dp_t^2} &= \frac{\alpha^2}{xy\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L} \\
&\quad +\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad +\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}+\eta_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\,\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad +\eta_L\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad +\eta_t\Big[\varepsilon\,\sin(\phi_h+\phi_\eta)\,F_{UT}^{\sin(\phi_h+\phi_\eta)}+\sin(\phi_h-\phi_\eta)\,F_{UT,T}^{\sin(\phi_h-\phi_\eta)} \\
&\quad +\varepsilon\,\sin(\phi_h-\phi_\eta)\,F_{UT,L}^{\sin(\phi_h-\phi_\eta)}+\varepsilon\,\sin(3\phi_h-\phi_\eta)\,F_{UT}^{\sin(3\phi_h-\phi_\eta)} \\
&\quad +\sqrt{2\,\varepsilon(1+\varepsilon)}\left(\sin\phi_\eta\,F_{UT}^{\sin\phi_\eta}+\sin(2\phi_h-\phi_\eta)\,F_{UT}^{\sin(2\phi_h-\phi_\eta)}\right)\Big] \\
&\quad +\eta_t\,\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_\eta)\,F_{LT}^{\cos(\phi_h-\phi_\eta)} \\
&\quad +\sqrt{2\,\varepsilon(1-\varepsilon)}\left(\cos\phi_\eta\,F_{LT}^{\cos\phi_\eta}+\cos(2\phi_h-\phi_\eta)\,F_{LT}^{\cos(2\phi_h-\phi_\eta)}\right)\Big]\Bigg\},
\end{aligned}
$$

A. Bacchetta et al. JHEP 0702 (2007) 093

# Born cross section

$$\frac{d^6\sigma_B}{dx\,dy\,d\phi\,dz\,d\phi_h\,dp_t^2} = \frac{\alpha^2}{xy\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}$$

$$+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}+\eta_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\,\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+\eta_L\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+\eta_t\Big[\varepsilon\,\sin(\phi_h+\phi_\eta)\,F_{UT}^{\sin(\phi_h+\phi_\eta)}+\sin(\phi_h-\phi_\eta)\,F_{UT,T}^{\sin(\phi_h-\phi_\eta)}$$

$$+\varepsilon\,\sin(\phi_h-\phi_\eta)\,F_{UT,L}^{\sin(\phi_h-\phi_\eta)}+\varepsilon\,\sin(3\phi_h-\phi_\eta)\,F_{UT}^{\sin(3\phi_h-\phi_\eta)}$$

$$+\sqrt{2\,\varepsilon(1+\varepsilon)}\left(\sin\phi_\eta\,F_{UT}^{\sin\phi_\eta}+\sin(2\phi_h-\phi_\eta)\,F_{UT}^{\sin(2\phi_h-\phi_\eta)}\right)\Big]$$

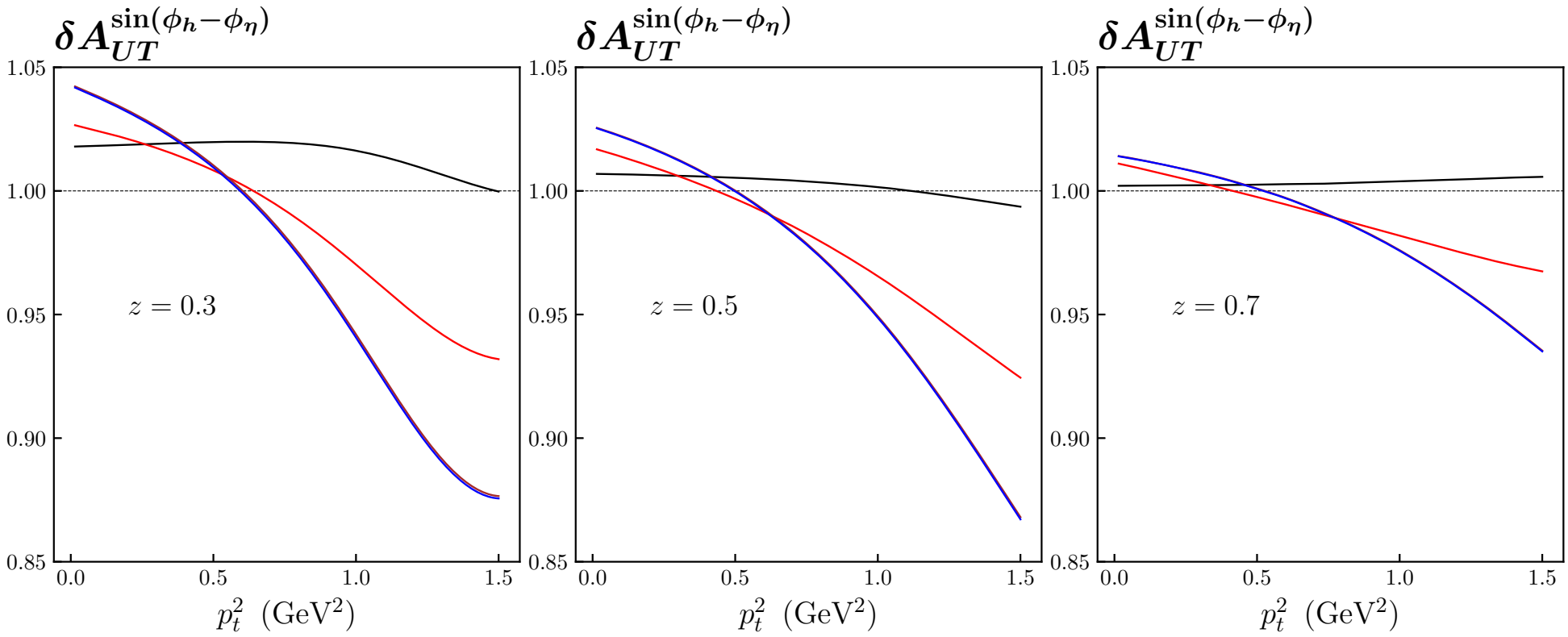
$$+\eta_t\,\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_\eta)\,F_{LT}^{\cos(\phi_h-\phi_\eta)}$$

$$+\sqrt{2\,\varepsilon(1-\varepsilon)}\left(\cos\phi_\eta\,F_{LT}^{\cos\phi_\eta}+\cos(2\phi_h-\phi_\eta)\,F_{LT}^{\cos(2\phi_h-\phi_\eta)}\right)\Big]\Big\},$$

A. Bacchetta et al. JHEP 0702 (2007) 093

# Sivers asymmetries for JLab

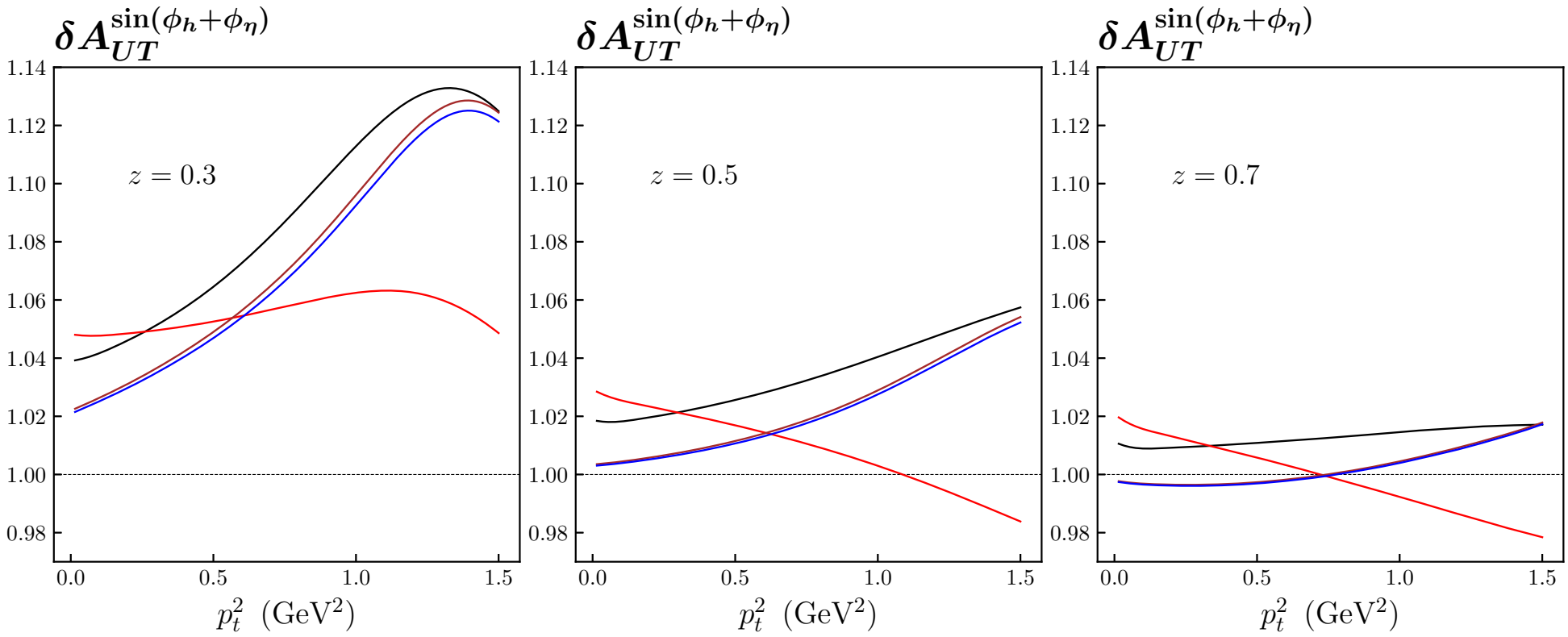
$E_{beam} = 11$ GeV, $Q^2 = 3$ GeV$^2$, $x = 0.2$




$$\delta A_{UT}^{\sin(\phi_h-\phi_\eta)} = 1 + \frac{A_{UT,\,RC}^{\sin(\phi_h-\phi_\eta)}}{A_{UT,\,B}^{\sin(\phi_h-\phi_\eta)}}$$

# Collins asymmetries for Jlab

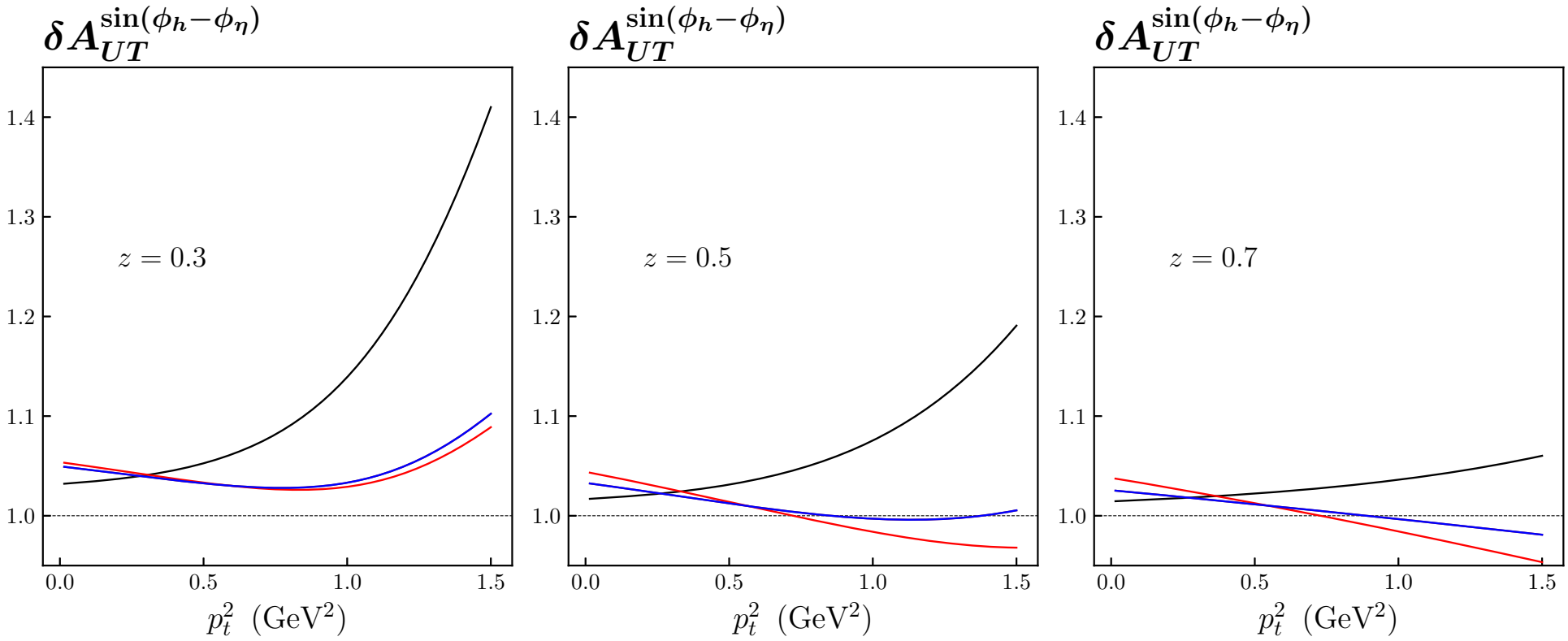
$E_{beam} = 11$ GeV, $Q^2 = 3$ GeV$^2$, $x = 0.2$




$$\delta A_{UT}^{\sin(\phi_h+\phi_\eta)} = 1 + \frac{A_{UT,\,RC}^{\sin(\phi_h+\phi_\eta)}}{A_{UT,\,B}^{\sin(\phi_h+\phi_\eta)}}$$

# Sivers asymmetries for EIC

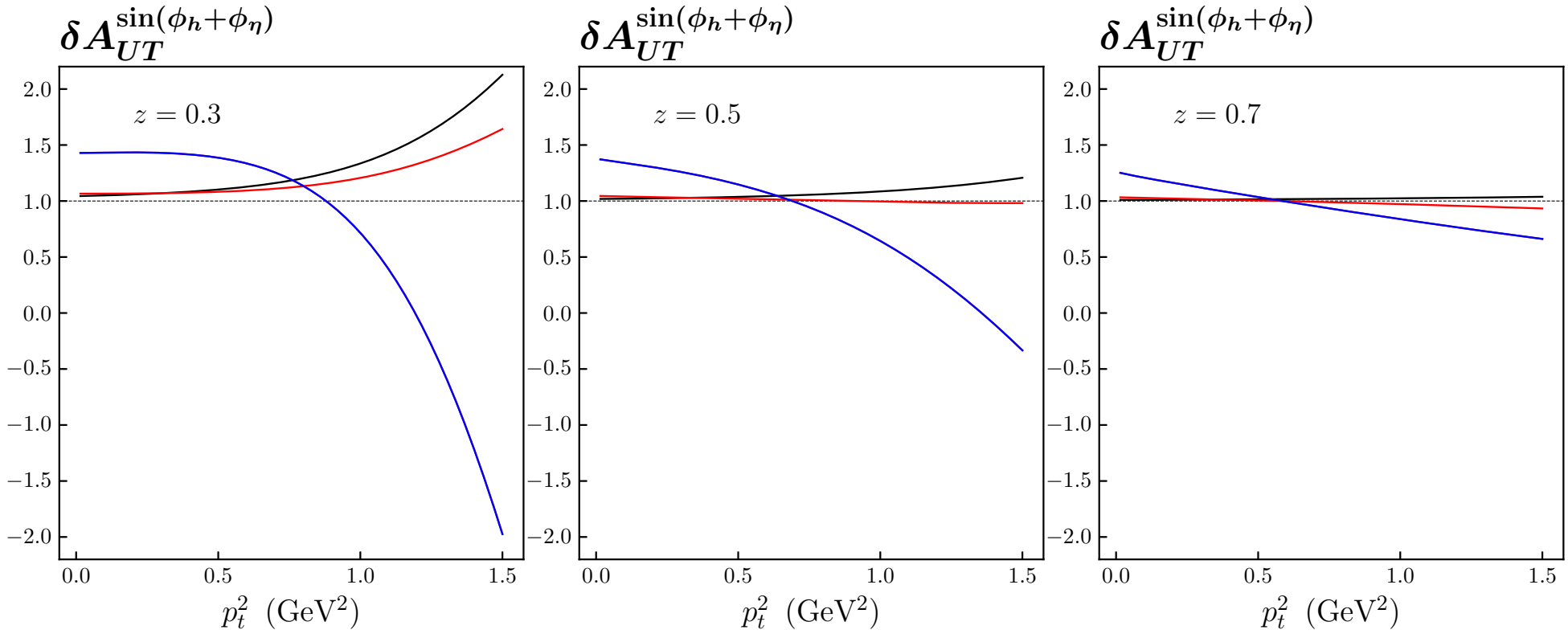
$\sqrt{S} = 28.6$ GeV, $Q^2 = 3$ GeV$^2$, $x = 0.0073$




$$\delta A_{UT}^{\sin(\phi_h-\phi_\eta)} = 1 + \frac{A_{UT,\,RC}^{\sin(\phi_h-\phi_\eta)}}{A_{UT,\,B}^{\sin(\phi_h-\phi_\eta)}}$$

# Collins asymmetries for EIC

$\sqrt{S} = 28.6$ GeV, $Q^2 = 3$ GeV$^2$, $x = 0.0073$




$$\delta A_{UT}^{\sin(\phi_h+\phi_\eta)} = 1 + \frac{A_{UT,\,RC}^{\sin(\phi_h+\phi_\eta)}}{A_{UT,\,B}^{\sin(\phi_h+\phi_\eta)}}$$

Part V:

# Open Theoretical Questions and Future Directions

# Open Points, Previous Version

**Exlusive SFs for polarized case** *We are working on it, but we will need several models to test model-dependence. Any help will be appreciated.*

**SIDIS SFs** *We currently implemented WW approach, however, additional cross checks are required for specific regions. Bounds of applicability of WW are needed to be defined and discussed. Additional models would be helpful as well.*

**Iteration procedure of RC** *We need to decide whether iteration procedure of RC will be used in data analyses. We strongly recommend this (especially for asymmetry measurements) to avoid an additional bias due to RC procedure. The bias is proportional to the difference between the values of Born asymmetries in the given bin: i) finally extracted and ii) used in RC codes. These values coincide by definition when the iteration procedure is used.*

**Comparison and agreement between theoretical calculations** *We need to complete analytic and numeric comparison to leptonic RC calculated by Tianbo Liu, W. Melnitchouk, Jian-Wei Qiu, and N. Sato (Physical Review D, 104(9), 094033; Journal of High Energy Physics, 2021(11), 1).*

**Hadronic corrections** *Discuss and decide whether and how non-leptonic corrections (including box diagrams and emission by hadrons) will be calculated.*

**Higher order corrections** *We need to discuss approaches for higher order corrections, such as exponentiation, electron SFs, etc. Pay specific attention to the region with small* $M_x^2$.

**Monte Carlo** *Approaches to MC generators has to be discussed and implemented.*

# Open Points, Current Version

**Development of the best approximation** *Construction of the best approximation in which: i) the order $O(\alpha)$ corrections are calculated exactly, ii) the exclusive RCs are calculated separately, iii) the higher order RCs are computed using leading- and next-to-leading-log contributions, and iv) radiation from hadrons and quarks is incorporated along with radiation from leptons.*

**Further development of the hybrid approach** *Continued theoretical development of the hybrid approach, including further clarification of the origin of the $\log(1-z)$ term and development of EPM factorization scheme.*

**Effects of radiation-induced azimuthal mixing** *Detailed numerical investigation of radiation-induced azimuthal mixing for a wide range of asymmetries and other observables in modern SIDIS experiments.*

**Evaluation of radiative-correction-induced systematic uncertainties in SIDIS** *Quantitative assessment of SIDIS systematic error due to RC, focusing on hadronic SF modeling. A key source is the extension of QCD-based parameterizations into the resonance region, where nonperturbative dynamics prevail, potentially leading to significant biases in RC calculations.*

**Electroweak radiative corrections** *Extension of the current framework to include electroweak effects, development of a best-approximation formalism incorporating both QED and electroweak radiative corrections, and detailed numerical evaluations for future EIC measurements.*

**Extension to additional SIDIS channels** *Extension of the theoretical framework and its practical implementation to $\pi^0$ and $\pi^-$-SIDIS measurements.*

**Software development for SIDIS radiative-correction studies** *Further development of computational tools for SIDIS radiative-correction analyses, including both point-by-point calculation codes (e.g., HAPRAD 3.0) and Monte Carlo event generators suitable for realistic experimental applications.*

# Conclusion

- Newly achieved accuracies at JLab and the precision goals of the EIC require renewed attention to radiative-correction calculations and their implementation in data-analysis software.
- The Hybrid approach combines the strengths of traditional and factorized formalisms and provides a pathway toward the most complete SIDIS radiative-correction framework currently achievable.
- Theoretical differences between traditional and factorized approaches are now understood, and the mechanisms responsible for these differences have been identified.
- The best approximation currently possible from a theoretical perspective can be constructed by combining the key advantages of the traditional and factorized frameworks.
- Radiative azimuthal mixing demonstrates that QED radiation can modify the relation between measured asymmetries and underlying SIDIS structure functions and should be accounted for in precision analyses.

# Perspectives

- Completion of a Hybrid radiative-correction framework combining exact $O(\alpha)$ calculations, exclusive radiative tails, higher-order LL/NLL contributions, and radiation from quarks/hadrons within a unified theoretical and computational framework.
- Further development of the Hybrid framework, including higher-order corrections, studies of factorization-scheme dependence, and construction of the best approximation of RC in SIDIS.
- Development of electroweak radiative-correction frameworks for EIC measurements, where electroweak effects become an essential component of high-precision phenomenology.
- Development and implementation of radiative unfolding procedures for spin-dependent SIDIS observables. Since radiative corrections depend on the structure functions being extracted, the problem requires solving an integral equation through iterative reconstruction.
- Quantitative evaluation of radiative-correction-induced systematic uncertainties, with particular emphasis on SIDIS and exclusive structure-function modeling in resonance and other beyond-QCD regions.
- Development of next-generation computational tools for SIDIS radiative-correction studies, including analytical point-by-point calculation codes, SIDIS-specific Monte Carlo generators, and realistic radiated-photon event generation.