



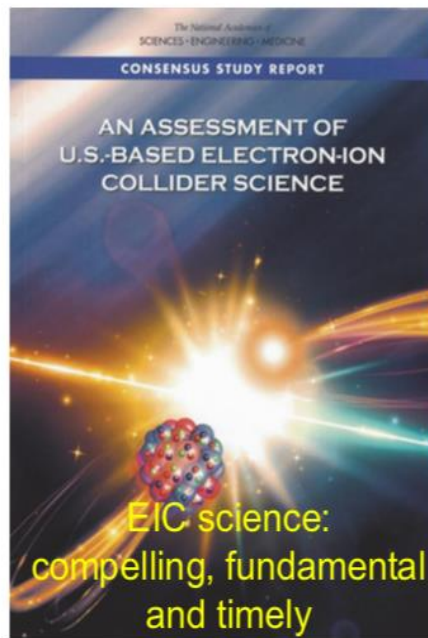
# hQCD @ Jlab @ eIC

I. Zahed

JLAB 26'



# US National Academy of Sciences, Engineering and Medicine: Assessment & Consensus Report



## Physics of EIC

Emergence of Spin

Emergence of Mass

Physics of high-density gluon fields

## Machine Design Parameters:

High luminosity: up to  $10^{33}$ - $10^{34}$  cm<sup>-2</sup>sec<sup>-1</sup>

Broad range in center-of-mass energy: ~20-100 (upgrade 140) GeV

Polarized beams e-, p, and light ion beams

Broad range in hadron species: protons.... Uranium

Up to two detectors well-integrated detector(s) with machine lattice



# Outline

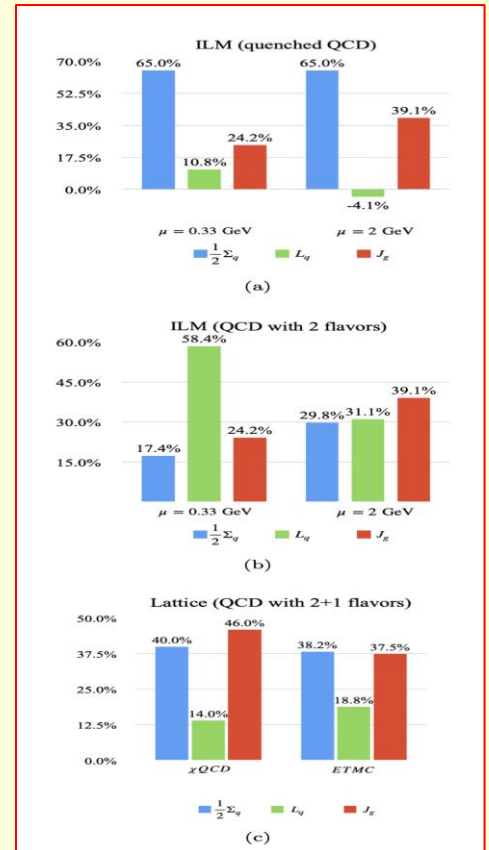
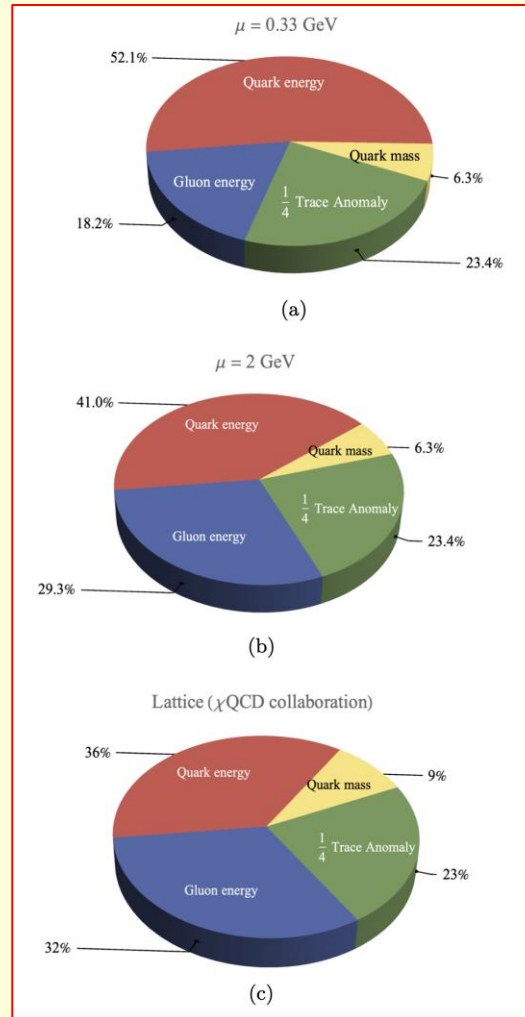
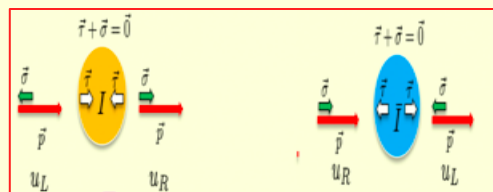
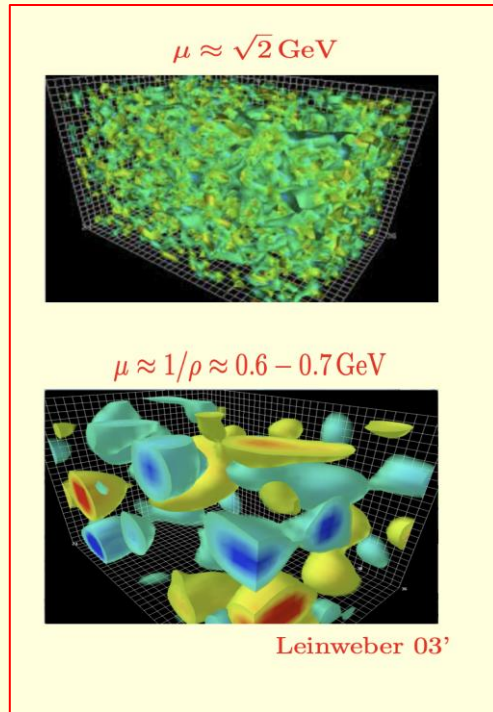
- Why strings and hQCD
- EMT, Jpsi, GPDs, low-x

## 4D topological vacuum: ILM mass and spin

Ji mass SR



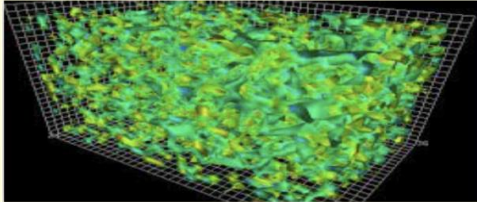
Ji spin SR



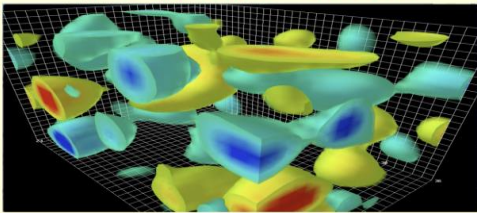
Liu-Shuryak-Zahed 23'

## Topology: 2D, 3D, 4D

$$\mu \approx \sqrt{2} \text{ GeV}$$

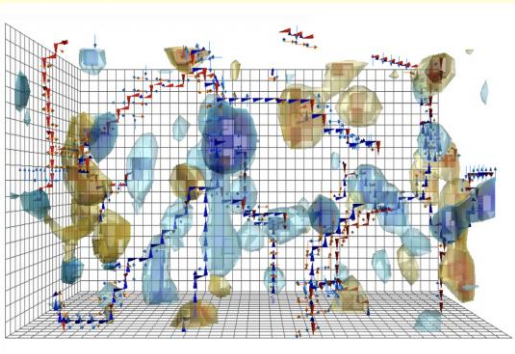


$$\mu \approx 1/\rho \approx 0.6 - 0.7 \text{ GeV}$$



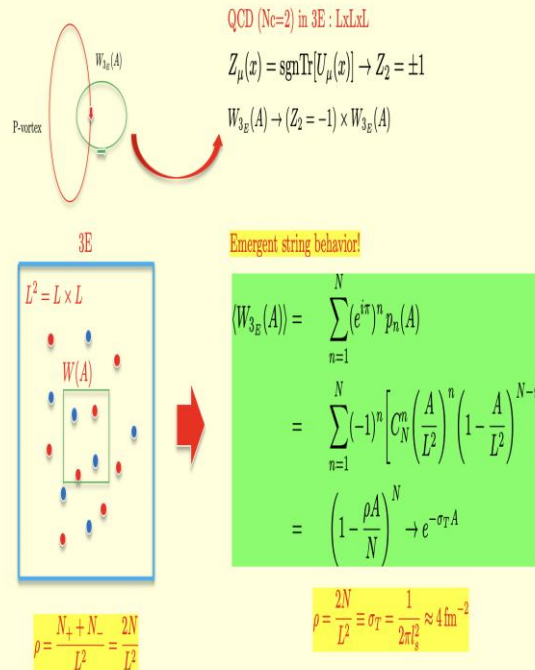
Leinweber 03'

Instantons and anti-instantons and P-vortices



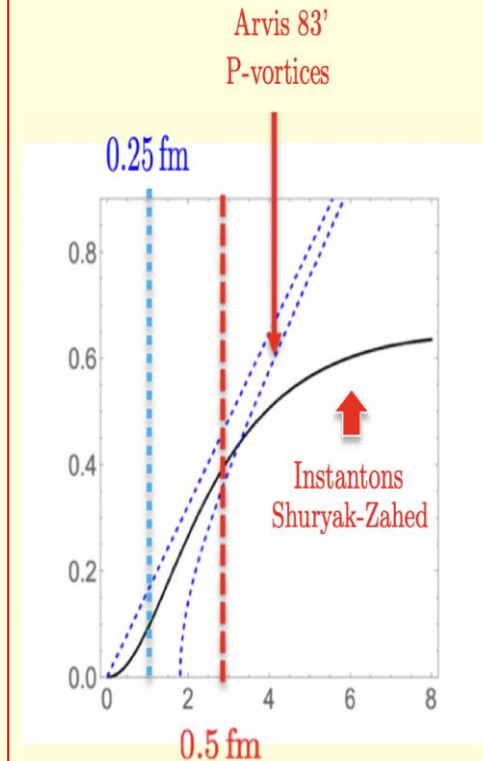
Biddle-Kamleh-Leinweber 20'

## Mechanism of Confinement

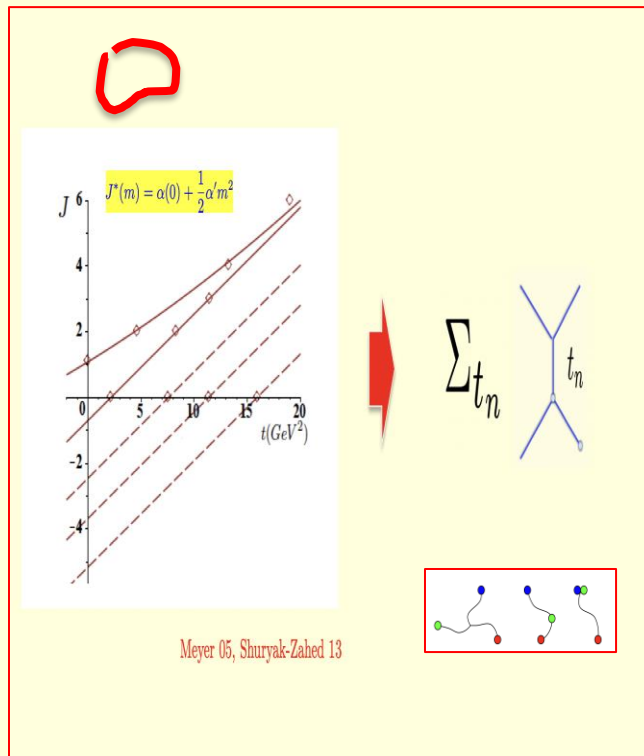


Holography provides a string based approach dual to YM  $N_c > \lambda = g^2 N_c > 1$

## Genealogy of linear Potential



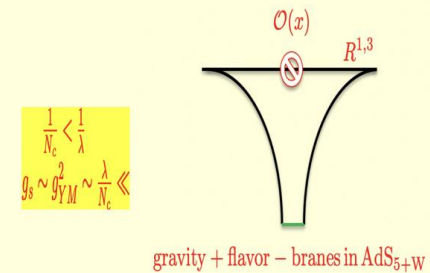
## Linear Regge spectra: Resonance dual model 2x2



## hQCD as an effective string field theory nxm

top-down: Maldacena 98, Witten 98

bottom-up: Polshinski-Strassler 02, Erlich et al. 05



$$\mathcal{O}(x) = R^\mu(x), T^{\mu\nu}(x) \rightarrow R^{MN}, h^{MN}(x)$$

$$\Delta = 2 + \sqrt{4 + m_5^2}$$

$$\frac{1}{\sqrt{g}} \partial_M (\sqrt{g} e^{-\kappa_N z^2} R^{MN}) + m_5^2 R^N = 0$$

$$m_n^2 = 4\kappa_N \left( n + \frac{\Delta}{2} \right)$$

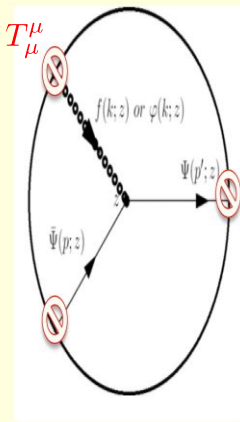
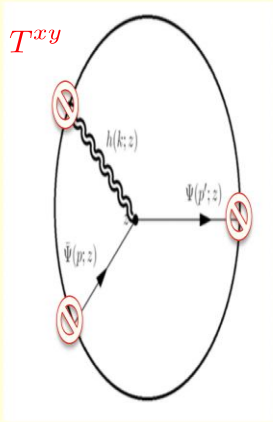
# Outline

- Why strings and hQCD
- EMT, Jpsi, GPDs, low-x



## GFs from hQCD 2x1

$$\langle p_2 | T^{\mu\nu}(0) | p_1 \rangle = \bar{u}(p_2) \left( A(k) \gamma^{(\mu} p^{\nu)} + B(k) \frac{i p^{(\mu} \sigma^{\nu)\alpha} k_\alpha}{2m_N} + C(k) \frac{k^\mu k^\nu - \eta^{\mu\nu} k^2}{m_N} \right) u(p_1)$$



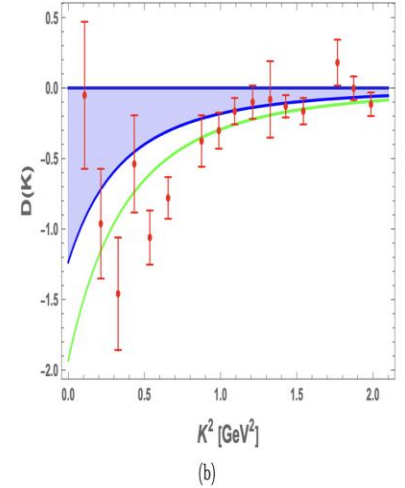
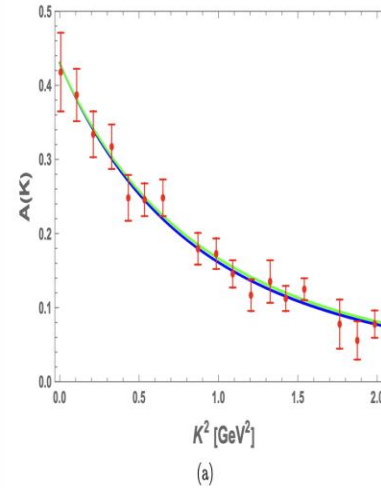
Spin-2:  $\langle p_2 | T^{xy}(0) | p_1 \rangle$

Spin-0:  $\langle p_2 | T_{\mu}^{\mu}(0) | p_1 \rangle$

$$h_{\mu\nu}(k, z) = \left[ \epsilon_{\mu\nu}^{TT} h(k, z) + k_\mu k_\nu H(k, z) \right] + \left[ k_\mu A_\nu^\perp(k, z) + k_\nu A_\mu^\perp(k, z) \right] + \left[ \frac{1}{3} \eta_{\mu\nu} f(k, z) \right]$$

Mamo-Zahed 20

## GFs holography vs Lattice



$$A(k, \kappa_T) = A(0) \times 6 \times \frac{\Gamma(2 + \frac{a_K}{2})}{\Gamma(4 + \frac{a_K}{2})} \times {}_2F_1\left(3, \frac{a_K}{2}; \frac{a_K}{2} + 4; -1\right),$$

$$a_K = k^2 / 8\kappa_T^2 \rightarrow k^2 / 8 (0.350) \text{ GeV}^2$$

Lattice: Pefkou, Hackett, Shanahan 22'

$$D(k, \kappa_T, \kappa_S) = \frac{4 m_N^2}{3 k^2} \left( A(k, \kappa_T) - A_S(k, \kappa_S) \right)$$

$$A_S(K, \kappa_S) = A(0) \times 6 \times \frac{\Gamma(2 + \frac{\hat{a}_K}{2})}{\Gamma(4 + \frac{\hat{a}_K}{2})} \times {}_2F_1\left(3, \frac{\hat{a}_K}{2}; \frac{\hat{a}_K}{2} + 4; -1\right),$$

Mamo-Zahed 22'

# Genealogy of D=4C-term and radial Force

$$\overset{0^{++}}{\frac{1}{3} \langle p_2 | T_\mu^\mu(0) | p_1 \rangle} = \overset{2^{++}}{\bar{u}(p_2)} \left( A(k) \frac{m_N}{3} + \frac{k^2}{12m_N} B(k) - \frac{k^2}{m_N} C(k) \right) u(p_1)$$

$$C(k) = \frac{1}{3} \frac{m_N^2}{k^2} \left( \underset{2^{++}}{A(k)} - \underset{0^{++}}{A_S(k)} \right) + \frac{1}{12} B(k) \rightarrow \frac{1}{3} \frac{m_N^2}{k^2} \left( A(k) - A_S(k) \right)$$

Negative!

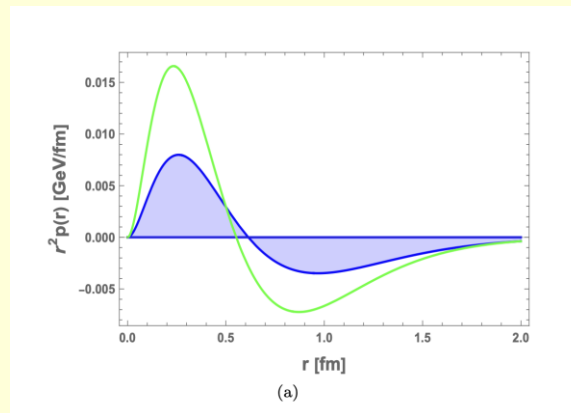


$$D(K) = 4C(K) \approx -\frac{4m_N^2}{3K^2} \left[ \underset{2^{++}}{\frac{A(0)}{\left(1 + \frac{K^2}{m_T^2}\right)^2}} - \underset{0^{++}}{\frac{A(0)}{\left(1 + \frac{K^2}{m_S^2}\right)^2}} \right] = D(0) \left[ \frac{1 + \frac{K^2}{4} \left( \frac{1}{m_S^2} + \frac{1}{m_T^2} \right)}{\left(1 + \frac{K^2}{m_T^2}\right)^2 \left(1 + \frac{K^2}{m_S^2}\right)^2} \right]$$

Tripole!

$$p(r) = \frac{1}{3} \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d}{dr} \tilde{D}(r) \right)$$

Polyakov 03, Polyakov-Schweitzer 18



Mamo-Zahed 20

# Gravitational radii

Ji 22'

$$A_S(K) \equiv A(K) - \frac{K^2}{4m_N^2} B(K) + \frac{3K^2}{4m_N^2} D(K)$$

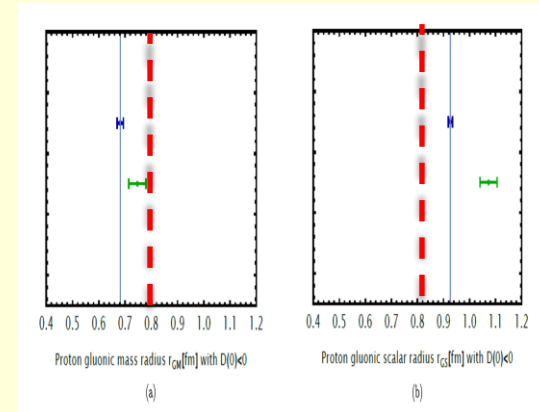
$$A_M(K) \equiv A(K) - \frac{K^2}{4m_N^2} B(K) + \frac{K^2}{4m_N^2} D(K)$$

$$\langle r_{GS}^2 \rangle = -\frac{6}{A_S(0)} \left( \frac{dA_S(K)}{dK^2} \right)_0 \hbar^2 c^2$$

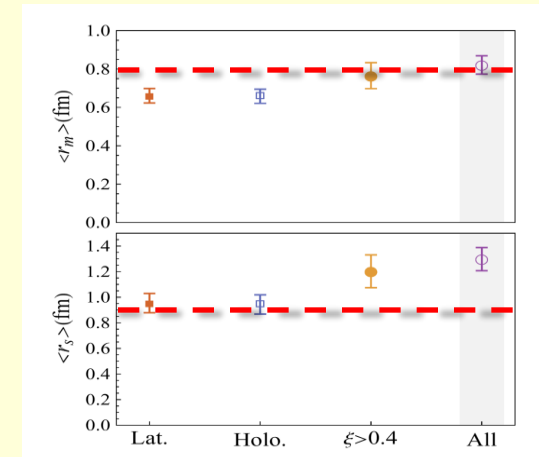
$$\langle r_{GM}^2 \rangle = -\frac{6}{A_M(0)} \left( \frac{dA_M(K)}{dK^2} \right)_0 \hbar^2 c^2$$

|                                   | $\langle r_{GS}^2 \rangle \text{ (fm}^2\text{)}$                                            | $\langle r_{GM}^2 \rangle \text{ (fm}^2\text{)}$                                                                        |
|-----------------------------------|---------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|
| Holographic QCD [?] ]             | $\frac{1.04}{\kappa_S^2} = 0.926^2 \text{ (or } 0.519^2 \text{ with } \kappa_S = \kappa_T)$ | $\frac{0.693}{\kappa_T^2} + \frac{0.347}{\kappa_S^2} = 0.682^2 \text{ (or } 0.519^2 \text{ with } \kappa_S = \kappa_T)$ |
| Holographic QCD (tripole approx.) | $-\frac{D(0)}{A(0)} \times \frac{4.5}{m_N^2} + \frac{18}{m_{TT}^2} = 0.926^2$               | $-\frac{D(0)}{A(0)} \times \frac{1.5}{m_N^2} + \frac{18}{m_{TT}^2} = 0.682^2$                                           |
| Lattice QCD (tripole fit) [20]    | $-\frac{D(0)}{A(0)} \times \frac{4.5}{m_N^2} + \frac{18}{m_{TT}^2} = 1.073^2$               | $-\frac{D(0)}{A(0)} \times \frac{1.5}{m_N^2} + \frac{18}{m_{TT}^2} = 0.747^2$                                           |
| Lattice QCD (dipole fit) [30]     | $-\frac{D(0)}{A(0)} \times \frac{4.5}{m_N^2} + \frac{12}{m_{TT}^2} = 1.945^2$               | $-\frac{D(0)}{A(0)} \times \frac{1.5}{m_N^2} + \frac{12}{m_{TT}^2} = 1.227^2$                                           |

Mamo-Zahed 22'



Mamo-Zahed 22'



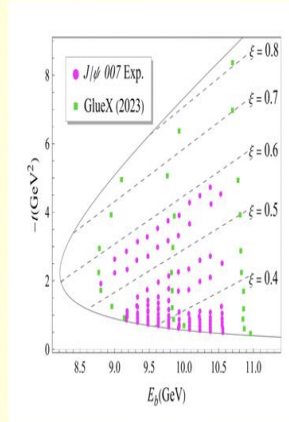
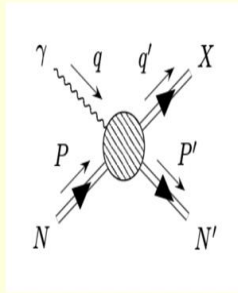
Guo-Ji-Liu-Yang 23'

# Outline

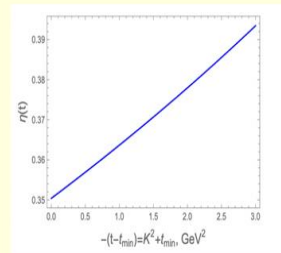
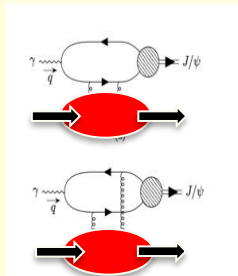
- Why strings and hQCD
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## J/psi photoproduction at Jlab



Guo-Ji-Liu-Yang 23'

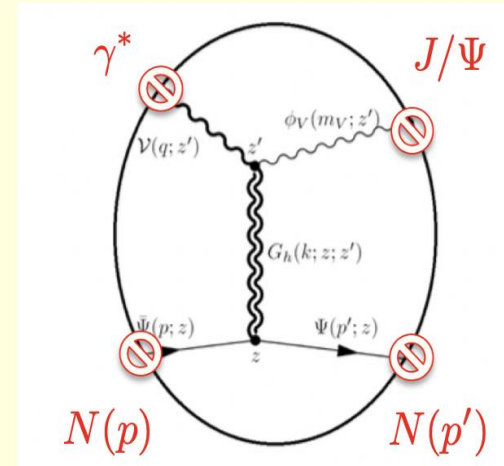


Mamo-Zahed 22'

$$\xi(t) = \eta(t) = -\frac{q \cdot (P' - P)}{q \cdot (P' + P)}$$

## hQCD 2x2

$$\gamma^*(q, \epsilon) + N(p) \rightarrow V(q', \epsilon') + N(p')$$

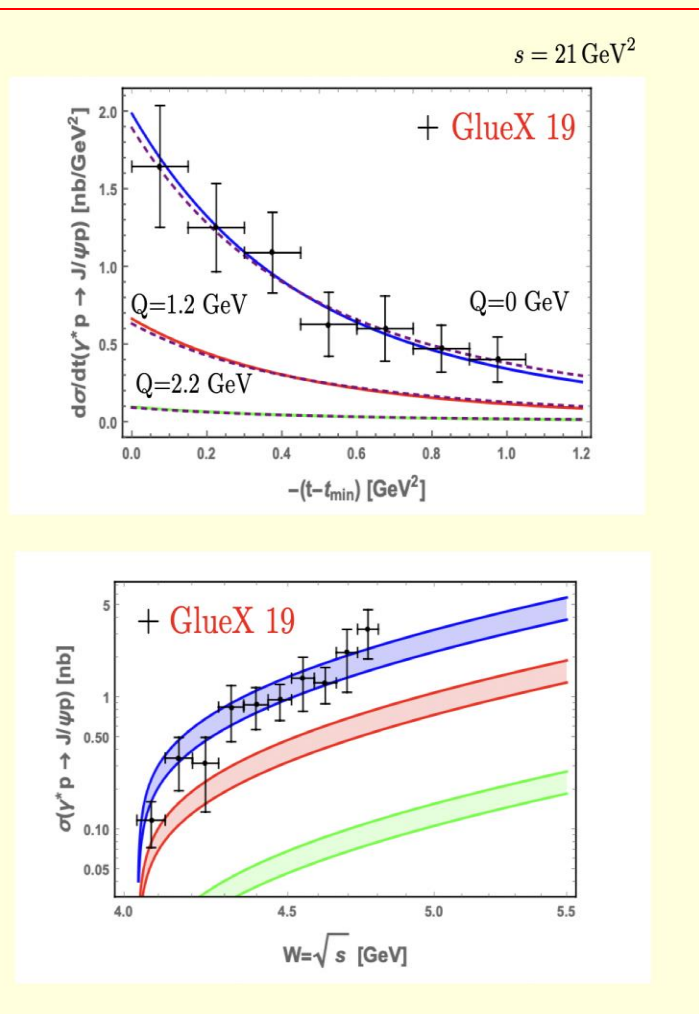


$$\left(\frac{d\sigma}{dt}\right) = \mathcal{N}^2 \times \frac{e^2}{64\pi(s - m_N^2)^2} \times \frac{[A(K, \kappa_T) + \eta^2 D(K, \kappa_T, \kappa_S)]^2}{A^2(0)} \times \tilde{F}(s) \times 8,$$

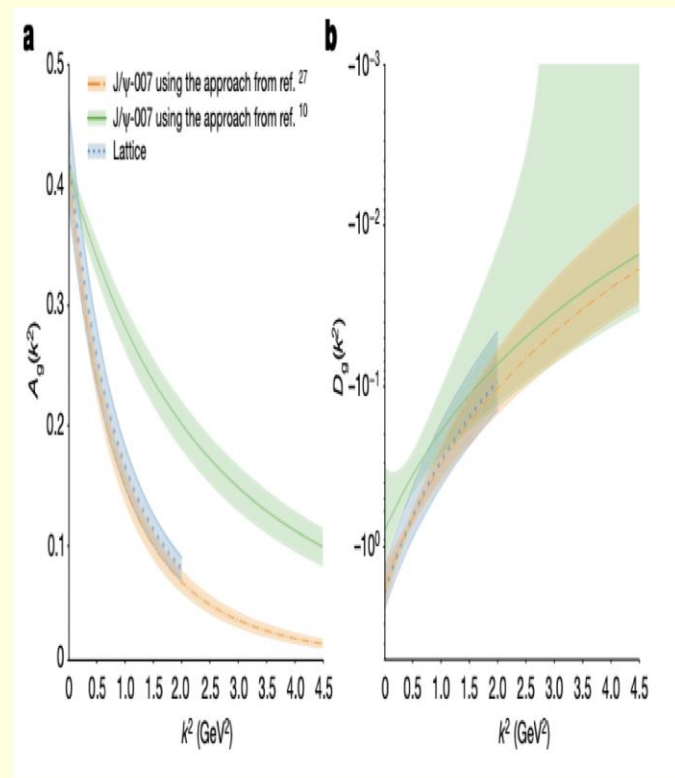
Mamo-Zahed 20

See also Kharzeev 96',  
Hatta et al. 19, 20',  
Guo, Ji and Liu 21',  
Yuan et al. 21, 22'

## A-FF measured by GlueX 19'

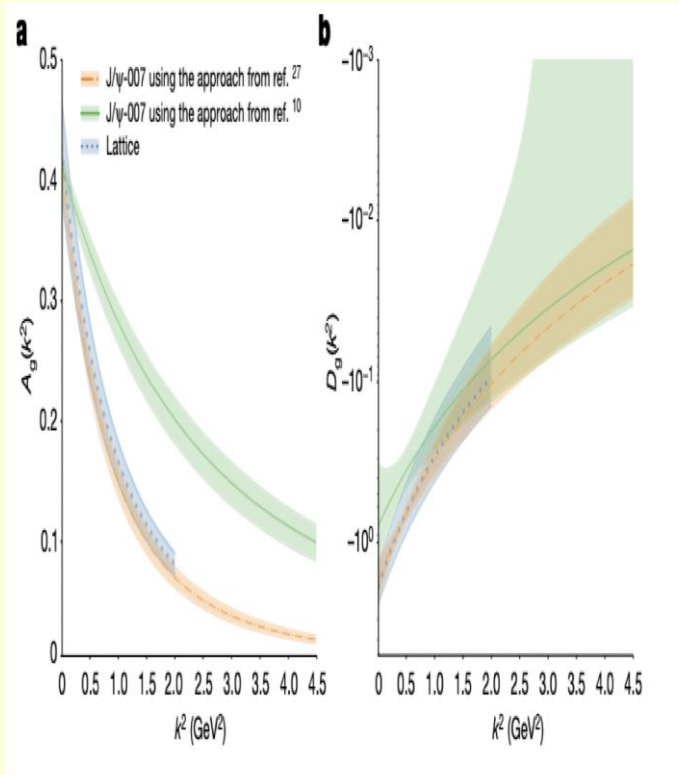


## A-FF and D-FF measured by 007 23'



J/Psi-007 23'

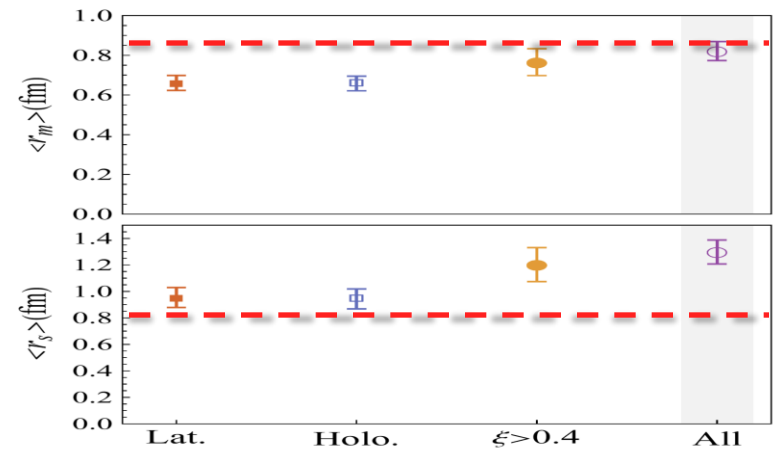
## A-FF and D-FF measured by 007 23'



J/Psi-007 23'

## Extracted mass radii by 007 23'

| Theoretical approach | $\chi^2/\text{n.d.f}$ | $m_A$ (GeV)       | $m_C$ (GeV)     | $C_g(0)$           | $\sqrt{\langle r_m^2 \rangle_g}$ (fm)   | $\sqrt{\langle r_s^2 \rangle_g}$ (fm)   |
|----------------------|-----------------------|-------------------|-----------------|--------------------|-----------------------------------------|-----------------------------------------|
| GFF functional form  |                       |                   |                 |                    |                                         |                                         |
| Holographic QCD      | 0.925                 | $1.575 \pm 0.059$ | $1.12 \pm 0.21$ | $-0.45 \pm 0.132$  | $0.755 \pm 0.035$                       | $1.069 \pm 0.056$                       |
| Tripole-tripole      |                       |                   |                 |                    |                                         |                                         |
| GPD                  | 0.924                 | $2.71 \pm 0.19$   | $1.28 \pm 0.50$ | $-0.20 \pm 0.11$   | <del><math>0.472 \pm 0.042</math></del> | <del><math>0.695 \pm 0.071</math></del> |
| Tripole-tripole      |                       |                   |                 |                    |                                         |                                         |
| Lattice              |                       | $1.641 \pm 0.043$ | $1.07 \pm 0.12$ | $-0.483 \pm 0.133$ | $0.7464 \pm 0.025$                      | $1.073 \pm 0.066$                       |
| Tripole-tripole      |                       |                   |                 |                    |                                         |                                         |



Guo-Ji-Liu-Yang 23'

## Experimental proposal for reconstruction of the trace anomaly FF

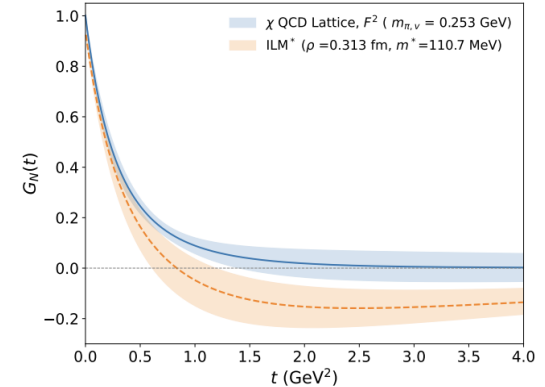
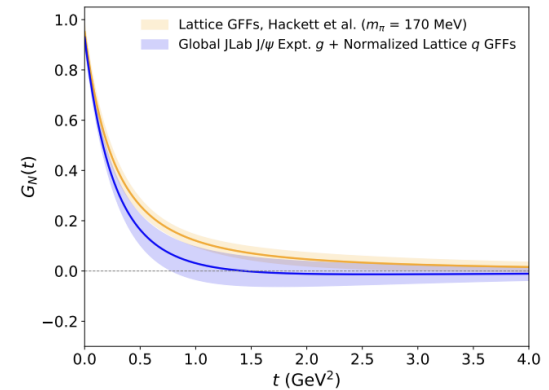
$$T_{\mu}^{\mu} = \frac{\beta(g)}{2g} F_{\mu\nu}^a F^{a\mu\nu} + \sum_f (1 + \gamma_m) m_f \bar{\psi}_f \psi_f$$

$$G_{s,a}(t) = A_a(t) + \frac{t}{4M^2} B_a(t) - \frac{3t}{M^2} C_a(t), \quad a = q, g$$

$$G_N(t) \equiv \frac{\langle p' | T_{g\mu}^{\mu} | p \rangle}{M \bar{U}(p') U(p)} = G_{s,g}(t) + G_{s,q}(t) - \frac{\sigma(t)}{M}$$

$$G_N(0) = 1 - \frac{\sigma(0)}{M_N}$$

$$\text{DVCS}(q) + \text{threshold } J/\psi(g) + \sigma_{\pi N} \Rightarrow \text{trace anomaly}$$

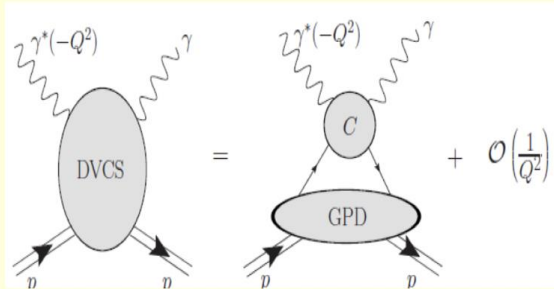


# Outline

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## GPDS and light rays in DVCS



$$\mathcal{H}^a(\eta, t, Q^2) = \int dx C^a(x, \eta, Q^2/Q_0^2) H^a(x, \eta, t, Q_0^2)$$



$H, E, \tilde{H}, \tilde{E}$

$$\begin{aligned} H^q(z^-, t) &= \langle p_2 | \bar{\psi}(z_1^-) [z_1^-, z_2^-] \gamma^+ \psi(z_2^-) | p_1 \rangle, \\ H^g(z^-, t) &= \langle p_2 | F^{+\mu} a(z_1^-) [z_1^-, z_2^-]^{ab} g_{\mu\nu} F_b^{\nu+}(z_2^-) | p_1 \rangle, \end{aligned}$$

Mueller et al 94', Ji 96', Radyushkin 96'

## Light rays/string vertices in hQCD

$$F^{+\mu}(z_1^-) [z_1^-, z_2^-] F_\mu^+(z_2^-) = \sum_{j=1}^{\infty} \frac{1}{j!} (-iz^-)^j n_{\mu_1} \dots n_{\mu_j} F^{+\mu} i D^{\mu_1} \dots i D^{\mu_j} F_\mu^+$$

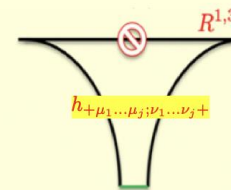
String theory



$$\bar{h}_{+\mu_1 \dots \mu_j; \nu_1 \dots \nu_j +} \partial X^+ \partial X^{\mu_1} \dots \partial X^{\mu_j} \bar{\partial} X^{\nu_1} \dots \bar{\partial} X^{\nu_j} \bar{\partial} X^+ e^{ik \cdot X}$$

$$h = \bar{h} = j + 1 + \frac{\alpha' k^2}{4} = 1 : \text{BRST}$$

SUGRA



gravity + flavor – branes in AdS<sub>5</sub>+w

## Unpolarized and polarized GPDs

$$\begin{aligned}
 \langle p_2 | \bar{\psi}_q(-z^-) [-, +] \gamma^+ \psi_q(z^-) | p_1 \rangle &= \int_{-1}^1 dx e^{-ixP^+z^-} \left( \tilde{h}^+ H_q(x, \eta, t; \mu) + e^+ E_q(x, \eta, t; \mu) \right), \\
 \langle p_2 | \bar{\psi}_q(-z^-) [-, +] \gamma^+ \gamma^5 \psi_q(z^-) | p_1 \rangle &= \int_{-1}^1 dx e^{-ixP^+z^-} \left( \tilde{h}^+ \tilde{H}_q(x, \eta, t; \mu) + \tilde{e}^+ \tilde{E}_q(x, \eta, t; \mu) \right), \\
 \langle p_2 | F_a^{\mu+}(-z^-) [-, +] F_{a\mu}^+(z^-) | p_1 \rangle &= \int_{-1}^1 dx e^{-ixP^+z^-} \left( h^+ H_g(x, \eta, t; \mu) + e^+ E_g(x, \eta, t; \mu) \right), \\
 \langle p_2 | F_a^{\mu+}(-z^-) [-, +] \tilde{F}_{\mu a}^+(z^-) | p_1 \rangle &= \int_{-1}^1 dx e^{-ixP^+z^-} \left( \tilde{h}^+ \tilde{H}_g(x, \eta, t; \mu) + \tilde{e}^+ \tilde{E}_g(x, \eta, t; \mu) \right),
 \end{aligned}$$

$$h^+ = \bar{N}(p_2) \gamma^+ N(p_1), \quad e^+ = \frac{1}{2m_N} \bar{N}(p_2) i \sigma^{\nu+} q_\nu N(p_1), \quad (1a)$$

$$\tilde{h}^+ = \bar{N}(p_2) \gamma^+ \gamma^5 N(p_1), \quad \tilde{e}^+ = -\frac{q^+}{2m_N} \bar{N}(p_2) \gamma^5 N(p_1). \quad (1b)$$

Radyushkin-Beletsky 05'

## GPDs conformal expansion in hQCD

$$\mathbb{H}_a(j, \eta, t; \mu_0) = \mathcal{H}_a(j, t; \mu_0) + \mathcal{H}_{a\eta}(j, \eta, t; \mu_0)$$

$$\mathcal{H}_a(j, t; \mu_0) = \int_0^1 dx \frac{f_a(x, \mu_0)}{x^{j_a - j + \alpha'_a} t} \quad \xrightarrow{\text{MSTW and AAC}}$$

$$\mathcal{H}_{a\eta}(j, \eta, t; \mu_0) = [\hat{d}_j(\eta, t) - 1] \left[ \mathcal{H}_a(j, t; \mu_0) w - \mathcal{H}_a^S(j, t; \mu_0) \right]$$

$$\hat{d}_j(\eta, t) = {}_2F_1\left(-\frac{j}{2} - \frac{j-1}{2}, \frac{1}{2} - j; -\frac{4m_N^2 \eta^2}{t}\right)$$

Mamo-zahed 22', 24'  
Hechenberger-Mamo-Zahed 25'

$$H_a(x, \eta, t) = \sum_{n \in S_a} (-1)^{n+1} P_n(x, \eta) \mathbb{H}_a(n, t, \eta),$$

$$P_n(x, \eta) = \frac{(1 + x/\eta)^{n+1}}{(1 - x/\eta)^{n+2}} C_n^{\frac{3}{2}}\left(\frac{x}{\eta}\right)$$

$$\mathbb{H}_a(n, t, \eta) = \int_{-1}^1 dx P_n(x, \eta) H_a(x, \eta, t)$$

Mellin – Barnes :

$$H_a(x, \eta, t) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{dj}{\sin(\pi j)} P_j(x, \eta) \mathbb{H}_a(j, t, \eta).$$

$\sin(\pi j) \mathbb{H}_a(j, t, \eta)$  : pole free

Muller-Schafer 06'  
Guo-Ji-Santiago-Shiells-Yang 23'

## String based GPDs vs Lattice GPDs

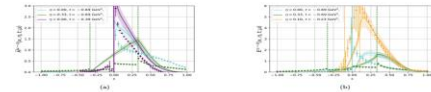


FIG. 4. (a) Evolved non-singlet vector axial GPD at a resolution of  $\mu = 2$  GeV compared to [4] and (b) non-singlet vector GPD  $E$  compared to [4] (green, transparent), [24] (orange) at  $\mu = 2$  GeV and [4] (purple) at  $\mu = 3$  GeV. The dashed line separates the ERBL region (inner) from the DGLAP region (outer).

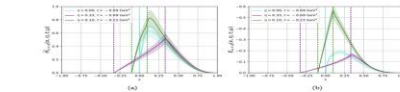


FIG. 5. (a) Evolved non-singlet axial tensor  $\hat{H}$  GPD at a resolution of  $\mu = 3$  GeV. (b) Evolved non-singlet tensor  $\hat{E}$  GPD at a resolution of  $\mu = 2$  GeV. The dashed line separates the ERBL region (inner) from the DGLAP region (outer).



FIG. 6. Evolved singlet axial sea quark (a) and gluon (b) GPDs at a resolution of  $\mu = 2$  GeV. The dashed line separates the ERBL region (inner) from the DGLAP region (outer).

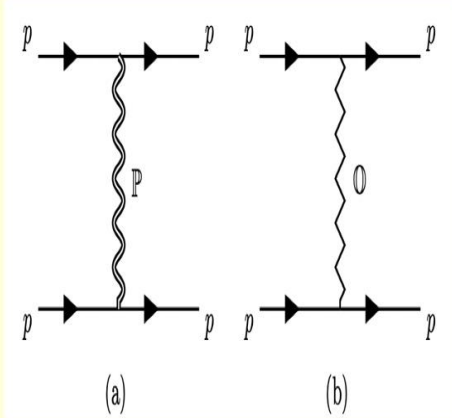
Hechenberger-Mamo-Zahed 25'

# Outline

- Why strings and hQCD
- EMT, Jpsi, GPDs, low-x



## pp at large s



**2<sup>++</sup>**

**1<sup>--</sup>**

$$2^{++} \rightarrow (2+j)^{++} \quad 1^{--} \rightarrow (1+j)^{--}$$

$$\mathcal{A}_{pp} = \mathcal{A}_{pp}^{\mathbb{P}} + \mathcal{A}_{pp}^{\mathbb{O}}$$

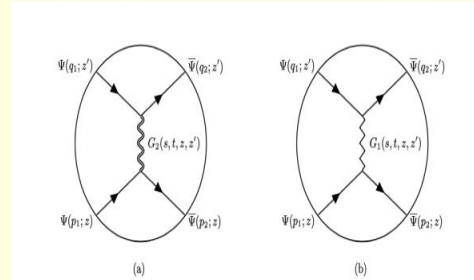
$$\mathcal{A}_{p\bar{p}} = \mathcal{A}_{pp}^{\mathbb{P}} - \mathcal{A}_{pp}^{\mathbb{O}}$$

## pp in hQCD

$$h^{\mu\nu}[2^{++}] : G^{a\mu\alpha} G_{\alpha}^{a\nu}$$

$$B^{\mu\nu}[1^{+-}] : d^{abc} G^{a\alpha\beta} G_{\alpha\beta}^b G^{c\mu\nu}$$

$$C^{\mu\nu}[1^{--}] : d^{abc} G^{a\alpha\beta} G_{\alpha\beta}^b \tilde{G}^{c\mu\nu}$$



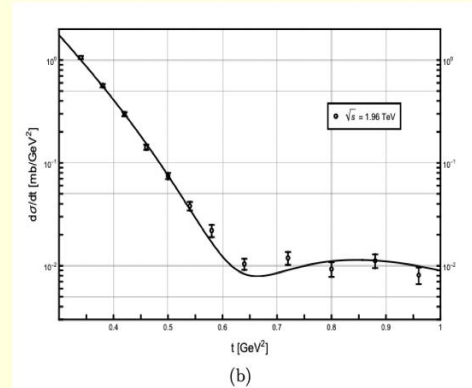
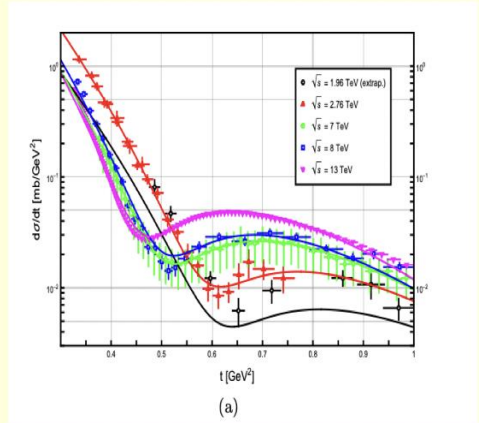
$$h_j^{\mu\nu}[(2+j)^{++}] : G^{a\mu\alpha} D_{\alpha_1} \dots D_{\alpha_j} G_{\alpha}^{a\nu}$$

$$B_j^{\mu\nu}[(1+j)^{+-}] : d^{abc} G^{a\alpha\beta} D_{\alpha_1} \dots D_{\alpha_j} G_{\alpha\beta}^b G^{c\mu\nu}$$

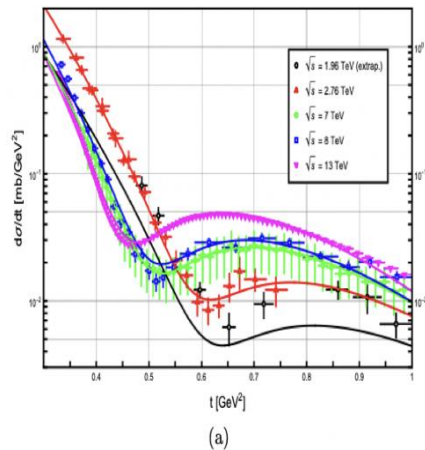
$$C_j^{\mu\nu}[(1+j)^{--}] : d^{abc} G^{a\alpha\beta} D_{\alpha_1} \dots D_{\alpha_j} G_{\alpha\beta}^b \tilde{G}^{c\mu\nu}$$

| $\sqrt{s}$ | $\alpha' [\text{GeV}^{-2}]$ | $g_{\mathbb{P}\bar{\Psi}\Psi}$ | $\mathcal{N}_{d\sigma}$ |
|------------|-----------------------------|--------------------------------|-------------------------|
| 1.96 TeV   | 0.640(21)                   | 1.071(15)                      | 0.003                   |
| 2.76 TeV   | 0.715(27)                   | 1.009(3)                       | 0.007                   |
| 7 TeV      | 0.607(5)                    | 1.089(3)                       | 0.002                   |
| 8 TeV      | 0.626(15)                   | 1.046(9)                       | 0.003                   |
| 13 TeV     | 0.587(5)                    | 1.0782(3)                      | 0.002                   |

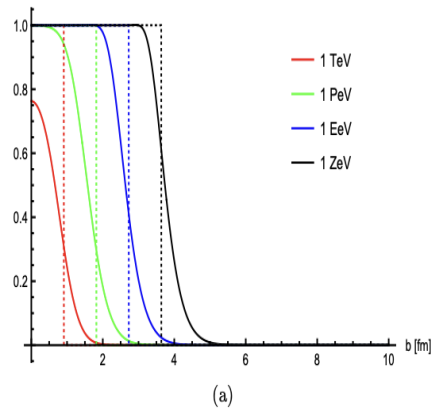
## hQCD @ TOTEM/DO



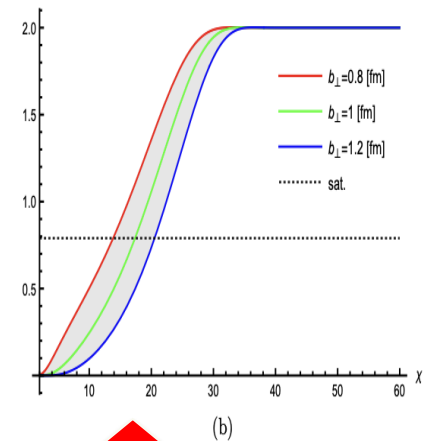
## hQCD/TOTEM/DO



## hQCD Black-disc



## hQCD Saturation: Low-x 'matter' ★



↑  
PTev

$$\frac{1}{2} \frac{d\sigma_+}{d^2b_\perp} = \langle \text{Re}(1 - e^{i(\chi_{\mathbb{P}} \pm \chi_{\mathbb{O}})}) (s, b_\perp) \rangle \equiv \langle \text{Re} \mathcal{T}_+(s, b_\perp) \rangle \rightarrow \theta(b(s) - b_\perp)$$

$$\left. \frac{d\sigma_+}{d^2b_\perp} \right|_S = 2(1 - e^{-\frac{1}{2}}) = 0.79$$

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# Summary

- Vacuum : 2D,3D,4D
- 4D : spin-mass
- 2D,3D : Reggeized string

