William & Mary

Overview of Resonances, K-Matrix Formalism, and Coupled-Channel Analysis in the Context of Hadron Spectroscopy

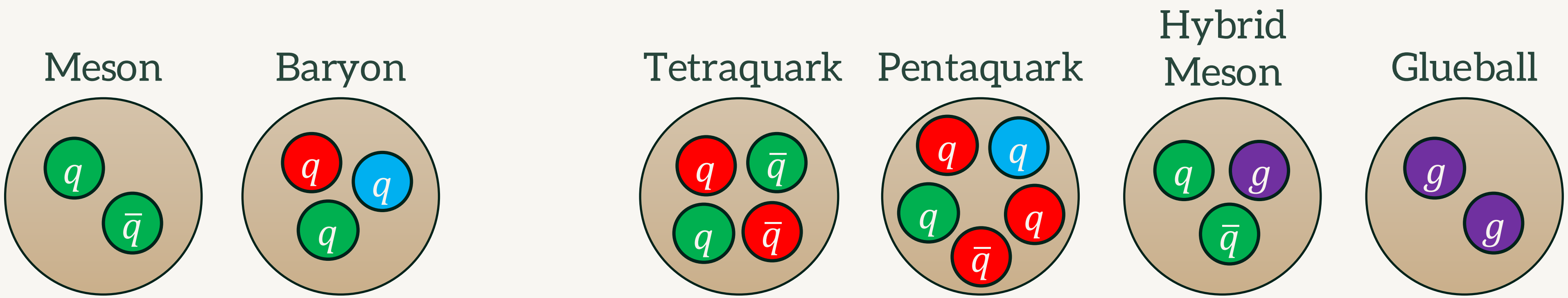
Addison Kovats-Bernat

Outline

- Overview of Resonances
- Breit-Wigner Parameterization and its limitations
- K-Matrix Formalism
- Coupled-Channel Analysis
- GlueX Coupled-Channel Analysis and K^+K^-

Overview of Resonances

- Resonances are short lived bound states
  - Can be mesons or baryons
  - Can also be exotic → Unable to be described by 2 or 3 quarks (tetra/pentaquarks, hybrid mesons, glueballs)



- Usually appear as peaks in mass spectra

Overview of Resonances

- Formally, resonances are complex poles of the scattering matrix $\mathcal{S}$:

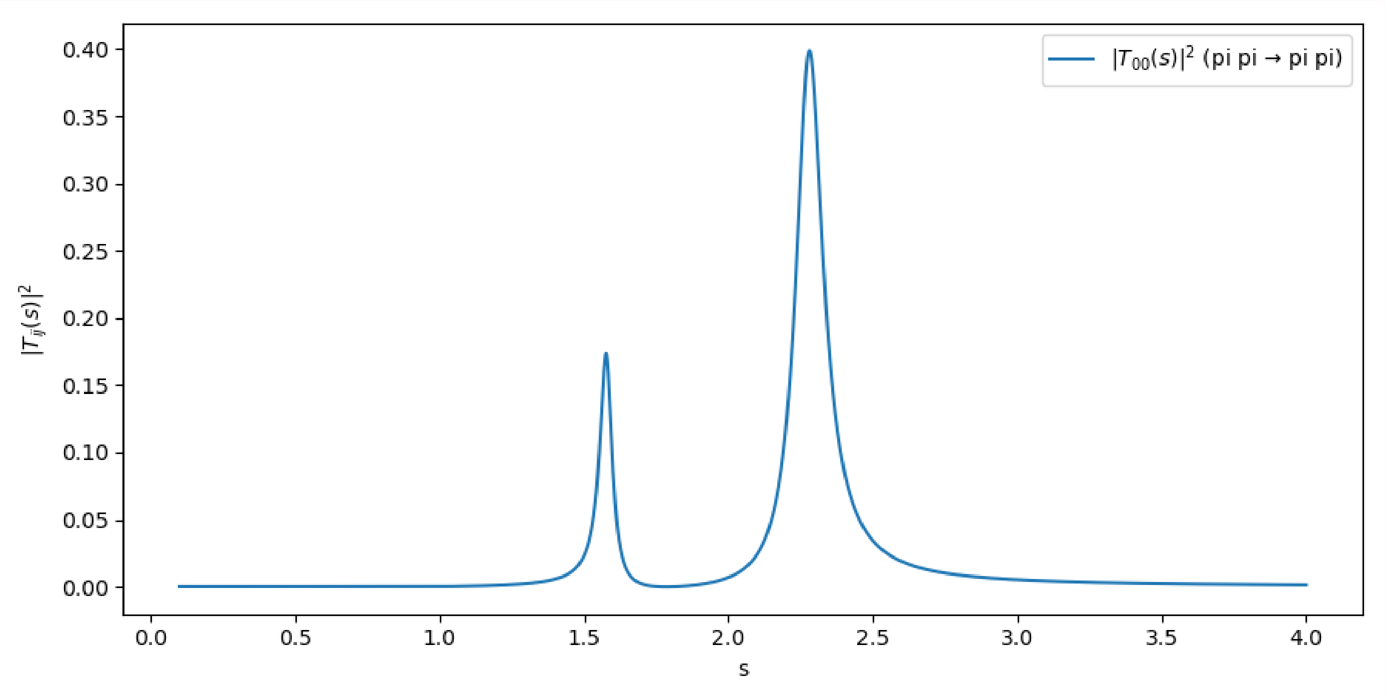
$$i(2\pi)^4\delta^4(p_1 + p_2 - p_1' - p_2')\mathcal{M}(p_1, p_2; p', p_2')_{ba} = {}_{out}\langle p_1' p_2', b|\mathcal{S} - 1|p_1 p_2, a\rangle_{in}$$

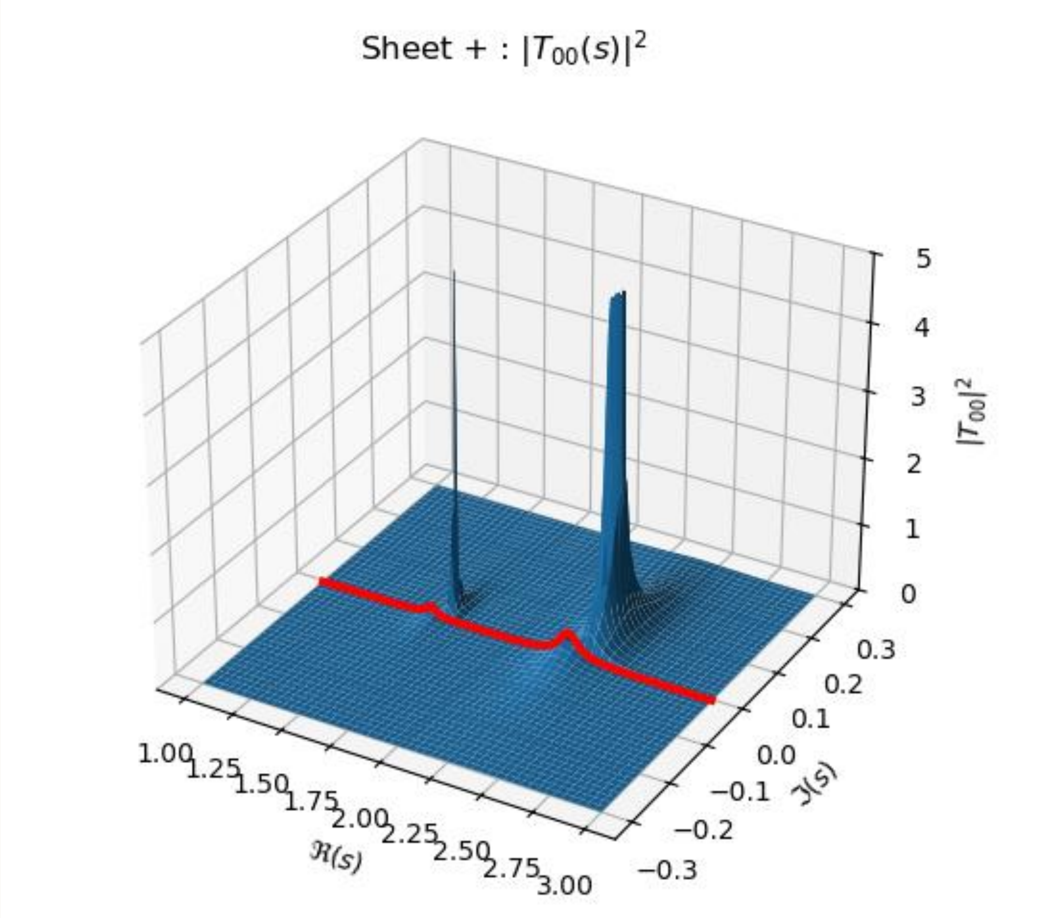
- Mass and width are directly related to pole position:

$$\sqrt{s_R} = M_R - i\Gamma_R/2$$

- What we see in mass spectra is the real line → resonance pole will affect this

Resonance Visualization





<u>Note: Not actual data!</u>

Breit-Wigner Parameterization

- Often, resonances are described w/ a Breit-Wigner Function:

$$\mathrm{BW}(s) = \frac{1}{m_{\mathrm{BW}}^2 - s - i m_{\mathrm{BW}} \Gamma(s)}$$

- Can describe scattering amplitude and production amplitude in terms of BW:

$$\mathcal{M}_{ab}(s) = g_a n_a(s) \mathrm{BW}(s) g_b n_b(s)$$

$$\mathcal{A}_a(s) = \mathcal{N}_a(s) BW(s)$$

  - g_a : bare coupling of bare pole to channel a, related to partial width
  - $n_a(s)$: function of breakup momentum for channel a
  - $\mathcal{N}_a(s)$: numerator factor, covers kinematics and couplings to production and decay

Limitations of BW Parameterization

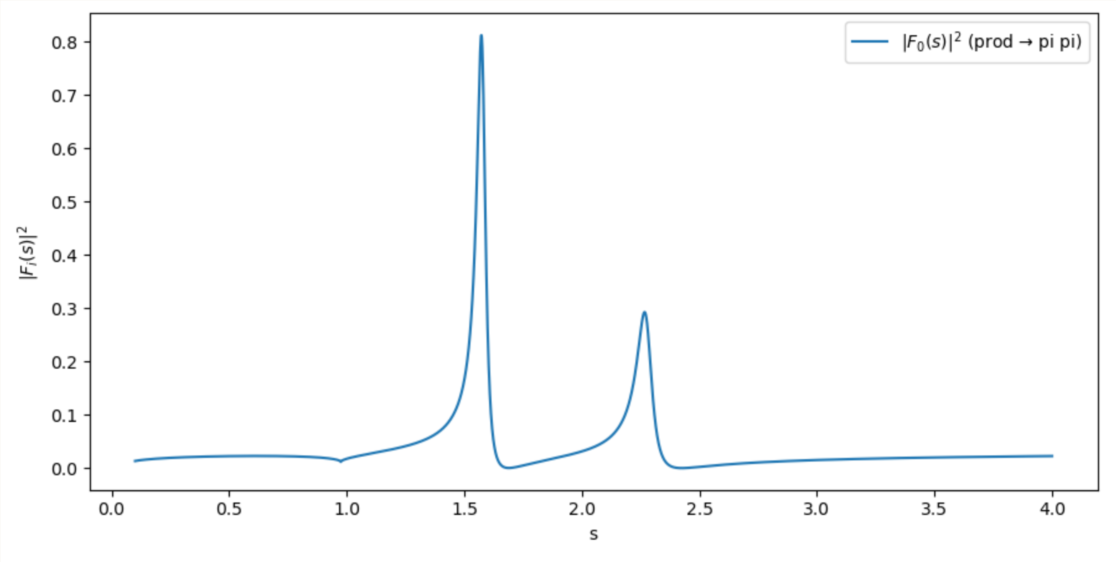
1. Only valid in $\Gamma/\Delta \to 0$ limit
   - BW parameterization is only accurate for small width and isolated resonances.
2. Pole position obtained from BW does not necessarily match the actual pole position
   - Must have small width, and can have significant non-resonant contributions
3. If multiple similar resonances couple to the same channel, using BWs may violate unitarity greatly

K-Matrix Formalism

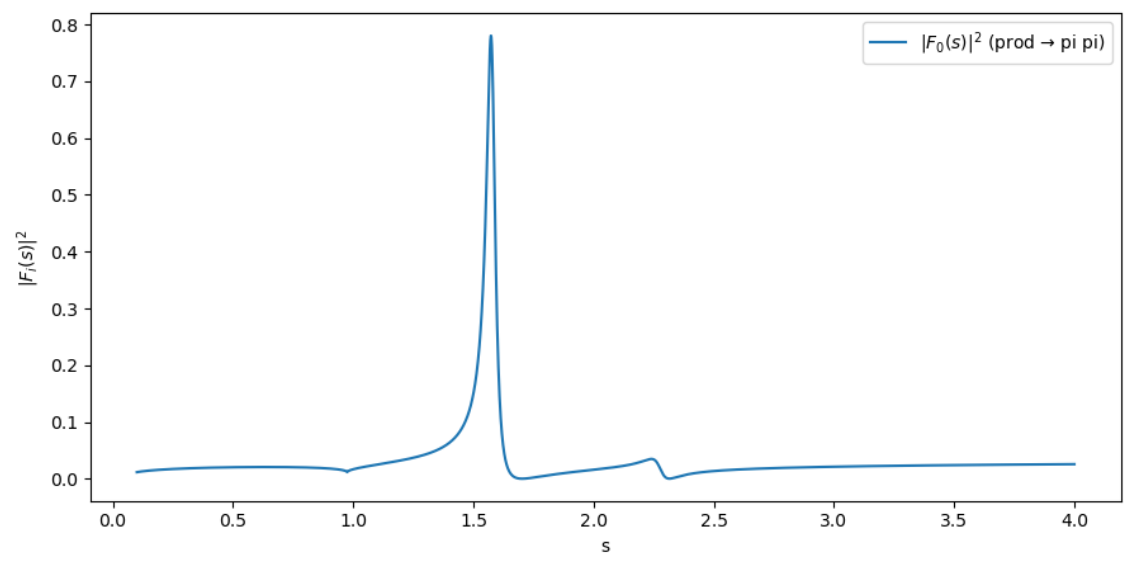
- The defining equation for the K-Matrix is
$$n_b \mathcal{M}_{ba}^{-1} n_a = \mathcal{K}_{ba}^{-1} - i\delta_{ba}\rho_a n_a^2$$
  - ρ_a: phase space factor
- The most common parameterization is
$$\mathcal{K}_{ba}(s) = \sum_r \frac{g_b^r g_a^r}{m_r^2 - s} + \sum_n c_{ba}^{(n)} s^n$$
  - $c_{ba}^{(n)}$: non-pole contributions
- K-Matrix formalism can handle multiple resonances as well as multiple channels without issue*

K-Matrix Formalism: Rescattering

- K-Matrix can account for rescattering: no direct production from a channel, but indirect production from secondary interactions



Direct Production of m=1.525 resonance



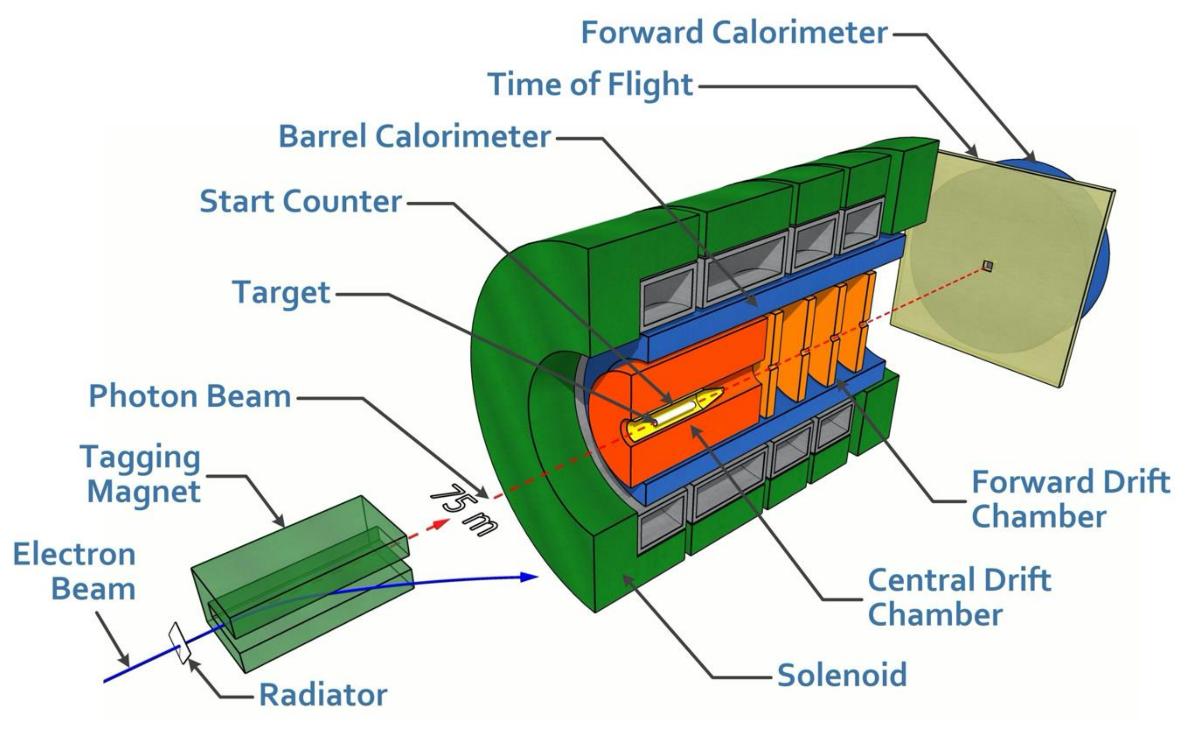
Indirect Production of m=1.525 resonance!

Coupled-Channel Analysis

- K-matrix allows for a coupled-channel analysis, where multiple different production mechanisms and decays are considered simultaneously
  - Can even use channels from different production mechanisms
- Can use well-known channels to constrain less confident ones, resulting in better fitting overall

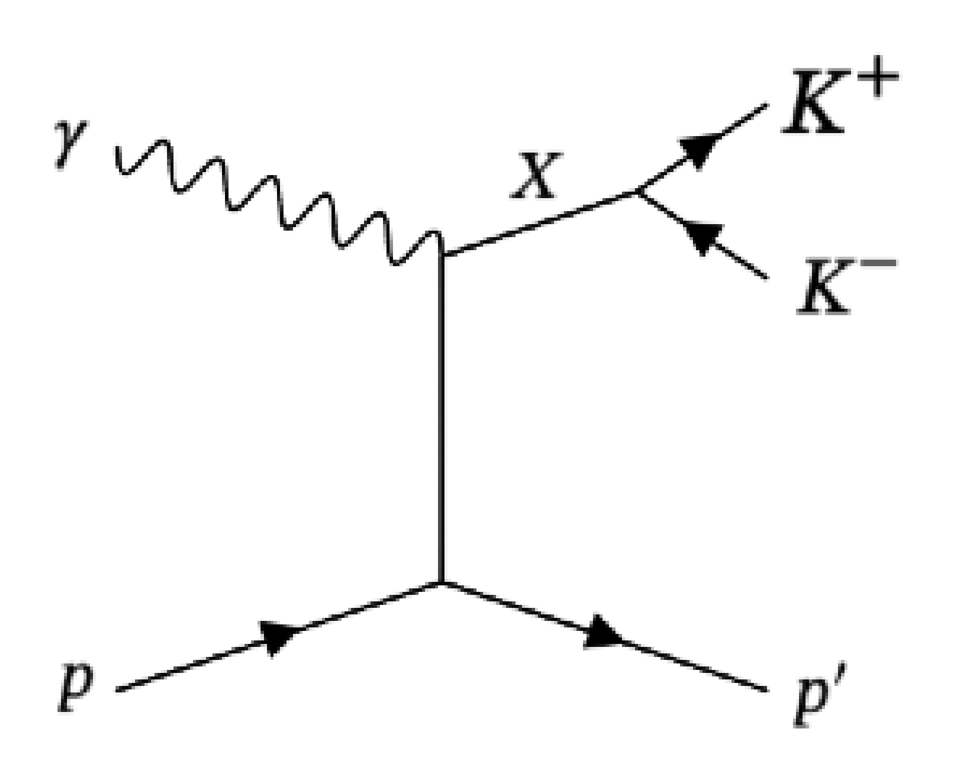
GlueX Coupled-Channel Analysis

- GlueX is an experiment going on in Hall D focused on studying exotic mesons
  - Linearly polarized photon beam aimed at liquid hydrogen target
- Channels we plan to include in analysis:
  - $\pi\pi, K\bar{K}, \eta\eta, \eta\eta', \pi\eta, \pi\eta'$
  - (all also have recoil proton in final state)



K^+K^-

- Possible quantum numbers of X align with some exotics:
  - Scalar glueball (0++)
  - Light tetraquark (0++, 1++, 1+-, 2++)
- K^+K^- serves as a well-known channel to constrain others
  - Two charged particles → easier to measure at GlueX
  - Better reconstruction from DIRC



Bibliography

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- Chung, S. U., Brose, J., Hackmann, R., Klempt, E., Spanier, S., and Strassburger, C. (1995). Partial wave analysis in K matrix formalism. *Annalen Phys.*, 4:404–430.

Questions?

Various Equations

$$\Gamma(s) = \sum_b \Gamma_b(s) \text{ with } \Gamma_b(s) = \frac{1}{m_{\text{BW}}} g_b^2 \rho_b(s) n_b^2(s)\,.$$

$$\rho_c(s) = \frac{(2\pi)^4}{2} \int \mathrm{d}\Phi_2 = \frac{1}{16\pi} \frac{2|\vec{q}_c|}{\sqrt{s}}\,,$$

$$n_a = (q_a/q_0)^{l_a} F_{l_a}(q_a/q_0)\,,$$

$$F_0^2(z) = 1\,,$$
$$F_1^2(z) = 1/(1+z^2)\,,$$
$$F_2^2(z) = 1/(9+3z^2+z^4)\,,$$

Blatt-Weisskopf form factors

F-Vector

$$F_a(s) = \left[P(s) \cdot (I + iK(s)C(s))^{-1}\right]_a ,$$

$$P_a(s) = \sum_\alpha \frac{g_{\alpha a}\beta_\alpha}{m_\alpha^2 - s} B_{\alpha a}(s) + \sum_j d_a^{(j)} s^j ,$$

$$B_{\alpha a}(s) = \frac{F_{\ell_\alpha}(qR)}{F_{\ell_\alpha}(q_0 R)} ,$$

Blatt-Weisskopf barrier factors

$C(s)$: Chew-Mandelstam phase space matrix