CNUGS

CHRISTOPHER NEWPORT UNIVERSITY GRADUATE SUMMER WORKSHOP AT JEFFERSON LAB 2026

formerly The HUGS Program

QCD factorization and global analysis

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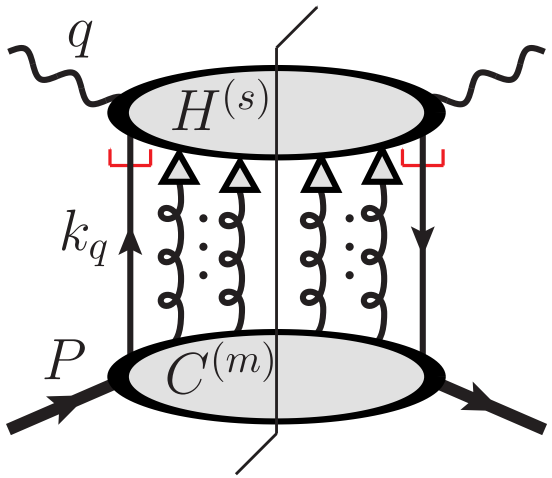
- Introduction to QCD factorization
- Parton distribution functions and global analysis
- From inclusive to exclusive processes
- Generalized parton distributions and global analysis

CNUGS Summer Workshop, May 26 -- June 13, 2026

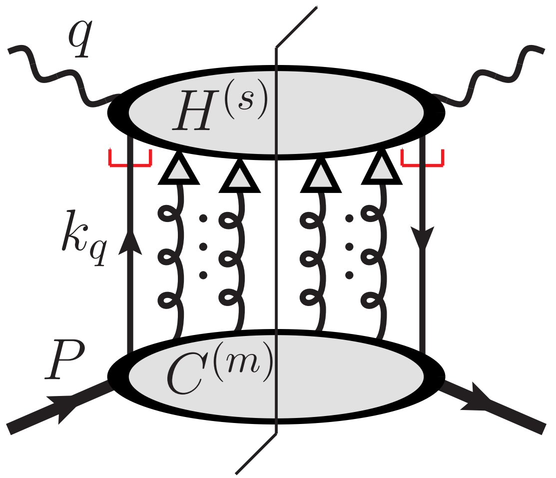
From DIS to Drell-Yan



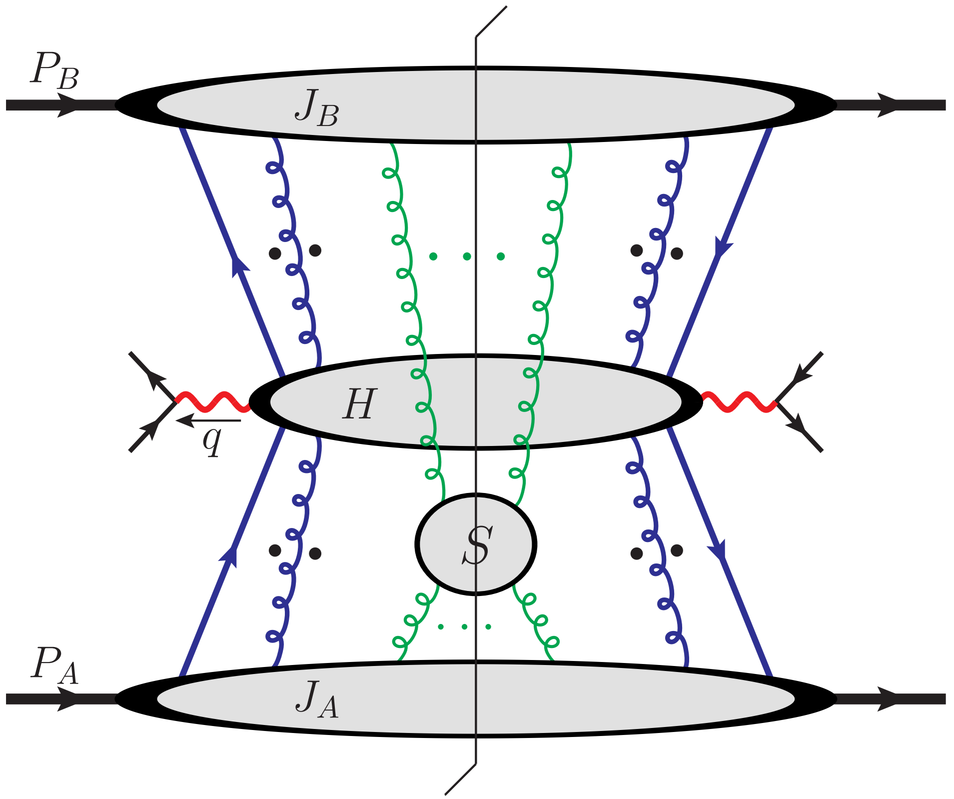
$e + p \to e + X$

- **DIS factorization:** **factorize collinear pinches.**

From DIS to Drell-Yan



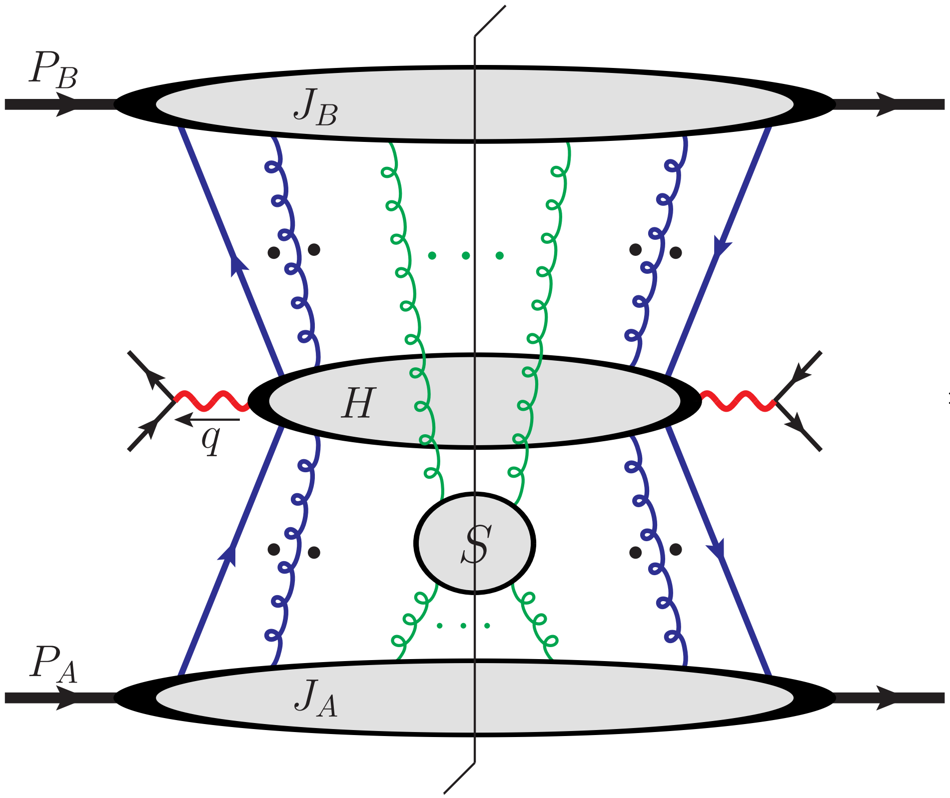
$e + p \to e + X$



$H_A + H_B \to \gamma^*(Q)\,[\to \mu^+\mu^-] + X$

- **DIS factorization: factorize collinear pinches.**
- **Goal of Drell-Yan factorization:**
  - Collinear sectors from two hadrons are separated **from each other**
  - They are also factorized **from the hard part**
  - Connected to the PDF in DIS

Drell-Yan factorization



$$= \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \, f_{a/H_A}(x_a, \mu) \, f_{b/H_B}(x_b, \mu) \, C_{ab}(x_a, x_b; Q/\mu)$$

- **Key: inclusiveness $\Longrightarrow$ unitarity cancels soft gluons**
- **PDFs from Drell-Yan factorization are the same as in DIS**
- **Hard part involves subtraction for collinear pinches**

Global analysis of PDFs

- **Parametrize PDFs at a given scale μ_0**

E.g., JAM20-SIDIS $f_i(x;\mu_0) = T_i(x;\boldsymbol{a}_i) = M_i\, x^{\alpha_i}(1-x)^{\beta_i}\left(1+\gamma_i\sqrt{x}+\delta_i\, x\right)$

$$i = g,\ u,\ d,\ s,\ c,\ \bar{u},\ \bar{d},\ \bar{s},\ \bar{c}$$

JAM, PRD 104, 016015 (2021)

9 independent flavors. Typically, μ_0 is just above m_c

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- **Solve DGLAP evolution to obtain PDFs at scale Q**

$$\frac{d}{d\ln\mu^2} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(x/z)\, f_j(z,\mu)$$

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- **Use factorization to make prediction for observables at scale Q**

E.g., DIS structure function

$$F_1^h(x_B,\, Q^2) = \sum_i \int_{x_B}^1 \frac{dx}{x} f_{i/h}(x,\mu)\, \hat{F}_1^i\left(\frac{x_B}{x},\, \frac{Q^2}{\mu^2}\right)$$

Global analysis of PDFs

➢ Parametrize PDFs at a given scale μ_0

E.g., JAM20-SIDIS $$f_i(x;\mu_0) = T_i(x;\boldsymbol{a}_i) = M_i\, x^{\alpha_i}(1-x)^{\beta_i}\left(1+\gamma_i\sqrt{x}+\delta_i\, x\right)$$

$$i = g,\ u,\ d,\ s,\ c,\ \bar{u},\ \bar{d},\ \bar{s},\ \bar{c}$$

JAM, PRD 104, 016015 (2021)

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➢ Solve DGLAP evolution to obtain PDFs at scale Q

$$\frac{d}{d\ln\mu^2} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(x/z)\, f_j(z,\mu)$$

➢ Use factorization to make prediction for observables at scale Q

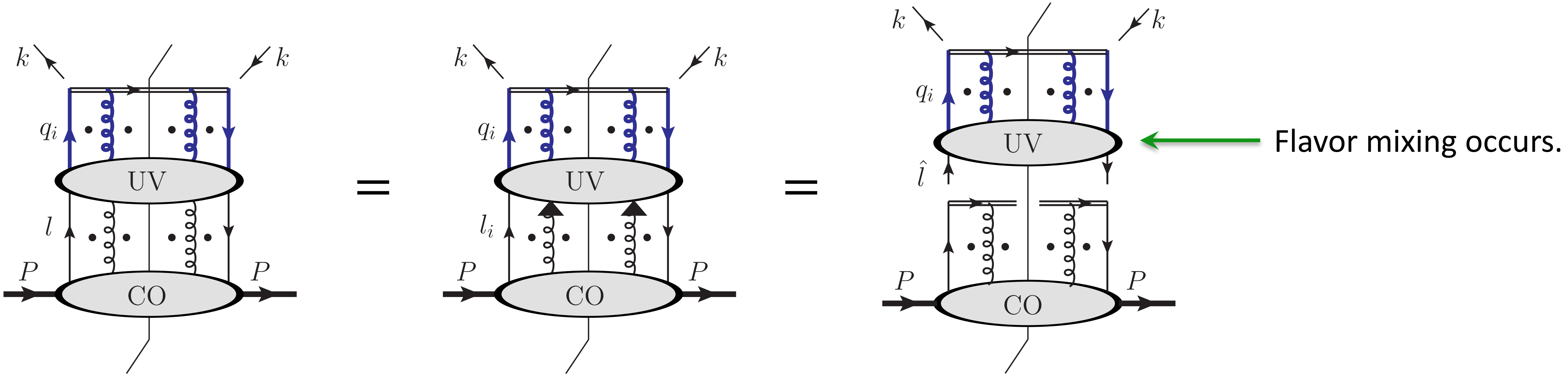
E.g., DIS structure function $$F_1^h(x_B, Q^2) = \sum_i \int_{x_B}^1 \frac{dx}{x} f_{i/h}(x,\mu)\, \hat{F}_1^i\left(\frac{x_B}{x}, \frac{Q^2}{\mu^2}\right)$$

➢ Compare with experimental measurements

- Design loss function, e.g., $\chi^2(\vec{a})$, and minimize it to find the best fit
- Quantify uncertainties

DGLAP equation

Recall: DGLAP arises from **UV renormalization** of bare PDFs



$$f_{i/H}(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} Z_{ij}(x/z, 1/\epsilon; \mu)\, f^{\text{bare}}_{j/H}(z, 1/\epsilon)$$

- Z_{ij} collects all UV poles. Scheme dependent. Typically, $\overline{\text{MS}}$ scheme.
- **UV property of operator**. Does not depend on target state or **parton mass**.
- Perturbatively calculated, order by order.

Symmetry of DGLAP kernel

Replace H by a parton state.

$$f_{i/k}(x,\mu) = \sum_j \int_x^1 \frac{\mathrm{d}z}{z}\, Z_{ij}(x/z, 1/\epsilon;\mu)\, f_{j/k}^{\text{bare}}(z, 1/\epsilon) \qquad i,j,k = q,\bar{q},g$$

❖ **Charge conjugation symmetry**

$$f_{i/k}^{\text{bare}} = f_{\bar{i}/\bar{k}}^{\text{bare}} \longrightarrow \mathrm{UV}\left[f_{i/k}^{\text{bare}}\right] = \mathrm{UV}\left[f_{\bar{i}/\bar{k}}^{\text{bare}}\right] \longrightarrow Z_{ij} = Z_{\bar{i}\bar{j}} \longrightarrow f_{i/k} = f_{\bar{i}/\bar{k}}$$

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$$f_{i/k}(x,\mu) = \sum_j \int_x^1 \frac{\mathrm{d}z}{z}\, Z_{ij}(x/z, 1/\epsilon;\mu)\, f^{\text{bare}}_{j/k}(z, 1/\epsilon) \qquad i,j,k = q,\bar{q},g$$

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$$\frac{d}{d\ln\mu^2} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z}\, P_{ij}(x/z)\, f_j(z,\mu) \qquad P_{ij} = P_{\bar{i}\bar{j}}$$

Symmetry of DGLAP kernel

Replace H by a parton state.

$$f_{i/k}(x,\mu) = \sum_j \int_x^1 \frac{\mathrm{d}z}{z}\, Z_{ij}(x/z, 1/\epsilon;\mu)\, f^{\rm bare}_{j/k}(z,1/\epsilon) \qquad i,j,k = q,\bar{q},g$$

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❖ **Together with SU(n_f) flavor symmetry**

$$P_{q_i g} = P_{\bar{q}_i g} \equiv P_{qg}, \qquad P_{g q_i} = P_{g\bar{q}_i} \equiv P_{gq}$$

$$P_{ud} = P_{\bar{u}\bar{d}} = P_{us} = P_{\bar{u}\bar{s}} = \ldots \qquad P_{uu} = P_{\bar{u}\bar{u}} = P_{dd} = P_{\bar{d}\bar{d}} = \ldots$$

$$P_{u\bar{d}} = P_{\bar{u}d} = P_{u\bar{s}} = P_{\bar{u}s} = \ldots \qquad P_{u\bar{u}} = P_{\bar{u}u} = P_{d\bar{d}} = P_{\bar{d}d} = \ldots$$

Seven independent DGLAP kernels

$$P_{q_i g} = P_{\bar{q}_i g} \equiv P_{qg}, \quad P_{g q_i} = P_{g \bar{q}_i} \equiv P_{gq}$$

$$P_{ud} = P_{\bar{u}\bar{d}} = P_{us} = P_{\bar{u}\bar{s}} = \ldots \qquad P_{uu} = P_{\bar{u}\bar{u}} = P_{dd} = P_{\bar{d}\bar{d}} = \ldots$$

$$P_{u\bar{d}} = P_{\bar{u}d} = P_{u\bar{s}} = P_{\bar{u}s} = \ldots \qquad P_{u\bar{u}} = P_{\bar{u}u} = P_{d\bar{d}} = P_{\bar{d}d} = \ldots$$

$$P_{q_i q_j} = P_{\bar{q}_i \bar{q}_j} = P^v_{qq}\,\delta_{ij} + P^s_{qq} = \begin{cases} P^v_{qq} + P^s_{qq} & \text{if } i = j \\ P^s_{qq} & \text{if } i \neq j \end{cases}$$

$$P_{q_i \bar{q}_j} = P_{\bar{q}_i q_j} = P^v_{q\bar{q}}\,\delta_{ij} + P^s_{q\bar{q}} = \begin{cases} P^v_{q\bar{q}} + P^s_{q\bar{q}} & \text{if } i = j \\ P^s_{q\bar{q}} & \text{if } i \neq j \end{cases}$$

$$P_{q_i g} = P_{\bar{q}_i g} = P_{qg},$$

$$P_{g q_i} = P_{g \bar{q}_i} = P_{gq}$$

$$P_{gg}$$

Flavor decomposition of DGLAP equation

Notation: Momentum convolution $(A \otimes B)(x) \equiv \int_x^1 \frac{\mathrm{d}z}{z} A(x/z)\, B(z)$

Flavor index $i, j = u, d, s, \ldots$ $\bar{i}, \bar{j} = \bar{u}, \bar{d}, \bar{s}, \ldots$ & g

$$P_{q_i q_j} = P_{\bar{q}_i \bar{q}_j} = P^v_{qq}\,\delta_{ij} + P^s_{qq}$$
$$P_{q_i \bar{q}_j} = P_{\bar{q}_i q_j} = P^v_{q\bar{q}}\,\delta_{ij} + P^s_{q\bar{q}}$$
$$P_{q_i g} = P_{\bar{q}_i g} = P_{qg}$$
$$P_{g q_i} = P_{g \bar{q}_i} = P_{gq}$$
$$P_{gg}$$

$$\frac{\mathrm{d}\, f_i}{\mathrm{d} \ln \mu^2} = \sum_j P_{ij} \otimes f_j + \sum_{\bar{j}} P_{i\bar{j}} \otimes f_{\bar{j}} + P_{ig} \otimes f_g$$

$$\frac{\mathrm{d}\, f_{\bar{i}}}{\mathrm{d} \ln \mu^2} = \sum_j P_{\bar{i}j} \otimes f_j + \sum_{\bar{j}} P_{\bar{i}\bar{j}} \otimes f_{\bar{j}} + P_{\bar{i}g} \otimes f_g$$

$$\frac{\mathrm{d}\, f_g}{\mathrm{d} \ln \mu^2} = \sum_j P_{gj} \otimes f_j + \sum_{\bar{j}} P_{g\bar{j}} \otimes f_{\bar{j}} + P_{gg} \otimes f_g$$

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$$= P^v_{qq} \otimes f_i + P^v_{q\bar{q}} \otimes f_{\bar{i}} + P^s_{qq} \otimes \sum_j f_j + P^s_{q\bar{q}} \otimes \sum_{\bar{j}} f_{\bar{j}} + P_{qg} \otimes f_g,$$

$$\frac{\mathrm{d}\, f_{\bar{i}}}{\mathrm{d}\ln\mu^2} = \sum_j P_{\bar{i}j} \otimes f_j + \sum_{\bar{j}} P_{\bar{i}\bar{j}} \otimes f_{\bar{j}} + P_{\bar{i}g} \otimes f_g$$
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$$\frac{\mathrm{d}\, f_g}{\mathrm{d}\ln\mu^2} = \sum_j P_{gj} \otimes f_j + \sum_{\bar{j}} P_{g\bar{j}} \otimes f_{\bar{j}} + P_{gg} \otimes f_g$$
$$= P_{gg} \otimes f_g + P_{gq} \otimes \sum_j (f_j + f_{\bar{j}})\,.$$

How to disentangle flavors?

Flavor decomposition of DGLAP equation

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Flavor index $i, j = u, d, s, \ldots$ & g
$\bar{i}, \bar{j} = \bar{u}, \bar{d}, \bar{s}, \ldots$

$$P_{q_i q_j} = P_{\bar{q}_i \bar{q}_j} = P^v_{qq}\, \delta_{ij} + P^s_{qq}$$
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$$\frac{\mathrm{d}\, f_{\bar{i}}}{\mathrm{d} \ln \mu^2} = \sum_j P_{\bar{i}j} \otimes f_j + \sum_{\bar{j}} P_{\bar{i}\bar{j}} \otimes f_{\bar{j}} + P_{\bar{i}g} \otimes f_g$$
$$= P^v_{qq} \otimes f_{\bar{i}} + P^v_{q\bar{q}} \otimes f_i + P^s_{qq} \otimes \sum_{\bar{j}} f_{\bar{j}} + P^s_{q\bar{q}} \otimes \sum_j f_j + P_{qg} \otimes f_g,$$

$$\frac{\mathrm{d}\, f_g}{\mathrm{d} \ln \mu^2} = \sum_j P_{gj} \otimes f_j + \sum_{\bar{j}} P_{g\bar{j}} \otimes f_{\bar{j}} + P_{gg} \otimes f_g$$
$$= P_{gg} \otimes f_g + P_{gq} \otimes \sum_j (f_j + f_{\bar{j}}).$$

How to disentangle flavors?

f_i and $f_{\bar{i}}$ mix with f_g in the same way

Charge conjugation symmetry!

Define $f_i^{\pm} \equiv f_i \pm f_{\bar{i}}$

Flavor decomposition of DGLAP equation

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Flavor index $i, j = u, d, s, \ldots$ & g

$\bar{i}, \bar{j} = \bar{u}, \bar{d}, \bar{s}, \ldots$

$$P_{q_i q_j} = P_{\bar{q}_i \bar{q}_j} = P^v_{qq}\, \delta_{ij} + P^s_{qq}$$

$$P_{q_i \bar{q}_j} = P_{\bar{q}_i q_j} = P^v_{q\bar{q}}\, \delta_{ij} + P^s_{q\bar{q}}$$

$$P_{q_i g} = P_{\bar{q}_i g} = P_{qg}$$

$$P_{g q_i} = P_{g \bar{q}_i} = P_{gq}$$

$$P_{gg}$$

Task (3mins): Work out the evolution equation of $f_i^{\pm}$

$$\frac{\mathrm{d}\, f_i}{\mathrm{d} \ln \mu^2} = P^v_{qq} \otimes f_i + P^v_{q\bar{q}} \otimes f_{\bar{i}} + P^s_{qq} \otimes \sum_j f_j + P^s_{q\bar{q}} \otimes \sum_{\bar{j}} f_{\bar{j}} + P_{qg} \otimes f_g,$$

$$\frac{\mathrm{d}\, f_{\bar{i}}}{\mathrm{d} \ln \mu^2} = P^v_{qq} \otimes f_{\bar{i}} + P^v_{q\bar{q}} \otimes f_i + P^s_{qq} \otimes \sum_{\bar{j}} f_{\bar{j}} + P^s_{q\bar{q}} \otimes \sum_j f_j + P_{qg} \otimes f_g,$$

$$\frac{\mathrm{d}\, f_g}{\mathrm{d} \ln \mu^2} = P_{gg} \otimes f_g + P_{gq} \otimes \sum_j (f_j + f_{\bar{j}}).$$

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Define $f_i^{\pm} \equiv f_i \pm f_{\bar{i}}$

Flavor decomposition of DGLAP equation

$$\frac{\mathrm{d}\, f_i}{\mathrm{d}\ln\mu^2} = P^v_{qq} \otimes f_i + P^v_{q\bar{q}} \otimes f_{\bar{i}} + P^s_{qq} \otimes \sum_j f_j + P^s_{q\bar{q}} \otimes \sum_{\bar{j}} f_{\bar{j}} + P_{qg} \otimes f_g,$$

$$\frac{\mathrm{d}\, f_{\bar{i}}}{\mathrm{d}\ln\mu^2} = P^v_{qq} \otimes f_{\bar{i}} + P^v_{q\bar{q}} \otimes f_i + P^s_{qq} \otimes \sum_{\bar{j}} f_{\bar{j}} + P^s_{q\bar{q}} \otimes \sum_j f_j + P_{qg} \otimes f_g,$$

$$\frac{\mathrm{d}\, f_g}{\mathrm{d}\ln\mu^2} = P_{gg} \otimes f_g + P_{gq} \otimes \sum_j (f_j + f_{\bar{j}}).$$

$$f_i^{\pm} \equiv f_i \pm f_{\bar{i}}$$

$$\frac{\mathrm{d}\, f_i^+}{\mathrm{d}\ln\mu^2} = \left(P^v_{qq} + P^v_{q\bar{q}}\right) \otimes f_i^+ + \left(P^s_{qq} + P^s_{q\bar{q}}\right) \otimes \sum_j f_j^+ + 2P_{qg} \otimes f_g,$$

$$\frac{\mathrm{d}\, f_i^-}{\mathrm{d}\ln\mu^2} = \left(P^v_{qq} - P^v_{q\bar{q}}\right) \otimes f_i^- + \left(P^s_{qq} - P^s_{q\bar{q}}\right) \otimes \sum_j f_j^-,$$

$$\frac{\mathrm{d}\, f_g}{\mathrm{d}\ln\mu^2} = P_{gg} \otimes f_g + P_{gq} \otimes \sum_j f_j^+.$$

- f_i^+ and f_i^- don’t mix
- f_i^+ mixes with their flavor sum, and gluon
- f_i^- only mixes with their flavor sum

Flavor singlet and non-singlet

$$\frac{\mathrm{d}\, f_i}{\mathrm{d}\ln\mu^2} = P^v_{qq}\otimes f_i + P^v_{q\bar{q}}\otimes f_{\bar{i}} + P^s_{qq}\otimes\sum_j f_j + P^s_{q\bar{q}}\otimes\sum_{\bar{j}} f_{\bar{j}} + P_{qg}\otimes f_g,$$

$$\frac{\mathrm{d}\, f_{\bar{i}}}{\mathrm{d}\ln\mu^2} = P^v_{qq}\otimes f_{\bar{i}} + P^v_{q\bar{q}}\otimes f_i + P^s_{qq}\otimes\sum_{\bar{j}} f_{\bar{j}} + P^s_{q\bar{q}}\otimes\sum_j f_j + P_{qg}\otimes f_g,$$

$$\frac{\mathrm{d}\, f_g}{\mathrm{d}\ln\mu^2} = P_{gg}\otimes f_g + P_{gq}\otimes\sum_j (f_j + f_{\bar{j}}).$$

$$f_i^{\pm} \equiv f_i \pm f_{\bar{i}}$$

$$\frac{\mathrm{d}\, f_i^+}{\mathrm{d}\ln\mu^2} = \left(P^v_{qq} + P^v_{q\bar{q}}\right)\otimes f_i^+ + \left(P^s_{qq} + P^s_{q\bar{q}}\right)\otimes\sum_j f_j^+ + 2P_{qg}\otimes f_g,$$

$$\frac{\mathrm{d}\, f_i^-}{\mathrm{d}\ln\mu^2} = \left(P^v_{qq} - P^v_{q\bar{q}}\right)\otimes f_i^- + \left(P^s_{qq} - P^s_{q\bar{q}}\right)\otimes\sum_j f_j^-,$$

$$\frac{\mathrm{d}\, f_g}{\mathrm{d}\ln\mu^2} = P_{gg}\otimes f_g + P_{gq}\otimes\sum_j f_j^+.$$

$$f_{\mathrm{s}} \equiv \sum_j f_j^+$$

$$f_{\mathrm{ns}} \equiv \sum_j f_j^-$$

Task (2mins): Work out the evolution equation of f_s and f_{ns} ,

assuming there are n_f active quark flavors.

Flavor singlet and non-singlet

$$\frac{\mathrm{d}\, f_i^+}{\mathrm{d}\ln\mu^2} = \left(P_{qq}^v + P_{q\bar{q}}^v\right) \otimes f_i^+ + \left(P_{qq}^s + P_{q\bar{q}}^s\right) \otimes f_{\mathrm{s}} + 2P_{qg} \otimes f_g,$$

$$\frac{\mathrm{d}\, f_i^-}{\mathrm{d}\ln\mu^2} = \left(P_{qq}^v - P_{q\bar{q}}^v\right) \otimes f_i^- + \left(P_{qq}^s - P_{q\bar{q}}^s\right) \otimes f_{\mathrm{ns}},$$

$$\frac{\mathrm{d}\, f_g}{\mathrm{d}\ln\mu^2} = P_{gg} \otimes f_g + P_{gq} \otimes f_{\mathrm{s}}.$$

$$f_{\mathrm{s}} \equiv \sum_j f_j^+$$

$$f_{\mathrm{ns}} \equiv \sum_j f_j^-$$

3 equations

$$\frac{\mathrm{d}\, f_{\mathrm{ns}}}{\mathrm{d}\ln\mu^2} = \left[P_{qq}^v - P_{q\bar{q}}^v + n_f\left(P_{qq}^s - P_{q\bar{q}}^s\right)\right] \otimes f_{\mathrm{ns}}$$

Evolves by itself

$$\frac{\mathrm{d}\, f_{\mathrm{s}}}{\mathrm{d}\ln\mu^2} = \left[P_{qq}^v + P_{q\bar{q}}^v + n_f\left(P_{qq}^s + P_{q\bar{q}}^s\right)\right] \otimes f_{\mathrm{s}} + 2n_f P_{qg} \otimes f_g$$

$$\frac{\mathrm{d}\, f_g}{\mathrm{d}\ln\mu^2} = P_{gg} \otimes f_g + P_{gq} \otimes f_{\mathrm{s}}$$

Coupled together

Flavor singlet and non-singlet

$$\frac{d\,f_i^+}{d\ln\mu^2} = \left(P_{qq}^v + P_{q\bar{q}}^v\right) \otimes f_i^+ + \left(P_{qq}^s + P_{q\bar{q}}^s\right) \otimes f_s + 2P_{qg} \otimes f_g,$$

$$\frac{d\,f_i^-}{d\ln\mu^2} = \left(P_{qq}^v - P_{q\bar{q}}^v\right) \otimes f_i^- + \left(P_{qq}^s - P_{q\bar{q}}^s\right) \otimes f_{ns},$$

$$\frac{d\,f_g}{d\ln\mu^2} = P_{gg} \otimes f_g + P_{gq} \otimes f_s.$$

$$f_s \equiv \sum_j f_j^+$$

$$f_{ns} \equiv \sum_j f_j^-$$

3 equations

$$\frac{d\,f_{ns}}{d\ln\mu^2} = \left[P_{qq}^v - P_{q\bar{q}}^v + n_f\left(P_{qq}^s - P_{q\bar{q}}^s\right)\right] \otimes f_{ns}$$

Evolves by itself

$$\frac{d\,f_s}{d\ln\mu^2} = \left[P_{qq}^v + P_{q\bar{q}}^v + n_f\left(P_{qq}^s + P_{q\bar{q}}^s\right)\right] \otimes f_s + 2n_f P_{qg} \otimes f_g$$

$$\frac{d\,f_g}{d\ln\mu^2} = P_{gg} \otimes f_g + P_{gq} \otimes f_s$$

Coupled together

Remaining $2(n_f - 1)$ equations are for relative differences among f_i^+ and f_i^-

E.g., $f_{ij}^{\pm} \equiv f_i^{\pm} - f_j^{\pm}$ evolve by themselves

$$\frac{d\,f_{ij}^{\pm}}{d\ln\mu^2} = \left(P_{qq}^v \pm P_{q\bar{q}}^v\right) \otimes f_{ij}^{\pm}$$

Flavor singlet and non-singlet

$$\frac{d\, f_i^+}{d\ln\mu^2} = \left(P_{qq}^v + P_{q\bar{q}}^v\right) \otimes f_i^+ + \left(P_{qq}^s + P_{q\bar{q}}^s\right) \otimes f_s + 2P_{qg} \otimes f_g,$$

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Coupled together

Remaining $2(n_f - 1)$ equations are for relative differences among f_i^+ and f_i^-

Or,
$$\Delta f_i^+ = f_s - n_f f_i^+$$
$$\Delta f_i^- = f_{ns} - n_f f_i^-$$
evolve by themselves, too, by the same kernels
$$\frac{d\,\Delta f_i^\pm}{d\ln\mu^2} = \left(P_{qq}^v \pm P_{q\bar{q}}^v\right) \otimes \Delta f_i^\pm$$

Flavor decomposed DGLAP equations

$$\frac{\mathrm{d}\, f_{\mathrm{ns}}}{\mathrm{d}\ln\mu^2} = \left[P^v_{qq} - P^v_{q\bar q} + n_f\left(P^s_{qq} - P^s_{q\bar q}\right)\right]\otimes f_{\mathrm{ns}}$$

$$\frac{\mathrm{d}\, f_{\mathrm{s}}}{\mathrm{d}\ln\mu^2} = \left[P^v_{qq} + P^v_{q\bar q} + n_f\left(P^s_{qq} + P^s_{q\bar q}\right)\right]\otimes f_{\mathrm{s}} + 2n_f P_{qg}\otimes f_g$$

$$\frac{\mathrm{d}\, f_g}{\mathrm{d}\ln\mu^2} = P_{gg}\otimes f_g + P_{gq}\otimes f_{\mathrm{s}}$$

3 equations

$$\frac{\mathrm{d}\,\Delta f_i^{\pm}}{\mathrm{d}\ln\mu^2} = \left(P^v_{qq} \pm P^v_{q\bar q}\right)\otimes \Delta f_i^{\pm}$$

$2(n_f - 1)$ independent equations

$$f_{\mathrm{s}} \equiv \sum_j f_j^{+}$$

$$f_{\mathrm{ns}} \equiv \sum_j f_j^{-}$$

$$f_g$$

$$\Delta f_i^{+} = f_{\mathrm{s}} - n_f f_i^{+}$$

$$\Delta f_i^{-} = f_{\mathrm{ns}} - n_f f_i^{-}$$

$$f_i^{+} = \frac{1}{n_f}\left(f_{\mathrm{s}} - \Delta f_i^{+}\right)$$

$$f_i^{-} = \frac{1}{n_f}\left(f_{\mathrm{ns}} - \Delta f_i^{-}\right)$$

Proper matching at quark thresholds is needed as μ goes up.

DGLAP kernels: Are they all nonzero and independent?

Question (1min): Which kernels are nonzero at LO? $\mathcal{O}(\alpha_s)$

$$P_{q_i q_j} = P_{\bar{q}_i \bar{q}_j} = P^v_{qq}\,\delta_{ij} + P^s_{qq}$$

$$P_{q_i \bar{q}_j} = P_{\bar{q}_i q_j} = P^v_{q\bar{q}}\,\delta_{ij} + P^s_{q\bar{q}}$$

$$P_{q_i g} = P_{\bar{q}_i g} = P_{qg}$$

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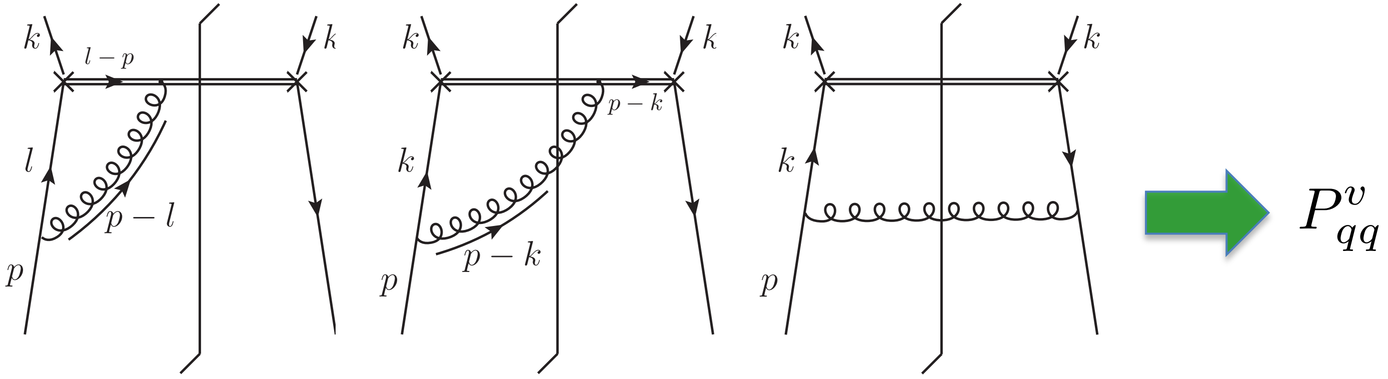
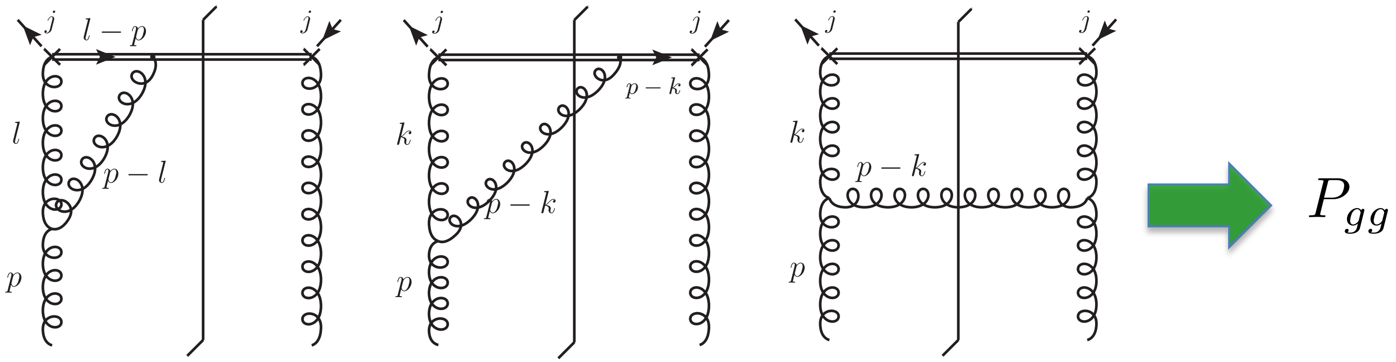
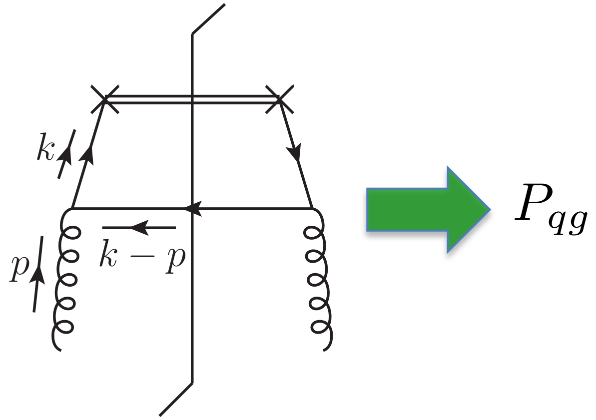
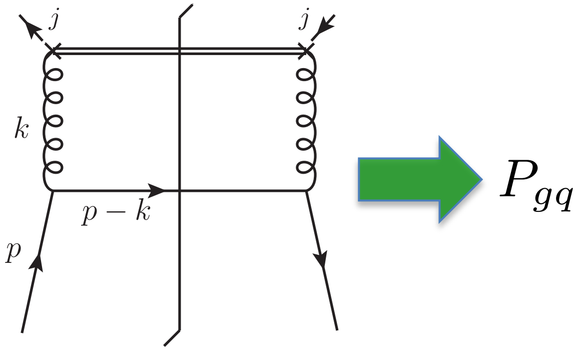
P_{gg}









But $P^v_{q\bar{q}} = P^s_{qq} = P^s_{q\bar{q}} = 0$ at LO

DGLAP kernels: Are they all nonzero and independent?

Question (3mins): Any ***new*** kernels occur at NLO? $\mathcal{O}(\alpha_s^2)$

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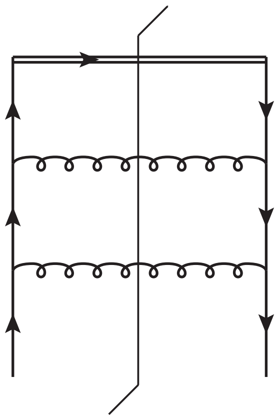
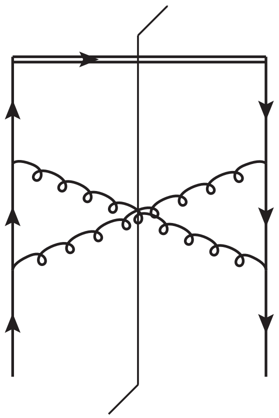
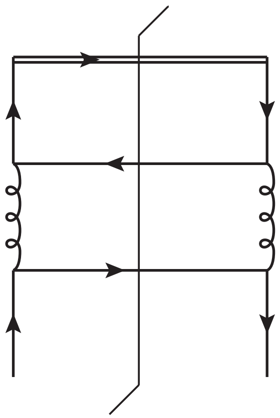
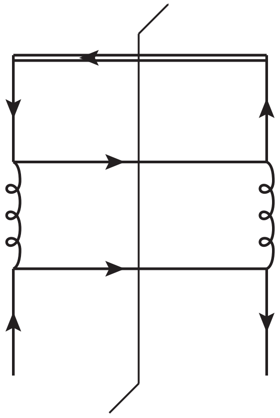
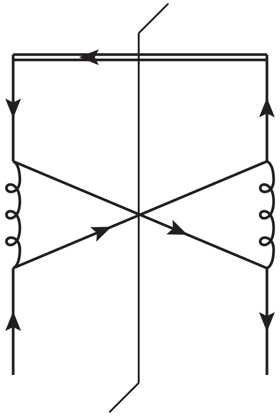



$\Rightarrow$ P^v_{qq}

$\Rightarrow$ $P^s_{qq} = P^s_{q\bar{q}}$



$\Rightarrow$ $P^v_{q\bar{q}}$

- All the seven kernels are nonzero.
- But there is a degeneracy.

DGLAP kernels: Are they all nonzero and independent?

Question (3mins): Does the degeneracy retain at NNLO? $\mathcal{O}(\alpha_s^3)$

$$P_{q_i q_j} = P_{\bar{q}_i \bar{q}_j} = P^v_{qq}\,\delta_{ij} + P^s_{qq}$$

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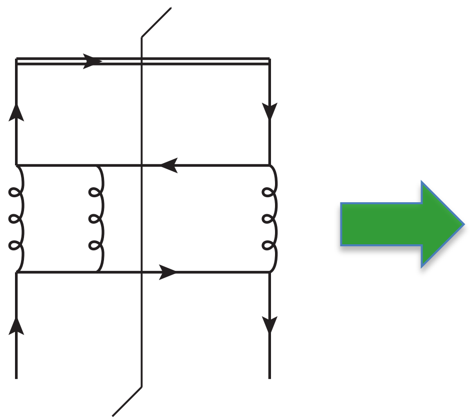
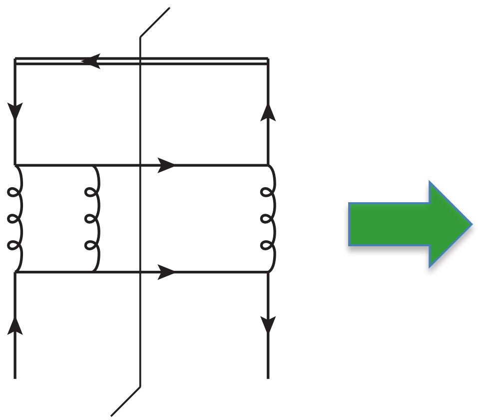


P^s_{qq}



$P^s_{q\bar{q}}$

DGLAP kernels: They **ARE** all nonzero and independent!

$$P_{q_i q_j} = P_{\bar{q}_i \bar{q}_j} = P^v_{qq}\,\delta_{ij} + P^s_{qq}$$
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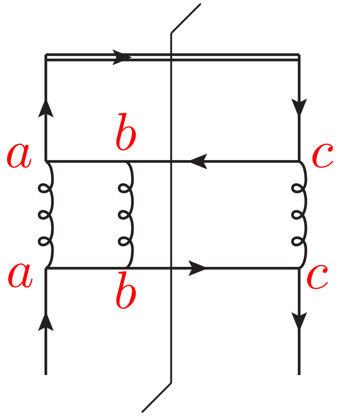
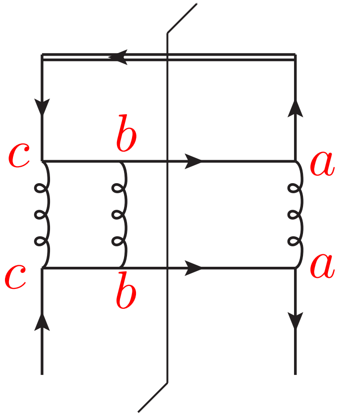


$\Rightarrow P^s_{qq}$



$\Rightarrow P^s_{q\bar{q}}$

$$-\frac{1}{N_c}\,\mathrm{tr}\{t^a t^b t^c\}\,\mathrm{tr}\{t^c t^b t^a\} = \frac{-1}{16N_c}(d^2_{abc} + f^2_{abc})$$

$-$

$$\frac{1}{N_c}\,\mathrm{tr}\{t^a t^b t^c\}\,\mathrm{tr}\{t^a t^b t^c\} = \frac{1}{16N_c}(d^2_{abc} - f^2_{abc})$$

$$= -\frac{d^2_{abc}}{8N_c}$$

$\Rightarrow$ $P^s_{qq} \neq P^s_{q\bar{q}}$ starting at NNLO, due to **Abelian quantum interference** effect!

What shall we expect?

At LO:

$$\frac{\mathrm{d}\, f_i}{\mathrm{d}\ln\mu^2} = P_{qq}^v \otimes f_i + P_{qg} \otimes f_g,$$
$$\frac{\mathrm{d}\, f_{\bar{i}}}{\mathrm{d}\ln\mu^2} = P_{qq}^v \otimes f_{\bar{i}} + P_{qg} \otimes f_g,$$
$$\frac{\mathrm{d}\, f_g}{\mathrm{d}\ln\mu^2} = P_{gg} \otimes f_g + P_{gq} \otimes f_s.$$

$$P_{qg}(z) = \frac{\alpha_s}{2\pi} T_F \left[z^2 + (1-z)^2\right] > 0$$
$$P_{gq}(z) = \frac{\alpha_s}{2\pi} C_F \left[\frac{1+(1-z)^2}{z}\right] > 0$$
$$P_{gg}(z) = \frac{\alpha_s}{2\pi} \left[2C_A \left(\frac{z}{(1-z)_+} + z(1-z) + \frac{1-z}{z}\right) + \left(\frac{11}{6}C_A - \frac{2}{3}n_f T_F\right)\delta(1-z)\right]$$
$$P_{qq}^v(z) = \frac{\alpha_s}{2\pi} C_F \left[\frac{2}{(1-z)_+} - 1 - z + \frac{3}{2}\delta(1-z)\right]$$

Property of the convolution:

$$(P \otimes f)(x) \equiv \int_x^1 \frac{\mathrm{d}z}{z}\, P(x/z)\, f(z) = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2\, \delta(x - z_1 z_2)\, P(z_1) f(z_2),$$

➡ Change in $f(x)$ is determined by the product $P(z_1)\, f(z_2)$ with $z_1, z_2 \in [x, 1]$

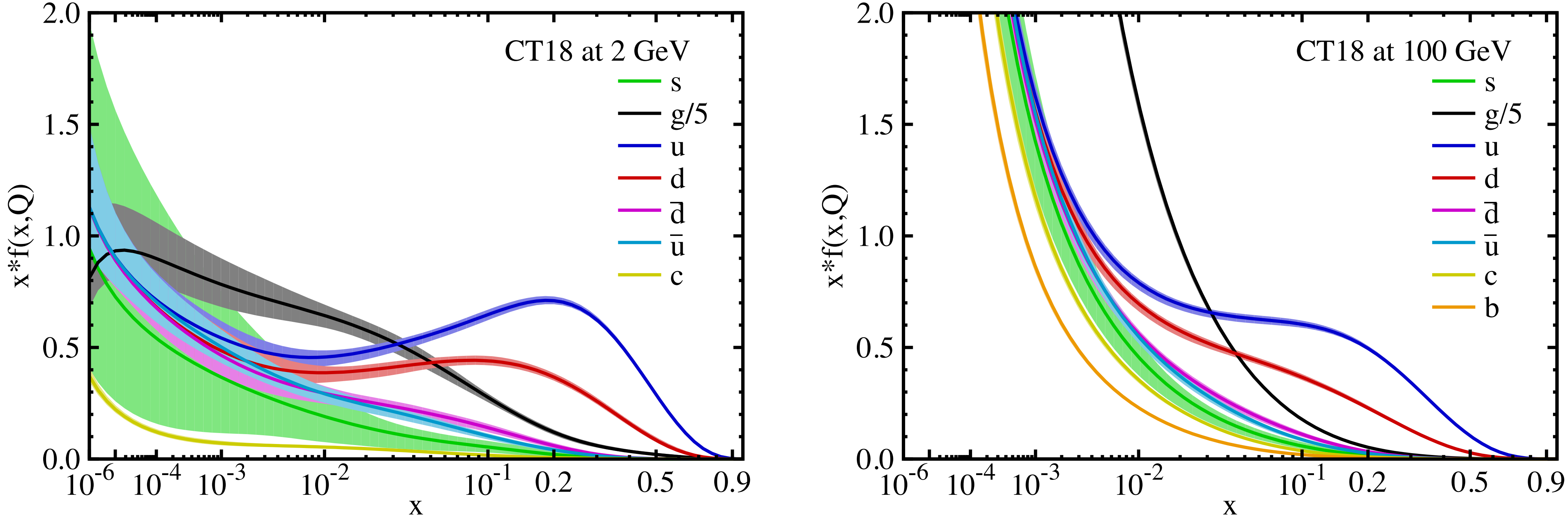
- **At large x:** evolution is dominated by singular pieces in P_{qq}^v and P_{gg}, which are negative.
- **At small x:** evolution is dominated by $1/z$ pieces from gluon evolution, which are positive.

➡ **Evolution generally brings PDFs from large x down to small x.**

What shall we expect?

At LO:

Ref: CT18 (arXiv:1912.10053)



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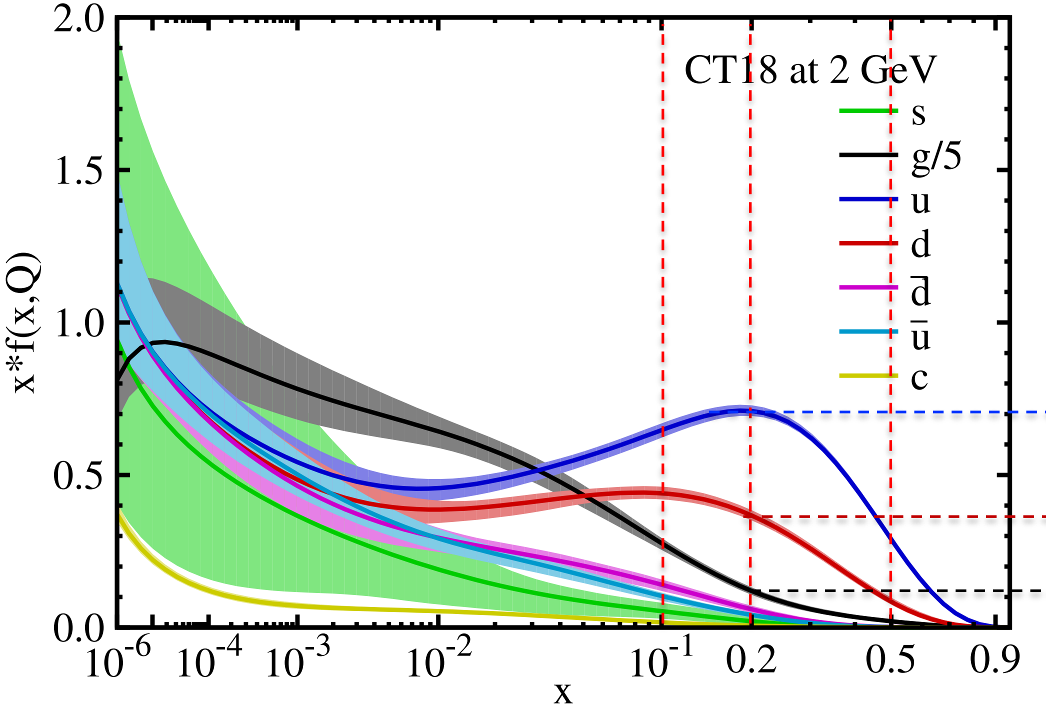
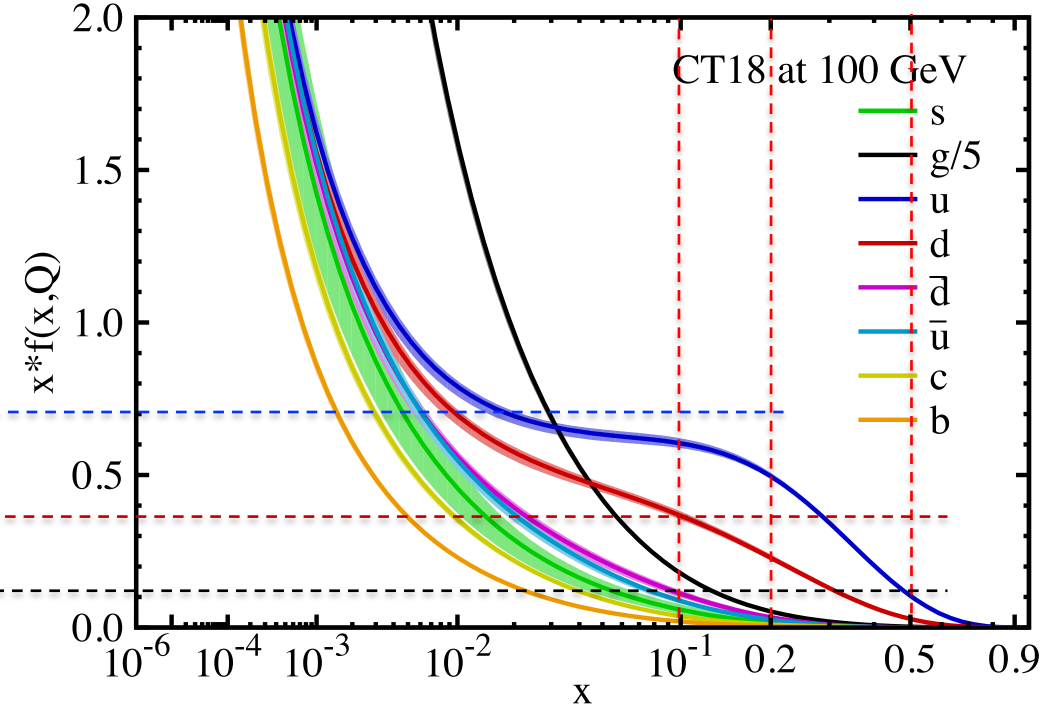


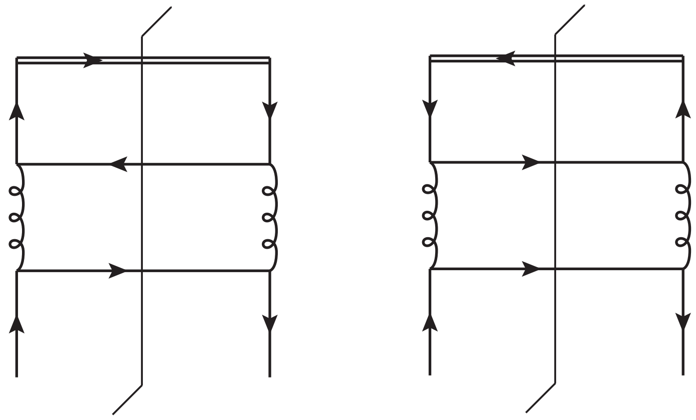


This overall property should NOT be altered by high-order corrections.

Evolution generally brings PDFs from large x down to small x.

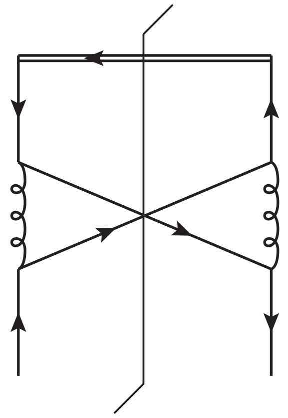
What shall we expect?

At NLO: 3 new kernels



$$P^s_{qq} = P^s_{q\bar{q}} > 0$$

u has equal probability to split into $d/\bar{d}$, $s/\bar{s}$, $c/\bar{c}$, etc.



$$P^v_{q\bar{q}} \propto \frac{1}{N_c} \mathrm{tr}\{t^a t^b t^a t^b\} = C_F^2 - \frac{1}{2} C_F C_A = -\frac{2}{9} < 0$$ negative due to color factor

$u \to \bar{u}$ is **suppressed** as compared to $u \to \bar{d},\ \bar{s},$ etc.

Flavor difference between antiquarks.

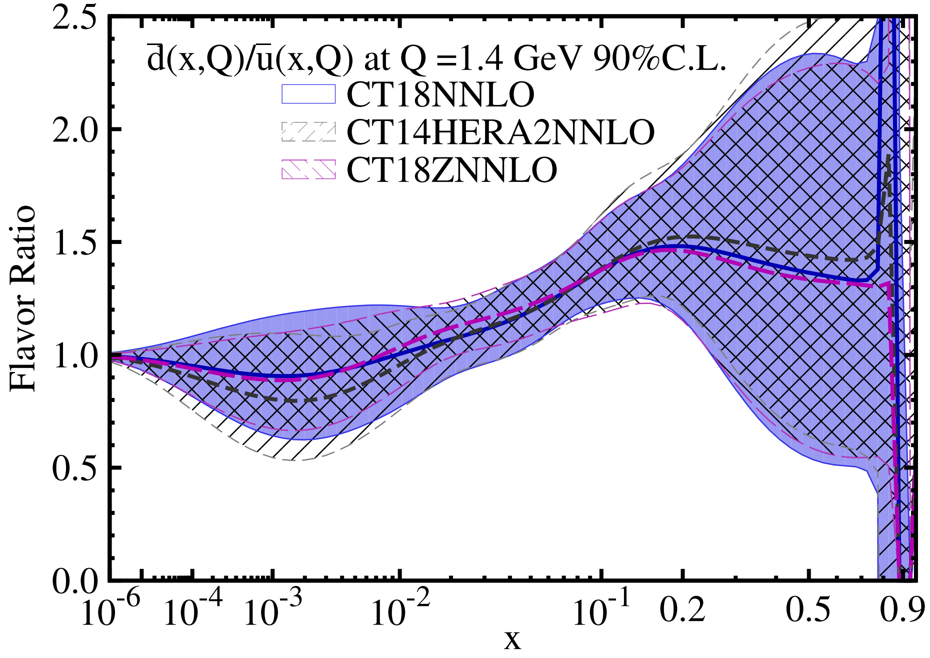
What shall we expect?

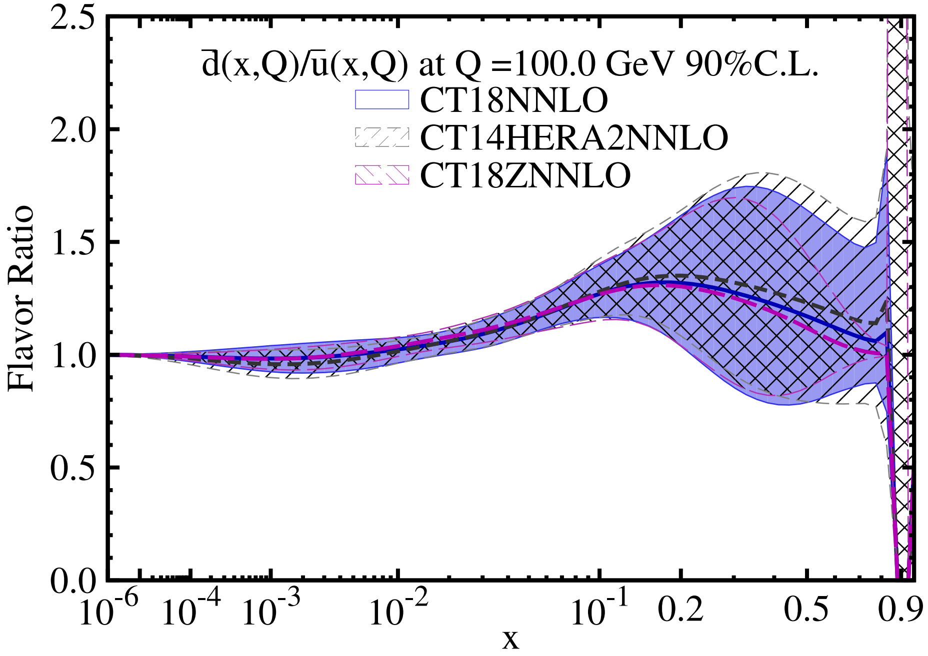
At NLO: Antiquark flavor asymmetry

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}(u-d) = P^v_{qq}\otimes(u-d) + P^v_{q\bar{q}}\otimes(\bar{u}-\bar{d})\,,$$
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}(\bar{u}-\bar{d}) = P^v_{qq}\otimes(\bar{u}-\bar{d}) + P^v_{q\bar{q}}\otimes(u-d)$$

→ $\bar{d}$ **tends to** $> \bar{u}$ **at large** x**.**

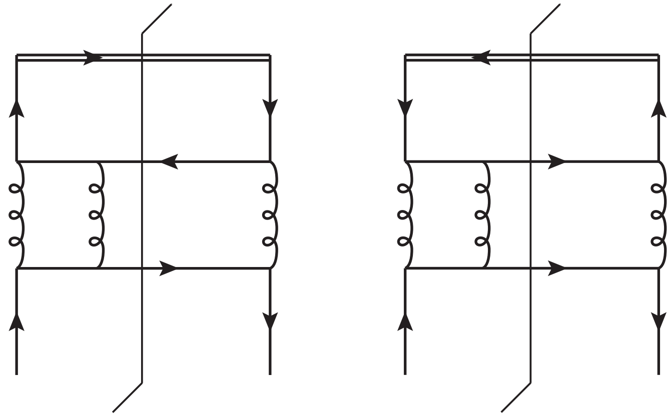
Ref: CT18 (arXiv:1912.10053)





What shall we expect?

At NNLO: Sea quark-antiquark asymmetry



$$P^s_{qq} \neq P^s_{q\bar{q}}, \quad \textbf{but} \quad \int_0^1 dz \left[P^s_{qq}(z) - P^s_{q\bar{q}}(z)\right] = 0$$

Equal splitting between $u \to s$ and $u \to \bar{s}$ is broken

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}(s-\bar{s}) = \left(P^v_{qq} - P^v_{q\bar{q}}\right) \otimes (s-\bar{s}) + \left(P^s_{qq} - P^s_{q\bar{q}}\right) \otimes f_{\mathrm{ns}}$$

$s(x) \neq \bar{s}(x)$ generally, although $\int_0^1 dx\,(s-\bar{s})(x) = 0$

Similarly, $c(x) \neq \bar{c}(x)$ and $b(x) \neq \bar{b}(x)$

S. Catani, et al., PRL 93, 152003 (2004)

So, all “naïve flavor symmetries” of PDFs are broken

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- **How to separate u from d?**

DIS $F_{1,2}$ structure functions $\propto e_u^2\,(u+\bar{u}) + e_d^2\,(d+\bar{d}) + \cdots$ (in proton target by default)

With a neutron target (from deuteron) $\to e_u^2\,(d+\bar{d}) + e_d^2\,(u+\bar{u}) + \cdots$ (isospin symmetry)

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  Drell-Yan production cross section, charge asymmetry, A_{FB}, etc. $q(x_1)\,\bar{q}'(x_2) \pm \bar{q}'(x_1)\,q(x_2)$

  DIS F_3 structure function $\propto e_u^2\,(u-\bar{u}) + e_d^2\,(d-\bar{d}) + \cdots$

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- **How to constrain gluon? [1min]**

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- **How to separate u from d?**

  DIS $F_{1,2}$ structure functions $\propto e_u^2\,(u+\bar{u}) + e_d^2\,(d+\bar{d}) + \cdots$ (in proton target by default)

  With a neutron target (from deuteron) $\rightarrow e_u^2\,(d+\bar{d}) + e_d^2\,(u+\bar{u}) + \cdots$ (isospin symmetry)

- **How to separate $\bar{q}$ from q?**

  Drell-Yan production cross section, charge asymmetry, A_{FB}, etc. $q(x_1)\,\bar{q}'(x_2) \pm \bar{q}'(x_1)\,q(x_2)$

  DIS F_3 structure function $\propto e_u^2\,(u-\bar{u}) + e_d^2\,(d-\bar{d}) + \cdots$

- **How to constrain gluon?**

  $u, d, \bar{u}, \bar{d}, g$

  Jet production at hadron colliders $q + g \rightarrow q + g, \ \ldots$

So, all “naïve flavor symmetries” of PDFs are broken

Since they are broken perturbatively, no reason they are reserved at their initial scale μ_0.

But in an actual fit, one needs data to constrain and separate different flavors!

- **How to probe strange quark? (2mins)**

So, all “naïve flavor symmetries” of PDFs are broken

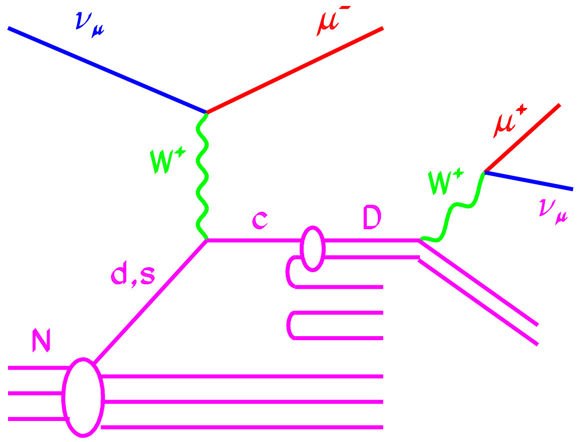
Since they are broken perturbatively, no reason they are reserved at their initial scale μ_0.

But in an actual fit, one needs data to constrain and separate different flavors!

➢ How to probe strange quark?

- It’s all about precision …
- By flavor tagging, e.g., dimuon data from CCFR NuTeV SIDIS

$$\nu_\mu + s \to \mu^- + c\,(\to \mu^+ + \nu_\mu) + X$$
$$\bar{\nu}_\mu + \bar{s} \to \mu^+ + \bar{c}\,(\to \mu^- + \bar{\nu}_\mu) + X$$



CCFR, PRD 64 (2001), 112006

- $W + c$ production with charm jet tagging at hadron collider

$$s + g \to W^- + c, \quad \bar{s} + g \to W^+ + \bar{c}$$

- Simultaneous fit of PDF and fragmentation function, by tagging kaon production JAM, PRD 101, 074020 (2020)
- Lattice QCD constraint …

Summary

DGLAP kernels and flavor decomposition

$$\frac{d\, f_{ns}}{d \ln \mu^2} = \left[P^v_{qq} - P^v_{q\bar{q}} + n_f \left(P^s_{qq} - P^s_{q\bar{q}} \right) \right] \otimes f_{ns}$$

$$\frac{d\, f_{s}}{d \ln \mu^2} = \left[P^v_{qq} + P^v_{q\bar{q}} + n_f \left(P^s_{qq} + P^s_{q\bar{q}} \right) \right] \otimes f_{s} + 2 n_f P_{qg} \otimes f_g$$

$$\frac{d\, f_{g}}{d \ln \mu^2} = P_{gg} \otimes f_g + P_{gq} \otimes f_s$$

$$\frac{d\, \Delta f_i^{\pm}}{d \ln \mu^2} = \left(P^v_{qq} \pm P^v_{q\bar{q}} \right) \otimes \Delta f_i^{\pm}$$

$$P_{q_i q_j} = P_{\bar{q}_i \bar{q}_j} = P^v_{qq}\, \delta_{ij} + P^s_{qq}$$

$$P_{q_i \bar{q}_j} = P_{\bar{q}_i q_j} = P^v_{q\bar{q}}\, \delta_{ij} + P^s_{q\bar{q}}$$

$$P_{q_i g} = P_{\bar{q}_i g} = P_{qg}$$

$$P_{g q_i} = P_{g \bar{q}_i} = P_{gq}$$

$$P_{gg}$$

This is a great example of

- How high-order corrections bring qualitative impacts;
- How perturbation theory guides nonperturbative “intuition” or modeling.

Further high order corrections only give quantitative changes.

How this plays a role in actual global analysis