Light-Front Description of Tensor Structure Functions of Deuteron

Bheemsehan Gurjar
USTC, Hefei

Based on: arXiv:2606:xxxxx (In prepration)

In collaboration with: X. Cao, Y. Li (USTC), and V.A. Karmanov (Lebedev Inst.)

Date : June 5, 2026

Outline

- Motivation
- One Boson Exchange model of Deuteron
- Results
  - (a) Longitudinal Momentum Distributions (LMDs)
  - (b) Deuteron structure functions : LMDs $\otimes$ PDFs
  - (c) Light flavor sea asymmetry in Deuteron
- Conclusion and outlook

The deuteron is the lightest nuclei → a simplest nuclear bound system of a proton and neutron.

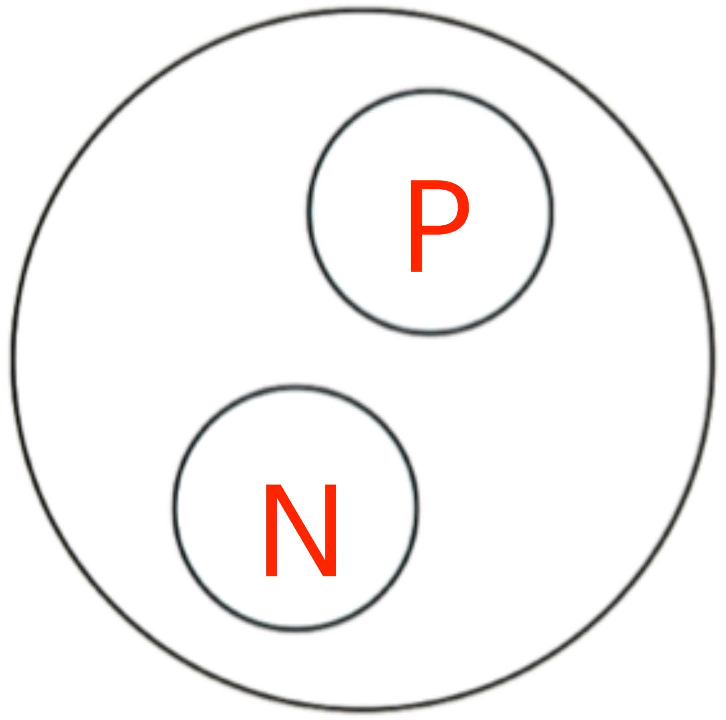
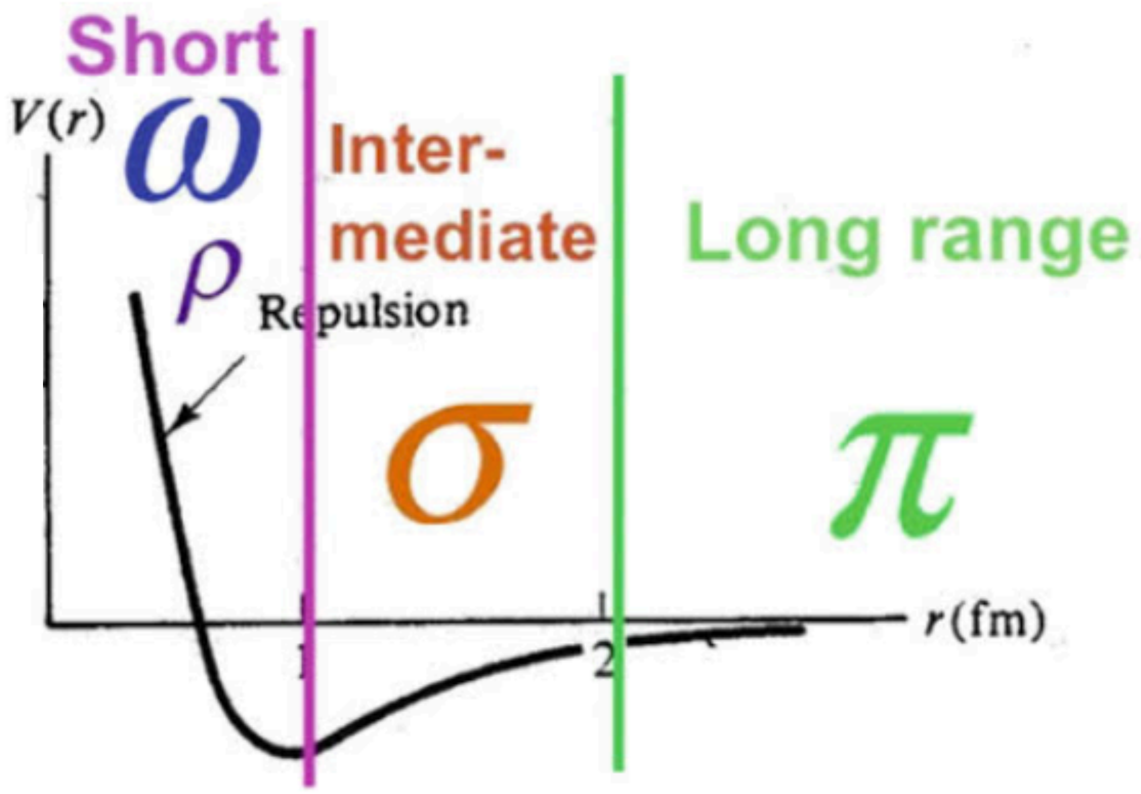


Table 3-1 Ground state properties of deuteron.

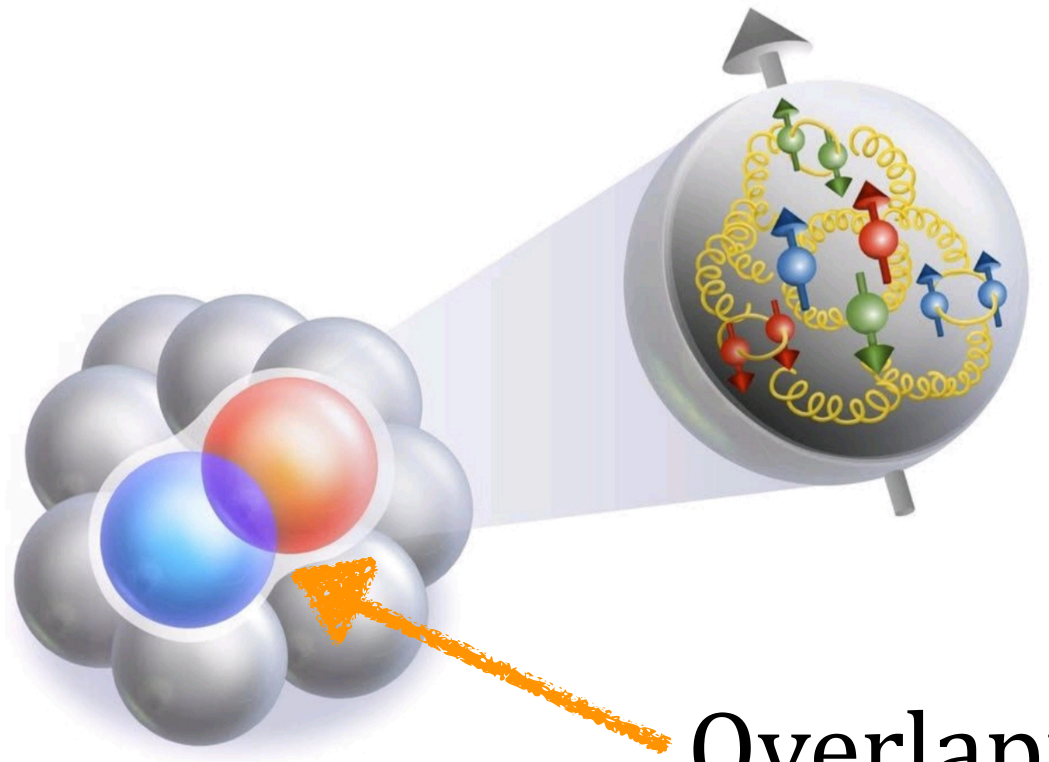
Ground State Property		Value	
Binding energy	E_B	2.22457312(22) MeV	
Spin and parity	J^π	1^+	
Isospin	T	0	
Magnetic dipole moment	μ_d	0.857406(1)	μ_N
Electric quadrupole moment	Q_d	0.28590(30)	efm^2
Radius	r_d	1.963(4)	fm

At **large distances** (r>2 fm): Meson exchanges b/w nucleons (Yukawa Model)



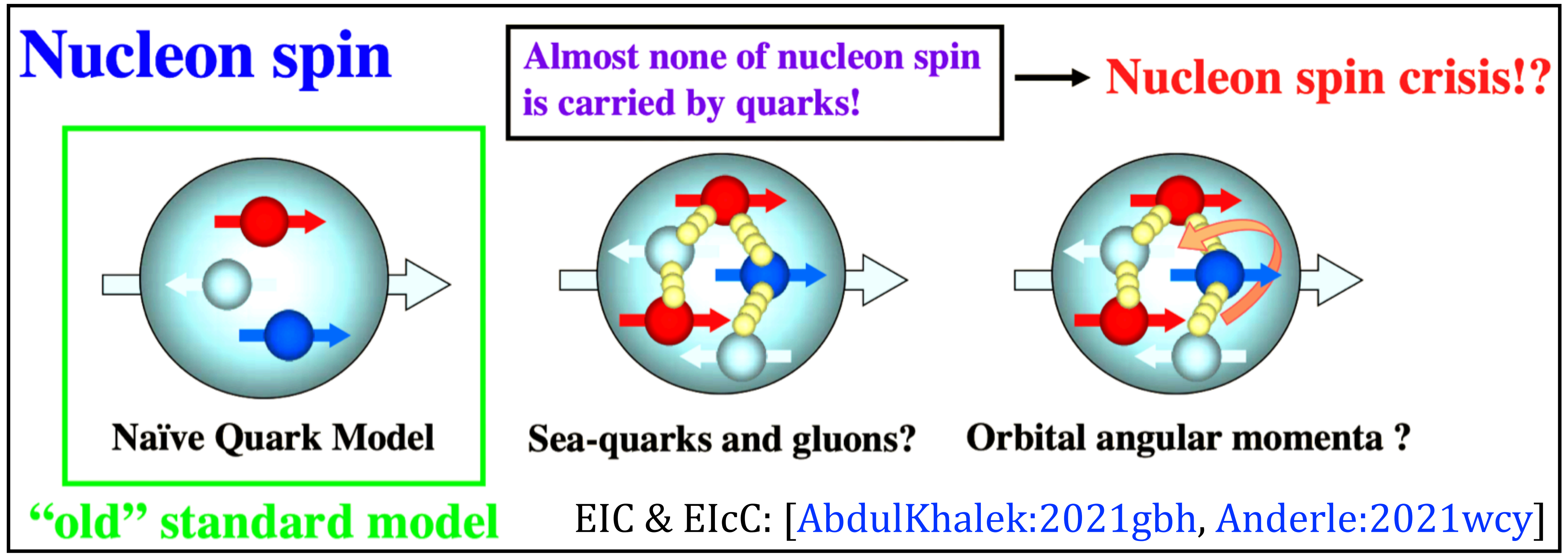
Deuteron

At the **short distances** (r< 1 fm): Strong interactions b/w quarks and gluons in shaping the deuteron.



Overlapping of nucleons

The Deuteron serves as the ultimate laboratory bridge between QCD and nuclear physics !!



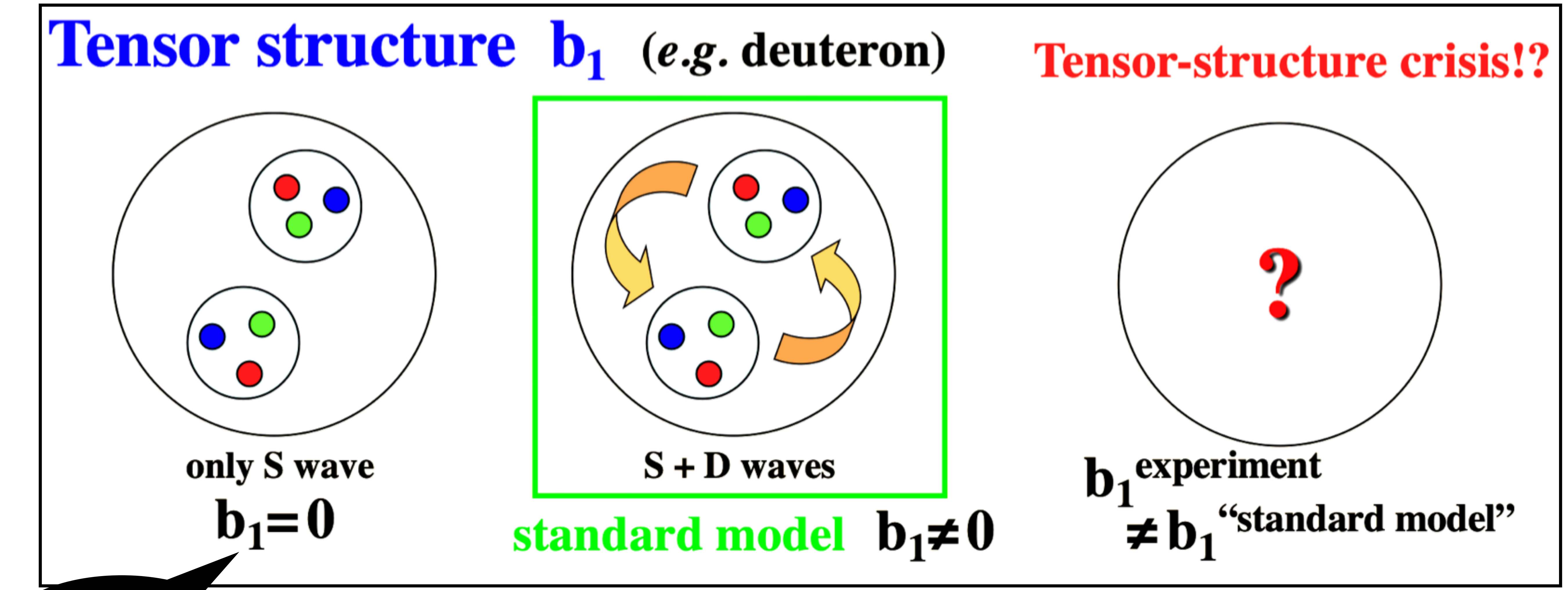
[Kumano:2024fpr]

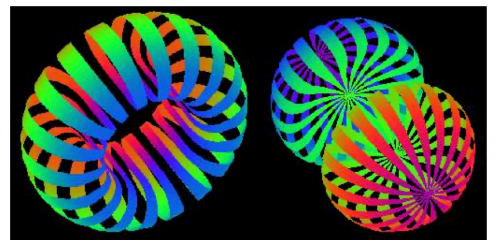
The Structure functions: F_1, F_2, g_1, and g_2 are common for spin-1/2 nucleons and spin-1 deuteron.

➡ Deuteron contains information inaccessible in the nucleons.

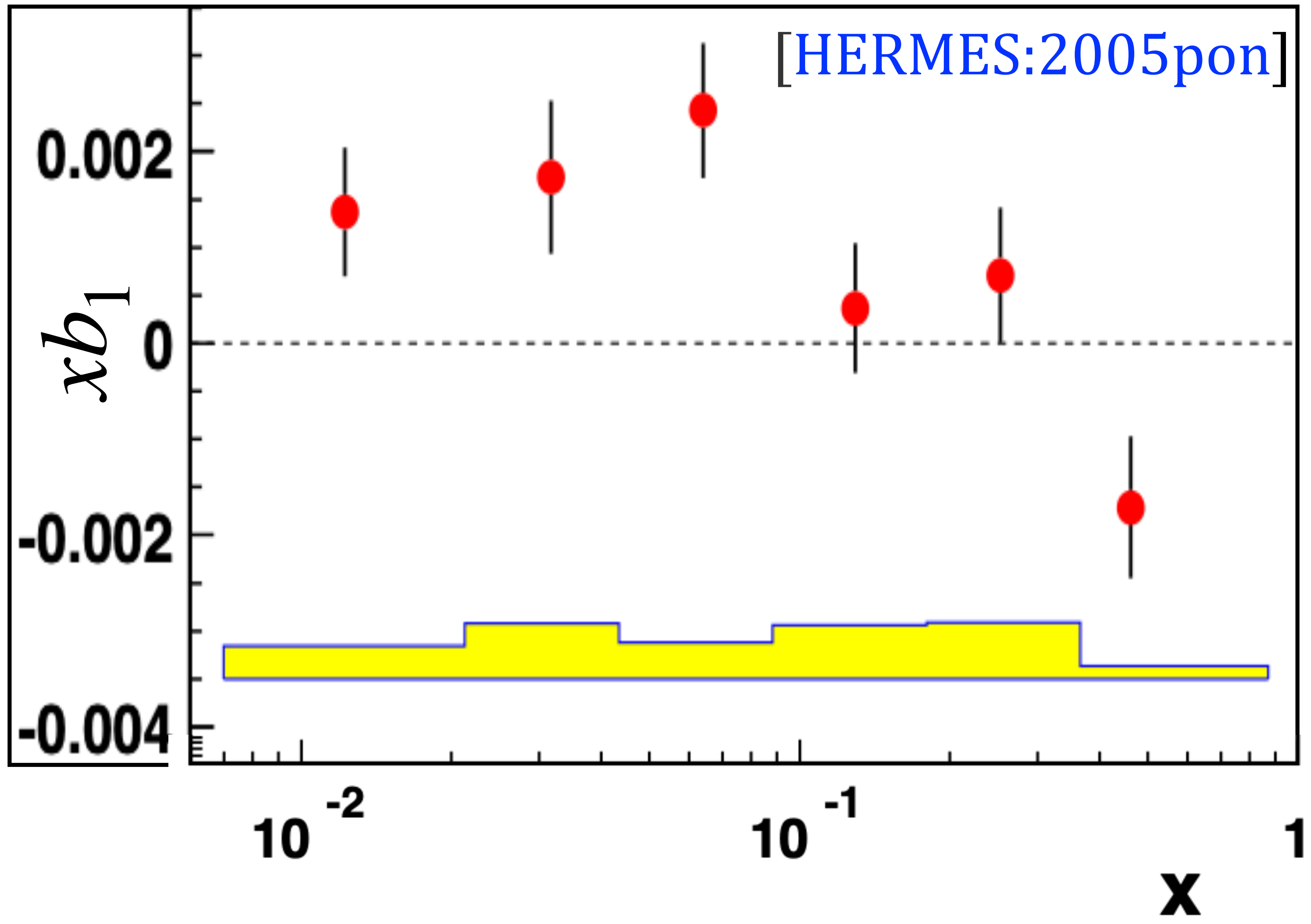
➡ Unique spin-1 nature of the deuteron → Introduces new structure functions (b_1, b_2, b_3, b_4) with $2xb_1 = b_2$ and b_3, b_4 are twist-4.

[Hoodbhoy:1988am]





Directly sensitive to the D-state admixture → nonzero electric quadrupole moment



First measurement of the tensor structure function b_1 of the Deuteron at HERMES

Tensor structure crisis → The standard conventional deuteron convolution models are not able to describe the HERMES data in the small x region.

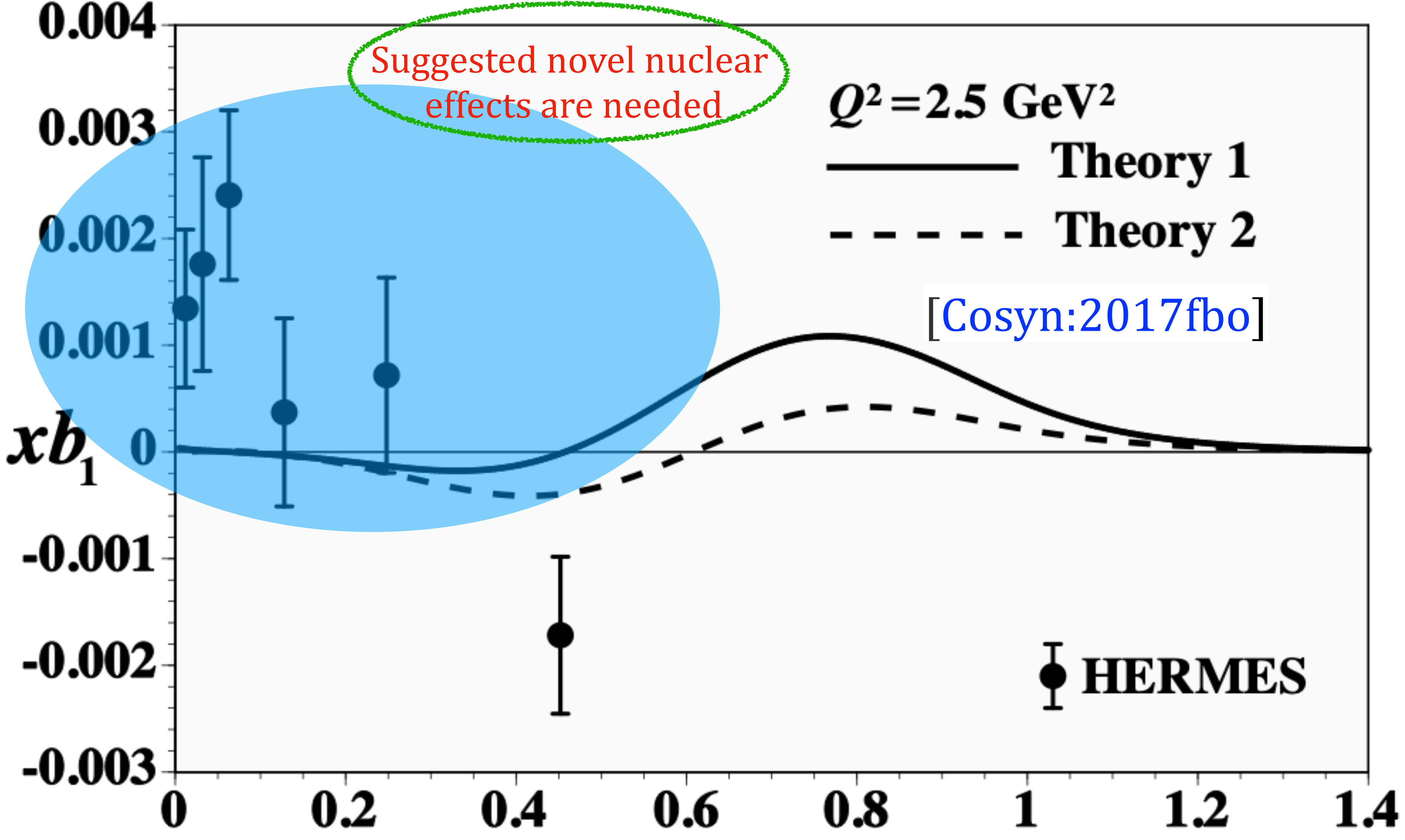
Theoretical Challenges

➡ **Convolution Model**: Deuteron → a **simple bound system of two nucleons.**

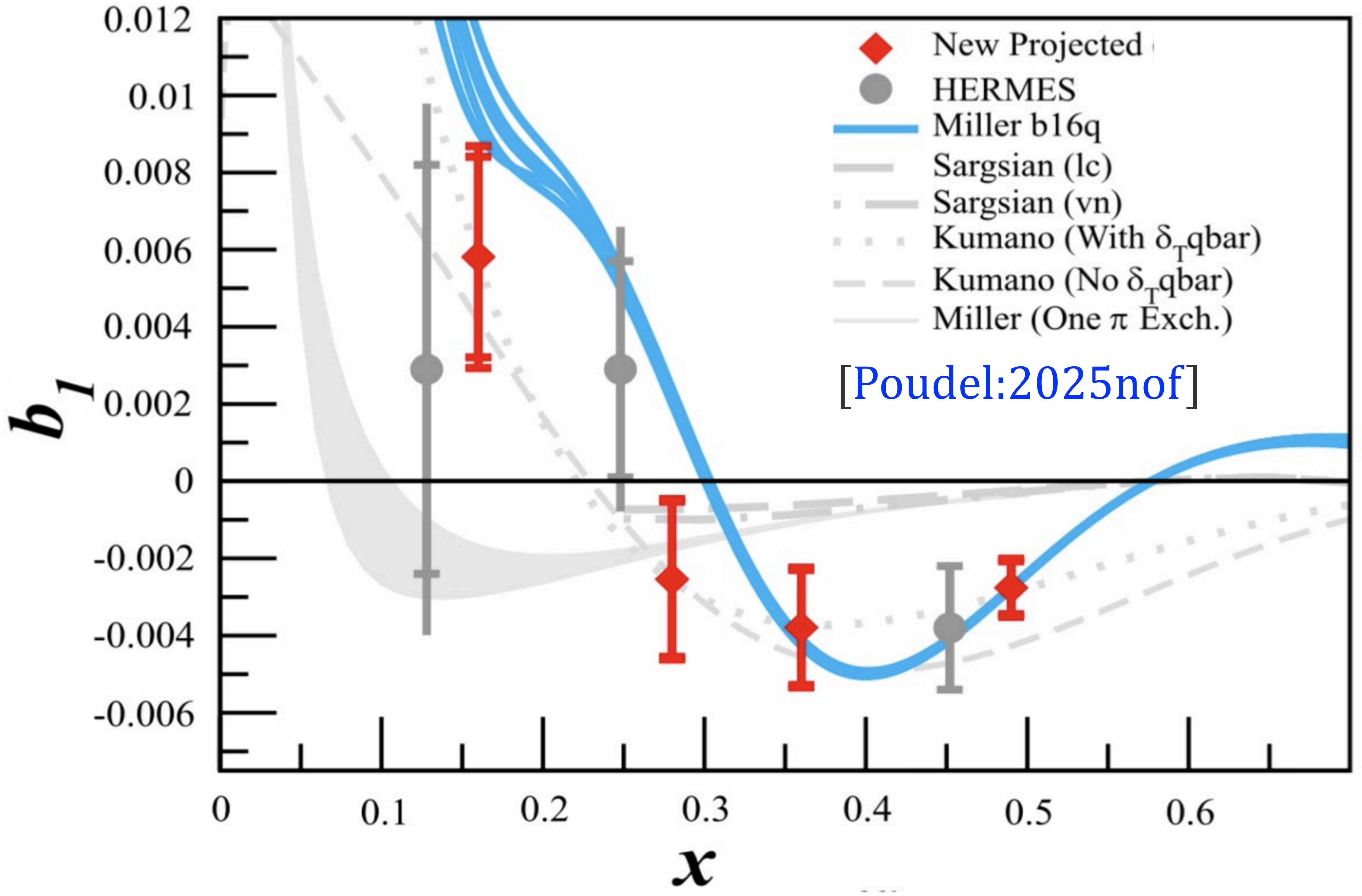
➡ **Shadowing effects:** At low x, multiple scattering of virtual photons lead to **shadowing** which **contributes to the rise of b_1** observed by he HERMES.

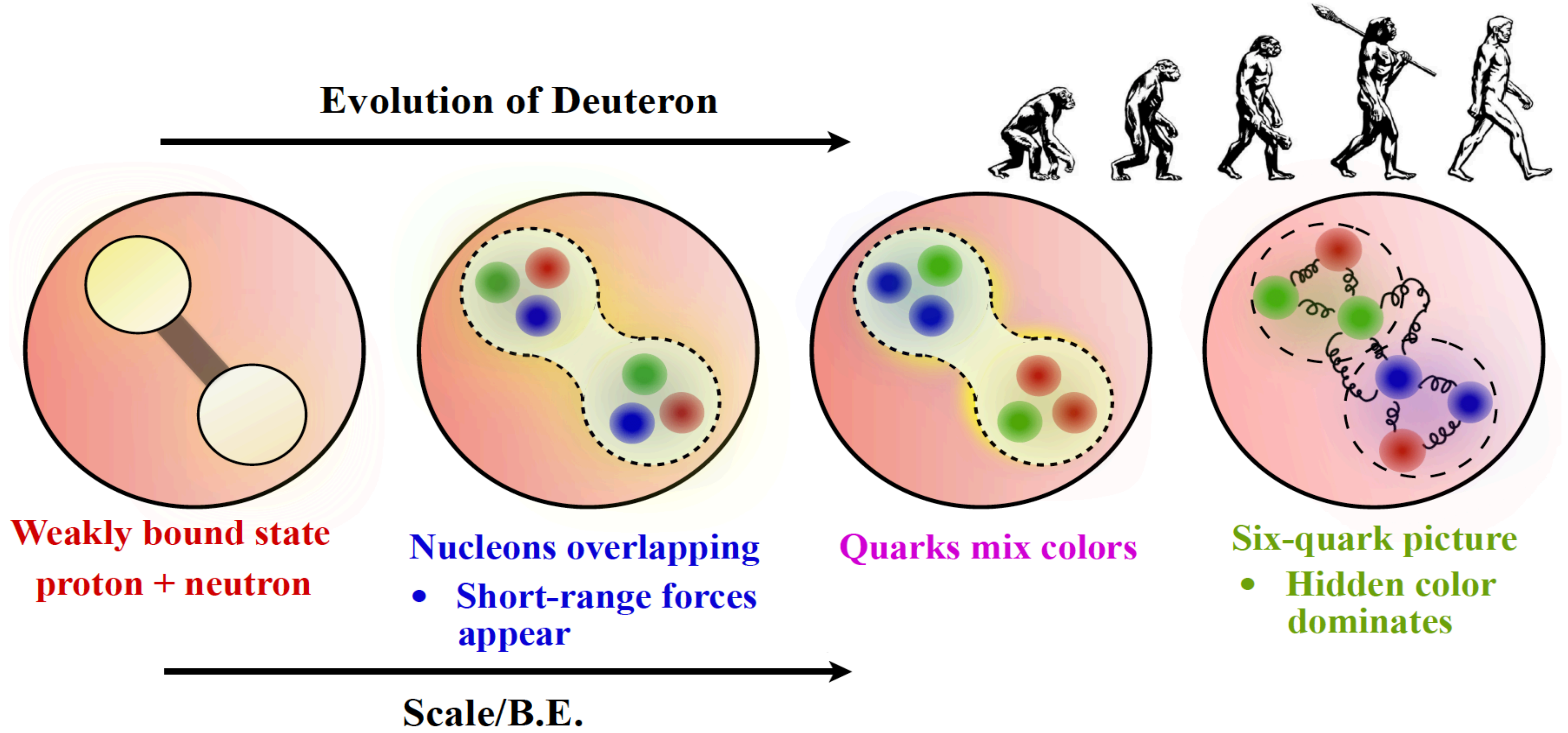
➡ **Higher twist effects:** The power surpassed corrections $(1/Q^2)$ become **important at low energy** scales → potentially explaining the discrepancy.

Discrepancy : $|b_1|(\text{theory}) \ll |b_1|(\text{experiment})$ at $x < 0.5$



Projected result of b_1 experiment at CLAS12 @ JLab(Hall-C)



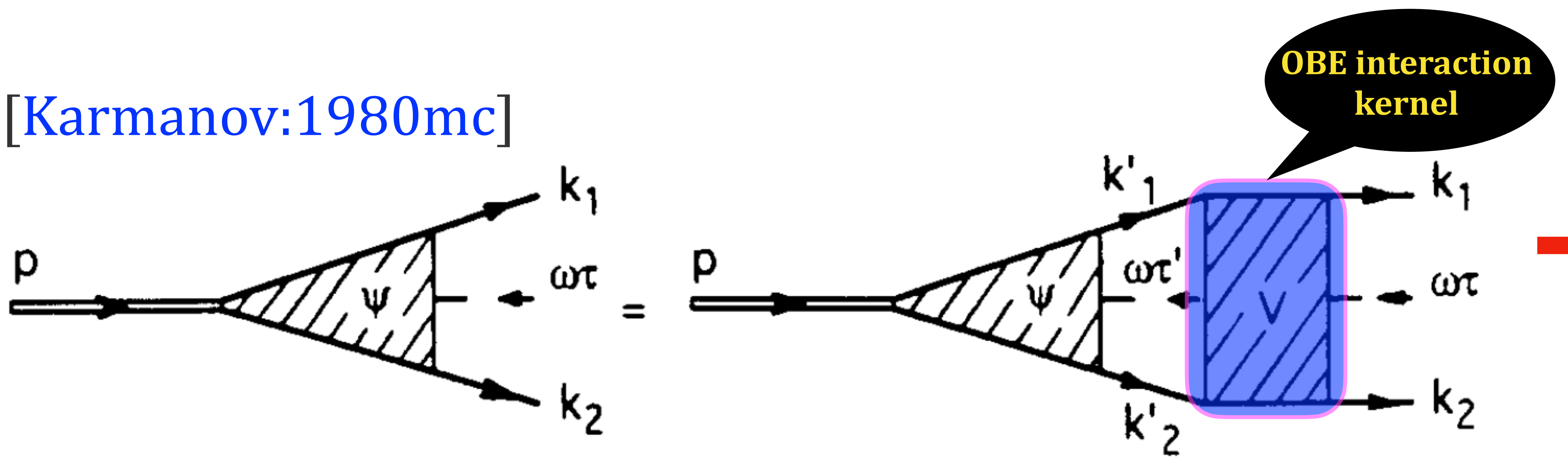


[Hoodbhoy:1988am, Khan:1991qk, Cosyn:2017fbo, Kumano:2024fpr]

[Ji:1985ky, Miller:2013hla, Kaur:2025css]

One Boson Exchange model in LF dynamics

[Karmanov:1980mc]



$k_1, k_2 \to$ Nucleon momentum, $p \to$ Deuteron momentum

- The two particle Fock-state expansion of deuteron Eigen state on the light-front plane

$$|p,\lambda\rangle = \sum_{\sigma_1,\sigma_2} \int_0^1 \frac{dx}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} \Phi^\lambda_{\sigma_1\sigma_2} a^\dagger_{\sigma_1}(k_1) a^\dagger_{\sigma_2}(k_2)|0\rangle$$

LFWFs

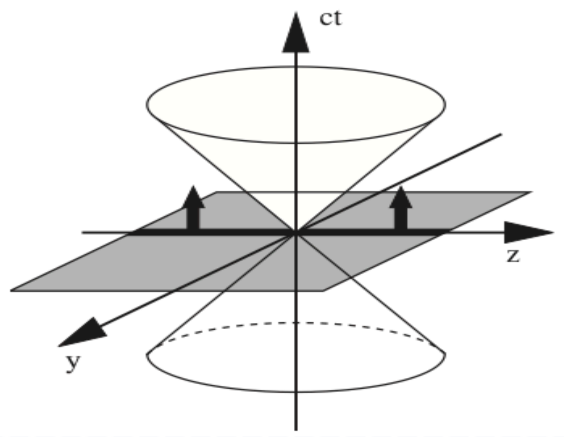
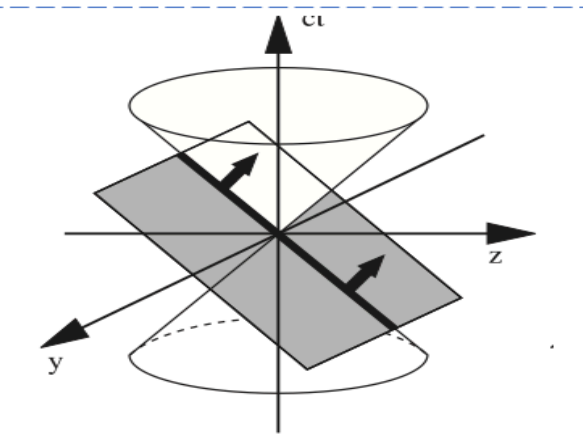
$$\Phi^\lambda_{\sigma_1\sigma_2} = (w^*_{\sigma_1})^T \psi^\lambda(\vec{q},\vec{n})\sigma_y w^*_{\sigma_2},$$

Infinite momentum frame variable transform

[Carbonell:1995yi]
$$q^2 = \frac{k_\perp^2 + m^2}{4x(1-x)} - m^2, \quad \vec{q}\cdot\vec{\omega} = \sqrt{\frac{k_\perp^2+m^2}{x(1-x)}}\left(\frac{1}{2} - x\right)$$

Light-front Quantization

[Dirac:1949cp]

	Equal time quantization	Light-front quantization
Time variable	$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
Quantization surface		
Coordinate space	x^1, x^2, x^3	$x^- = x^0 - x^3$, $x^\perp = x^{1,2}$
Momentum Space	$P^0, \vec{P}$	$P^- = P^0 - P^3$, $P^+ = P^0 + P^3$, $P^\perp = P^{1,2}$
	$i\frac{\partial}{\partial t}\|\varphi(t)\rangle = H\|\varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}\|\varphi(x^+)\rangle = \frac{1}{2}P^-\|\varphi(x^+)\rangle$
Dispersion relation	$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

- The relativistic deuteron WFs → **six spin components** ($f_1, f_2, \ldots . f_6$)
- Non-relativistic case → **two components** (f_1, f_2)

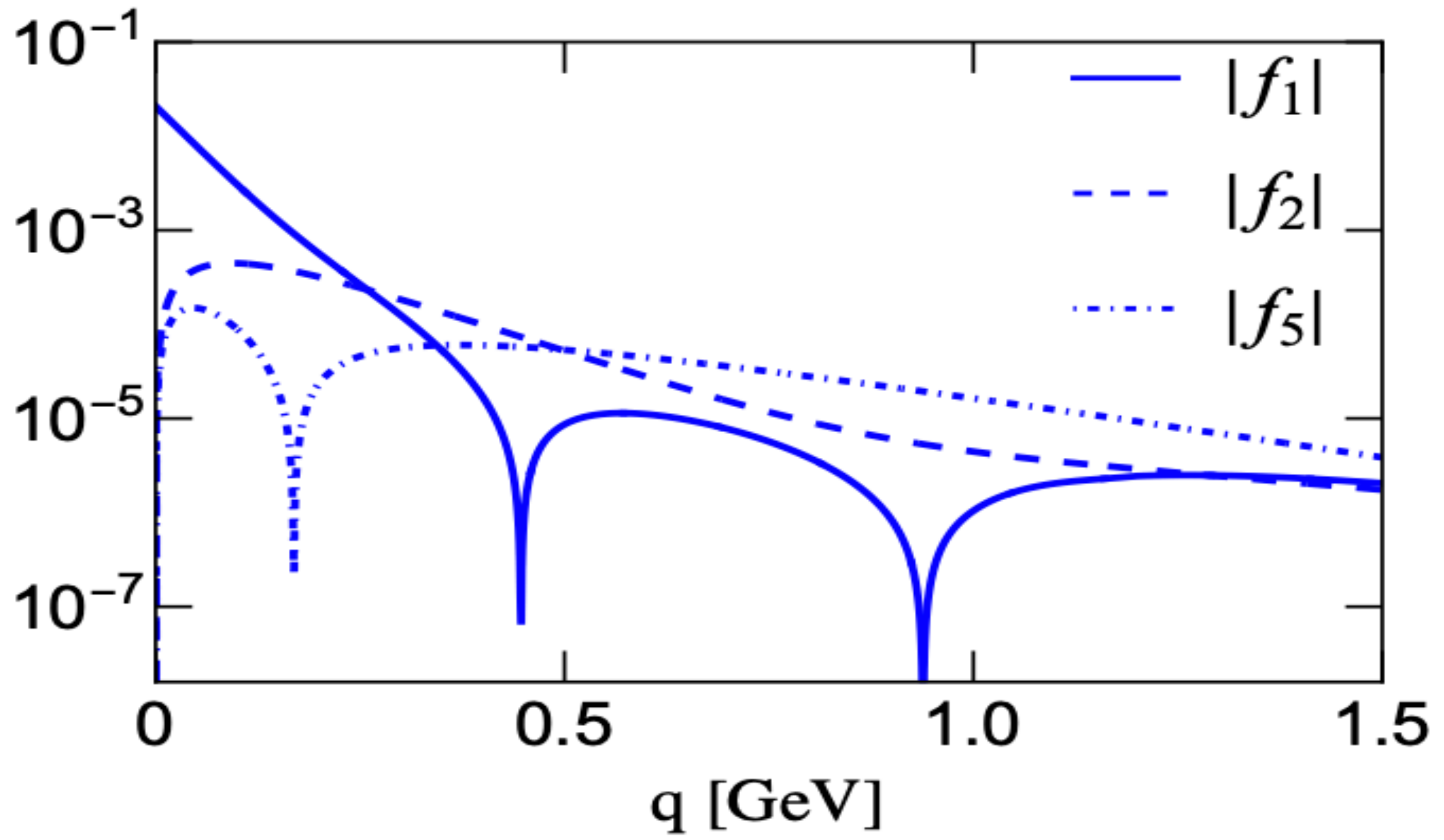
S-wave D-wave

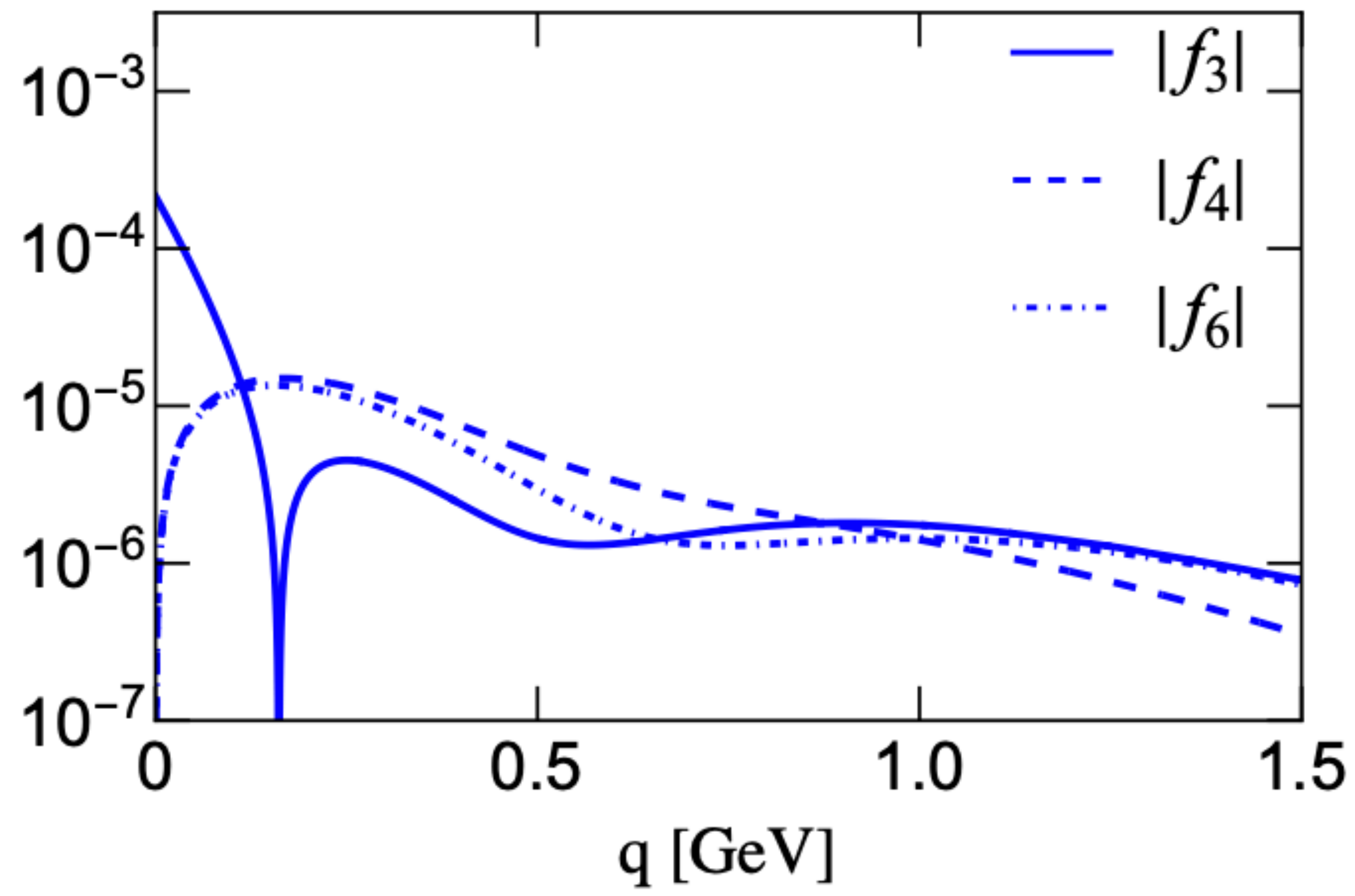
$$\vec{\psi}(\vec{q},\vec{n}) = f_1 \frac{1}{\sqrt{2}}\vec{\sigma} + f_2 \frac{1}{2}\left(\frac{3\vec{q}(\vec{q}\cdot\vec{\sigma})}{\vec{q}^2} - \vec{\sigma}\right) + f_3\frac{1}{2}\left(3\vec{n}(\vec{n}\cdot\vec{\sigma}) - \vec{\sigma}\right)$$
$$+ f_4 \frac{1}{2q}\left(3\vec{q}(\vec{n}\cdot\vec{\sigma}) + 3\vec{n}(\vec{q}\cdot\vec{\sigma}) - 2(\vec{q}\cdot\vec{n})\vec{\sigma}\right)$$
$$+ f_5 \frac{3}{2}\frac{i}{q}[\vec{q}\times\vec{n}] + f_6 \frac{\sqrt{3}}{2q}[[\vec{q}\times\vec{n}]\times\vec{\sigma}],$$

[Carbonell:1998rj]

Electromagnetic properties of Deuteron

- Deuteron wave functions components in LF dynamics





- Only f_1, f_5 and f_5 contributes others are negligible

- **EMFFs**: Hadronic matrix elements of electromagnetic current

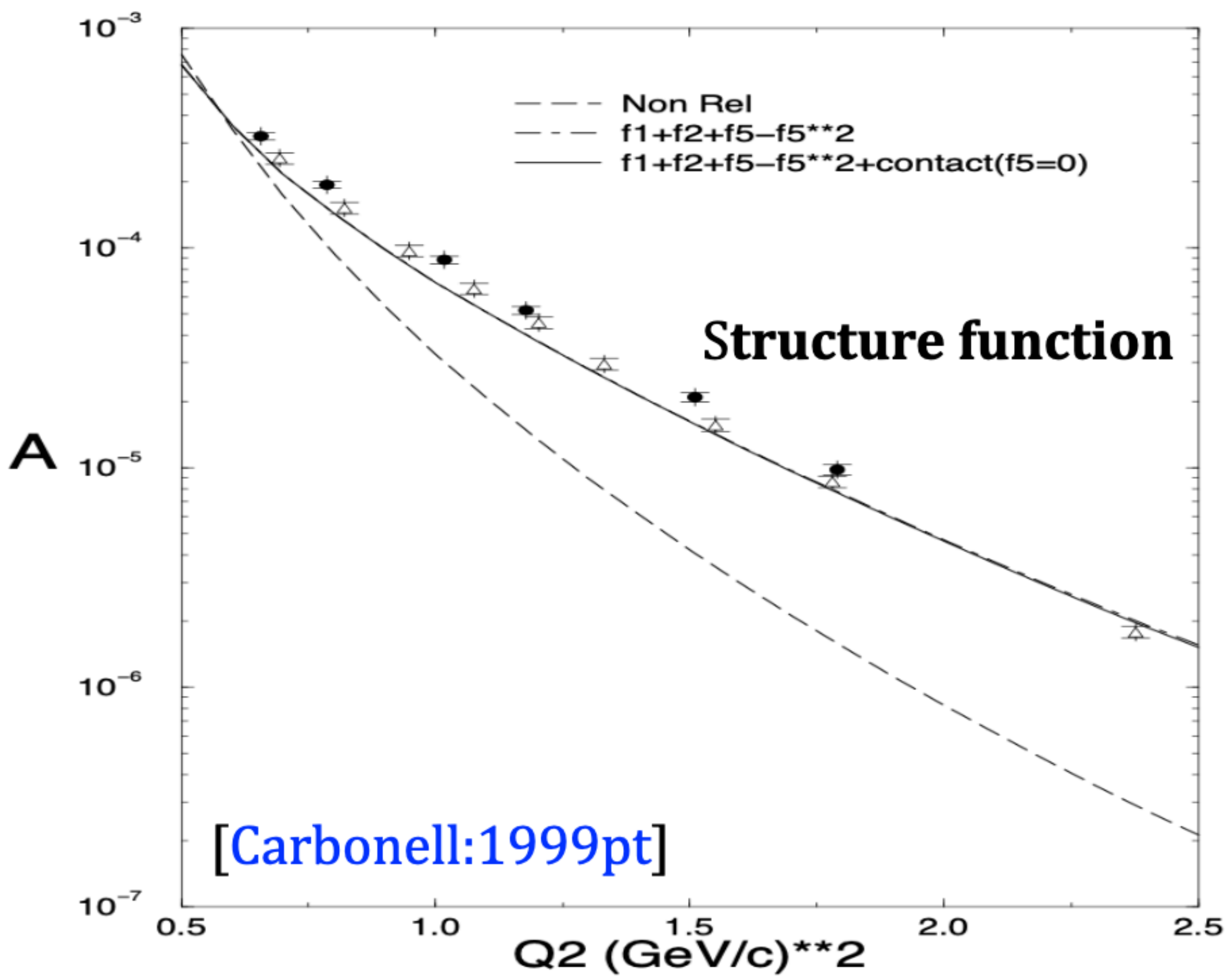
$$\langle V(P',\Lambda')\,|J^\mu|\,V(P,\Lambda)\rangle = -\,\epsilon^*_{\Lambda'}\cdot\epsilon_\Lambda (P+P')^\mu F_1(Q^2) + \left(\epsilon^\mu_\Lambda q\cdot\epsilon^*_{\Lambda'} - \epsilon^{*\mu}_{\Lambda'} q\cdot\epsilon_{\Lambda'}\right) F_2(Q^2) + \frac{(\epsilon^*_{\Lambda'}\cdot q)(\epsilon_\Lambda\cdot q)}{2M_V^2}(P+P')^\mu F_3(Q^2)$$

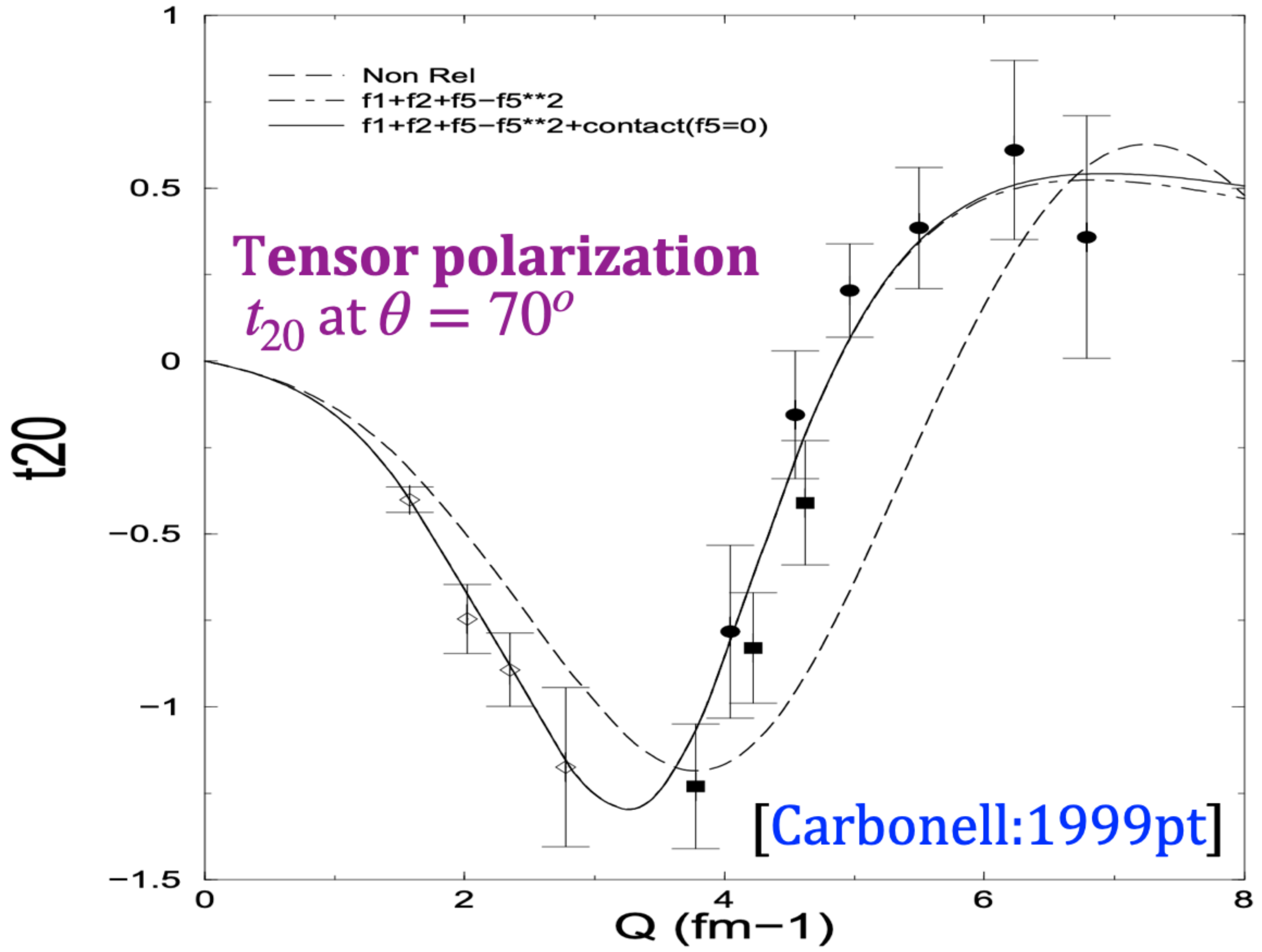
[Brodsky:1992px, Arnold:1979cg]

- F_1, F_2 and F_3 : **physical** form factors → The **charge** (G_c), **magnetic** (G_M) and **quadrupole** (G_Q) can be obtained through the physical FFs.

- The mesons interactions and other parameter values are used same as the Bonn potential. [Machleidt:1987hj]

- Model predictions of EMFFs are consistent with the JLab → the contributions beyond the S- and D-wave components.





Longitudinal Momentum Distributions (LMDs)

Deuteron longitudinal momentum distribution correlation function

$$\Phi^{\lambda}_{\beta\alpha}(z) = \int \frac{dy^-}{2\pi} e^{\frac{i}{2}k^+\cdot y^-} \langle P\lambda|\overline{\psi}_\alpha(0)U_{[0,y^-]}\psi_\beta(y^-)|P\lambda\rangle,$$

[Mulders:1995dh]

$$f_1(z) = \frac{1}{3}[\Phi_0^{[\gamma^+]}(z) + \Phi_{+1}^{[\gamma^+]}(z) + \Phi_{-1}^{[\gamma^+]}(z)]\,,$$

Unpolarized nucleons in an unpolarized deuteron

$$g_{1L}(z) = \Phi_0^{[\gamma^+\gamma_5]}(z)\,,$$

Longitudinally polarized nucleons in A longitudinally polarized deuteron

$$f_{1LL}(z) = \frac{1}{2}\left[2\Phi_0^{[\gamma^+]}(z) - \Phi_{+1}^{[\gamma^+]}(z) - \Phi_{-1}^{[\gamma^+]}(z)\right]$$

Unpolarized nucleons in a tensor polarized deuteron

Quark / Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					
L			g_{1L} (g_1)			
T					[h_1]	
LL	f_{1LL} (b_1)					
LT						*1 [h_{1LT}]
TT						

Unpolarized LMD

Longitudinal polarized LMD

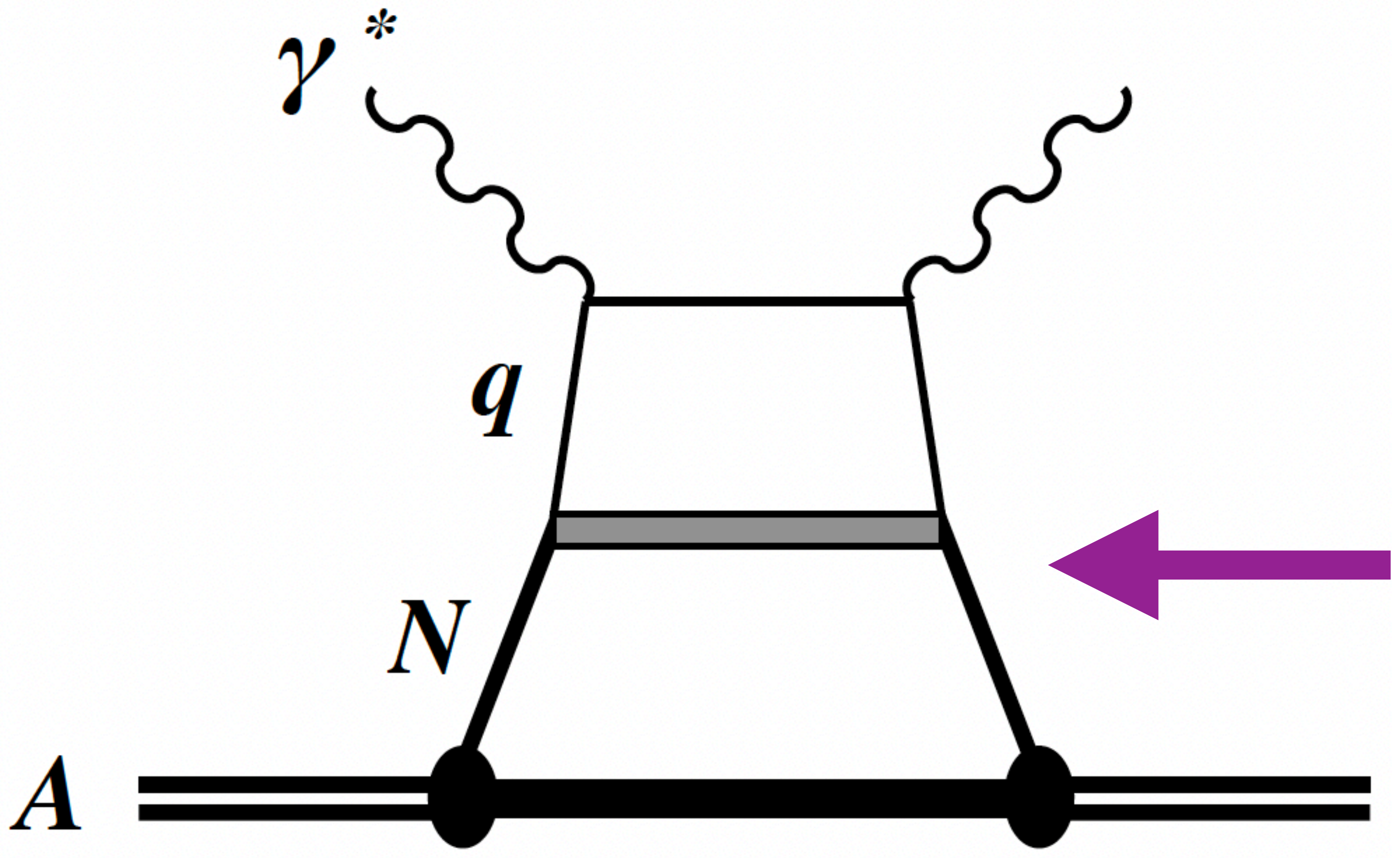
Tensor polarized LMD

[Kumano:2024fpr]

Model predictions satisfy the QCD sum rules: $\int_0^1 dz\, f_1(z) = 1$ and $\int_0^1 dz\, f_{1LL}(z) = 0$ [Bacchetta:2000jk]

- **Sharp peak**: weak interaction between nucleons (with binding energy ~**2 MeV**).
- **Peak at 0.5**: Nucleons equally share the deuteron energy

Deuteron Structure Functions



Convolution description for Deuteron structure functions in DIS

The SFs are dominated by the quark distributions of nucleon in small-x region !

Quark distribution inside the deuteron

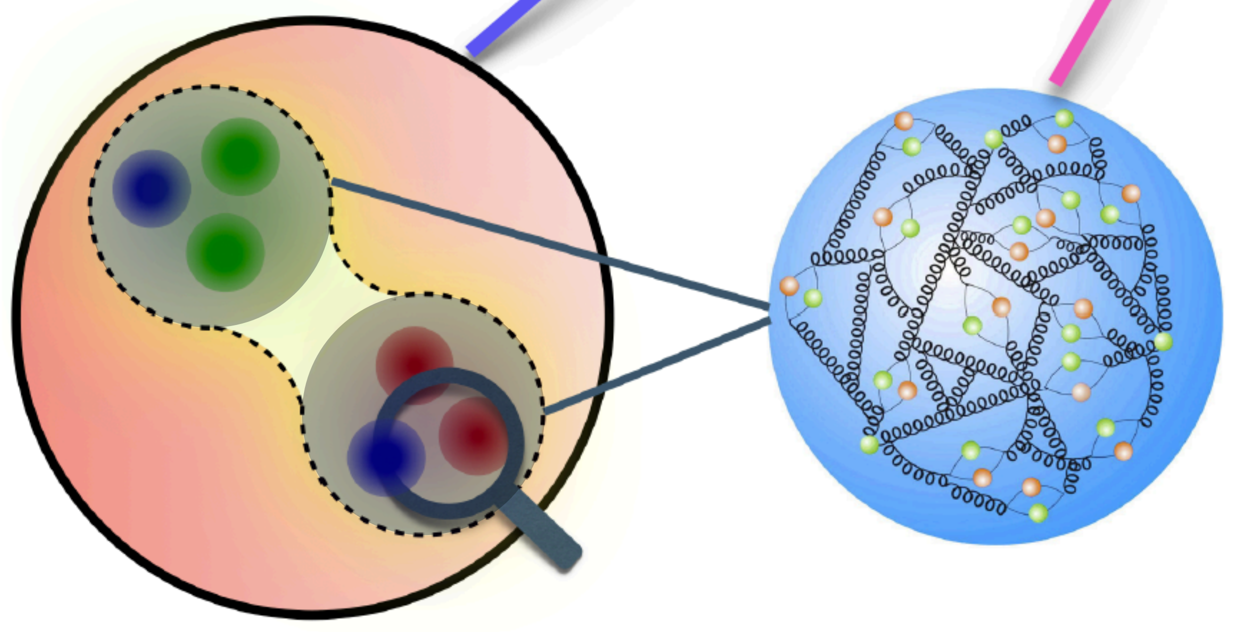
$$\mathrm{SF}^D(x,Q^2) = x\sum_q e_q^2 \mathcal{F}_q^D(x,Q^2),$$

[Hoodbhoy:1988am]

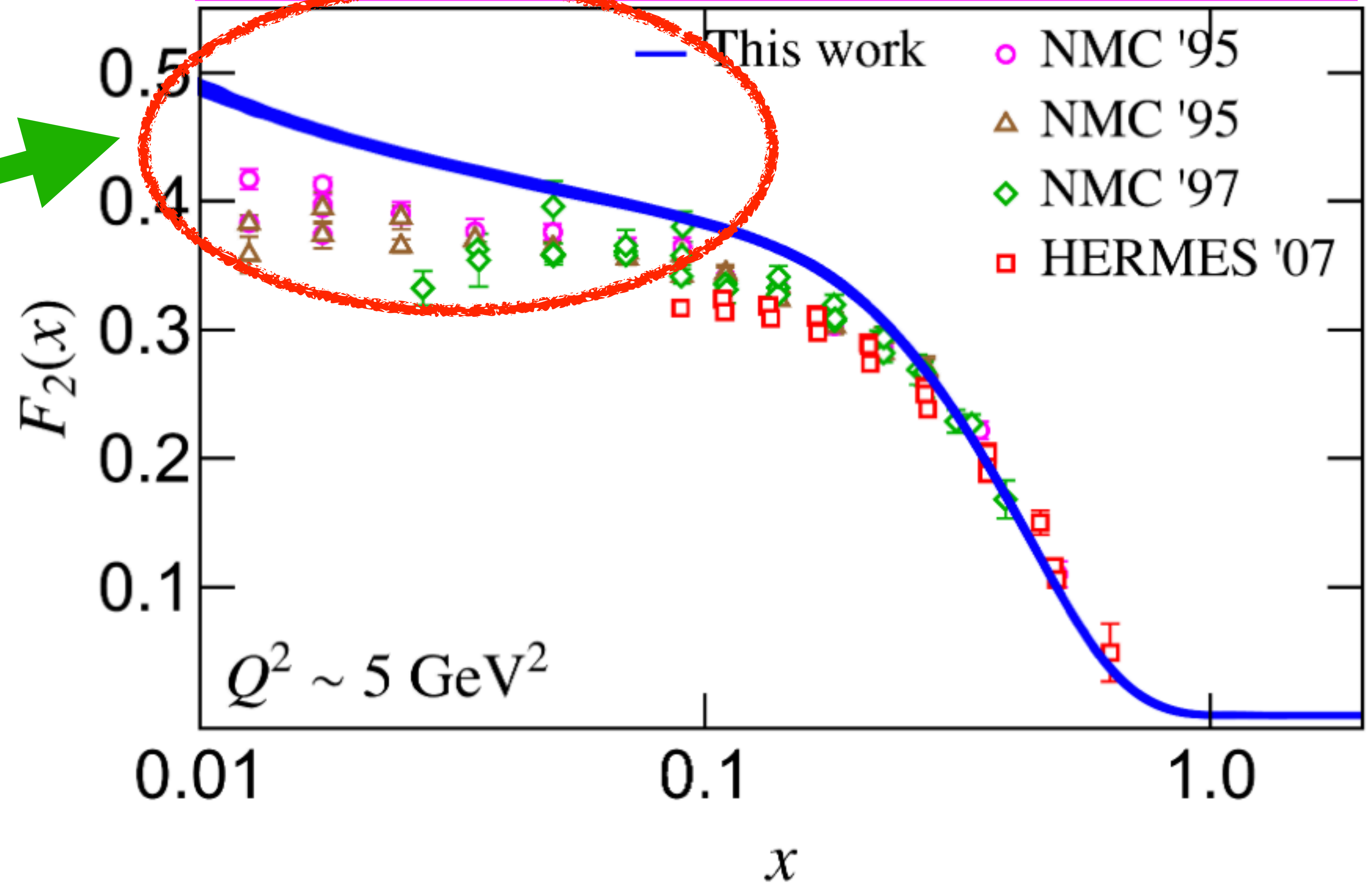
Factorization of Structure Functions

$$\mathcal{F}_q^D(x,Q^2) = \frac{1}{2}\sum_N \int_x^1 \frac{dz}{z}\,\mathrm{LMDF}(z)\,\mathcal{F}_q^N\left(\frac{x}{z},Q^2\right)$$

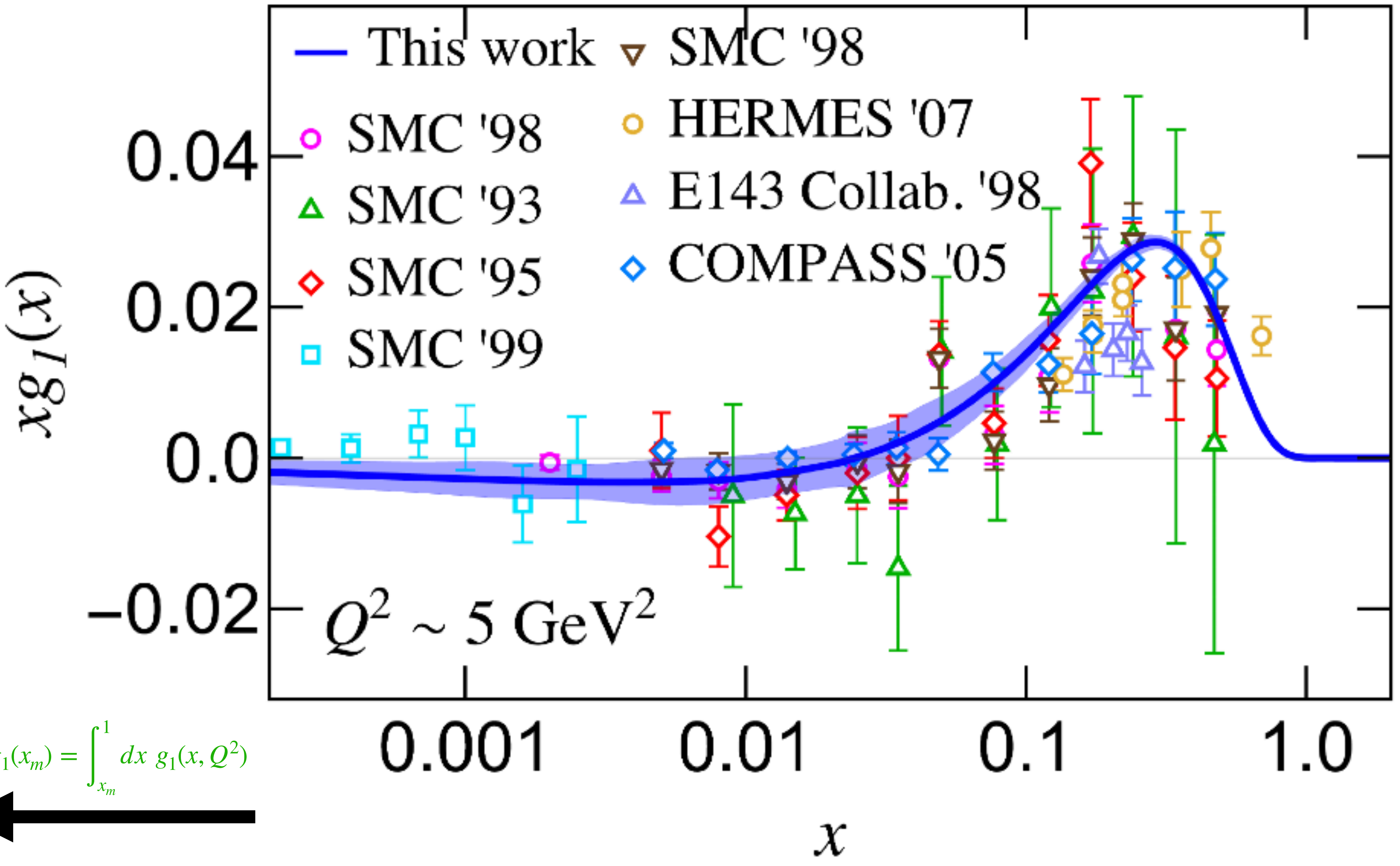
[Khan:1991qk]

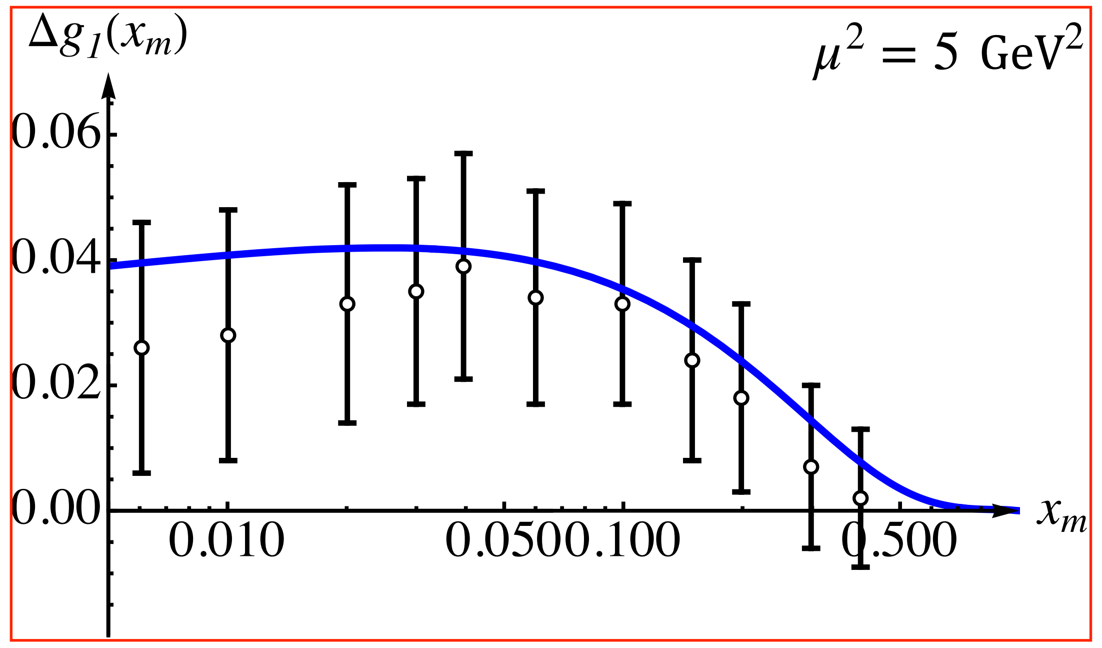


$F_2 \rightarrow f_1 \otimes$ unpolarized NNPDF



$g_1 \rightarrow g_{1L} \otimes$ helicity NNPDF





$\Delta g_1(x_m) = \int_{x_m}^1 dx\, g_1(x,Q^2)$

Tensor-polarized Structure Function

Same sign of S-D and D-D

No pure S-wave: $f_1^2 \to 0$ 	Pure D-wave < S-D wave overlap

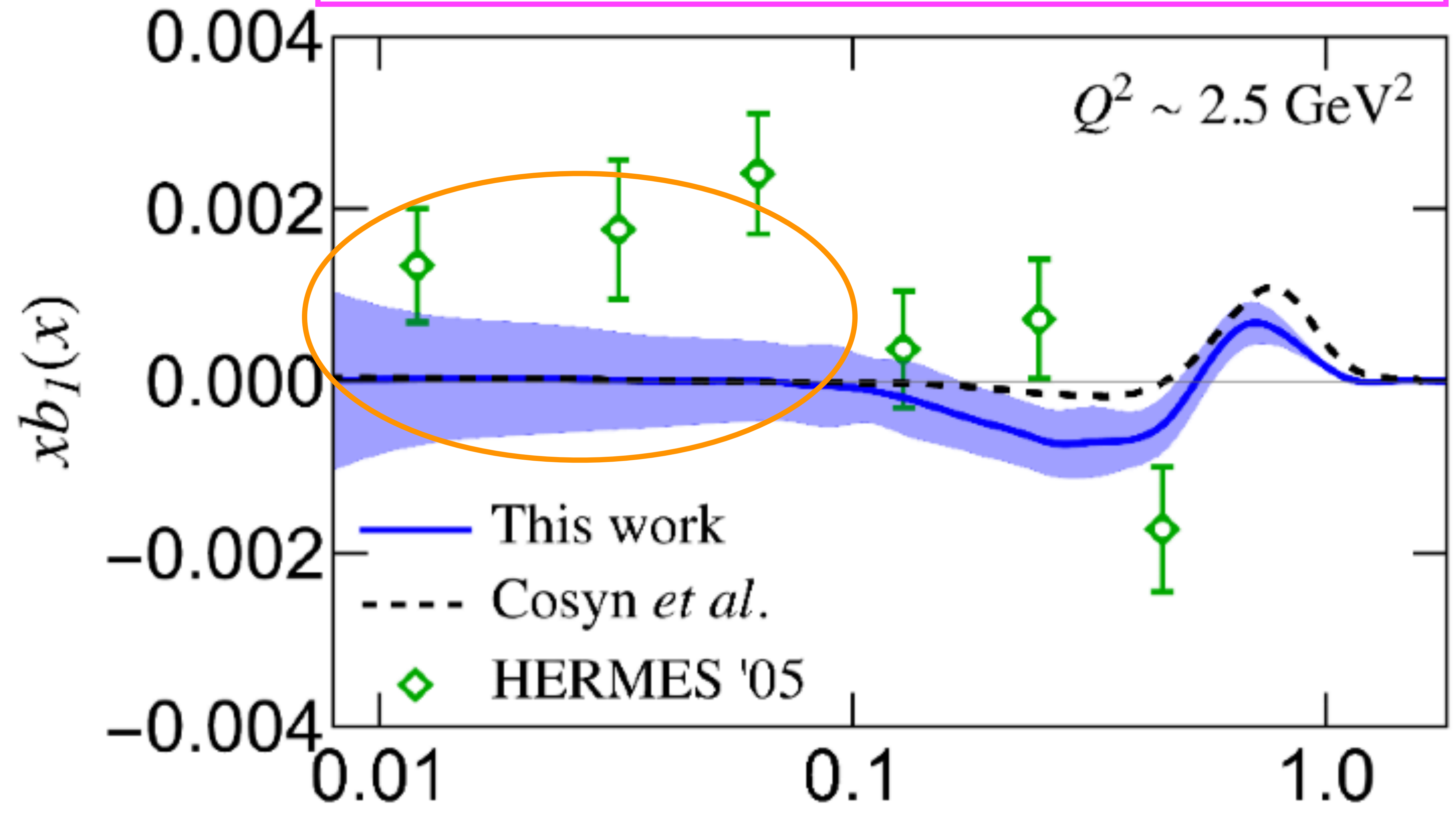
$$\frac{2}{3}\sum_{s_1,s_2}\left[2\Phi^{[\gamma^+]}_{\lambda=0}(z) - \Phi^{[\gamma^+]}_{\lambda=1} - \Phi^{[\gamma^+]}_{\lambda=-1}\right] = (3z^2-1)f_2^2 + 2f_3^2 + (5z^2+3)f_4^2 + 2(z^2-1)f_5^2 + (1-z^2)f_6^2$$

$$f_{1LL}(z) \quad + 2\sqrt{2}(3z^2-1)f_1f_2 + 4\sqrt{2}f_1f_3 + 8\sqrt{2}zf_1f_4 + 2(3z^2-1)f_2f_3$$

$$+ 2z(3z^2+1)f_2f_4 - 6\sqrt{3}z(z^2-1)f_2f_6 + 8zf_3f_4 - 6\sqrt{3}(z^2-1)f_4f_6$$

- Our model predictions in small x ($x < 0.1$) are negligible → consistent with other convolution models. [Cosyn:2017fbo, Kumano:2024fpr]

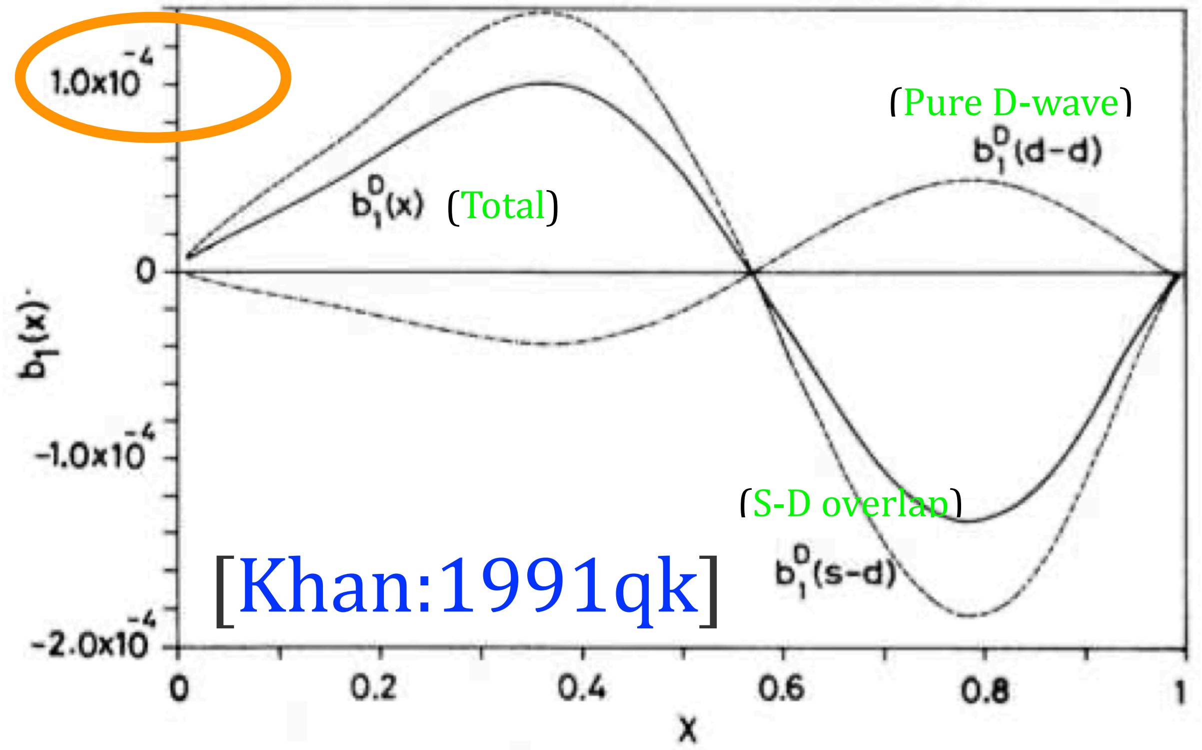
$b_1 \quad \to \quad f_{1LL} \quad \otimes \quad$ unpolarized NNPDF

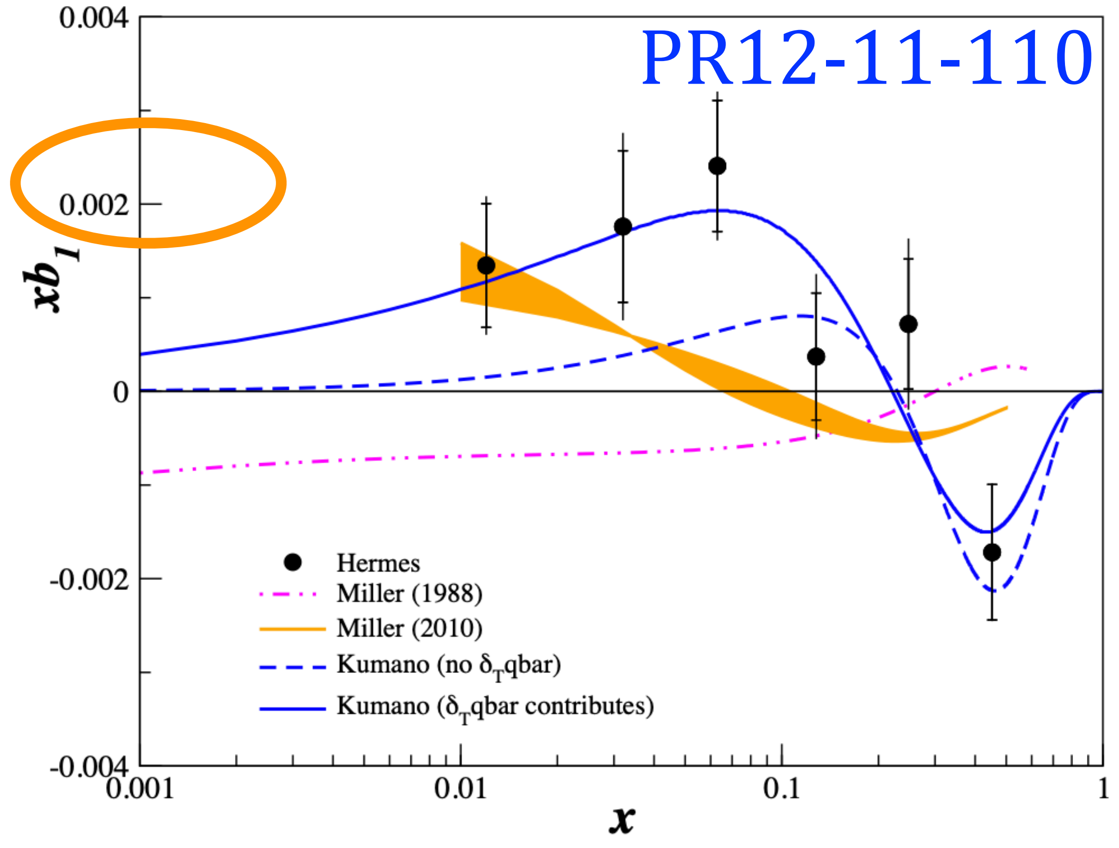


$xb_1 \sim 10^{-4}$

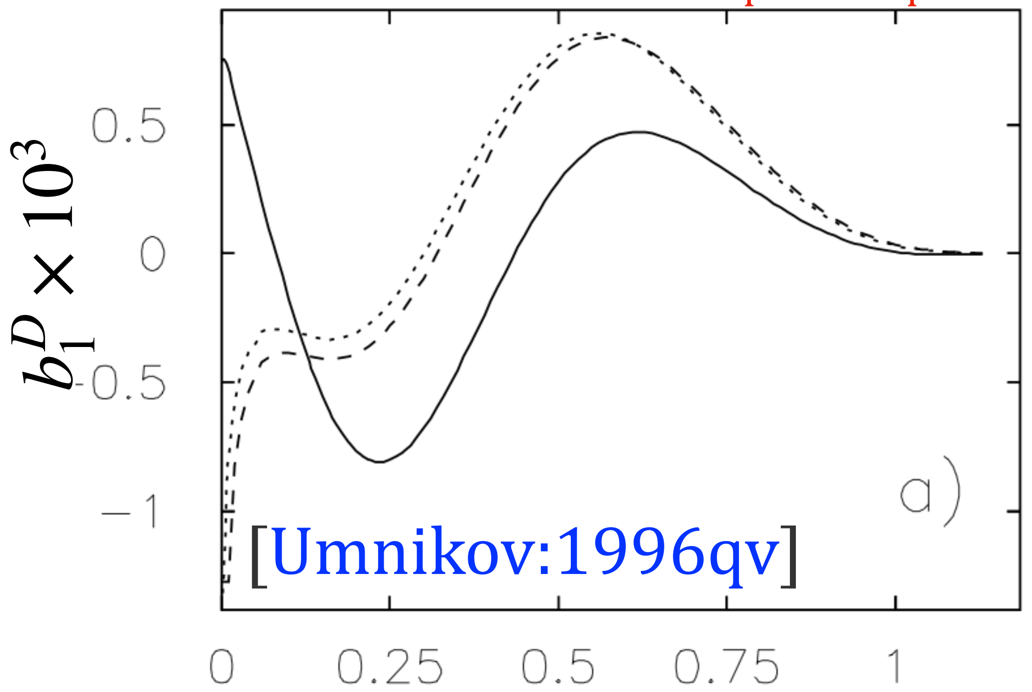
↕ **Order of magnitude difference**

$xb_1 \sim 10^{-3}$ **in HERMES data**

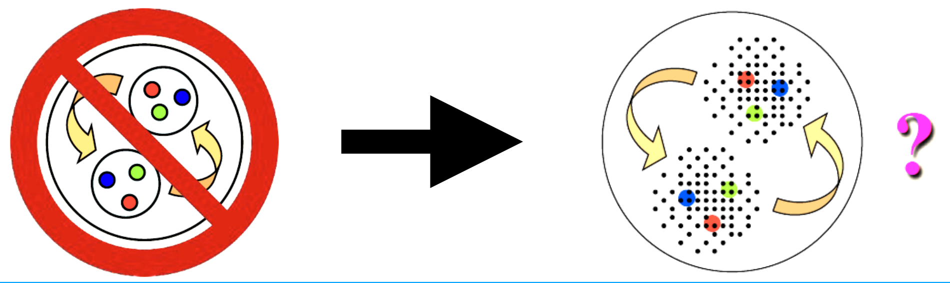




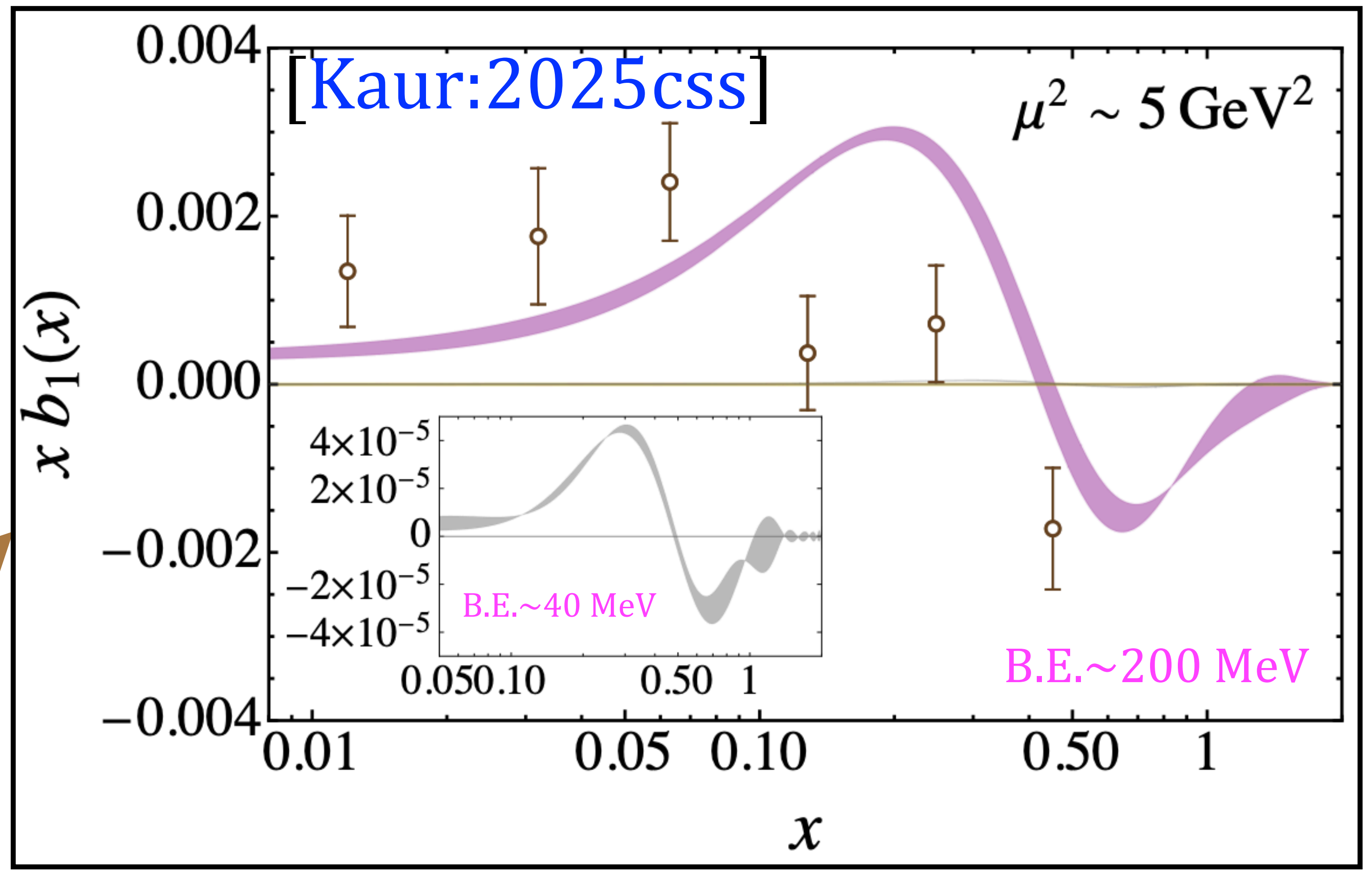
Realistic relativistic Bethe-Salpeter amplitude



G. A. Miller, PRC 89 (2014) 045203,
Interesting suggestions:
hidden-color, 6-quark, · · ·
$|6q\rangle = |NN\rangle + |\Delta\Delta\rangle + |CC\rangle + \cdots$



Hidden-color dof as a singlet-singlet and octet-octet color-cluster mixture using LF-holographic QCD & `t-Hooft equation

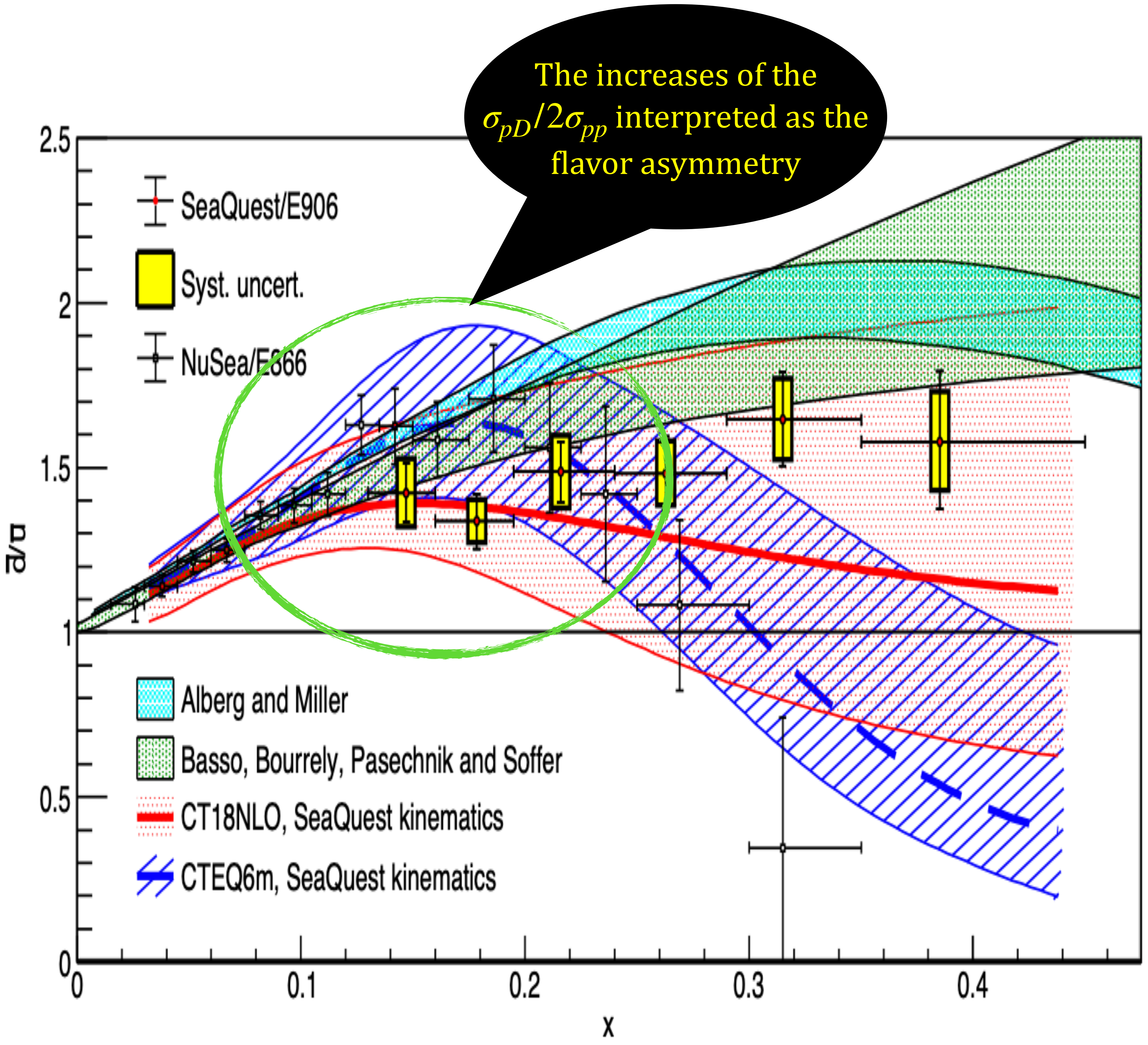


Requires very large BE~200 MeV → hidden-color dof, which become more significant at higher B.E.

SU(2) flavor asymmetry of the light quark sea

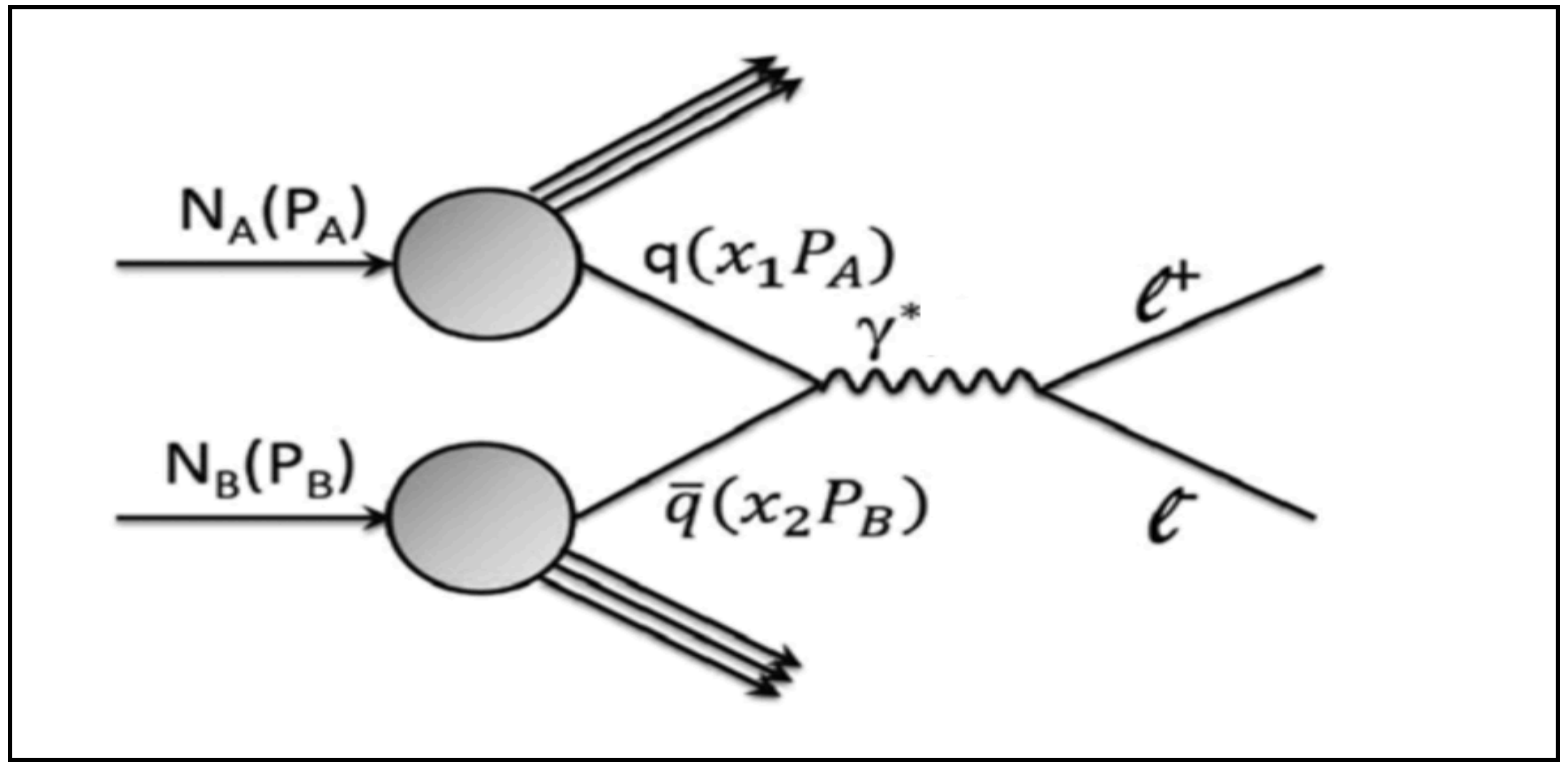
How sea quarks generated inside the proton?

- Perturbative mechanism (gluon splitting): consistent light anti-quark distributions $\bar{d}(x) \approx \bar{u}(x)$.
- Non-perturbative mechanism: SU(2) flavor asymmetry $\bar{d}(x) \neq \bar{u}(x)$.

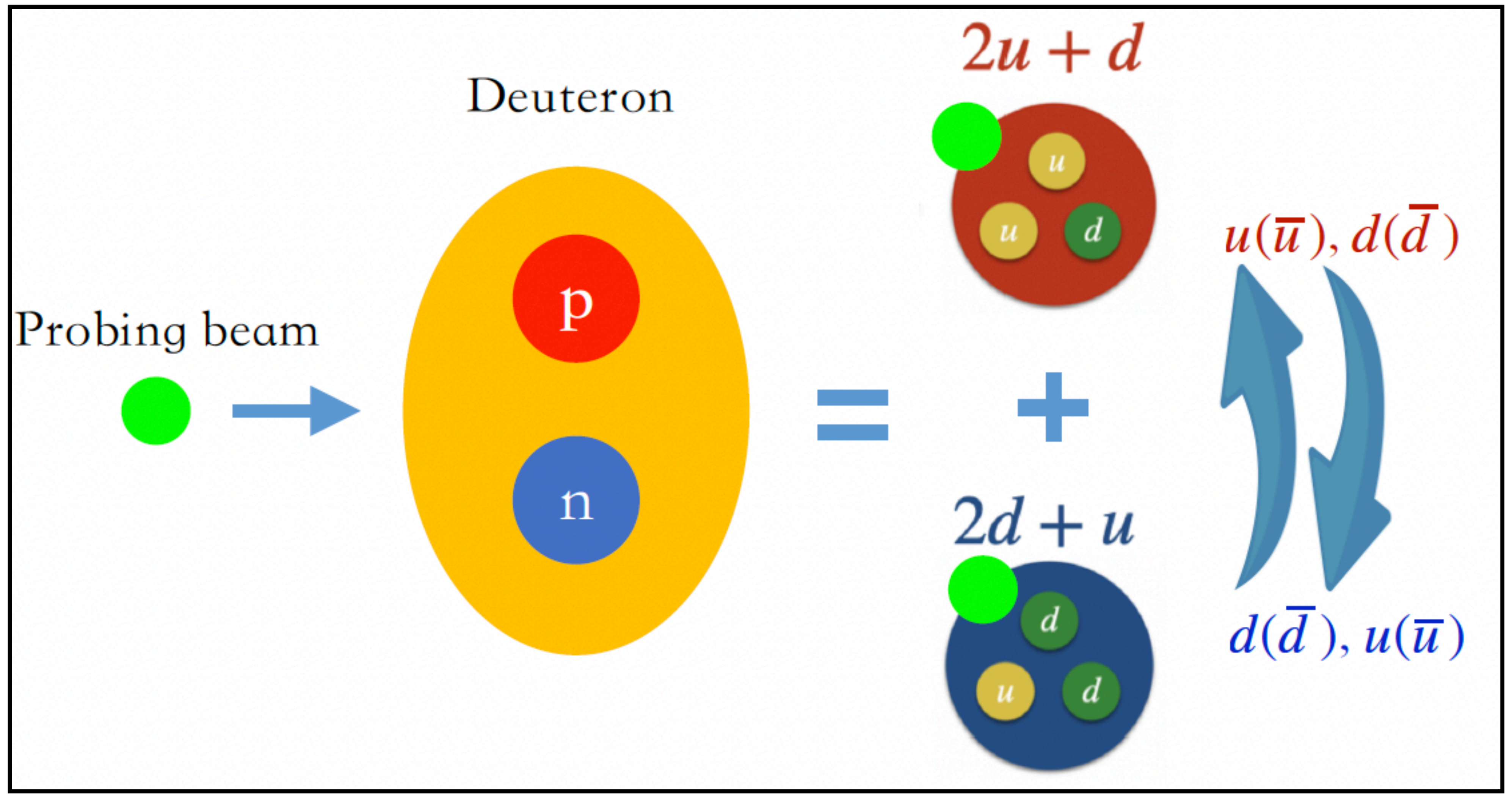


- **Pauli Exclusion**: Proton has 2 u-quarks vs 1 d-quark. Pauli blocking suppresses $g \to \bar{u}u$ pairs relative to $g \to \bar{d}d$, generating $\bar{d} > \bar{u}$. [Zhang:2001gna]

- One can probe the anti-quark distributions in Drell-Yan process cleanly.



$$\frac{d^2\sigma}{dx_b dx_t} = \frac{4\pi\alpha_{EM}^2}{9 s x_b x_t} \sum_q e_q^2 \left[q(x_b)\bar{q}(x_t) + \bar{q}(x_b)q(x_t)\right]$$



Proton-neutron to proton-proton scattering

$$\frac{\sigma_{pn}}{\sigma_{pp}} = \frac{8\bar{d}_p(x_t) + \bar{u}_p(x_t)}{8\bar{u}_p(x_t) + \bar{d}_p(x_t)} \approx \frac{\bar{d}_p(x_t)}{\bar{u}_p(x_t)}.$$

Proton-deuteron to proton-proton scattering

$$\frac{\sigma_{pD}}{\sigma_{pp}} = \frac{8\bar{u}_D(x_t^D) + \bar{d}_D(x_t^D)}{8\bar{u}_p(x_t) + \bar{d}_p(x_t)} = \frac{9\bar{u}_D(x_t^D)}{8\bar{u}_p(x_t) + \bar{d}_p(x_t)}$$

Nuclear corrections are neglected [NuSea:2001idv]

$$\frac{\sigma_{pD}}{\sigma_{pH}} \approx \frac{\sigma_{pp} + \sigma_{pn}}{\sigma_{pp}} \approx 1 + \frac{\bar{d}_p(x_t)}{\bar{u}_p(x_t)}.$$

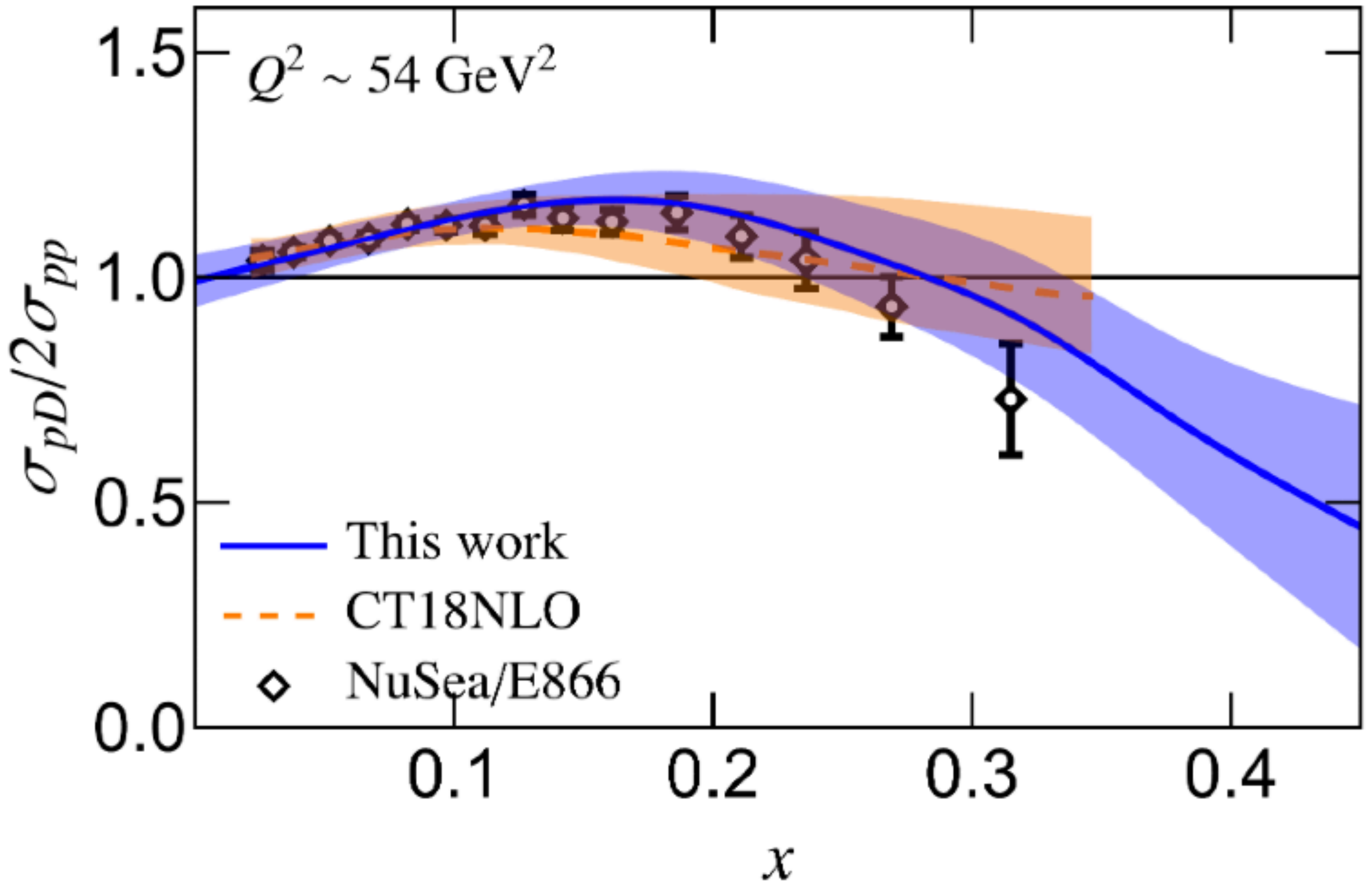
$$\frac{\sigma_{pD}}{\sigma_{pH}} \approx 1 + \frac{d_D(x_t)}{\bar{u}_D(x_t)}$$

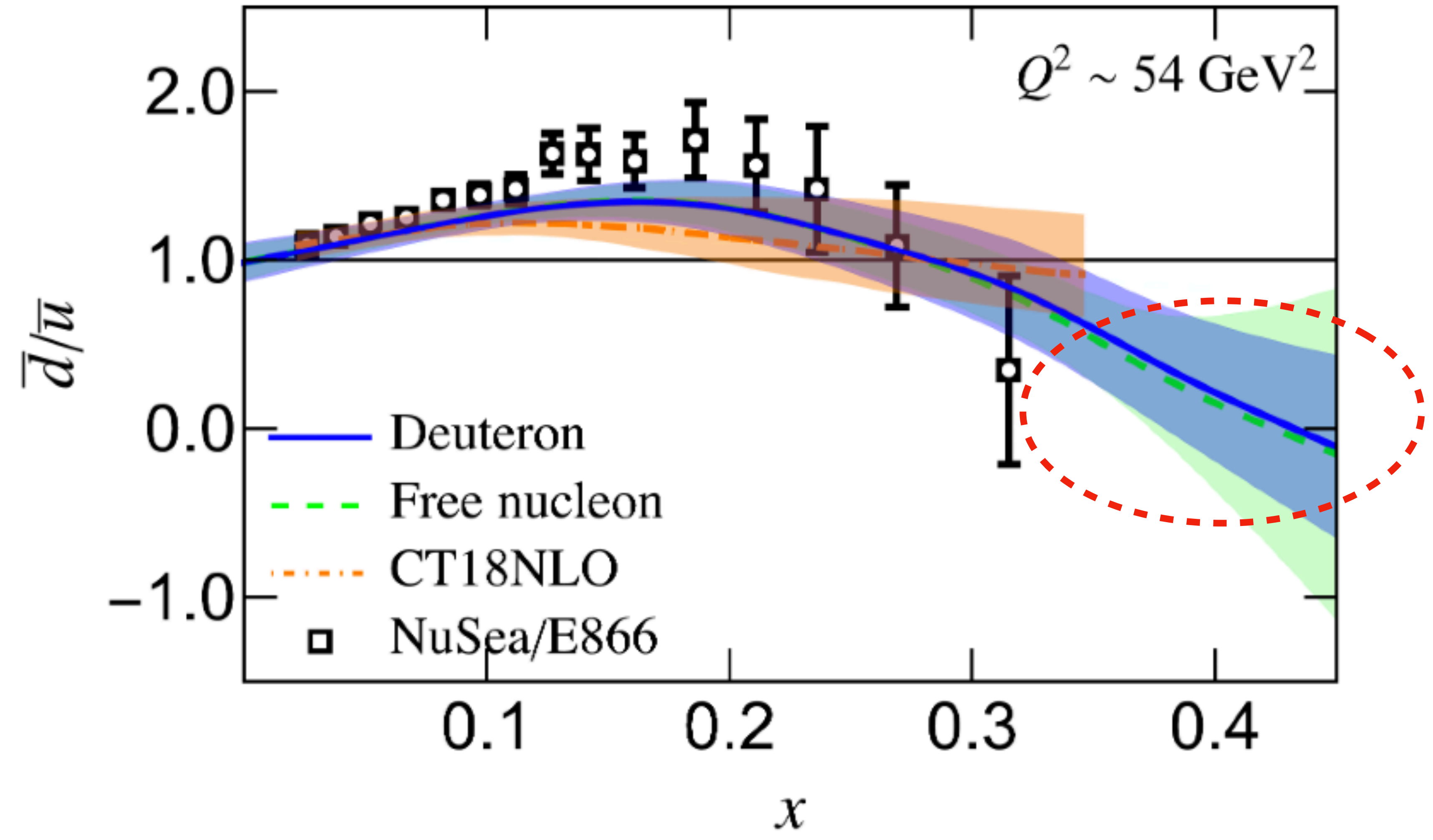
Direct extractions of light sea quark flavor asymmetry inside the deuteron

$\bar{u}_D$ and $\bar{d}_D$ → the quark distributions within deuteron: Deuteron LMDs ⊗ quark PDFs in nucleons

- **Isospin Filtering in Drell-Yan:** Flavor symmetry → $\bar{d}_D = \bar{u}_D$ and $u_v \approx 2d_v$.

$$\mathcal{F}_i^D(x, Q^2) = \frac{1}{2}\sum_q \int \frac{dz}{z}\text{LMD}(z)\mathcal{F}_i^N\left(\frac{x}{z}, Q^2\right).$$





- Deuteron B.E.~2 MeV: EMC effect is negligible, leaving the intrinsic nucleon sea unperturbed aside from minor Fermi-motion smearing.

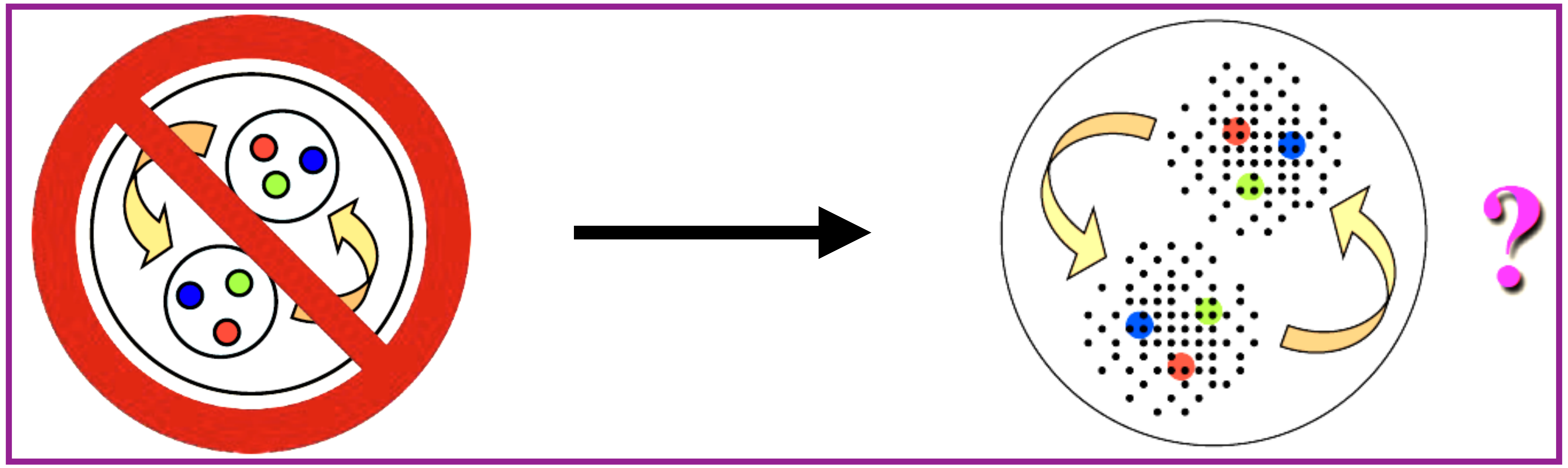
[EuropeanMuon:1983wih]

Conclusion

- Relativistic OBE models reveal 6 spin components, far exceeding non-relativistic 2-component descriptions.
- Consistent agreement found between model and data for unpolarized F_2^D and helicity-dependent g_1^D structure functions.
- The elastic EMFFs, DY cross section and flavor asymmetries are also consistent with the experimental data.
- Significant discrepancy in b_1^D relative to HERMES data suggests physics beyond standard nucleon-based descriptions.

Outlooks

- Exploration of non-nucleonic degrees of freedom by treating the deuteron as a six-quark (6q) system.
- Integration of contributions from hidden-color configurations in current models.
- Development of future high-precision measurements from JLab and upcoming EICs to map exotic internal dynamics.



THANKS FOR YOUR KIND ATTENTION !!