CNUGS

CHRISTOPHER NEWPORT UNIVERSITY
GRADUATE SUMMER WORKSHOP
AT JEFFERSON LAB 2026

formerly The HUGS Program

QCD factorization and global analysis

Zhite Yu (BNL, High Energy Physics)

- Introduction to factorization
- Parton distribution functions and global analysis
- From inclusive to exclusive processes
- Generalized parton distributions and global analysis

CNUGS Summer Workshop, June 5, 2026

Outline

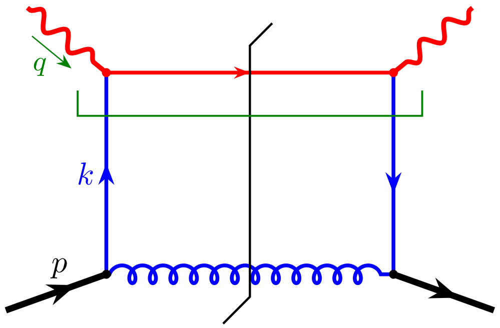
- ❑ **From inclusive to exclusive processes**
- ❑ **Generalized parton distribution (GPD)**
- ❑ **Phenomenology of GPD**
- ❑ **Inverse problem and global analysis of GPD**

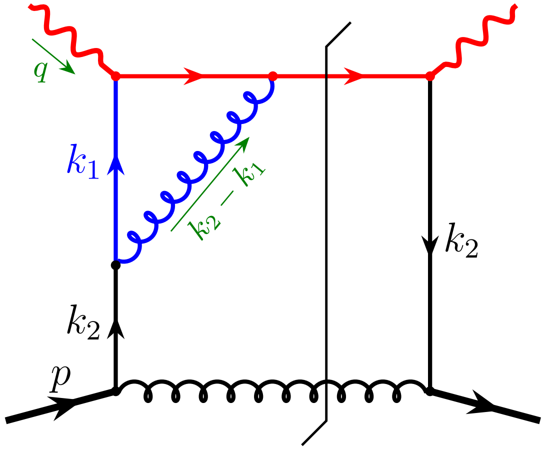
Outline

- ❑ From inclusive to exclusive processes
- ❑ Generalized parton distribution (GPD)
- ❑ Phenomenology of GPD
- ❑ Inverse problem and global analysis of GPD

Inclusiveness and DIS factorization

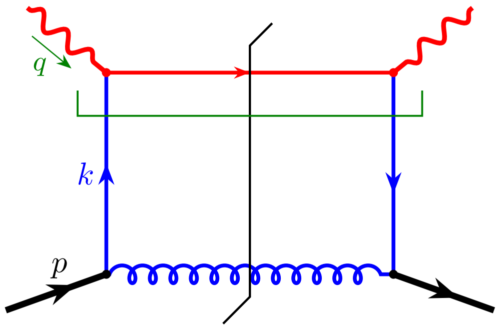
- Why can we say the upper quark lines are "hard"?

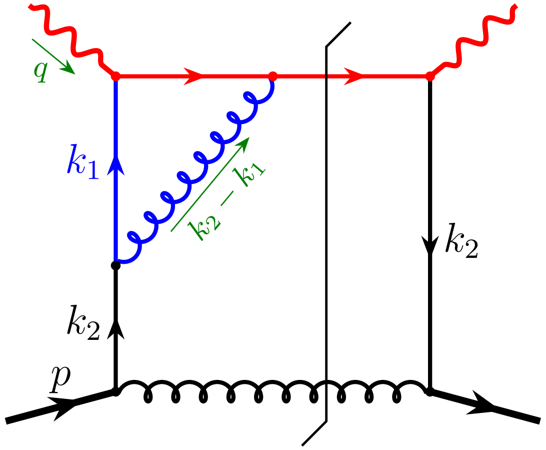


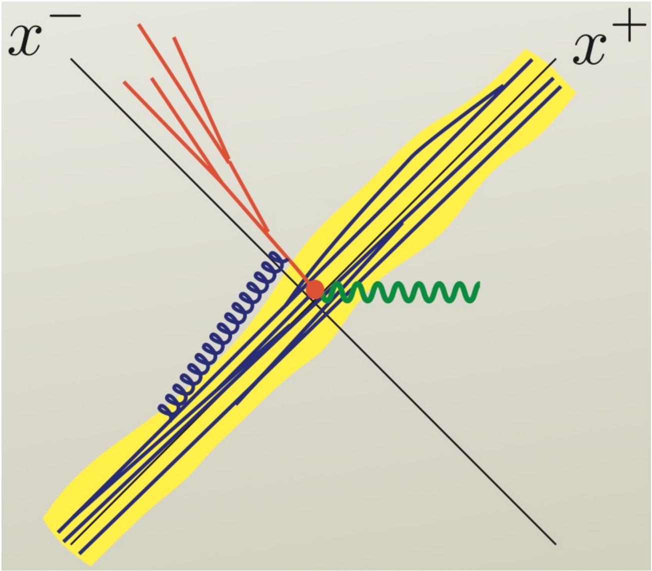


Inclusiveness and DIS factorization

❑ Why can we say the upper quark lines are “hard”?





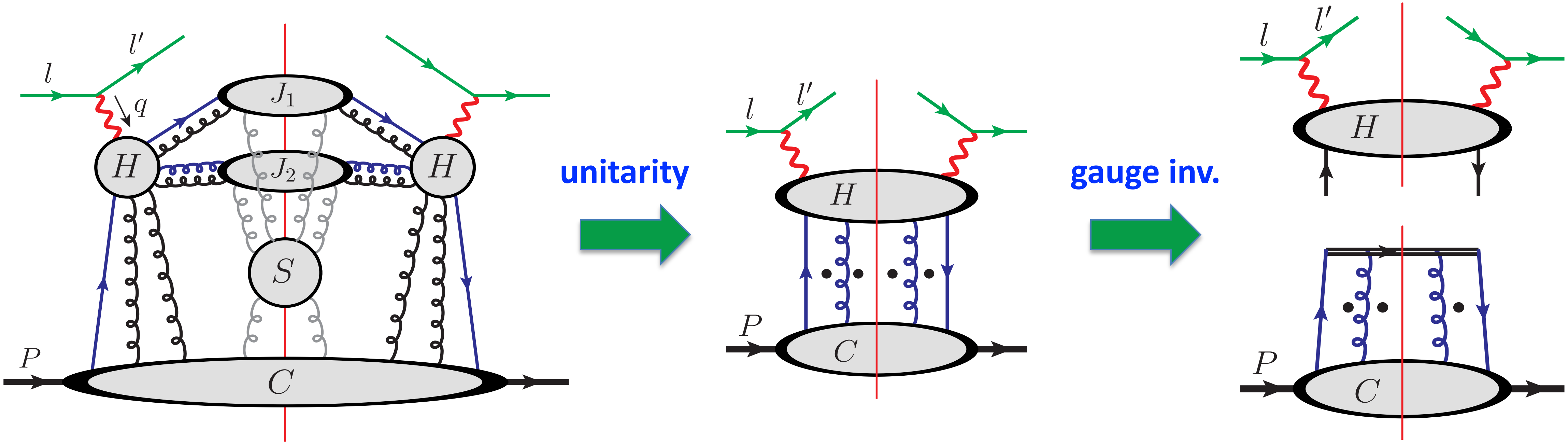


Picture from D. Soper

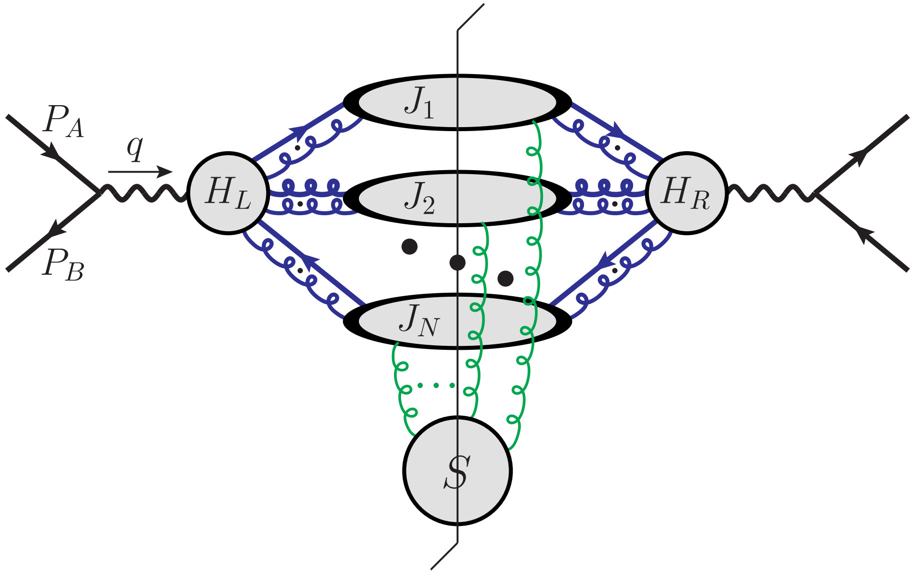
- Struck parton moves on-shell or closely on-shell, for a **long** distance.
- But this happens way **after** the hard interaction.
- Since we do ***not* observe** it, cross section is unchanged.

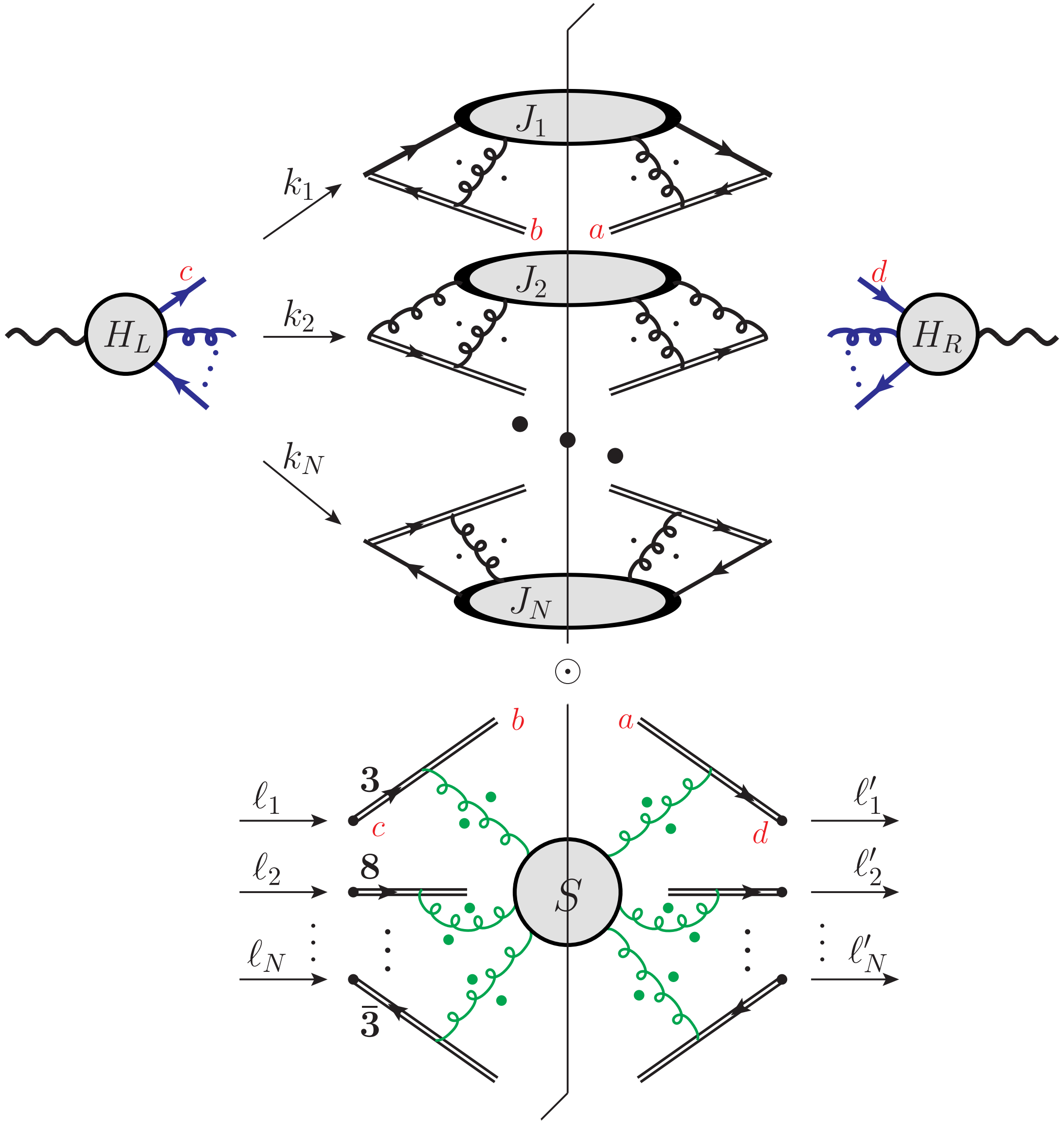
Inclusiveness and DIS factorization

❑ Inclusiveness and unitarity

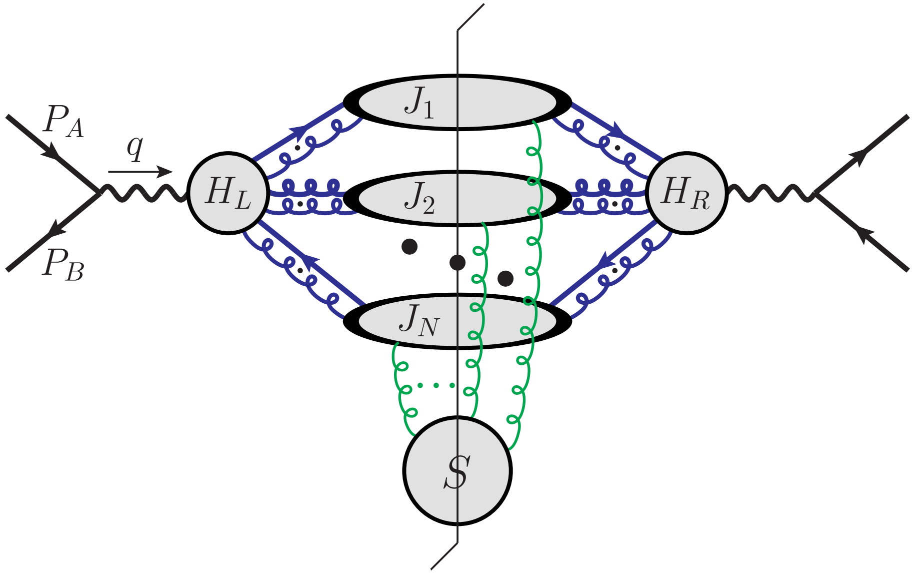


Inclusive hadron production at e^+e^- annihilation

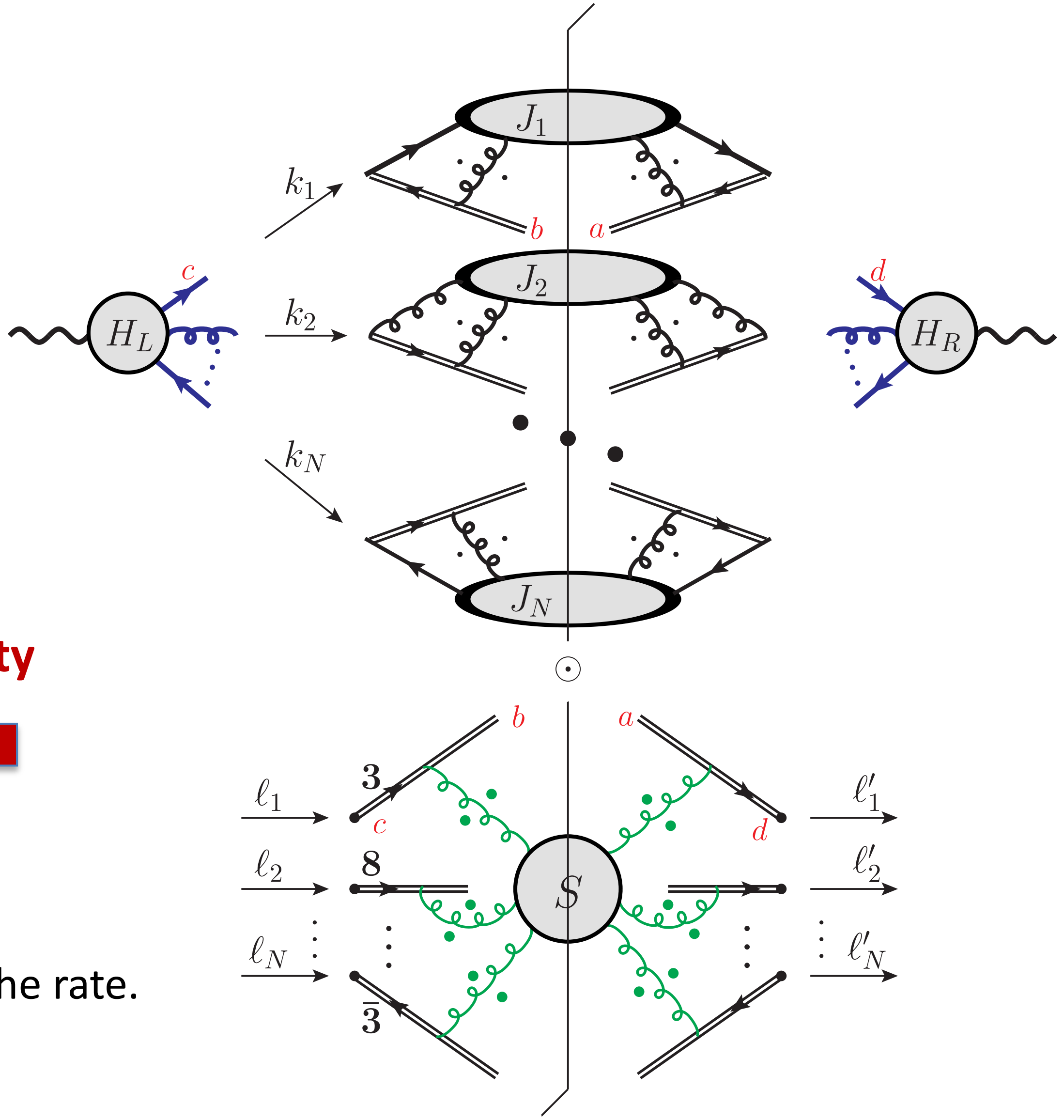




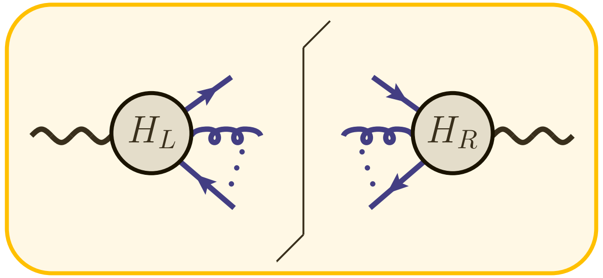
Inclusive hadron production at e^+e^- annihilation



Ward id.



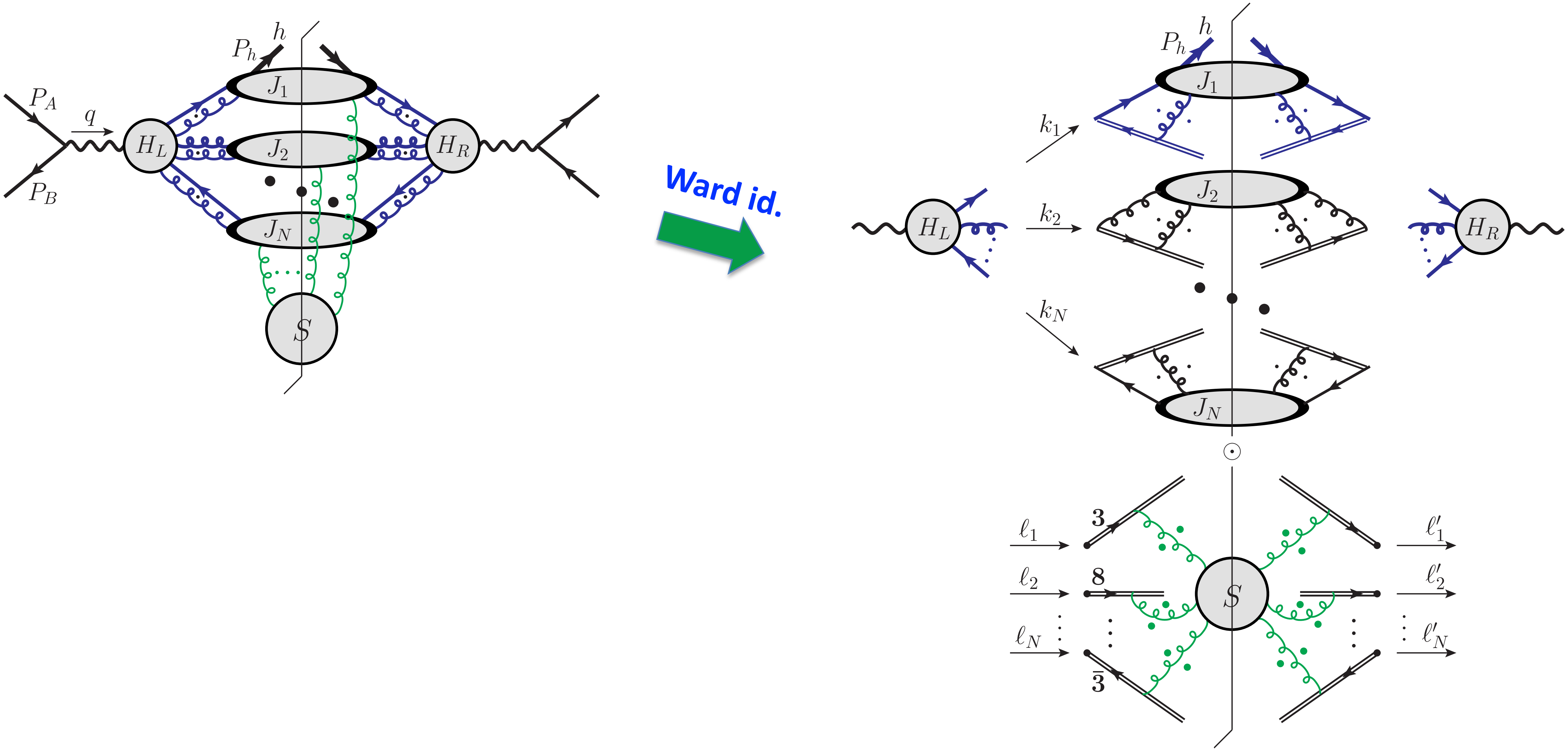
unitarity



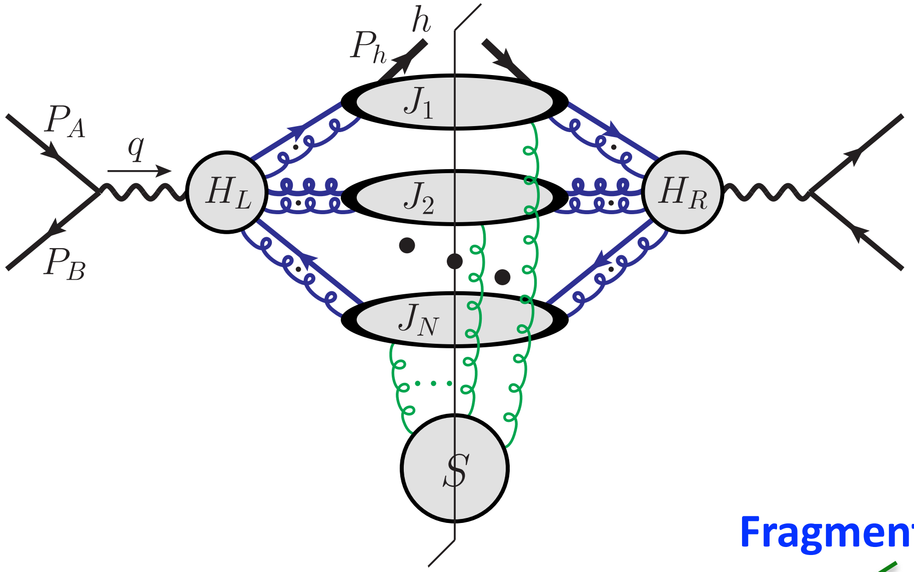
- All soft and collinear pinch singularities are canceled!
- Long-distance effects after hard interaction does not change the rate.

$$\sigma_{\text{tot}}(e^+e^- \to \text{hadrons}) = \sigma_{\text{tot}}(e^+e^- \to \text{partons}) + \mathcal{O}\left(\frac{1}{Q}\right)$$

One hadron inclusive production at e^+e^- annihilation



One hadron inclusive production at e^+e^- annihilation



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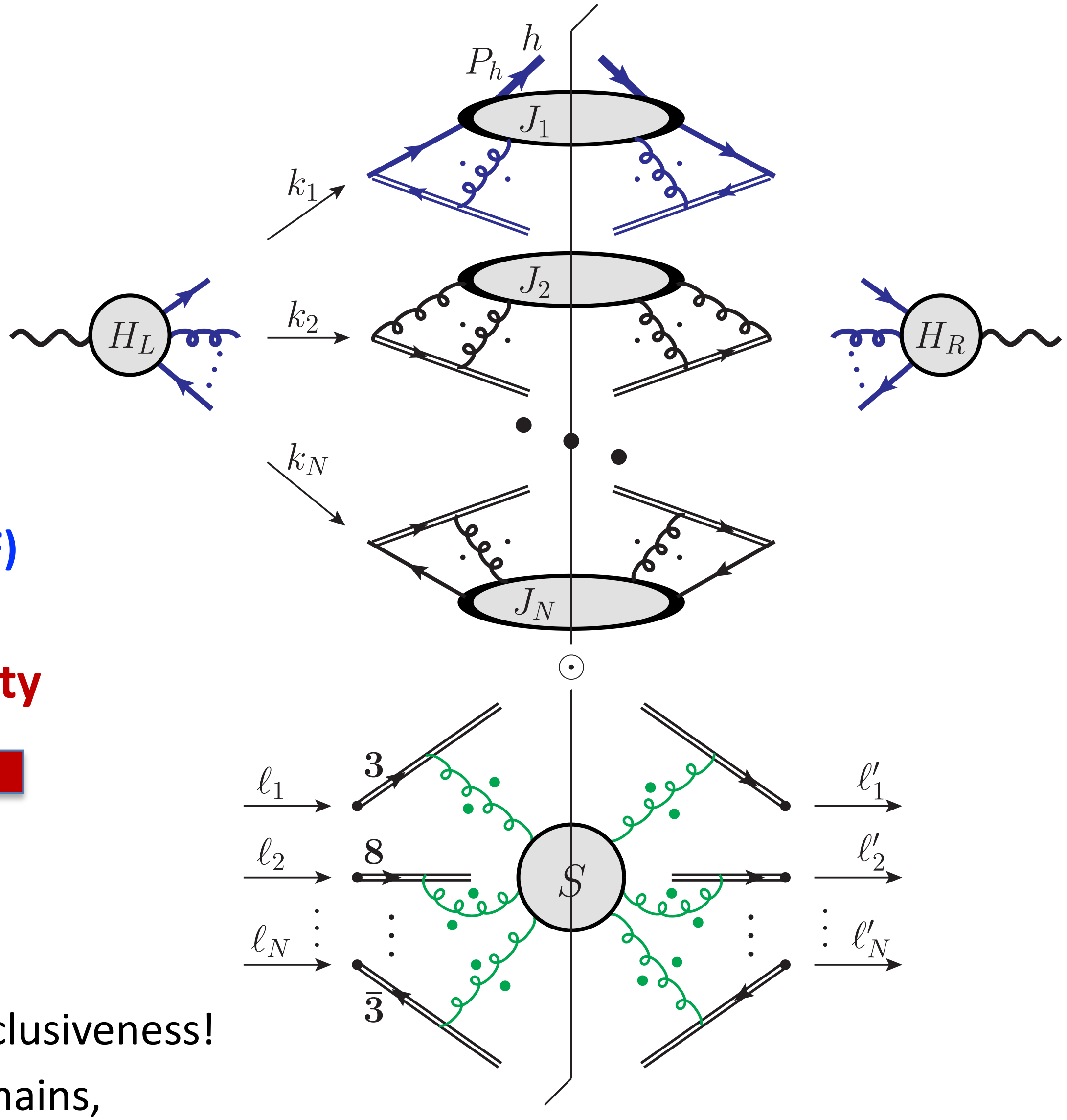
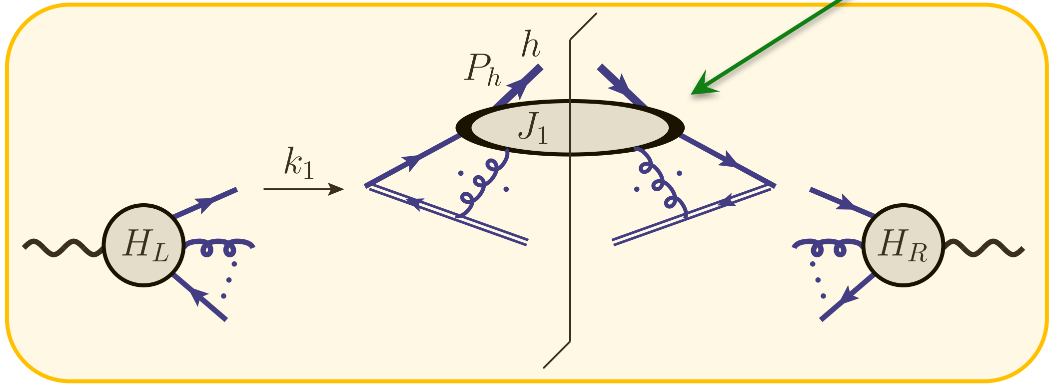


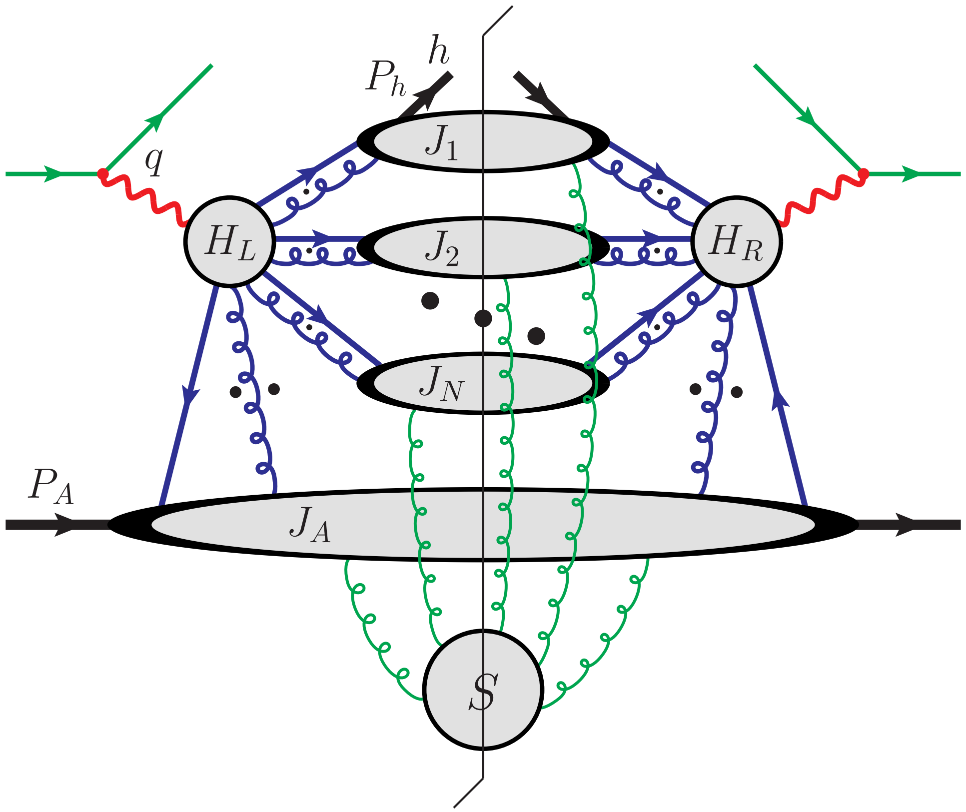
Fragmentation function (FF)

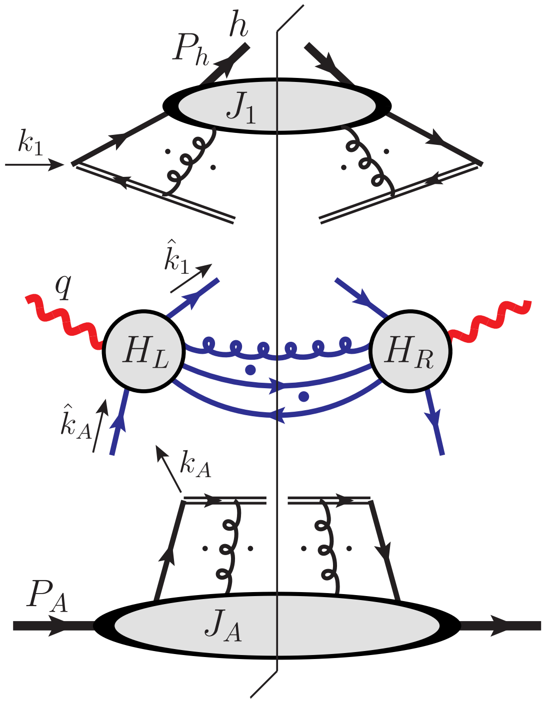


unitarity

- Soft and unobserved collinear pinches are canceled due to inclusiveness!
- Long-distance effect associated with the observed hadron remains,
- but is factorized from the hard interaction

One hadron inclusive production at *ep* collision

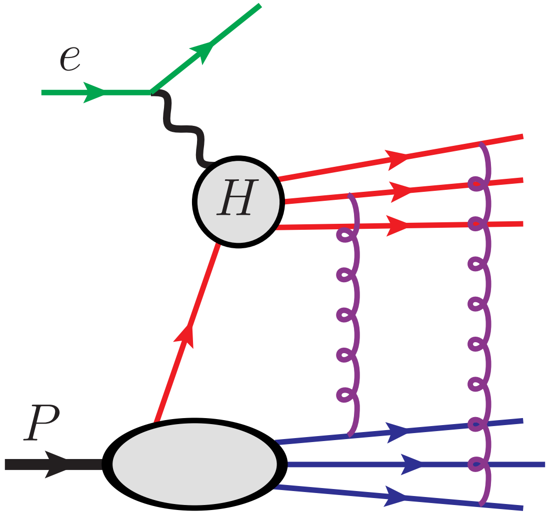




- Initial-state long-distance effect $\Longrightarrow$ **PDF**
- Final-state long-distance effect $\Longrightarrow$ **FF**
- Their communication is power suppressed

Inclusive processes vs. Exclusive processes

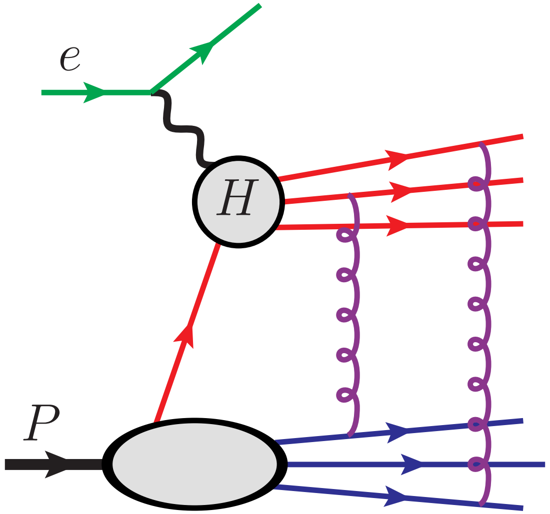
Inclusive process



- **Hadron is broken**
- **Bare color**
- **Inclusiveness**
- **One active parton line**

Inclusive processes vs. Exclusive processes

Inclusive process



- Hadron is broken
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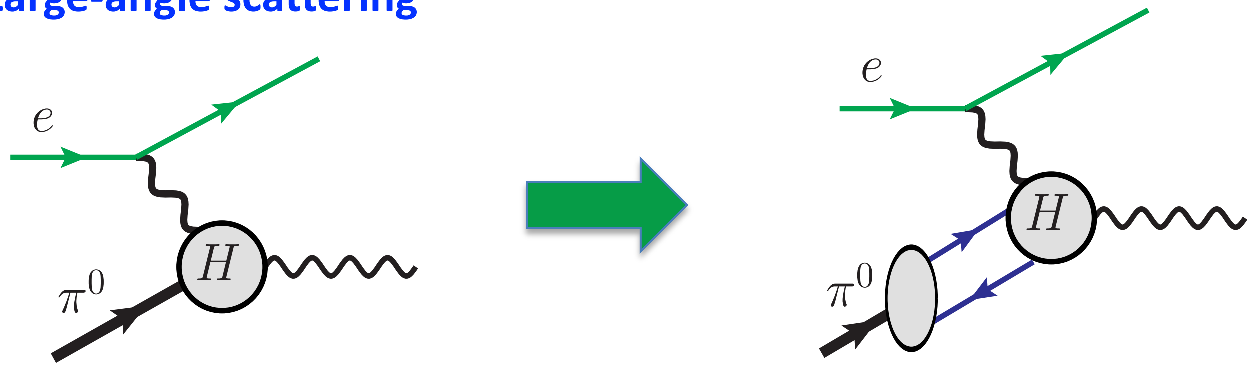
- Inclusiveness
- One active parton line

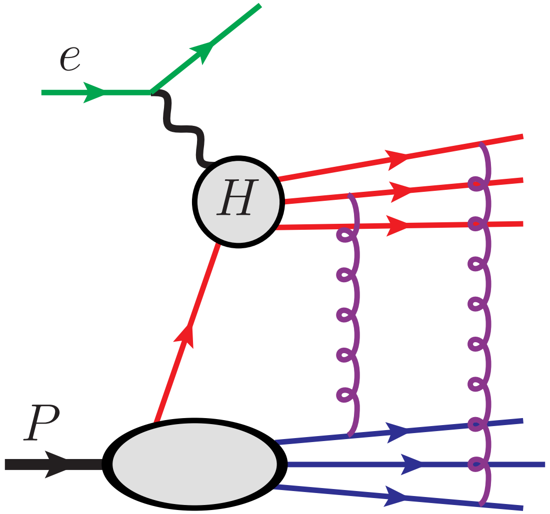
Exclusive process

- Large-angle scattering



Inclusive processes vs. Exclusive processes

Inclusive process



- Hadron is broken
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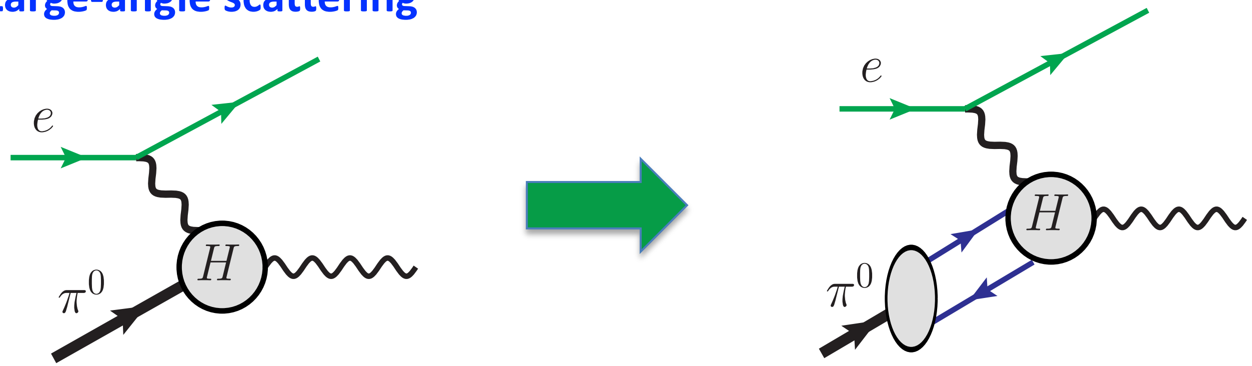
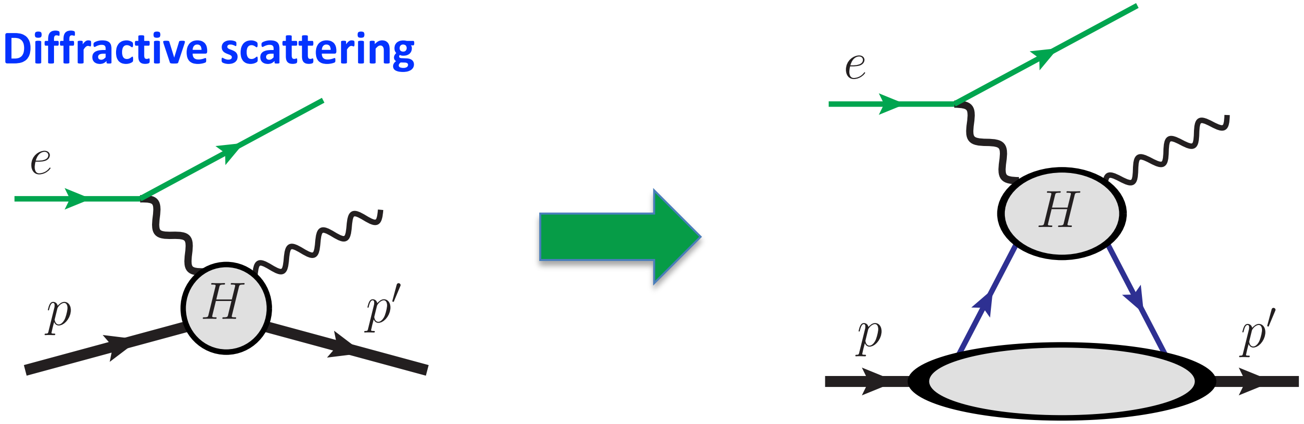
Exclusive process

- Large-angle scattering



- Diffractive scattering

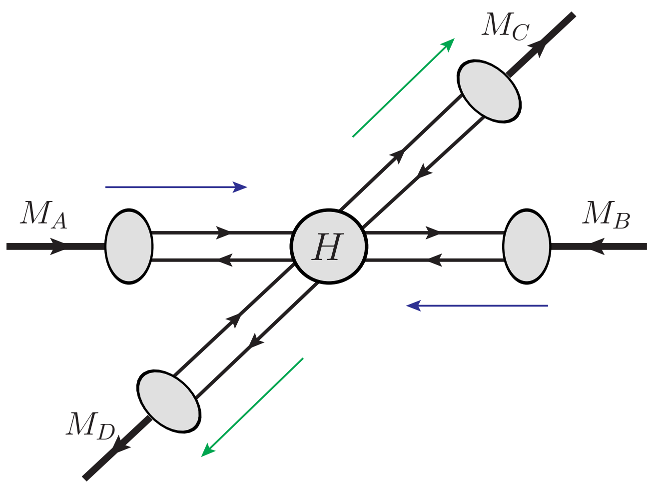


Not to break the hadron!

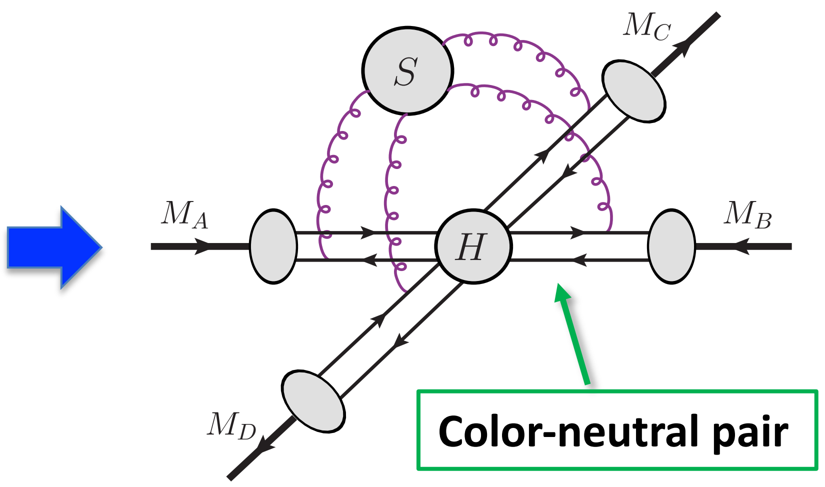
At least **two** active parton lines → Lower rate

Exclusive processes: large-angle $2 \to 2$ hadron scattering

Born level

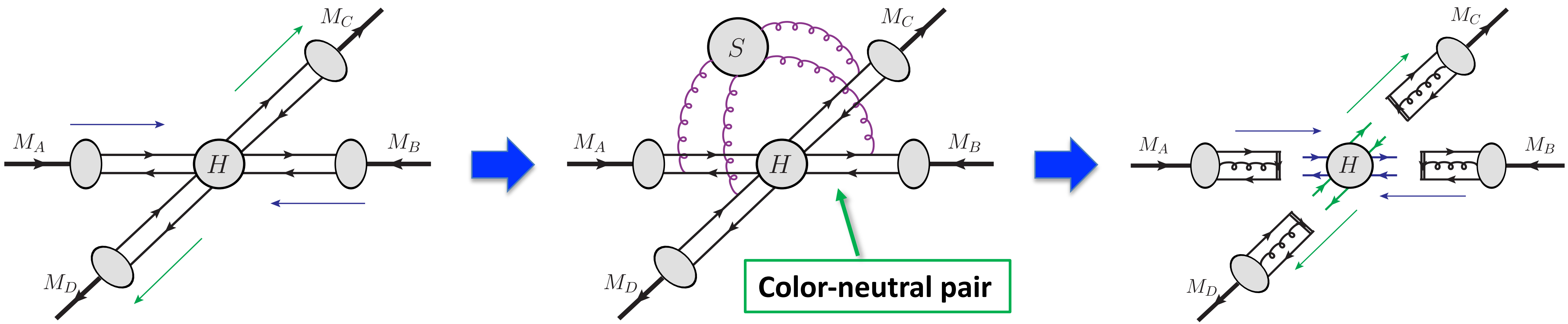


+ soft connection



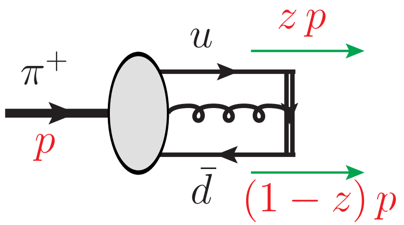
Exclusive processes: large-angle 2 → 2 hadron scattering

Born level | **+ soft connection** | **Factorization**



Meson distribution amplitude (DA)

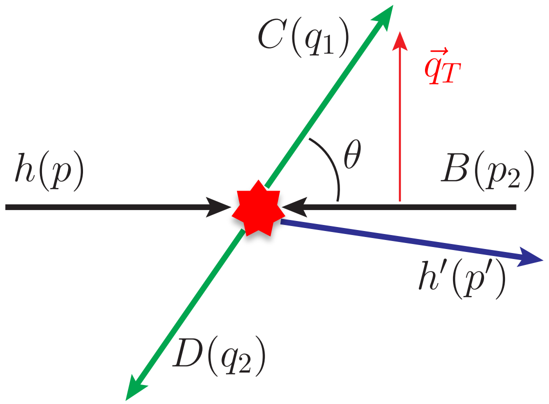
[Lepage & Brodsky, PRD 1980;
Adv. Ser. Direct. High Energy Phys. 5, 93 (1989)]



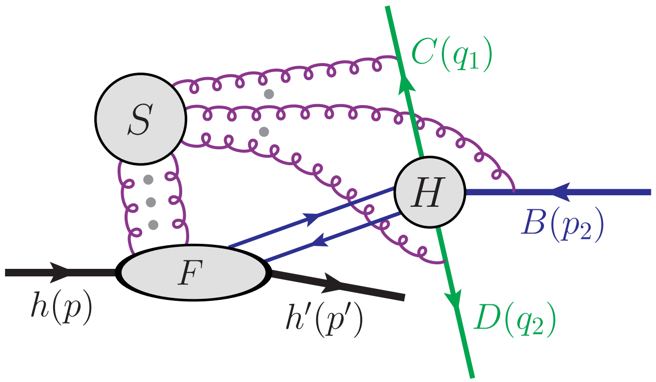
$$= D_{u/\pi^+}(z) = \int_{-\infty}^{\infty} \frac{dy^-}{4\pi}\, e^{izp^+y^-} \langle 0|\bar{d}(0)\gamma^+\gamma_5 W_n(0,y^-)u(y^-)|\pi^+(p)\rangle$$

Defined at amplitude level: "Lightcone wavefunction"

Exclusive processes: single-diffractive hard scattering



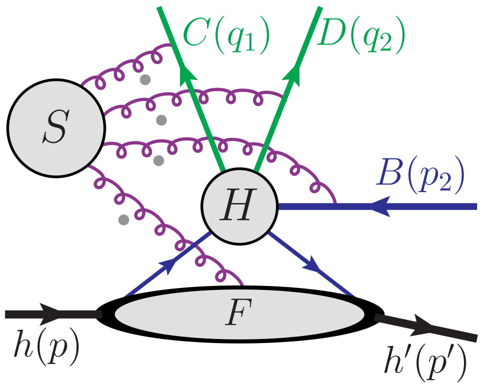
$2 \to 3$



ERBL region

$|x| < \xi$

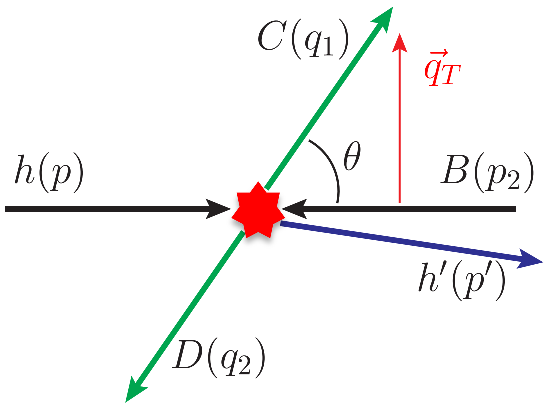
$+$



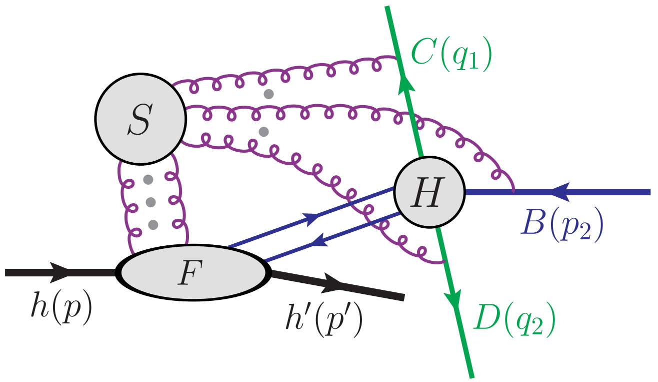
DGLAP region

$\xi < |x| < 1$

Exclusive processes: single-diffractive hard scattering



$2 \to 3$



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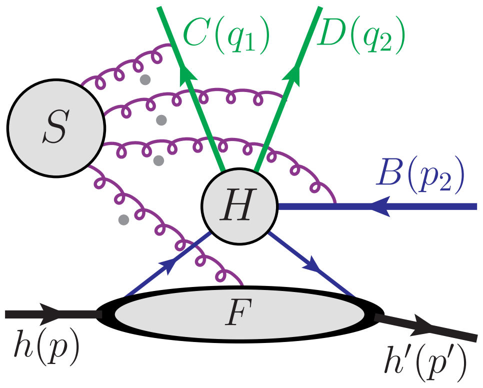
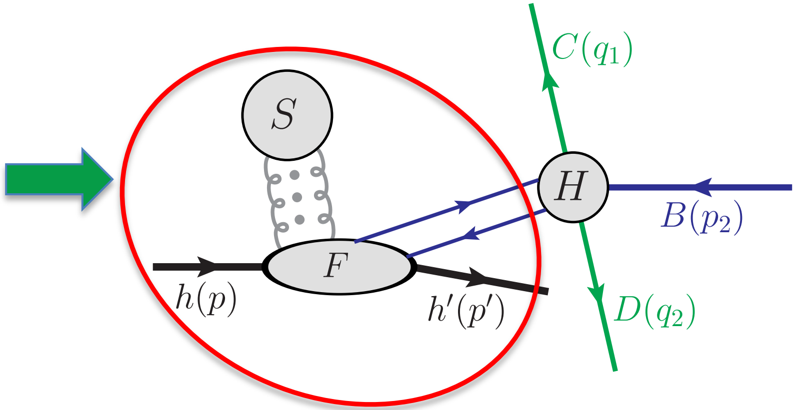


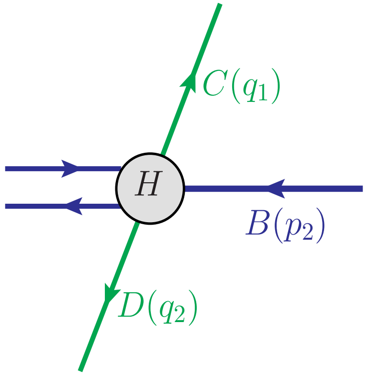
DGLAP region

$\xi < |x| < 1$

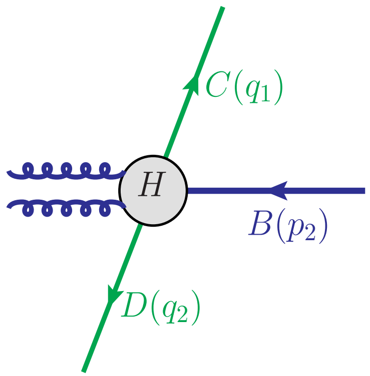
Soft gluons cancel when coupled to mesons!



$\text{GPD} \otimes$



or



Generalized parton distribution (GPD)

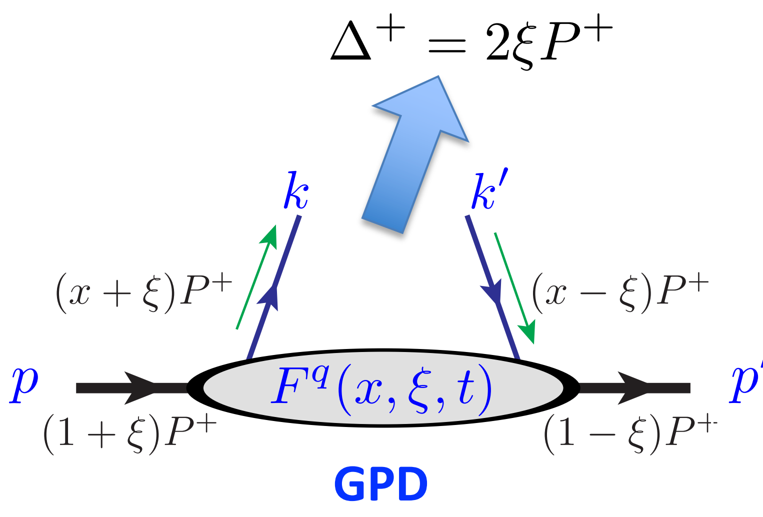
[Qiu & Yu, PRD 107 (2023), 014007]

Outline

- ❑ From inclusive to exclusive processes
- ❑ **Generalized parton distribution (GPD)**
- ❑ Phenomenology of GPD
- ❑ Inverse problem and global analysis of GPD

Generalized parton distribution (GPD)

D. Muller, et al., Fortsch. Phys. 42, 101 (1994)
X.-D. Ji, PRL 78, 610 (1997)
A. V. Radyushkin, PRD 56, 5524 (1997)



$$P = \frac{p+p'}{2}$$
$$\Delta = p - p'$$
$$t = \Delta^2$$
$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

Hadron diffraction $p \to p'$

$$x = \frac{(k+k')^+}{(p+p')^+}$$

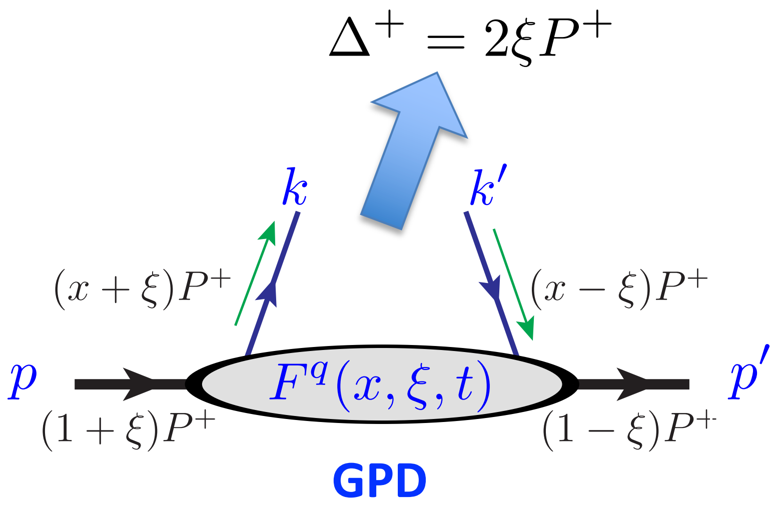
← **parton momentum**

$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2)\gamma^+ q(-z^-/2) | p \rangle$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

Generalized parton distribution (GPD)

❑ Unpolarized and polarized GPDs

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\,\bar{q}(z^-/2)\,\gamma^+ q(-z^-/2)|p\rangle$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}\,(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}\,(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$
$$\widetilde{F}^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\,\bar{q}(z^-/2)\,\gamma^+\gamma_5\, q(-z^-/2)|p\rangle$$
$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}\,(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}\,(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right]$$



Generalized parton distribution (GPD)

Unpolarized and polarized GPDs

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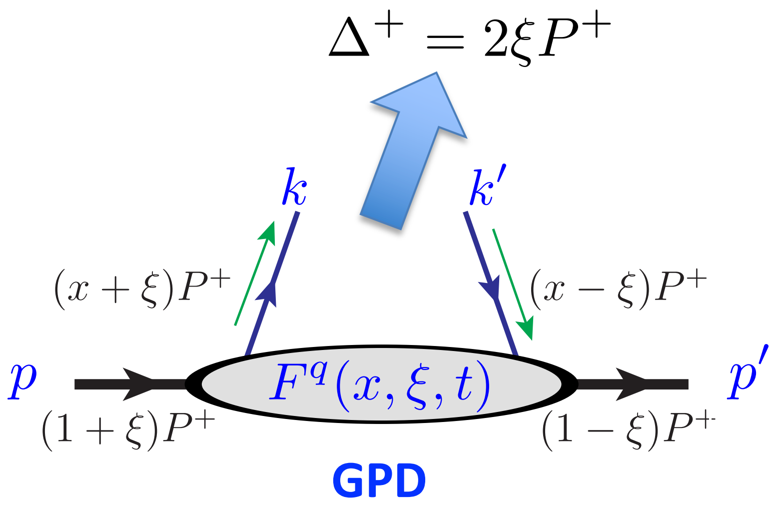

GPDs combine PDF and Distribution Amplitude (DA)

Forward limit: $H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x)$

Generalized parton distribution (GPD)

❑ Unpolarized and polarized GPDs

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\,\bar{q}(z^-/2)\,\gamma^+ q(-z^-/2)|p\rangle$$

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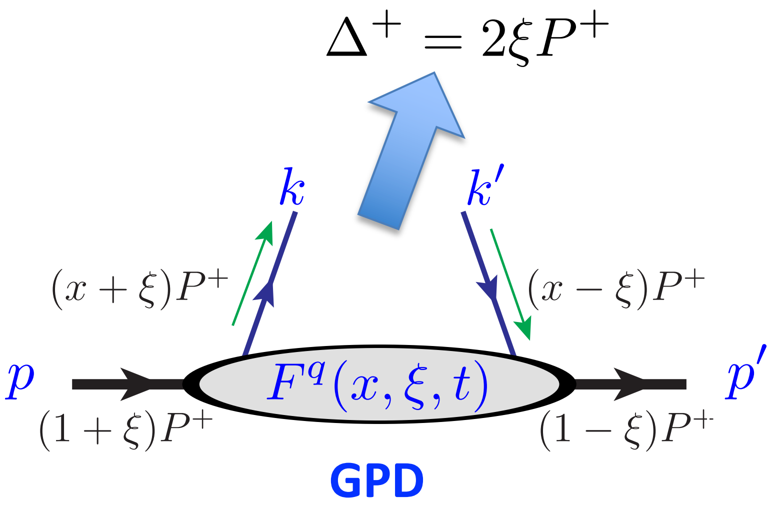
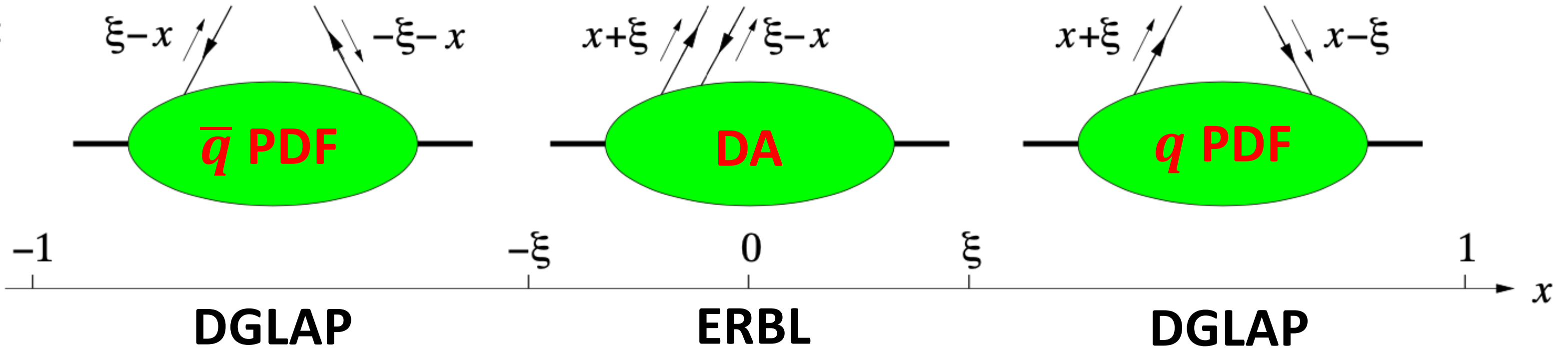


❑ GPDs combine PDF and Distribution Amplitude (DA)

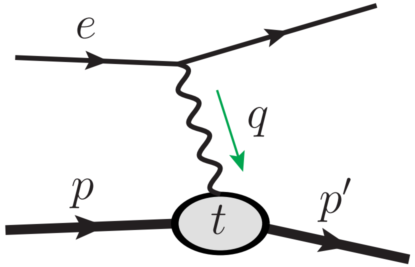
Forward limit: $H^q(x,0,0) = q(x), \quad \widetilde{H}^q(x,0,0) = \Delta q(x)$

***x*-regions:**



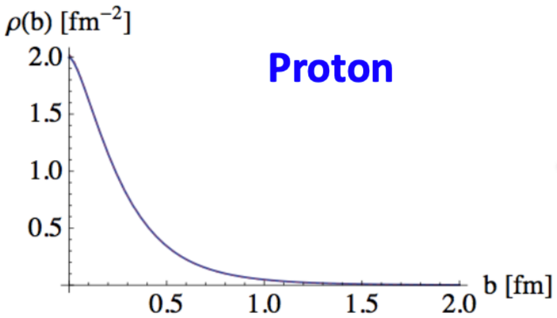
GPD and 3D tomography

❑ **Diffraction probes spatial density**



F. T.

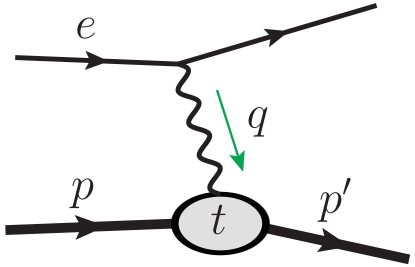
$F_{1,2}(t)$



Electric charge radius

GPD and 3D tomography

- Diffraction probes spatial density



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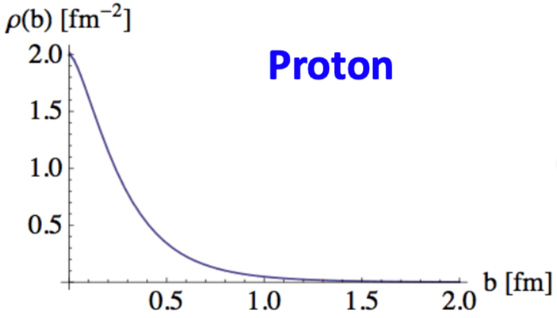
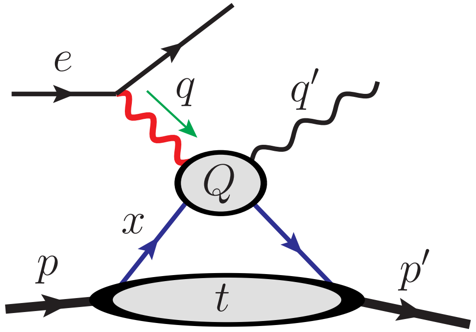


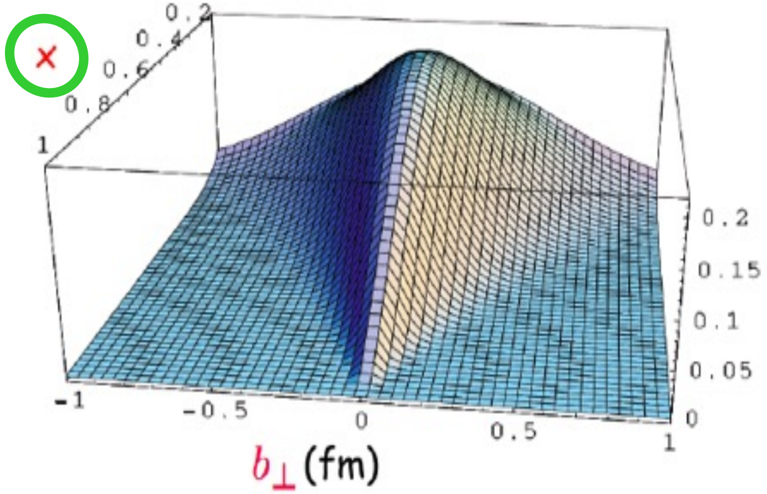
Electric charge radius

- ***Two-scale*** diffraction probes 3D tomography



F. T.

3D image



Proton radius in terms of parton contents

$$f_i(x, \boldsymbol{b}_T) = \int d^2\boldsymbol{\Delta}_T\, e^{i\boldsymbol{\Delta}_T\cdot\boldsymbol{b}_T} F_i(x, 0, -\Delta_T^2)$$

Parton density in $dx\, d^2\boldsymbol{b}_T$

[M. Burkardt, 2000, 2003]

GPD and mechanical form factor

❑ PDF and momentum fraction

$$\int_0^1 dx\, x\, f_{i/P}(x) =$$ **momentum fraction of P carried by parton i**

more formally

$$= \frac{1}{2(P^+)^2} \langle P| T_i^{++}(0) |P\rangle = A_i \qquad \sum_{i=q,g} A_i = 1$$

energy-momentum tensor $T^{\mu\nu}$

GPD and mechanical form factor

❑ PDF and momentum fraction

$$\int_0^1 dx\, x\, f_{i/P}(x) = \text{ momentum fraction of } P \text{ carried by parton } i$$

more formally

$$= \frac{1}{2(P^+)^2} \langle P| T_i^{++}(0) |P\rangle = A_i \qquad \sum_{i=q,g} A_i = 1$$

energy-momentum tensor $T^{\mu\nu}$

❑ GPD and mechanical/gravitational form factor

$$\int_{-1}^1 dx\, x\, F_i(x,\xi,t) = \frac{1}{2(P^+)^2}\langle p'|T_i^{++}(0)|p\rangle \quad \propto \quad \bar{u}(p')\left[\underbrace{\left(A_i(t)+\xi^2 D_i(t)\right)}_{\int_{-1}^1 dx\, x\, H_i(x,\xi,t)}\gamma^+ - \underbrace{\left(B_i(t)-\xi^2 D_i(t)\right)}_{\int_{-1}^1 dx\, x\, E_i(x,\xi,t)}\frac{i\sigma^{+\Delta}}{2m}\right]u(p)$$

Angular momentum sum rule

$$J_i = \lim_{t\to 0}\int_{-1}^1 dx\, x\,[H_i(x,\xi,t)+E_i(x,\xi,t)]$$

$$i = q, g$$

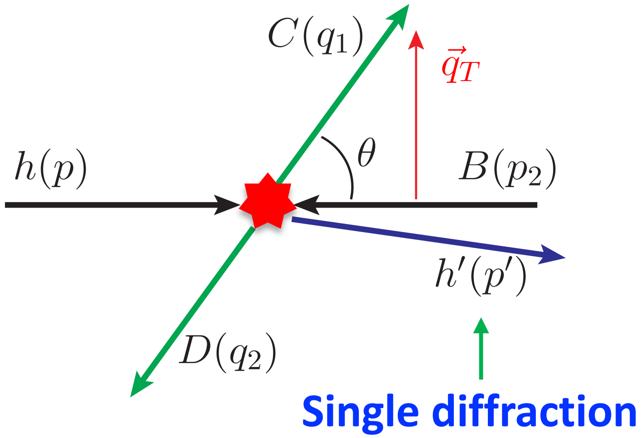
[X.-D. Ji, 1997]

Outline

- ❑ From inclusive to exclusive processes
- ❑ Generalized parton distribution (GPD)
- ❑ **Phenomenology of GPD**
- ❑ Inverse problem and global analysis of GPD

Single-Diffractive Hard Exclusive Process (SDHEP)

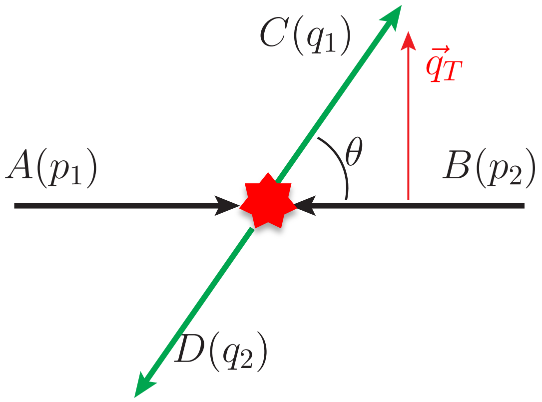
$$h(p) + B\,(p_2) \to h'\,(p') + C\,(q_1) + D\,(q_2)$$



Two scales:

- **Hard** $\boldsymbol{q_T}$
- **Soft** $t = (p - p')^2$

$$A(p_1) + B(p_2) \to C(q_1) + D(q_2)$$

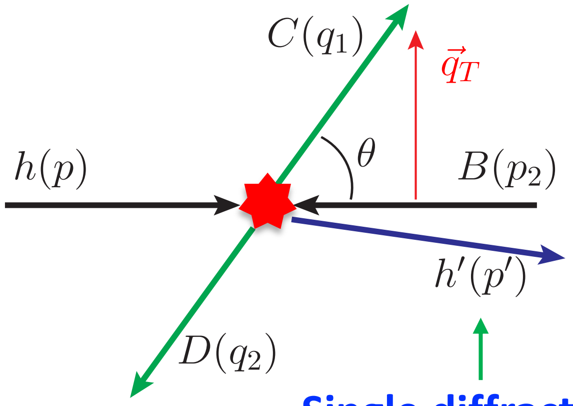


Large-angle 2-to-2 exclusive scattering

$2 \to 3$: *minimal*
kinematic configuration!

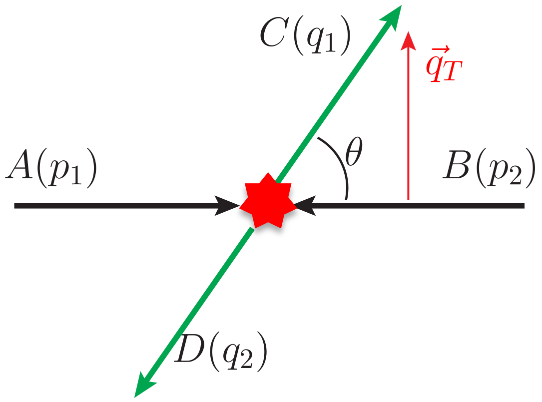
Single-Diffractive Hard Exclusive Process (SDHEP)

$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$



Single diffraction

$A(p_1) + B(p_2) \to C(q_1) + D(q_2)$



Large-angle 2-to-2 exclusive scattering

Two scales:

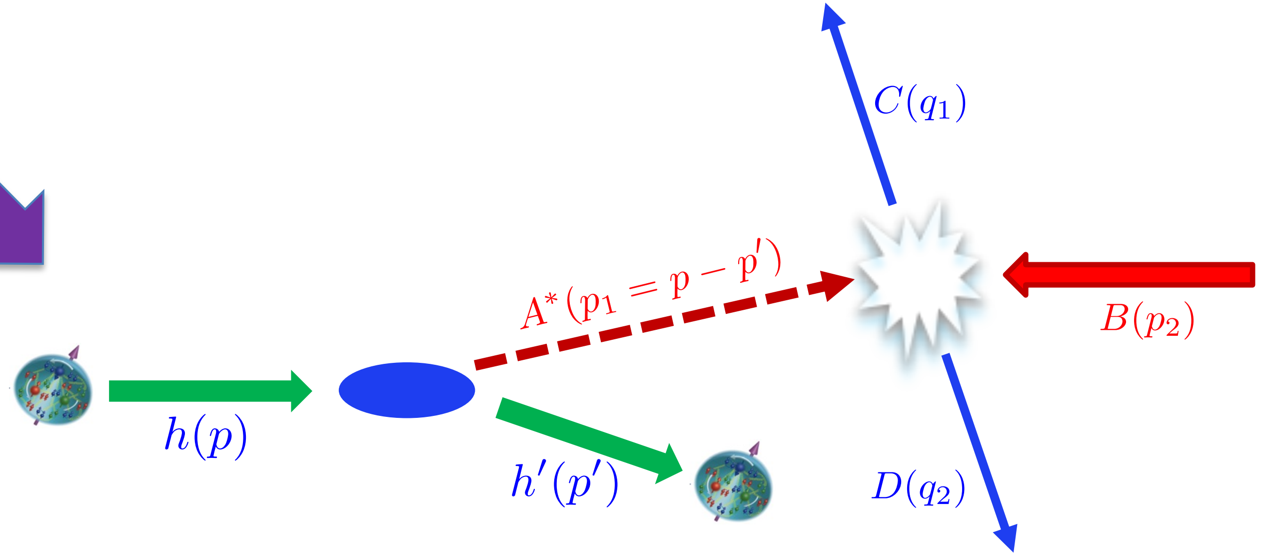
- **Hard q_T**
- **Soft $t = (p - p')^2$**

➢ Two-stage paradigm

$$N(p) \to N(p') + A^*(p_1 = p - p')$$

factorize

$$A^*(p_1) + B(p_2) \to C(q_1) + D(q_2)$$

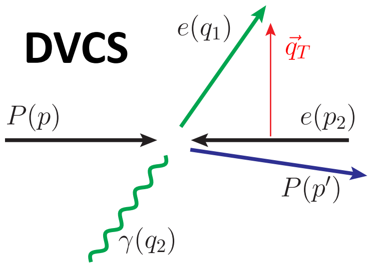


***Necessary* for factorization:** $q_T \gg \sqrt{-t} \simeq \Lambda_{QCD}$

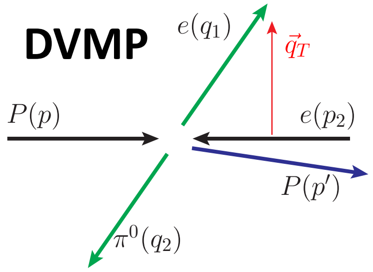
Classification of SDHEPs

❑ Electro-production (JLab, EIC, …)

DVCS

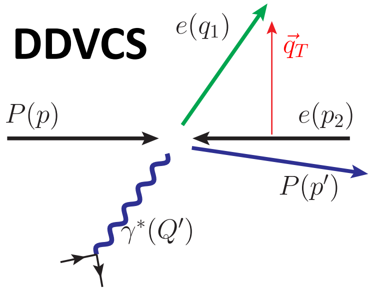


DVMP



add virtuality →

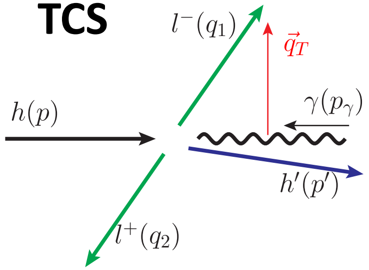
DDVCS

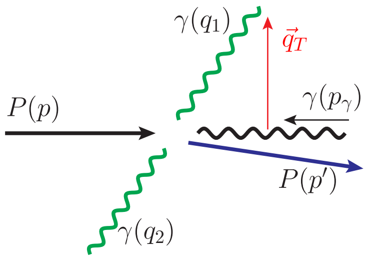


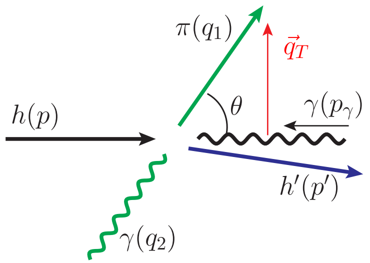
DHQP…

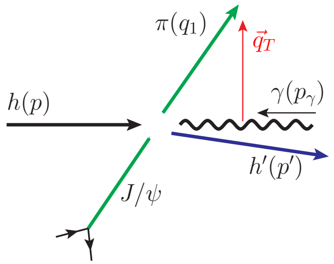
❑ Photo-production (JLab, EIC, …)

TCS



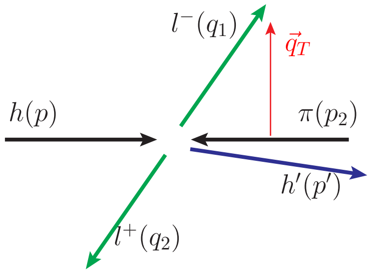


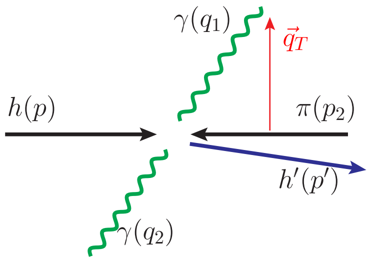




…

❑ Meso-production (AMBER, J-PARC, …)





…

Generic discussion

[Qiu, Yu, PRD 107 (2023), 014007]

Classification of SDHEPs

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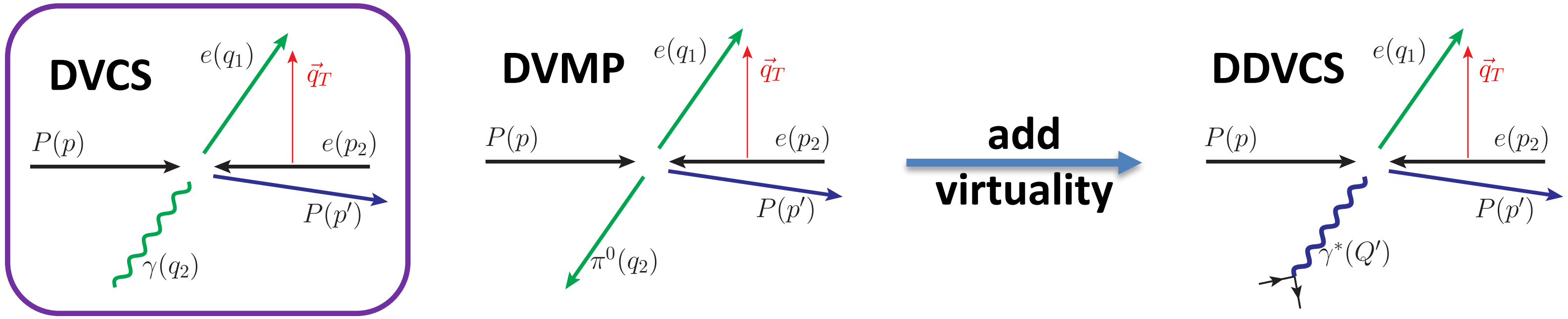

DHQP…

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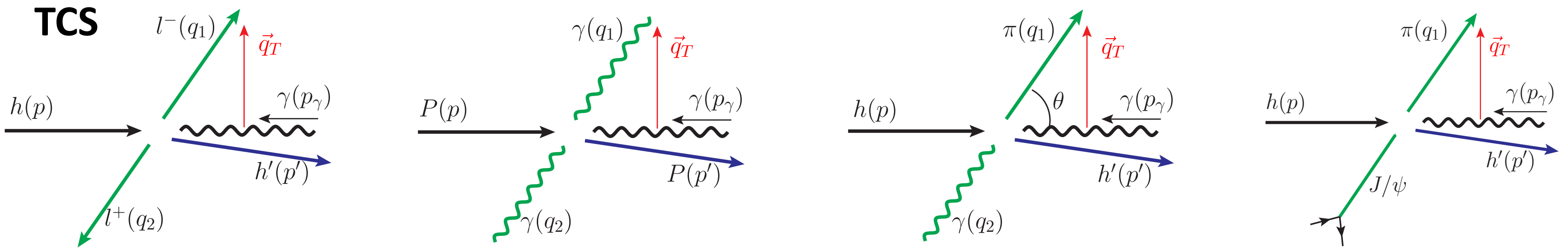

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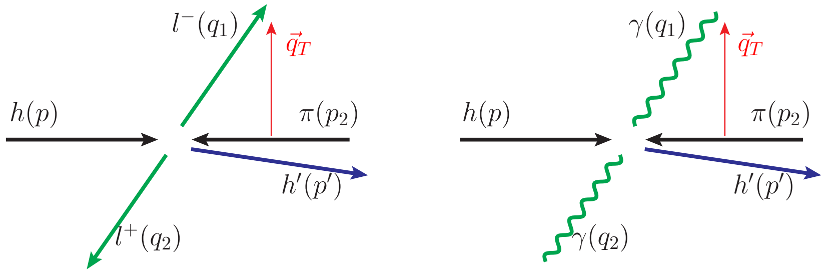


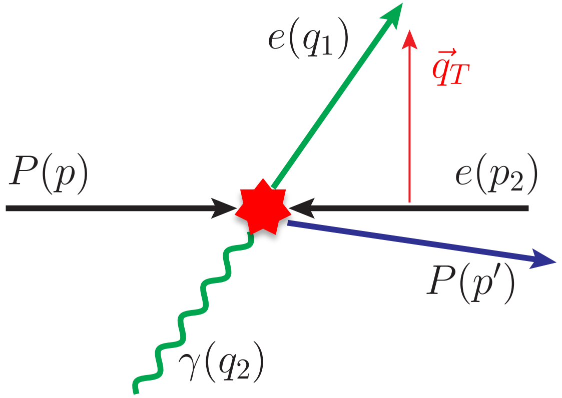
…

Generic discussion

[Qiu, Yu, PRD 107 (2023), 014007]

Deeply Virtual Compton Scattering (DVCS) as an SDHEP

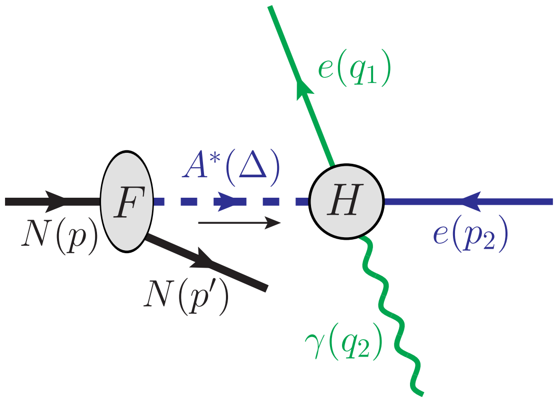
$$N(p) + e(p_2) \to N(p') + e(q_1) + \gamma(q_2)$$



Photon electroproduction

$q_T \gg \sqrt{-t}$

two-stage paradigm

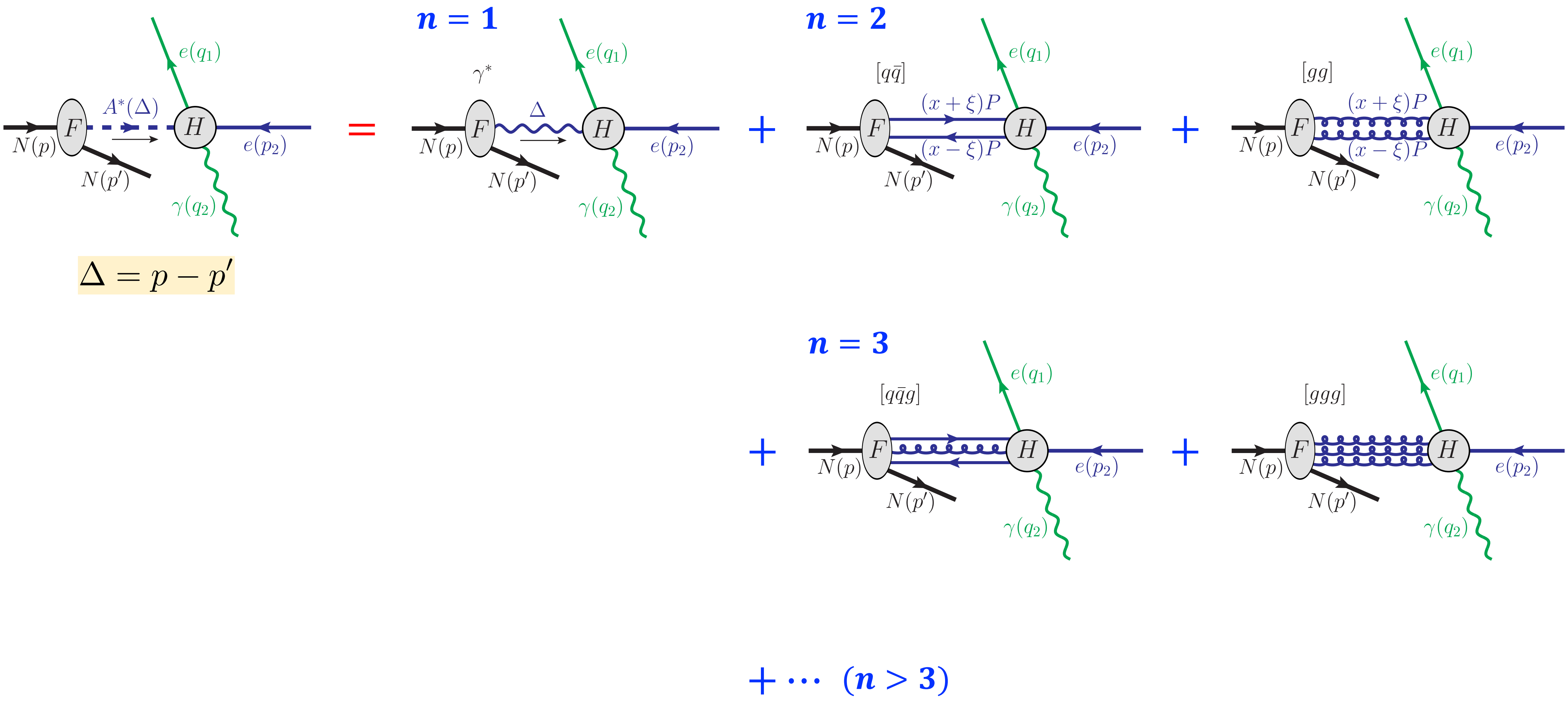


$\Delta = p - p'$

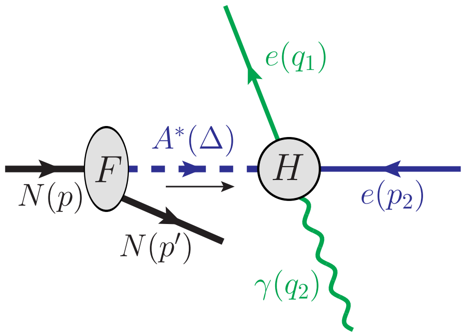
- **Focus on the full physical process**
- **Highlight the two-scale feature**

Qiu, Sato, Yu, PRD 2025, 2026

What is the A^*? --- channel expansion



Channel expansion and power counting

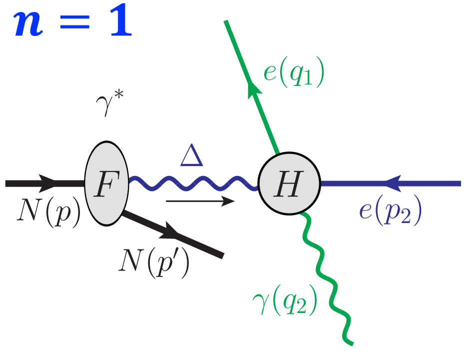


$\Delta = p - p'$

=

Bethe-Heitler (BH)

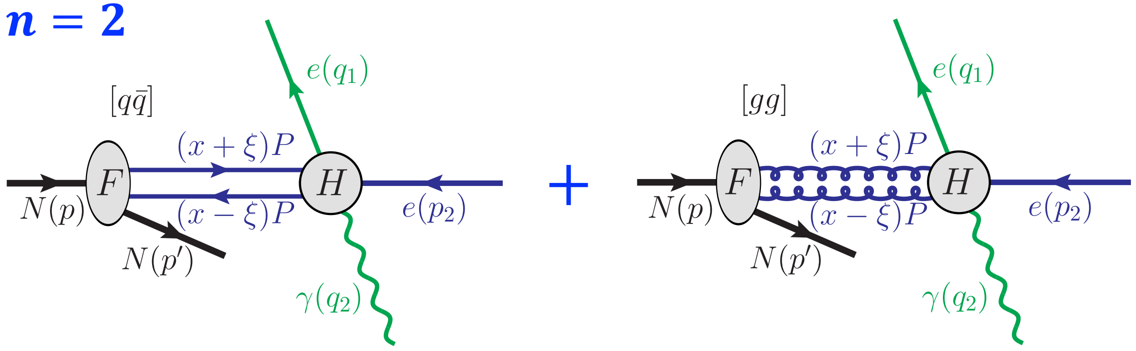
$n = 1$



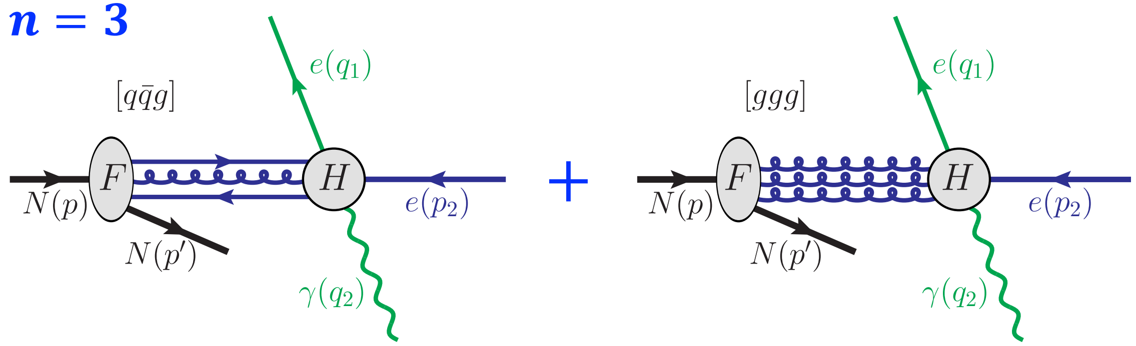
+

Deeply virtual Compton scattering (DVCS)

$n = 2$



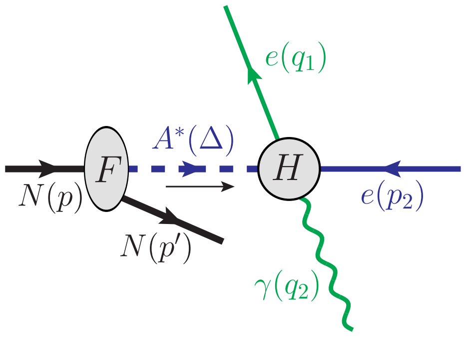
+ $n = 3$



$+ \cdots \quad (n > 3)$

BH and DVCS are treated on the *same* ground!

Channel expansion and power counting

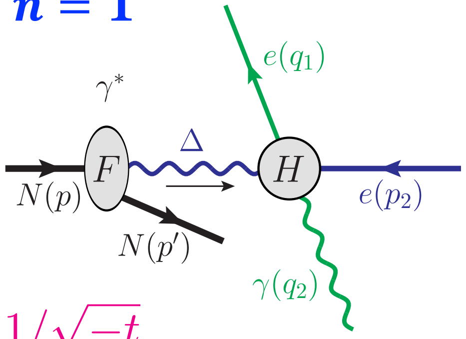


$\Delta = p - p'$

=

Leading power (LP)

$\boldsymbol{n = 1}$

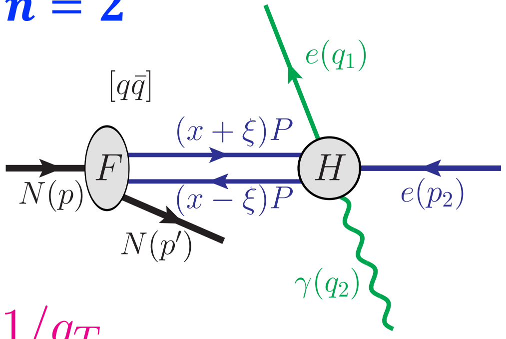


$1/\sqrt{-t}$

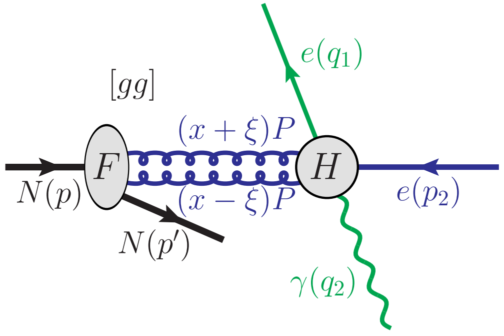
+

Next-to-leading power (NLP)

$\boldsymbol{n = 2}$



+

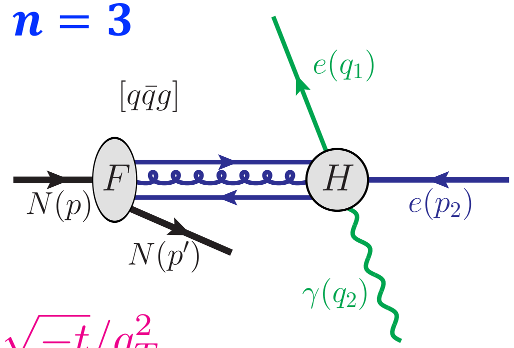


$1/q_T$

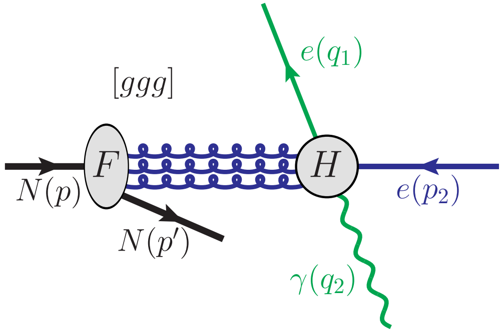
NNLP

+

$\boldsymbol{n = 3}$



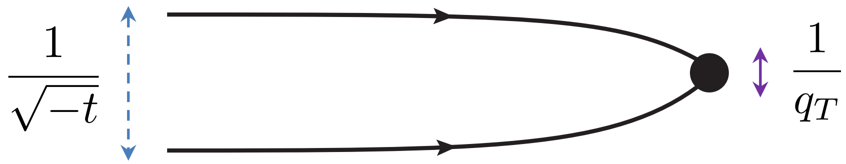
+



$\sqrt{-t}/q_T^2$

One more physically polarized parton in A^*

one more suppression of $\sqrt{-t}/q_T$



- **Consistent power counting**
- **Channel expansion = power expansion**

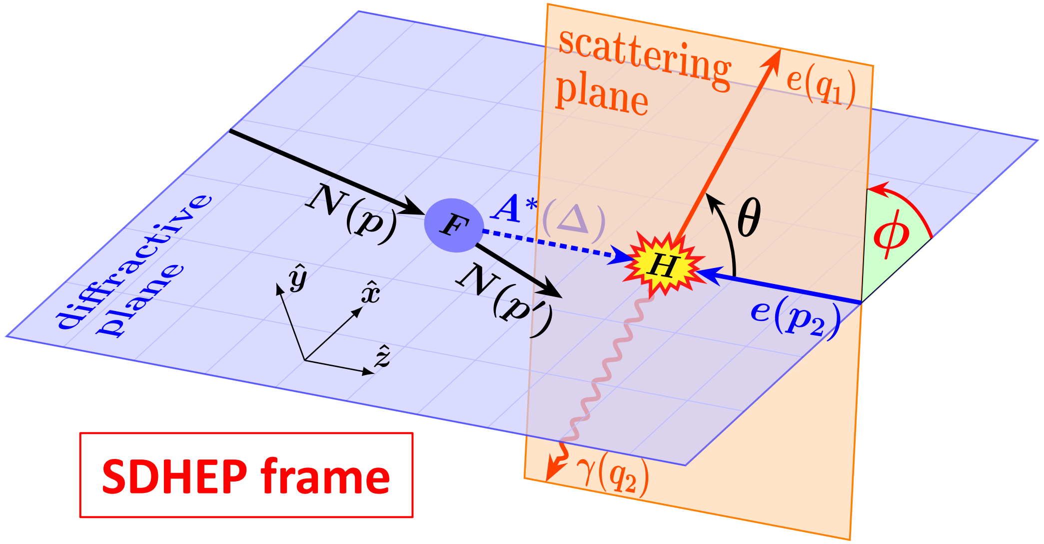
NNNLP…

$+ \cdots \; (\boldsymbol{n > 3}) \quad \mathcal{O}\left(-t/q_T^3\right)$

SDHEP frame and ϕ distribution

SDHEP frame observables: $(t,\ \xi,\ \theta,\ \phi)$

$$\mathcal{M}\left(t, \xi, \phi_S, \theta, \phi\right) = \sum_{A^*} e^{i(\lambda_A - \lambda_e)\phi} F_{N \to N A^*} \otimes G_{A^* e \to e\gamma}$$



SDHEP frame

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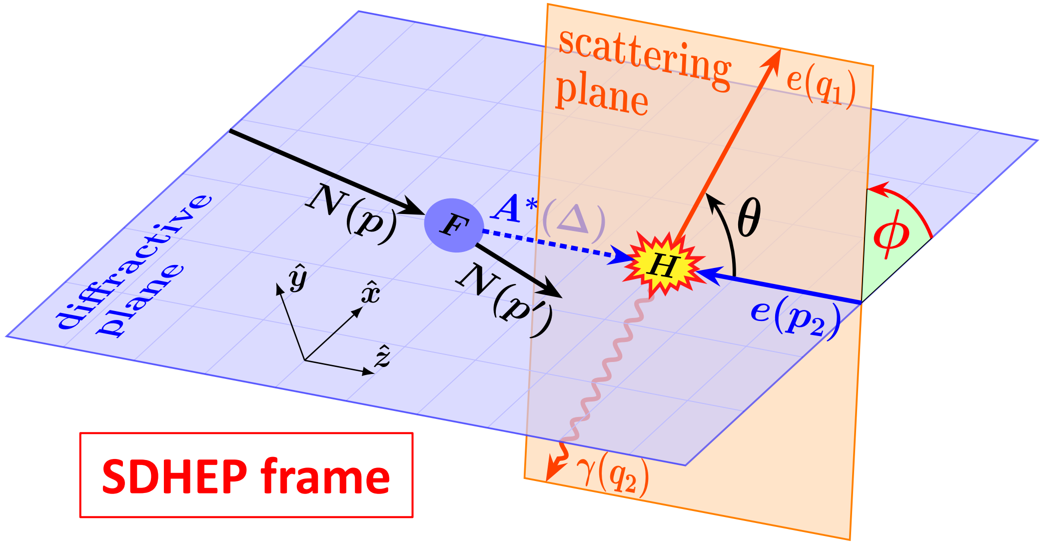
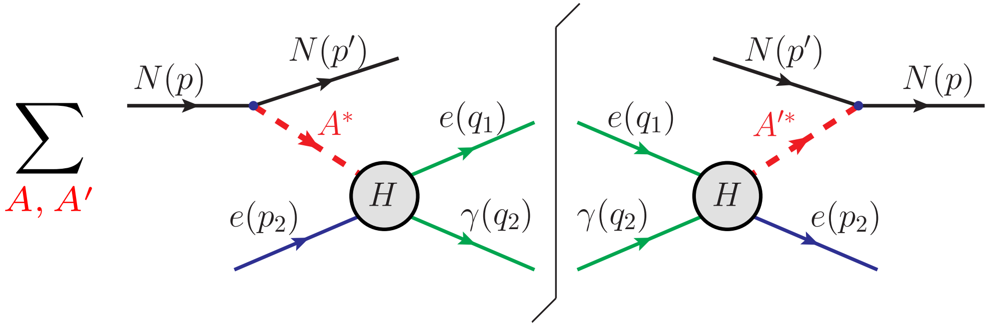


SDHEP frame

$|\mathcal{M}|^2$



Interference of $(\lambda_A,\ \lambda'_A)$ channels

$e^{i(\Delta\lambda_A)\phi}$ $\quad \Delta\lambda_A = \lambda_A - \lambda'_A$

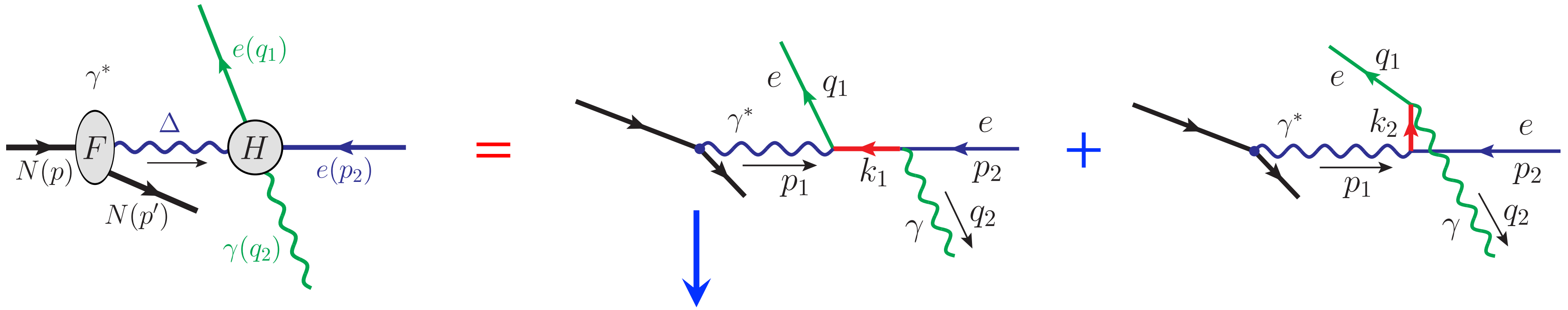
$\cos[(\Delta\lambda_A)\phi]$

$\sin[(\Delta\lambda_A)\phi]$

ϕ distribution is determined by the quantum interference of **different A^* spin states**!

$n = 1$: γ^* channel --- BH subprocess

- **Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$**



$$F_N^{\mu}(p,p') = \langle N(p')|J^{\mu}(0)|N(p)\rangle = \bar{u}(p',s')\left[F_1(t)\,\gamma^{\mu} - F_2(t)\,\frac{i\sigma^{\mu\nu}\Delta_{\nu}}{2m}\right]u(p,s)$$

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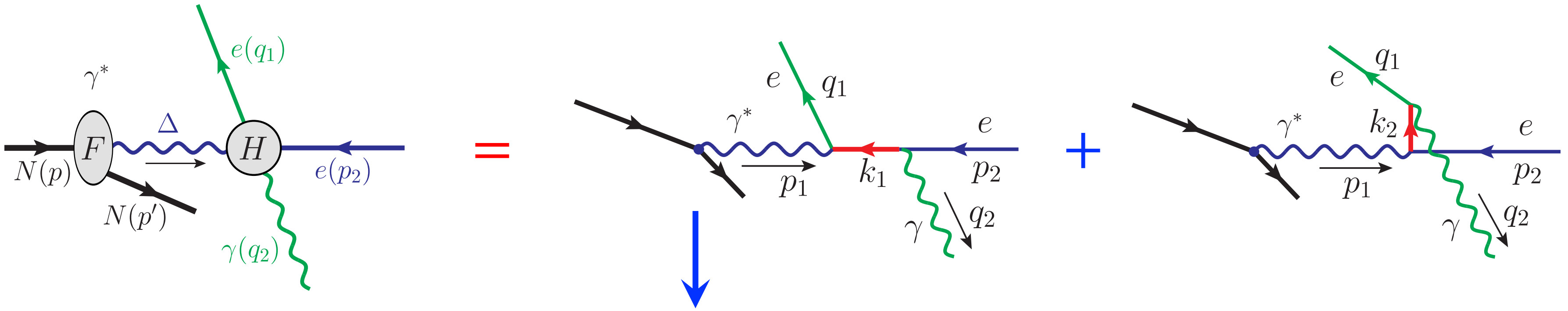


$$F_N^\mu(p,p') = \langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p',s')\left[F_1(t)\,\gamma^\mu - F_2(t)\,\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}\right]u(p,s)$$

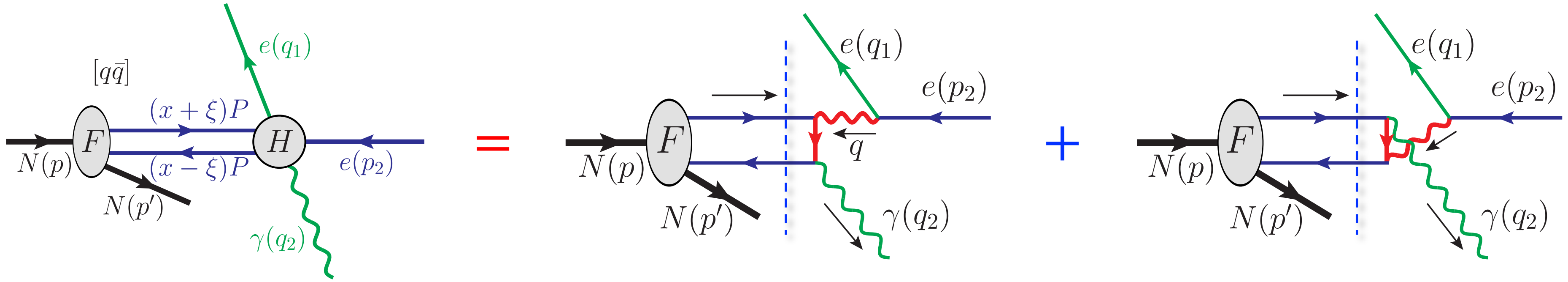
$$\mathcal{M}^{[1]} = \frac{-e}{t}\,F_N^\mu(p,p')\,G_\mu^\gamma(\Delta,p_2,q_1,q_2) = \frac{e}{t}\left[\sum_{\lambda=\pm1}(F_N\cdot\epsilon_\lambda^*)(\epsilon_\lambda\cdot G^\gamma) - 2(F_N\cdot n)(\bar{n}\cdot G^\gamma)\right]$$

$\lambda_A^\gamma = \pm 1$ $\qquad$ $\lambda_A^\gamma = 0$

- Only the transverse polarization γ_T^* is at LP $\quad \mathcal{O}(1/\sqrt{-t})$
- The longitudinal polarization γ_L^* is at NLP $\quad \mathcal{O}(1/q_T)$ ⟷ Combine with $n = 2$ (DVCS)

$n = 2$: $[q\bar{q}]$ channel --- DVCS (twist-2)

- **Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$**



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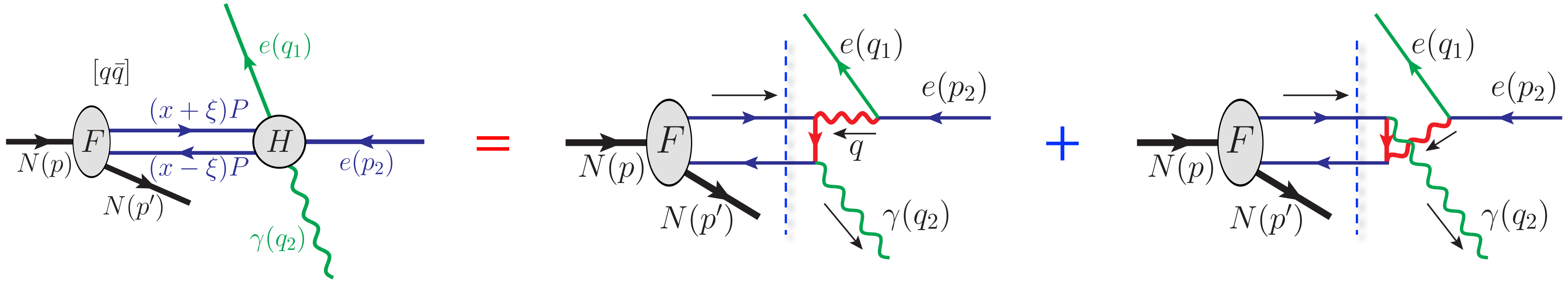

$$\mathcal{M}^{[2]} \simeq \sum_q \int_{-1}^{1} dx \left[F^q(x,\xi,t)\, G^q(x,\xi;\hat{s},\theta,\phi) + \widetilde{F}^q(x,\xi,t)\, \widetilde{G}^q(x,\xi;\hat{s},\theta,\phi) \right] + \mathcal{O}\left(\sqrt{-t}/q_T^2\right)$$

GPDs $(\boldsymbol{H}, \boldsymbol{E})$: defined with $\boldsymbol{\gamma}^+$

$(\widetilde{\boldsymbol{H}}, \widetilde{\boldsymbol{E}})$: defined with $\boldsymbol{\gamma}^+\boldsymbol{\gamma}_5$

$$\lambda_A^{q\bar{q}} = 0$$

$$G^q = g^q(\theta,\phi)\left(\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon}\right)$$

$$\widetilde{G}^q = \tilde{g}^q(\theta,\phi)\left(\frac{1}{x-\xi+i\epsilon} - \frac{1}{x+\xi-i\epsilon}\right)$$

Combine $n = 1$ and $n = 2$ channels

- **Amplitude level**

LP $\mathcal{M}_{\rm I}: \quad A^* = \gamma_T^* \; (\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_{\rm II}: \quad (1)\; A^* = \gamma_L^* \; (\lambda_A^\gamma = 0); \quad (2)\; A^* = [q\bar{q}] \; (\lambda_A^q = 0)$ $+ [gg]$ (high order)

NNLP: …

Combine $n = 1$ and $n = 2$ channels

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NNLP: …

- **Cross section level**

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \cdots$$

$\cos[(\Delta\lambda_A)\phi]$

$\sin[(\Delta\lambda_A)\phi]$

$$\Delta\lambda_A = \lambda_A - \lambda'_A$$

Combine $n = 1$ and $n = 2$ channels

❑ Amplitude level

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NNLP: …

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$\cos[(\Delta\lambda_A)\phi]$

$\sin[(\Delta\lambda_A)\phi]$

$$\Delta\lambda_A = \lambda_A - \lambda'_A$$

LP $|\mathcal{M}|^2_{\rm LP} = |\mathcal{M}_{\rm I}|^2$ No ϕ modulation. $\lambda_A^\gamma = +1$ and $\lambda_A^\gamma = -1$ do NOT interfere until NNLP.

Combine $n = 1$ and $n = 2$ channels

❑ Amplitude level

LP $\mathcal{M}_{\rm I}: \quad A^* = \gamma_T^* \; (\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_{\rm II}: \quad (1)\; A^* = \gamma_L^* \; (\lambda_A^\gamma = 0); \quad (2)\; A^* = [q\bar{q}] \; (\lambda_A^q = 0)$ $+ [gg]$ (high order)

NNLP: …

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$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \cdots$$

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NLP $|\mathcal{M}|^2_{\rm NLP} = 2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)$ ⇨ $\cos\phi$ or $\sin\phi$ modulation.

Combine $n = 1$ and $n = 2$ channels

❑ Amplitude level

LP $\mathcal{M}_{\rm I}: \quad A^* = \gamma_T^* \; (\lambda_A^\gamma = \pm 1)$

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NNLP: …

❑ Cross section level

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \cdots$$

$\cos[(\Delta\lambda_A)\phi]$

$\sin[(\Delta\lambda_A)\phi]$

$$\Delta\lambda_A = \lambda_A - \lambda_A'$$

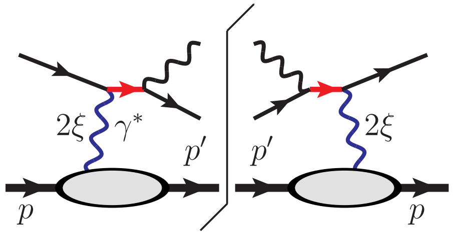
LP $|\mathcal{M}|^2_{\rm LP} = |\mathcal{M}_{\rm I}|^2$ No ϕ modulation. $\lambda_A^\gamma = +1$ and $\lambda_A^\gamma = -1$ do NOT interfere until NNLP.

NLP $|\mathcal{M}|^2_{\rm NLP} = 2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)$ ⇒ $\cos\phi$ or $\sin\phi$ modulation.

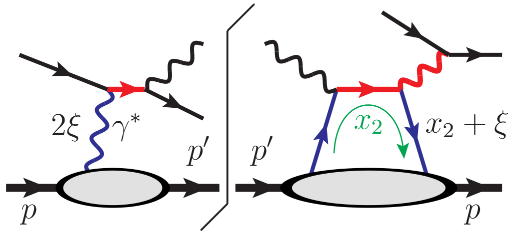
⇒ Interference of different numbers of particles.

⇓

Unique signal of GPDs.



"twist-2"

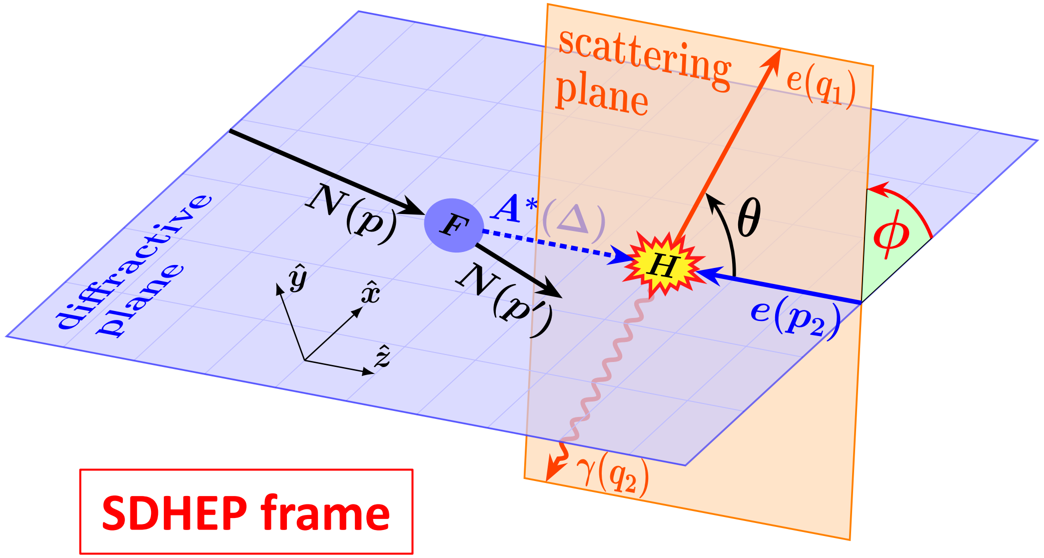


"twist-3"

Cross section within NLP: unpolarized proton

$$\frac{d\sigma}{dt\, d\xi\, d\cos\theta\, d\phi} = \frac{1}{2\pi} \frac{d\sigma^{\text{unpol}}}{dt\, d\xi\, d\cos\theta} \cdot \left[1 + A_{UU}^{\text{NLP}}(t,\xi,\cos\theta)\cos\phi + \lambda_e A_{UL}^{\text{NLP}}(t,\xi,\cos\theta)\sin\phi\right]$$

Interference of γ_T^* and GPD moments



SDHEP frame

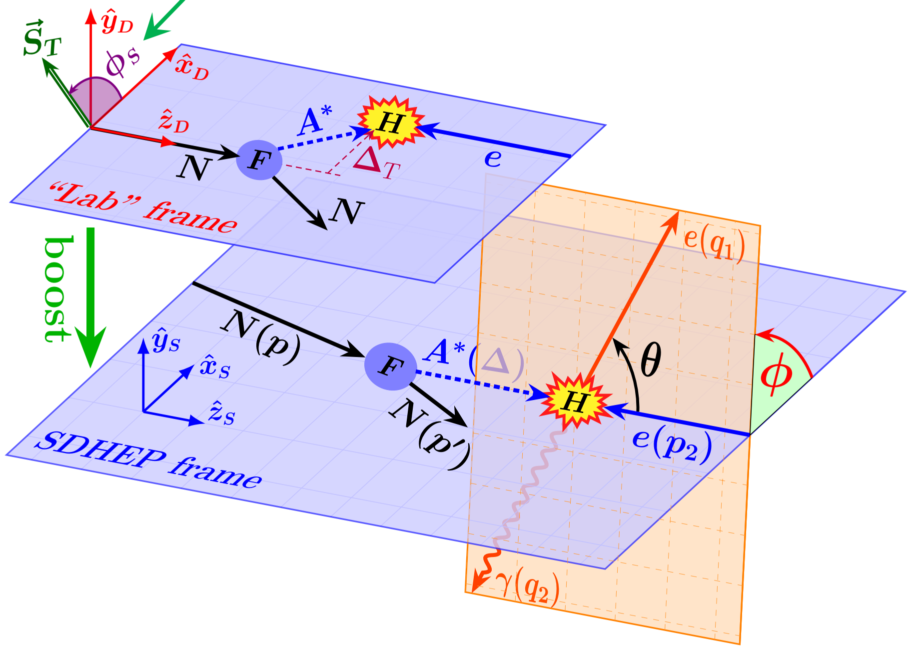
Cross section within NLP: introducing proton spin

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e\lambda_N A_{LL}^{\rm LP} + \lambda_e s_T A_{TL}^{\rm LP}\cos\phi_S$$
$$+ \left(A_{UU}^{\rm NLP} + \lambda_e\lambda_N A_{LL}^{\rm NLP}\right)\cos\phi + \left(\lambda_e A_{UL}^{\rm NLP} + \lambda_N A_{LU}^{\rm NLP}\right)\sin\phi$$
$$+ s_T\left(A_{TU,1}^{\rm NLP}\cos\phi_S\sin\phi + A_{TU,2}^{\rm NLP}\sin\phi_S\cos\phi\right)$$
$$+ \lambda_e s_T\left(A_{TL,1}^{\rm NLP}\cos\phi_S\cos\phi + A_{TL,2}^{\rm NLP}\sin\phi_S\sin\phi\right)\Big]$$

In the experimental setting (**fixed lab frame**),

- Nucleon spin vector $\vec{s}_N = (s_T, 0, \lambda_N)$
- Electron spin vector $\vec{s}_e = (0, 0, \lambda_e)$

ϕ_S: of nucleon transverse spin in the "Lab" frame



Subscripts: (nucleon, electron)

U = Unpolarized

L = Longitudinally polarized

T = Transversely polarized

Cross section within NLP: full polarization

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\text{unpol}}}{dt\, d\xi\, d\cos\theta}\cdot\Big[1+\lambda_e\lambda_N A_{LL}^{\text{LP}}+\lambda_e s_T A_{TL}^{\text{LP}}\cos\phi_S$$

$$+\left(A_{UU}^{\text{NLP}}+\lambda_e\lambda_N A_{LL}^{\text{NLP}}\right)\cos\phi+\left(\lambda_e A_{UL}^{\text{NLP}}+\lambda_N A_{LU}^{\text{NLP}}\right)\sin\phi$$

$$+\,s_T\left(A_{TU,1}^{\text{NLP}}\cos\phi_S\sin\phi+A_{TU,2}^{\text{NLP}}\sin\phi_S\cos\phi\right)$$

$$+\,\lambda_e s_T\left(A_{TL,1}^{\text{NLP}}\cos\phi_S\cos\phi+A_{TL,2}^{\text{NLP}}\sin\phi_S\sin\phi\right)\Big]$$

NLP: from γ_T^*-γ_L^* and γ_T^*-$[q\bar{q}]$ interference

$$A_{XX}^{\text{NLP}} = \frac{1}{\Sigma_{UU}^{\text{LP}}}\cdot\left(\frac{-t}{m\sqrt{\hat{s}}}\right)\Sigma_{XX}^{\text{NLP}}$$

$$\Sigma_{UU}^{\text{NLP}} = \frac{\Delta_T}{2m}\frac{1+\xi}{\xi}\left[\frac{2\sin\theta}{\xi}\left(F_1^2-\frac{t}{4m^2}F_2^2\right)-\frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_1\cdot\text{Re}\,V_{\mathcal{F}}\right)\right],$$

$$\Sigma_{LL}^{\text{NLP}} = -\frac{\Delta_T}{m}\left[\sin\theta(F_1+F_2)\left(\frac{1+\xi}{\xi}F_1+F_2\right)+\frac{3-\cos\theta}{\sin\theta}\left(M_2\cdot\text{Re}\,V_{\mathcal{F}}\right)\right],$$

$$\Sigma_{TL,1}^{\text{NLP}} = 2\sin\theta\,(F_1+F_2)\left[F_1+\left(\frac{\xi}{1+\xi}+\frac{t}{4\xi m^2}\right)F_2\right]+\frac{2(3-\cos\theta)}{\sin\theta}\left(M_3\cdot\text{Re}\,V_{\mathcal{F}}\right),$$

$$\Sigma_{TL,2}^{\text{NLP}} = 2\sin\theta\,(F_1+F_2)\left(F_1+\frac{t}{4m^2}F_2\right)-\frac{2(3-\cos\theta)}{\sin\theta}\left(M_4\cdot\text{Re}\,V_{\mathcal{F}}\right),$$

$$\Sigma_{UL}^{\text{NLP}} = -\frac{\Delta_T}{m}\frac{1+\xi}{\xi}\frac{3-\cos\theta}{\sin\theta}\left(M_1\cdot\text{Im}\,V_{\mathcal{F}}\right),$$

$$\Sigma_{LU}^{\text{NLP}} = -\frac{\Delta_T}{2m}\frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_2\cdot\text{Im}\,V_{\mathcal{F}}\right),$$

$$\Sigma_{TU,1}^{\text{NLP}} = \frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_3\cdot\text{Im}\,V_{\mathcal{F}}\right),$$

$$\Sigma_{TU,2}^{\text{NLP}} = \frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_4\cdot\text{Im}\,V_{\mathcal{F}}\right).$$

- **Linear in GPD moments** $V_{\mathcal{F}} = (\mathcal{H},\mathcal{E},\widetilde{\mathcal{H}},\widetilde{\mathcal{E}})^T$

$$\{\mathcal{H},\mathcal{E}\}(\xi,t)\equiv\sum_q e_q^2\int_{-1}^{1}dx\,\{H^q,E^q\}(x,\xi,t)\left[\frac{1}{x-\xi+i\epsilon}+\frac{1}{x+\xi-i\epsilon}\right]$$

$$\{\widetilde{\mathcal{H}},\widetilde{\mathcal{E}}\}(\xi,t)\equiv\sum_q e_q^2\int_{-1}^{1}dx\,\{\widetilde{H}^q,\widetilde{E}^q\}(x,\xi,t)\left[\frac{1}{x-\xi+i\epsilon}-\frac{1}{x+\xi-i\epsilon}\right]$$

- **8 asymmetries ⇔ 8 (real) GPD moments**

Cross section within NLP: full polarization

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\text{unpol}}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}} \cos\phi_S$$

$$+ \left(A_{UU}^{\text{NLP}} + \lambda_e \lambda_N A_{LL}^{\text{NLP}}\right) \cos\phi + \left(\lambda_e A_{UL}^{\text{NLP}} + \lambda_N A_{LU}^{\text{NLP}}\right) \sin\phi$$

$$+ s_T \left(A_{TU,1}^{\text{NLP}} \cos\phi_S \sin\phi + A_{TU,2}^{\text{NLP}} \sin\phi_S \cos\phi\right)$$

$$+ \lambda_e s_T \left(A_{TL,1}^{\text{NLP}} \cos\phi_S \cos\phi + A_{TL,2}^{\text{NLP}} \sin\phi_S \sin\phi\right)\Big]$$

NLP: from γ_T^*-γ_L^* and γ_T^*-$[q\bar{q}]$ interference

$$M = \begin{bmatrix} F_1 & -\frac{t}{4m^2} F_2 & \xi(F_1+F_2) & 0 \\ (1+\xi)(F_1+F_2) & \xi(F_1+F_2) & \frac{1+\xi}{\xi} F_1 & -\xi F_1 - (1+\xi)\frac{t}{4m^2} F_2 \\ \xi(F_1+F_2) & \left(\frac{\xi^2}{1+\xi} + \frac{t}{4m^2}\right)(F_1+F_2) & -\xi F_1 + \frac{t}{4m^2}\frac{1-\xi^2}{\xi} F_2 & -\left(\frac{\xi^2}{1+\xi} + \frac{t}{4m^2}\right) F_1 - \frac{\xi t}{4m^2} F_2 \\ \xi(F_1+F_2) & \frac{\xi t}{4m^2}(F_1+F_2) & -\xi F_1 + \frac{t}{4m^2}\frac{1-\xi^2}{\xi} F_2 & -\left(\xi + \frac{t}{4\xi m^2}\right) F_1 - \frac{\xi t}{4m^2} F_2 \end{bmatrix} \begin{matrix} \leftarrow M_1 \\ \leftarrow M_2 \\ \leftarrow M_3 \\ \leftarrow M_4 \end{matrix}$$

$$\Rightarrow \quad M \cdot \begin{bmatrix} \mathcal{H} \\ \mathcal{E} \\ \widetilde{\mathcal{H}} \\ \widetilde{\mathcal{E}} \end{bmatrix} = \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \hat{V}_3 \\ \hat{V}_4 \end{bmatrix} \leftarrow$$

Reconstructed from experiments
(complex valued)

$\det M \neq 0$ → **Unique solution for GPD moments!**

Outline

- ❑ From inclusive to exclusive processes
- ❑ Generalized parton distribution (GPD)
- ❑ Phenomenology of GPD
- ❑ **Inverse problem and global analysis of GPD**

Scaling in DVCS

What are constrained by DVCS: "Scaling integral"

Commonly called **"Compton form factors" (CFFs)**

$$\{\mathcal{H},\mathcal{E}\}(\xi,t) \equiv \sum_q e_q^2 \int_{-1}^{1} dx\, \{H^q, E^q\}(x,\xi,t) \left[\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon}\right]$$

$$\{\widetilde{\mathcal{H}},\widetilde{\mathcal{E}}\}(\xi,t) \equiv \sum_q e_q^2 \int_{-1}^{1} dx\, \{\widetilde{H}^q, \widetilde{E}^q\}(x,\xi,t) \left[\frac{1}{x-\xi+i\epsilon} - \frac{1}{x+\xi-i\epsilon}\right]$$

independent of Q, q_T, or θ at leading order

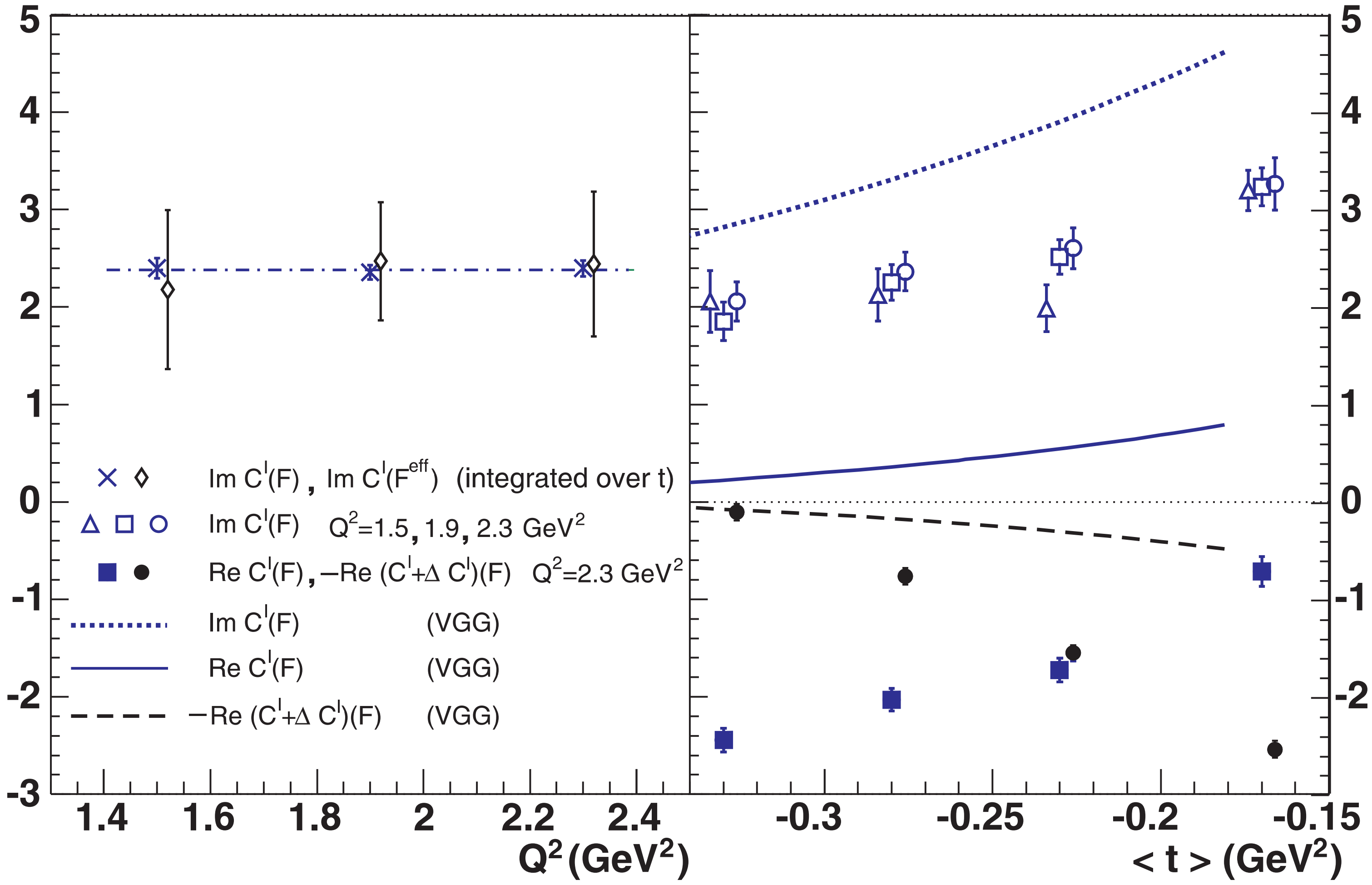
Predictable θ shape. E.g., $A_{UL}^{\rm NLP} \propto \frac{3-\cos\theta}{\sin\theta}\left[F_1\,{\rm Im}\,\mathcal{H} - \frac{t}{4m^2}F_2\,{\rm Im}\,\mathcal{E} + \xi\,(F_1+F_2)\,{\rm Im}\,\tilde{\mathcal{H}}\right]$

Unknown but does not affect θ shape

- **Advantage: Helps to experimentally confirm parton-dominated dynamics (i.e., parton model)**

Scaling in DVCS

Confirmation of scaling phenomenon for the “CFFs” extracted from DVCS data!



JLab Hall A, PRL 97 (2006) 262002

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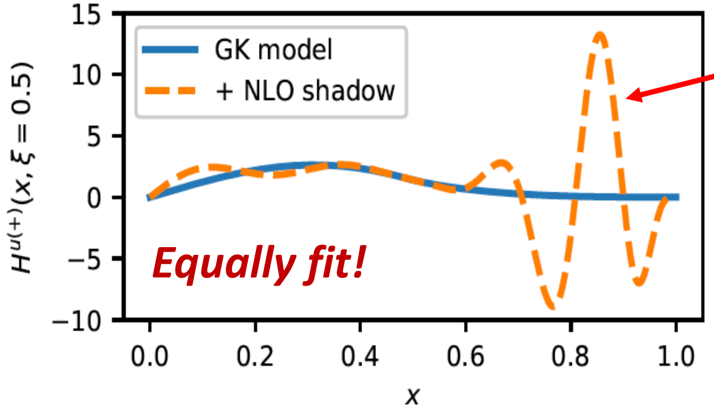
independent of Q, q_T, or θ at leading order

- **Disadvantage: Difficult to extract x-dependence of GPDs**

Shadow GPD problem

$$\int_{-1}^{1} dx \frac{S(x,\xi,t)}{x-\xi+i\epsilon} = 0$$

$$S(\pm\xi,\xi,t) = S(x,0,0) = 0$$



Shadow GPD

[Bertone et al. PRD `21]

Scaling in DVCS

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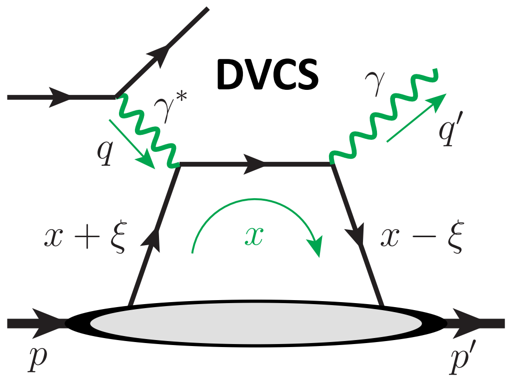
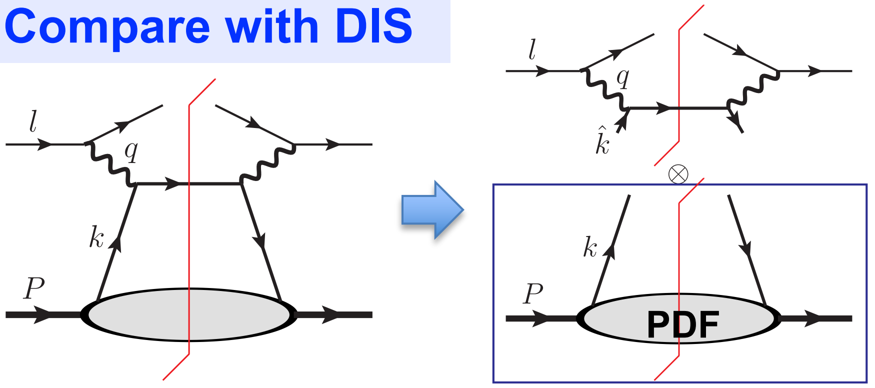


$x \sim$ loop momentum

never pin down to some x

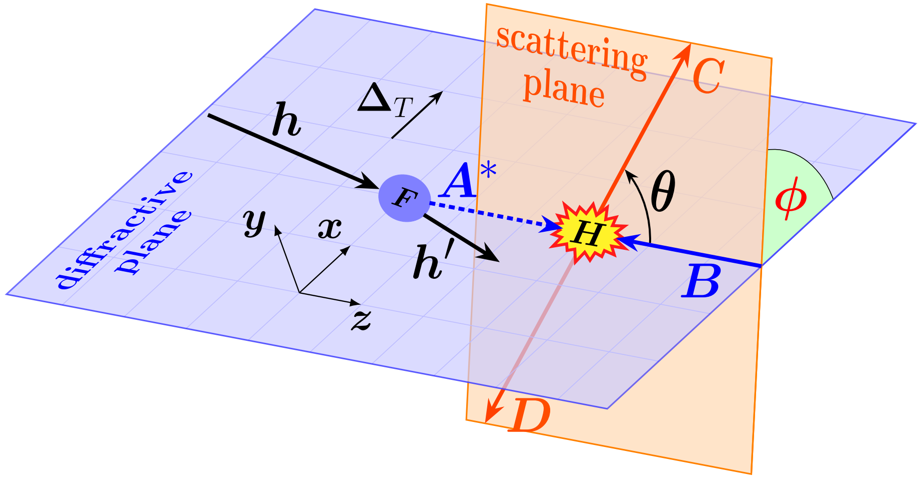
Compare with DIS



***cross section*: cut diagram**

$$\frac{d\sigma_{\rm DIS}}{dx_B\, dQ^2} = \int_{x_B}^{1} dx\, f(x)\, \hat{\sigma}(x_B/x; Q^2)$$

Where does the x-sensitivity come from?



- x-sensitivity ⇔ 2 → 2 hard scattering

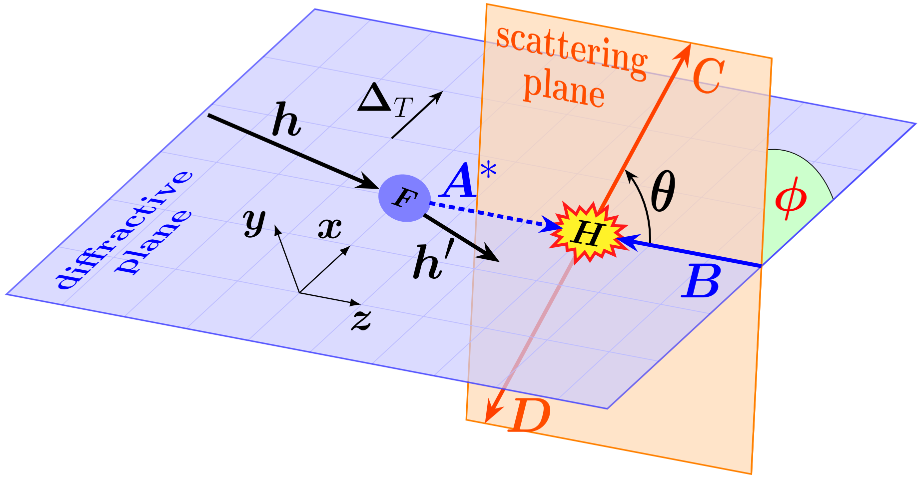
Kinematics:

1. $\hat{s} = 2\,\xi\, s\,/\,(1+\xi)$ ⇐ ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2)\sin\theta$ ⇔ x
3. ϕ ⇐ $(A^* B)$ spin states

$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A - \lambda_B)\phi} \cdot \int_{-1}^{1} dx\, F_A(x)\, C_A(x;Q) \qquad (Q = \theta \text{ or } q_T)$$

[suppressing t and ξ dependence]

Where does the *x*-sensitivity come from?



❑ ***x*-sensitivity ⇔ 2 → 2 hard scattering**

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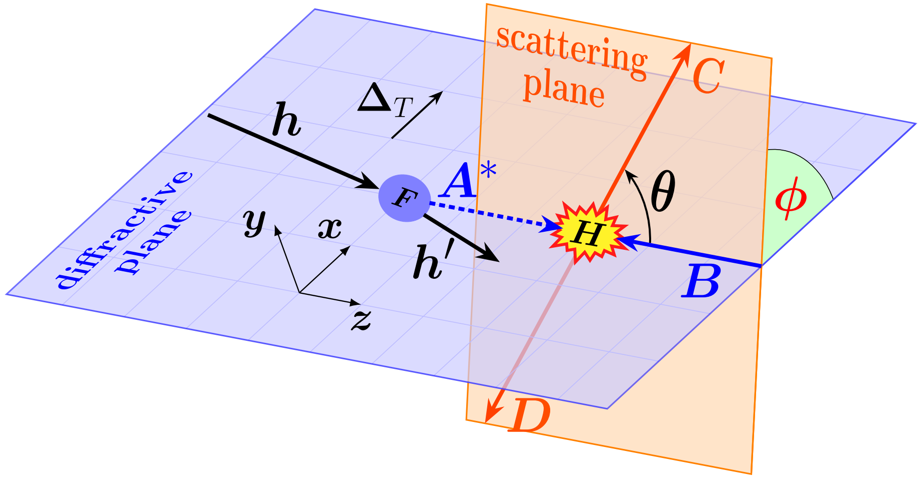
➢ **Moment-type sensitivity** $C(x;Q) = G(x)\cdot T(Q)$ ⇒ $F_G = \int_{-1}^{1} dx\, G(x)\, F(x,\xi,t)$ **Independent of Q. Scaling for F_G.**

⇒ **Inversion problem: *shadow GPD*** $S_G = \int_{-1}^{1} dx\, G(x)\, S(x,\xi) = 0$ [Bertone et al. PRD `21]

Where does the x-sensitivity come from?



❑ **x-sensitivity $\Leftrightarrow$ 2 → 2 hard scattering**

Kinematics:

1. $\hat{s} = 2\,\xi\,s\,/\,(1+\xi)$ ⇦ ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2)\sin\theta$ ⇔ x
3. ϕ ⇦ $(A^* B)$ spin states

$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A-\lambda_B)\phi}\cdot\int_{-1}^{1} dx\,F_A(x)\,C_A(x;Q) \qquad (Q=\theta \text{ or } q_T)$$

➢ **Moment-type sensitivity** $C(x;Q) = G(x)\cdot T(Q)$ ➡ $F_G = \int_{-1}^{1} dx\,G(x)\,F(x,\xi,t)$ **Independent of Q. Scaling for F_G.**

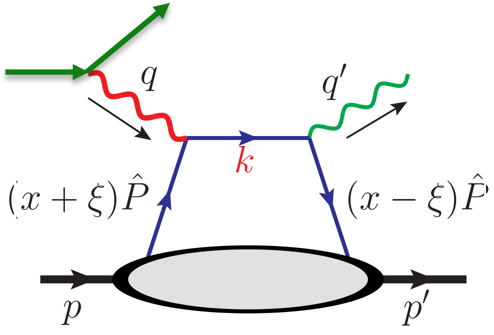
➡ **Inversion problem: *shadow GPD*** $S_G = \int_{-1}^{1} dx\,G(x)\,S(x,\xi) = 0$ [Bertone et al. PRD `21]

➢ **Enhanced sensitivity** $C(x;Q) \neq G(x)\cdot T(Q)$ ➡ $d\sigma/dQ \sim |C(x;Q)\otimes_x F(x,\xi,t)|^2$

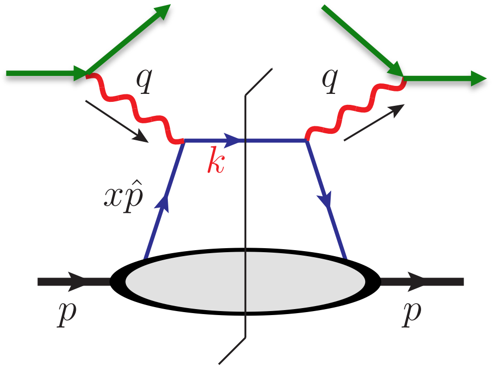
Scaling *breaking* at LO

Scaling kernels and moment sensitivity

Origin of scaling: massless parton approximation + massless external states.



DVCS



DIS

$q'^2 = 0$

$$k^2 = \left[q' + (x-\xi)\hat{P}\right]^2 = (x-\xi)\,(2\hat{P}\cdot q')$$

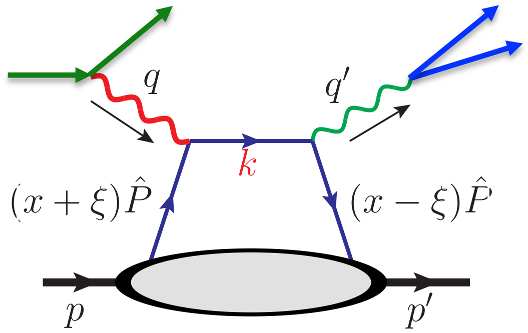
$$\Rightarrow \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x-\xi+i\epsilon}$$

$$k^2 = [q + x\hat{p}]^2 = x\,(2\hat{p}\cdot q) - Q^2 = (2\hat{p}\cdot q)\,(x - x_B)$$

$$\Rightarrow \int dx f(x)\delta(x-\xi) = f(x_B)$$

Enhancing sensitivity by breaking the scaling

Origin of scaling: massless parton approximation + massless external states.



$q'^2 = 0$

$$k^2 = \left[q' + (x-\xi)\hat{P}\right]^2 = (x-\xi)\,(2\hat{P}\cdot q')$$

$$\Rightarrow \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x-\xi+i\epsilon}$$

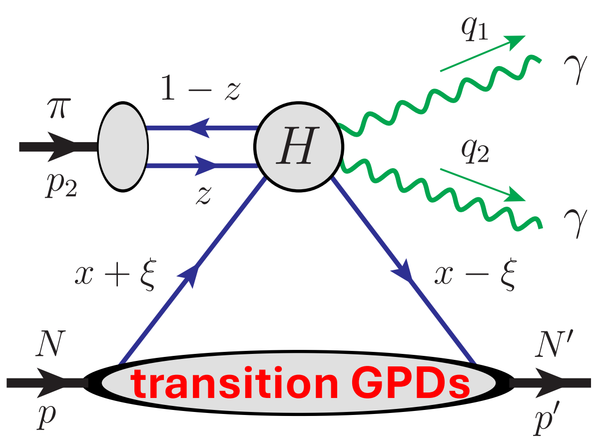
$\Rightarrow$ **DDVCS** $q'^2 = Q'^2 > 0$

$$k^2 = \left[q' + (x-\xi)\hat{P}\right]^2$$
$$= (x-\xi)\,(2\hat{P}\cdot q') + Q'^2$$
$$= \frac{Q^2+Q'^2}{2\xi}\left[x - \xi\left(\frac{Q^2-Q'^2}{Q^2+Q'^2}\right)\right]$$

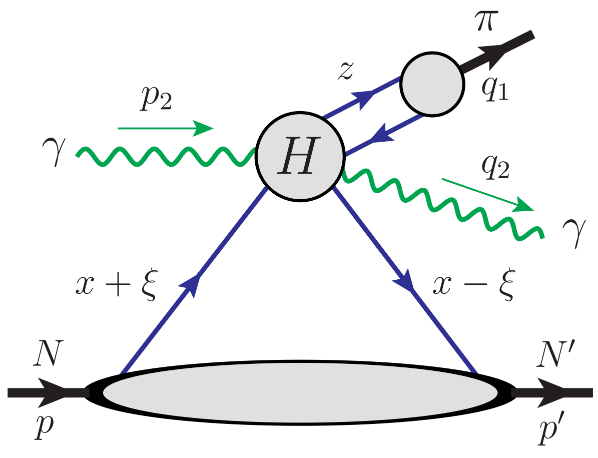
$$\Rightarrow \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x-\xi\left(\frac{Q^2-Q'^2}{Q^2+Q'^2}\right)+i\epsilon}$$ **Scaling violation**

The rate of DDVCS is likely too low to be useful…

Scaling breaking at $2 \to 3$ level



J-PARC, AMBER



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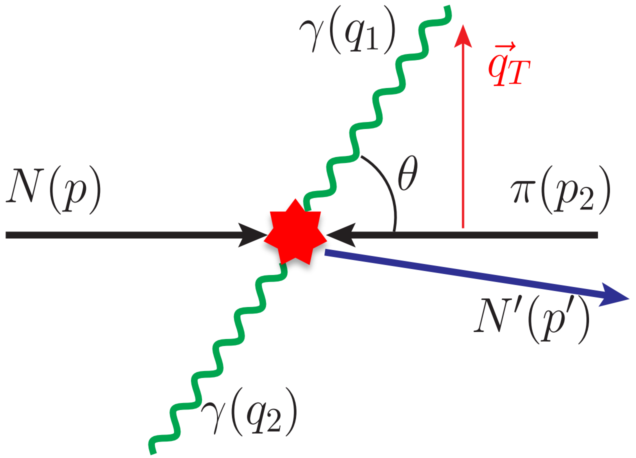
Qiu & Yu, JHEP 08 (2022) 103
Qiu & Yu, PRD 109 (2024) 074023

G. Duplancic et al., JHEP 11 (2018) 179
G. Duplancic et al., JHEP 03 (2023) 241
G. Duplancic et al., PRD 107 (2023), 094023
Qiu & Yu, PRD 107 (2023), 014007
Qiu & Yu, PRL 131 (2023), 161902

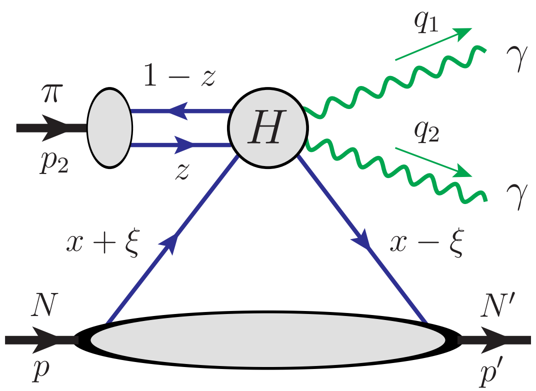
Non-scaling 2 → 3 process: (1) diphoton mesoproduction

[Qiu & Yu, JHEP 08 (2022) 103;
PRD 109 (2024) 074023]



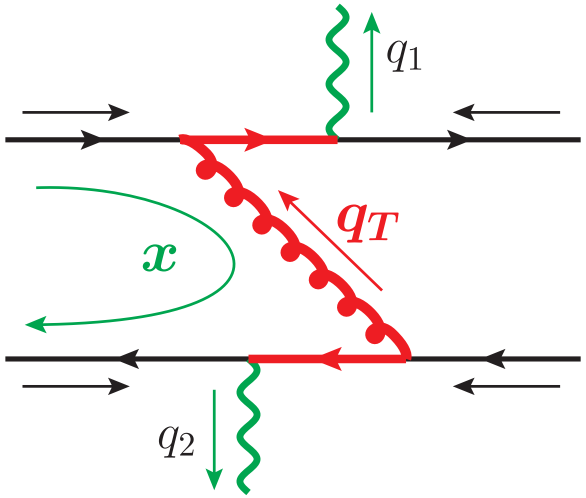
In addition to

$$F_0(\xi,t) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x-\xi+i\epsilon}$$

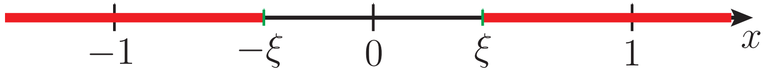


$i\mathcal{M}$ **also contains**

$$I(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x-\rho(z;\theta)+i\epsilon\,\mathrm{sgn}\left[\cos^2(\theta/2)-z\right]}$$



$$\rho(z;\theta) = \xi\cdot\left[\frac{1-z+\tan^2(\theta/2)\,z}{1-z-\tan^2(\theta/2)\,z}\right] \in (-\infty,-\xi]\cup[\xi,\infty)$$



Non-scaling 2 → 3 process: (1) diphoton mesoproduction

[Qiu & Yu, PRD 109 (2024) 074023]

- **Diphoton process:** $N\pi \to N'\gamma\gamma$: (1) $p\pi^- \to n\gamma\gamma$; (2) $n\pi^+ \to p\gamma\gamma$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = 2\pi\left(\alpha_e\alpha_s\frac{C_F}{N_c}\right)^2\frac{1}{\xi^2 s^3}\cdot\left[(1-\xi^2)\sum_{\alpha=\pm}\left(|\mathcal{M}_\alpha^{[\widetilde{H}]}|^2+|\widetilde{\mathcal{M}}_\alpha^{[H]}|^2\right)-\left(\xi^2+\frac{t}{4m^2}\right)\sum_{\alpha=\pm}|\widetilde{\mathcal{M}}_\alpha^{[E]}|^2\right.$$
$$\left.-\frac{\xi^2 t}{4m^2}\sum_{\alpha=\pm}|\mathcal{M}_\alpha^{[\widetilde{E}]}|^2-2\xi^2\sum_{\alpha=\pm}\mathrm{Re}\left(\widetilde{\mathcal{M}}_\alpha^{[H]}\widetilde{\mathcal{M}}_\alpha^{[E]*}+\mathcal{M}_\alpha^{[\widetilde{H}]}\mathcal{M}_\alpha^{[\widetilde{E}]*}\right)\right]$$

Non-scaling 2 → 3 process: (1) diphoton mesoproduction

[Qiu & Yu, PRD 109 (2024) 074023]

❑ **Diphoton process:** $N\pi \to N'\gamma\gamma:\ (1)\ p\pi^- \to n\gamma\gamma;\ (2)\ n\pi^+ \to p\gamma\gamma$

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Nucleon transition GPDs

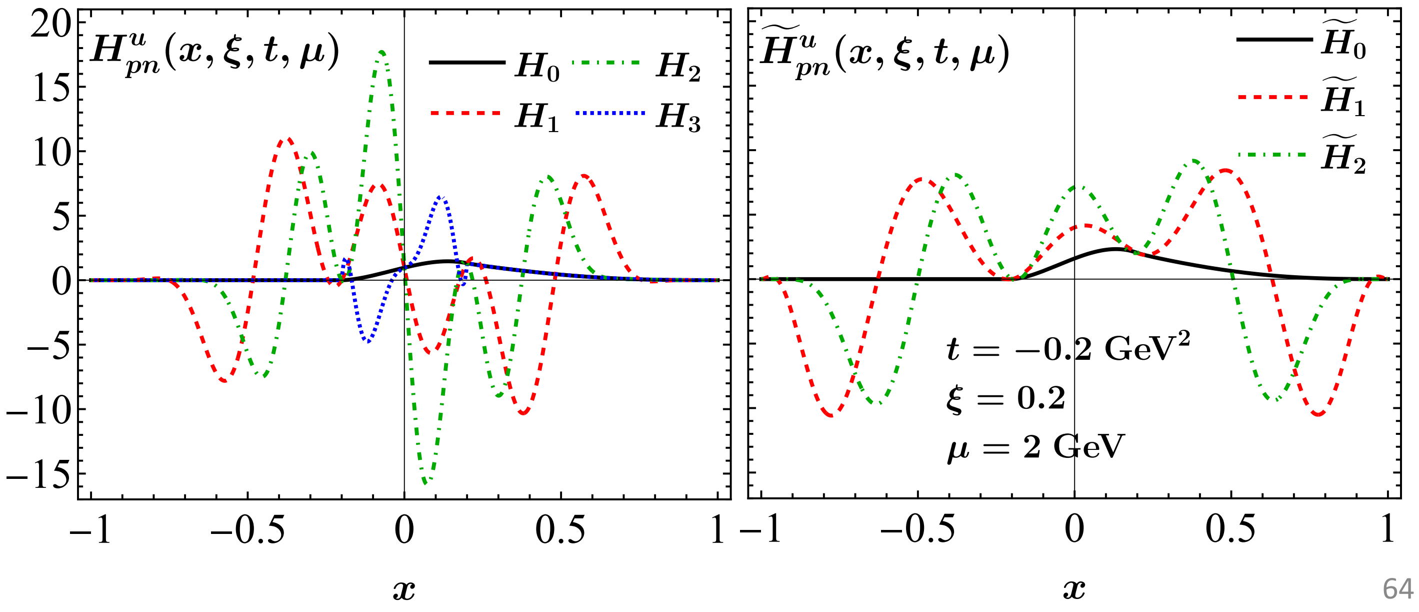
$$H^u_{pn} = H^u_p - H^d_p,\ \text{etc.}$$

GPD models = GK model + shadow GPDs

Goloskokov & Kroll, `05, `07, `09

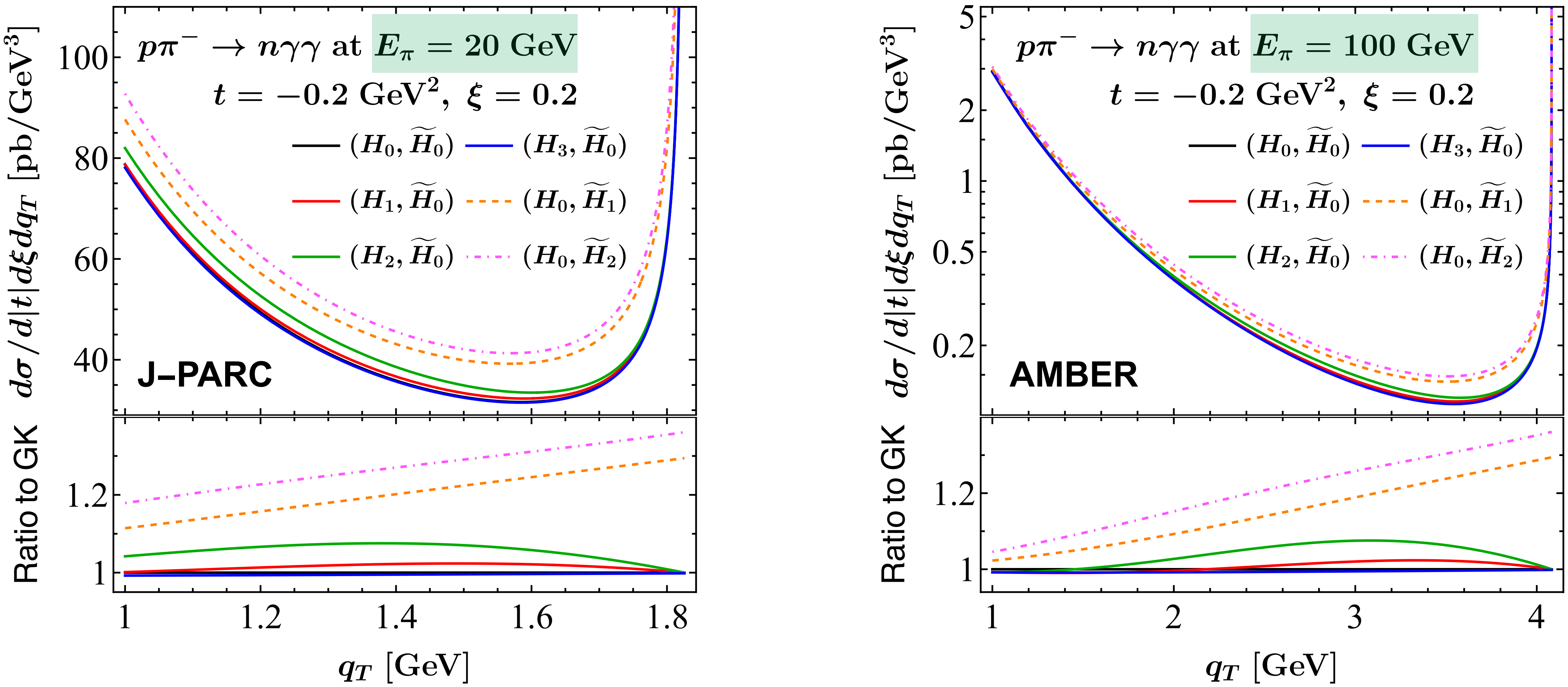
Bertone et al. `21
Moffat et al. `23

$$\int_{-1}^{1}\frac{dx\,S(x,\xi)}{x-\xi\pm i\epsilon}=0$$



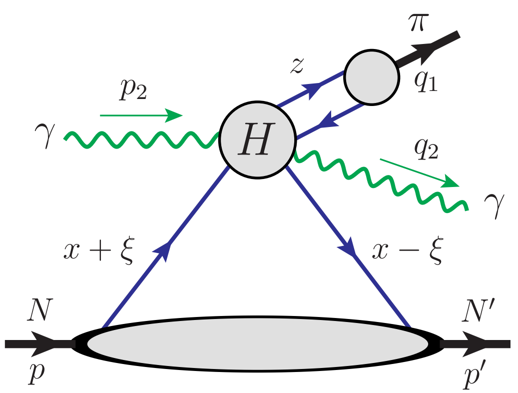
Non-scaling 2 → 3 process: (1) diphoton mesoproduction

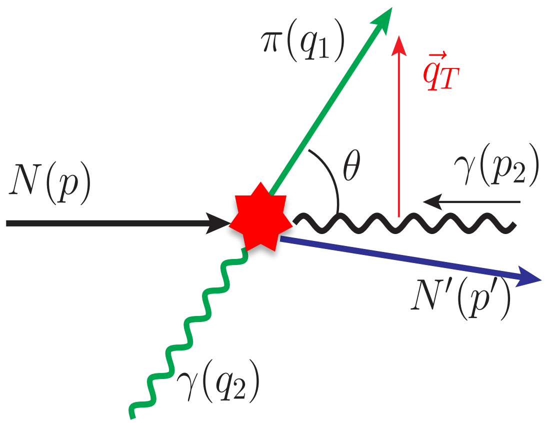
[Qiu & Yu, PRD 109 (2024) 074023]



Non-scaling 2 → 3 process: (2) γ-π pair photoproduction

[Qiu & Yu, PRL 131 (2023) 161902]

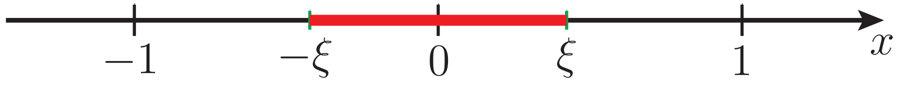




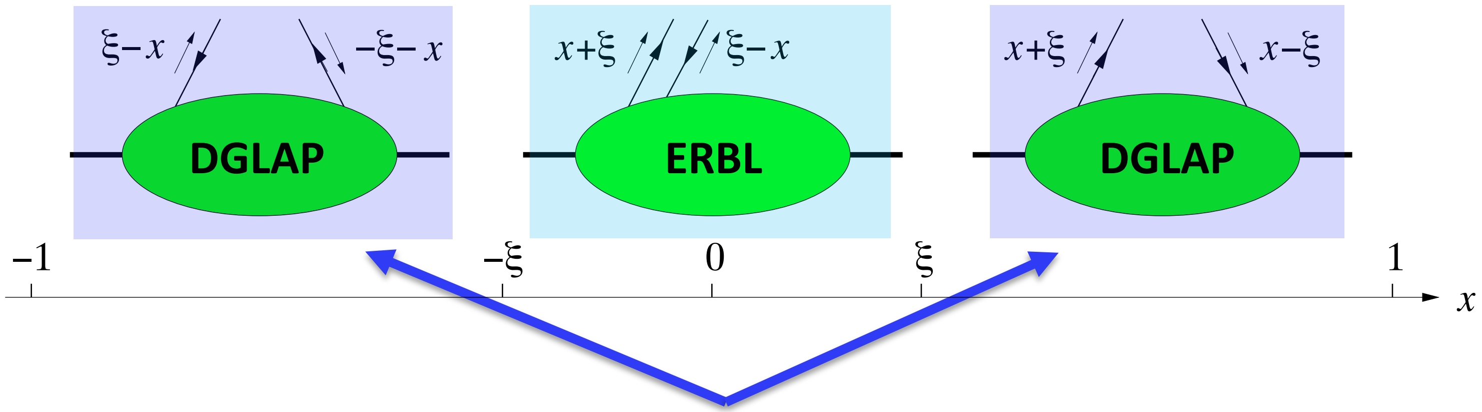
$i\mathcal{M}$ **also contains the special integral**

$$I'(t, \xi; z, \theta) = \int_{-1}^{1} \frac{dx\, F(x, \xi, t)}{x - \rho'(z; \theta) + i\epsilon}$$

$$\rho'(z; \theta) = \xi \cdot \left[\frac{\cos^2(\theta/2)\,(1-z) - z}{\cos^2(\theta/2)\,(1-z) + z} \right] \in [-\xi, \xi]$$

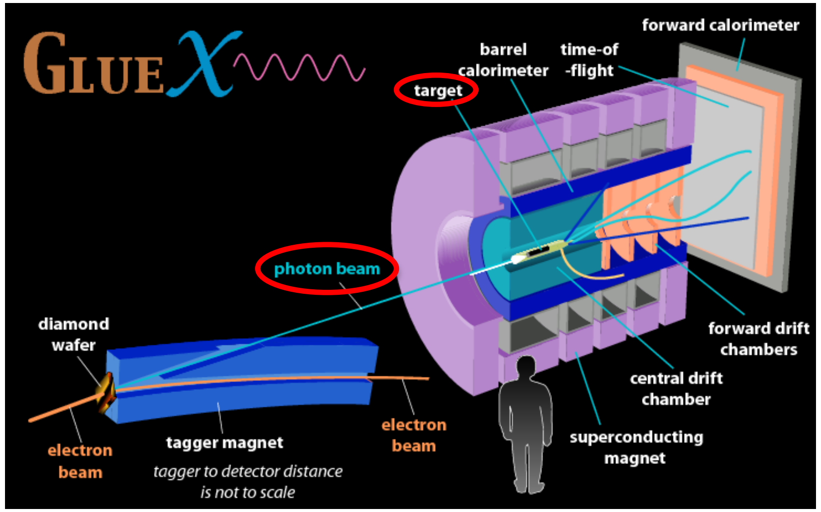


➡ ***Complementary sensitivity***

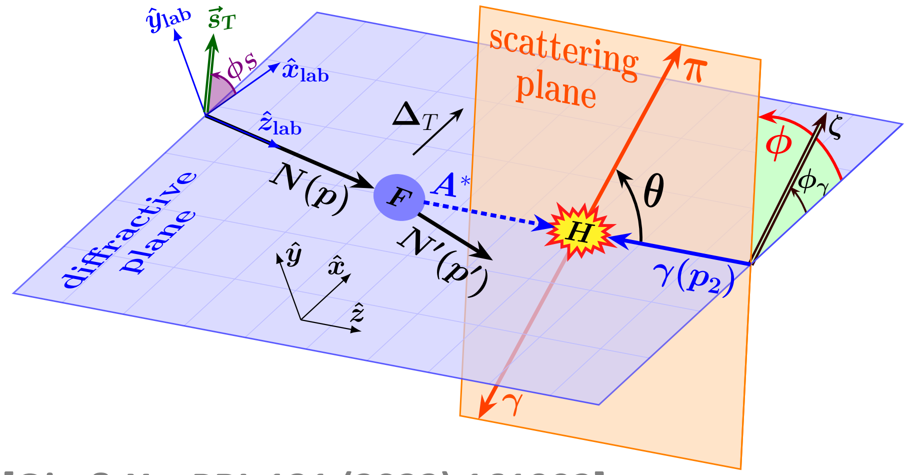


$N\,\pi \to N'\,\gamma\,\gamma$

Non-scaling 2 → 3 process: (2) γ-π pair photoproduction



JLab Hall D



[Qiu & Yu, PRL 131 (2023) 161902]

Polarization asymmetries

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi}\frac{d\sigma}{d|t|d\xi d\cos\theta}\cdot\left[1+\lambda_N\lambda_\gamma A_{LL} + \zeta\, A_{UT}\cos 2(\phi-\phi_\gamma) + \lambda_N\zeta\, A_{LT}\sin 2(\phi-\phi_\gamma)\right]$$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = \pi\,(\alpha_e\alpha_s)^2\left(\frac{C_F}{N_c}\right)^2\frac{1-\xi^2}{\xi^2 s^3}\Sigma_{UU}$$

$$\Sigma_{UU} = |\mathcal{M}_+^{[\widetilde{H}]}|^2 + |\mathcal{M}_-^{[\widetilde{H}]}|^2 + |\widetilde{\mathcal{M}}_+^{[H]}|^2 + |\widetilde{\mathcal{M}}_-^{[H]}|^2,$$

$$A_{LL} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*}\right],$$

$$A_{UT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\widetilde{\mathcal{M}}_+^{[H]}\,\widetilde{\mathcal{M}}_-^{[H]*} - \mathcal{M}_+^{[\widetilde{H}]}\,\mathcal{M}_-^{[\widetilde{H}]*}\right],$$

$$A_{LT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Im}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*}\right].$$

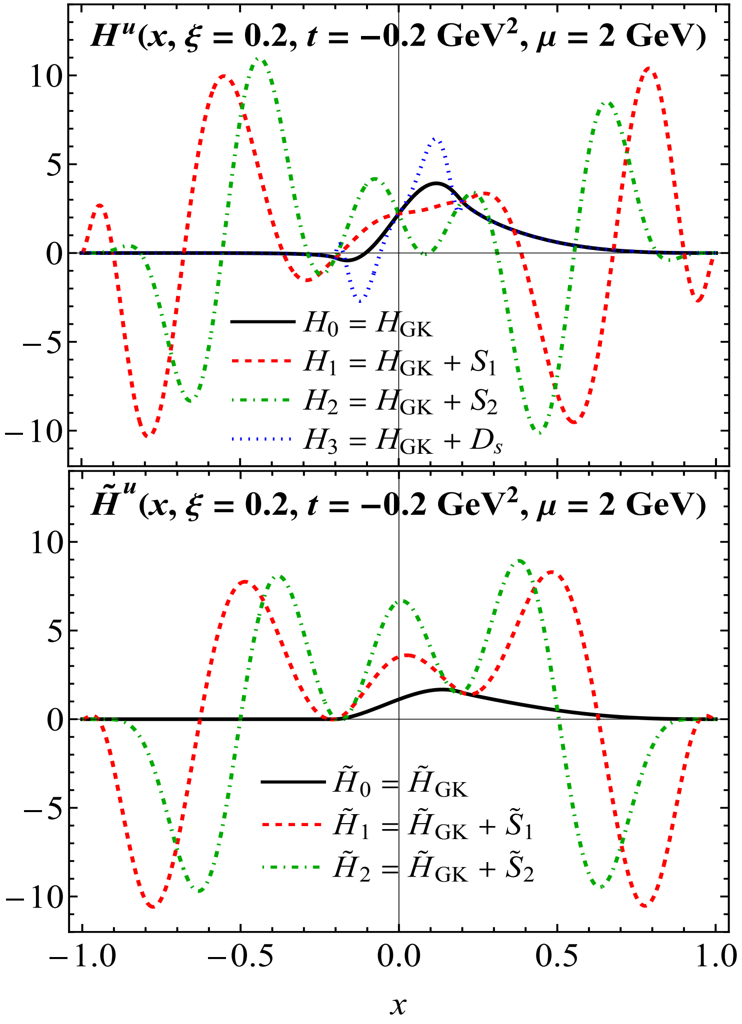
Neglecting: (1) E and $\widetilde{E}$; (2) gluon channel

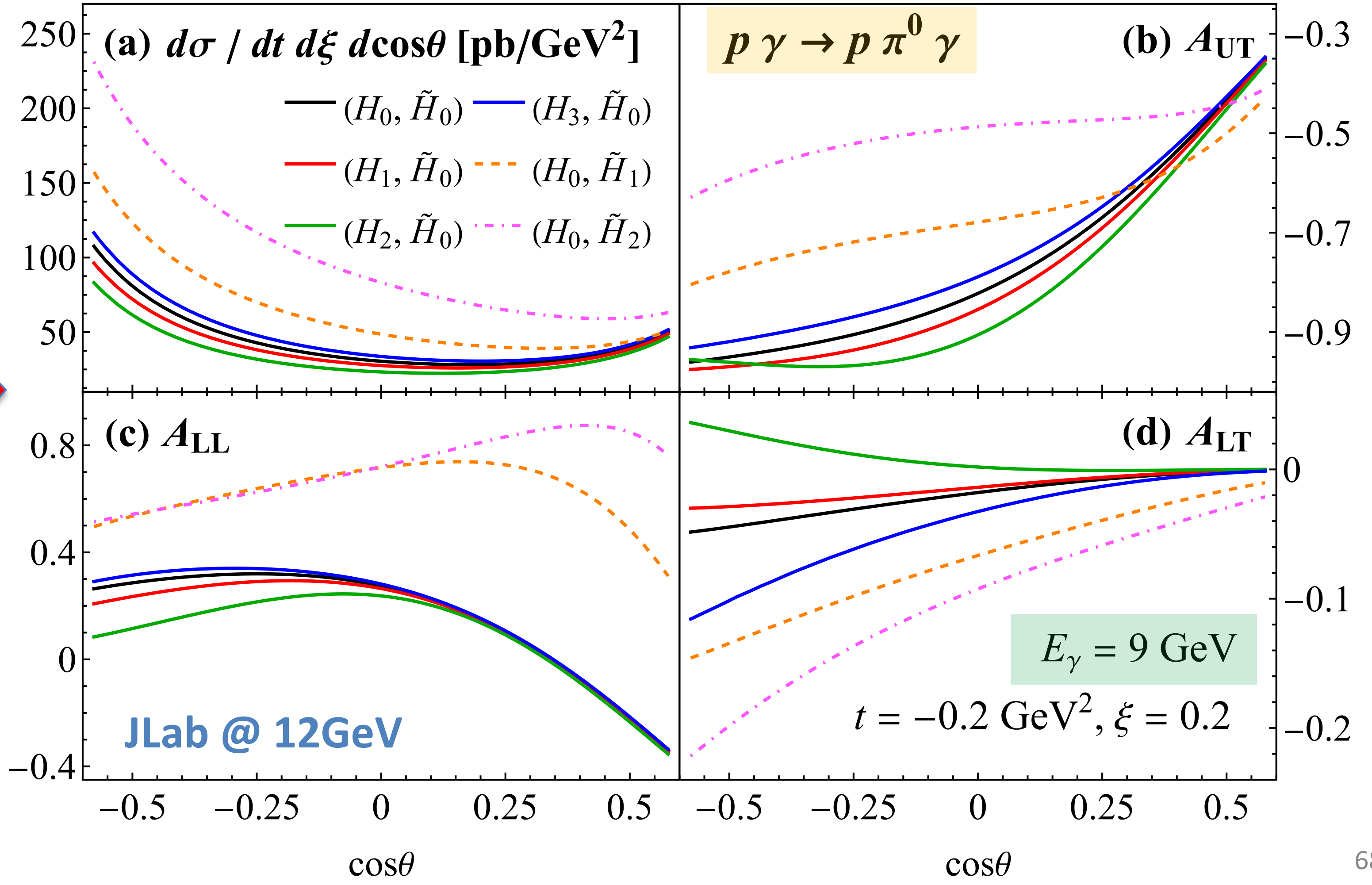
Non-scaling 2 → 3 process: (2) γ-π pair photoproduction

GPD models = GK model + shadow GPDs

$$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$$

Goloskokov, Kroll, `05, `07, `09
Bertone et al. `21
Moffat et al. `23





Non-scaling 2 → 3 process: (2) γ-π pair photoproduction

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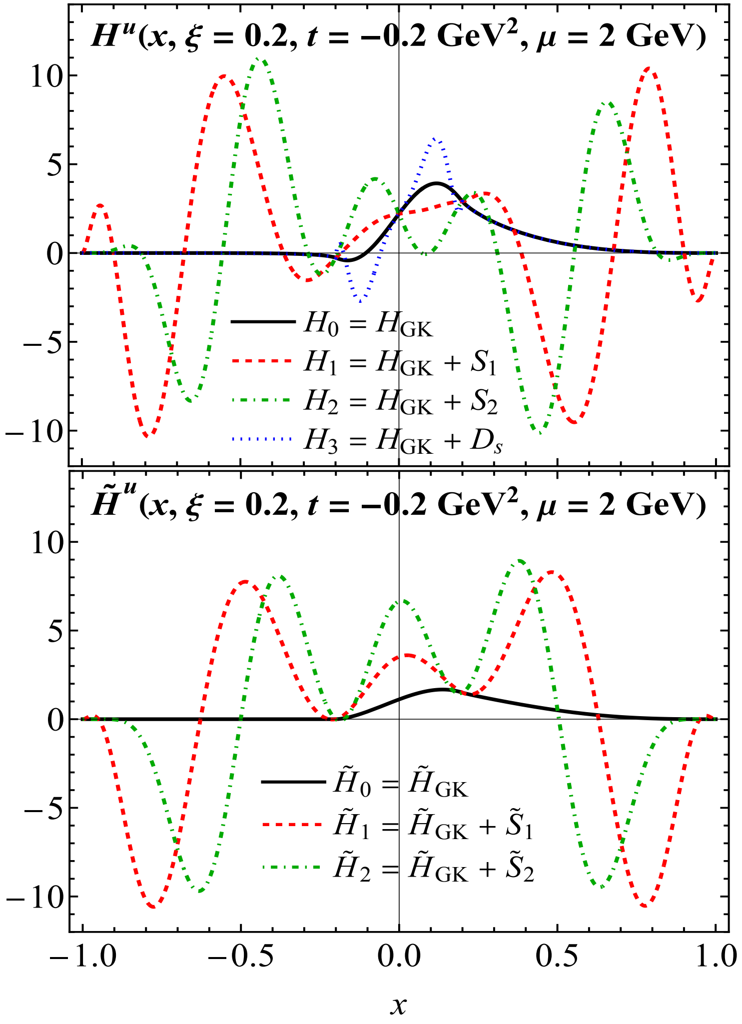
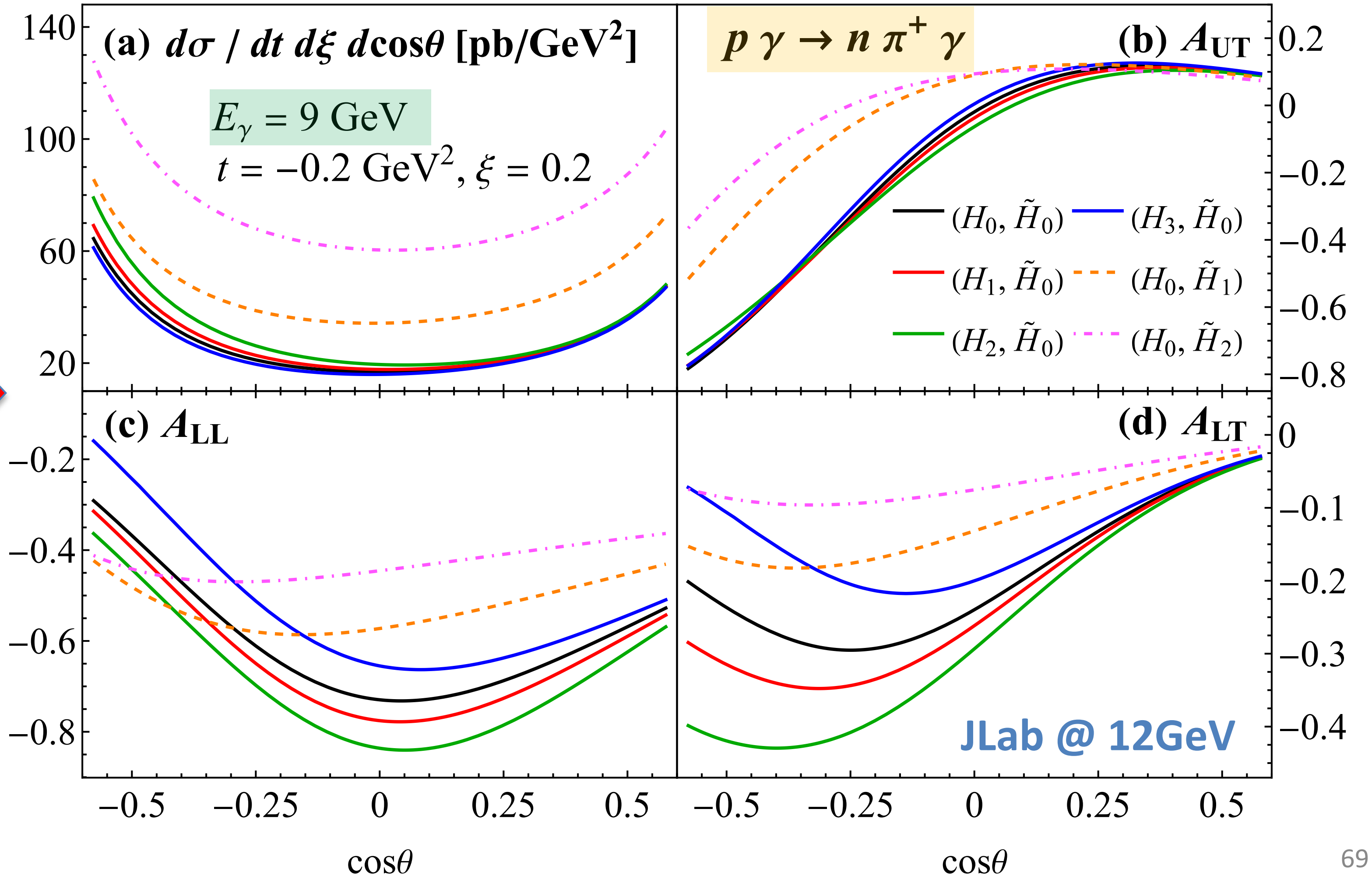
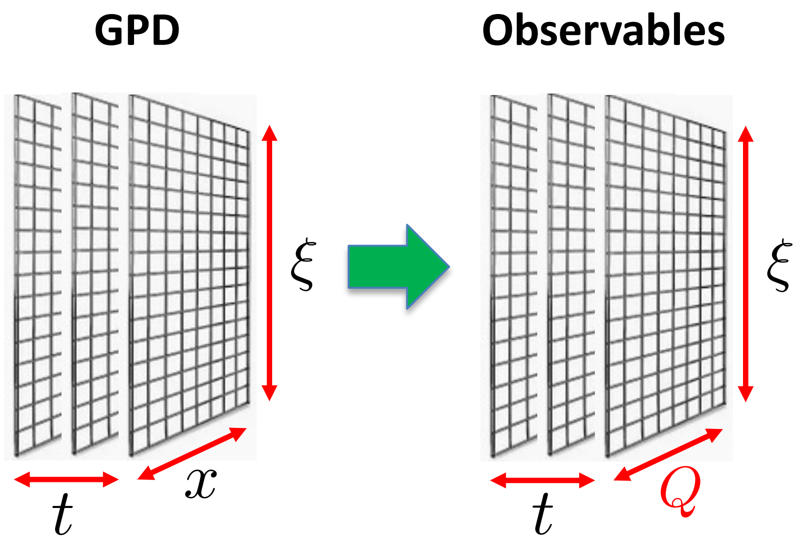




GPD inversion from neural network

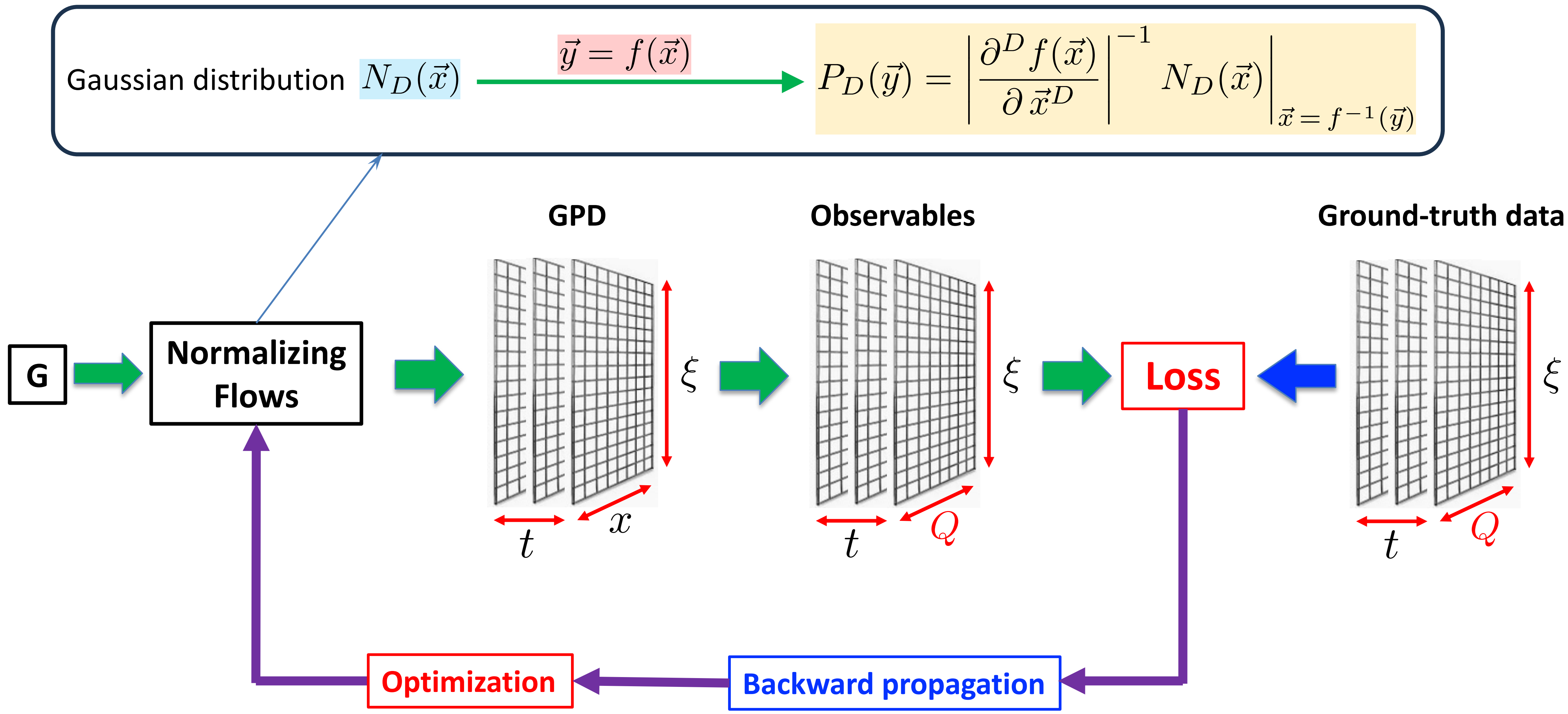
Challenge: Shadow GPDs make parametric method *biased*

➡ **Construct GPDs from most flexible pixelation method**

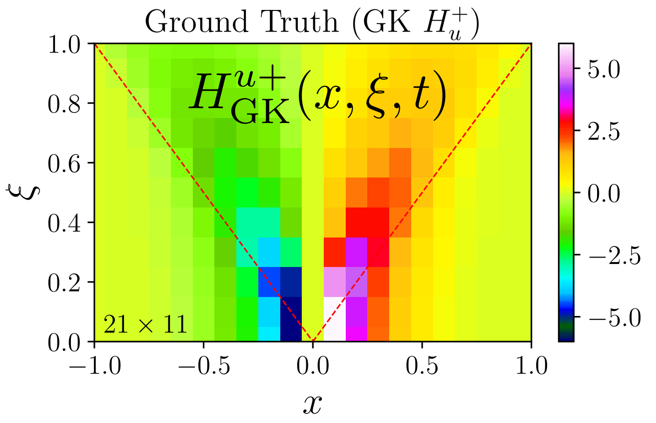


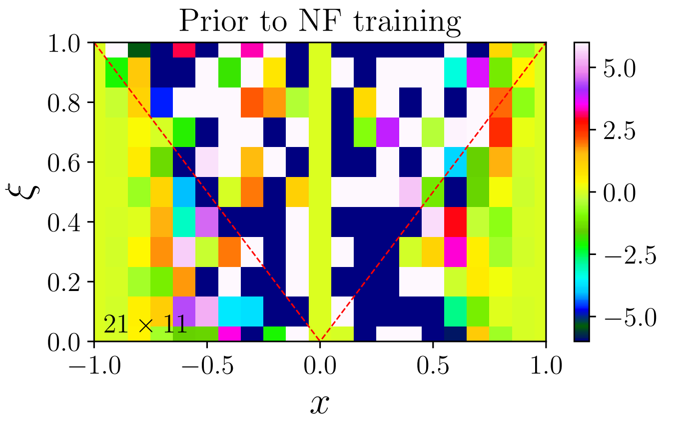
GPD inversion from neural network

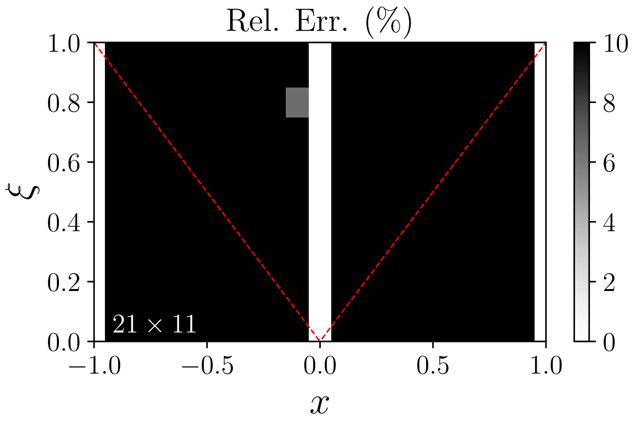
Normalizing flows = A set of *differentiable and invertible* changes of variable.



Reconstruct with only scaling DVCS integral







- **Neural network generator**

$$H^u_{\rm NF}(x,\xi,t) = H^u_{\rm GK}(x,\xi,t) * \epsilon(x,\xi,t)$$

$\epsilon(x,\xi,t)$ is generated by NF

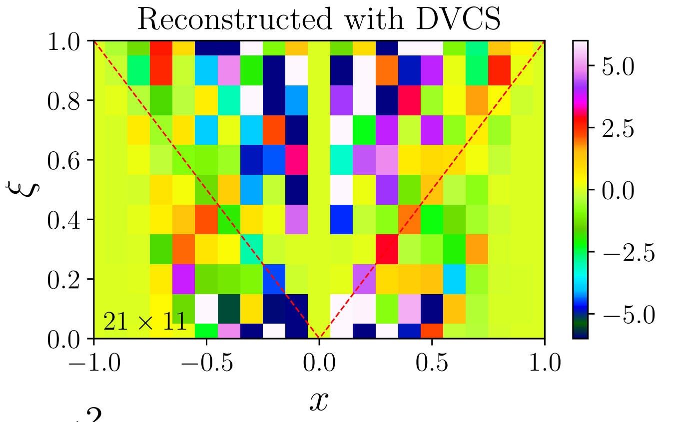
- **Observable: DVCS integral**

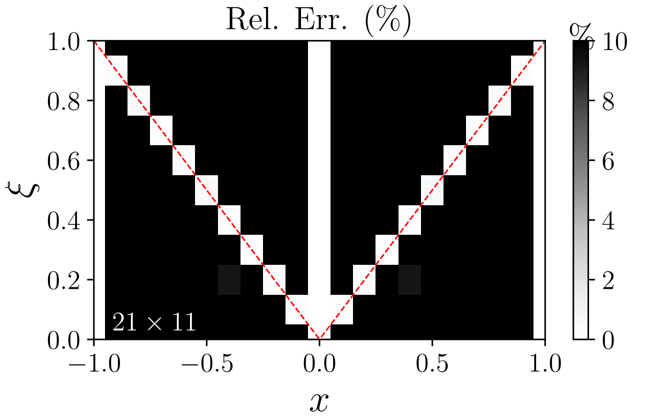
$$\mathcal{M}^{[H]}(\xi,t) = \int_{-1}^{1} dx \, \frac{H^+(x,\xi,t)}{x-\xi+i\epsilon}$$

- **Optimize with a loss function**

$$\chi^2(H_{\rm NF}, H_{\rm GK}) = \sum_{\xi,t} \left| \frac{\mathcal{M}^{[H_{\rm NF}]}(\xi,t) - \mathcal{M}^{[H_{\rm GK}]}(\xi,t)}{r \cdot \mathcal{M}^{[H_{\rm GK}]}(\xi,t)} \right|^2$$

Training with DVCS moment. $r = 0.01$





Sensitivity only on the ridge.

Shadow GPDs from pixelation approach

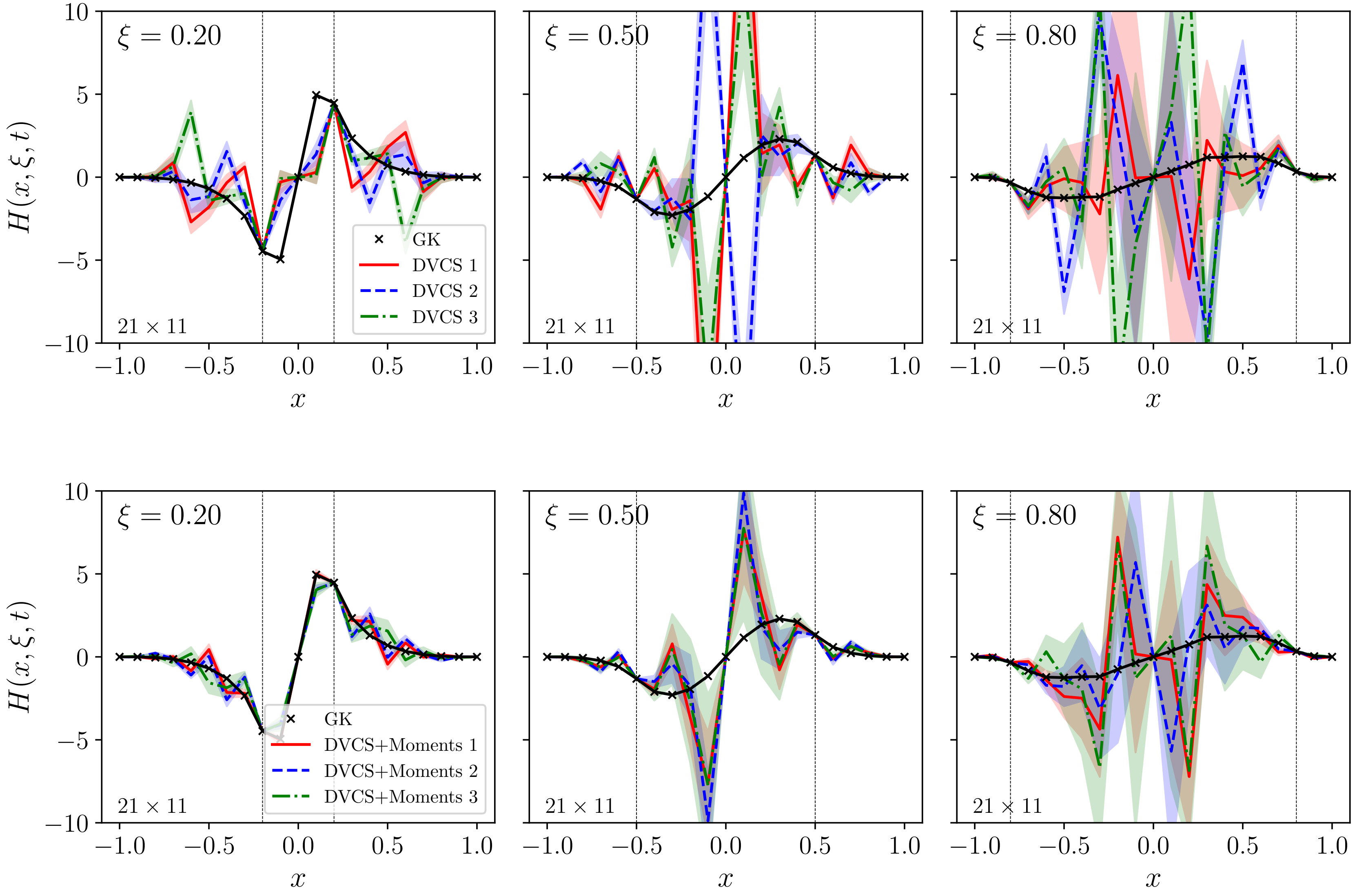
DVCS integral

$$\mathcal{M}^{\text{DVCS}}(\xi,t) = \int_{-1}^{1} dx\, \frac{H^+(x,\xi,t)}{x-\xi+i\epsilon}$$

DVCS integral
+ first two x-moments

$$H_n^+(\xi,t) = \int_{-1}^{1} dx\, x^{n-1} H^+(x,\xi,t)$$

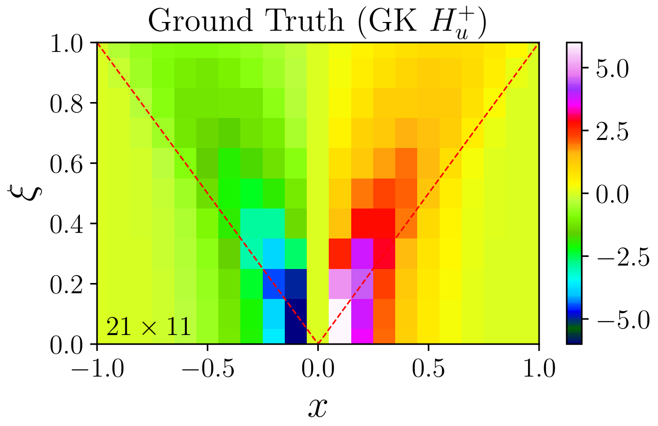
$n = 2, 4$



Improve GPD extraction with non-scaling photoproduction

Non-scaling integral

$$\mathcal{M}^{[H]}(\xi, t, \theta) = \int_{-1}^{1} dx \, \frac{H^{+}(x, \xi, t)}{x - x_p(\xi, \theta) + i\epsilon}$$



NF

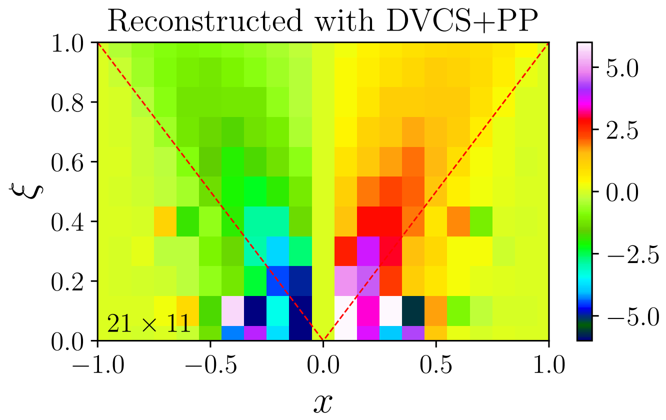
DVCS +

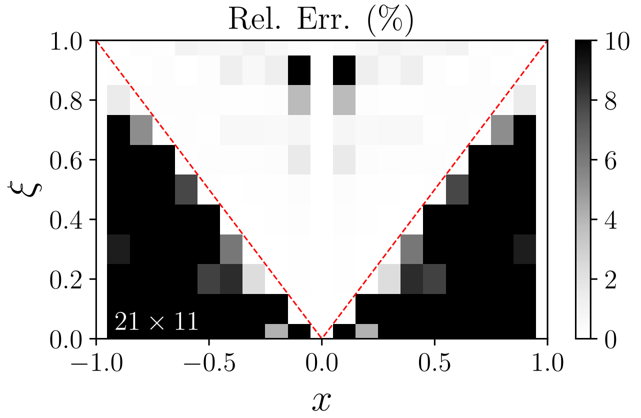
Photoproduction

(PP)

$\cos\theta \in [-0.9, 0.9]$

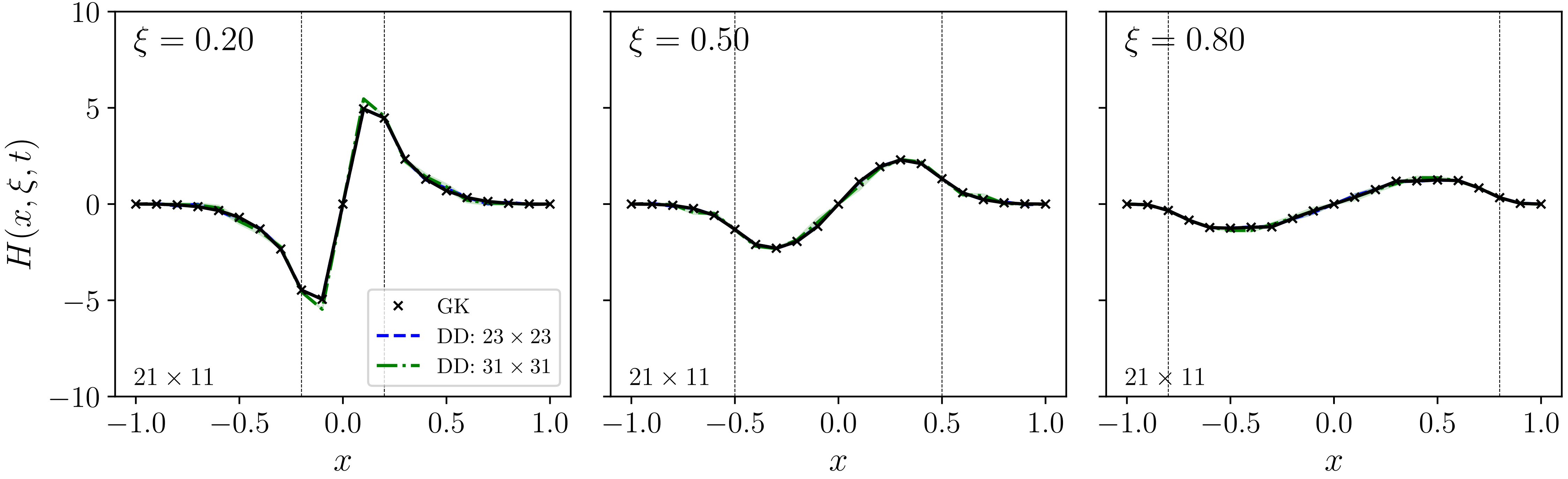
$r = 0.01$





Reconstructed at GPD level

With both non-scaling mesoproduction and photoproduction

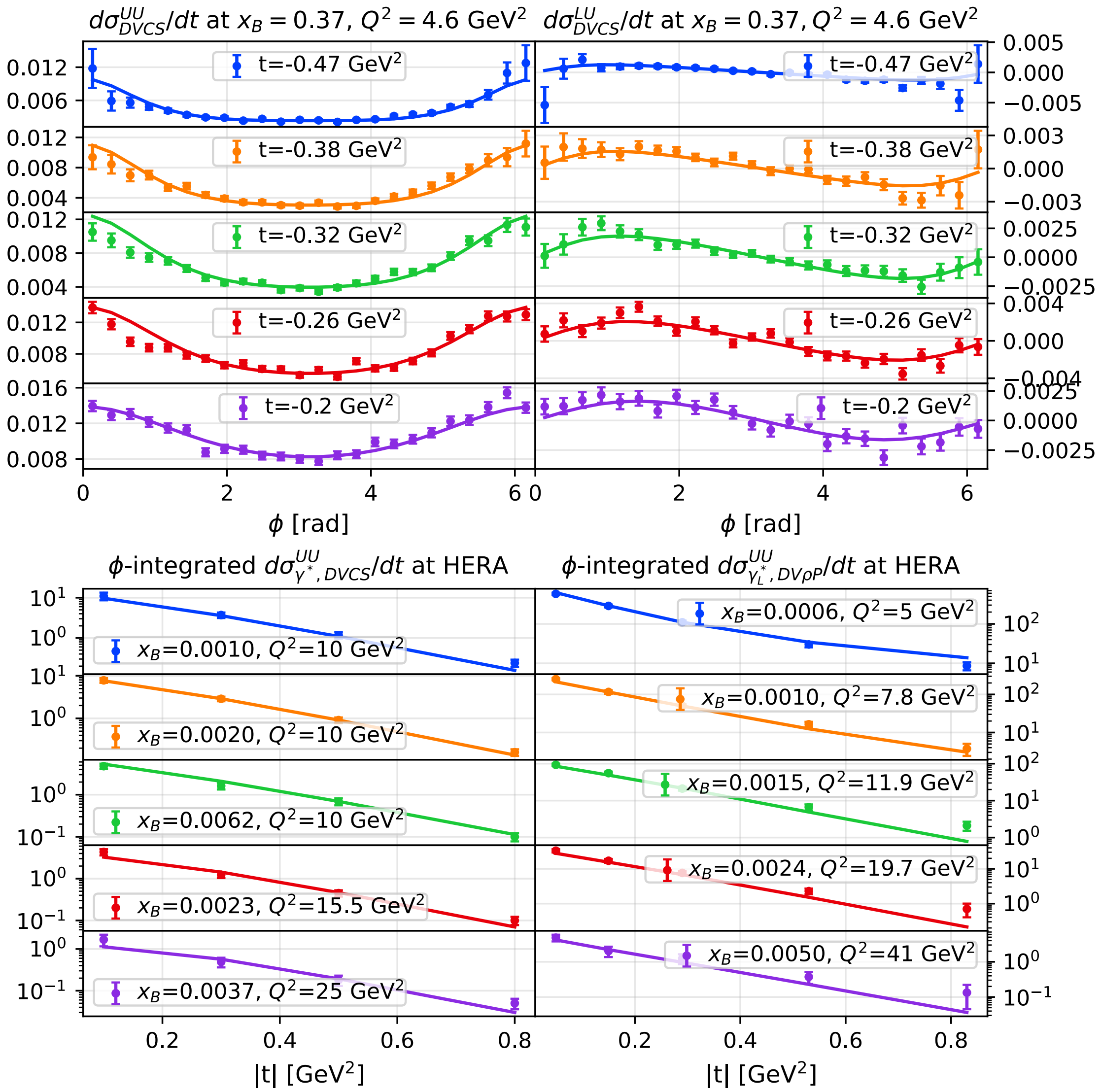


Reconstructed at double distribution (DD) level

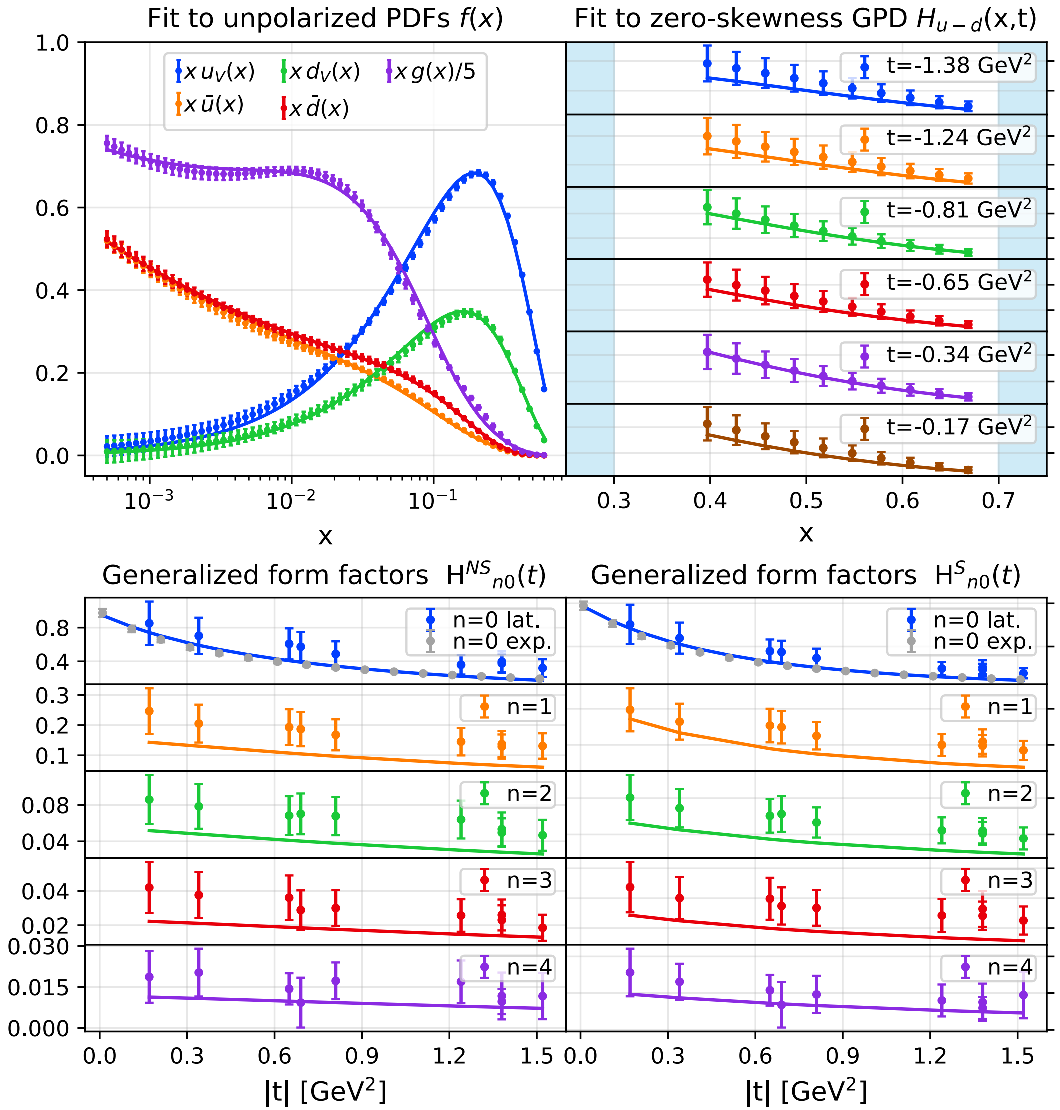
Recent progress: global fit from GUMP

Y. Guo, et al., PRL 135 (2025) 26, 261903

Fit to DVCS and DVMP data



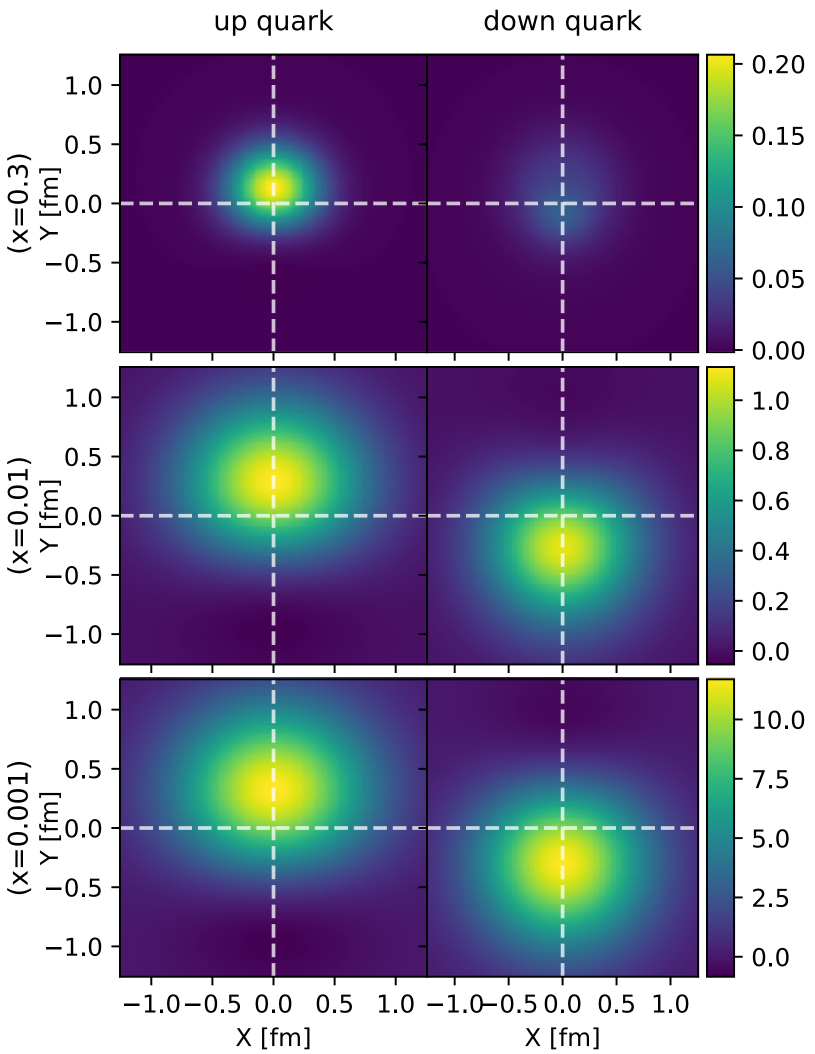
Fit to PDF and lattice moments

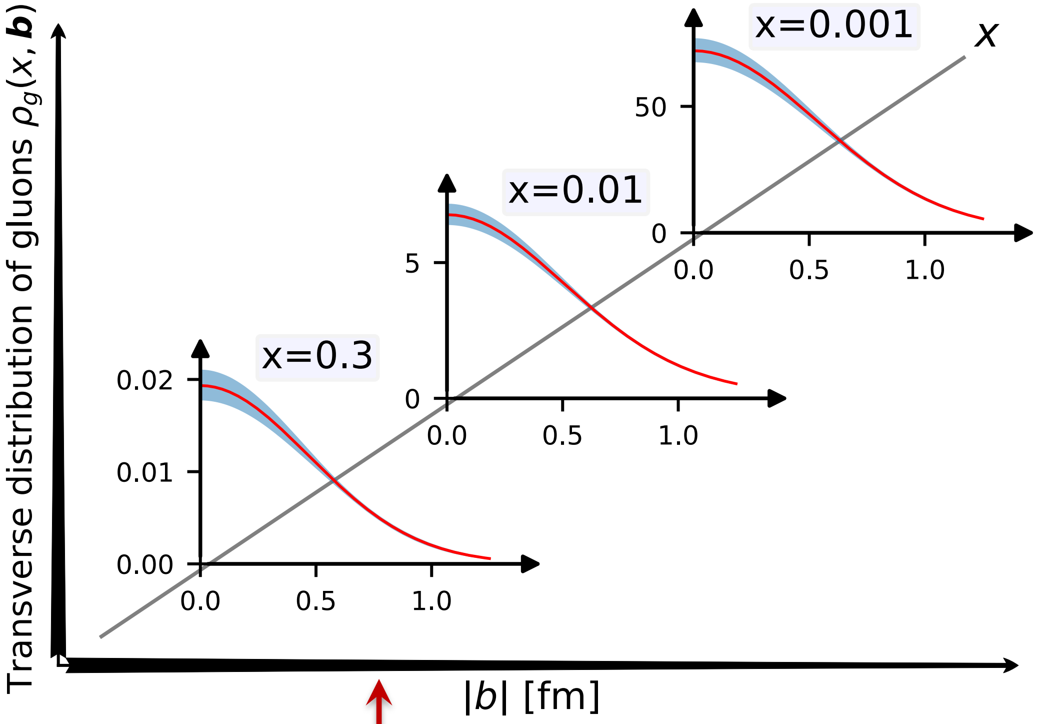


Recent progress: global fit from GUMP

Y. Guo, et al., PRL 135 (2025) 26, 261903

Partonic images

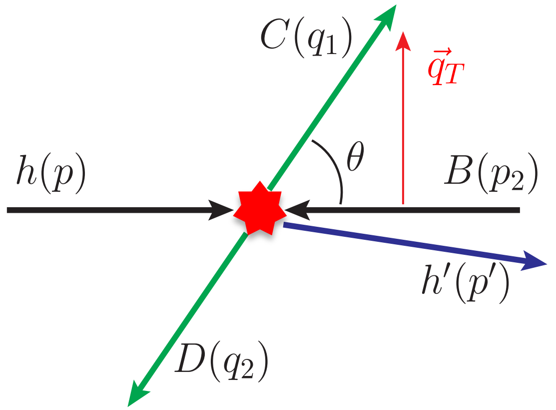




Unpolarized proton

Proton transversely polarized along $\vec{x}$ direction

Summary and outlook



- Factorization is extended from inclusive to exclusive processes
- New correlation functions: DAs and GPDs
- GPDs provide tomographic images
- Single-diffractive hard exclusive processes (SDHEPs) for GPDs
- Two-stage picture of SDHEPs entails a unique quantum interference pattern
- Inversion problem makes GPD extraction very difficult
- Two new processes to give enhanced x-sensitivity
- So far, looks consistent. But still not there yet.
  - Need more processes, more observables
  - Need input from lattice QCD

3D image