Solving Rate Equations for Spin-1 Vector and Tensor Polarization

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UNH NPG

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- **What is Tensor Spin/Polarization?**
  - Vector vs. Tensor polarization
- **Why do I (read: non-tensor-spin nuclear physicist) care? (Brief)**
  - Experimental history
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- **How do we do this?**
  - DNP
  - ssRF
- **What do you (Aden) actually do?**
  - Math
  - More math
  - Even more math

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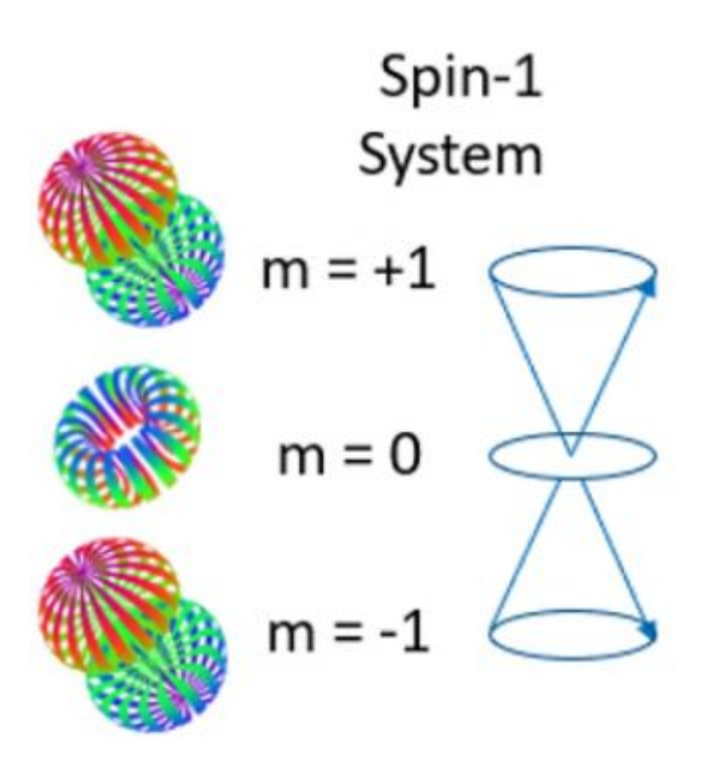
Long Lab



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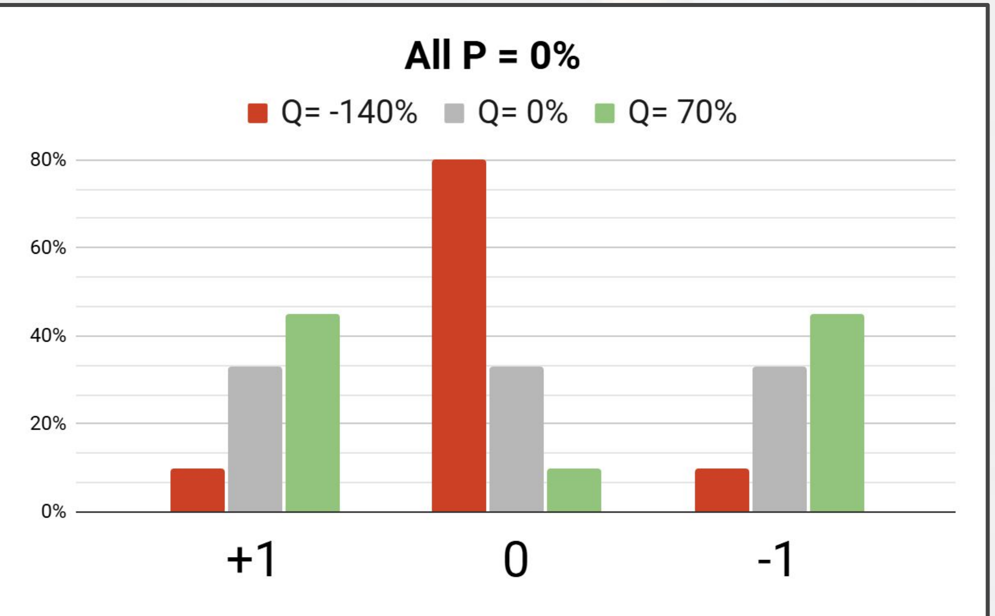
  - Not found in spin-½ particles
  - Spin-0 state can be filled *or* emptied



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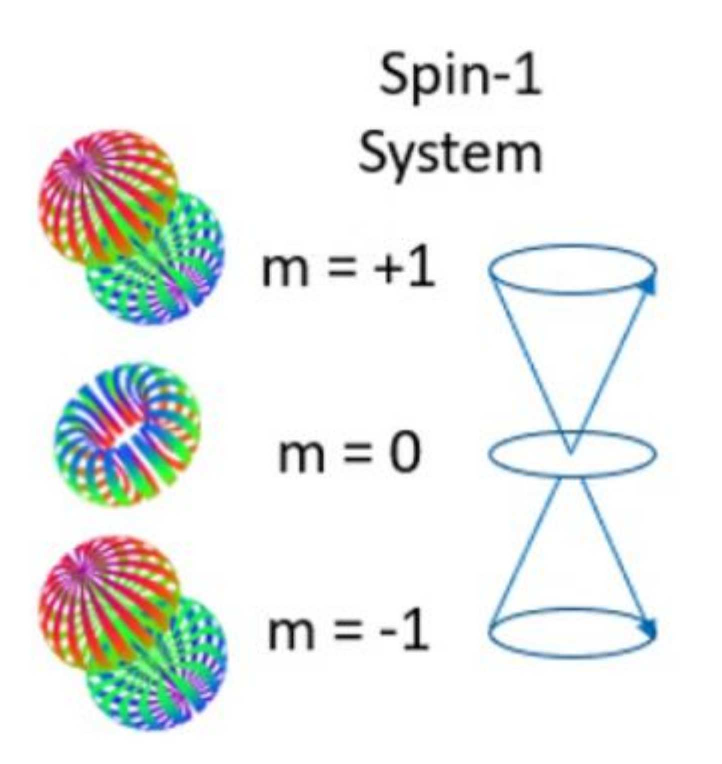

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- Lack of data
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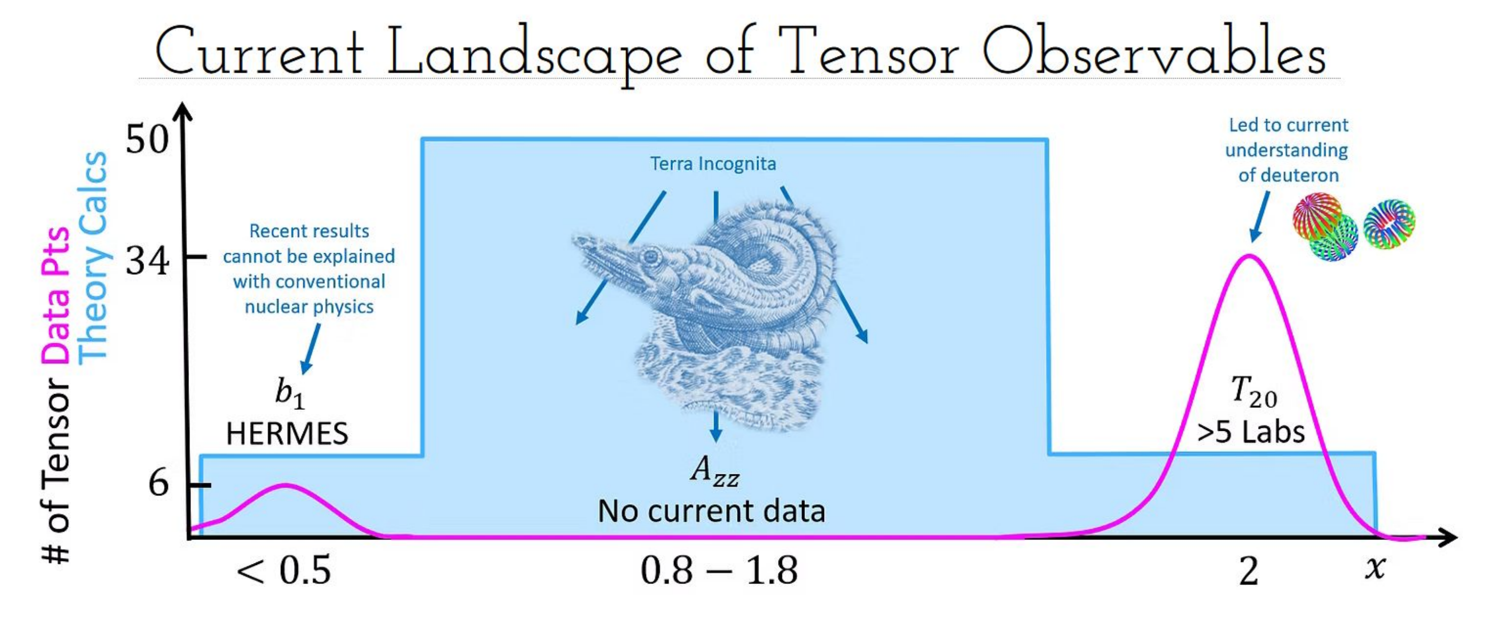
- Lack of data
  - Impactful degrees of tensor polarization very difficult to achieve in solid targets until very recently
  - Modern experiments have not probed certain bjorken-x regions
- Measuring observables
  - T_{20} already investigated
  - A_{zz} and b_1 unmapped

Long Lab



What Even is b_1?

- Special nuclear structure function that only arises under tensor polarization
- Describes what happens to quark structure when particle is tensor-polarized
- Probed with low-x ($x \leq 0.5$) scattering
- Related to another structure function (b_2) through Callan-Gross relation
  - $b_2 = 2 \cdot x \cdot b_1$

$$W_{\mu\nu}^{\lambda_f \lambda_i} = -F_1 \hat{g}_{\mu\nu} + \frac{F_2}{M\nu}\hat{p}_\mu \hat{p}_\nu + \frac{ig_1}{\nu}\epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + \frac{ig_2}{M\nu^2}\epsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma)$$
$$- b_1 r_{\mu\nu} + \frac{1}{6} b_2 (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) + \frac{1}{2} b_3 (s_{\mu\nu} - u_{\mu\nu}) + \frac{1}{2} b_4 (s_{\mu\nu} - t_{\mu\nu}), \qquad (3)$$

where $r_{\mu\nu}$, $s_{\mu\nu}$, $t_{\mu\nu}$, and $u_{\mu\nu}$ are defined by

$$r_{\mu\nu} = \frac{1}{\nu^2}\left[q \cdot E^*(\lambda_f) q \cdot E(\lambda_i) - \frac{1}{3}\nu^2 \kappa \right] \hat{g}_{\mu\nu}, \qquad s_{\mu\nu} = \frac{2}{\nu^2}\left[q \cdot E^*(\lambda_f) q \cdot E(\lambda_i) - \frac{1}{3}\nu^2 \kappa \right] \frac{\hat{p}_\mu \hat{p}_\nu}{M\nu},$$
$$t_{\mu\nu} = \frac{1}{2\nu^2}\left[q \cdot E^*(\lambda_f) \left\{ \hat{p}_\mu \hat{E}_\nu(\lambda_i) + \hat{p}_\nu \hat{E}_\mu(\lambda_i) \right\} + \left\{ \hat{p}_\mu \hat{E}^*_\nu(\lambda_f) + \hat{p}_\nu \hat{E}^*_\mu(\lambda_f) \right\} q \cdot E(\lambda_i) - \frac{4\nu}{3M}\hat{p}_\mu \hat{p}_\nu \right],$$
$$u_{\mu\nu} = \frac{M}{\nu}\left[\hat{E}^*_\mu(\lambda_f) \hat{E}_\nu(\lambda_i) + \hat{E}^*_\nu(\lambda_f) \hat{E}_\mu(\lambda_i) + \frac{2}{3}\hat{g}_{\mu\nu} - \frac{2}{3M^2}\hat{p}_\mu \hat{p}_\nu \right]. \qquad (4)$$

S. Kumano, 2014

Need a Hint?

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S. Kumano, 2014

A Cleaner Simplification

$$\mathbf{W} = \alpha \mathbf{F}_1 + \beta \mathbf{F}_2$$

$$+ \gamma \mathbf{g}_1 + \delta \mathbf{g}_2$$

$$+ \varepsilon \mathbf{b}_1 + \zeta \mathbf{b}_2 + \eta \mathbf{b}_3 + \lambda \mathbf{b}_4$$

□ 1st Order Leading Terms

□ Higher Order Terms, Smaller Contribution

□ Unpolarized Structure

□ **Polarized Structure**

□ **Tensor Polarized Structure**

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E. Long, *et al.*, Proposal, 2013

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- Tells us about the change in cross-section when target is tensor-polarized
  - Suppresses bulk effects from unpolarized cross-section
- Can be measured in different kinematic regions to learn about different structure functions

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$$A_{zz} = \frac{2}{P_{zz}}\left(\frac{\sigma_p - \sigma_u}{\sigma_u}\right)$$

$$b_1^d = -\frac{3}{2}A_{zz}\left(\frac{F_1^d}{A_D}\right) = -\frac{3}{2}A_{zz}\left(\frac{F_1^d}{2}\right)$$

Why Should We Measure These Things?

- A_{zz} tells us how the cross-section (and thus structure) of the nucleus changes when tensor-polarized
  - Necessary to extract other useful quantities, like…

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- A_{zz} tells us how the cross-section (and thus structure) of the nucleus changes when tensor-polarized
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- b_1 tells us about how partons behave in the nucleus under tensor-polarized conditions
  - Can teach us about six-quark states, hidden color effects, short-range correlations, and a lot of other buzzword-y things

Tensor-Polarized Deuteron Recipe

1. Pour ~8 grams of irradiated D-Butanol or Deuterated Ammonia into a target cup. Make sure it's nice and cold (<77K)!

Karl Slifer, Presentation, Tensor Collaboration Meeting

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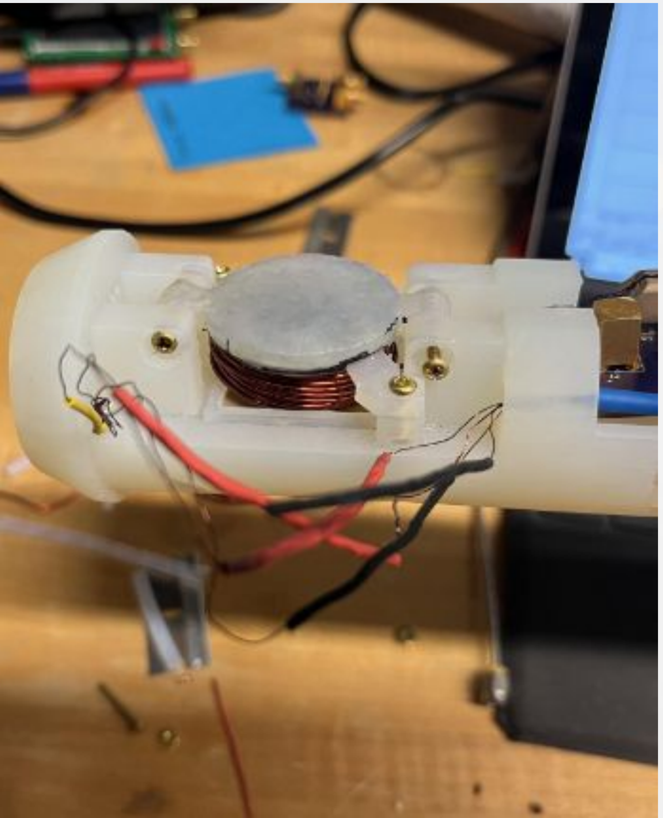
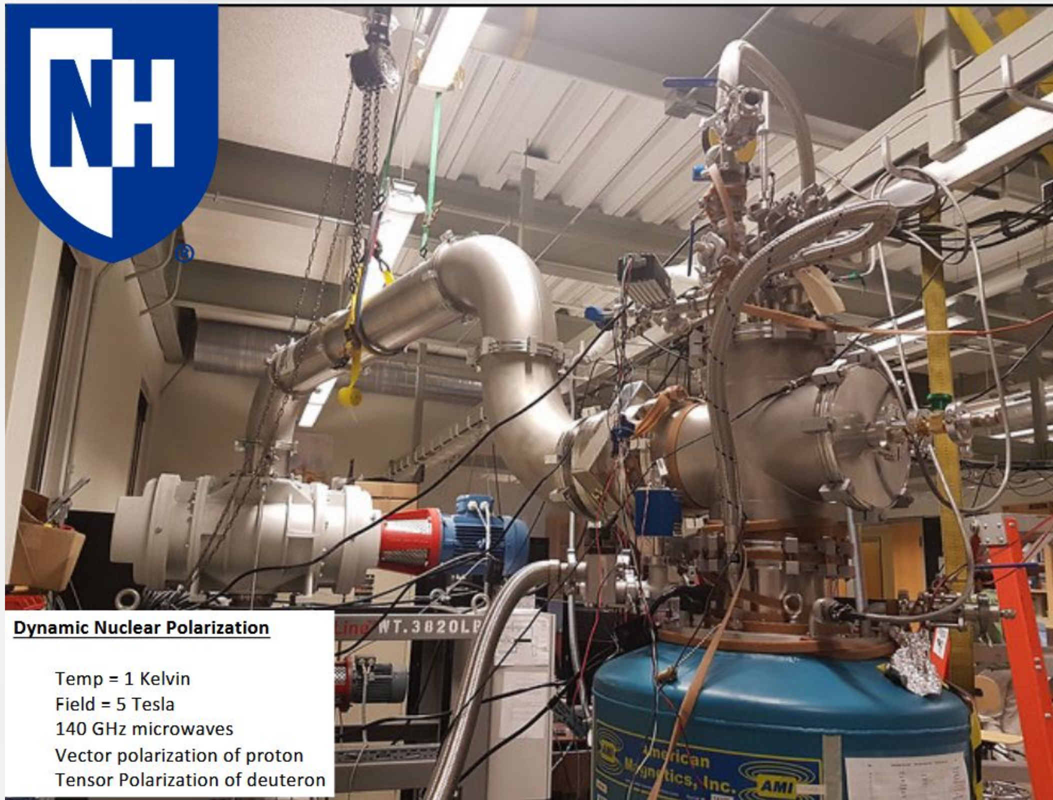


Anchit Arora, Presentation, Tensor Collaboration Meeting

Karl Slifer, Presentation, Tensor Collaboration Meeting



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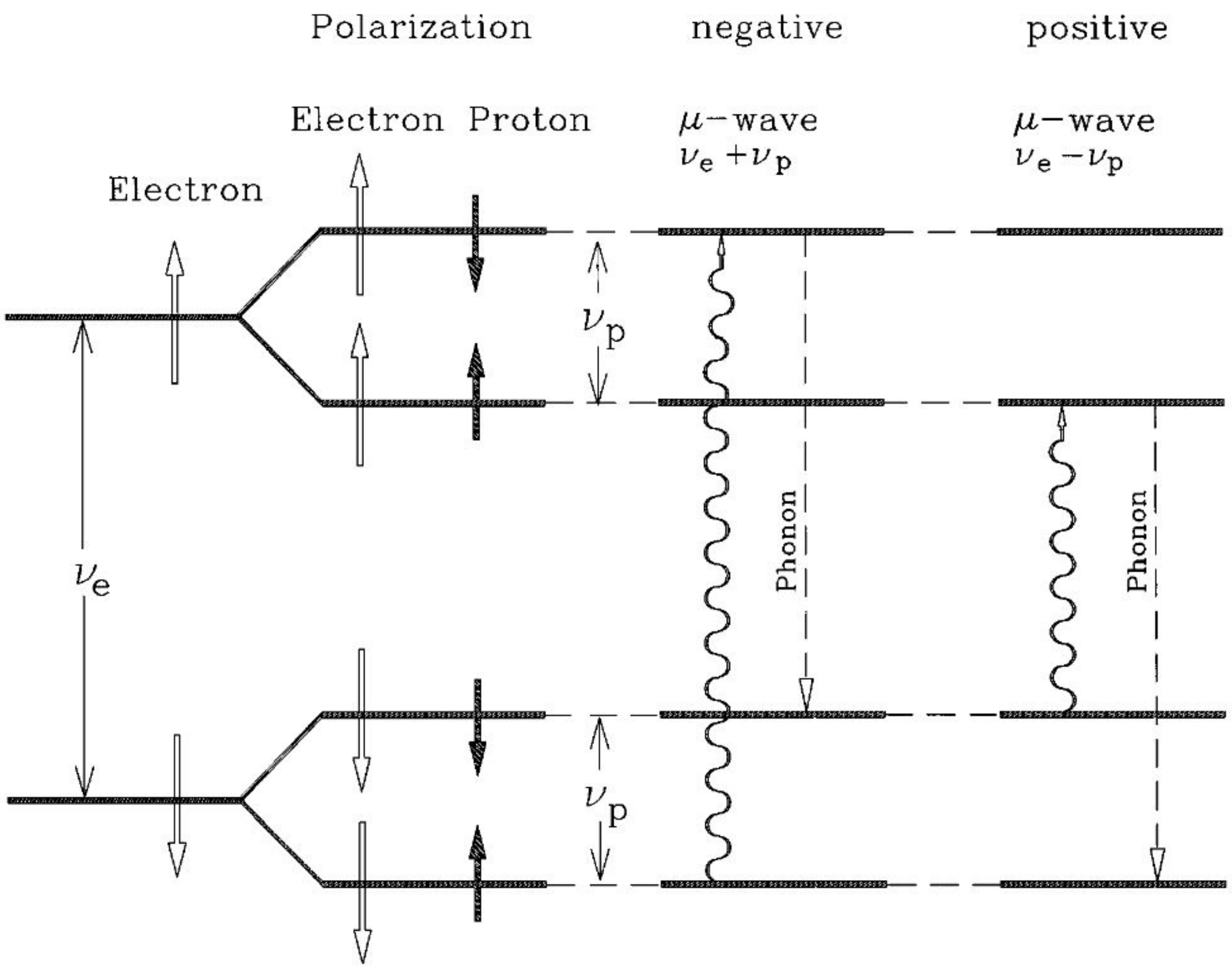
6. Run a McClellan fit on your NMR signal to calculate tensor polarization, and enjoy! (Tensor-polarized deuteron best served cold.)

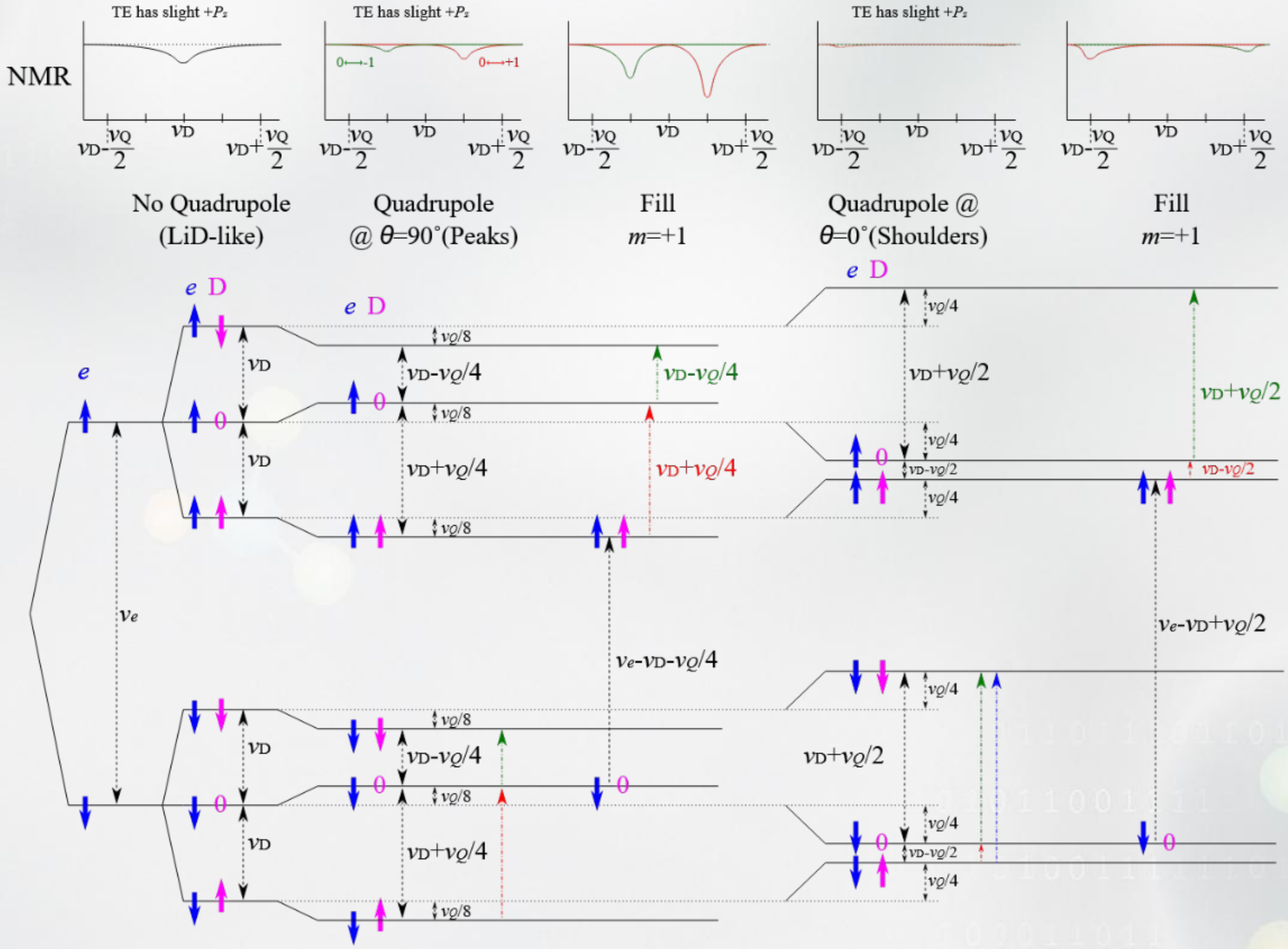
Dynamic Nuclear Polarization

- ❖ Pictured: DNP in electron-proton system (energy splits not to scale)
- ❖ Procedure (for deuteron):
  - ➢ Electron spin-couples to nucleus
  - ➢ Microwave matching the Larmor frequency of electron ± deuteron
  - ➢ Electron flips spin, takes deuteron with it
  - ➢ Electron uncouples, relaxes quickly
  - ➢ Deuteron relaxes slowly, stays in polarized state
- ❖ Creates a 6-state system



Crabb & Meyer, 1997

DNP in a Deuteron



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- In order to run accurate simulations, we need math to support the code!
  - Enter the Spin-1 Polarization rate change equations

- Luckily for us, Fedders & Souers (1987) already solved the equations for us!

Fedders & Souers, 1987

$$\frac{dP_e}{dt} = -\beta\omega_1[P_e(\tfrac{4}{3} - \tfrac{1}{3}Q_n) + P_n] - 2(1+\sigma)\omega_1(P_e - P_0) ,$$

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$$-c(\sigma+\theta)\omega_1(P_e - P_0)P_n , \qquad (13)$$

$$\frac{dQ_n}{dt} = -\tfrac{3}{2}c\beta\omega_1(Q_n + P_e P_n) - \phi_2\omega_1 Q_n$$

$$-3c(\sigma+\theta)\omega_1(P_e - P_0)Q_n .$$

If we neglect the relaxation rates σ and θ, we obtain the rather simple set of equations

$$T_{1e}\frac{dP_e}{dt} = -\tfrac{1}{2}\beta[P_e(\tfrac{4}{3} - \tfrac{1}{3}Q_n) + P_n] - (P_e - P_0) ,$$

$$T_{1n}\frac{dP_n}{dt} = -(\beta/4f_1)[P_n + P_e(\tfrac{4}{3} - \tfrac{1}{3}Q_n)] - P_n ,$$

$$T_{1q}\frac{dQ_n}{dt} = -(3\beta/4f_2)(Q_n + P_e P_n) - Q_n ,$$

$$T_{1n} = 1/\phi_1\omega_1 , \qquad (14)$$

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- Liao JiZhi published another solution in 1999, dropping some assumptions that Fedders & Souers made and fixing some algebraic errors
- So we can just use his solution then!

After a lengthy algebraic calculations we obtain the rate equations for $I = 1$,

$$T'_{1e}\frac{\mathrm{d}P_e}{\mathrm{d}t} = -\left(1+\frac{2\beta}{3f_e}\right)P_e - \frac{\beta}{2f_e}P_n + \frac{1}{6f_e}[\beta+(1+r)\sigma]P_eQ_n - \frac{\sigma}{6f_e}(1+r)P_0Q_n + P_0\,,$$

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$$T'_{1q}\frac{\mathrm{d}Q_n}{\mathrm{d}t} = -\left(1+\frac{3\beta}{4f_2}\right)Q_n - \frac{3\beta}{4f_2}P_eP_n - \frac{3\theta}{4f_2}(1+r)P_0P_n\,, \tag{18}$$

where

$$\frac{1}{T'_{1e}} = \frac{1}{T_{1e}}\left(1+\frac{4}{3}\sigma\right), \qquad \frac{1}{T_{1e}} = (1+r)\omega_1\,, \qquad f_e = \frac{(1+r)T_{1e}}{2T'_{1e}}\,,$$

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It should be pointed out that the equation set (18) is a nonlinear coupling differential equation set, which cannot be analytically solved.

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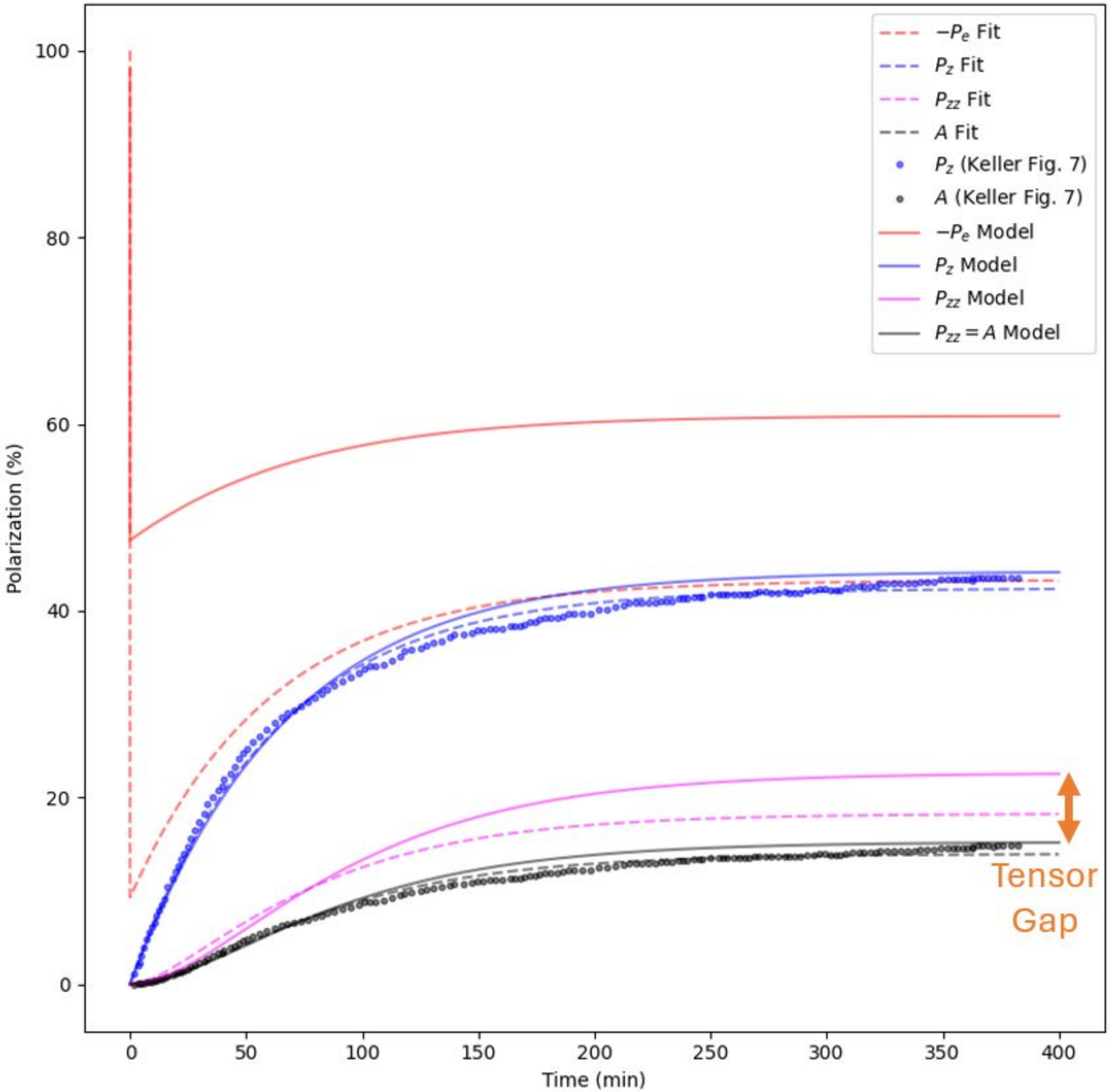
- With the empirical data gathered by the University of New Hampshire's Nuclear and Particle Group (UNH NPG, us), we can see that both JiZhi's and Fedders' & Souers' solutions don't match up with reality
  - Specifically, it fails to describe tensor polarization during a normal DNP spin-up
  - Kind of extremely very important for the specific research we're conducting

- According to Boltzmann statistics, the tensor polarization should settle into the solid black line
- Fit created by UVA's Dustin Keller and recreated by UNH's Elena Long, based on rate equations from JiZhi, Fedders & Souers, and the equations from *Dynamic Nuclear Orientation* (C.D. Jeffries, 1963), yields solid pink line
- Difference is so-called "tensor gap"
- Leaves us only one choice…



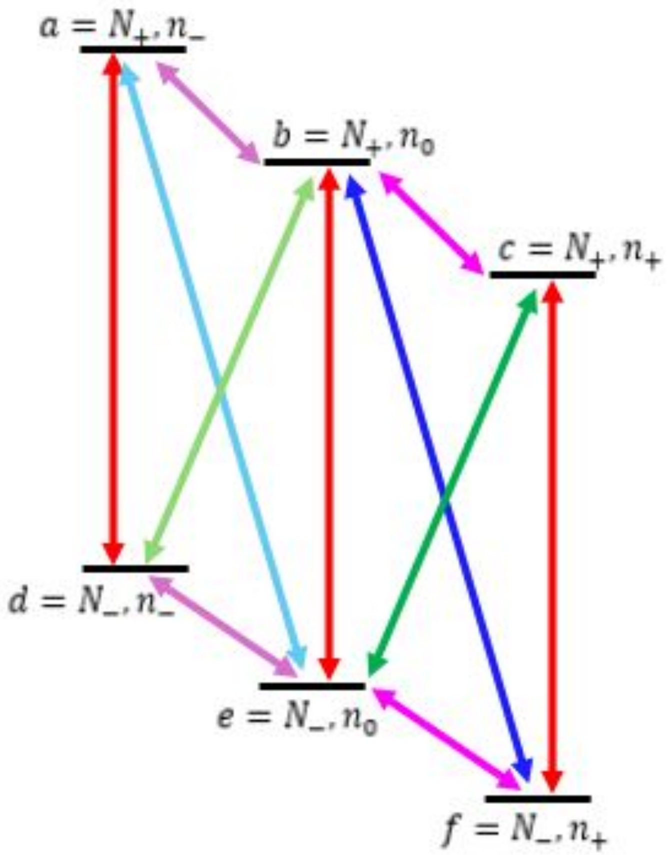
The Shadow Realm, Part 4 — One Choice Left

- Goal: Have three equations, one for each polarization change, with nothing but rate coefficients and polarizations
- Problem: Equations can be easily formulated in terms of state populations (see below), but converting to purely polarizations is difficult
- Solution: Do it anyway

$$P_e = a + b + c - d - e - f$$

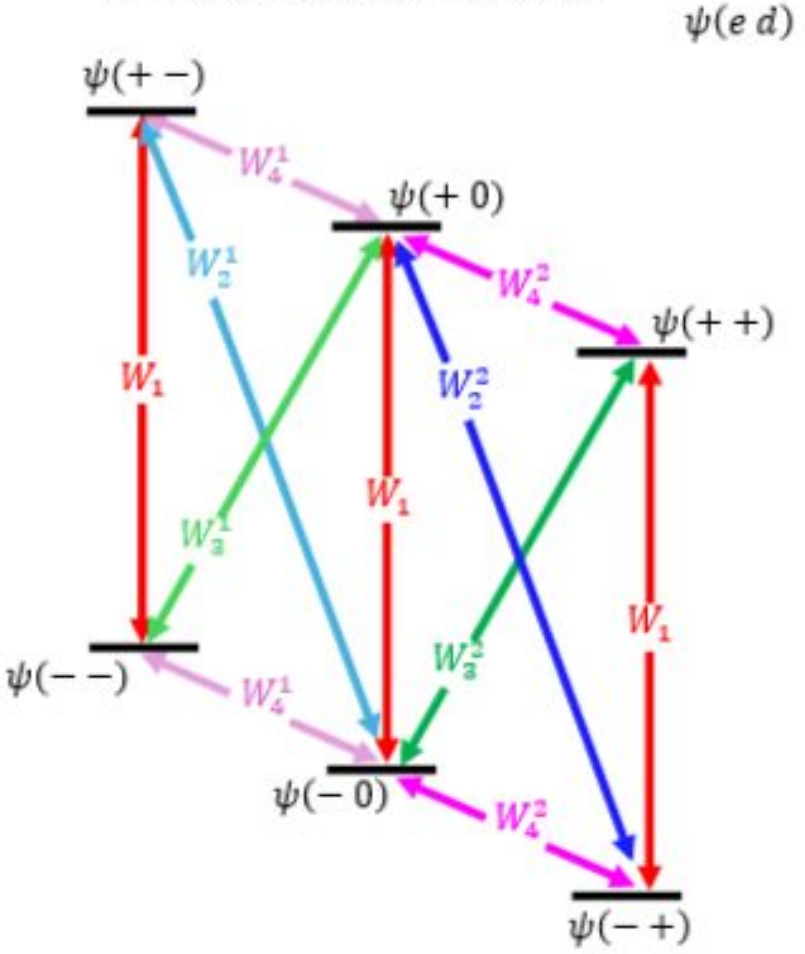
$$P_n = c + f - a - d$$

$$Q_n = c + f + a + d - 2b - 2e$$



PRELIMINARY

RF-induced Transitions



$W_1 = s\omega_1$

$W_2^1 = \alpha_1 \omega_1$

$W_2^2 = \alpha_2 \omega_1$

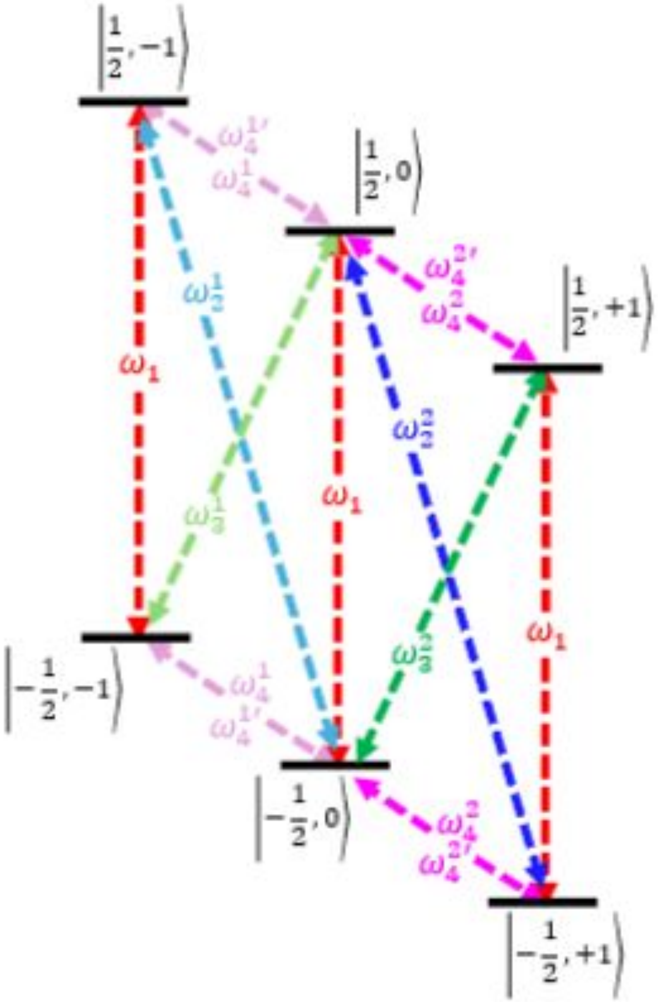
$W_3^1 = \beta_1 \omega_1$

$W_3^2 = \beta_2 \omega_1$

$W_4^1 = \zeta_1 \omega_1$

$W_4^2 = \zeta_2 \omega_1$

Thermal Relaxations



ω_1 = Electron Thermal Relaxation (All times scaled to ω_1)

$\omega_2^1 \approx \sigma_1 \omega_1$

$\omega_2^2 \approx \sigma_2 \omega_1$

$\omega_3^1 \approx \sigma_1 \omega_1$

$\omega_3^2 \approx \sigma_2 \omega_1$

$\omega_4^1 = \theta_1 \omega_1$ = Electron/Nuclear Relaxation

$\omega_4^2 = \theta_2 \omega_1$

$\omega_4^{1\prime} = \phi_1 \omega_1$ = Nuclear Lattice Relaxation

$\omega_4^{2\prime} = \phi_2 \omega_1$

PRELIMINARY

$$
\begin{aligned}
\dot a = &(b-a)\zeta_1\omega_1 - 2a\phi_1\omega_1\left(\frac{b_0}{a_0+b_0+c_0}\right) + 2b\phi_1\omega_1\left(\frac{a_0}{a_0+b_0+c_0}\right)\\
&-a\left[s+2\left(\frac{d_0}{a_0+d_0}\right) + C_n 2\theta_1\left(\frac{b_0}{a_0+b_0+c_0}\right) + \alpha_1 + C_n\sigma_1(1-2(a_0-e_0))\right]\omega_1\\
&+b\left[C_n 2\theta_1\left(\frac{a_0}{a_0+b_0+c_0}\right)\right]\omega_1\\
&+d\left[s+2\left(\frac{a_0}{a_0+d_0}\right)\right]\omega_1\\
&+e[\alpha_1 + C_n\sigma_1(1+2(a_0-e_0))]\omega_1
\end{aligned}
$$

$$
\begin{aligned}
\dot b = &-(b-a)\zeta_1\omega_1 + 2a\phi_1\omega_1\left(\frac{b_0}{a_0+b_0+c_0}\right) - 2b\phi_1\omega_1\left(\frac{a_0}{a_0+b_0+c_0}\right)\\
&+(c-b)\zeta_2\omega_1 - 2b\phi_2\omega_1\left(\frac{c_0}{a_0+b_0+c_0}\right) + 2c\phi_2\omega_1\left(\frac{b_0}{a_0+b_0+c_0}\right)\\
&+a\left[C_n 2\theta_1\left(\frac{b_0}{a_0+b_0+c_0}\right)\right]\omega_1\\
&-b\left[s+2\left(\frac{e_0}{b_0+e_0}\right) + C_n 2\theta_1\left(\frac{a_0}{a_0+b_0+c_0}\right) + C_n 2\theta_2\left(\frac{c_0}{a_0+b_0+c_0}\right) + \beta_1 + C_n\sigma_1(1-2(b_0-d_0)) + \alpha_2 + C_n\sigma_2(1-2(b_0-f_0))\right]\omega_1\\
&+c\left[C_n 2\theta_2\left(\frac{b_0}{a_0+b_0+c_0}\right)\right]\omega_1\\
&+d[\beta_1 + C_n\sigma_1(1+2(b_0-d_0))]\omega_1\\
&+e\left[s+2\left(\frac{b_0}{b_0+e_0}\right)\right]\omega_1\\
&+f[\alpha_2 + C_n\sigma_2(1+2(b_0-f_0))]\omega_1
\end{aligned}
$$

$$
\begin{aligned}
\dot c = &-(c-b)\zeta_2\omega_1 + 2b\phi_2\omega_1\left(\frac{c_0}{a_0+b_0+c_0}\right) - 2c\phi_2\omega_1\left(\frac{b_0}{a_0+b_0+c_0}\right)\\
&+b\left[C_n 2\theta_2\left(\frac{c_0}{a_0+b_0+c_0}\right)\right]\omega_1\\
&-c\left[s+2\left(\frac{f_0}{c_0+f_0}\right) + C_n 2\theta_2\left(\frac{b_0}{a_0+b_0+c_0}\right) + \beta_2 + C_n\sigma_2(1-2(c_0-e_0))\right]\omega_1\\
&+e[\beta_2 + C_n\sigma_2(1+2(c_0-e_0))]\omega_1\\
&+f\left[s+2\left(\frac{c_0}{c_0+f_0}\right)\right]\omega_1
\end{aligned}
$$

$$
\begin{aligned}
\dot d = &(e-d)\zeta_1\omega_1 - 2d\phi_1\omega_1\left(\frac{e_0}{d_0+e_0+f_0}\right) + 2e\phi_1\omega_1\left(\frac{d_0}{d_0+e_0+f_0}\right)\\
&+a\left[s+2\left(\frac{d_0}{a_0+d_0}\right)\right]\omega_1\\
&+b[\beta_1 + C_n\sigma_1(1-2(b_0-d_0))]\omega_1\\
&-d\left[s+2\left(\frac{a_0}{a_0+d_0}\right) + C_n 2\theta_1\left(\frac{e_0}{d_0+e_0+f_0}\right) + \beta_1 + C_n\sigma_1(1+2(b_0-d_0))\right]\omega_1\\
&+e\left[C_n 2\theta_1\left(\frac{d_0}{d_0+e_0+f_0}\right)\right]\omega_1
\end{aligned}
$$

$$
\begin{aligned}
\dot e = &-(e-d)\zeta_1\omega_1 + 2d\phi_1\omega_1\left(\frac{e_0}{d_0+e_0+f_0}\right) - 2e\phi_1\omega_1\left(\frac{d_0}{d_0+e_0+f_0}\right)\\
&+(f-e)\zeta_2\omega_1 - 2e\phi_2\omega_1\left(\frac{f_0}{d_0+e_0+f_0}\right) + 2f\phi_2\omega_1\left(\frac{e_0}{d_0+e_0+f_0}\right)\\
&+a[\alpha_1 + C_n\sigma_1(1-2(a_0-e_0))]\omega_1\\
&+b\left[s+2\left(\frac{e_0}{b_0+e_0}\right)\right]\omega_1\\
&+c[\beta_2 + C_n\sigma_2(1-2(c_0-e_0))]\omega_1\\
&+d\left[C_n 2\theta_1\left(\frac{e_0}{d_0+e_0+f_0}\right)\right]\omega_1\\
&-e\left[s+2\left(\frac{b_0}{b_0+e_0}\right) + C_n 2\theta_1\left(\frac{d_0}{d_0+e_0+f_0}\right) + C_n 2\theta_2\left(\frac{f_0}{d_0+e_0+f_0}\right) + \alpha_1 + C_n\sigma_1(1+2(a_0-e_0)) + \beta_2 + C_n\sigma_2(1+2(c_0-e_0))\right]\omega_1\\
&+f\left[C_n 2\theta_2\left(\frac{e_0}{d_0+e_0+f_0}\right)\right]\omega_1
\end{aligned}
$$

$$
\begin{aligned}
\dot f = &-(f-e)\zeta_2\omega_1 + 2e\phi_2\omega_1\left(\frac{f_0}{d_0+e_0+f_0}\right) - 2f\phi_2\omega_1\left(\frac{e_0}{d_0+e_0+f_0}\right)\\
&+b[\alpha_2 + C_n\sigma_2(1-2(b_0-f_0))]\omega_1\\
&+c\left[s+2\left(\frac{f_0}{c_0+f_0}\right)\right]\omega_1\\
&+e\left[C_n 2\theta_2\left(\frac{f_0}{d_0+e_0+f_0}\right)\right]\omega_1\\
&-f\left[s+2\left(\frac{c_0}{c_0+f_0}\right) + C_n 2\theta_2\left(\frac{e_0}{d_0+e_0+f_0}\right) + \alpha_2 + C_n\sigma_2(1+2(b_0-f_0))\right]\omega_1
\end{aligned}
$$

- Beginning step – all non-canceling letter terms for dP_e/dt

PRELIMINARY

$$
\begin{aligned}
\dot{P}_e(a,b,c,d,e,f) = -2\omega_1 \Bigg(& a\left[s + 2\left(\frac{d_0}{a_0+d_0}\right) + \alpha_1 + C_n\sigma_1(1-2(a_0-e_0))\right] \\
& + b\Bigg[s + 2\left(\frac{e_0}{b_0+e_0}\right) + \beta_1 + C_n\sigma_1(1-2(b_0-d_0)) + \alpha_2 \\
& \qquad + C_n\sigma_2(1-2(b_0-f_0))\Bigg] \\
& + c\left[s + 2\left(\frac{f_0}{c_0+f_0}\right) + \beta_2 + C_n\sigma_2(1-2(c_0-e_0))\right] \\
& - d\left[s + 2\left(\frac{a_0}{a_0+d_0}\right) + \beta_1 + C_n\sigma_1(1+2(b_0-d_0))\right] \\
& - e\Bigg[s + 2\left(\frac{b_0}{b_0+e_0}\right) + \alpha_1 + C_n\sigma_1(1+2(a_0-e_0)) + \beta_2 \\
& \qquad + C_n\sigma_2(1+2(c_0-e_0))\Bigg] \\
& - f\left[s + 2\left(\frac{c_0}{c_0+f_0}\right) + \alpha_2 + C_n\sigma_2(1+2(b_0-f_0))\right] \\
\Bigg) & \qquad (70)
\end{aligned}
$$

- Pictured here: first step with only polarizations that fits on one page

PRELIMINARY

$$
\begin{aligned}
&\dot{P}_e(a,b,c,d,e,f) \qquad (110)\\
&= -2\omega_1 \Bigg(P_e s + P_e - P_{e0} + \alpha_1(\frac{1}{12}[P_e(4 - Q_n - 3P_n) - 3(P_n - Q_n)])\\
&\qquad + \alpha_2(\frac{1}{12}[P_e(4 - Q_n + 3P_n) - 3(P_n + Q_n)])\\
&\qquad + \beta_1(\frac{1}{12}[P_e(4 - Q_n - 3P_n) + 3(P_n - Q_n)])\\
&\qquad + \beta_2(\frac{1}{12}[P_e(4 - Q_n + 3P_n) + 3(P_n + Q_n)])\\
&\qquad + C_n \Bigg[\sigma_1(\frac{1}{12}[P_e(4 - Q_n - 3P_n) - 3(P_n - Q_n)])\\
&\qquad - \sigma_1(\frac{1}{12}[P_e(4 - Q_n - 3P_n) + 3(P_n - Q_n)])\\
&\qquad + \sigma_2(\frac{1}{12}[P_e(4 - Q_n + 3P_n) - 3(P_n + Q_n)])\\
&\qquad - \sigma_2(\frac{1}{12}[P_e(4 - Q_n + 3P_n) + 3(P_n + Q_n)])\\
&\quad + \sigma_1 \Bigg[-P_{e0} \left(\frac{2}{3}(\frac{1}{12}(4 - Q_n - 3P_n)) + \frac{2}{3}(\frac{1}{12}(4 - Q_n - 3P_n)) \right)\\
&+ P_{e0}Q_{n0} \left(\frac{1}{6}(\frac{1}{12}(4 - Q_n - 3P_n)) + \frac{1}{6}(\frac{1}{12}(4 - Q_n - 3P_n)) \right)\\
&+ P_{e0}P_{n0} \left(\frac{1}{2}(\frac{1}{12}(4 - Q_n - 3P_n)) + \frac{1}{2}(\frac{1}{12}(4 - Q_n - 3P_n)) \right)\\
&+ P_{n0} \left(\frac{1}{2}(\frac{1}{12}[(4 - Q_n - 3P_n) - 3P_e(P_n - Q_n)]) - \frac{1}{2}(\frac{1}{12}[(4 - Q_n - 3P_n) + 3P_e(P_n - Q_n)]) \right)\\
&+ Q_{n0} \left(-\frac{1}{2}(\frac{1}{12}[(4 - Q_n - 3P_n) - 3P_e(P_n - Q_n)]) + \frac{1}{2}(\frac{1}{12}[(4 - Q_n - 3P_n) + 3P_e(P_n - Q_n)]) \right) \Bigg]\\
&\quad + \sigma_2 \Bigg[-P_{e0} \left(\frac{2}{3}(\frac{1}{12}(4 - Q_n + 3P_n)) + \frac{2}{3}(\frac{1}{12}(4 - Q_n + 3P_n)) \right)\\
&+ P_{e0}Q_{n0} \left(\frac{1}{6}(\frac{1}{12}(4 - Q_n + 3P_n)) + \frac{1}{6}(\frac{1}{12}(4 - Q_n + 3P_n)) \right)\\
&- P_{e0}P_{n0} \left(\frac{1}{2}(\frac{1}{12}(4 - Q_n + 3P_n)) + \frac{1}{2}(\frac{1}{12}(4 - Q_n + 3P_n)) \right)\\
&+ P_{n0} \left(\frac{1}{2}(\frac{1}{12}[(4 - Q_n + 3P_n) - 3P_e(P_n + Q_n)]) - \frac{1}{2}(\frac{1}{12}[(4 - Q_n + 3P_n) + 3P_e(P_n + Q_n)]) \right)\\
&+ Q_{n0} \left(\frac{1}{2}(\frac{1}{12}[(4 - Q_n + 3P_n) - 3P_e(P_n + Q_n)]) - \frac{1}{2}(\frac{1}{12}[(4 - Q_n + 3P_n) + 3P_e(P_n + Q_n)]) \right) \Bigg] \Bigg] \Bigg)
\end{aligned}
$$

- Preliminary resulting equation
- Comparisons still necessary to verify

PRELIMINARY

$$
\begin{aligned}
\dot{P}_e(a,b,c,d,e,f) = {} & -2\omega_1 P_e s - 2\omega_1 P_e + 2\omega_1 P_{e0} \\
& - \frac{1}{6}\omega_1 \alpha_1 [P_e(4 - Q_n - 3P_n) - 3(P_n - Q_n)] \\
& - \frac{1}{6}\omega_1 \alpha_2 [P_e(4 - Q_n + 3P_n) - 3(P_n + Q_n)] \\
& - \frac{1}{6}\omega_1 \beta_1 [P_e(4 - Q_n - 3P_n) + 3(P_n - Q_n)] \\
& - \frac{1}{6}\omega_1 \beta_2 [P_e(4 - Q_n + 3P_n) + 3(P_n + Q_n)] \\
& - C_n \Big(\sigma_1 \Big[\omega_1 (P_n - Q_n) - \frac{2}{9}\omega_1 P_{e0} (4 - Q_n - 3P_n) \\
& + \frac{1}{18}\omega_1 P_{e0} Q_{n0} (4 - Q_n - 3P_n) + \frac{1}{6}\omega_1 P_{e0} P_{n0} (4 - Q_n - 3P_n) \\
& - \frac{1}{2}\omega_1 P_{n0} (P_e(P_n - Q_n)) + \frac{1}{2}\omega_1 Q_{n0} (P_e(P_n - Q_n)) \Big] \\
& + \sigma_2 \Big[\omega_1 (P_n + Q_n) - \frac{2}{9}\omega_1 P_{e0} (4 - Q_n + 3P_n) \\
& + \frac{1}{18}\omega_1 P_{e0} Q_{n0} (4 - Q_n + 3P_n) - \frac{1}{6}\omega_1 P_{e0} P_{n0} (4 - Q_n + 3P_n) \\
& - \frac{1}{2}\omega_1 P_{n0} (P_e(P_n + Q_n)) - \frac{1}{2}\omega_1 Q_{n0} (P_e(P_n + Q_n)) \Big] \Big)
\end{aligned} \tag{117}
$$

- Beginning step – all non-canceling letter terms for dP_n/dt

PRELIMINARY

$$\begin{aligned}
\dot{P}_n = {} & a\zeta_1\omega_1 + 2a\phi_1\omega_1\left(\frac{b_0}{a_0+b_0+c_0}\right) \\
& + a\left[C_n 2\theta_1\left(\frac{b_0}{a_0+b_0+c_0}\right) + \alpha_1 + C_n\sigma_1(1-2(a_0-e_0))\right]\omega_1 \\
& + b\zeta_2\omega_1 - b\zeta_1\omega_1 + 2b\phi_2\omega_1\left(\frac{c_0}{a_0+b_0+c_0}\right) \\
& - 2b\phi_1\omega_1\left(\frac{a_0}{a_0+b_0+c_0}\right) + b\left[C_n 2\theta_2\left(\frac{c_0}{a_0+b_0+c_0}\right)\right]\omega_1 \\
& + b\left[\alpha_2 + C_n\sigma_2(1-2(b_0-f_0))\right]\omega_1 - b\left[C_n 2\theta_1\left(\frac{a_0}{a_0+b_0+c_0}\right)\right]\omega_1 \\
& - b\left[\beta_1 + C_n\sigma_1(1-2(b_0-d_0))\right]\omega_1 - c\zeta_2\omega_1 - 2c\phi_2\omega_1\left(\frac{b_0}{a_0+b_0+c_0}\right) \\
& - c\left[C_n 2\theta_2\left(\frac{b_0}{a_0+b_0+c_0}\right) + \beta_2 + C_n\sigma_2(1-2(c_0-e_0))\right]\omega_1 \\
& + d\zeta_1\omega_1 + 2d\phi_1\omega_1\left(\frac{e_0}{d_0+e_0+f_0}\right) \\
& + d\left[C_n 2\theta_1\left(\frac{e_0}{d_0+e_0+f_0}\right) + \beta_1 + C_n\sigma_1(1+2(b_0-d_0))\right]\omega_1 \\
& + e\zeta_2\omega_1 - e\zeta_1\omega_1 + 2e\phi_2\omega_1\left(\frac{f_0}{d_0+e_0+f_0}\right) \\
& - 2e\phi_1\omega_1\left(\frac{d_0}{d_0+e_0+f_0}\right) - e\left[\alpha_1 + C_n\sigma_1(1+2(a_0-e_0))\right]\omega_1 \\
& + e\left[\beta_2 + C_n\sigma_2(1+2(c_0-e_0))\right]\omega_1 - e\left[C_n 2\theta_1\left(\frac{d_0}{d_0+e_0+f_0}\right)\right]\omega_1 \\
& + e\left[C_n 2\theta_2\left(\frac{f_0}{d_0+e_0+f_0}\right)\right]\omega_1 - f\zeta_2\omega_1 - 2f\phi_2\omega_1\left(\frac{e_0}{d_0+e_0+f_0}\right) \\
& - f\left[C_n 2\theta_2\left(\frac{e_0}{d_0+e_0+f_0}\right) + \alpha_2 + C_n\sigma_2(1+2(b_0-f_0))\right]\omega_1
\end{aligned} \tag{158}$$

Spin-1 Polarization Rate Change Equations, Part 3 — Nuclear Vector

- Current step – Letters almost eliminated

$$
\begin{aligned}
\dot{P}_n = &\frac{1}{2}(Q_n - P_n)\zeta_1\omega_1 - \frac{1}{2}(Q_n + P_n)\zeta_2\omega_1 \\
&+ \frac{1}{12}[P_e(4 - Q_n - 3P_n) - 3(P_n - Q_n)]\alpha_1\omega_1 \\
&+ \frac{1}{12}[P_e(4 - Q_n + 3P_n) - 3(P_n + Q_n)]\alpha_2\omega_1 \\
&- \frac{1}{12}[P_e(4 - Q_n - 3P_n) + 3(P_n - Q_n)]\beta_1\omega_1 \\
&- \frac{1}{12}[P_e(4 - Q_n + 3P_n) + 3(P_n + Q_n)]\beta_2\omega_1 \\
&+ \frac{1}{3}[(1 - P_n)(1 - Q_{n0}) - (1 - P_{n0})(1 - Q_n)]\phi_1\omega_1 \\
&+ \frac{1}{3}[(1 + P_{n0})(1 - Q_n) - (1 + P_n)(1 - Q_{n0})]\phi_2\omega_1 \\
&+ \frac{1}{3}C_n[(1 - P_n)(1 - Q_{n0}) - (1 - P_{n0})(1 - Q_n)]\theta_1\omega_1 \\
&+ \frac{1}{3}C_n[(1 + P_{n0})(1 - Q_n) - (1 + P_n)(1 - Q_{n0})]\theta_2\omega_1 \\
&+ a\,[C_n\sigma_1(1 - 2(a_0 - e_0))]\,\omega_1 \\
&+ b\,[C_n\sigma_2(1 - 2(b_0 - f_0)) - C_n\sigma_1(1 - 2(b_0 - d_0))]\,\omega_1 \\
&- c\,[C_n\sigma_2(1 - 2(c_0 - e_0))]\,\omega_1 + d\,[C_n\sigma_1(1 + 2(b_0 - d_0))]\,\omega_1 \\
&+ e\,[-C_n\sigma_1(1 + 2(a_0 - e_0)) + C_n\sigma_2(1 + 2(c_0 - e_0))]\,\omega_1 \\
&- f\,[C_n\sigma_2(1 + 2(b_0 - f_0))]\,\omega_1
\end{aligned}
$$

PRELIMINARY

Spin-1 Polarization Rate Change Equations, Part 4 — Next Up

- Will begin solving nuclear tensor equation after finishing nuclear vector equation
- Comparisons can be made with JiZhi's or Fedders' & Souers' spin-½ or spin-1 solutions
- After I finish solving, will compare results with Elena Long for a double-blind verification

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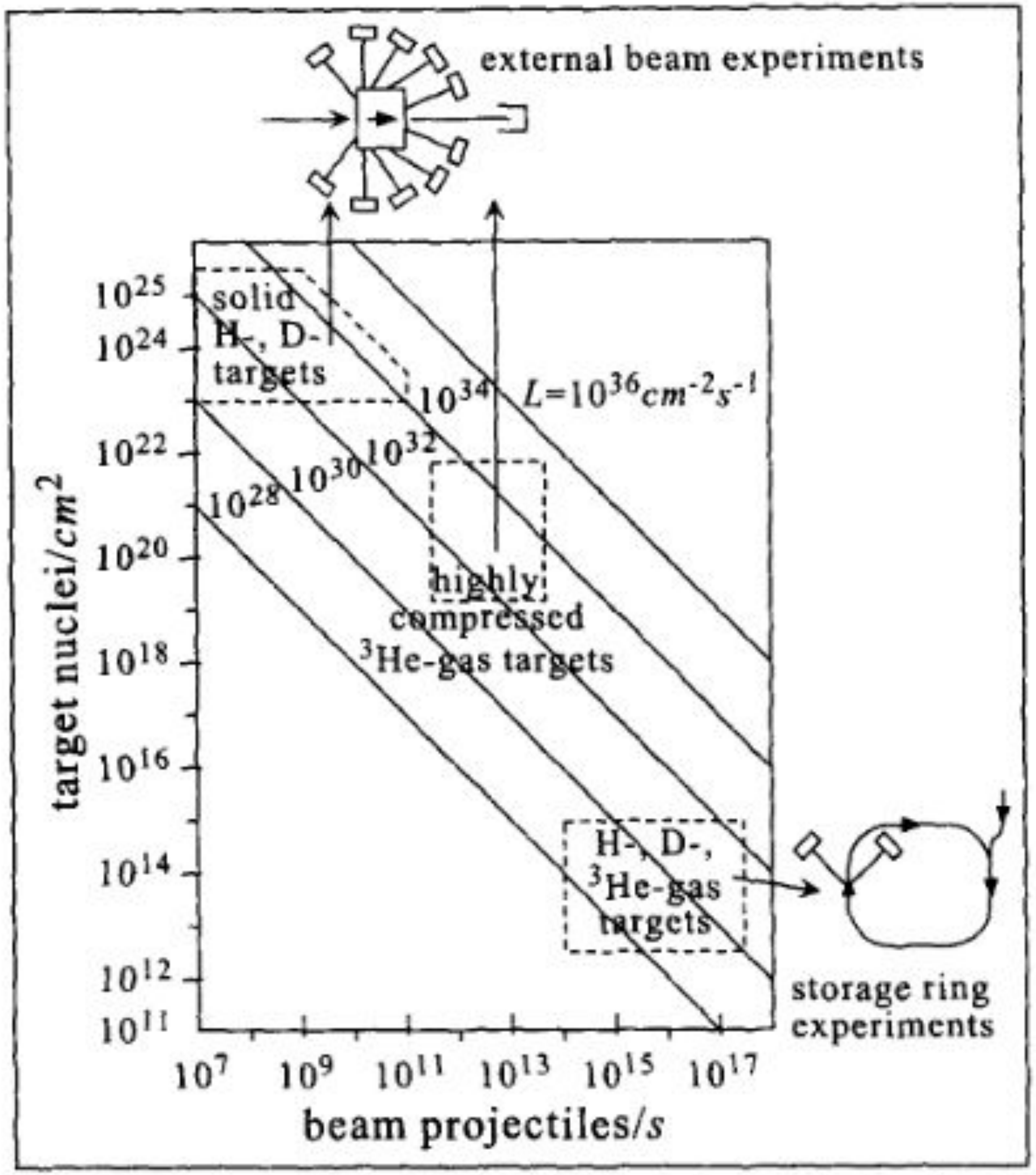
- St. Goertz, W. Meyer, and G. Reicherz, "Polarized H, D and ^{3}He targets for particle physics experiments", *Progress in Nuclear and Particle Physics*, 2002 https://www.sciencedirect.com/science/article/abs/pii/S014664100200159X?via%3Dihub
- Private communications with Prof. Elena Long

Extra Slide: Why Solid Target? (For Cavemen)

- Luminosity good
- Solid target = more luminosity
- More luminosity = run less time; save money for Jlab
- Beam target = much big tensor polarization
  - But small luminosity
  - Need run for very very long time
  - United States Department of Energy no spend money on this



Goertz, Meyer, and Reicherz, 2002