Probing the pion structure with CLAS12

First measurement of the DVCS beam spin asymmetry in the Sullivan process

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June 30, 2026

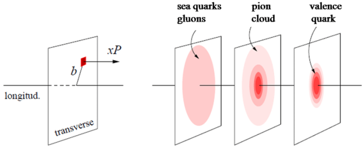
Probing the pion structure

Why the pion?

- **Lightest hadron**: quark-antiquark bound state.
- Goldstone boson of chiral symmetry breaking.
- **Spin-0**: only **two GPDs** at leading twist → simpler than the proton.
- FF and PDFs known, but **3D structure remains unexplored**.

Generalised Parton Distributions (GPDs)

- Generalise PDFs *and* FFs in a single object.
- $H_\pi(x, 0, 0) \to$ PDFs $\quad \int dx\, H_\pi \to$ FF
- Full $H_\pi(x, \xi, t)$: **3D tomography** of the pion.
- Encode the **mechanical properties** (pressure, energy density) via the gravitational FF.



Our goal

Access the **pion GPDs** via **Deeply Virtual Compton Scattering (DVCS)**: the most direct hard exclusive process to probe the partonic structure in 3D.

$$e\pi^+ \to e\gamma\pi^+$$

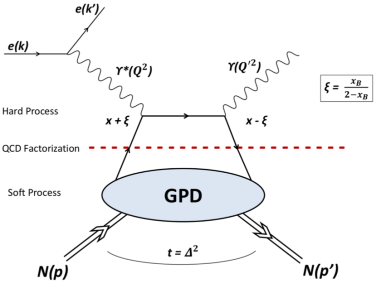
Accessing pion GPDs: DVCS and the Sullivan process

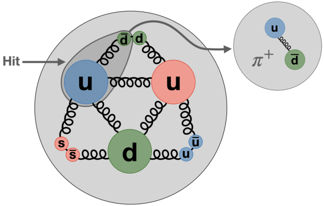
Deeply Virtual Compton Scattering

- Hard exclusive: $eX \to e\gamma X$.
- Large $Q^2 \Rightarrow$ collinear factorisation: $\mathcal{A} = C_{\text{hard}} \otimes H_\pi(x, \xi, t)$.

The Sullivan process

- No free π^+ target $\to$ use $p \to \pi^+ + n$ fluctuation.
- Small $|t|$: virtual π^+ is quasi-real.
- Validated for F_π at JLab [1].

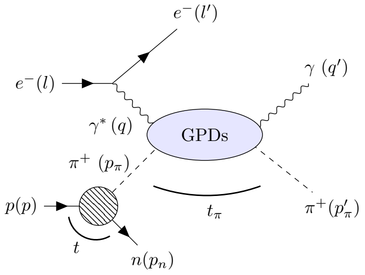




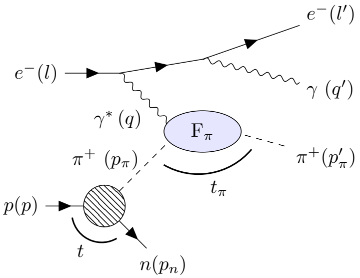
Virtual meson cloud as an effective pion target.

Sullivan DVCS: cross section

Sullivan DVCS



Sullivan Bethe–Heitler



Sullivan cross section: $d\sigma = |T_{\mathrm{BH}}|^2 + |T_{\mathrm{DVCS}}|^2 + \mathcal{I}_{\mathrm{unpol}} + \lambda_e \mathcal{I}_{\mathrm{pol}}$ [2]

The beam spin asymmetry (BSA): $A_{LU}(\phi) = \frac{1}{P_b}\frac{N^+(\phi)-N^-(\phi)}{N^+(\phi)+N^-(\phi)}$

$\Rightarrow$ Direct access to the pion GPDs.

A non-trivial background: baryon resonance transitions

Resonance contribution

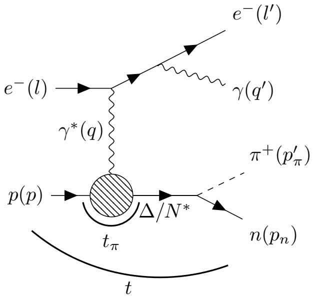
- The final state $e\gamma n\pi^+$ can also arise from:

$$ep \to e\gamma\,\Delta/N^* \to e\gamma\, n\pi^+$$

- Both DVCS and BH topologies contribute.
- **Irreducible** at JLab energies: cannot be removed by kinematics alone.

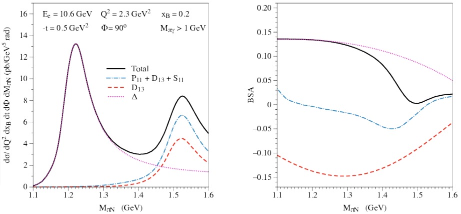
Impact on the analysis

- Contaminates the Sullivan signal.
- Must be **estimated** to extract pion GPDs reliably.



Cross section

BSA



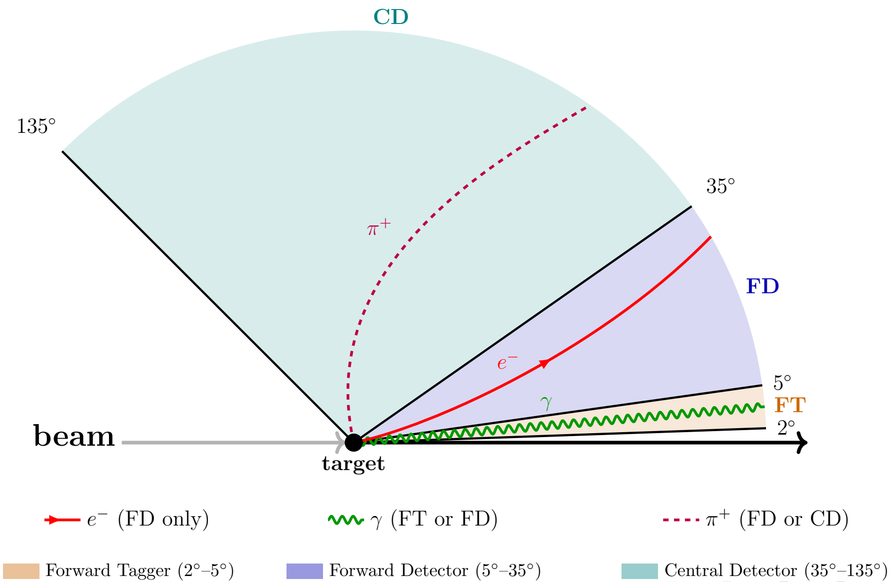
Semenov-Tian-Shansky & Vanderhaeghen [3]

Dataset

RGA Data Sample

- Fall 2018 Outbending run.
- Path: /rg-a/production/recon/fall2018/torus+1/pass2/train/nSidis/.
- Integrated luminosity: $\mathcal{L}_{\text{int}} = 42.589\ \text{fb}^{-1}$ (charge measured with QADB).
- Beam polarization: $P_B \sim 86\%$.
- Main cuts applied:
  - $Q^2 > 1\ \text{GeV}^2$.
  - $W > 2\ \text{GeV}$.

Sullivan DVCS event topology in CLAS12



Updates on this analysis — what's new ?

- **Momentum corrections for π^+ in the CD.**
- **MC normalisation scale factor for π^+.**
- π^0 **background estimation completed.**
- $N^\star(1440)$, $N^\star(1520)$, **and** $N^\star(1535)$ **contributions completed.**
- **New PID refinement cuts.**
- **Systematic uncertainties.**
- **Prediction for the BSA.**

New PID refinement cuts — method

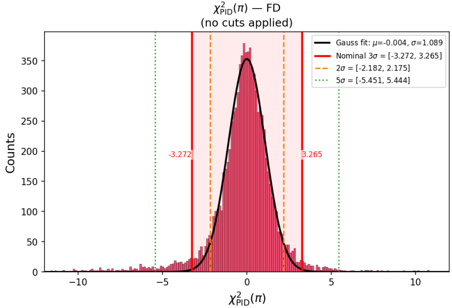
Gaussian fit strategy

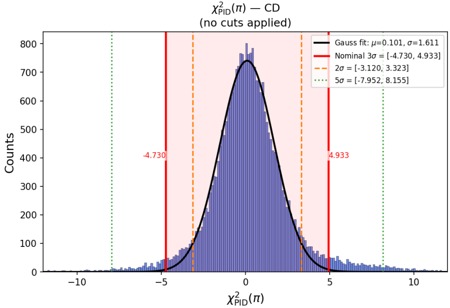
- For each PID variable, fit a Gaussian to the distribution with **all other cuts open** (kinematic cuts only).
- Nominal cut: $\mu \pm 3\sigma$ from the fit.
- FD/CD treated **separately** for $\chi^2_{\mathrm{PID}}(\pi)$ and Δv_z.
- 2σ and 5σ variants also derived from the same fit $\rightarrow$ used for the **systematic uncertainty study**.

Variables fitted

- $\chi^2_{\mathrm{PID}}(e^-)$ — single detector
- $\chi^2_{\mathrm{PID}}(\pi^+)$ — π^+ in FD and CD separately
- $\Delta v_z = v_{z,e} - v_{z,\pi}$ — π^+ in FD and CD separately

New PID refinement cuts — results

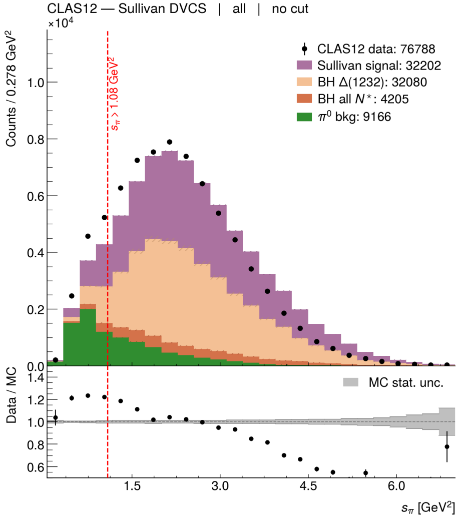




Summary of nominal cuts

Variable	FD	CD
$\chi^2_{\mathrm{PID}}(e^-)$	$[-2.81,\ 2.72]$	
$\chi^2_{\mathrm{PID}}(\pi^+)$	$[-3.27,\ 3.26]$	$[-4.72,\ 4.92]$
Δv_z [cm]	$[-6.31,\ 4.57]$	$[-5.57,\ 4.51]$

Cut on the pion-photon invariant mass s_π

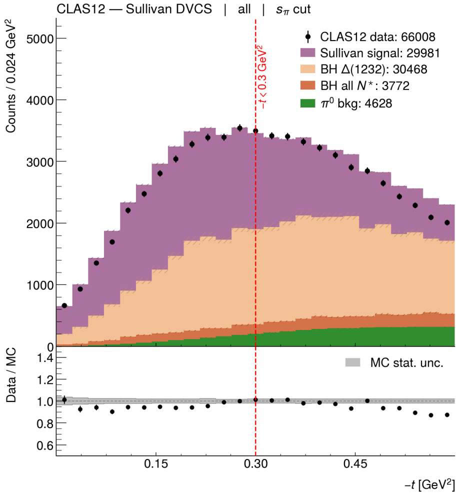


Spectrum of the pion-photon invariant mass s_π.

- **Purple:** MC events from the Sullivan signal.
- **Brown:** MC events from the BH on $\Delta^+(1232)$.
- **Orange:** MC events from the BH on all $N^\star$.
- **Green:** π^0 background.

- $s_\pi = (p_\pi + q')^2$.
- $s_\pi > 1.08\ \mathrm{GeV}^2$.
- Removes ρ^+.

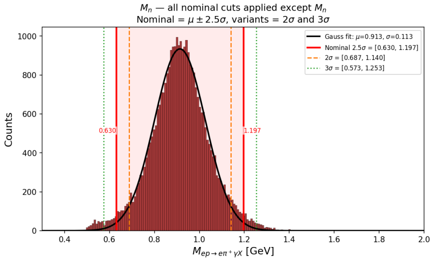
Cut on the pion virtuality $-t$



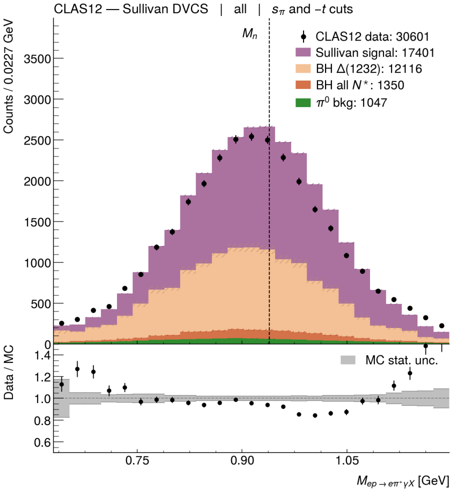
Spectrum of the pion virtuality $-t$ after the cut on s_π.

- $-t = -(p - p_n)^2$.
- $-t < 0.3$ GeV^2.
- Region where:
  - Sullivan process is expected to dominate,
  - interpretation in terms of pion GPDs is valid.

Exclusivity check: neutron missing mass



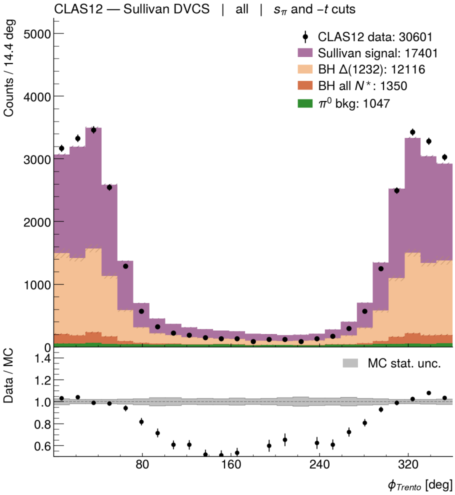
New cut on the missing mass.



- Missing mass computed as: $M_X = \sqrt{(p + l - l' - q' - p'_\pi)^2}$.
- New cut at 2.5σ.

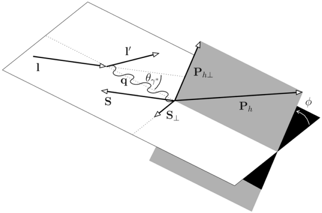
Distribution of the missing mass $M_{ep\to e\gamma\pi^+ X}$.

Azimuthal angle ϕ



Distribution of the azimuthal angle ϕ.

- The ϕ_{Trento} distribution is driven by the BH process.
- BH dominates the cross section.
- Most events cluster around $\phi = 0^\circ$ and $\phi = 360^\circ$, where the BH amplitude is maximal.



Systematic uncertainties — method

Cut variation strategy

- Refit each cut distribution with that cut **open**; build 2σ and 5σ variants (3σ for M_n).
- Re-extract $A_{LU}^{\sin\phi}$ for each variant.
- Variables: $\chi^2_{\mathrm{PID}}(e^-)$, $\chi^2_{\mathrm{PID}}(\pi^+)$, Δv_z, M_n.

$$\sigma_{k,i} = \sqrt{\frac{(\Delta A^{2\sigma}_{k,i})^2 + (\Delta A^{5\sigma}_{k,i})^2}{2}}$$

$$\Delta A^{\mathrm{var}}_{k,i} = \begin{cases} A_i^{\mathrm{var}} - A_i^{\mathrm{nom}} & r^{\mathrm{var}}_{k,i} \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow \quad \sigma_{\mathrm{syst}} = \sqrt{\sum_k \sigma^2_{k,i}}$$

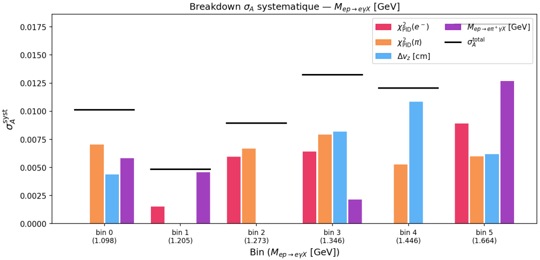
Barlow criterion

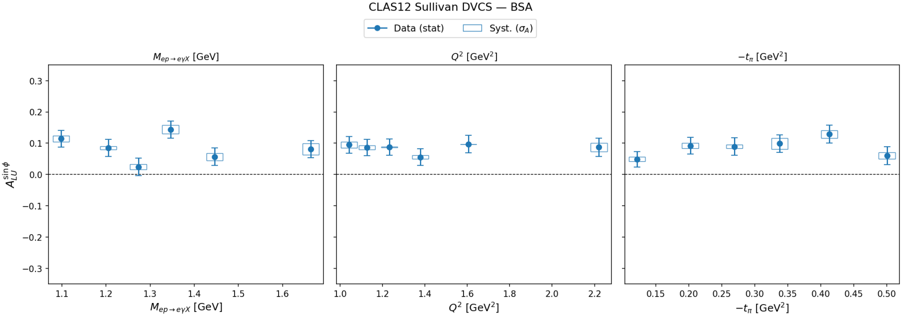
$$r^{\mathrm{var}}_{k,i} = \frac{|A_i^{\mathrm{var}} - A_i^{\mathrm{nom}}|}{\sqrt{|\sigma_i^{\mathrm{var}\,2} - \sigma_i^{\mathrm{nom}\,2}|}} \geq 1$$

$\Rightarrow$ genuine systematic

- Systematic uncertainties are **small compared to statistical uncertainties** across all kinematic variables.
- No single cut dominates the systematic budget across all bins.
- Most variations do not pass the Barlow test $\rightarrow$ conservative estimate.

Systematic uncertainties — results





Predicted BSA — construction of A_{pred}

Yield-weighted prediction

Each contribution to the $e\pi^+ n\gamma$ final state is weighted by its MC yield fraction $y_{c,i}$:

$$A_{LU,i}^{\sin\phi,\,\text{pred}} = y_{\text{Sull},i}\, A_{\text{Sull},i}^{\text{theo}} + y_{\Delta,i}\, A_{\Delta,i}^{\text{theo}} + y_{\pi^0,i}\, A_{\pi^0,i}^{\text{fit}}$$

- $y_{c,i} = W_{c,i} / \sum_{c'} W_{c',i}$ from MC weights.
- Sullivan: pion GPD model of Chavez et al. [4].
- Δ: DVCS$\times$BH interference — large-N_c GPD model [3, 5], computed on a discrete $(Q^2, t, M_{\pi N})$ grid by Sangyeong Son and Kirill M. Semenov-Tian-Shansky.
- π^0: BSA from $\pi 0$ estimation.
- $N^\star$: no calculation available $\rightarrow$ enters only as $A_{N^\star} = \pm 0.1$ uncertainty.

Uncertainty on A_{pred}

$$\sigma^2_{\text{tot}} = \left[\left(A_{\text{Sull}}^{\text{theo}} - A_{\Delta}^{\text{theo}}\right)\right]^2 \left(\sigma^2_{y_s} + \sigma^2_{y_d}\right) + \left[y_{\pi^0}\, \sigma_{A_{\pi^0}}\right]^2 + \left[y_{N^\star} \times 0.1\right]^2$$

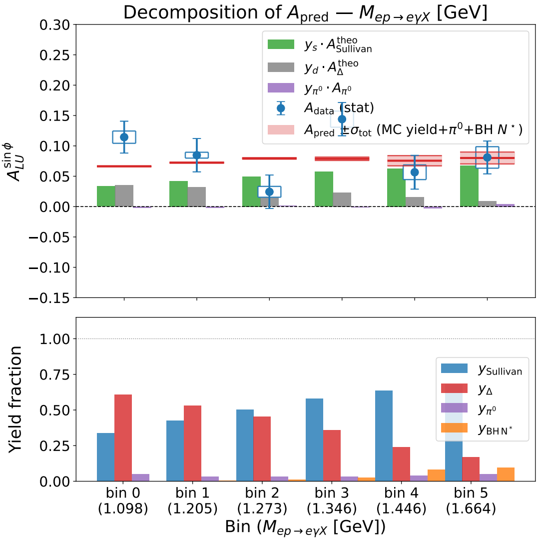
Naive approximation

- BH-only MC yields used for Δ and $N^\star$ (DVCS yield negligible).
- Cross-interference terms (e.g. Sullivan DVCS $\times$ BH-Δ) are **neglected**.

$\Delta(1232)$ grid limitations

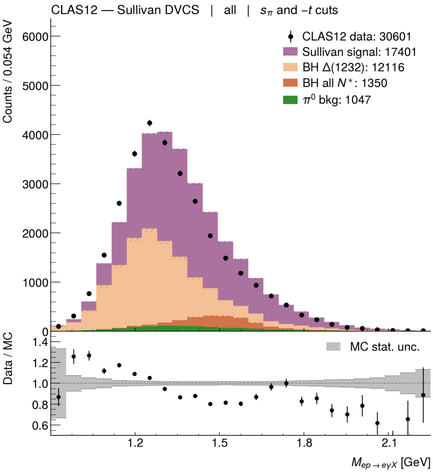
- Predictions provided without the $s_\pi > 1.08$ GeV^2 cut.
- Too few grid points $\rightarrow$ interpolation not smooth.
- Prediction **indicative only**.

Predicted BSA — decomposition by contribution

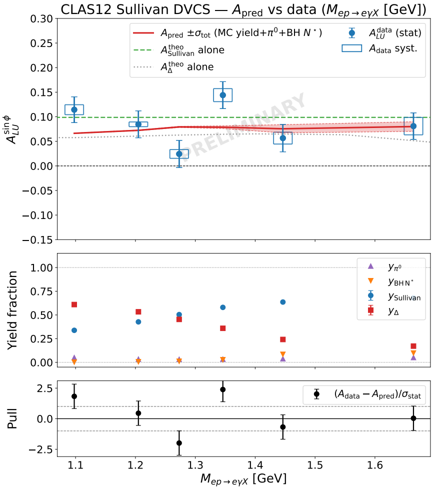


- Sullivan yield y_{Sullivan} increases with $M_{ep\to e\gamma X}$, as the Δ region is left behind — the prediction becomes increasingly dominated by the Sullivan contribution.
- y_{π^0} and $y_{N^\star}$ remain $\lesssim 0.1$ across all bins $\to$ sub-dominant corrections.
- The predicted band (red) is narrow: the dominant uncertainty comes from the $N^\star$ ± 0.1 assumption.

Baryon resonance background

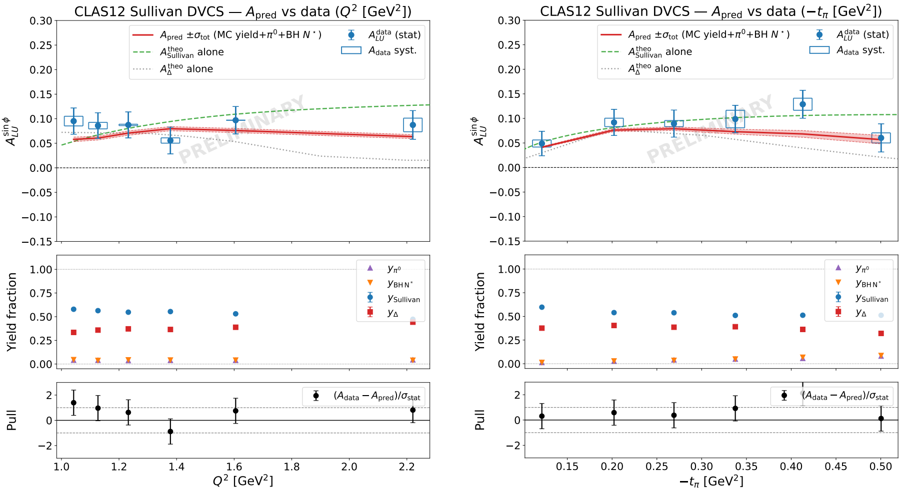


Invariant mass of the pion-neutron system.



Amplitude $A_{LU}^{\sin\phi}$ vs. invariant mass M_{π^+n}.

Results for $A_{LU}^{\sin\phi}$



Outlook and conclusion

Conclusion

- The MC simulations (Sullivan DVCS $+ \Delta + \pi^0$ background) reproduce reasonably well the observed data distributions.
- The predicted BSA, although only indicative given the model limitations, is found to be of the same sign and same order of magnitude as the measured BSA.
- However, a clean interpretation of these results in terms of pion GPDs is compromised by the sizeable contamination of the signal from the Δ contribution.

Publication roadmap

- **Analysis note** to be delivered to the CLAS12 collaboration, **before summer 2026**.

Analysis checks and validations

- **Closure test:** validate the full BSA extraction pipeline on simulated data.
- **Fit cross-check:** compare unbinned likelihood fits with standard binned fits for the BSA extraction.
- π^0 **background method:** test and validate on a dedicated control sample.
- **Resonance transition form factors:** evaluate sensitivity to different models.

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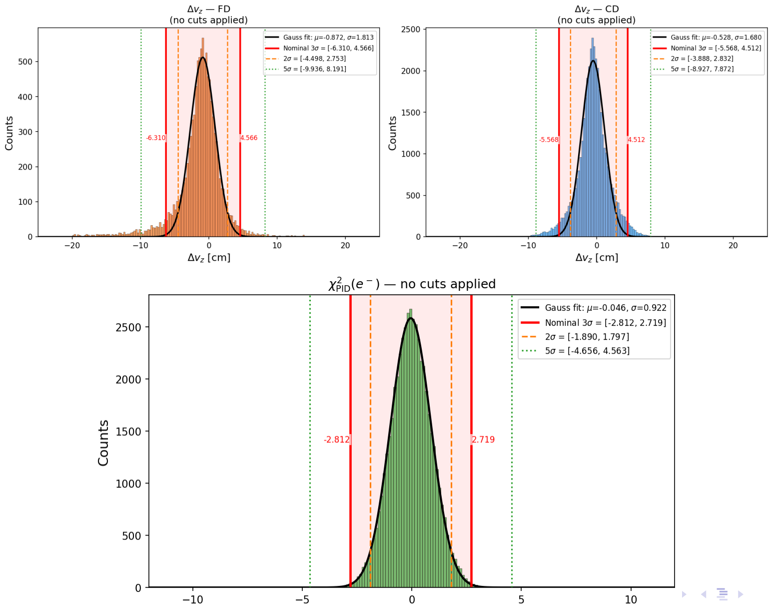
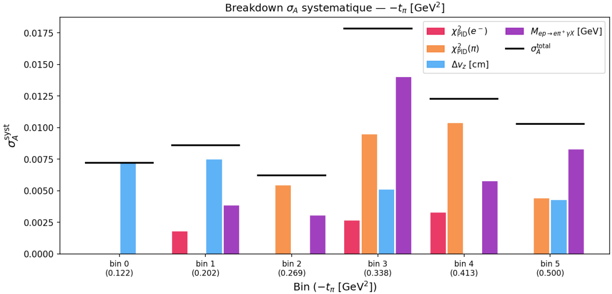
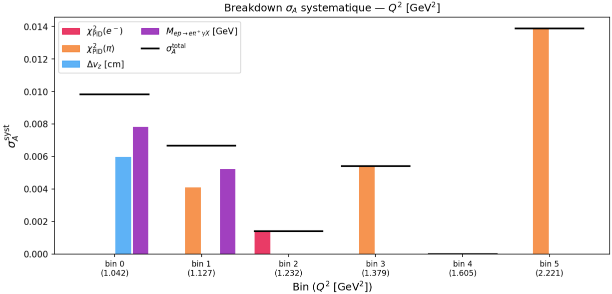
New PID refinement cuts — results bis

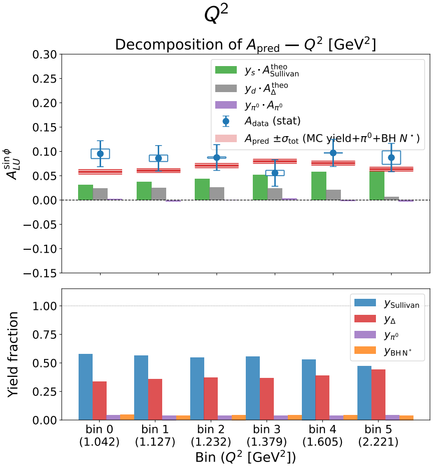


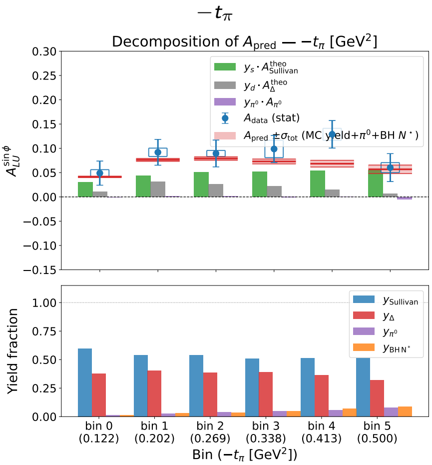
Systematic uncertainties — results bis





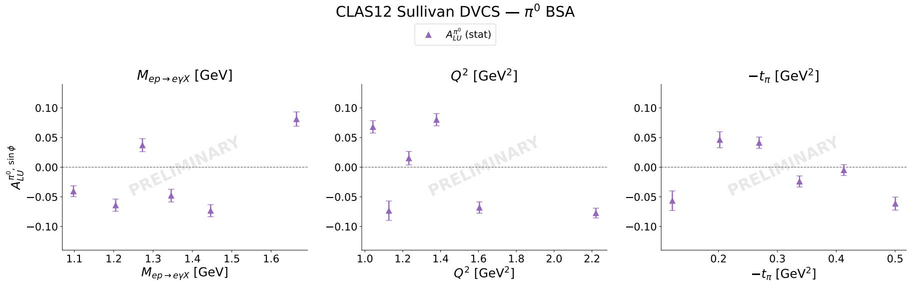
Predicted BSA — decomposition by contribution bis





Predicted BSA — π^0 contribution

The π^0 background BSA is extracted via a weighted maximum likelihood fit, identical to the data method. The log-likelihood is: $\mathcal{L}(A) = -2\sum_i w_i \ln(1 + h_i PA \sin\phi_i)$ where w_i are the π^0 contamination weights. The statistical uncertainty uses the sandwich estimator: $\sigma(A) = \sqrt{\sum w_i^2 g_i^2} \,/\, \sum w_i g_i^2$ with $g_i = h_i P \sin\phi_i / f_i$, which reduces to the standard Fisher error for uniform weights.



Event Selection

Electron (FD)

- Exactly one scattered electron.
- PID cuts: $E_{dep}^{PCAL} > 0.06$ GeV, EM Calorimeter sampling fraction [6], $v_z \in [-11, 1]$ cm, $E_e > 2.0$ GeV.

π^+ (FD or CD)

- Exactly one π^+ with PID cuts: χ^2_{PID}, Δv_z.

Photon (FD or FT)

- At least one photon.
- PID cuts: $\beta \in [0.9, 1.1]$, $\Delta\theta_{e\gamma} > 5^\circ$, $E_\gamma > 3.0$ GeV.

Neutron

Identified via missing mass reconstruction.

Fiducial Cuts, Momentum Corrections and MC Smearing

Fiducial Cuts

- Documented in: *"Fiducial cuts and PID refinements for RG-A pass 2 (Fa18 and Sp19)"* [6].

Momentum Corrections

- e^- and π^+ (FD) from Richard Capobianco [7].
- γ (FD) from Sebastian Alvarado [8].
- γ (FT) from Asli Acar [9] and Sebastian Alvarado [8].
- Momentum corrections for π^+ in the CD (see below).

Monte Carlo Smearing

- e^- smearing applied using the formula from V. Klimenko [10].
- γ smearing applied such that the neutron missing mass width in data matches that of the Monte Carlo signal.

Main cuts applied to the data

Virtuality of the photon Q^2

- $Q^2 > 1 \ \mathrm{GeV}^2$ with $Q^2 = -(l - l')^2$ to ensure the hard scattering regime.

Inelasticity W^2

- $W^2 > 4 \ \mathrm{GeV}^2$ with $W^2 = (p + l - l')^2$ to guarantee the deeply inelastic regime of the reaction.

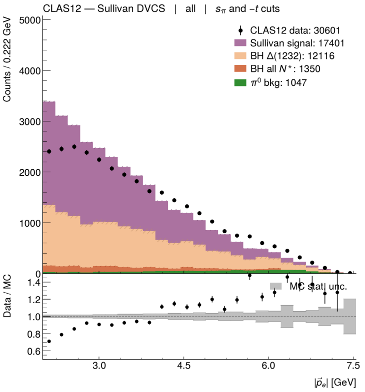
Momentum transfer to the recoil pion t_π

- $-t_\pi < 0.55 \ \mathrm{GeV}^2$ with $t_\pi = (p_\pi - p'_\pi)^2 = (p - p_n - p'_\pi)^2$ to select the dominant (leading-twist) contribution.

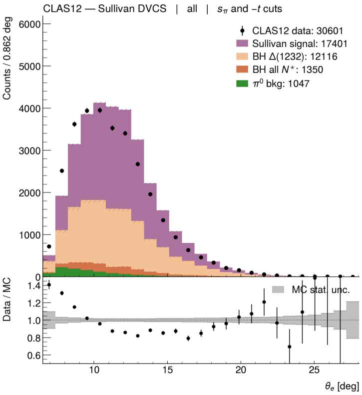
Neutron missing mass $M_{ep\to e\gamma\pi^+X}$

- $M_{ep\to e\gamma\pi^+X} \in [0.9, 2.25] \ \mathrm{GeV}$, the missing mass of the $ep \to e\gamma\pi^+X$ system, to select events consistent with a neutron in the final state.

Electron kinematics

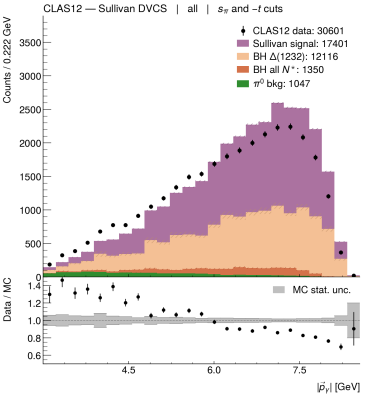


Momentum distribution of the scattered electron.

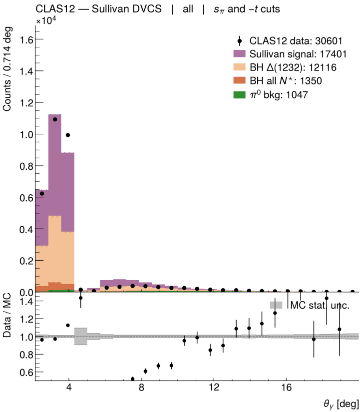


Polar angle distribution of the scattered electron.

Photon kinematics

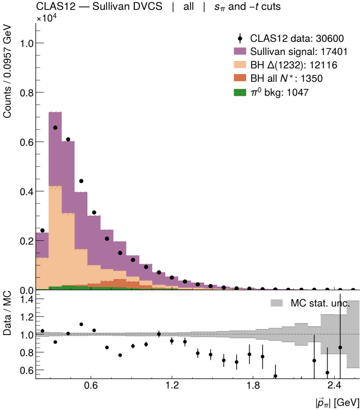


Momentum distribution of the photon.

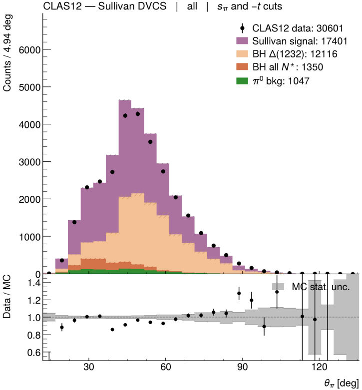


Polar angle distribution of the photon.

Pion kinematics

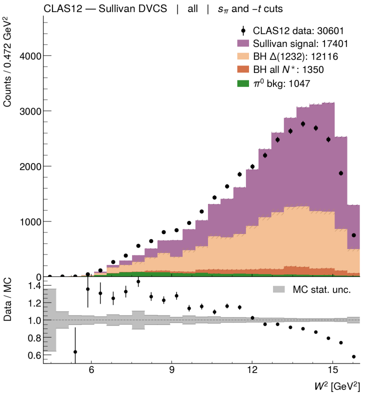


Momentum distribution of the pion.

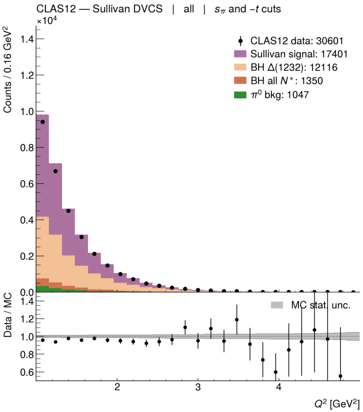


Polar angle distribution of the pion.

DIS kinematic variables

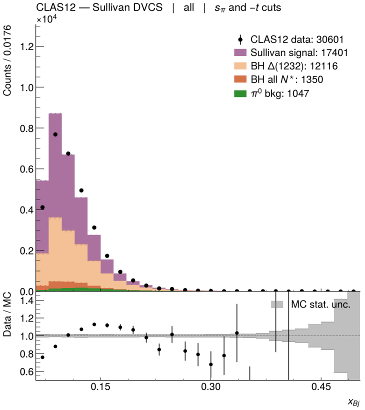


Distribution of W^2.

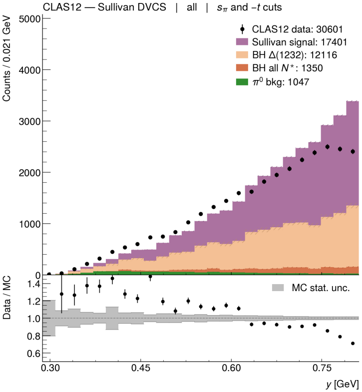


Distribution of Q^2.

DIS kinematic variables

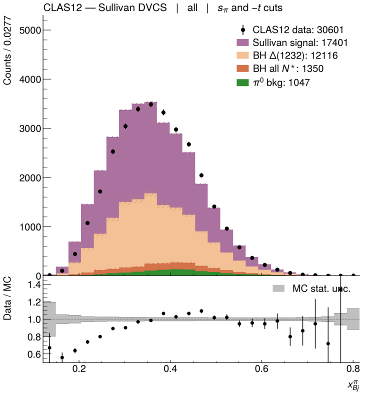


Distribution of x_{Bj}.

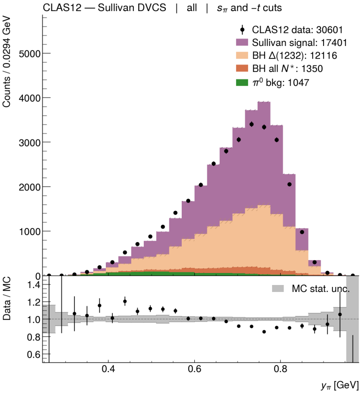


Distribution of y.

DVCS kinematic variables

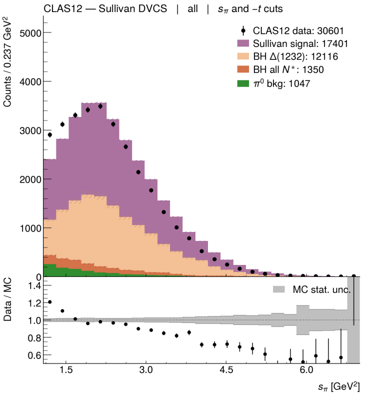


Distribution of x_{Bj}^{π}.

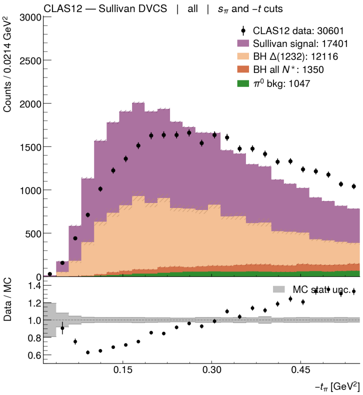


Distribution of y_π.

DVCS kinematic variables

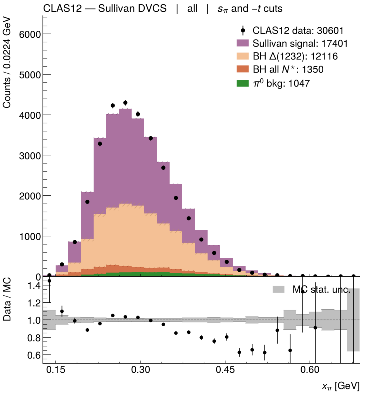


Distribution of s_π.

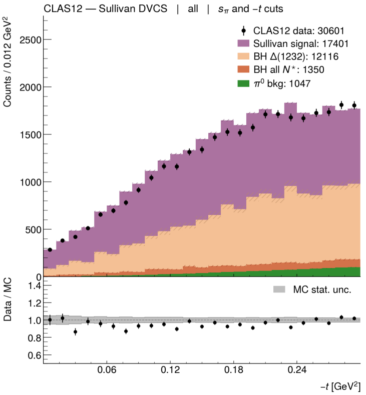


Distribution of $-t_\pi$.

Variables relative to the virtual pion emission



Distribution of x_π.



Distribution of $-t$.

Unbinned extraction of $A_{LU}^{\sin\phi}$ — Method

- Due to limited statistics and a non-uniform event distribution in ϕ, a binned extraction of A_{LU} can introduce statistical fluctuations and bin-width–dependent biases.
- To avoid this, an unbinned maximum likelihood fit is performed.

Event probability density function

$$f(h_i, \phi_i; A) = 1 + h_i \, P_B \, A \sin\phi_i$$

where $h_i = \pm 1$ is the beam helicity and P_B the effective beam polarization.

Log-likelihood function

$$\ln \mathcal{L}(A) = \sum_{i=1}^{N} \ln f(h_i, \phi_i; A)$$

The best-fit amplitude A_{fit} minimizes:

$$\chi^2 = -2 \ln \mathcal{L}(A)$$

Demonstration of the event probability density function

Event probability density function

For each event i with helicity $h_i = \pm 1$, the event probability is:

$$f(\phi_i, h_i; A) = 1 + h_i P_B A \sin \phi_i$$

Definition of asymmetry

The beam-spin asymmetry is defined as:

$$\mathcal{A}_{\mathrm{LU}}(\phi) = \frac{\mathrm{Prob}(h = +1|\phi) - \mathrm{Prob}(h = -1|\phi)}{\mathrm{Prob}(h = +1|\phi) + \mathrm{Prob}(h = -1|\phi)}$$

Conditional probabilities from $f(\phi_i, h_i; A)$

$$\mathrm{Prob}(h = \pm 1|\phi) = \frac{1 \pm P_B A \sin \phi}{2}$$

Demonstration of the event probability density function

Analytic expression for the asymmetry

$$\mathcal{A}_{\mathrm{LU}}(\phi) = \frac{(1 + P_B A \sin\phi) - (1 - P_B A \sin\phi)}{(1 + P_B A \sin\phi) + (1 - P_B A \sin\phi)} = P_B A \sin\phi$$

Normalized by the beam polarization P_B:

$$\mathcal{A}_{\mathrm{LU}}(\phi) = A \sin\phi$$

Uncertainty on $A_{LU}^{\sin\phi}$

The statistical uncertainty is obtained from the curvature of the log-likelihood at its minimum:

$$\sigma_A = \left(\left. \frac{\partial^2(-\ln\mathcal{L})}{\partial A^2} \right|_{A=A_{\mathrm{fit}}} \right)^{-1/2}$$

Method: π^0 Background

π^0 Background Estimation Method

- Select reconstructed π^0 in data that can contaminate the signal with $ep \to e\pi^0\pi^+ n$ final state.
- Each π^0 is decayed randomly $N_{\text{decay}} = 1000$ times and passed through GEMC.

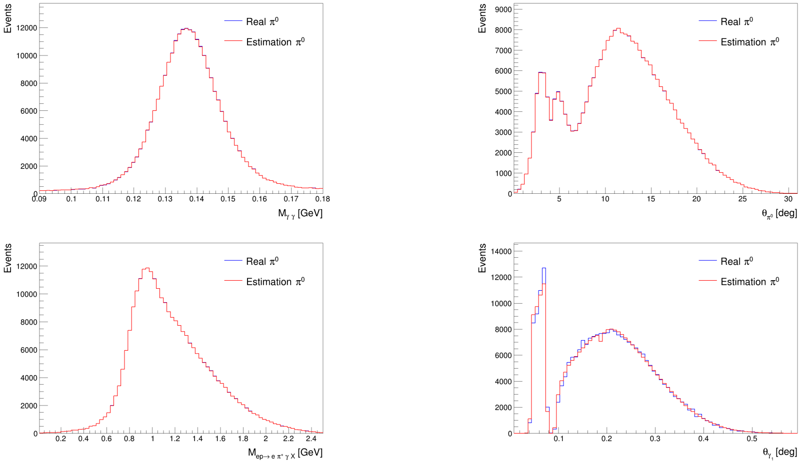
Weighting Procedure

For each data π^0_i:

- $N^i_{\pi^0}$: reconstructed π^0 from this parent.
- Weight assigned to each fake DVCS event from the decay of π^0_i:

$$w_i = \frac{1}{N^i_{\pi^0}}$$

Validation: π^0 Background



A cross-check with a generated π^0 sample is needed.

Method: π^+ CD Momentum Correction

Strategy

Use $ep \to e\,\pi^+\pi^- p$ via $\rho^0 \to \pi^+\pi^-$ (π^+ in CD, π^- and p in FD) and constrain the ρ^0 peak to $m_\rho^{\mathrm{PDG}} = 775.26$ MeV.

Correction parametrisation

Multiplicative scale factor (p in GeV, 3 sectors in ϕ):

$$\alpha(p,\theta) = 1 + a + b \cdot p + \frac{c}{p} + d \cdot p^2$$

Each coefficient is quadratic in $\tilde{\theta} = \dfrac{\theta - 65^\circ}{35^\circ}$:

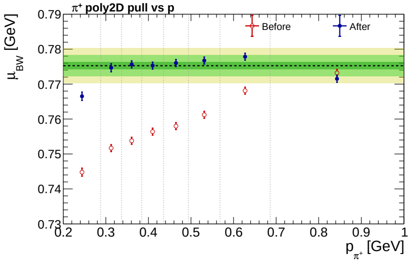
$$k(\tilde{\theta}) = k_0 + k_1 \cdot \tilde{\theta} + k_2 \cdot \tilde{\theta}^2$$

$\Rightarrow$ **12 free parameters per sector**

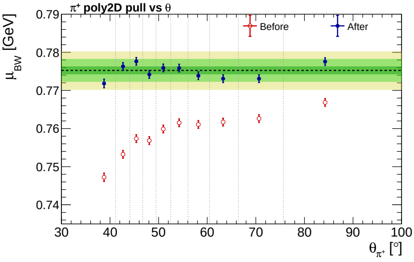
Multi-resolution optimisation

1. Bin data in (p,θ) at 4 resolution levels ($n_\theta \in \{2,3,4,5\}$, $n_p \in \{5\text{–}8\}$).
2. Fit $m_{\pi^+\pi^-}$ with Voigtian + Chebyshev in each bin.
3. Minimise $\chi^2 = \sum_{\mathrm{bins}} \dfrac{(\mu_{\mathrm{fit}} - m_\rho^{\mathrm{PDG}})^2}{\sigma_\mu^2}$ via Nelder–Mead.
4. Fit θ dependence of (a,b,c,d) by weighted least-squares quadratic.

Results: π^+ CD Momentum Correction



Fitted ρ^0 peak vs p_{π^+}



Fitted ρ^0 peak vs θ_{π^+}

Summary

Important reduction of the systematic bias in p and θ achieved with this method. **One remaining limitation:** the π^- momentum corrections used here (Fall18 outbending) were developed on pass-1 reconstruction and are not fully adapted to pass-2.

Method: π^+ Efficiency Corrections

Goal

Derive per-event scale factors (SF) to correct for the data/MC discrepancy in π^+ detection efficiency in both the CD and FD.

Tag-and-probe approach

Use $ep \to ep\,\rho^0 \to ep\,\pi^+\pi^-$ with π^-, p, and e^- in FD.

Four-momentum conservation fully predicts the π^+ kinematics *independently of its detection* $\Rightarrow$ unbiased efficiency measurement:

$$\varepsilon \times \mathrm{Acc} = \frac{N_{\pi^+\,\mathrm{detected}}}{N_{\pi^+\,\mathrm{predicted}}}$$

Applied **identically** to data and MC so that selection systematics cancel in the ratio:

$$\mathrm{SF}(p) = \frac{(\varepsilon \times \mathrm{Acc})_{\mathrm{data}}}{(\varepsilon \times \mathrm{Acc})_{\mathrm{MC}}}$$

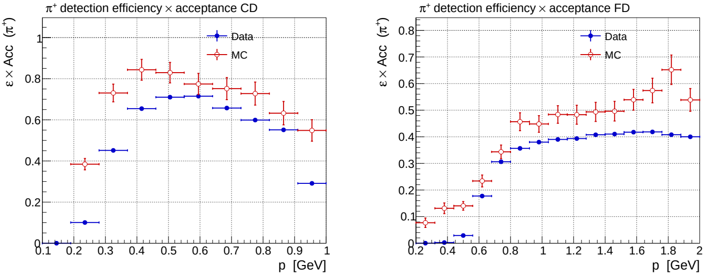
Event selection

1. $M_{\mathrm{miss}}(e,p) \in [0.55,\ 1.00]$ GeV (ρ^0 topology)
2. $\mathrm{MM}^2(e,p,\pi^-) \in [-0.05,\ 0.15]$ GeV^2 (missing π^+)
3. $M(\pi^-\pi^+_{\mathrm{det}}) \in [0.55,\ 1.00]$ GeV (numerator only)

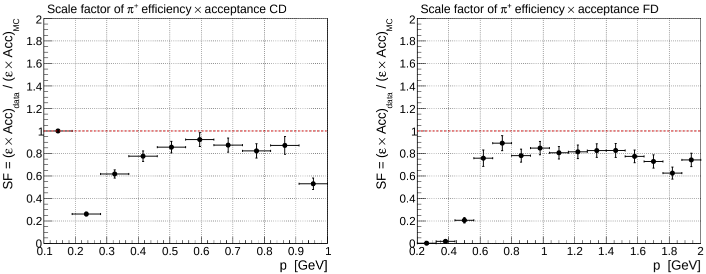
CD/FD classification by predicted polar angle: $\theta_{\mathrm{miss}} \gtrless 39^\circ$.

Efficiency binned in momentum p only: **10 bins** for CD, **15 bins** for FD.

Results: π^+ Efficiency Scale Factors

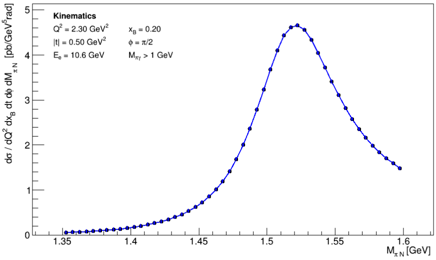


1D efficiency $\varepsilon \times \mathrm{Acc}$ vs p_{π^+} for the CD (left) and FD (right). Filled: data; open: MC.

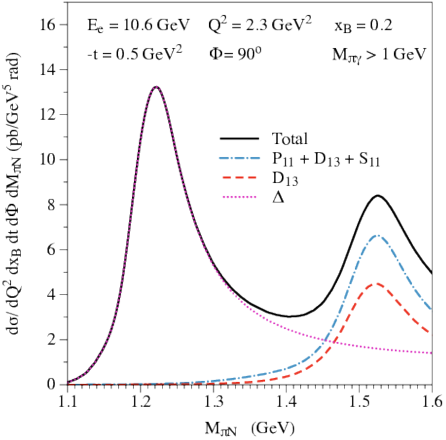


Scale factor $\mathrm{SF}(p)$ for the CD (left) and FD (right). Dashed line: $\mathrm{SF} = 1$.

Validation: $N^\star(1520)$ computation

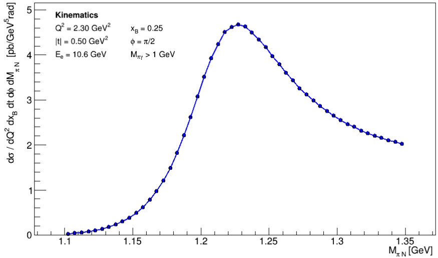


Our computation of $d\sigma/dM_{\pi N}$ for the $N^\star(1520)$ contribution at $-t = 0.5$ GeV^2.

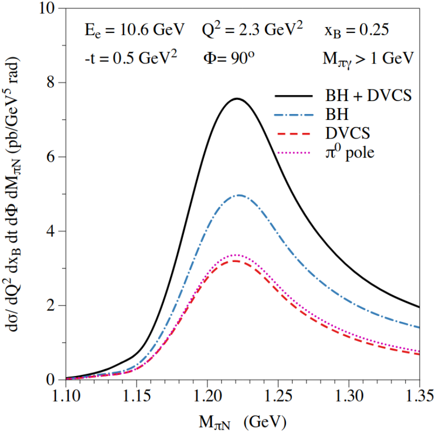


Prediction from Semenov et al. [3]. Our result is to be compared with the red dashed curve.

Validation: $\Delta^\star(1232)$ computation

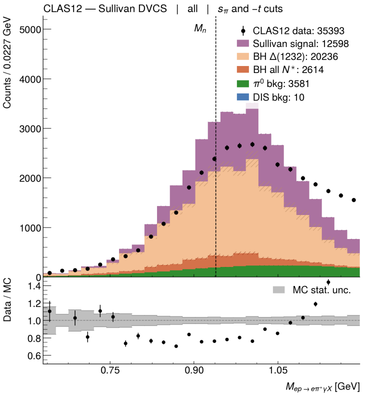


Our computation of $d\sigma/dM_{\pi N}$ for the $\Delta^\star(1232)$ contribution at $-t = 0.5$ GeV^2.

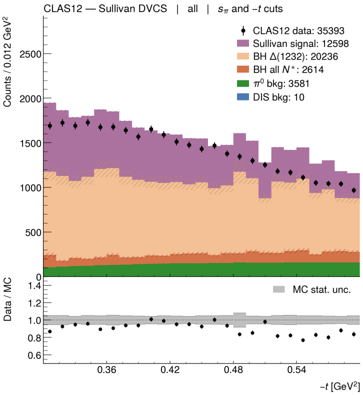


Prediction from Semenov et al. [3]. Our result is to be compared with the blue dashed curve.

MC simulation validity check — high $-t$ region



$M_{ep\to e\gamma\pi^+X}$ for $-t \in [0.3,\ 0.6]\ \mathrm{GeV}^2$.



$-t$ distribution for $-t \in [0.3,\ 0.6]\ \mathrm{GeV}^2$.