



Bayesian Inference of helicity amplitudes from vector meson electroproduction

High Luminosity Workshop

Bhawani Singh
Jefferson Lab

Monday, October 5, 2026

Based on B. Singh et al [arXiv:2608.27683](https://arxiv.org/abs/2608.27683) [nucl-ex] (2026)

- * Jefferson Lab: H. Avakian, V. Burkert, L. Elouadrhiri, and V. Kubarovsky,
- * ODU: Y. Li
- * MIT: R. G. Milner and Y. Wang,
- * UConn: S. Bhattacharya,
- * Glasgow University: D. Leahy and D. I. Glazier

✉ singh@jlab.org



Inverse problem

How do we infer observables which cannot be measured directly?

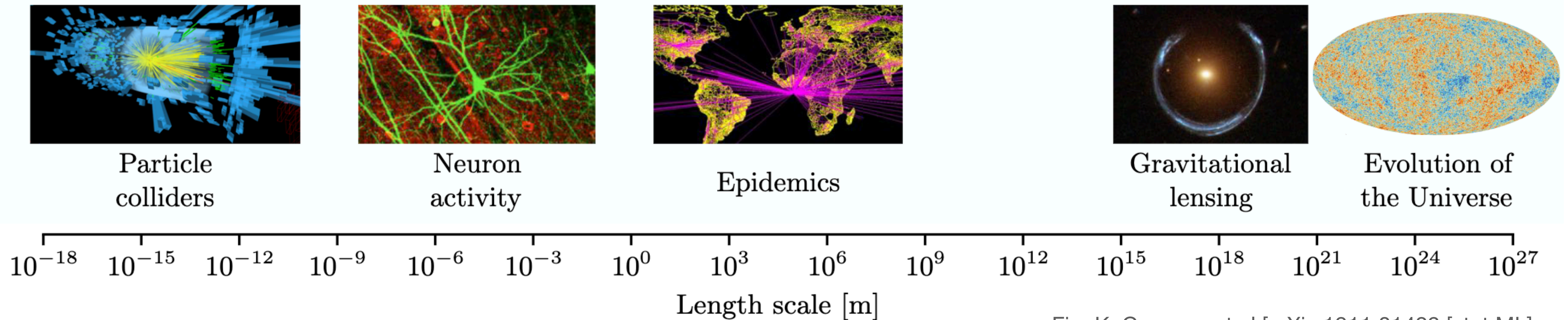
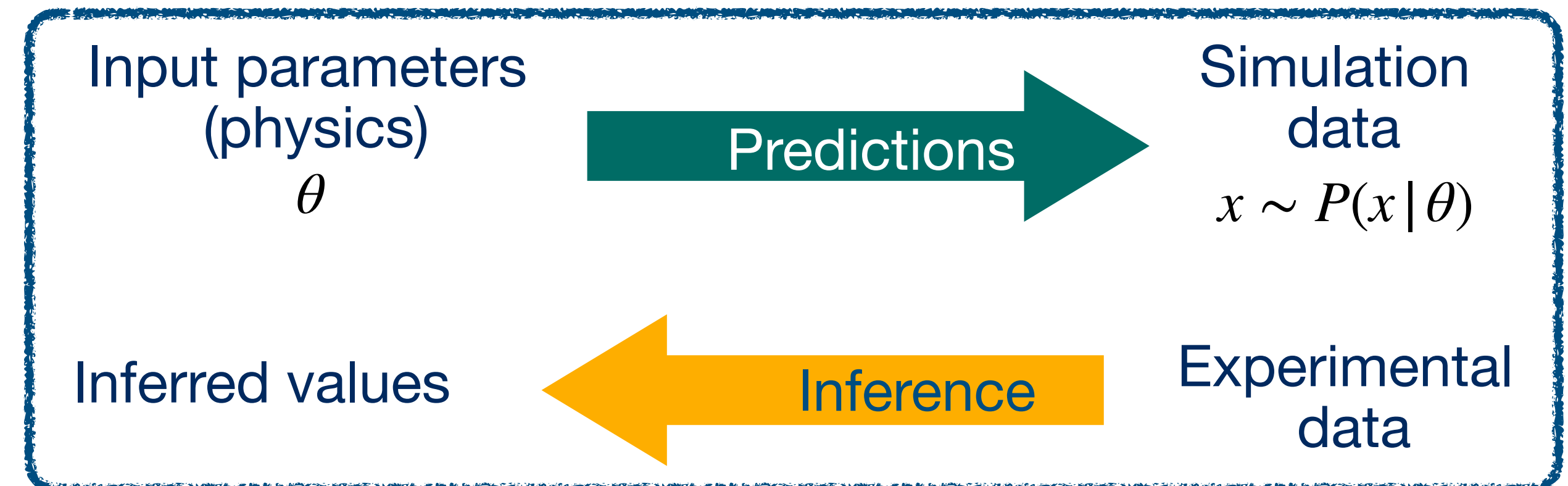


Fig: K. Cranmer et al [[arXiv:1911.01429](https://arxiv.org/abs/1911.01429)] [stat.ML]

Common across disciplines

- Access to high fidelity simulations
- Difficult to extract observables from measurements



Inverse problem

How do we infer observables which cannot be measured directly?

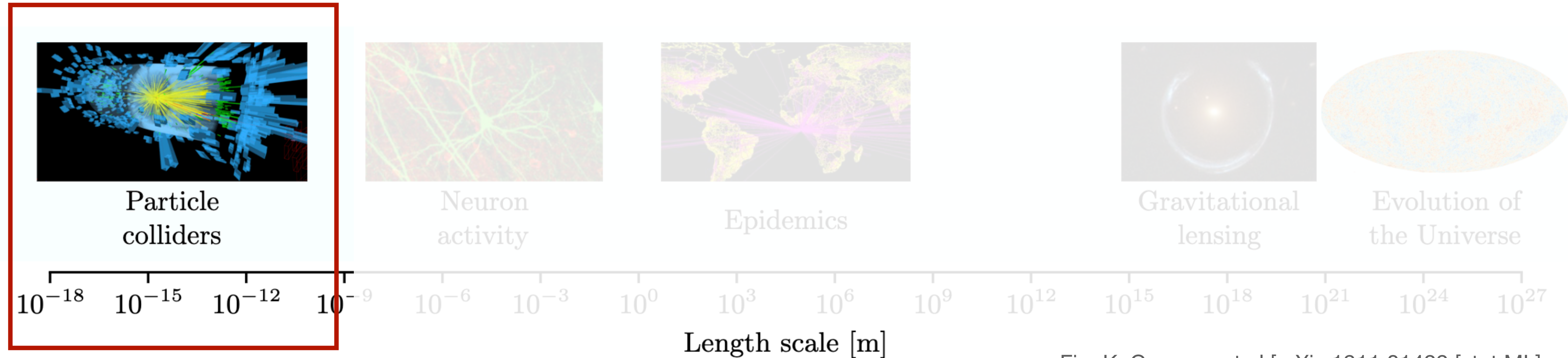


Fig: K. Cranmer et al [[arXiv:1911.01429](#)] [stat.ML]

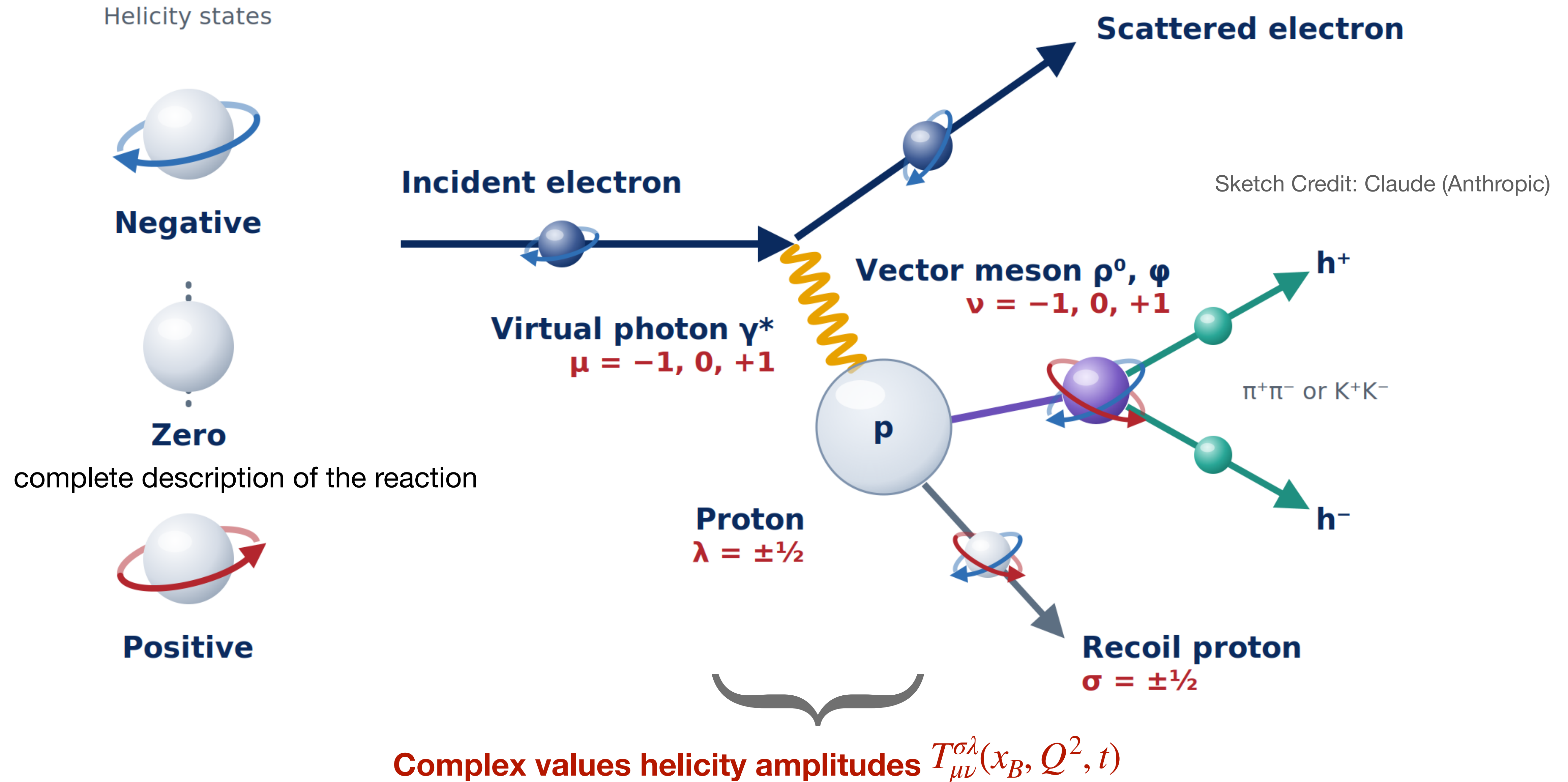
Experiments at the LHC/ Brookhaven, DESY and Jefferson Lab to learn direct physics with precision!

Our aim is to infer the physics of vector meson production in the exclusive electroproduction reaction in electron proton scattering experiment!

$$e + p \longrightarrow e' + p' + \phi(s\bar{s}) \quad \text{access to gluons..}$$

$$e + p \longrightarrow e' + p' + \rho^0(u\bar{u} - d\bar{d}/\sqrt{2}) \quad \text{the quarks..}$$

Exclusive vector-meson electroproduction



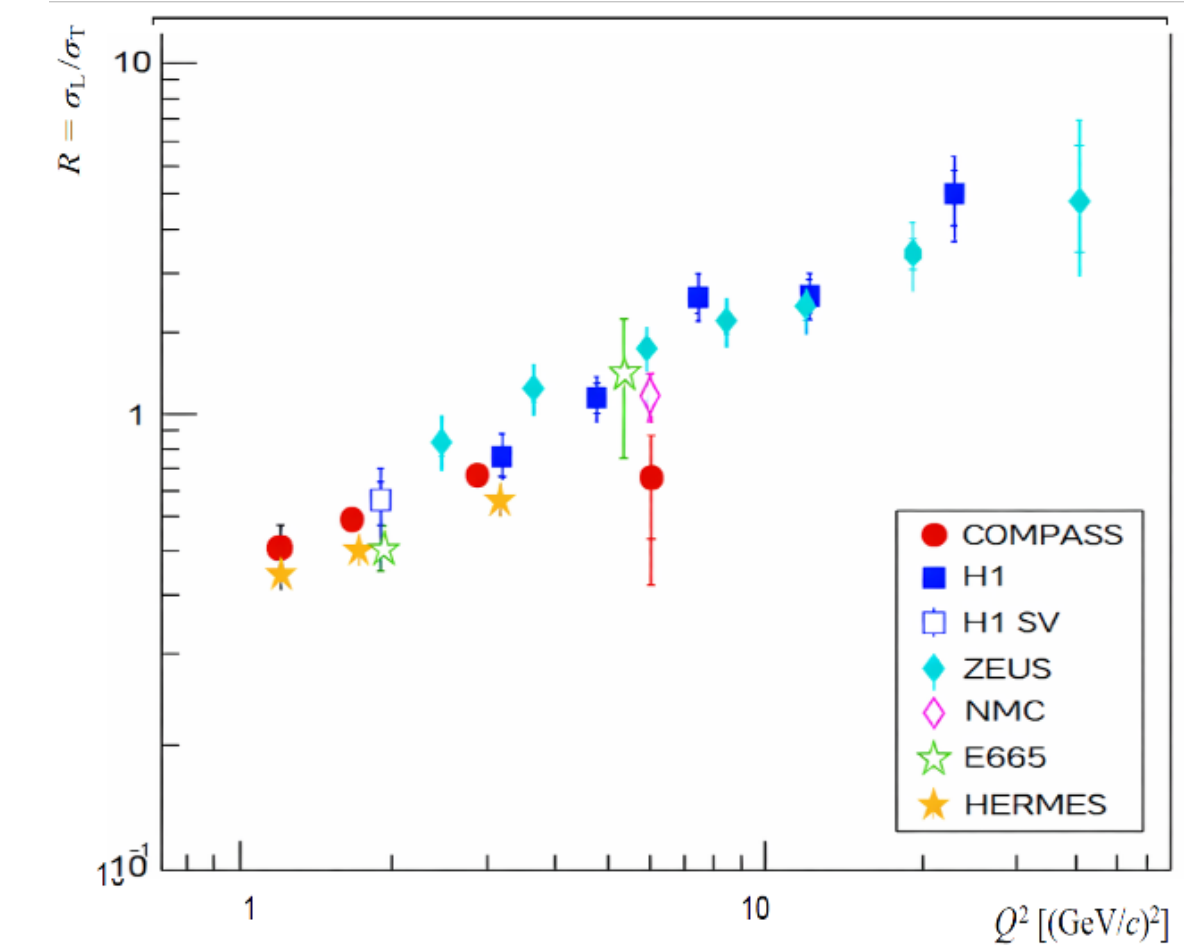
Why do helicity amplitudes matter?

- **Complete description of the reaction (bridge to the 3D proton)**
 - QCD factorization links them to GPDs: ϕ meson is a clean gluon probe, ρ^0 adds the quarks.. ($\sigma_L \sim 1/Q^6$, $\sigma_L/\sigma_T \sim Q^2$)
 - Polarization is the lever: target-spin terms open the nucleon-helicity-flip sector $\rightarrow$ GPD E $\rightarrow$ orbital angular momentum; beyond GPDs

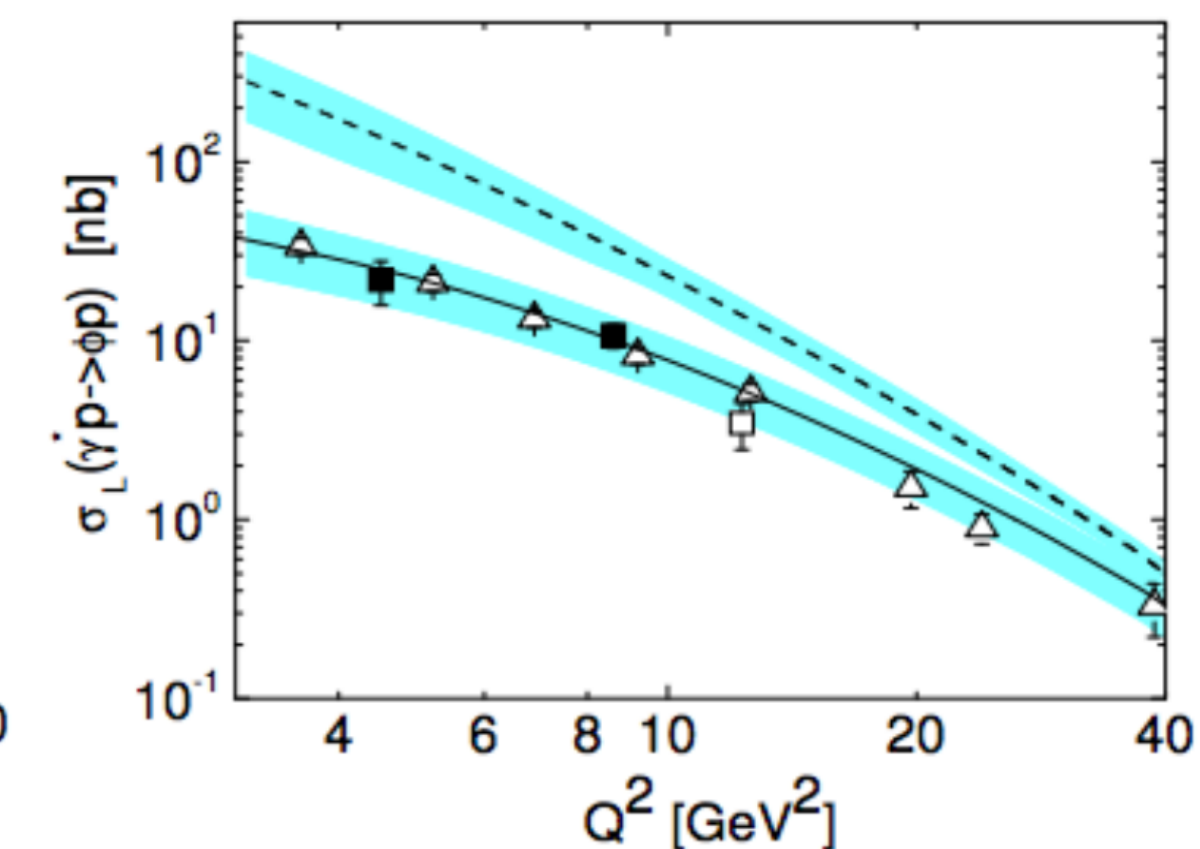
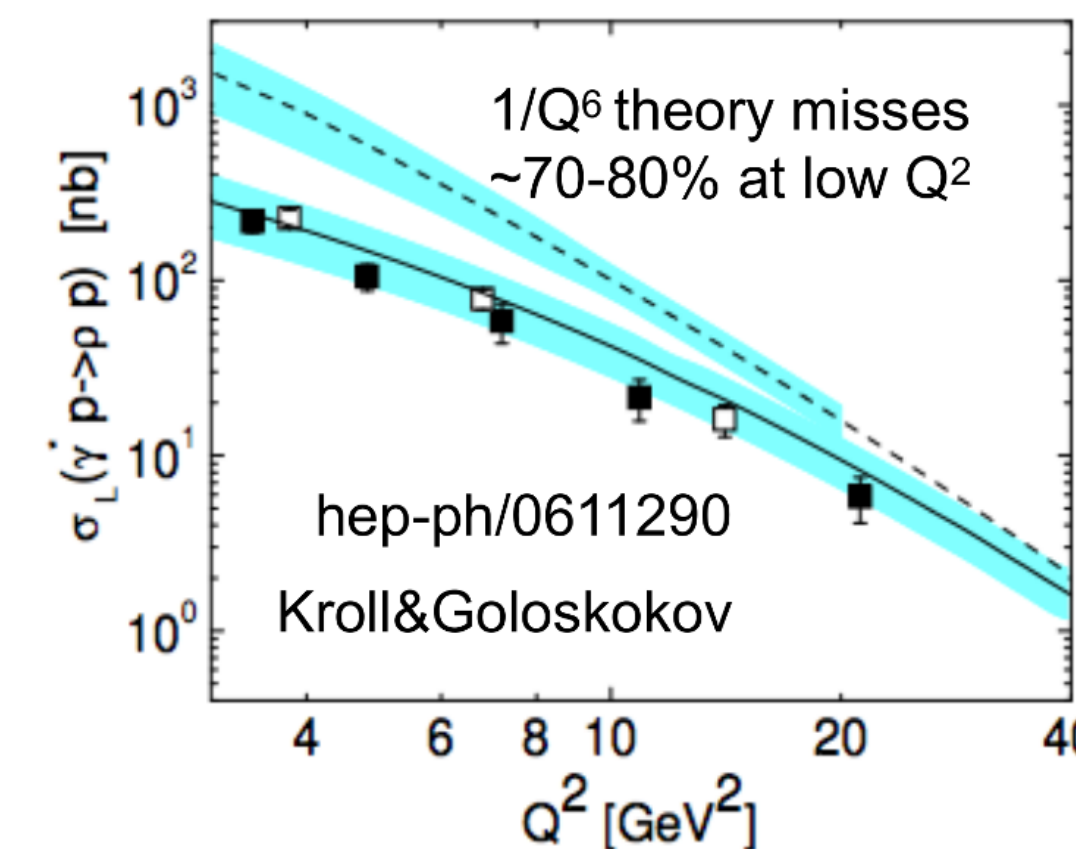
Why do helicity amplitudes matter?

- **Complete description of the reaction (bridge to the 3D proton)**

- QCD factorization links them to GPDs: ϕ meson is a clean gluon probe, ρ^0 adds the quarks.. ($\sigma_L \sim 1/Q^6$, $\sigma_L/\sigma_T \sim Q^2$)
- Polarization is the lever: target-spin terms open the nucleon-helicity-flip sector $\rightarrow$ GPD E $\rightarrow$ orbital angular momentum; beyond GPDs
- The ZEUS measurements, followed by others, indicated that the longitudinal photon contributions drop as $\sim 1/Q^4$
underestimate low Q^2 (To be checked both, extraction of σ_L cross-section and evaluation of σ_L/σ_T)



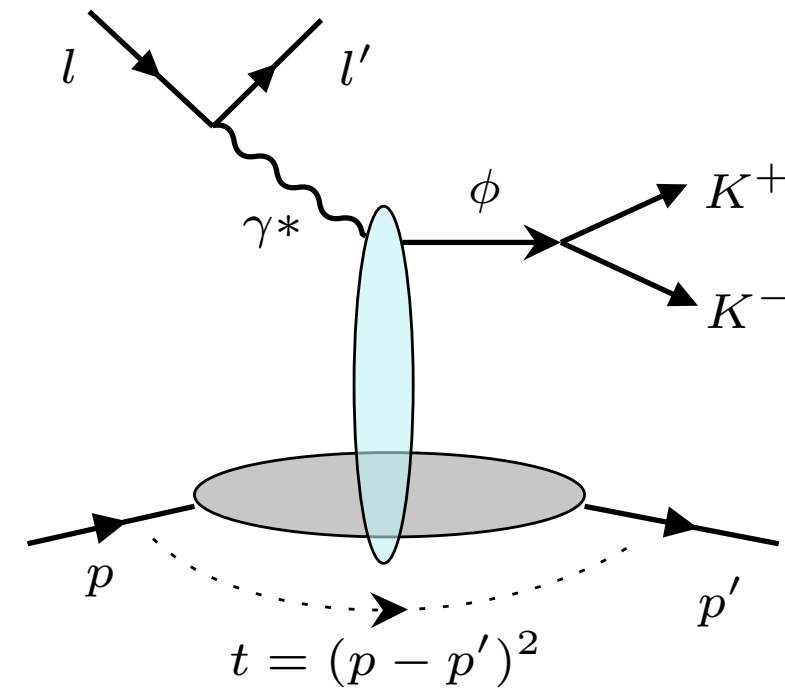
L/T separation assuming SCHC (helicity conservation)



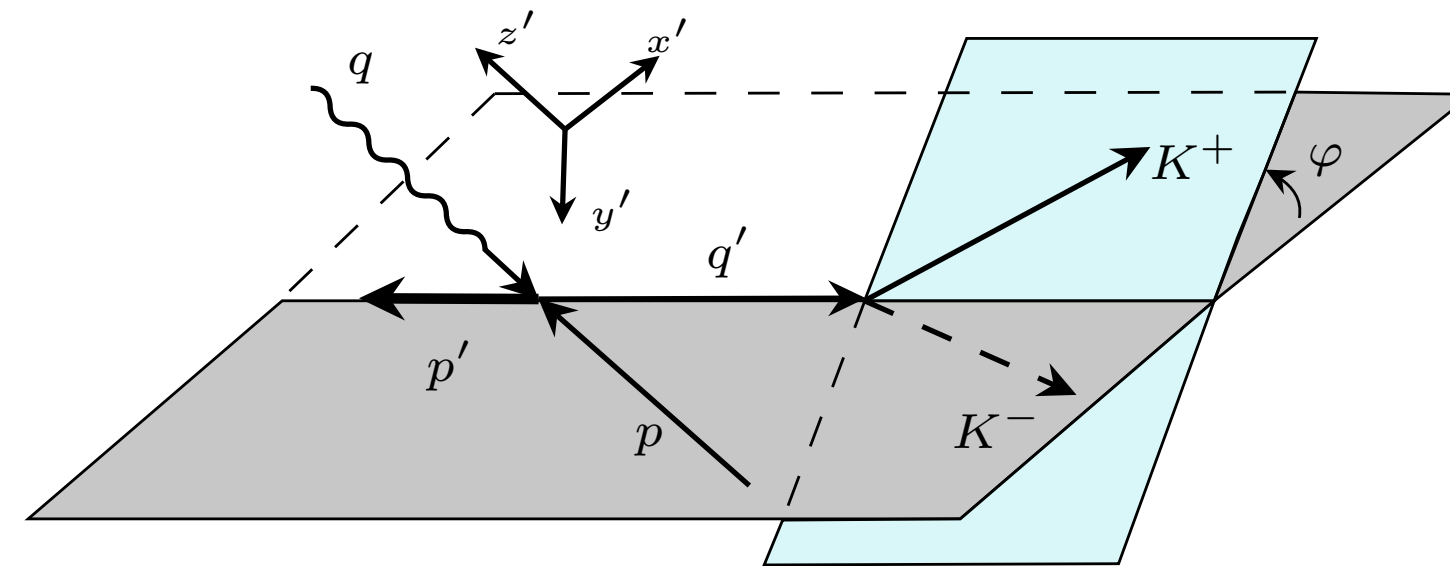
Shared goal of the JLab, COMPASS/AMBER and EIC
PDFs $\rightarrow$ GPDs $\rightarrow$ GTMDs

Longitudinally for ρ^0 and ϕ : data vs GPD handbag (solid) and leading twist (dashed).
Goloskokov, Kroll, hep-ph/0611290

From angular distributions to amplitudes

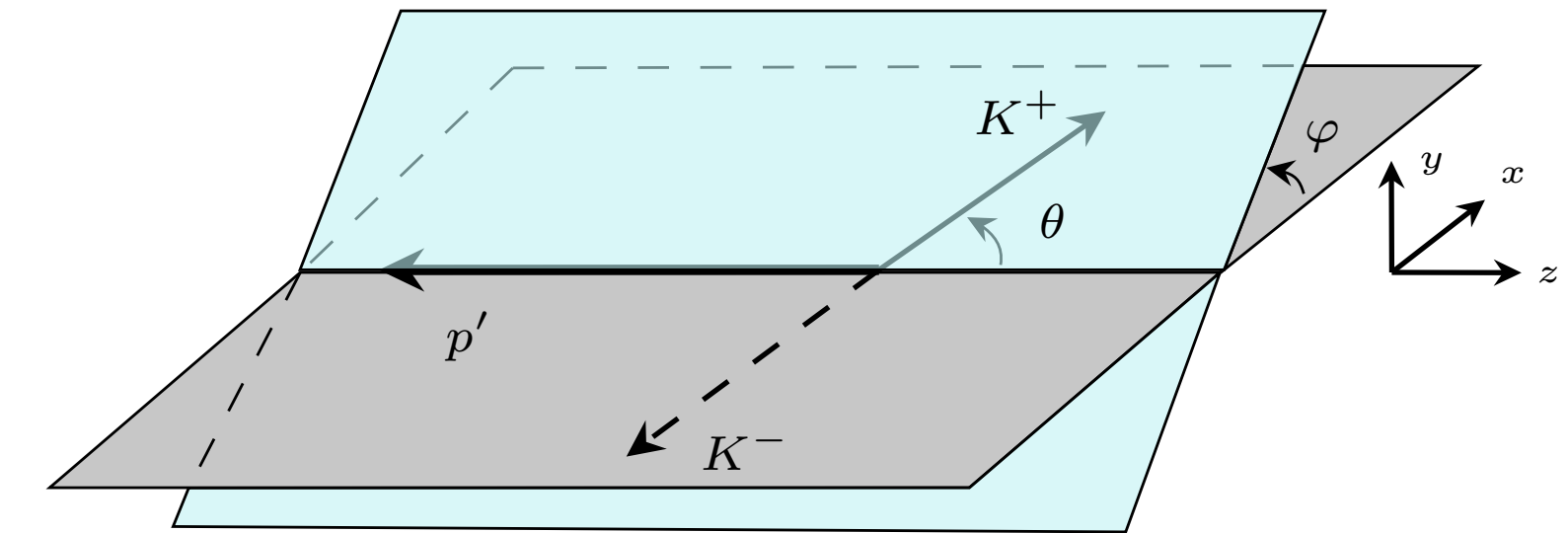


(a) gluon-GPD handbag



(b) γ^*p c.m. frame

boost



(c) ϕ rest frame

- Event rates, per kinematic bin, per beam energy
- Decay-angle distributions in $[\cos(\theta), \varphi, \Phi]$
- ... all of it shaped by acceptance, efficiency and finite resolution



- The complex production amplitudes $T_{\mu\nu}^{\sigma\lambda}(x_B, Q^2, t)$
- Their magnitudes AND their relative phases
- The separated cross sections σ_T and σ_L , not just their ratio

- Experiments measure bilinear combinations of amplitudes, folded through an instrument response
- Distinct amplitude configurations can produce identical data $\rightarrow$ statistics alone never yields a point inversion

Polarized target: the full amplitude set

- Full cross-section stages unpolarized (**U**) +Longitudinally polarized (**L**)+ Transversely Polarized (**T**)

$$\frac{d\sigma}{dx_B dQ^2 dt d\Omega} = \Gamma(x_B, Q^2, y) W(\Omega; \varepsilon, P_b, S_L, S_T)$$

- Angular intensities

$$W(\Omega; \varepsilon, P_b) = \frac{3}{4\pi} \left[\mathcal{U} + \varepsilon \mathcal{L}_2 + \sqrt{2\varepsilon(1+\varepsilon)} \mathcal{L}_1 + P_b \left(\sqrt{1-\varepsilon^2} \mathcal{B} + \sqrt{2\varepsilon(1-\varepsilon)} \mathcal{B}_1 \right) \right]$$

- Short-hand notations

$$\mathcal{U} = \left[\frac{1}{2} \left(|T_{11}|^2 + |T_{1-1}|^2 + |U_{11}|^2 + |U_{1-1}|^2 \right) + \varepsilon \left(|T_{10}|^2 + |U_{10}|^2 \right) \right] \sin^2 \theta + \left(|T_{01}|^2 + \varepsilon |T_{00}|^2 + |U_{01}|^2 \right) \cos^2 \theta + \dots \text{more angles}$$

- Including nucleon spin flip and non-flip cases $(3(\mu) \times 3(\nu) \times 2(\sigma) \times 2(\lambda) / 2 (\text{parity factor}) - 2(\text{global phase, real } T_{11}))$

$$T_{\mu\nu}^{--} = + T_{\mu\nu}^{++}, \quad U_{\mu\nu}^{--} = - U_{\mu\nu}^{++} \quad (\text{non-flip})$$

$$T_{\mu\nu}^{+-} = - T_{\mu\nu}^{-+}, \quad U_{\mu\nu}^{+-} = + U_{\mu\nu}^{-+} \quad (\text{flip})$$

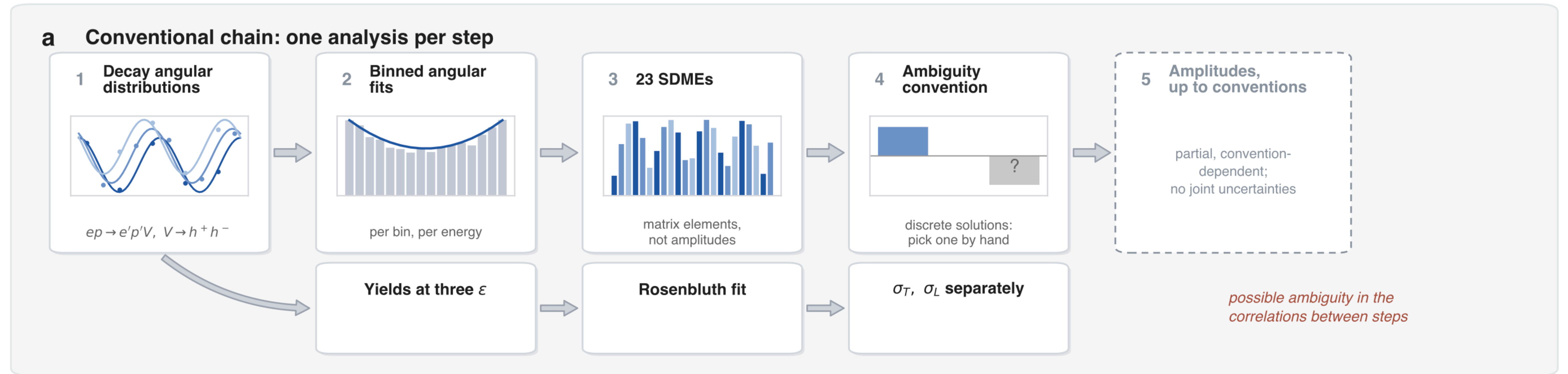
Spin Density Matrix Elements (SDMEs)

$$\rho_{\mu\mu',\lambda\lambda'}^{\nu\nu'} = \frac{1}{N_T + \varepsilon N_L} \sum_{\sigma} T_{\mu\lambda}^{\nu\sigma} \left(T_{\mu'\lambda'}^{\nu'\sigma} \right)^*$$

- Full Angular distributions: U+L+T (**34**)

$$W = W_{UU} + P_b W_{LU} + S_L (W_{UL} + P_b W_{LL}) + S_T (W_{UT} + P_b W_{LT})$$

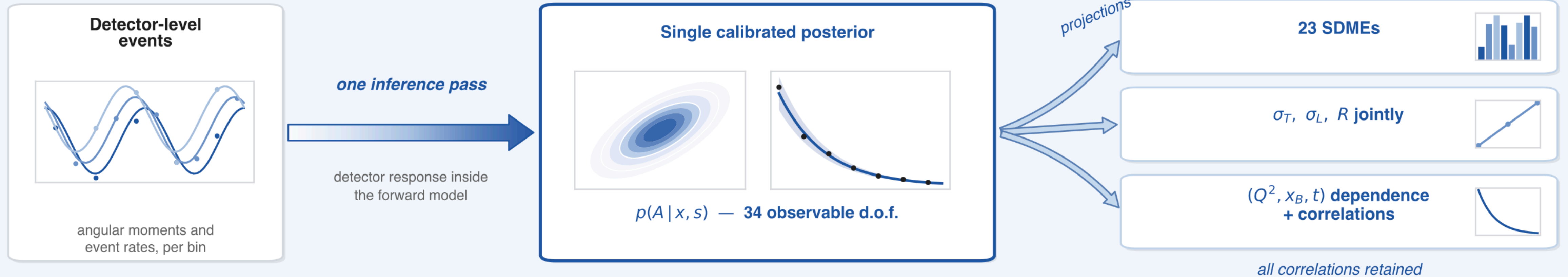
Traditionally: the extraction step-by-step process



- Angular fits $\rightarrow$ 23 SDMEs $\rightarrow$ hand-chosen ambiguity convention $\rightarrow$ amplitudes (up to conventions); a separate Rosenbluth fit gives σ_T and σ_L
- A highly successful strategy: HERMES, COMPASS, H1, ZEUS, CLAS, GlueX:
 - HERMES ρ^0 , H and D: 23 SDMEs, EPJC 62, 659 (2009); amplitude ratios, EPJC 71, 1609 (2011)
 - COMPASS ρ^0 : 23 SDMEs by unbinned maximum likelihood, EPJC 83, 924 (2023)
 - GlueX $\rho(770)$ photoproduction, linearly polarized 8.2–8.8 GeV, Phys. Rev. C 108, 055204 (2023)
 - GlueX $\phi(1020) \rightarrow K_0^s K_0^L$, $-t = 0.15\text{--}1.0 \text{ GeV}^2$, Phys. Rev. C 112, 025203 (2025)

Inference in one pass

b This work: one calibrated posterior



c Observable amplitude content grows with target polarization

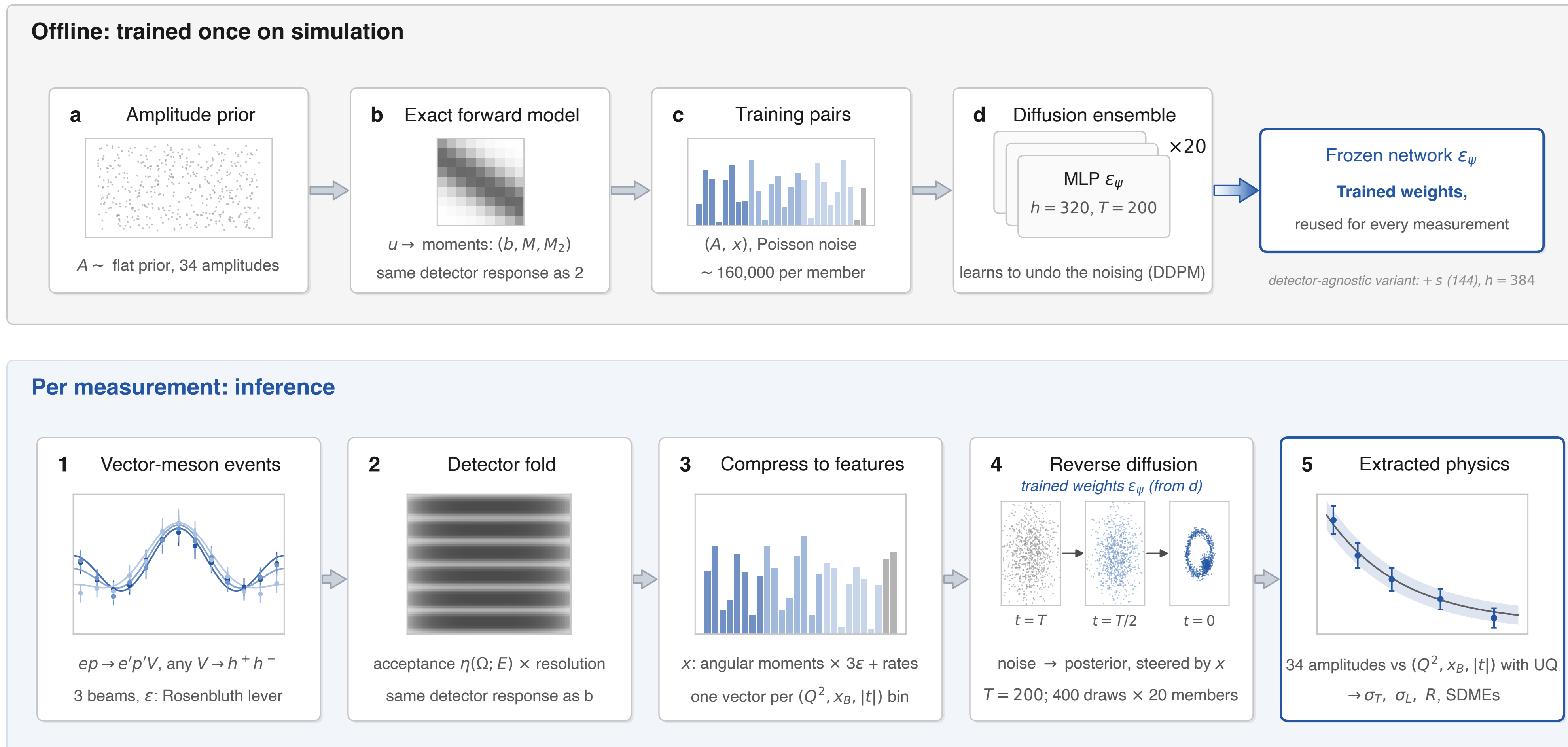


The detector response is carried by the forward model instead of being corrected out. The 23 SDMEs, the separated cross sections and their dependence on Q^2 , x_B and t all follow as projections of one posterior, with every correlation retained!

B. Singh et al [arXiv:2608.27683](https://arxiv.org/abs/2608.27683) [nucl-ex] (2026)

Work flow for extraction

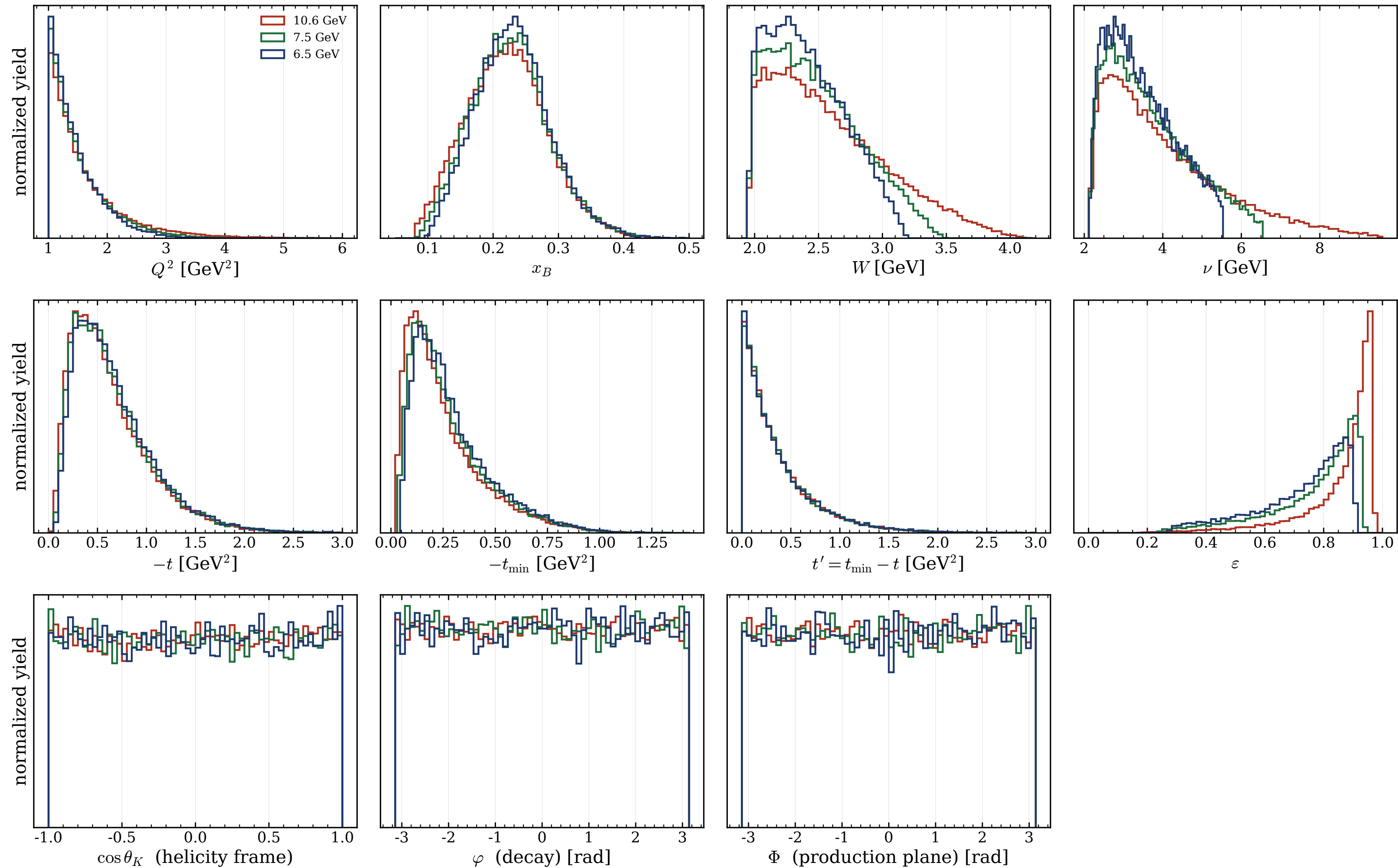
- **A conditional diffusion model:** $q_\psi(u | x)$ — the third neural route, twin of the normalizing-flow NPE.
- Different engine: iterative denoising instead of one invertible map (flexible, correlated posteriors).



B. Singh et al [arXiv:2608.27683](https://arxiv.org/abs/2608.27683) [nucl-ex] (2026)

Forward model

- **Amplitude level generator:** real physics case set of amplitudes $T_{\mu\nu}^{\sigma\lambda}(x_B, Q^2, t)$, give event distributions [**VecGen**]

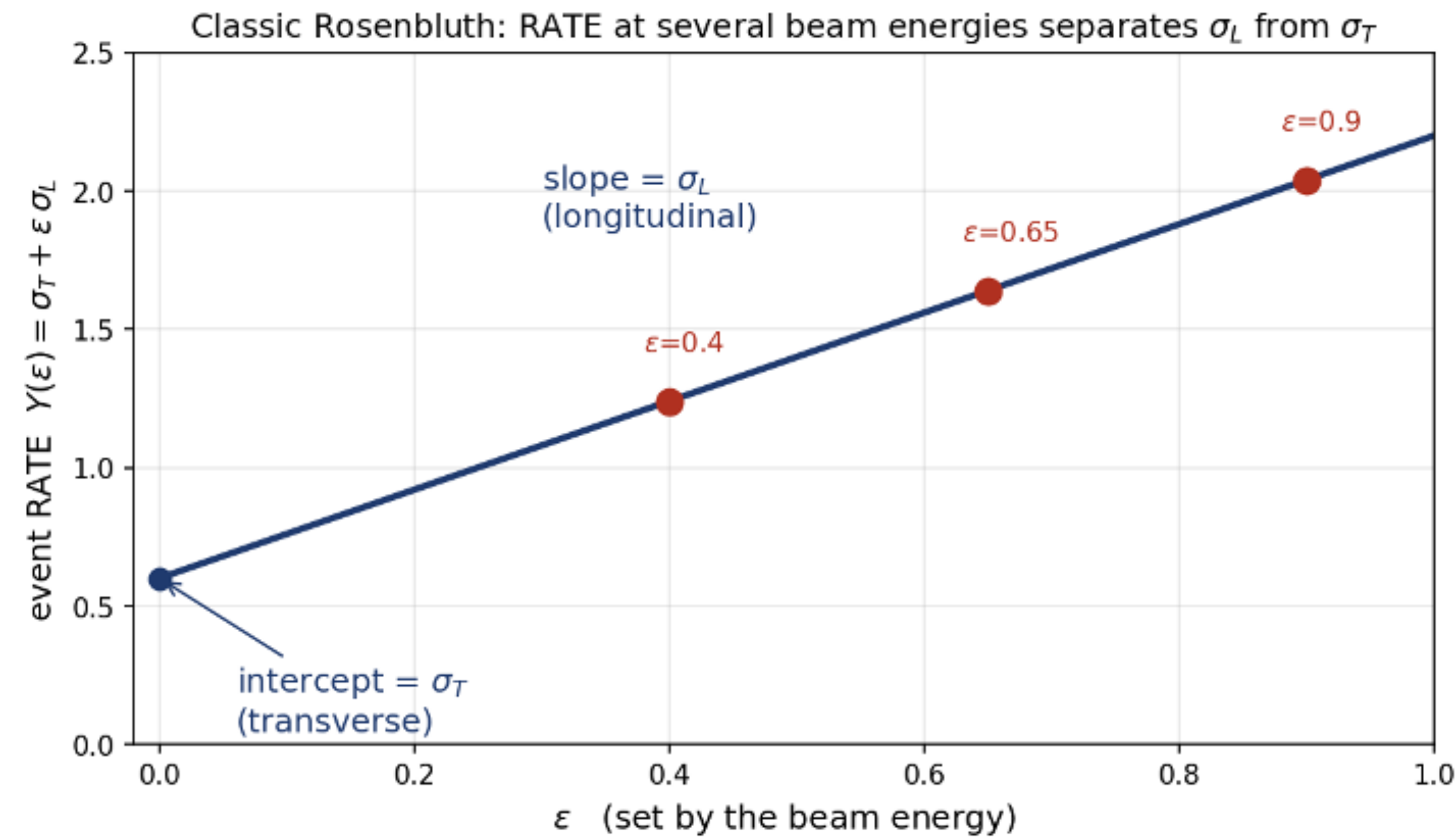


Forward model

- **Amplitude level generator:** real physics case set of amplitudes $T_{\mu\nu}^{\sigma\lambda}(x_B, Q^2, t)$, give event distributions [**VecGen**]
- **Collect in terms of moments** 106 (full polarized case) angular moments of basis functions over $[\cos(\theta), \varphi, \Phi]$

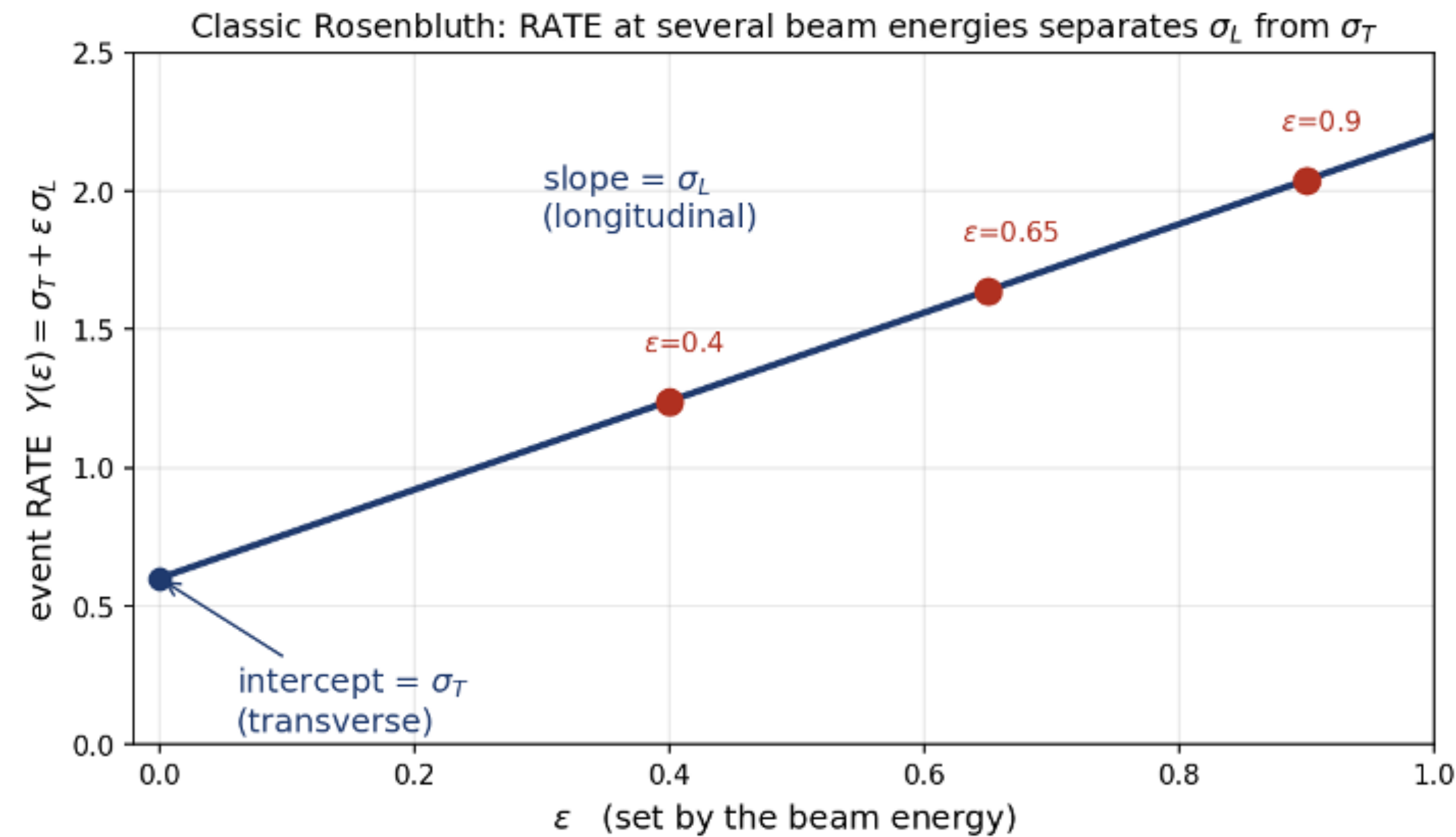
Forward model

- **Amplitude level generator:** real physics case set of amplitudes $T_{\mu\nu}^{\sigma\lambda}(x_B, Q^2, t)$, give event distributions [**VecGen**]
- **Collect in terms of moments** 106 (full polarized case) angular moments of basis functions over $[\cos(\theta), \varphi, \Phi]$
- **Rosenbluth separation:** $Y(\varepsilon) = \sigma_T + \varepsilon\sigma_L$ at 6.5, 7.5 and 10.6 GeV the energies of existing CLAS12
absolute size lives in the event **RATE**, not the angles. At each ε , the reduced yield is $Y(\varepsilon)$



Forward model

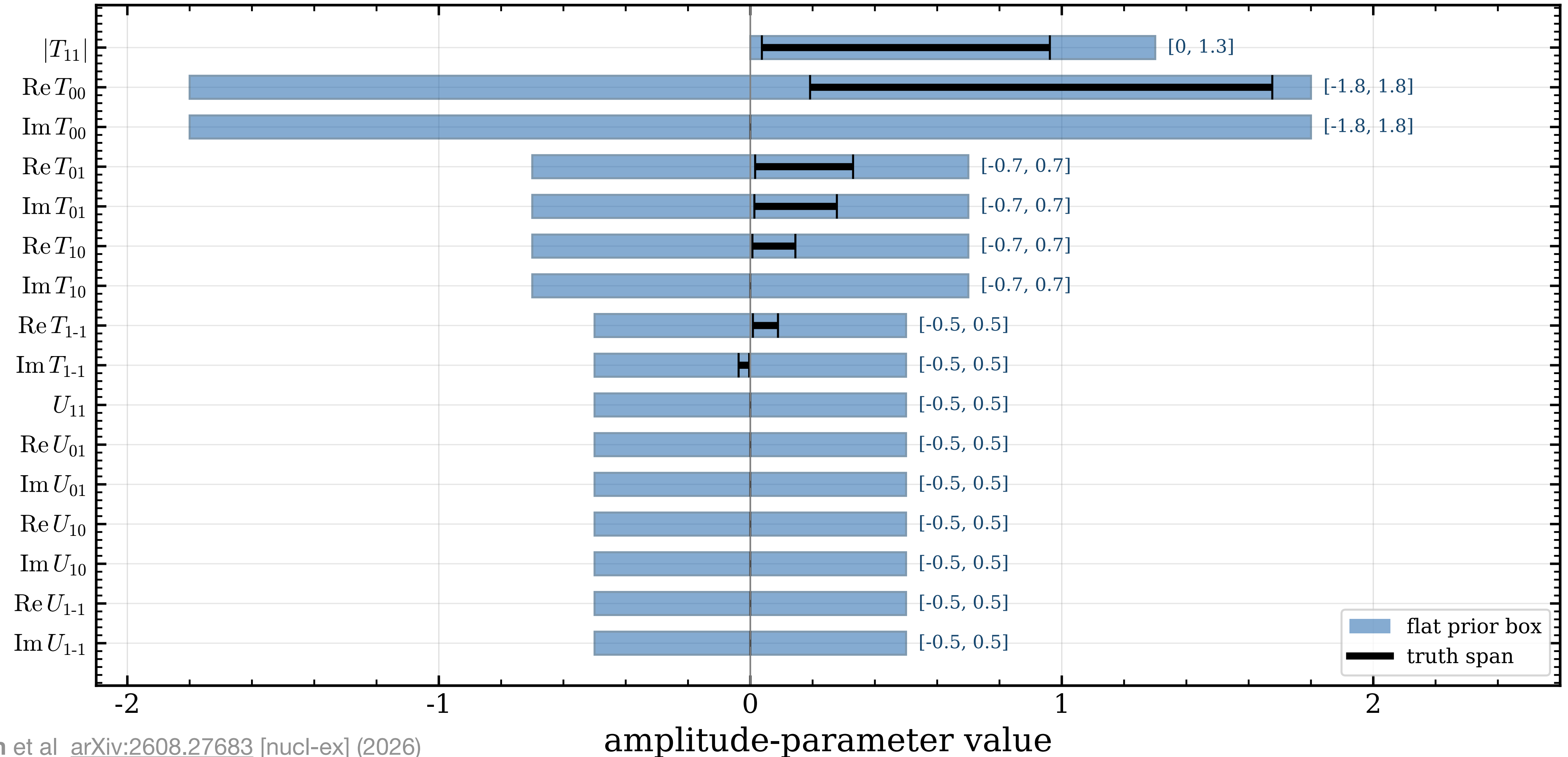
- **Amplitude level generator:** real physics case set of amplitudes $T_{\mu\nu}^{\sigma\lambda}(x_B, Q^2, t)$, give event distributions [**VecGen**]
- **Collect in terms of moments** 106 (full polarized case) angular moments of basis functions over $[\cos(\theta), \varphi, \Phi]$
- **Rosenbluth separation:** $Y(\varepsilon) = \sigma_T + \varepsilon\sigma_L$ at 6.5, 7.5 and 10.6 GeV the energies of existing CLAS12
absolute size lives in the event **RATE**, not the angles. At each ε , the reduced yield is $Y(\varepsilon)$



- **Realistic Detector model:** We include a detector model in **7 Dim acceptance** $\eta(Q^2, x_B, t, \varepsilon, \cos(\theta), \varphi, \Phi)$, crucial role of the acceptances into the raw data and the results (acceptance model)

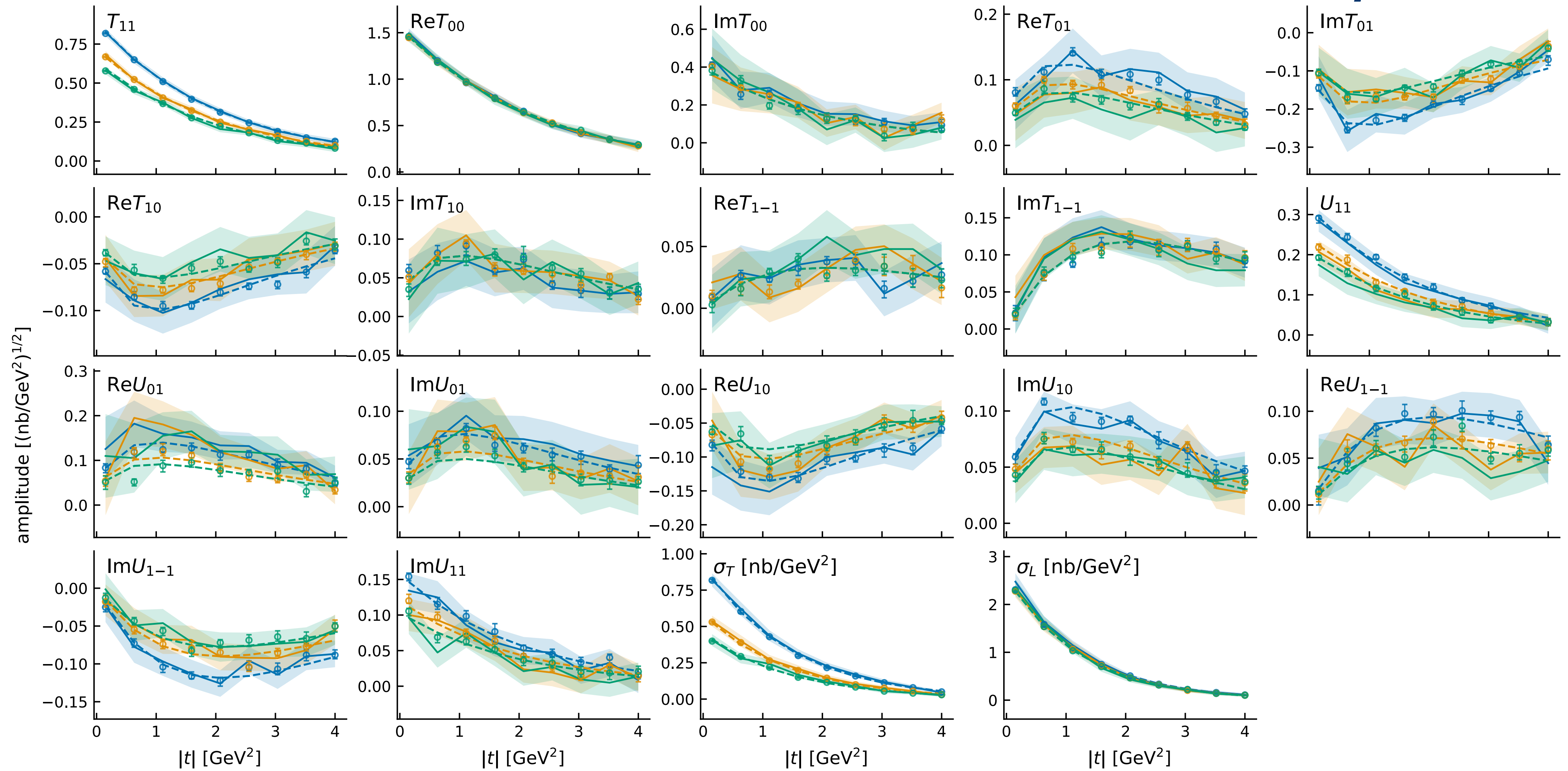
Training with uniform distributions

- The model does not know if there is any kinematics dependencies of the amplitudes



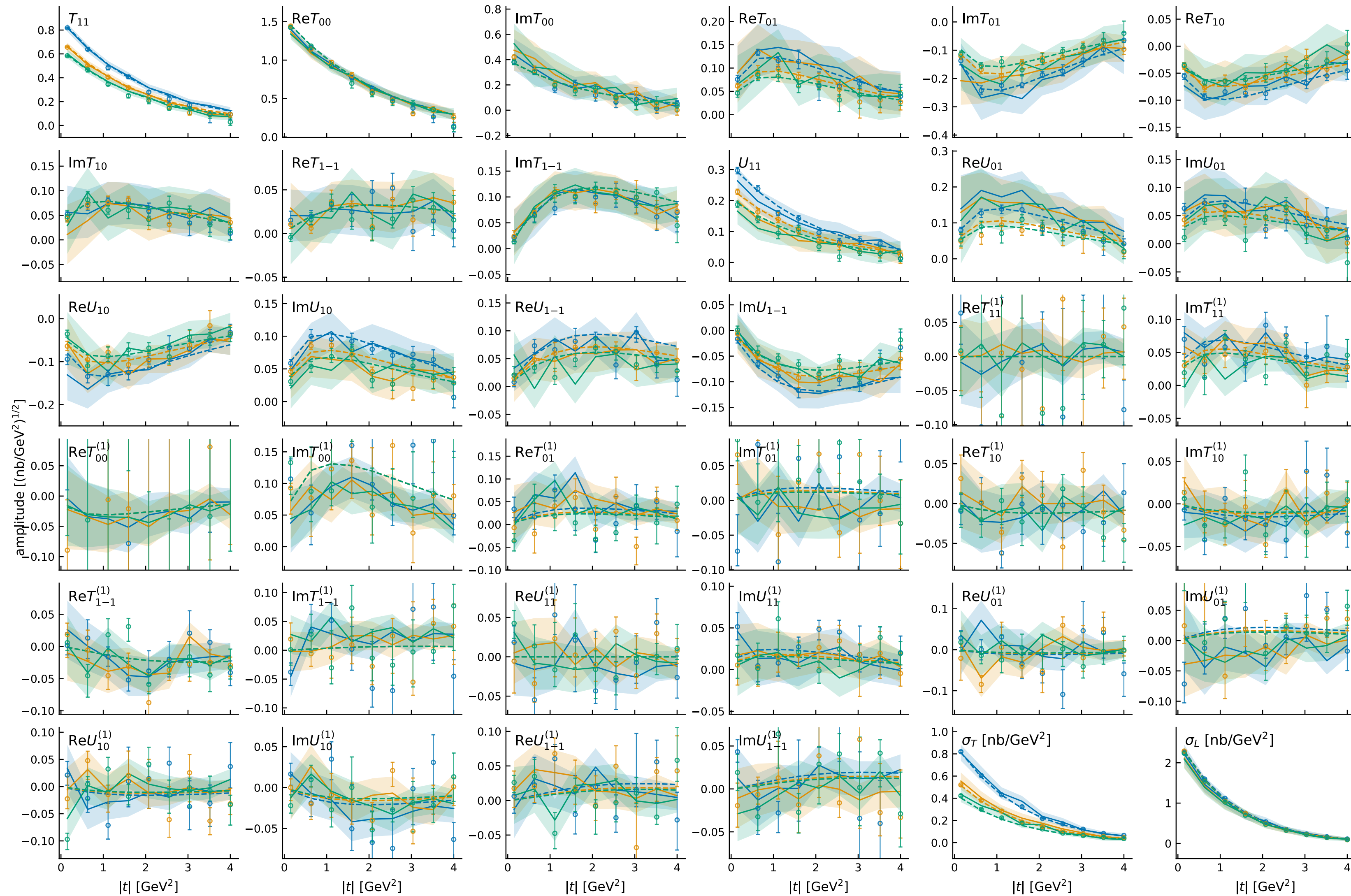
B. Singh et al [arXiv:2608.27683](https://arxiv.org/abs/2608.27683) [nucl-ex] (2026)

Results: longitudinally polarized target (ϕ)



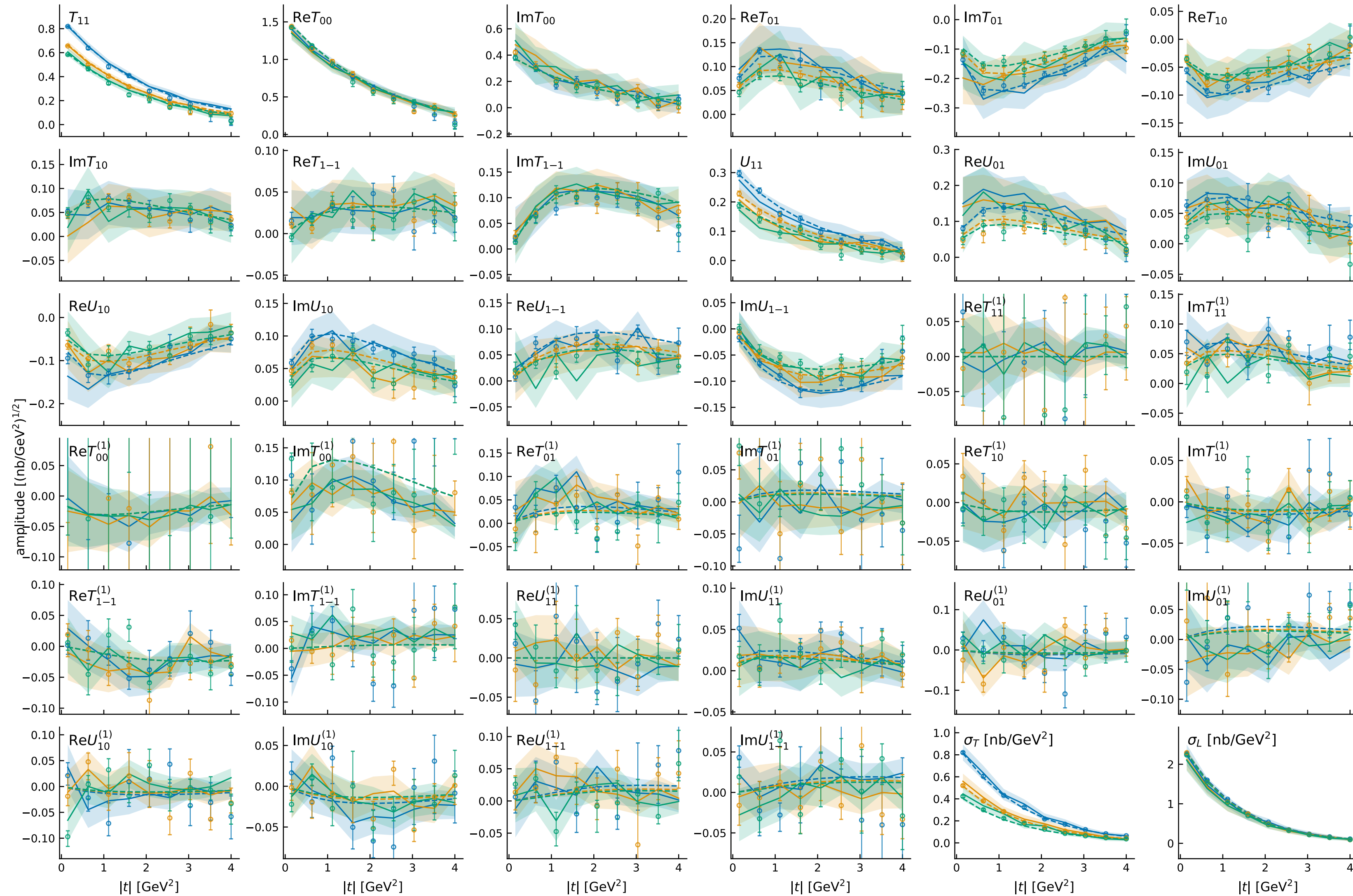
B. Singh et al [arXiv:2608.27683](https://arxiv.org/abs/2608.27683) [nucl-ex] (2026)

Results: Longitudinally+transversely polarized target (ϕ)



B. Singh et al [arXiv:2608.27683](https://arxiv.org/abs/2608.27683) [nucl-ex] (2026) — $1 < Q^2 < 2 \text{ GeV}^2$ — $2 < Q^2 < 3 \text{ GeV}^2$ — $3 < Q^2 < 6 \text{ GeV}^2$ — — truth — posterior $\pm 1\sigma$ ○ UML

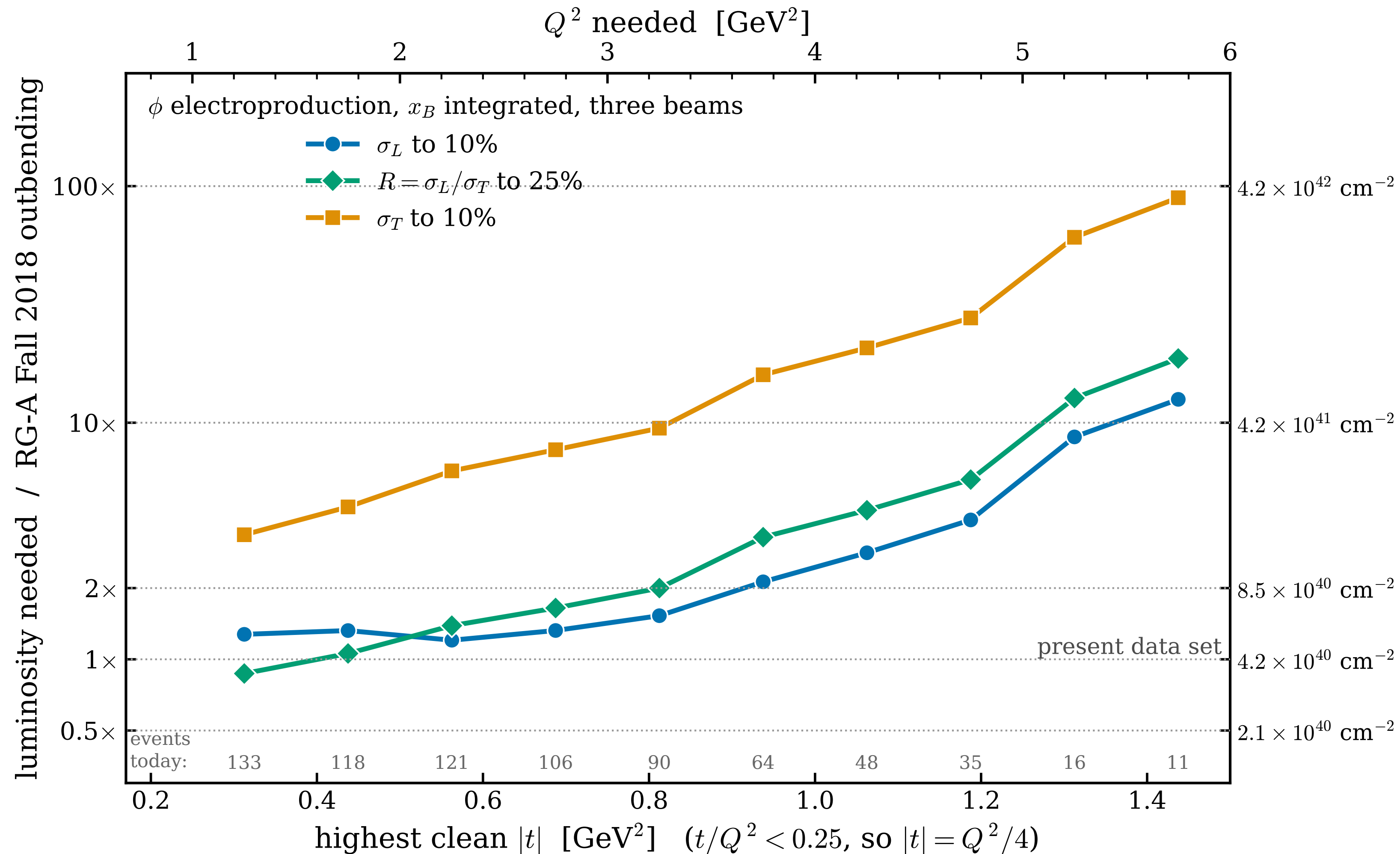
Results: Longitudinally+transversely polarized target (ρ^0)



B. Singh et al [arXiv:2608.27683](https://arxiv.org/abs/2608.27683) [nucl-ex] (2026) — $1 < Q^2 < 2 \text{ GeV}^2$ — $2 < Q^2 < 3 \text{ GeV}^2$ — $3 < Q^2 < 6 \text{ GeV}^2$ — — truth — posterior $\pm 1\sigma$ ○ UML

Projections: high luminosity

- Simulations based on CLAS12 (Jefferson Lab) analysis @ 10.6 GeV electron beam on proton target



Disclosure: no systematics of analysis included in this prediction

Summary

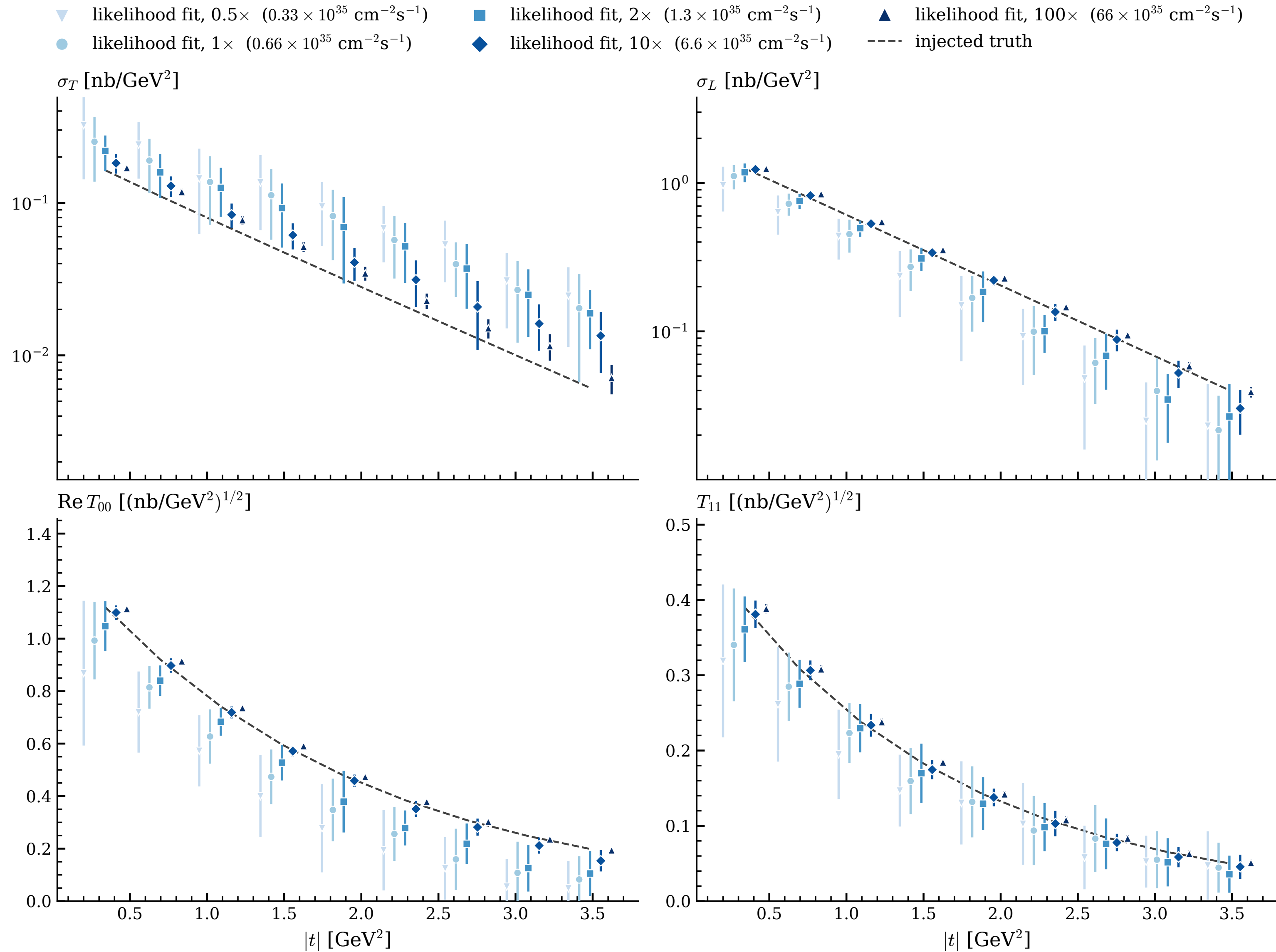
- A **new** approach to extract the helicity amplitudes with calibrated uncertainties and kinematics dependencies
- **Strong physics case:** A realistic Monte Carlo truth with known amplitudes, the extraction itself assumes **no model dependence**
- **Statistics:** calibration of the posterior widths, **high luminosity and polarized target** will help get all the amplitudes
- **Correct acceptance** mode will allow for reliable acceptance correction in the measurements
- Approach is **generalizable** unto a forward model [B. Singh et al [arXiv:2608.27683](https://arxiv.org/abs/2608.27683) [nucl-ex] (2026)]
- Next step: event-by-event extraction!

Summary

- A **new** approach to extract the helicity amplitudes with calibrated uncertainties and kinematics dependencies
- **Strong physics case:** A realistic Monte Carlo truth with known amplitudes, the extraction itself assumes **no model dependence**
- **Statistics:** calibration of the posterior widths, **high luminosity and polarized target** will help get all the amplitudes
- **Correct acceptance** mode will allow for reliable acceptance correction in the measurements
- Approach is **generalizable** unto a forward model [B. Singh et al [arXiv:2608.27683](https://arxiv.org/abs/2608.27683) [nucl-ex] (2026)]
- Next step: event-by-event extraction!

Thank you for your time!

Luminosity and extractions

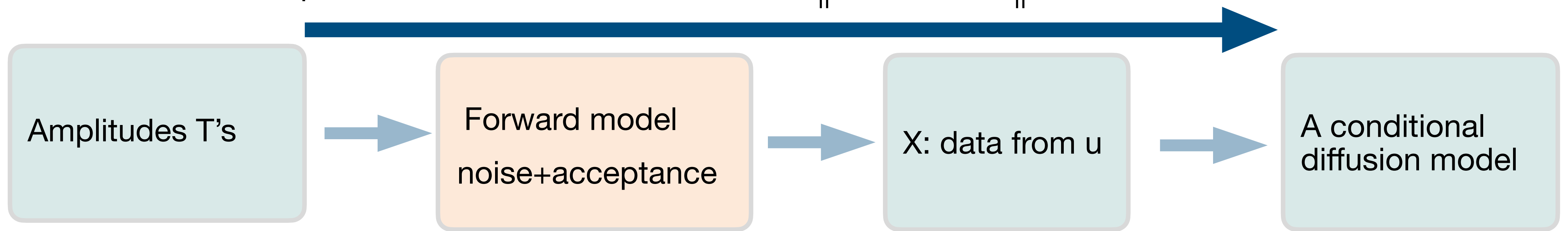


Forward model

- **Amplitude level generator:** real physics case set of amplitudes $T_{\mu\nu}^{\sigma\lambda}(x_B, Q^2, t)$, give event distributions [**VecGen**]
- **Collect in terms of moments** 106 (full polarized case) angular moments of basis functions over $[\cos(\theta), \varphi, \Phi]$
- **Rosenbluth separation:** $Y(\varepsilon) = \sigma_T + \varepsilon\sigma_L$ at 6.5, 7.5 and 10.6 GeV the energies of existing CLAS12
- **Realistic Detector model:** We include a detector model in **7 Dim acceptance** $\eta(Q^2, x_B, t, \varepsilon, \cos(\theta), \varphi, \Phi)$, crucial role of the acceptances into the raw data and the results (acceptance model)

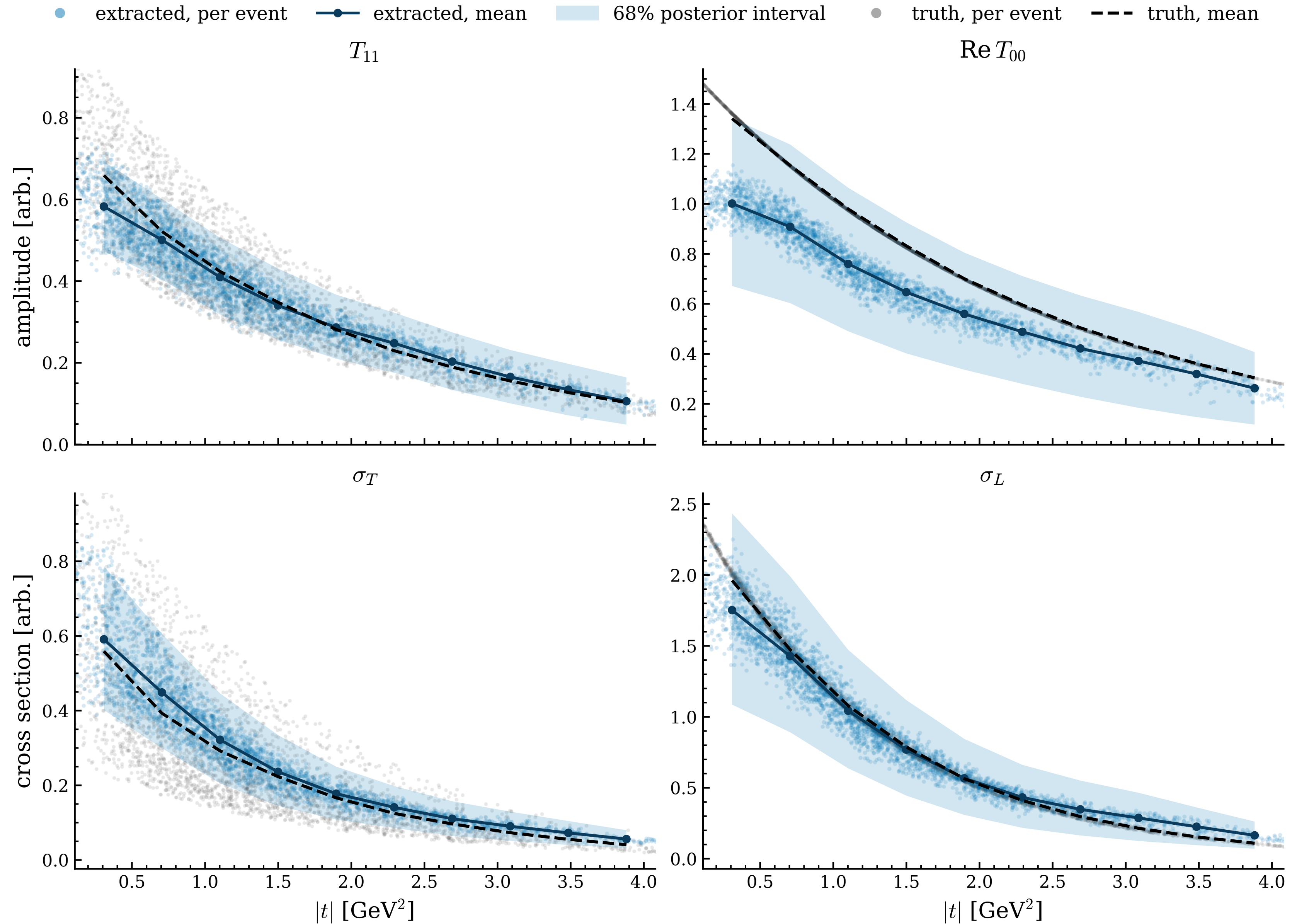
$$q(A_t | A_{t-1}) = \mathcal{N}\left(\sqrt{1 - \beta_t} A_{t-1}, \beta_t I\right), \quad A_t = \sqrt{\bar{\alpha}_t} A_0 + \sqrt{1 - \bar{\alpha}_t} z, \quad z \sim \mathcal{N}(0, I), \quad t = 1, \dots, 200 \text{ noise the amplitudes}$$

train a network to predict the noise: $\mathcal{L}(\psi) = \mathbb{E}_{A_0 \sim p(A), t, z} \left\| z - z_\psi(A_t, t, x) \right\|^2$



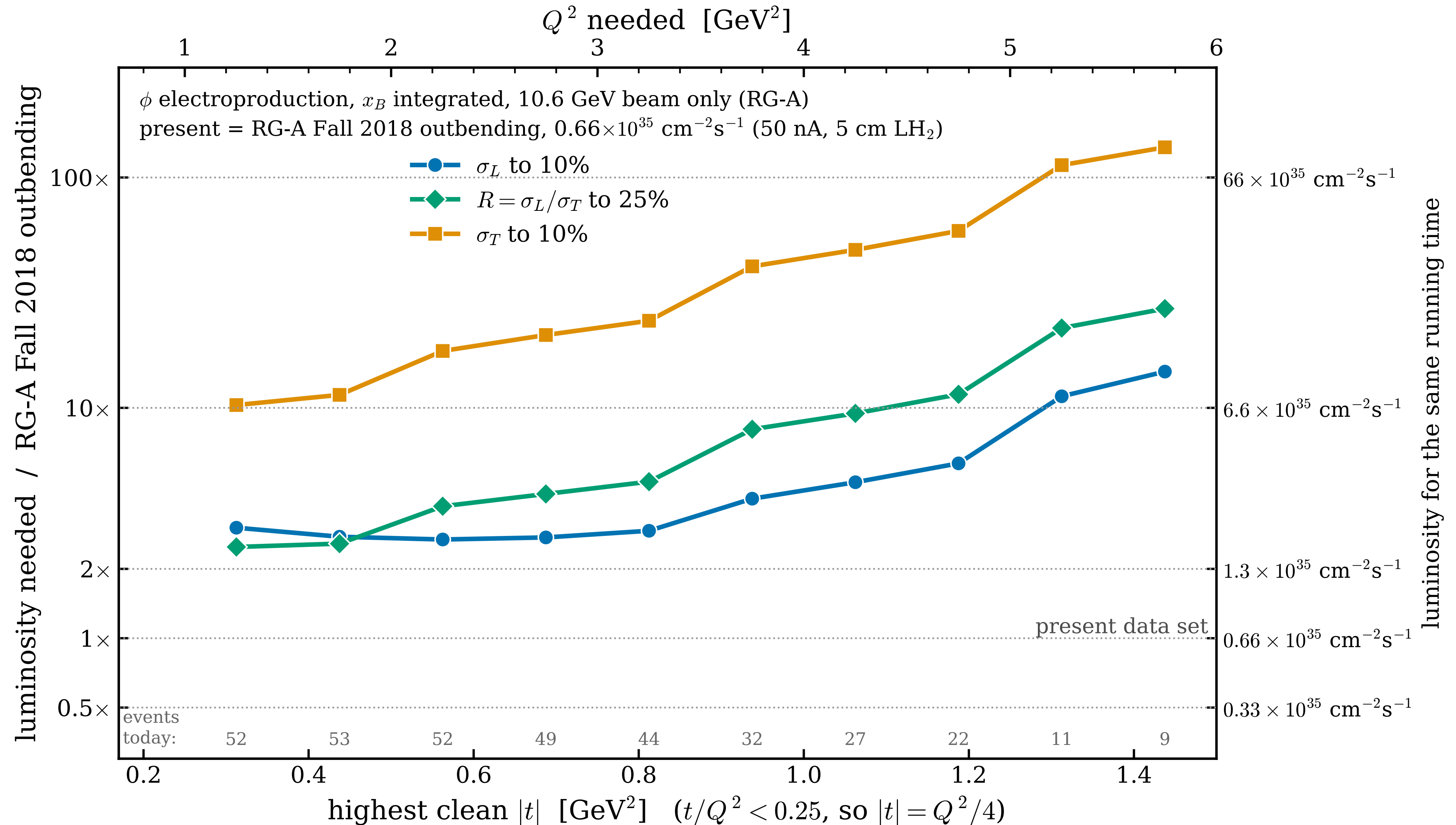
minimize the noise. A network that does this well has learned the score $\nabla_{A_t} \log q(A_t | x) = -z_\psi(A_t, t, x) / \sqrt{1 - \bar{\alpha}_t}$

Event-by-event analysis

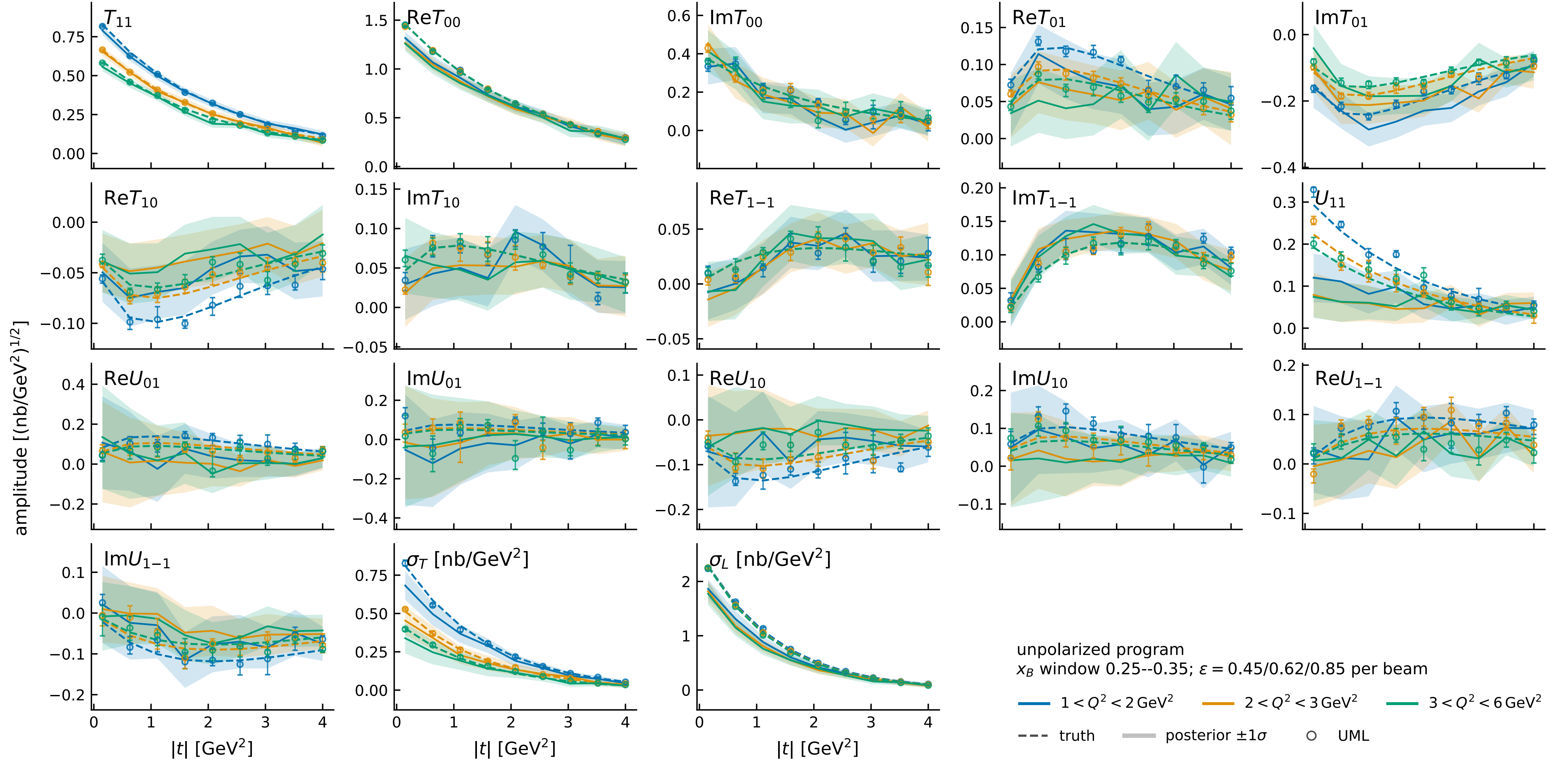


Projection for Instantaneous Luminosity

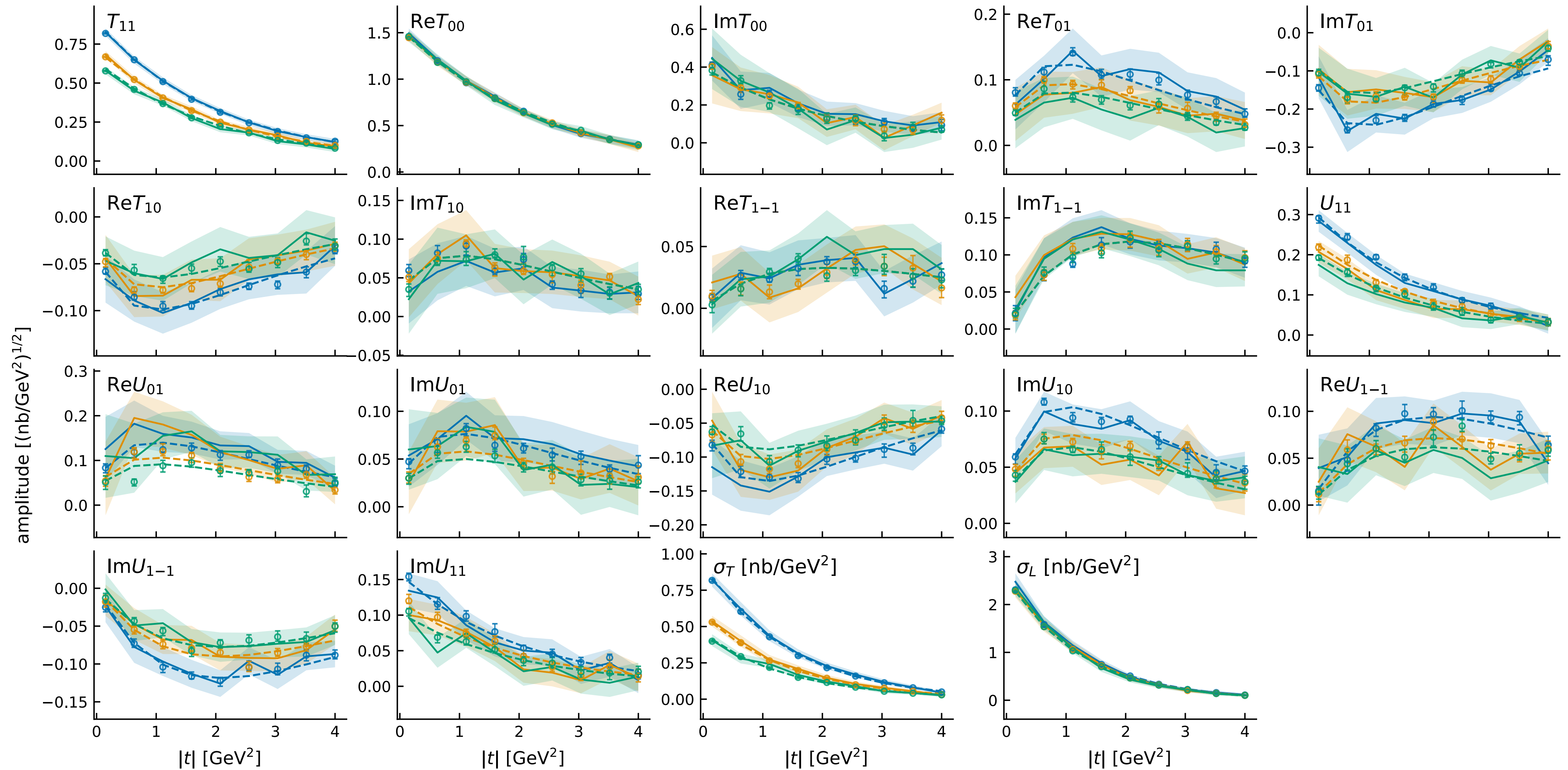
Reaching high $|t|$ with small higher twist needs high Q^2 , and high Q^2 needs luminosity



ϕ meson production on unpolarized target



ϕ meson production on longitudinally polarized target

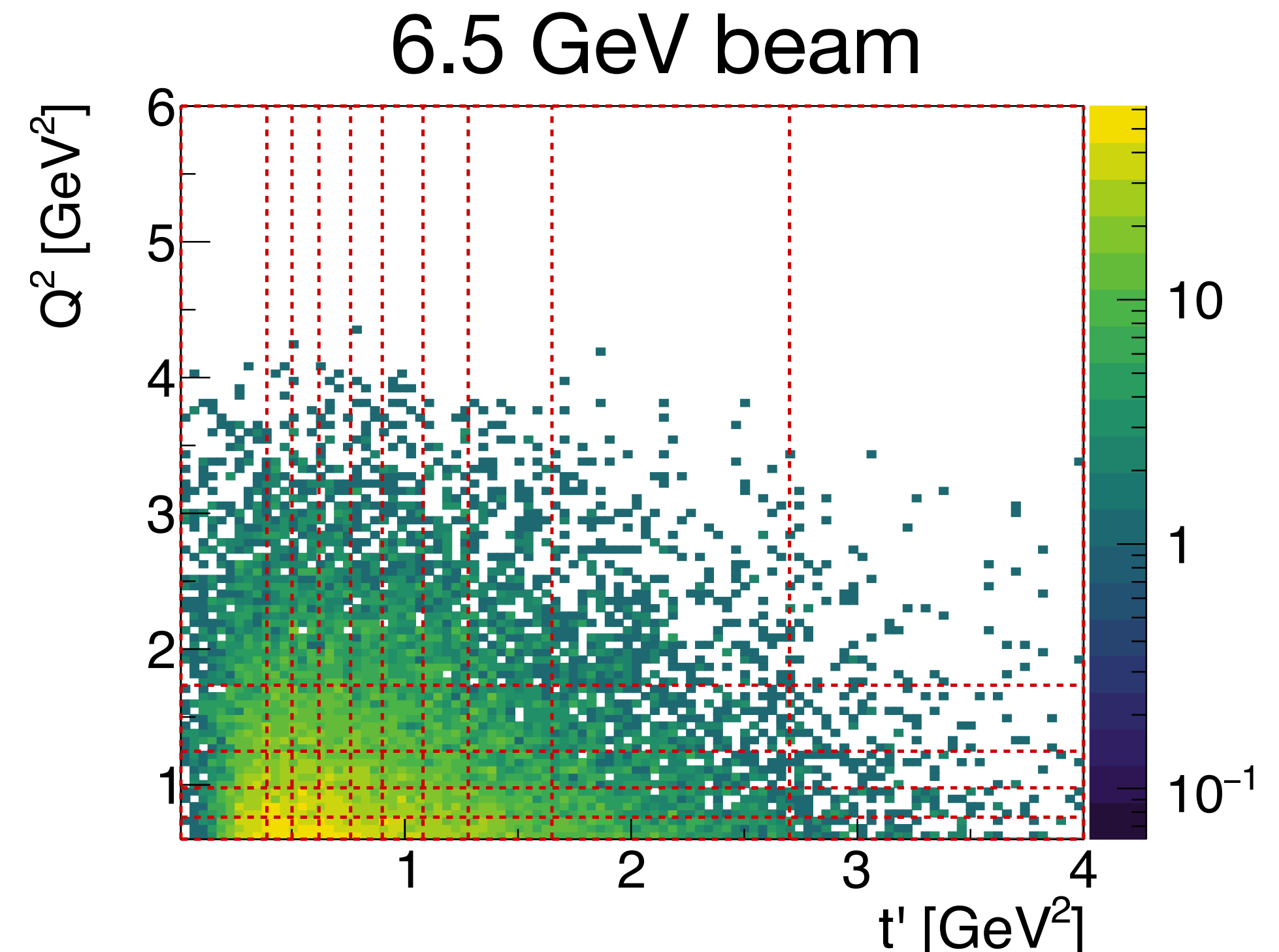
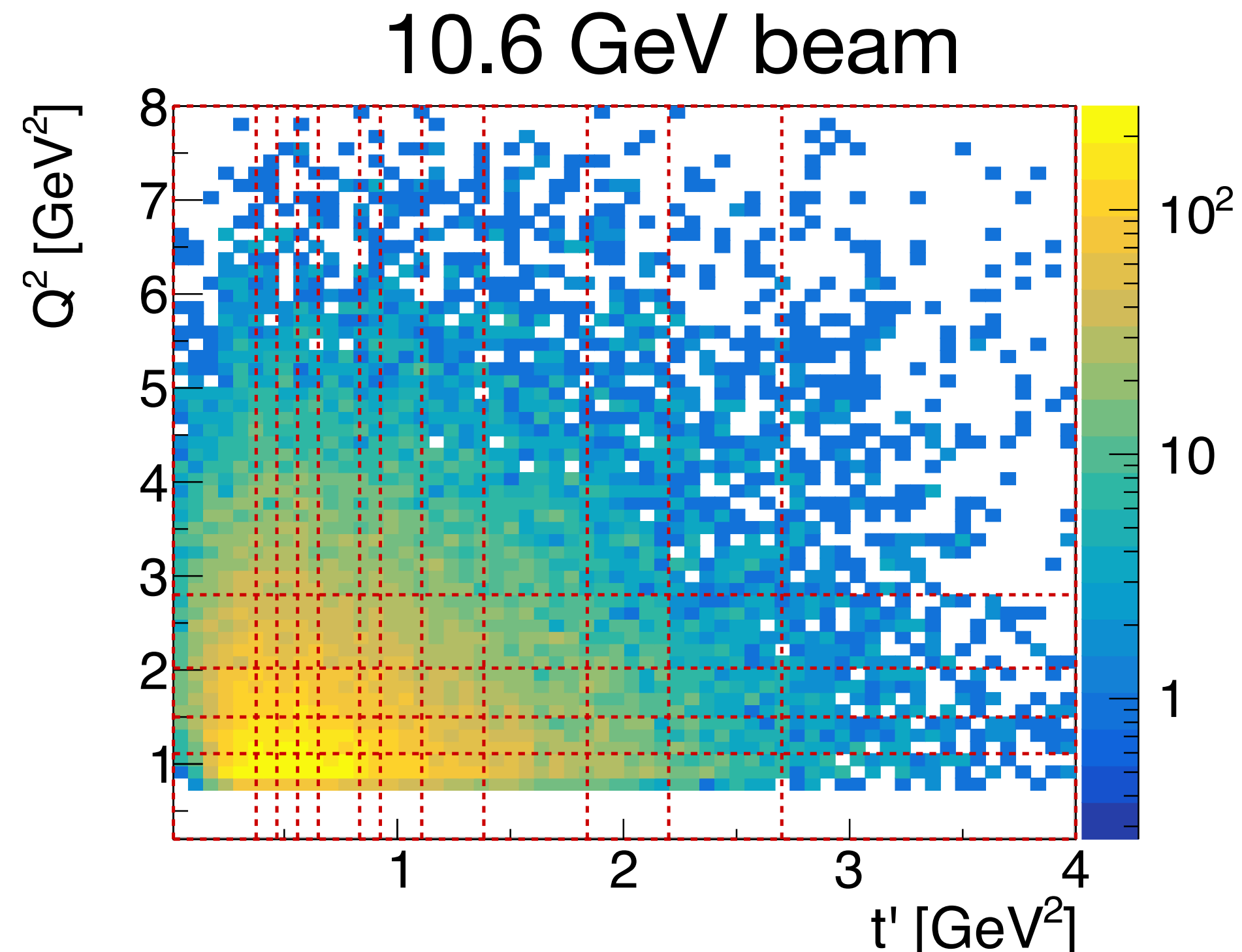


longitudinal-target program, $P = 0.8$
 x_B window 0.25--0.35; $\varepsilon = 0.45/0.62/0.85$ per beam

— 1 < Q^2 < 2 GeV² — 2 < Q^2 < 3 GeV² — 3 < Q^2 < 6 GeV² --- truth — posterior $\pm 1\sigma$ ○ UML

Towards high luminosity

- We want equal statistics in each Q^2 and t' bin from CLAS12
- Currently **5** Q^2 and **12** t' bins
- In each $-t'$ bin, we used a $InvM_{K^+K^-}$ distribution to extract the counts of ϕ



Model

- Method is agnostic to the process
 - It asks only for joint samples (A, x) from the prior and the forward map
 - The physics enters through the construction of x and the prior, never through the objective

Forward

A true amplitude vector is corrupted over T steps by repeatedly adding Gaussian noise. The injected noise is the regression target.

$$q(A_t | A_{t-1}) = \mathcal{N}(\sqrt{1 - \beta_t} A_{t-1}, \beta_t I) \quad A_t = \sqrt{\bar{\alpha}_t} A_0 + \sqrt{1 - \bar{\alpha}_t} z, \quad z \sim \mathcal{N}(0, I)$$

Reverse

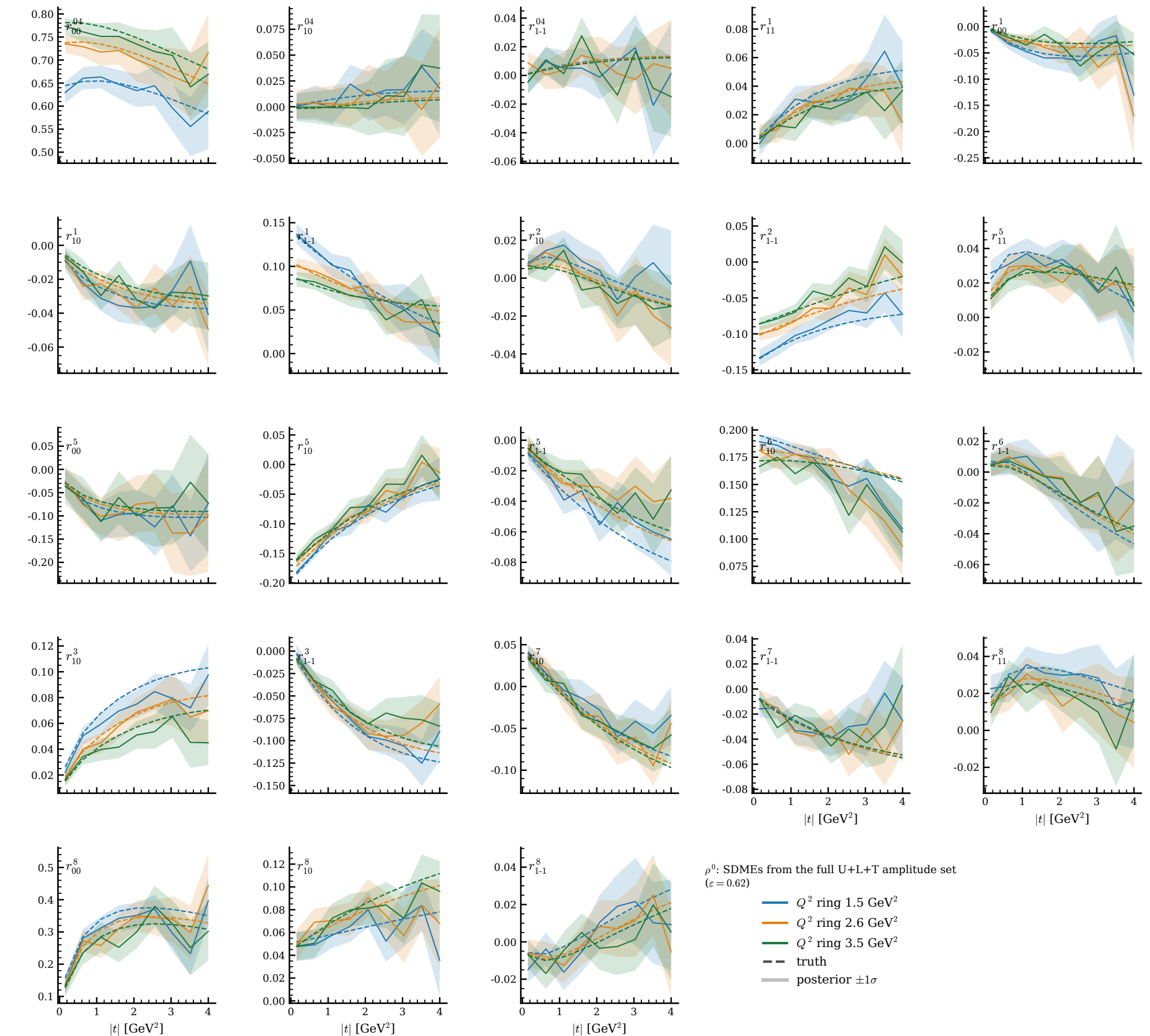
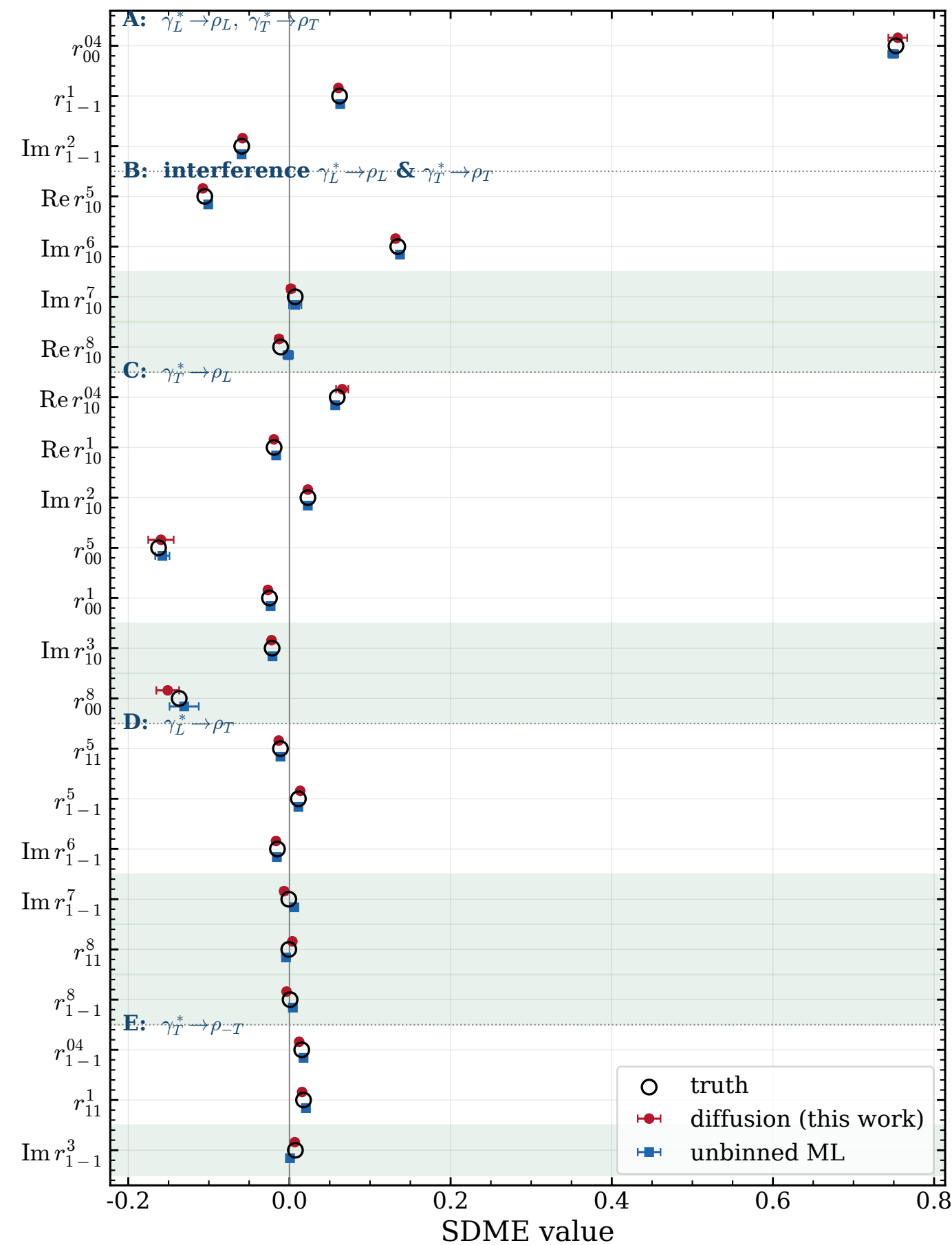
Predict the noise, conditioned on the measured features. Knowing the noise is equivalent to knowing the score.

$$\mathcal{L}(\psi) = \mathbb{E}_{A_0 \sim p(A), t, z} \|z - z_\psi(A_t, t, x)\|^2$$

- Total $T=200$ diffusion steps $K = 20$ ensemble of networks
- Noise: Knowing the noise is equivalent to knowing the score, $z_\psi(A_t, t, x)$ to predict noise
- Score: $\nabla_{A_t} \log q(A_t | x) = -z_\psi(A_t, t, x) / \sqrt{1 - \bar{\alpha}_t}$, at inference we take the measured x of a real bin, start from Gaussian noise, and apply the reverse step T times; the endpoint is one posterior sample

SDMEs just one case in the paper

- 23 SDMEs in the COMPASS/HERMES (A–E by helicity transition), at a representative bin ($Q_2=2.4$, $|t|=0.2$ GeV², $\varepsilon=0.8$)
- Three independent results overlap: the diffusion posterior (red, error bars = total statistical $\oplus$ model uncertainty), an unbinned maximum-likelihood fit of the same events (blue), and the injected truth (open circles).



Multi-Detector Model

- We train over a family, not a detector but 140 detectors
 - 16 random smooth fields: log-acceptance as a random low-order harmonic expansion through a logistic
 - a random resolution matrix
 - 12 hard-edged multi-sector geometries with random sector counts (4, 5, 7, 8), dead gaps and forward cuts
 - 16 deliberately hostile stress geometries beyond the nominal range
 - Rest smooth functions
- Network does not know which one it is: network learns from but through the measured response
 - through measured responses: how the detector folds the amplitudes into moments and rates

One network learns $q\psi(A | x, s)$ over the whole family. We try to go close to the true detector model, however it never says true detector is unnecessary

Angular modulations

- L block alone is very large

$$W_{UL}^{LL} = -2\sqrt{\varepsilon(1+\varepsilon)} \sin \Phi \operatorname{Im} l_{0+}^{00} - \varepsilon \sin 2\Phi \operatorname{Im} l_{-+}^{00}, \quad (\text{A11})$$

$$W_{UL}^{LT} = \sqrt{\varepsilon(1+\varepsilon)} \sin(\Phi - \varphi) \operatorname{Im} (l_{0+}^{0+} - l_{0+}^{-0}) + \sin \varphi \operatorname{Im} [(l_{++}^{0+} - l_{++}^{-0}) + 2\varepsilon l_{00}^{0+}] + \varepsilon \sin(2\Phi - \varphi) \operatorname{Im} l_{-+}^{0+} \\ - \sqrt{\varepsilon(1+\varepsilon)} \sin(\Phi + \varphi) \operatorname{Im} (l_{0+}^{0-} - l_{0+}^{+0}) + \varepsilon \sin(2\Phi + \varphi) \operatorname{Im} l_{-+}^{+0}, \quad (\text{A12})$$

$$W_{UL}^{TT} = \frac{\varepsilon}{2} \sin(2\Phi - 2\varphi) \operatorname{Im} l_{-+}^{-+} - \sqrt{\varepsilon(1+\varepsilon)} \sin \Phi \operatorname{Im} (l_{0+}^{++} + l_{0+}^{--}) + \sqrt{\varepsilon(1+\varepsilon)} \sin(\Phi - 2\varphi) \operatorname{Im} l_{0+}^{-+} \\ + \sin 2\varphi \operatorname{Im} [l_{++}^{-+} + \varepsilon l_{00}^{-+}] - \varepsilon \sin 2\Phi \operatorname{Im} l_{-+}^{++} + \sqrt{\varepsilon(1+\varepsilon)} \sin(\Phi + 2\varphi) \operatorname{Im} l_{0+}^{+-} \\ + \frac{\varepsilon}{2} \sin(2\Phi + 2\varphi) \operatorname{Im} l_{-+}^{+-}, \quad (\text{A13})$$

$$W_{LL}^{LL} = 2\sqrt{\varepsilon(1-\varepsilon)} \cos \Phi \operatorname{Re} l_{0+}^{00} - \sqrt{1-\varepsilon^2} \operatorname{Re} l_{++}^{00}, \quad (\text{A14})$$

$$W_{LL}^{LT} = -\sqrt{\varepsilon(1-\varepsilon)} \cos(\Phi - \varphi) \operatorname{Re} (l_{0+}^{0+} - l_{0+}^{-0}) + \sqrt{1-\varepsilon^2} \cos \varphi \operatorname{Re} (l_{++}^{0+} - l_{++}^{-0}) + \sqrt{\varepsilon(1-\varepsilon)} \cos(\Phi + \varphi) \operatorname{Re} (l_{0+}^{0-} - l_{0+}^{+0}), \quad (\text{A15})$$

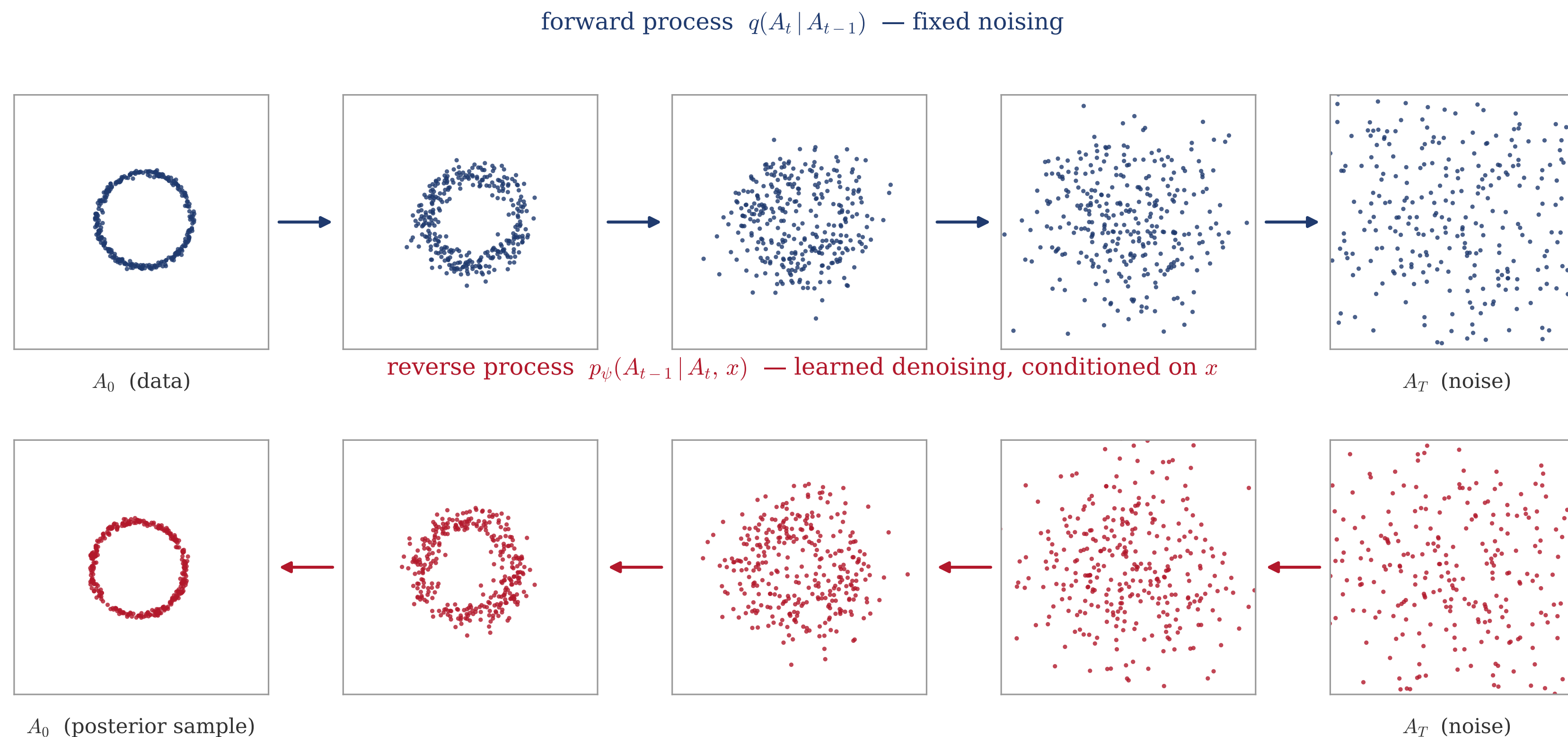
$$W_{LL}^{TT} = -\frac{\sqrt{1-\varepsilon^2}}{2} \operatorname{Re} [l_{++}^{++} + l_{++}^{--}] + \sqrt{\varepsilon(1-\varepsilon)} \cos \Phi \operatorname{Re} (l_{0+}^{++} + l_{0+}^{--}) - \sqrt{\varepsilon(1-\varepsilon)} \cos(\Phi - 2\varphi) \operatorname{Re} l_{0+}^{-+} \\ + \sqrt{1-\varepsilon^2} \cos 2\varphi \operatorname{Re} l_{-+}^{-+} - \sqrt{\varepsilon(1-\varepsilon)} \cos(\Phi + 2\varphi) \operatorname{Re} l_{0+}^{+-}. \quad (\text{A16})$$

All relations of structure function and amplitudes

coeff.	SW SDME	u -combination	amplitude content (T, U)
$\mathcal{U}$: unpolarized, Φ -independent			
A_{00}	r_{00}^{04}	$u_{++}^{00} + \varepsilon u_{00}^{00}$	$ T_{01} ^2 + \varepsilon T_{00} ^2 + U_{01} ^2$
A_{10}	$\text{Re } r_{10}^{04}$	$\frac{1}{2} \text{Re}(u_{++}^{0+} - u_{++}^{-0}) + \varepsilon \text{Re } u_{00}^{0+}$	$\varepsilon \text{Re } T_{00} T_{10}^* + \frac{1}{2} \text{Re } T_{01} T_{11}^* - \frac{1}{2} \text{Re } T_{01} T_{1-1}^* + \frac{1}{2} \text{Re } U_{01} U_{11}^* + \frac{1}{2} \text{Re } U_{01} U_{1-1}^*$
A_{1-1}	r_{1-1}^{04}	$\text{Re } u_{++}^{-+} + \varepsilon \text{Re } u_{00}^{-+}$	$\text{Re } T_{11} T_{1-1}^* - \varepsilon T_{10} ^2 - \text{Re } U_{11} U_{1-1}^* + \varepsilon U_{10} ^2$
$\mathcal{L}_2$: linear polarization ($\cos 2\Phi, \sin 2\Phi$)			
B_{11}^c	r_{11}^1	$\text{Re } u_{-+}^{++}$	$\text{Re } T_{11} T_{1-1}^* + \text{Re } U_{11} U_{1-1}^*$
B_{00}^c	r_{00}^1	u_{-+}^{00}	$- T_{01} ^2 + U_{01} ^2$
B_{10}^c	$\text{Re } r_{10}^1$	$\frac{1}{2} \text{Re}(u_{-+}^{0+} + u_{-+}^{+0})$	$-\frac{1}{2} \text{Re } T_{01} T_{11}^* + \frac{1}{2} \text{Re } T_{01} T_{1-1}^* + \frac{1}{2} \text{Re } U_{01} U_{11}^* + \frac{1}{2} \text{Re } U_{01} U_{1-1}^*$
B_{1-1}^c	r_{1-1}^1	$\frac{1}{2} (u_{-+}^{-+} + u_{-+}^{+-})$	$\frac{1}{2} (T_{11} ^2 + T_{1-1} ^2) - \frac{1}{2} (U_{11} ^2 + U_{1-1} ^2)$
B_{10}^s	$\text{Im } r_{10}^2$	$\frac{1}{2} \text{Re}(u_{-+}^{+0} - u_{-+}^{0+})$	$\frac{1}{2} \text{Re } T_{01} T_{11}^* + \frac{1}{2} \text{Re } T_{01} T_{1-1}^* - \frac{1}{2} \text{Re } U_{01} U_{11}^* + \frac{1}{2} \text{Re } U_{01} U_{1-1}^*$
B_{1-1}^s	$\text{Im } r_{1-1}^2$	$\frac{1}{2} (u_{-+}^{+-} - u_{-+}^{-+})$	$\frac{1}{2} (T_{1-1} ^2 - T_{11} ^2) + \frac{1}{2} (U_{11} ^2 - U_{1-1} ^2)$
$\mathcal{L}_1$: L-T interference ($\cos \Phi, \sin \Phi$)			
C_{11}^c	r_{11}^5	$-\frac{1}{\sqrt{2}} (u_{0+}^{++} + u_{0+}^{--})$	$\frac{1}{\sqrt{2}} (\text{Re } T_{10} T_{1-1}^* - \text{Re } T_{10} T_{11}^*) + \frac{1}{\sqrt{2}} (\text{Re } U_{10} U_{1-1}^* - \text{Re } U_{10} U_{11}^*)$
C_{00}^c	r_{00}^5	$-\sqrt{2} \text{Re } u_{0+}^{00}$	$-\sqrt{2} \text{Re } T_{00} T_{01}^*$
C_{10}^c	$\text{Re } r_{10}^5$	$\frac{1}{2\sqrt{2}} \text{Re}[(u_{0+}^{0-} - u_{0+}^{+0}) - (u_{0+}^{0+} - u_{0+}^{-0})]$	$\frac{1}{2\sqrt{2}} (\text{Re } T_{00} T_{1-1}^* - \text{Re } T_{00} T_{11}^* - 2\text{Re } T_{01} T_{10}^*)$
C_{1-1}^c	r_{1-1}^5	$-\frac{1}{\sqrt{2}} \text{Re}(u_{0+}^{-+} + u_{0+}^{+-})$	$\frac{1}{\sqrt{2}} (\text{Re } T_{10} T_{11}^* - \text{Re } T_{10} T_{1-1}^*) + \frac{1}{\sqrt{2}} (\text{Re } U_{10} U_{1-1}^* - \text{Re } U_{10} U_{11}^*)$
C_{10}^s	$\text{Im } r_{10}^6$	$\frac{1}{2\sqrt{2}} \text{Re}[(u_{0+}^{0-} - u_{0+}^{-0}) + (u_{0+}^{0-} - u_{0+}^{+0})]$	$\frac{1}{2\sqrt{2}} (\text{Re } T_{00} T_{11}^* + \text{Re } T_{00} T_{1-1}^*) - \frac{1}{\sqrt{2}} \text{Re } U_{01} U_{10}^*$
C_{1-1}^s	$\text{Im } r_{1-1}^6$	$\frac{1}{\sqrt{2}} \text{Re}(u_{0+}^{-+} - u_{0+}^{+-})$	$-\frac{1}{\sqrt{2}} (\text{Re } T_{10} T_{11}^* + \text{Re } T_{10} T_{1-1}^*) + \frac{1}{\sqrt{2}} (\text{Re } U_{10} U_{11}^* + \text{Re } U_{10} U_{1-1}^*)$
$\mathcal{B}$: beam spin, Φ -independent *			
D_{10}	$\text{Im } r_{10}^{3*}$	$-\frac{1}{2} \text{Im}(u_{++}^{0+} - u_{++}^{-0})$	$-\frac{1}{2} (\text{Im } T_{01} T_{11}^* + \text{Im } T_{01} T_{1-1}^*) - \frac{1}{2} \text{Im } U_{01} U_{11}^* + \frac{1}{2} \text{Im } U_{01} U_{1-1}^*$
D_{1-1}	$\text{Im } r_{1-1}^{3*}$	$-\text{Im } u_{++}^{-+}$	$\text{Im } T_{11} T_{1-1}^* - \text{Im } U_{11} U_{1-1}^*$
$\mathcal{B}_1$: beam spin $\times$ L-T interference *			
E_{10}^c	$\text{Im } r_{10}^{7*}$	$\frac{1}{2\sqrt{2}} \text{Im}[(u_{0+}^{0+} - u_{0+}^{-0}) + (u_{0+}^{0-} - u_{0+}^{+0})]$	$\frac{1}{2\sqrt{2}} (\text{Im } T_{00} T_{11}^* + \text{Im } T_{00} T_{1-1}^*) + \frac{1}{\sqrt{2}} \text{Im } U_{01} U_{10}^*$
E_{1-1}^c	$\text{Im } r_{1-1}^{7*}$	$\frac{1}{\sqrt{2}} \text{Im}(u_{0+}^{-+} - u_{0+}^{+-})$	$-\frac{1}{\sqrt{2}} (\text{Im } T_{10} T_{11}^* + \text{Im } T_{10} T_{1-1}^*) + \frac{1}{\sqrt{2}} (\text{Im } U_{10} U_{11}^* + \text{Im } U_{10} U_{1-1}^*)$
E_{11}^s	r_{11}^{8*}	$\frac{1}{\sqrt{2}} \text{Im}(u_{0+}^{++} + u_{0+}^{--})$	$\frac{1}{\sqrt{2}} (\text{Im } T_{10} T_{11}^* - \text{Im } T_{10} T_{1-1}^*) + \frac{1}{\sqrt{2}} (\text{Im } U_{10} U_{11}^* - \text{Im } U_{10} U_{1-1}^*)$
E_{00}^s	r_{00}^{8*}	$\sqrt{2} \text{Im } u_{0+}^{00}$	$\sqrt{2} \text{Im } T_{00} T_{01}^*$
E_{10}^s	$\text{Re } r_{10}^{8*}$	$\frac{1}{2\sqrt{2}} \text{Im}[(u_{0+}^{0+} - u_{0+}^{-0}) - (u_{0+}^{0-} - u_{0+}^{+0})]$	$\frac{1}{2\sqrt{2}} (\text{Im } T_{00} T_{11}^* - \text{Im } T_{00} T_{1-1}^* - 2\text{Im } T_{01} T_{10}^*)$
E_{1-1}^s	r_{1-1}^{8*}	$\frac{1}{\sqrt{2}} \text{Im}(u_{0+}^{-+} + u_{0+}^{+-})$	$-\frac{1}{\sqrt{2}} (\text{Im } T_{10} T_{11}^* - \text{Im } T_{10} T_{1-1}^*) + \frac{1}{\sqrt{2}} (\text{Im } U_{10} U_{11}^* - \text{Im } U_{10} U_{1-1}^*)$

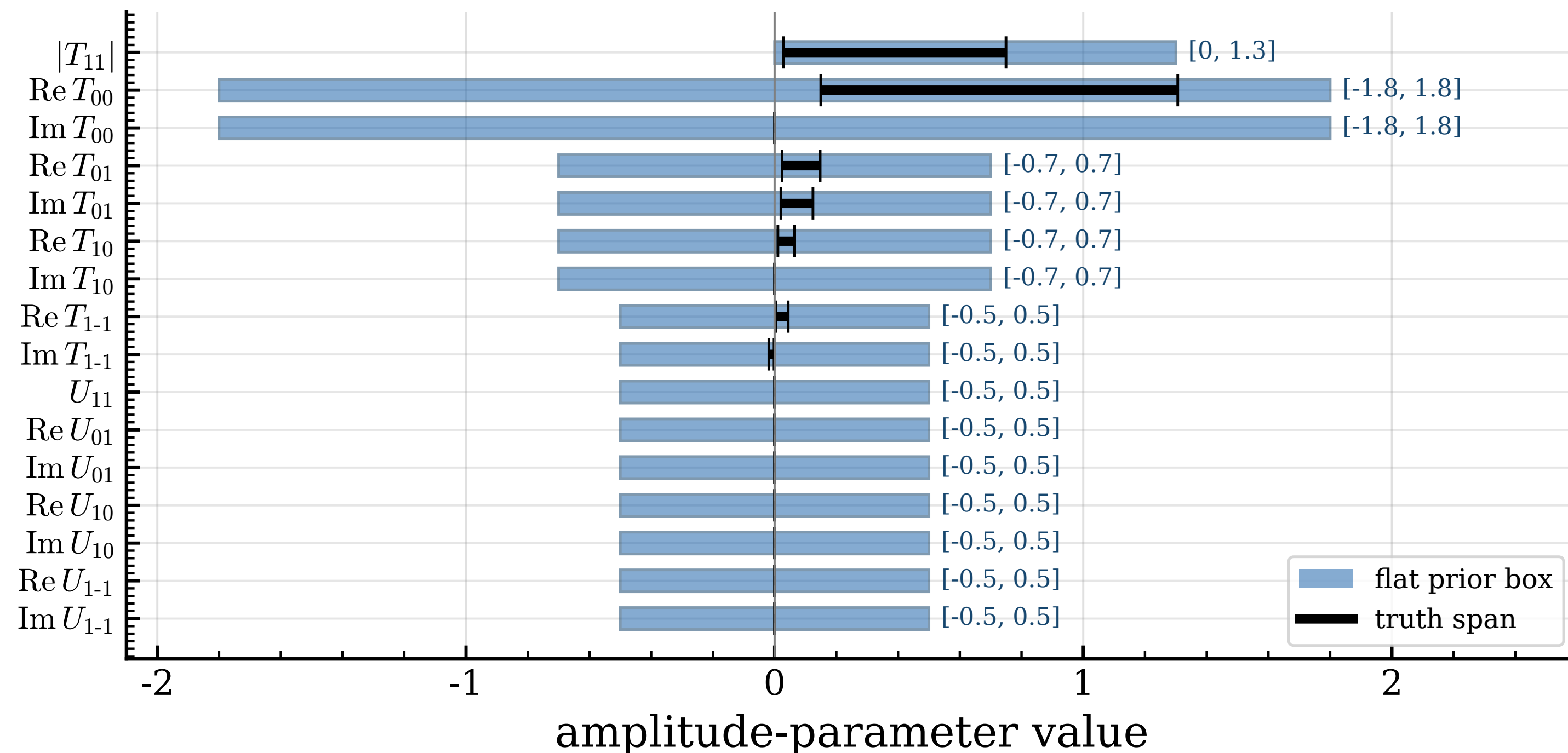
How diffusion model can help

- From those angles we want 34 numbers, the amplitude parameters that describe the spin state of the process
- Amplitudes $\rightarrow$ data is easy: we have the formula. [**New generator based on amplitudes VecGen**]
- Going data $\rightarrow$ Amplitudes is the hard INVERSE problem! A diffusion model is a tool that learns this inverse and gives honest error bars, we used **Denoising Diffusion Probabilistic Model!**



Training recipe

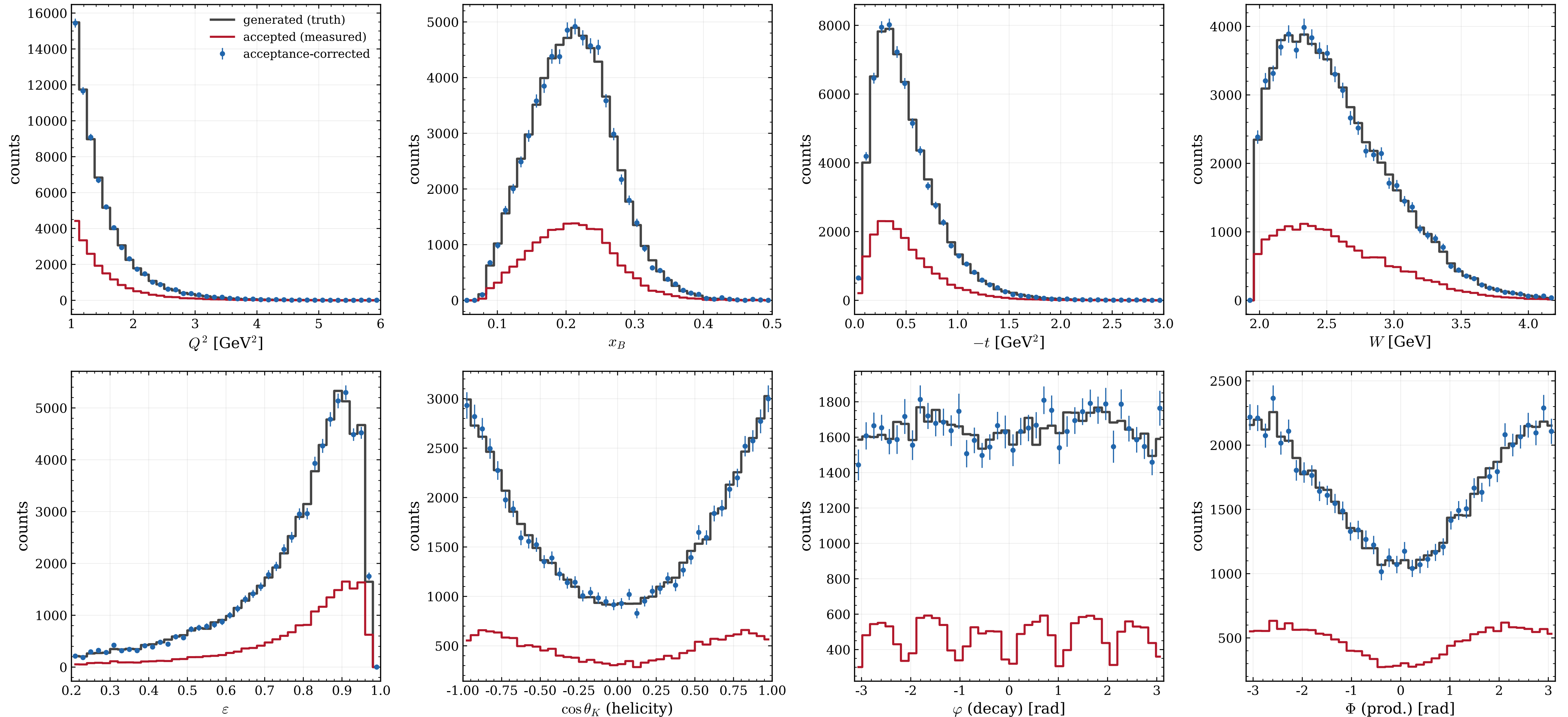
- Prior based training:
 - Cover the physically realized range. The injected truth lies well inside the prior for every parameter
 - Cross sections as function of amplitudes, the priors are kinematics independent
 - However real amplitudes are kinematics dependent, the learner network extracts the kinematics dependencies



- **Schedule:** variance-preserving, $T = 200$
- **Optimizer:** Adam, 400 epochs
- **Data:** 1.6×10^5 simulated pairs per ensemble member, 3 beam energies, 10.6, 6.5, and 7.5 GeV
- **Sampler:** T steps; 400 posterior draws per bin

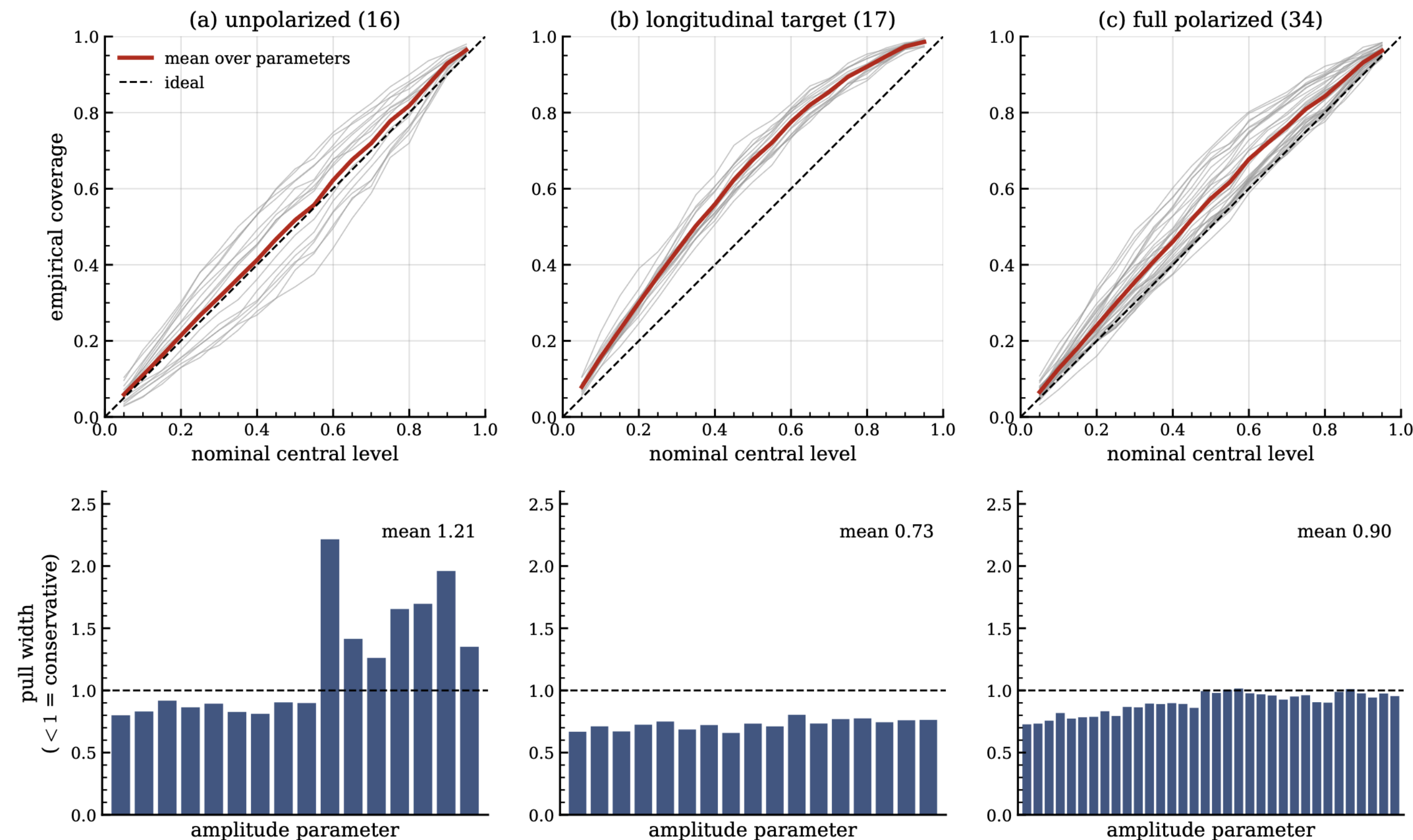
Acceptance correction to the data

- Once we extract the amplitudes we can apply inverse correction to the data and get proper acceptance corrected cross-section from the measurements

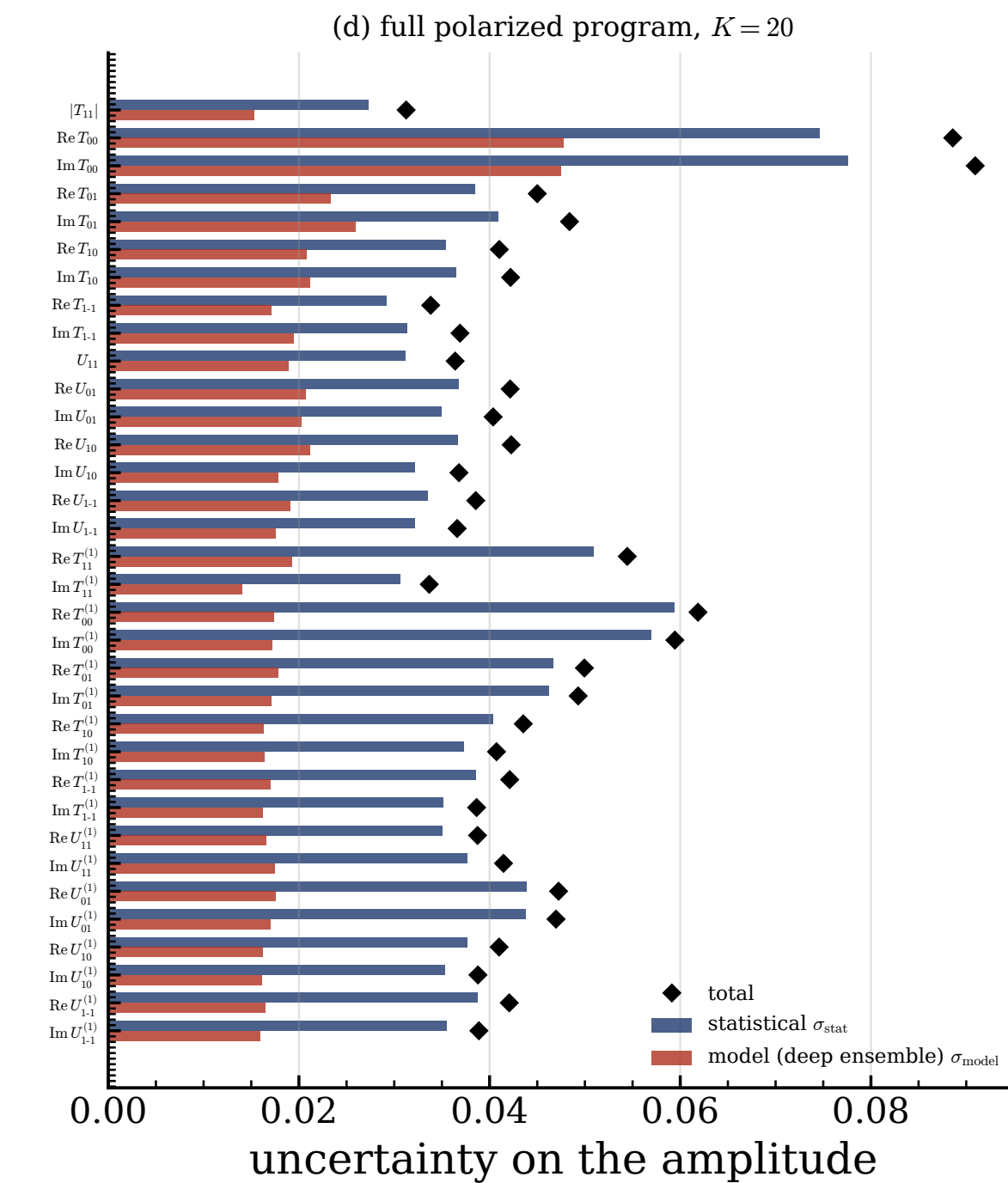
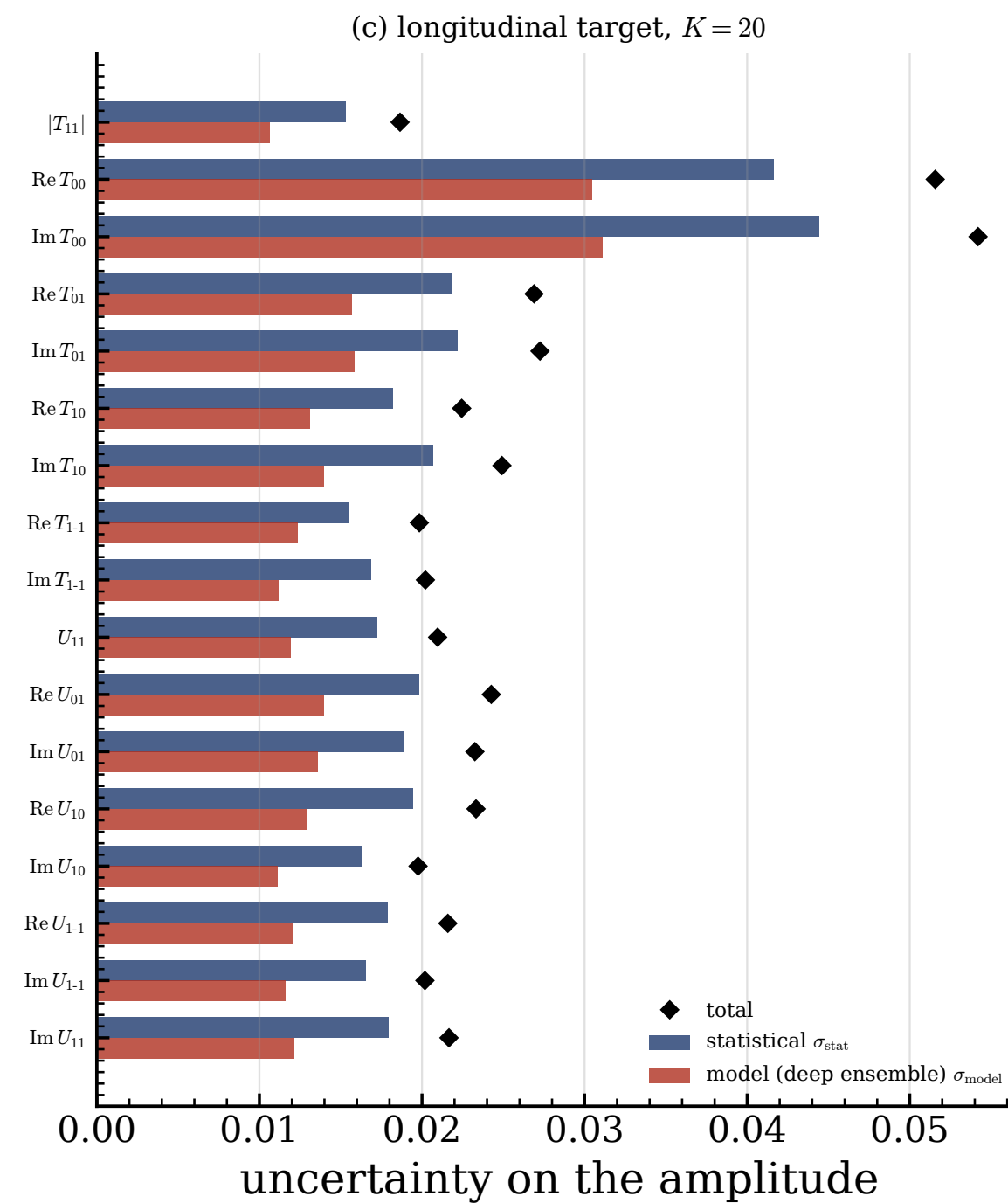
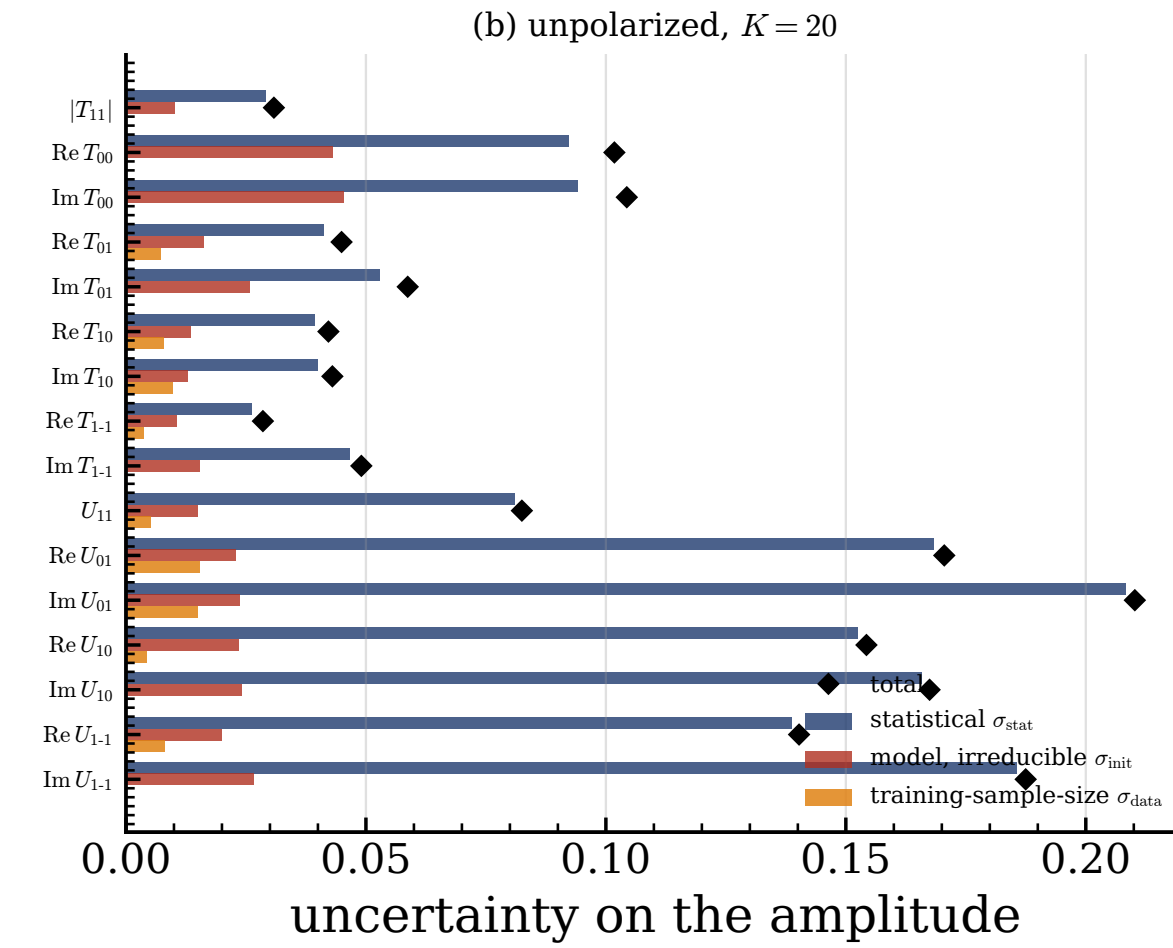
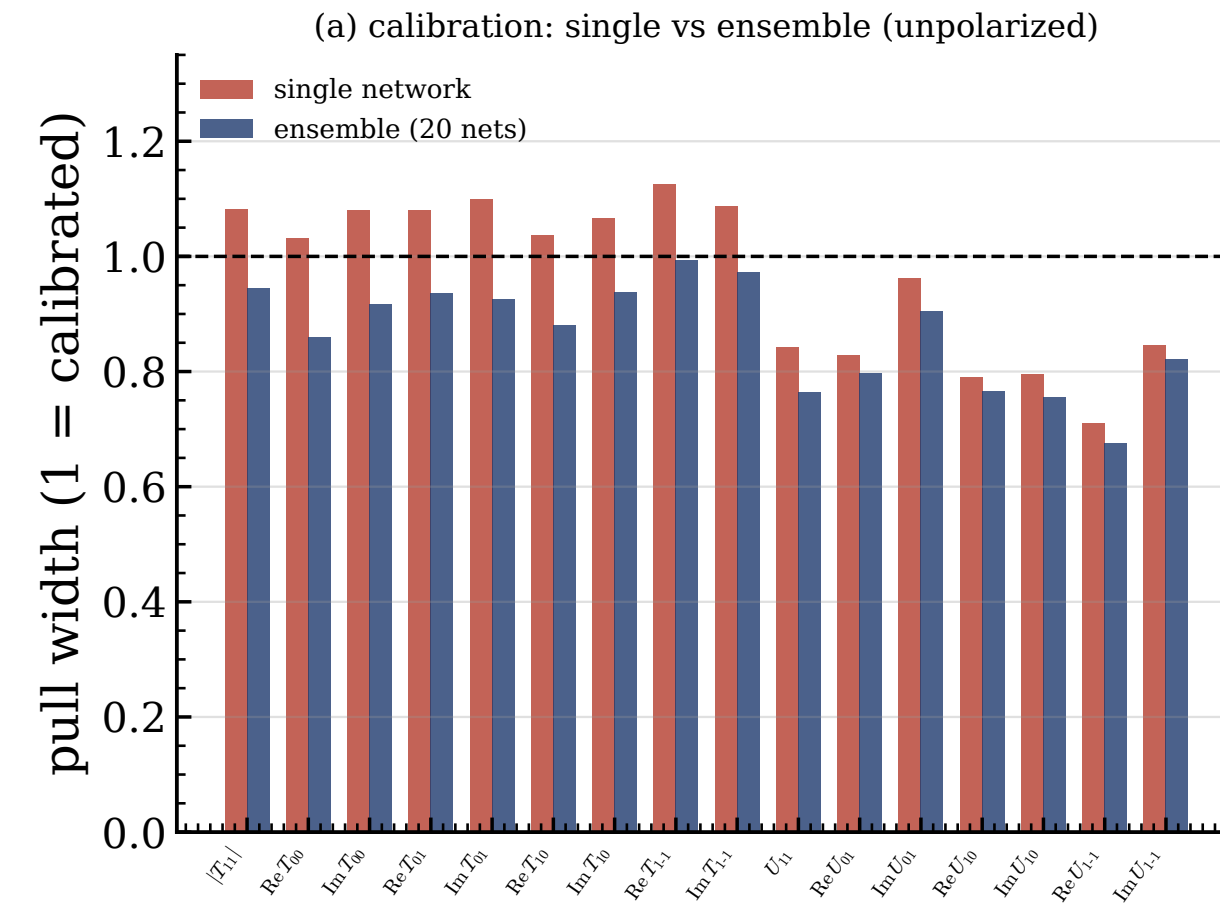


Calibration needed in case of diffusion approach

- A maximum-likelihood fit carries an asymptotic coverage guarantee through its parabolic errors
- A learned posterior $q_\psi(A|x)$ carries none, which is why the simulation-based calibration (SBC) is run before any error bar here is quoted
- SBC: draw a truth from the prior, simulate it, infer it, and rank that truth among its own posterior samples. The ranks are uniform if and only if the posterior is correctly calibrated

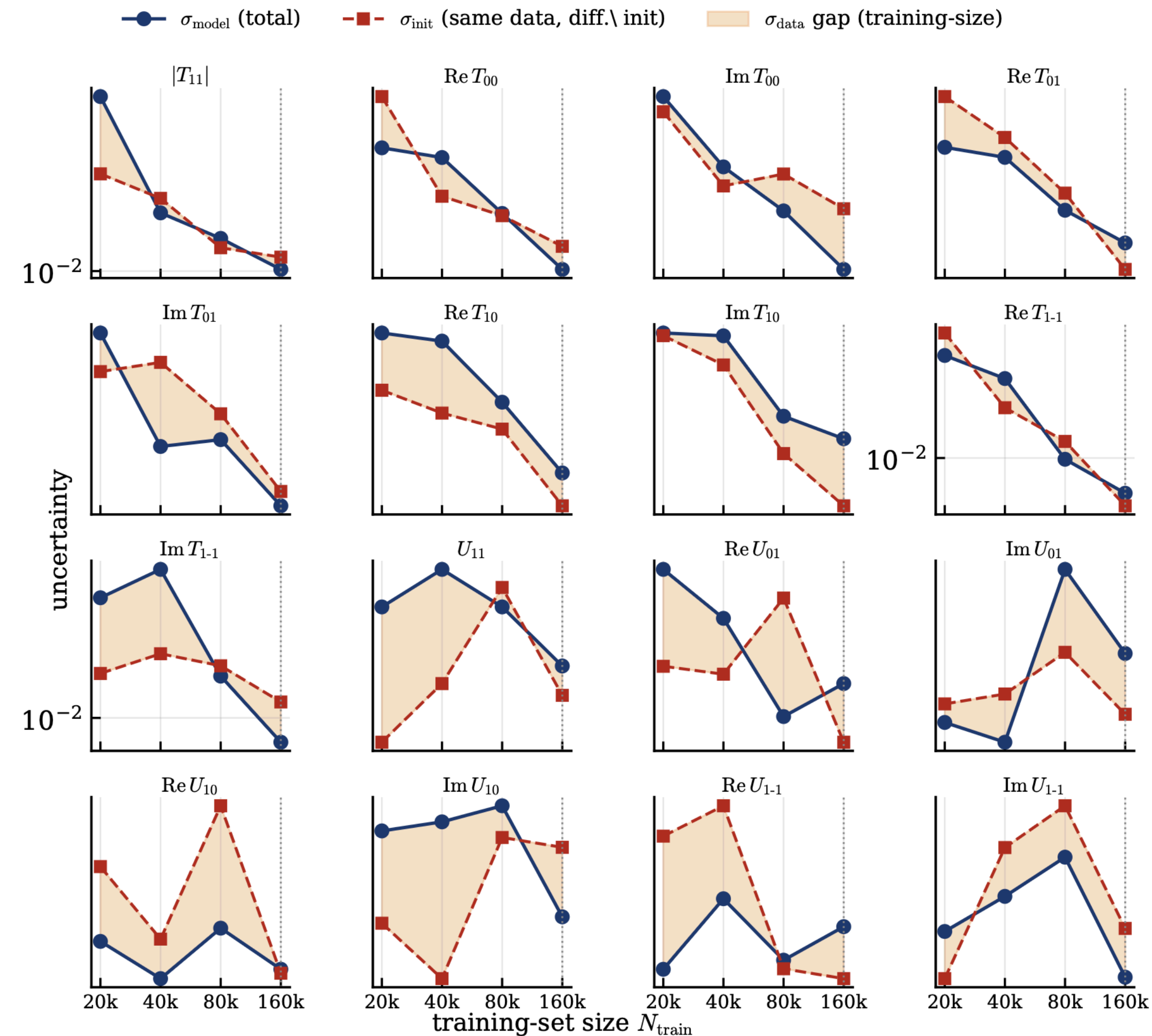


Uncertainty Quantification for all modes



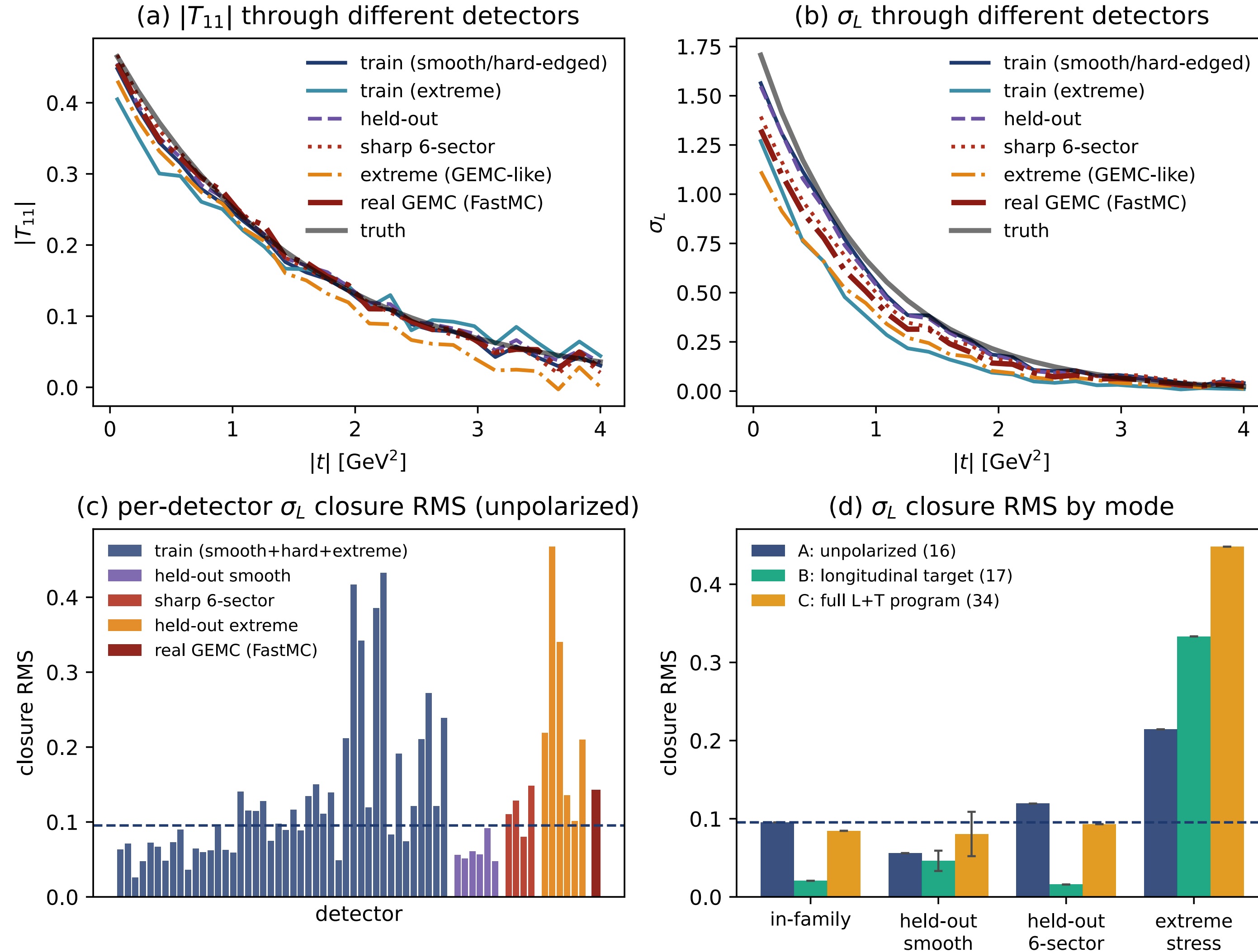
Training size uncertainties

- The dominant source is the statistical uncertainties



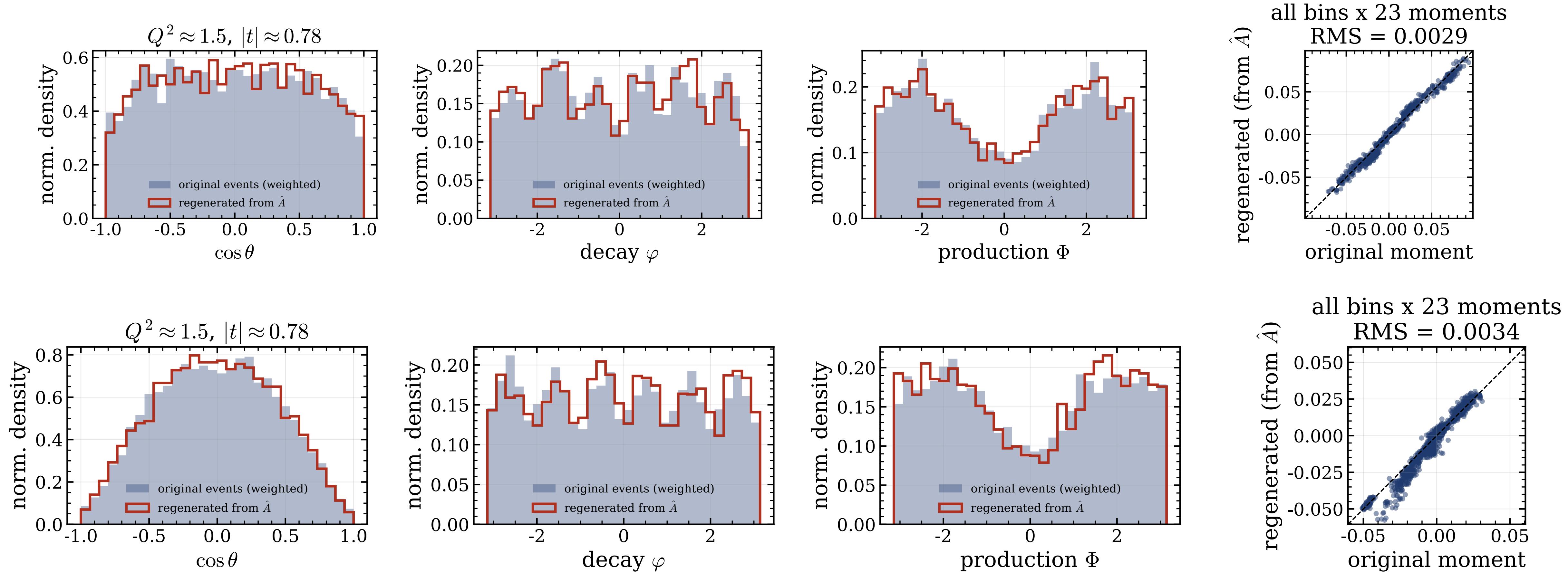
Detector amortization

- We take difference test cases for detectors which are not part of the training and extract the physics



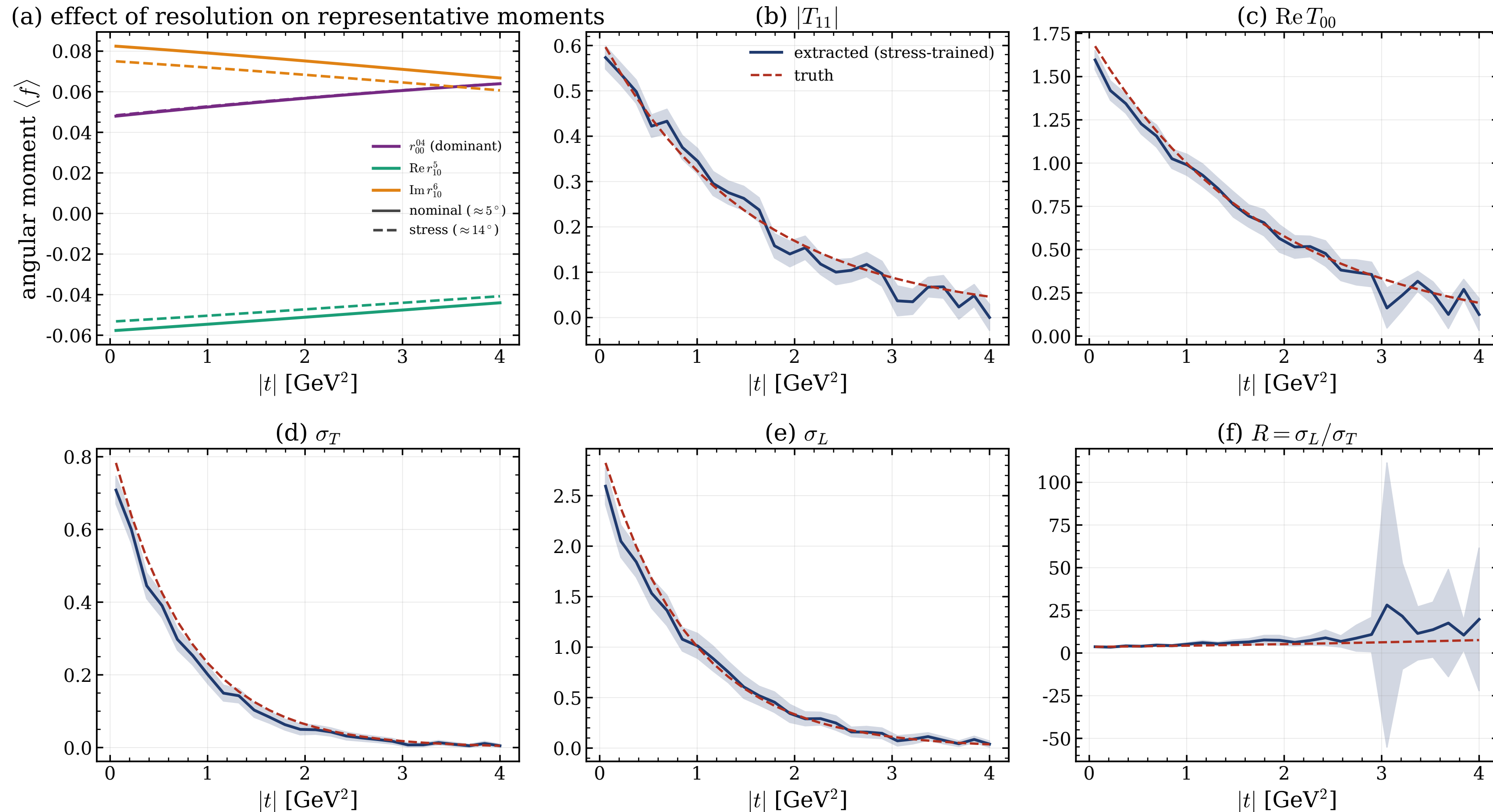
A closure loop test

- Consider N event sample for phi and rho0
- Pass it to inference stage, get the amplitudes, use those amplitudes to generate the events and compare



Effects of resolution of the detector: Resolution robustness

- A realistic detector model always have finite resolutions
- The same is taken into account in the toy approach to see if this can smear the extractions



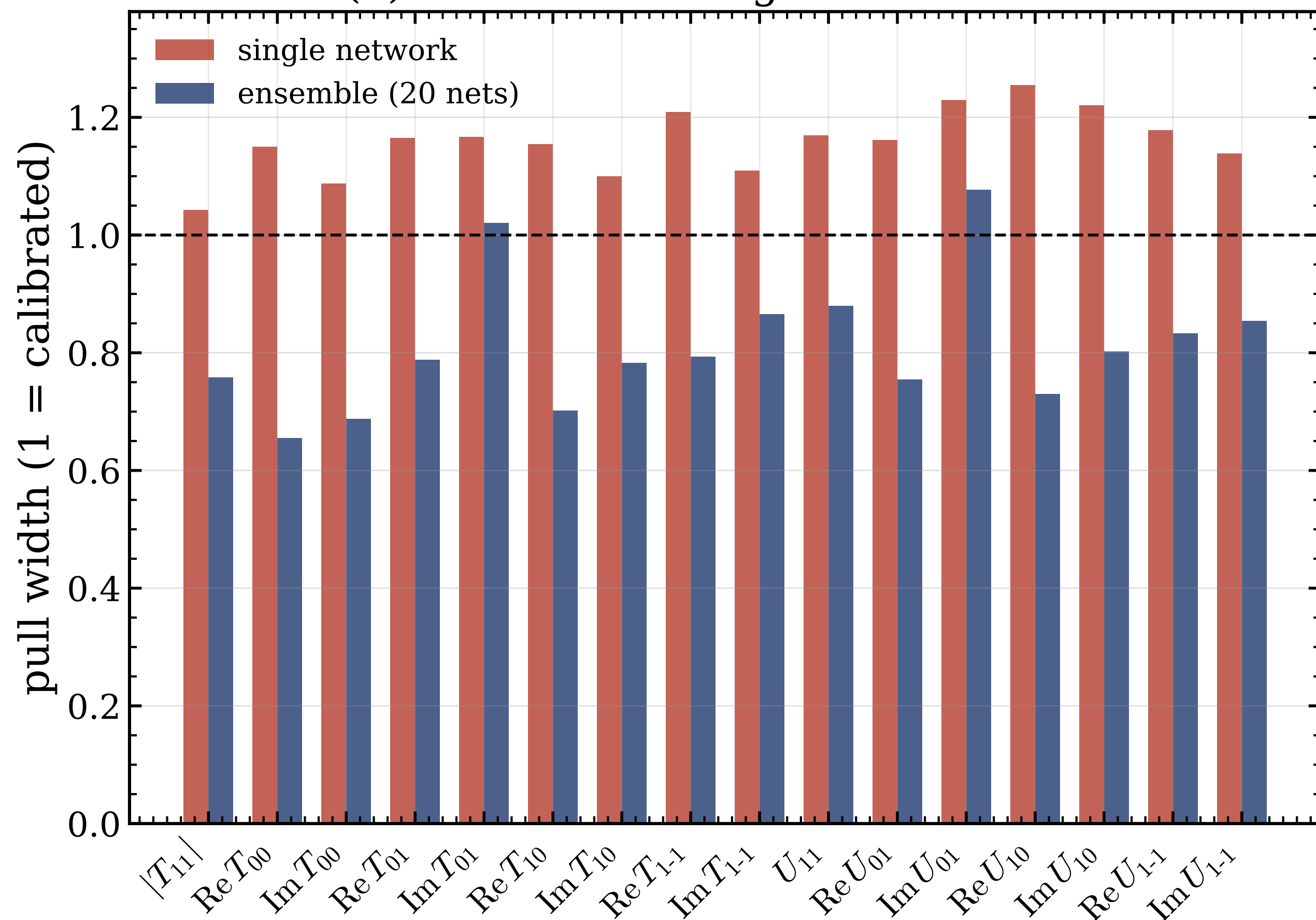
Architectural details:

- Extractor: estimator ε_ψ
 - Conditional DDPM denoiser, and deliberately a plain MLP — the conditioning vector
 - Inputs: A_t (34, amplitudes) - sinusoidal embedding · x (moments $\times 3\varepsilon + 3$ rates) · s (144, measured detector response)
 - x and s enter through a conditioner sub-network: Linear $\rightarrow$ SiLU $\rightarrow$ Linear
 - Trunk: $4 \times$ (Linear + SiLU), width $h = 320$ fixed-detector / 384 detector-amortized
 - Head: Linear $\rightarrow 34$. Output is $\hat{\varepsilon}$, the predicted noise — same dimension as A
- - ≈ 0.9 M parameters · loss = $\|\varepsilon - \varepsilon_\psi(A_t, t, x, s)\|^2$, denoising score matching · Adam 2×10^{-4} , gradient clipping, EMA weights · $K = 20$ members
- Fast MC
 - Acceptance: 5×512 SiLU $\rightarrow 1$ logit $\rightarrow P(\text{reconstructed})$ to resolve the six-sector ϕ edges
 - Resolutions: 3×384 SiLU $\rightarrow 10$ diagonal Gaussians over the momentum residuals (means, log-widths, mixture logits).

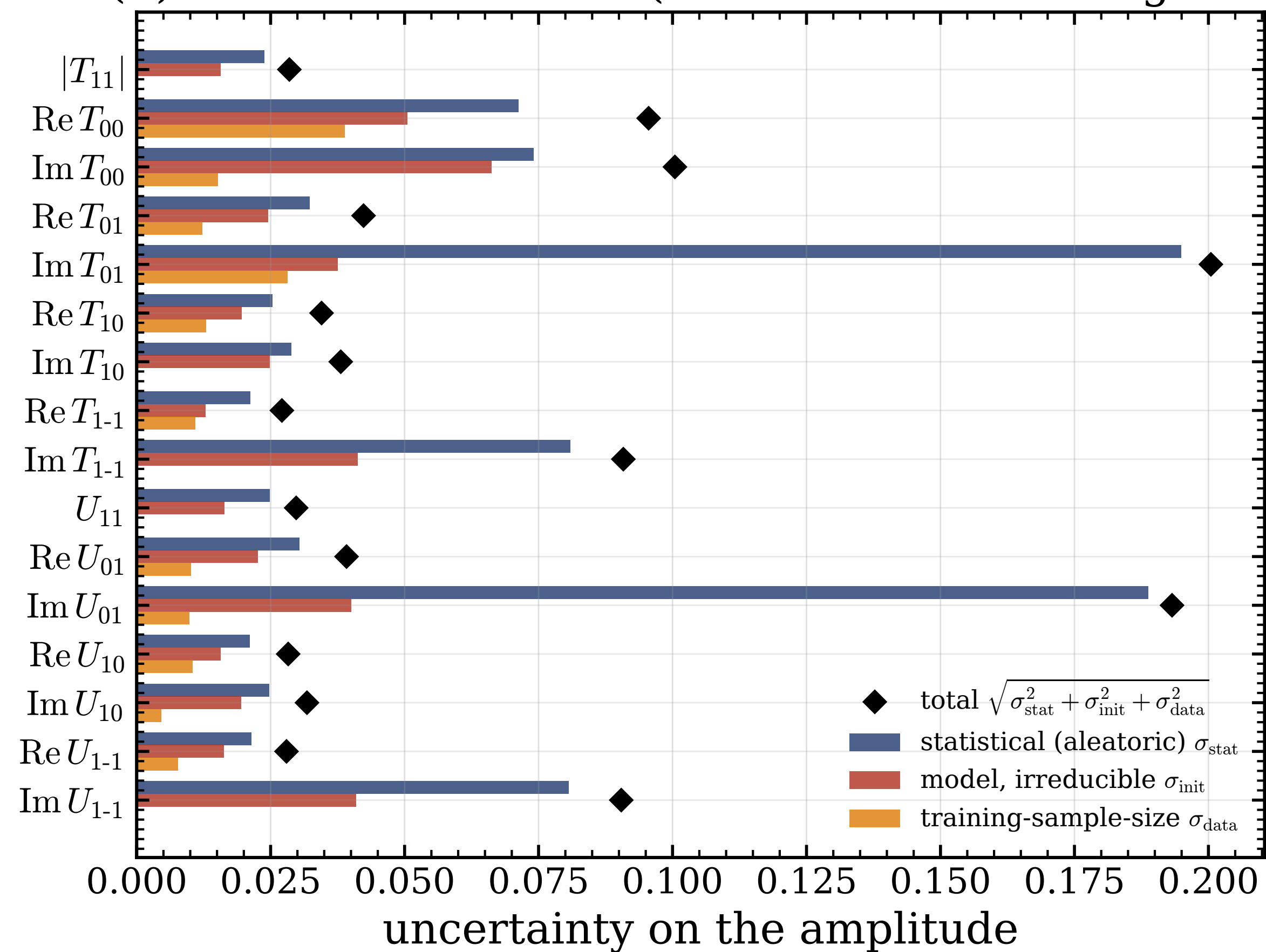
Diffusion model uncertainties

- Generally a AI based diffusion model always comes with uncertainties associated with trainings
- We try unto cluster of 5 different networks to get a conservative estimate on the uncertainties arising from it

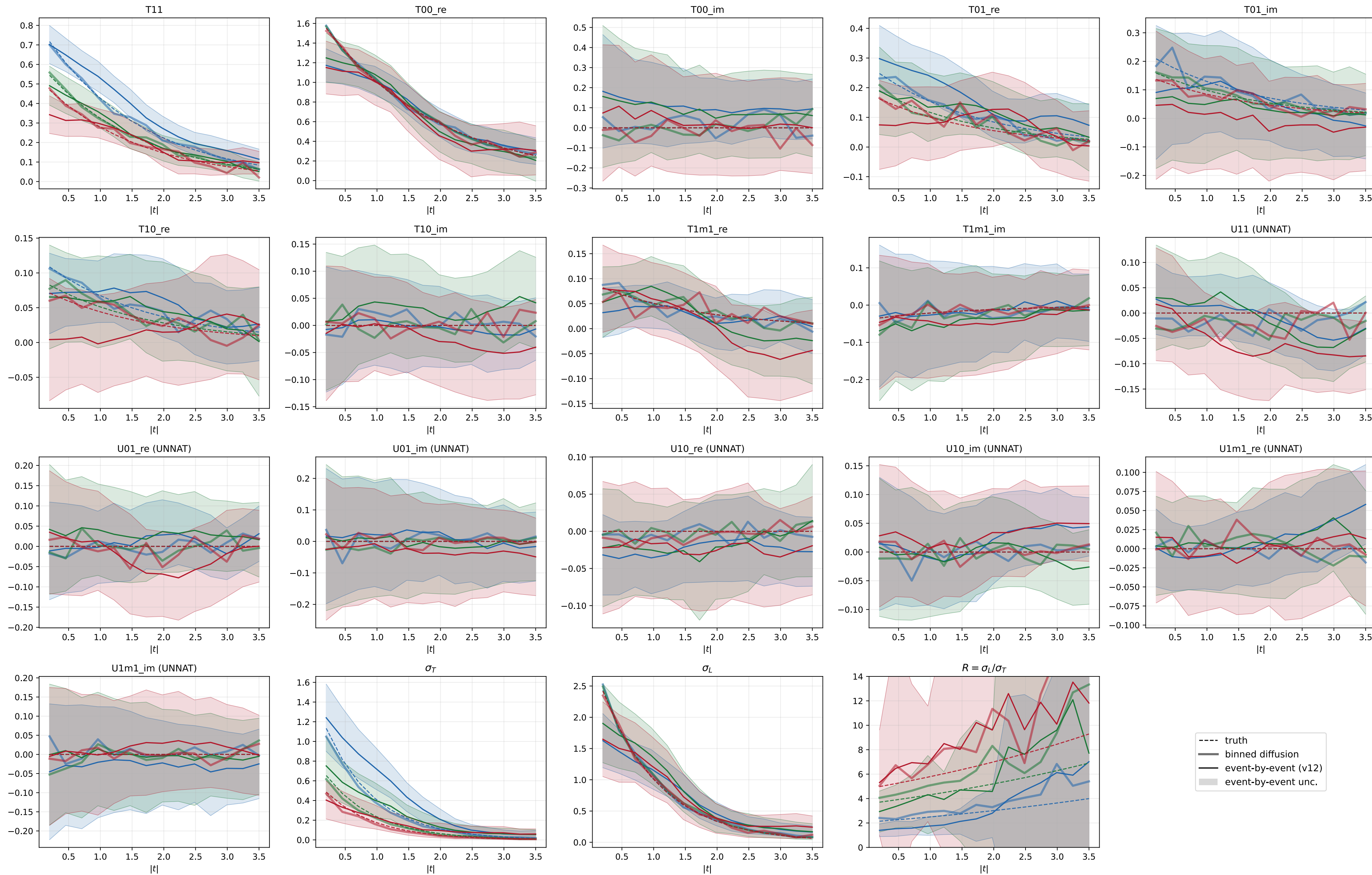
(a) calibration: single vs ensemble



(b) statistical vs model (irreducible + training-size)



More interesting: event-by-event extraction



Extracted parameters after Rosenbluth separation

- Only Diehl's notation here

