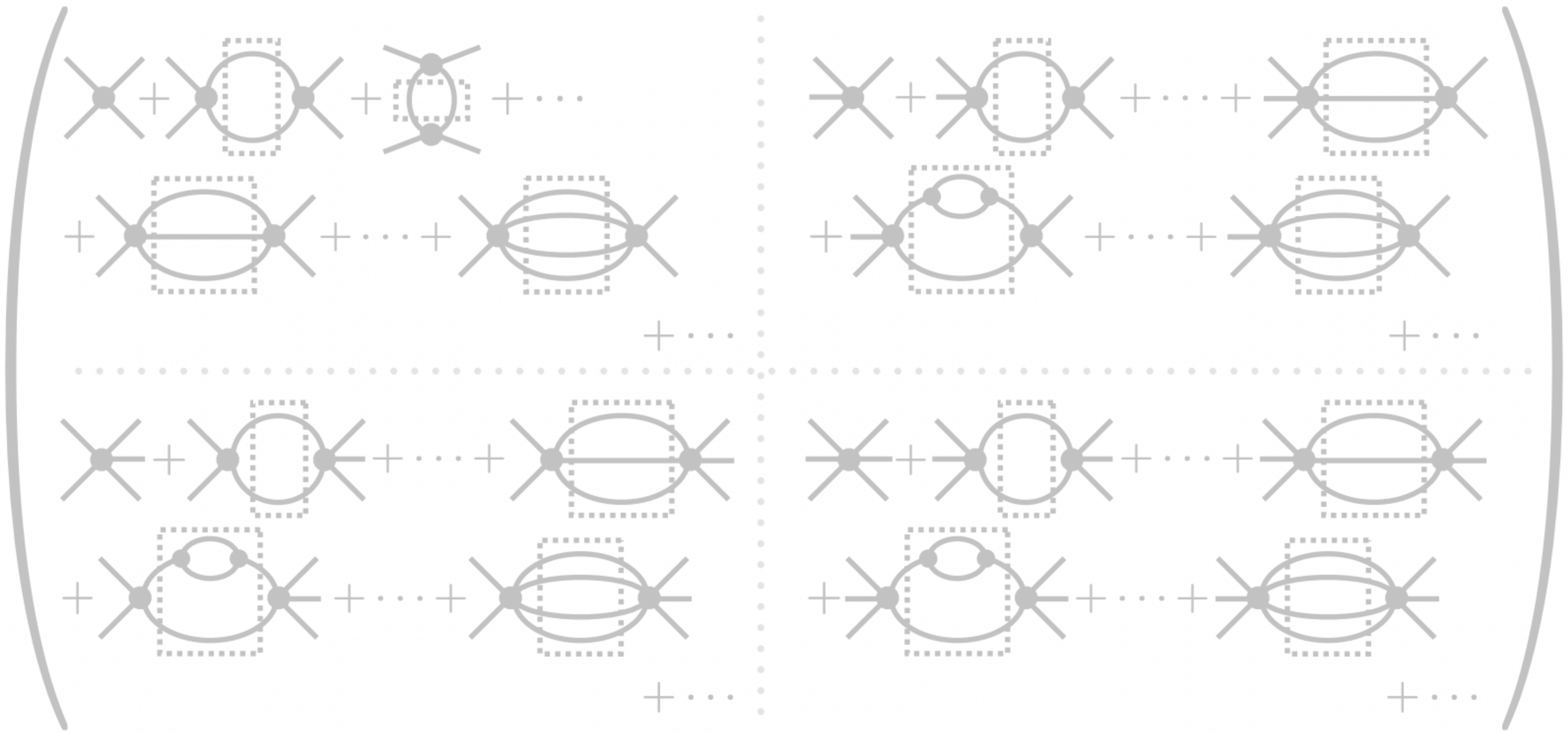
Coupled two- and three-body scattering amplitudes on an infinite volume

Rana Urek

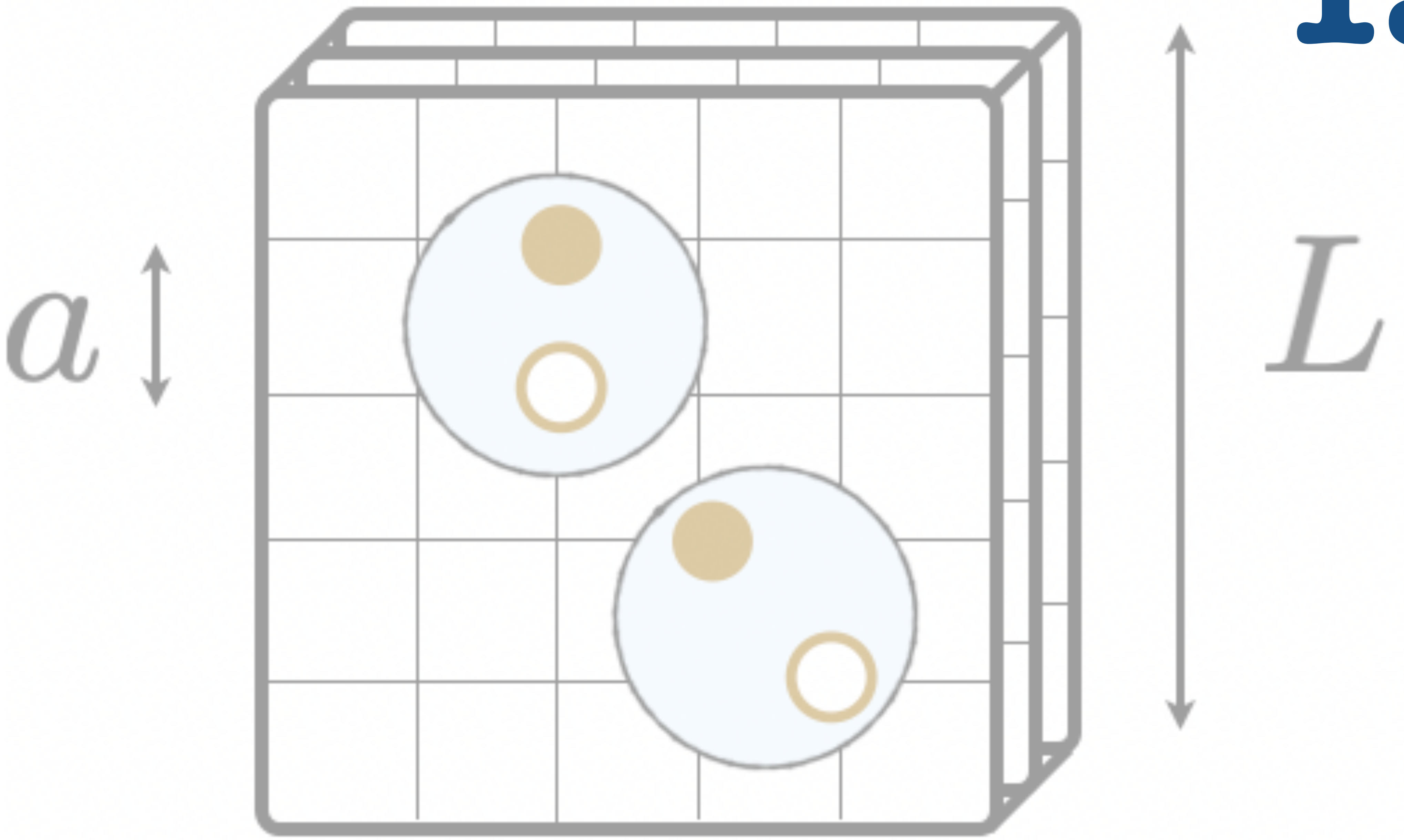


Berkeley
UNIVERSITY OF CALIFORNIA

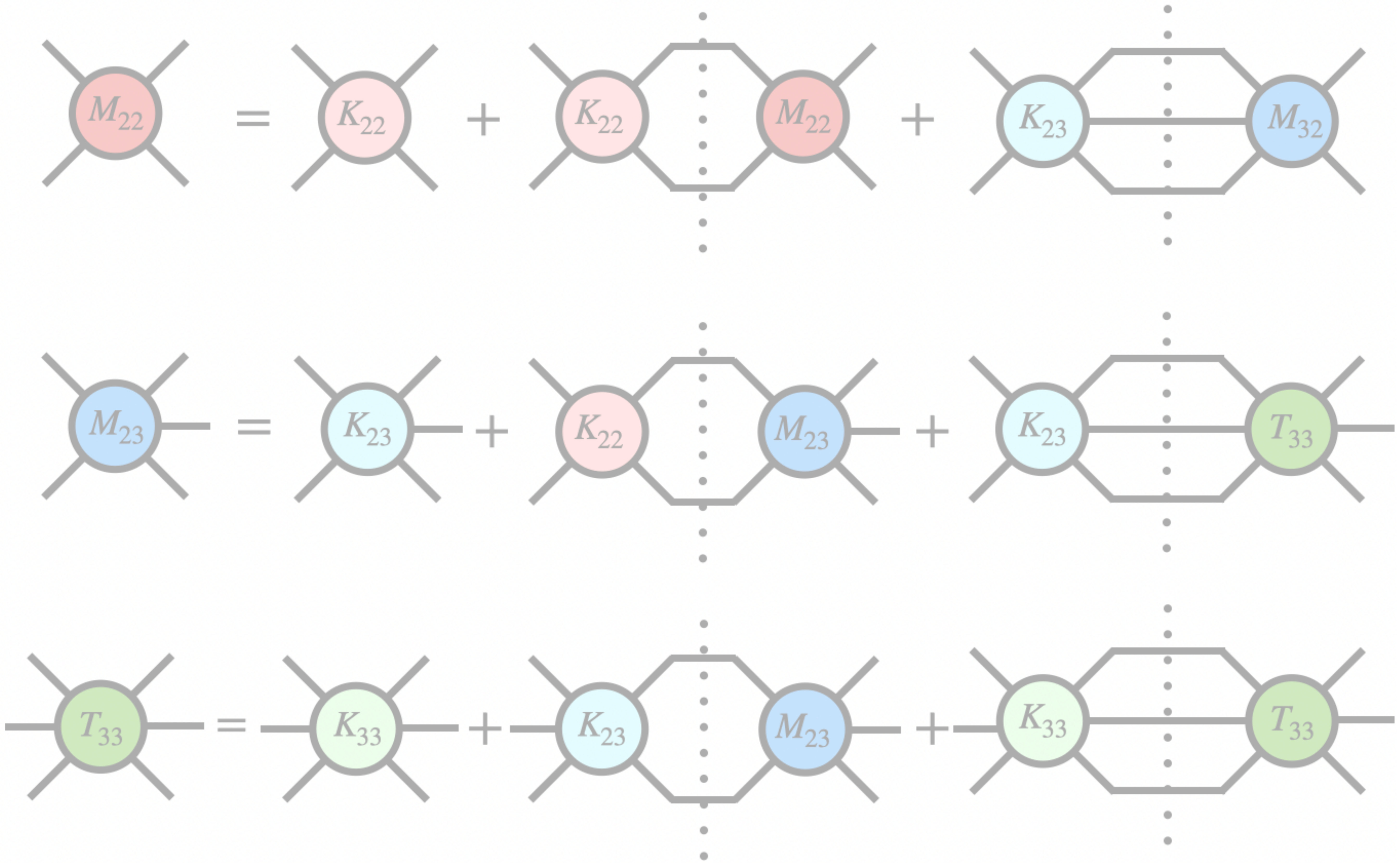
Shining Sun, Wyatt Smith, Raul Briceno, Andrew Jackura

CNUGS

Takeaways



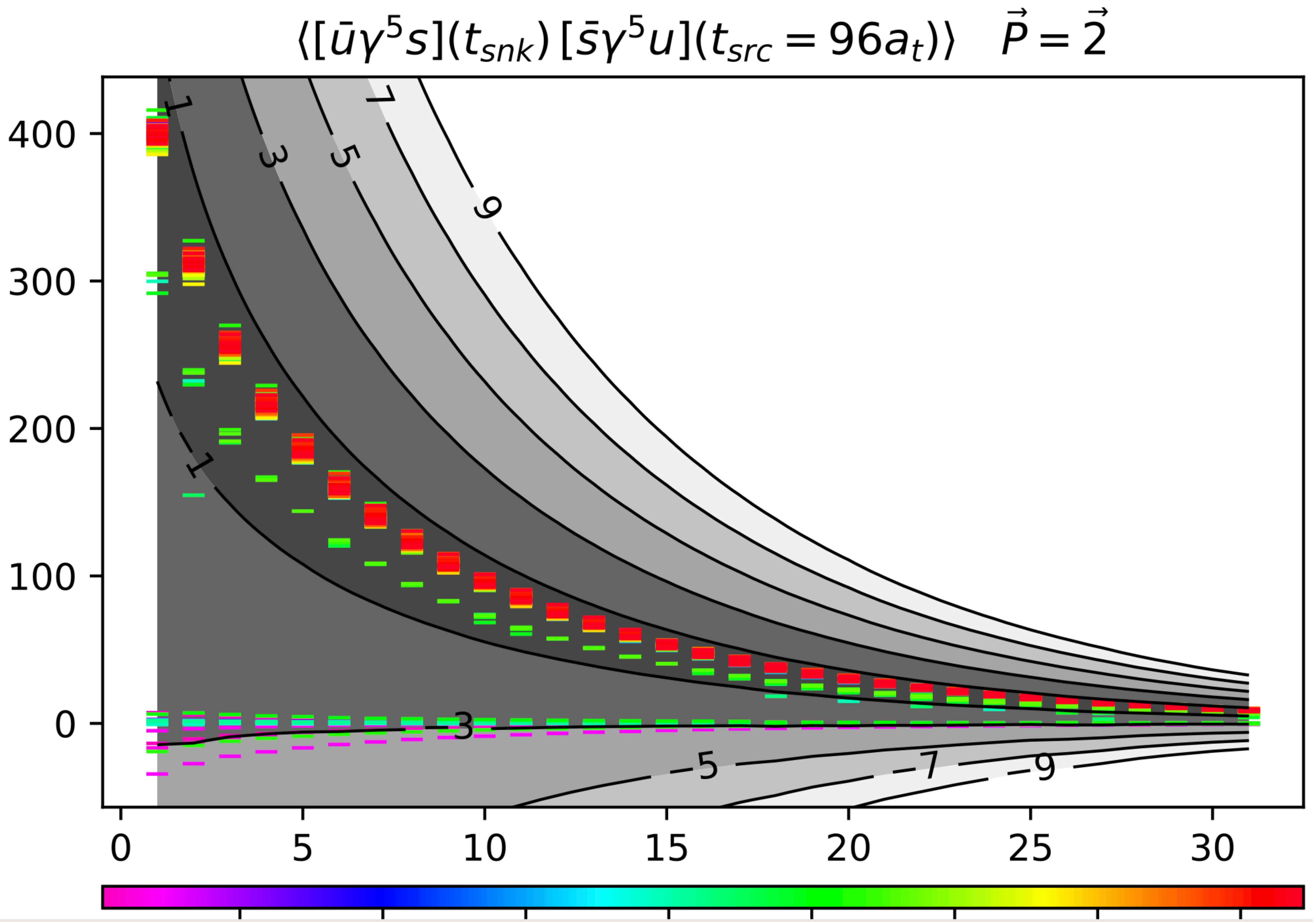
We can study scattering amplitudes in 2 to 3 coupled systems from the lattice

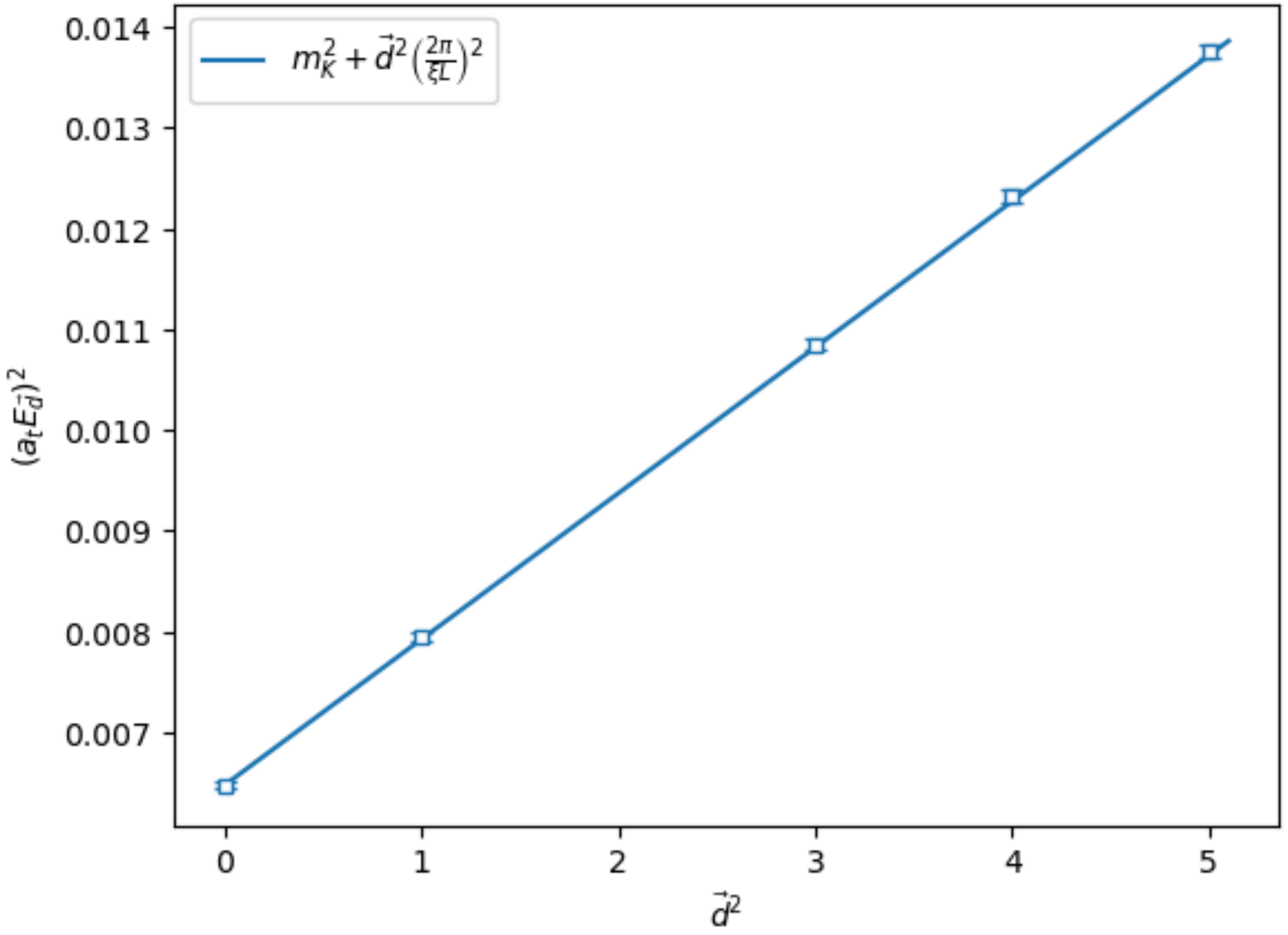


Integral equations involving scattering amplitudes can be parametrized with real functions (hint: K matrices)

Lattice QCD for Stable States

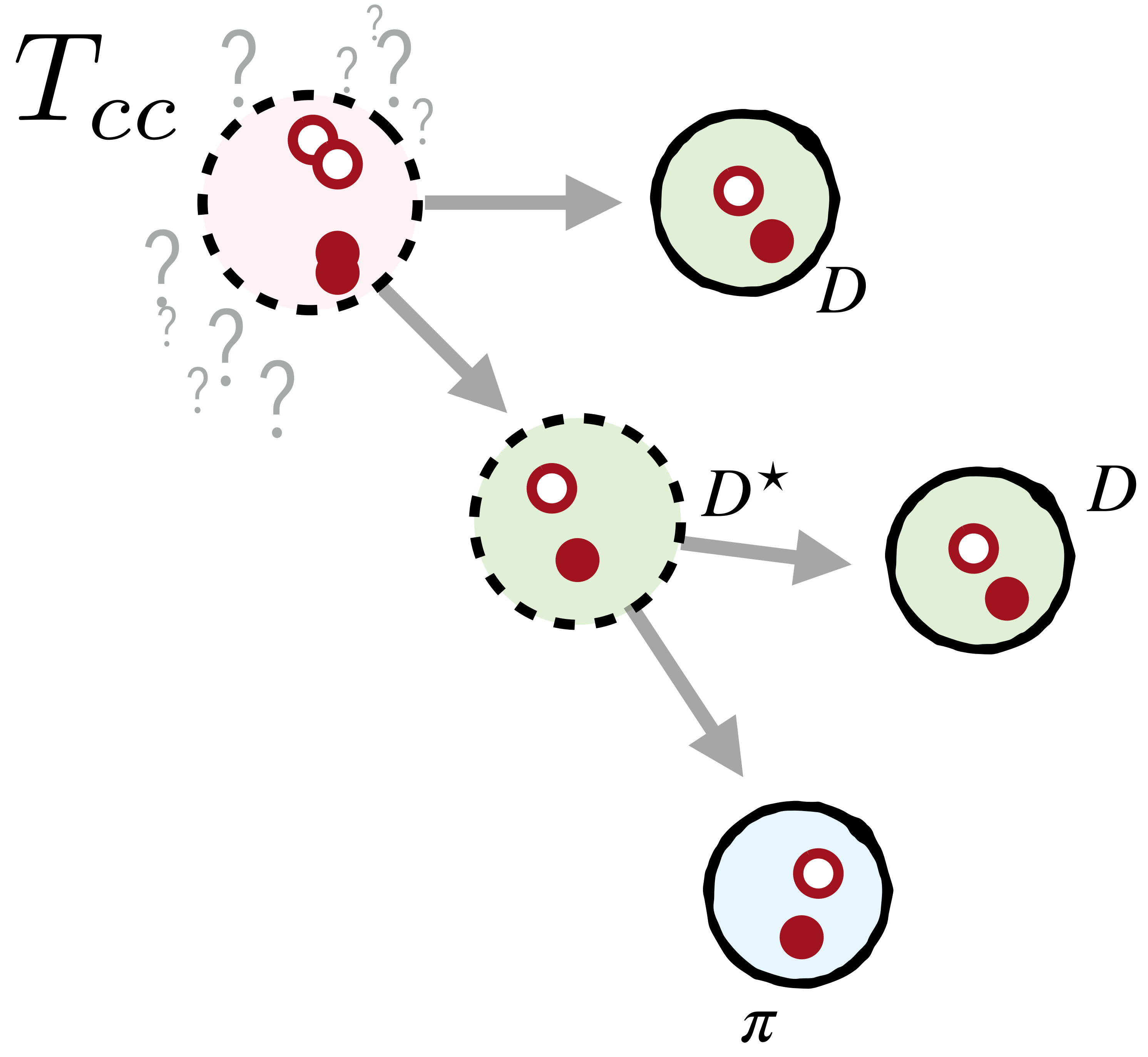
$$C^{2pt.}(t_E) \equiv \langle 0|\mathcal{O}(t_E)\mathcal{O}^\dagger(0)|0\rangle = \sum_n c_n e^{-E_n t_E}$$

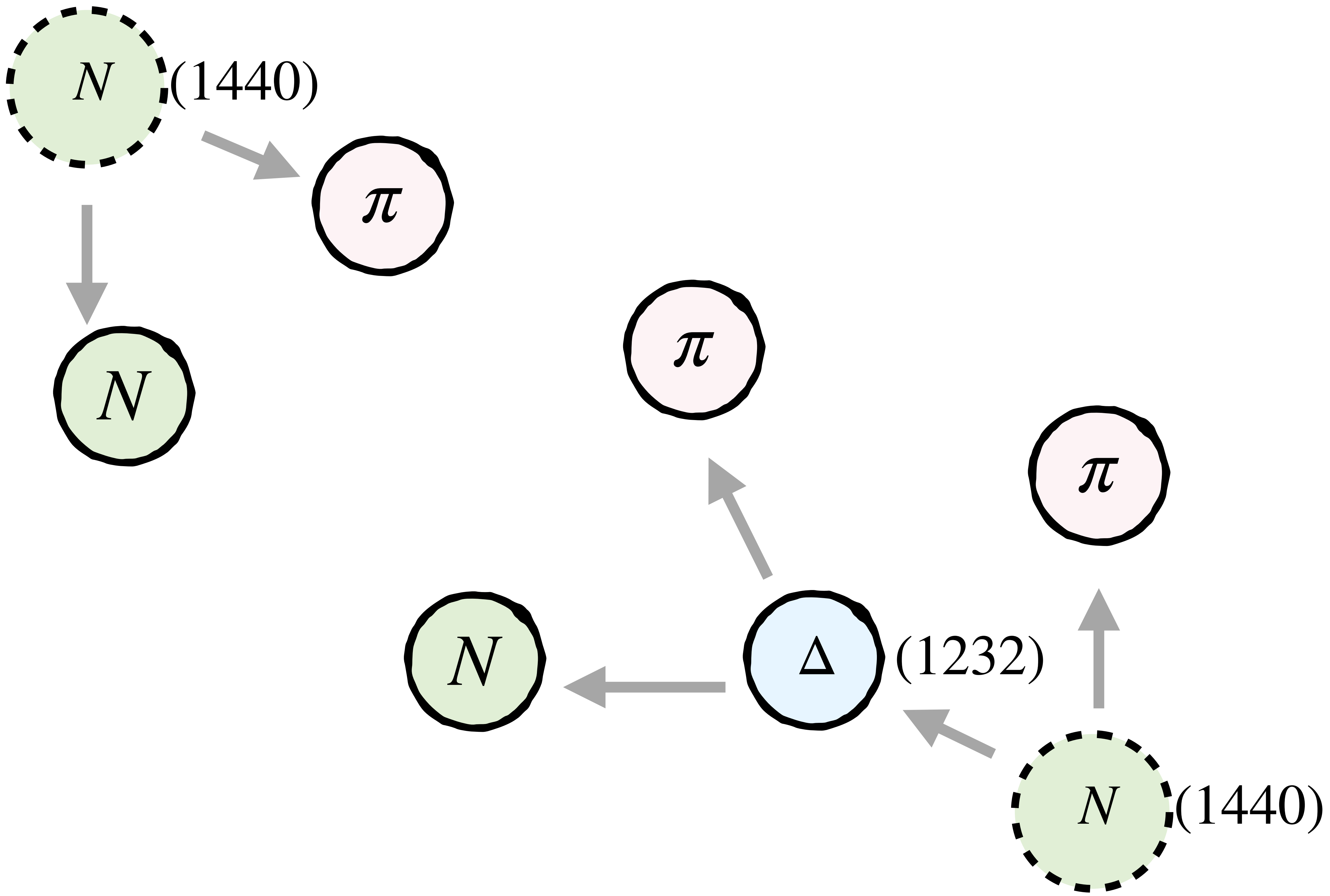




Why 2 to 3 coupled systems ?

- Exotic tetra quark decays



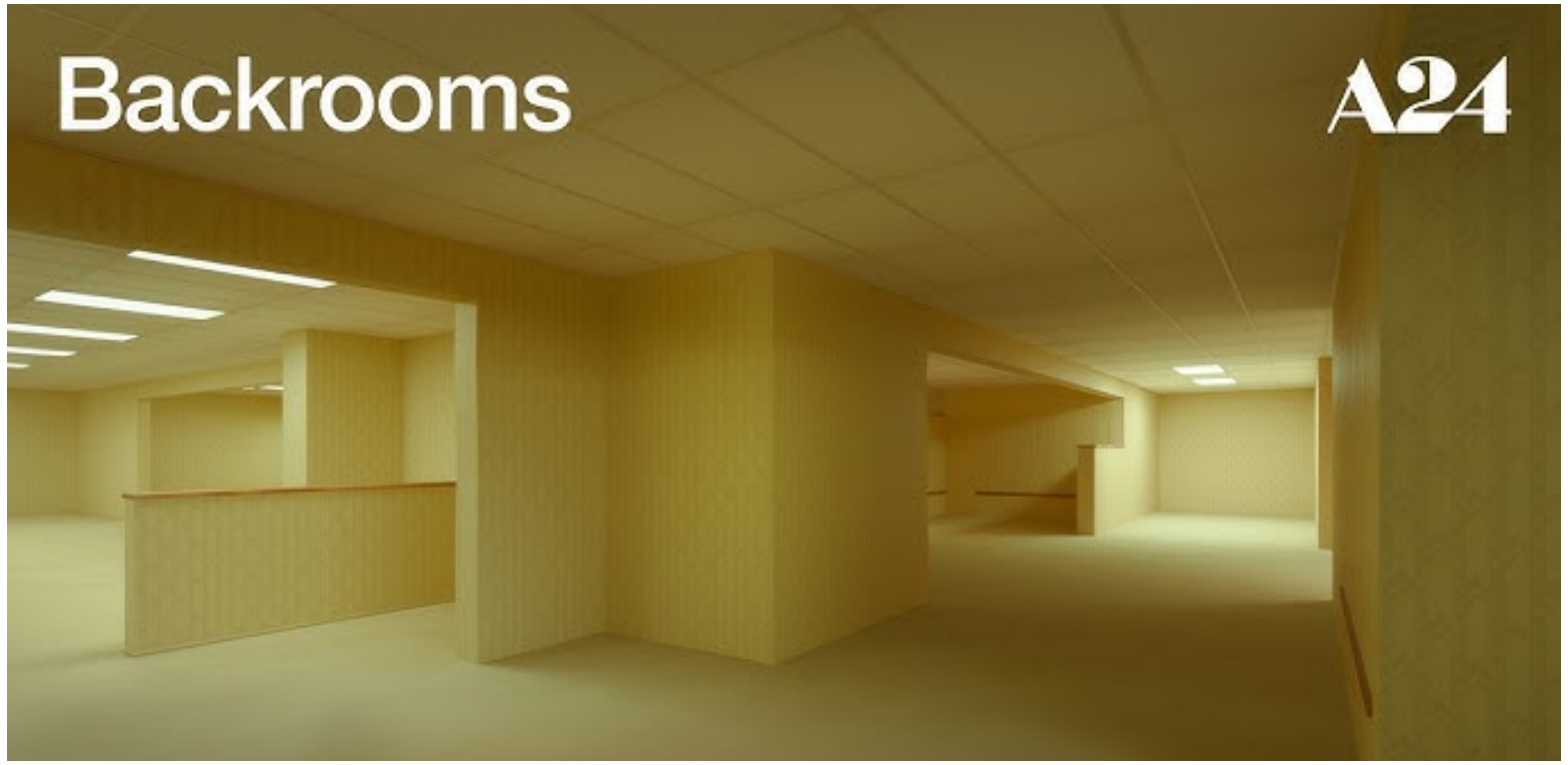


- Roper excitation

Unstable states !

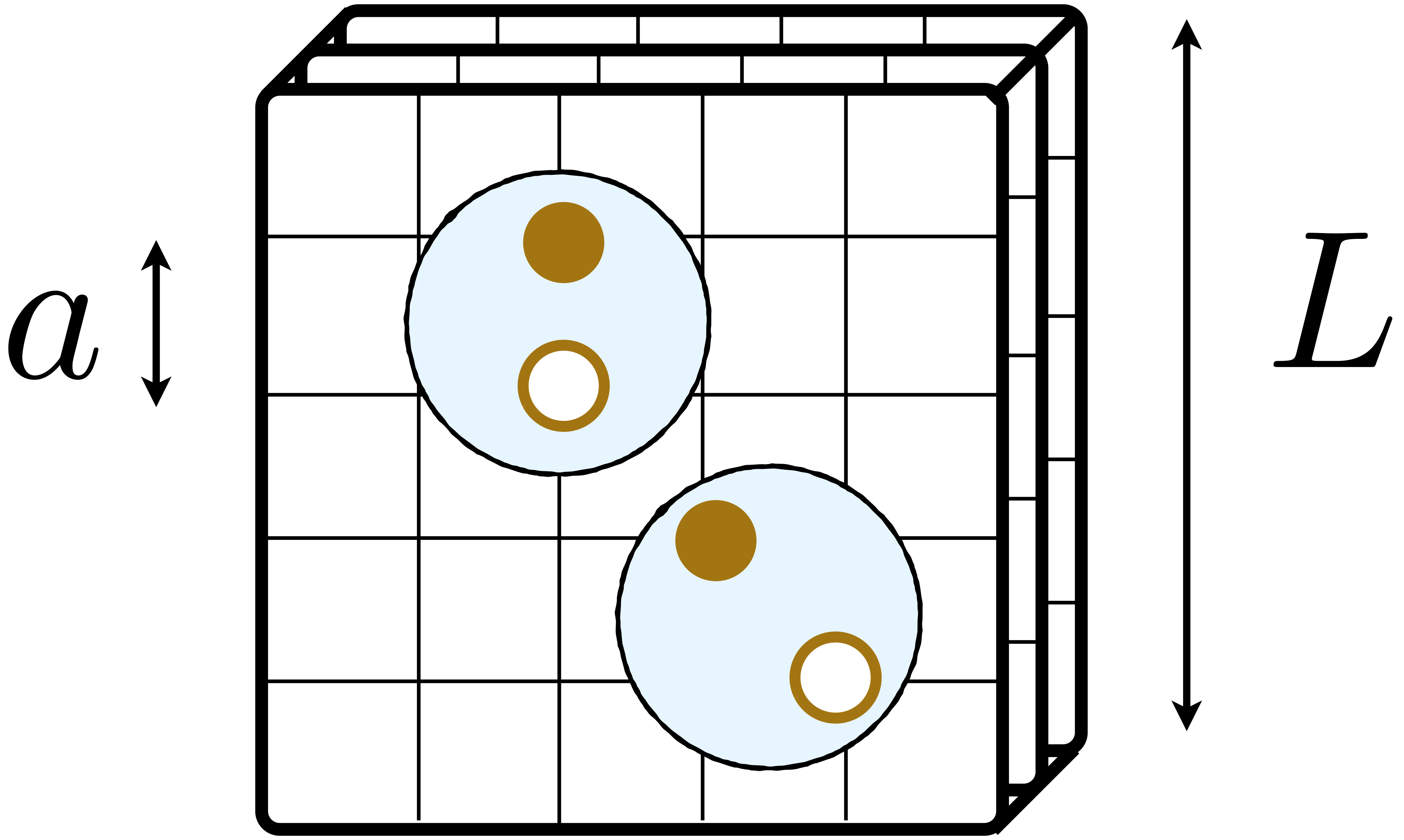
Infinite Volume

Scattering theory



Finite Volume

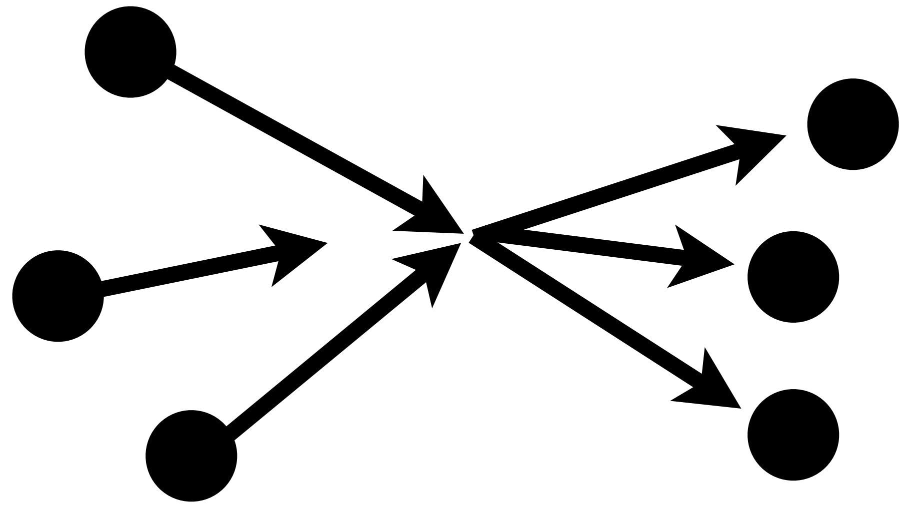
Lattice QCD



Infinite Volume

S matrix theory

Integral equations & scattering amplitudes from unitarity



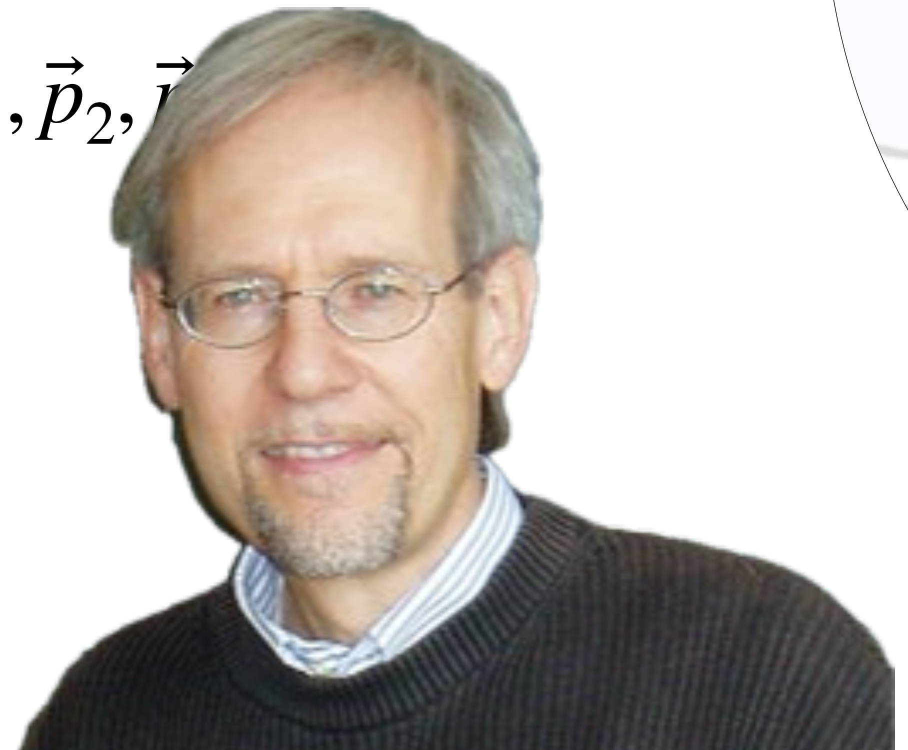
$\left|\vec{k}_1, \vec{k}_2, \vec{k}_3\right\rangle_{\text{in}}$ $\left|\vec{p}_1, \vec{p}_2, \vec{p}_3\right\rangle_{\text{out}}$

time

$-\infty$ → $+\infty$

$\left|\vec{k}_1, \vec{k}_2, \vec{k}_3\right\rangle$ $\left|\vec{p}_1, \vec{p}_2, \right.$

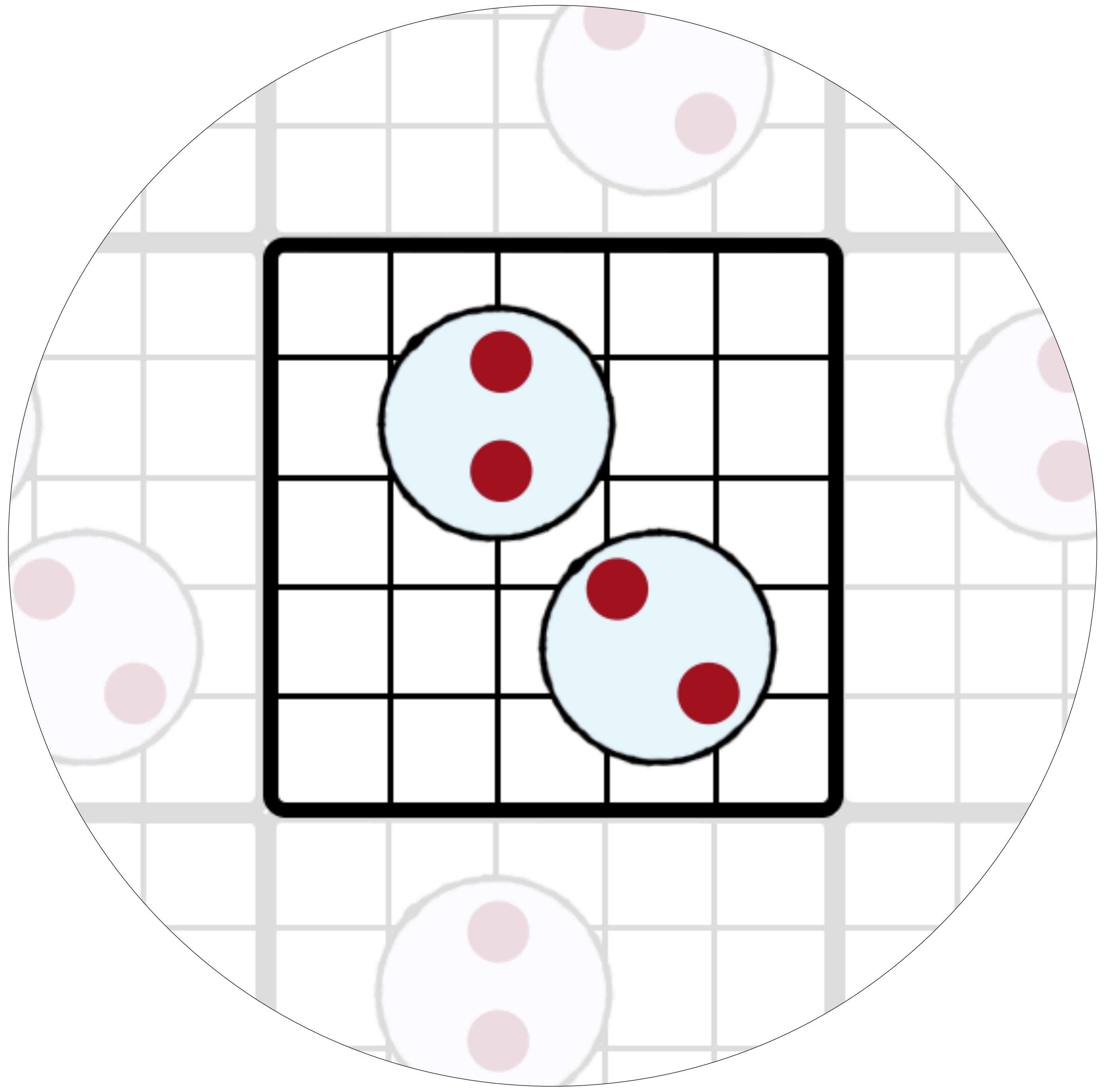
$$\langle \vec{p}_1, \vec{p}_2, \vec{p}_3 | S | \vec{k}_1, \vec{k}_2, \vec{k}_3 \rangle =_{out} \langle \vec{p}_1, \vec{p}_2, \vec{p}_3 | \vec{k}_1, \vec{k}_2, \vec{k}_3 \rangle_{in}$$



Finite Volume

S matrix formalism

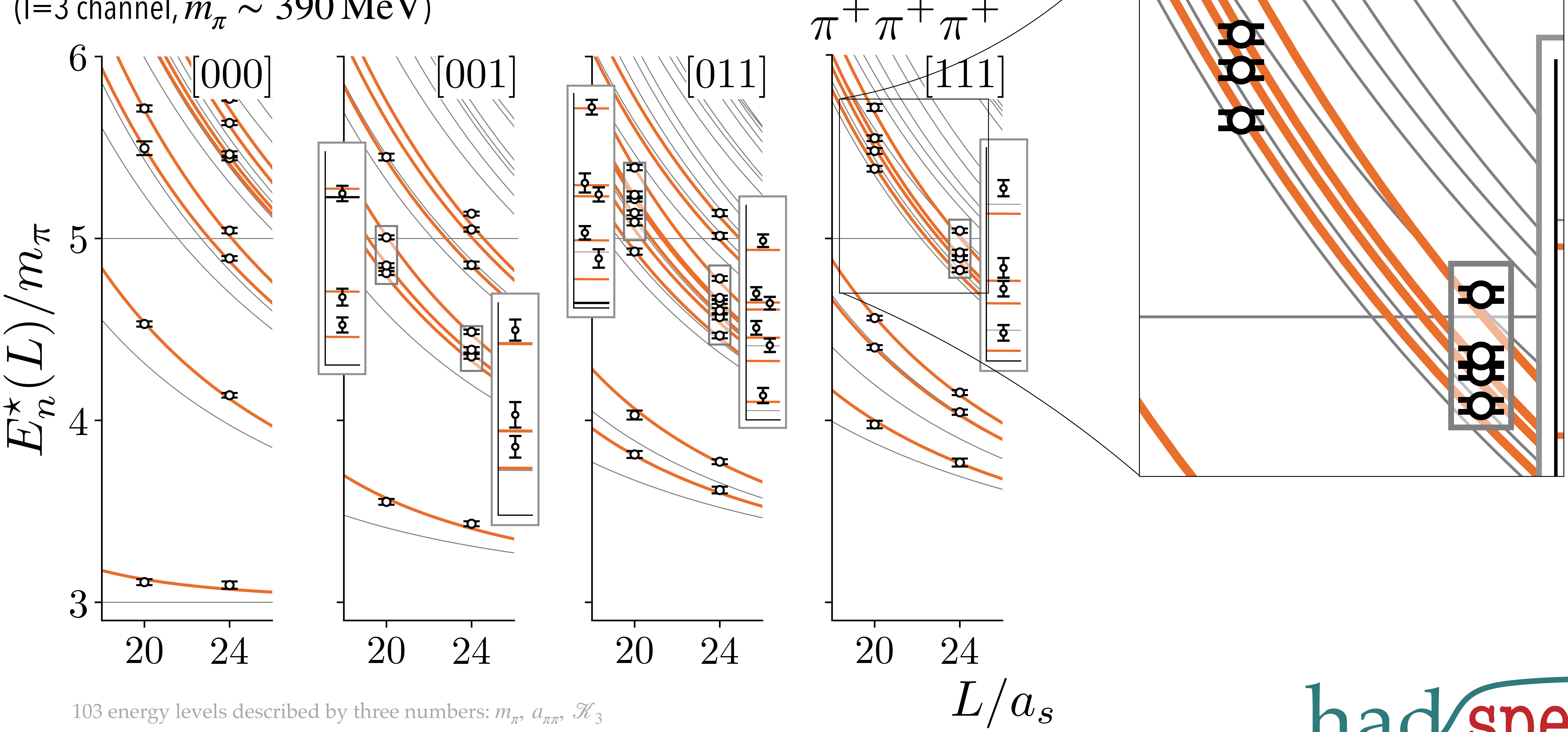
No asymptotic states



Uncoupled 3 body systems

$\pi\pi\pi$

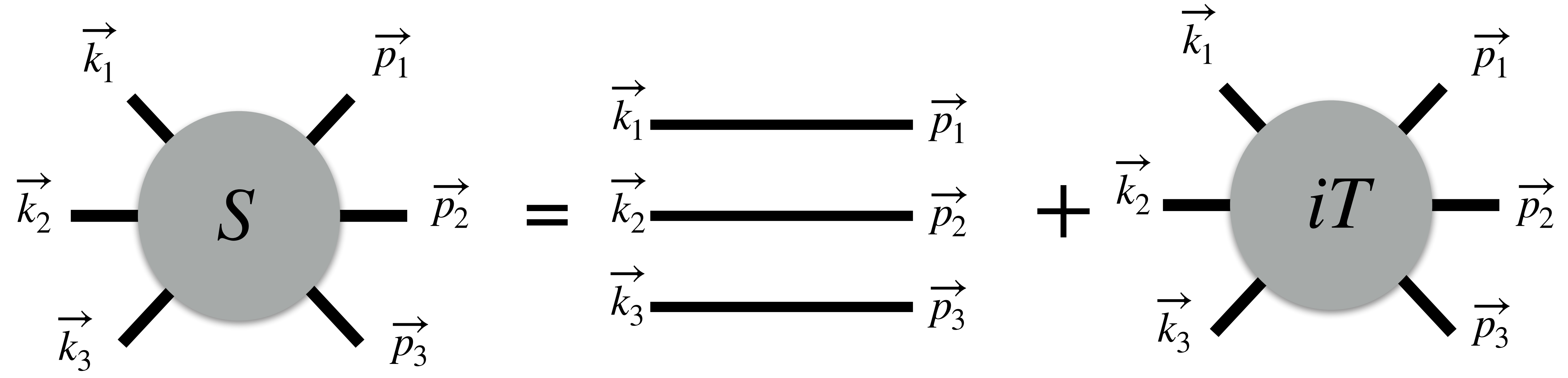
(I=3 channel, $m_\pi \sim 390$ MeV)

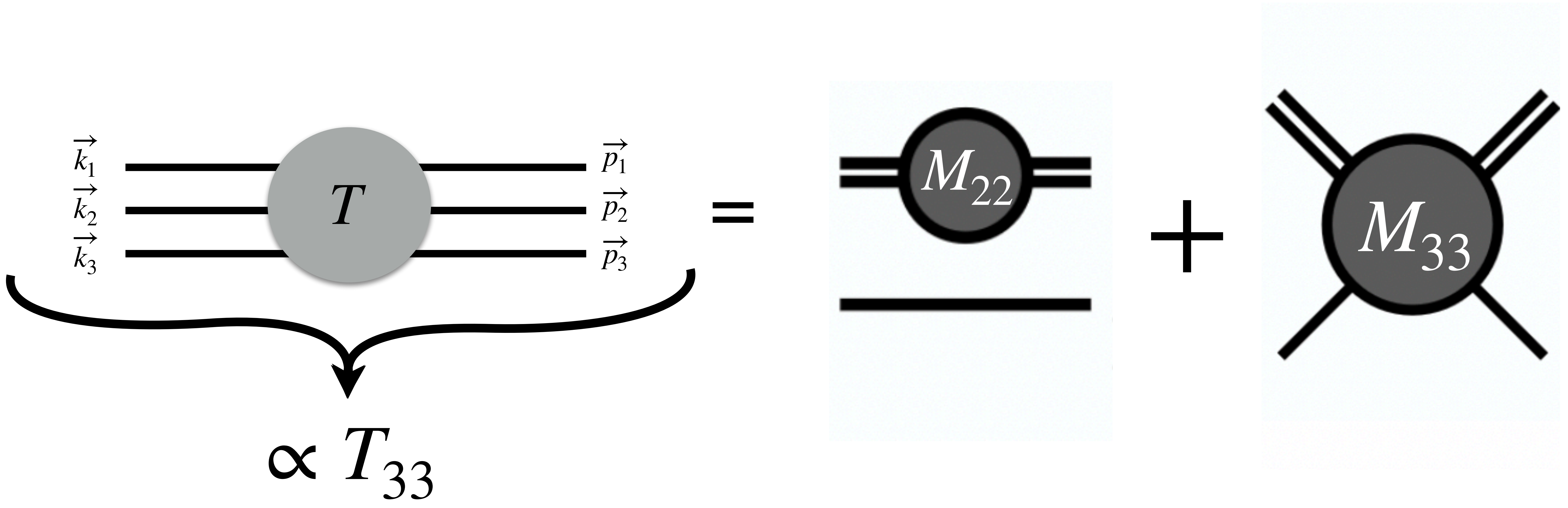


103 energy levels described by three numbers: m_π, $a_{\pi\pi}$, $\mathscr{K}_3$

Hansen, RB, Edwards, Thomas, & Wilson (2020)

S matrix Cluster Decomposition





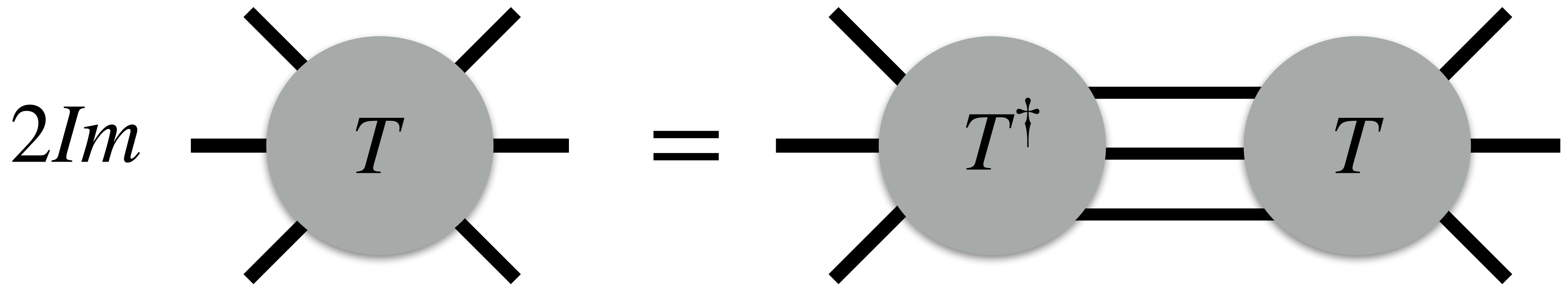
S matrix Unitarity

Conservation of probability

$$\Downarrow$$

$$S^\dagger S = 1$$

$$\Downarrow$$

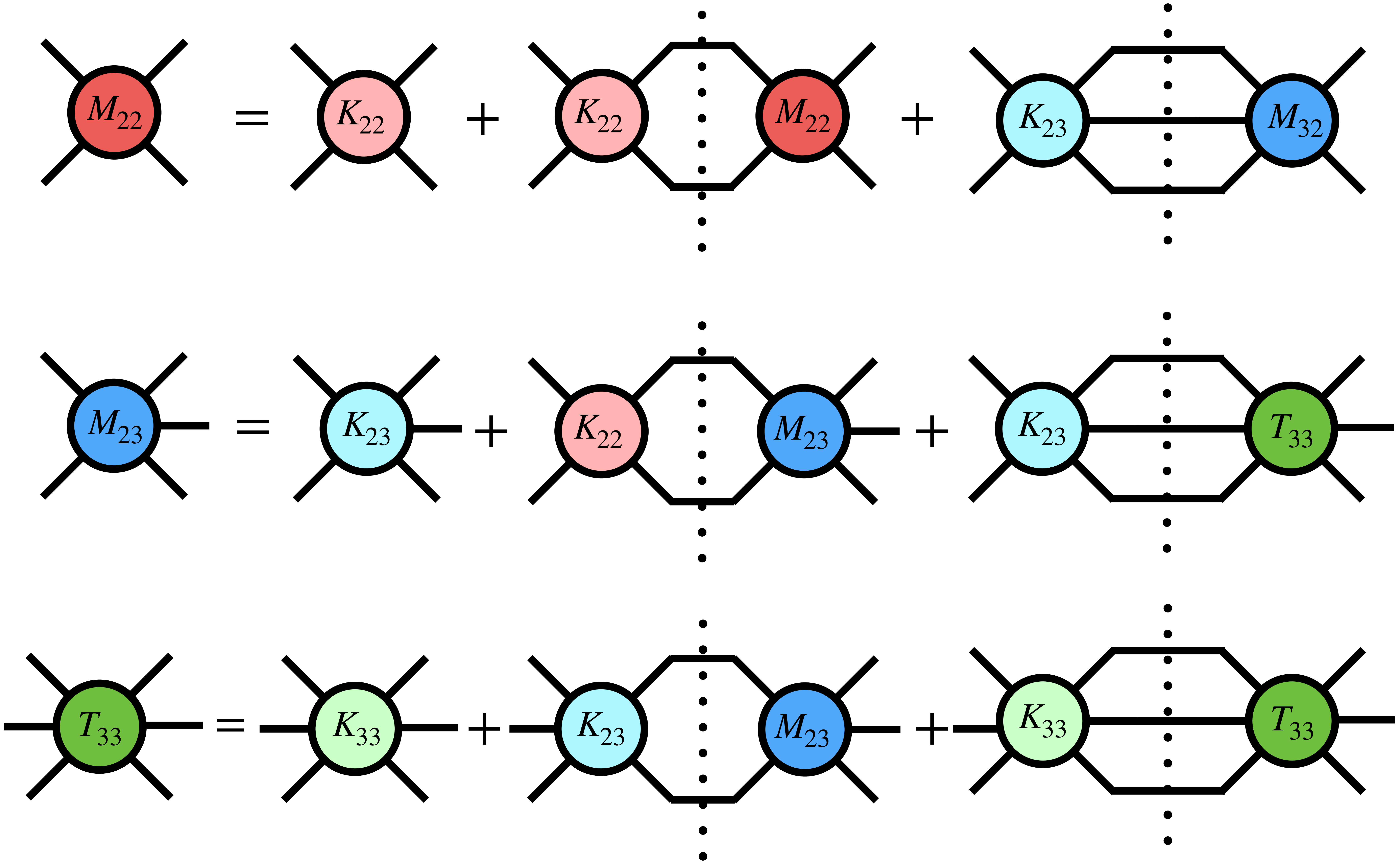


Solutions to Unitarity Constraints

$$
\begin{aligned}
\mathcal{M}_{22} &= \mathcal{K}_{22} - \mathcal{K}_{22} H \mathcal{I}\, \mathcal{M}_{22} - \int_{\mathbf{p}'} \int_{\mathbf{k}'} \mathcal{K}_{23}^{(u)}(\mathbf{p}')\, \Gamma(\mathbf{p}', \mathbf{k}')\, \mathcal{M}_{32}^{(u)}(\mathbf{k}')\,, \\
\mathcal{M}_{23}^{(u)}(\mathbf{k}) &= \mathcal{K}_{23}^{(u)}(\mathbf{k}) - \mathcal{K}_{22} H \mathcal{I}\, \mathcal{M}_{23}^{(u)}(\mathbf{k}) - \int_{\mathbf{p}'} \mathcal{K}_{23}^{(u)}(\mathbf{p}')\, \Gamma(\mathbf{p}', \mathbf{k})\, \mathcal{M}_{22}^{\mathrm{sub}}(\sigma_k) \\
&\quad - \int_{\mathbf{p}'} \int_{\mathbf{k}'} \mathcal{K}_{23}^{(u)}(\mathbf{p}')\, \Gamma(\mathbf{p}', \mathbf{k}')\, \mathcal{M}_{33}^{(u,u)}(\mathbf{k}', \mathbf{k})\,, \\
\mathcal{M}_{32}^{(u)}(\mathbf{p}) &= \mathcal{K}_{32}^{(u)}(\mathbf{p}) - \mathcal{K}_{32}^{(u)}(\mathbf{p}) H \mathcal{I}\, \mathcal{M}_{22} - \mathcal{K}_{22}^{\mathrm{sub}}(\sigma_p) \int_{\mathbf{k}'} \Gamma(\mathbf{p}, \mathbf{k}')\, \mathcal{M}_{32}^{(u)}(\mathbf{k}') \\
&\quad - \int_{\mathbf{p}'} \int_{\mathbf{k}'} \mathcal{K}_{\mathrm{df},3}(\mathbf{p}, \mathbf{p}')\, \Gamma(\mathbf{p}', \mathbf{k}')\, \mathcal{M}_{32}^{(u)}(\mathbf{k}')\,, \\
\mathcal{M}_{33}^{(u,u)}(\mathbf{p}, \mathbf{k}) &= \mathcal{K}_{\mathrm{df},3}(\mathbf{p}, \mathbf{k}) - \mathcal{K}_{32}^{(u)}(\mathbf{p}) H \mathcal{I}\, \mathcal{M}_{23}^{(u)}(\mathbf{k}) - \mathcal{K}_{22}^{\mathrm{sub}}(\sigma_p)\, \mathcal{G}(\mathbf{p}, \mathbf{k})\, \mathcal{M}_{22}^{\mathrm{sub}}(\sigma_k) \\
&\quad - \int_{\mathbf{p}'} \mathcal{K}_{\mathrm{df},3}(\mathbf{p}, \mathbf{p}')\, \Gamma(\mathbf{p}', \mathbf{k})\, \mathcal{M}_{22}^{\mathrm{sub}}(\sigma_k) - \mathcal{K}_{22}^{\mathrm{sub}}(\sigma_p) \int_{\mathbf{k}'} \Gamma(\mathbf{p}, \mathbf{k}')\, \mathcal{M}_{33}^{(u,u)}(\mathbf{k}', \mathbf{k}) \\
&\quad - \int_{\mathbf{p}'} \int_{\mathbf{k}'} \mathcal{K}_{\mathrm{df},3}(\mathbf{p}, \mathbf{p}')\, \Gamma(\mathbf{p}', \mathbf{k}')\, \mathcal{M}_{33}^{(u,u)}(\mathbf{k}', \mathbf{k})\,,
\end{aligned}
$$

Solutions to Unitarity Constraints

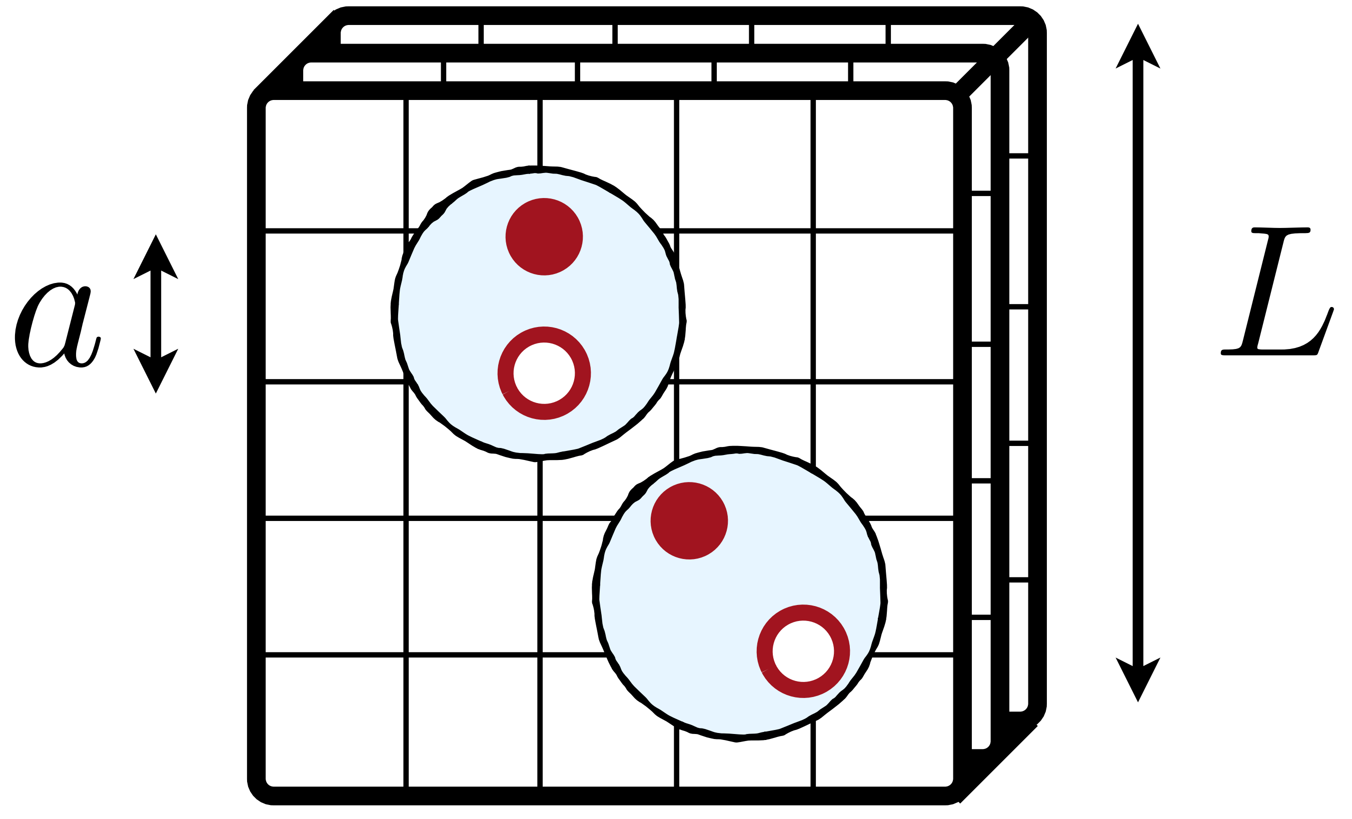
- $(3m)^2 \leq s < (4m)^2$
- 2 and 3 particles intermediate states are allowed to go on shell
- K matrices are real analytic functions that can be parametrized



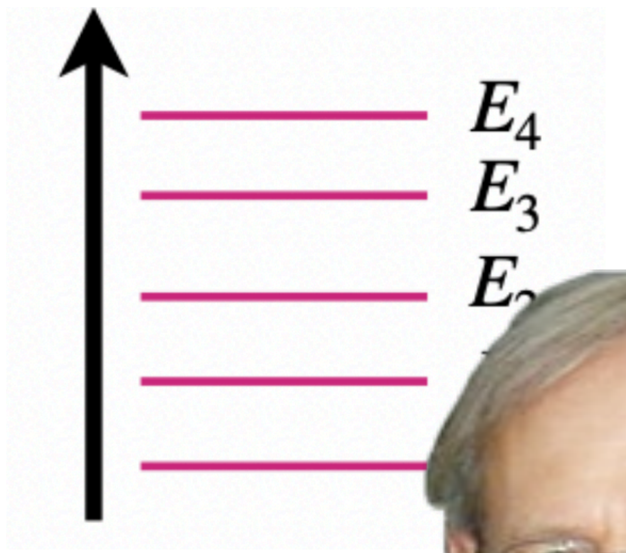
Finite Volume

Lattice QCD calculations
Two point correlation functions

$$C^{2pt.}(t_E) \equiv \langle 0|\mathcal{O}(t_E)\mathcal{O}^\dagger(0)|0\rangle = \sum_n c_n e^{-E_n t_E}$$



Lattice spectra



Infinite Volume

Energy spectra predictions dependent on K matrix parameters

$$\mathbf{K} = \begin{pmatrix} \mathcal{K}_{22} & \mathcal{K}_{23}^{(u)} \\ \mathcal{K}_{32}^{(u)} & \mathcal{K}_{33}^{(u,u)} \end{pmatrix}$$

Thank You !