

Study of Deuteron Structure in the Basis Light-Front Quantization Approach

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James P. Vary**

(BLFQ Collaboration)

Tensor SIDIS Workshop



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Outline

Motivation

Basis Light-Front Quantization

Electromagnetic Form Factors

PDFs and Structure Functions

Summary

Everything in the visible universe is made of atoms, and atoms themselves are built from nuclei.

Deuteron

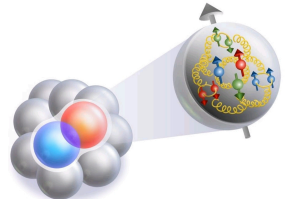
What do we know?

- Simplest nuclear bound state (p+n)
- Weakly bound, B.E. ~ 2.2 MeV
- At large distances: Meson exchange b/w nucleons



What happens at short distances?

- Overlapping of nucleons
- Strong interactions among quarks and gluons in shaping the deuteron.



Deuteron serves as a bridge between QCD and nuclear physics

Deuteron properties

- Charge Radius: 2.12799(74)fm
- Mass: 1875.61294257(57)MeV
- Binding Energy: 2.22452(20)MeV
- Quantum Number:

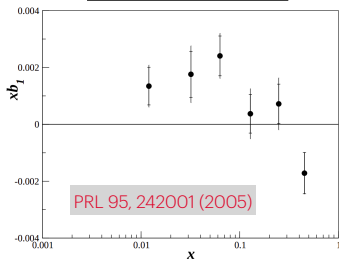
$$I = 0$$

$$J = 1$$

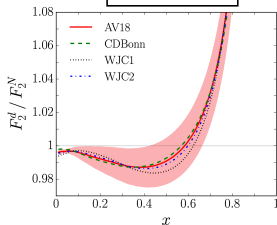
$$L = 0/2, S = 1$$

$$P = +$$

Tensor-polarized
Structure function



EMC effects

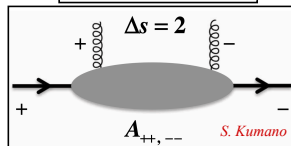


Accardi, Brady, WM, Owens, Sato (2016)

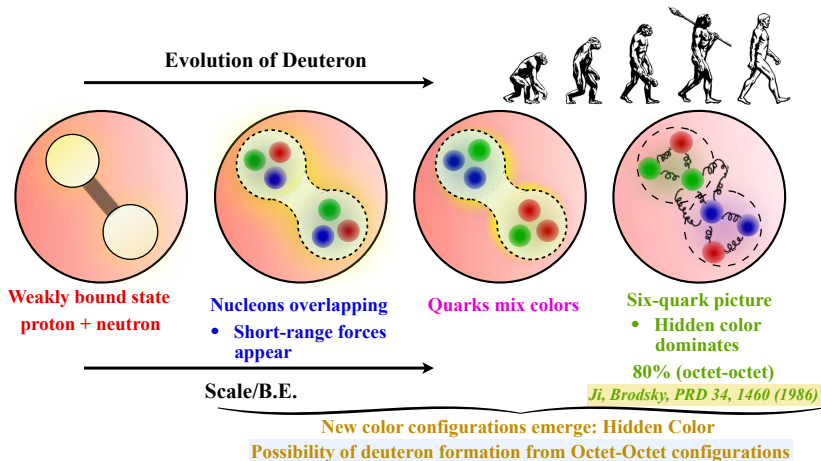
Exotic effects
in nuclei?

$$\frac{F_2^D(x)}{F_2^P(x) + F_2^N(x)} \neq 1$$

Gluon transversity:
double spin flip



Nucleonic description of deuteron is not sufficient!



- ▶ Deuteron can be formed from:
 - ▶ **1 Singlet-Singlet** → conventional 2-nucleon (colorless state)
 - ▶ **4 Octet-Octet** → hidden color components (mixed states)

¹ J.Phys.Conf.Ser.543, 012004 (2014); Prog.Part.Nucl.Phys. 74, 1 (2014); PRC 89, 045203 (2014)

Hidden Colors

In simple words, the interactions of the quarks of one nucleon with the other nucleons in a nuclear system.

$$\begin{array}{l}
 \text{3-quark system} \left\{ \begin{array}{l}
 \square \otimes \square \otimes \square = \square\square\square \oplus \begin{array}{|c|} \hline \square \\ \hline \end{array} \oplus \begin{array}{|c|} \hline \square \\ \hline \end{array} \oplus \begin{array}{|c|} \hline \square \\ \hline \end{array} \\
 \{3\} \otimes \{3\} \otimes \{3\} = 27 = \{10\} \oplus \{8\} \oplus \{8\} \oplus \{1\}
 \end{array} \right.
 \end{array}$$

Physical Nucleon State

$$\begin{array}{l}
 \text{6-quark system} \left\{ \begin{array}{l}
 \square \otimes \square \otimes \square \otimes \square \otimes \square \otimes \square = \square\square\square\square\square\square \oplus 5 \begin{array}{|c|} \hline \square\square\square\square \\ \hline \end{array} \oplus 9 \begin{array}{|c|} \hline \square\square\square \\ \hline \end{array} \oplus 10 \begin{array}{|c|} \hline \square\square \\ \hline \end{array} \\
 \oplus 5 \begin{array}{|c|} \hline \square\square \\ \hline \end{array} \oplus 16 \begin{array}{|c|} \hline \square \\ \hline \end{array} \oplus 5 \begin{array}{|c|} \hline \square \\ \hline \end{array} \\
 \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} = 729 \\
 = \{28\} \oplus 5 \{35\} \oplus 9 \{27\} \oplus 10 \{10\} \oplus 5 \{10\} \oplus 16 \{8\} \\
 \oplus 5 \{1\}
 \end{array} \right.
 \end{array}$$

Physical Deuteron States

For Deuteron, possible hidden color states: 4

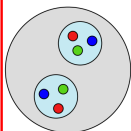
For Helium-3, possible hidden color states: 41

For Helium-4, possible hidden color states: 461

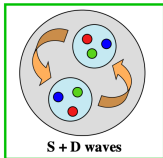
Hidden colors are very important in explaining the structure of a nuclear system.

¹ Prog.Part.Nucl.Phys. 74, 1 (2014)

Spin-1 deuteron

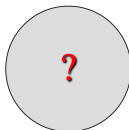


only S wave
 $b_1=0$



S + D waves
standard model $b_1 \neq 0$

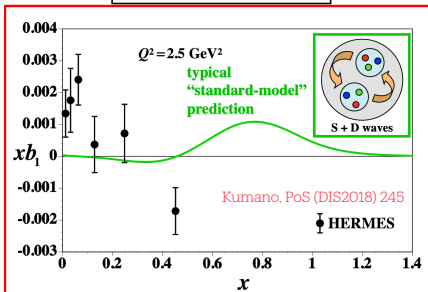
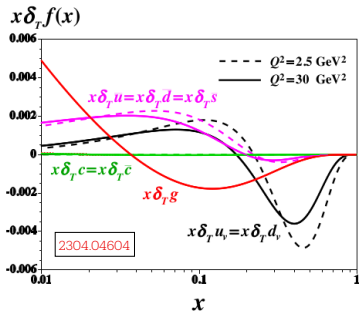
New hadronic mechanism!?

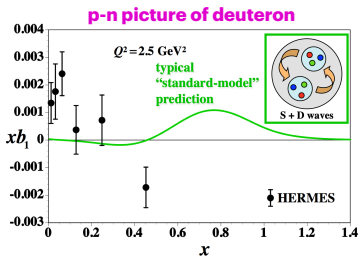
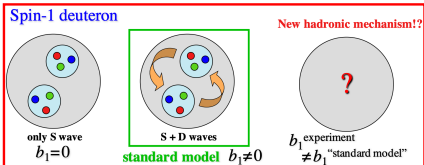


b_1 experiment
 $\neq b_1$ "standard model"

S. Kumano

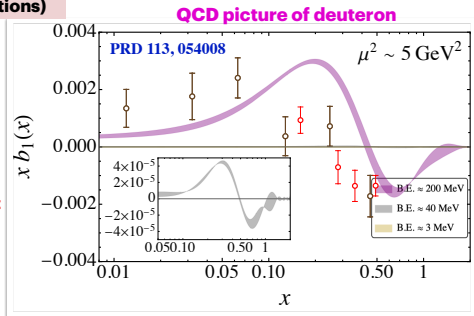
Cosyn et al. PRD 95, 074036 (2017)





S. Kaur, C. Mondal, X. Zhao, C.-R. Ji,
PRD 113, 054008 (2026)
 (included hidden color configurations)

- **At B.E. ~ 3 MeV (loosely bound),**
 $b_1(x) \approx 0$
- **At higher B.E.,**
hidden-color d.o.f. become significant



A Proposal to Jefferson Lab PAC-38
(Update to LOI-11-003)

LOI-11-003

J.-P. Chen (co-spokesperson), P. Solvignon (co-spokesperson),

K. Allada, A. Camsonne, A. Deur, D. Gaskell,
M. Jones, C. Keith, S. Wood, J. Zhang

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

FERMILAB-PUB-22-381-V

The Transverse Structure of the Deuteron with Drell-Yan

The SpinQuest Collaboration^a

FERMILAB-PUB-22-381-V

Eur. Phys. J. A (2016) 52: 208
DOI 10.1140/epja/i2016-16208-0

Review

THE EUROPEAN
PHYSICAL JOURNAL A

Electron-Ion Collider: The next QCD frontier

Understanding the glue that binds us all

A. Accardi^{1,2,8}, J.L. Albacete¹⁶, M. Anselmino²⁰, N. Armesto²⁷, E.C. Aschenauer^{2,8}, A. Bacchetta¹⁰, D. Boer³³, W.K. Brooks^{36,4}, T. Burton³, N.-B. Chang²⁵, W.-T. Deng^{15,22}, A. Deshpande^{26,3,3,3}, M. Diehl^{11,3}, A. Dumitriu², R. Dupré⁷, B. Ezer^{36,4}, S. Fazio¹⁷, H. Gao^{31,3}, V. Gayo²³, H. Hokenjyan²⁸, Y. Hao², D. Haseh¹⁹, R. Heh^{1,3}, T. Heron^{3,4}, M. Huang²¹, A. Hutton^{28,4}, C. Hyde²⁰, J. Jauchian-Maria²⁵, S. Klein¹⁷, B. Kopeliovich²⁶, V. Kochegorov^{19,8}, K. Kumar^{25,4}, K. Kumericki¹⁰, M.A.C. Lamont³, T. Lappi²⁴, J.-H. Lee³, Y. Lee³, E.M. Levin^{26,18}, F.-L. Lin³, V. Litvinenko², T.W. Ludlam^{3,1}, C. Marquet³, Z.-E. Meziani^{27,3,3,3}, R. McKeown^{36,4}, A. Metz²⁷, R. Miller¹⁸, V.S. Morozov²⁶, A.H. Mueller^{3,4}, B. Müller^{12,4}, D. Müller³, P. Nadez-Turonaki²⁸, H. Panikvar²⁶, A. Prokudin²⁸, V. Pitsyn¹, X. Qian⁴, J.-W. Qiu^{1,3,3,3}, M. Ramsey-Musul^{29,4}, T. Rose^{3,4}, F. Sabatia^{7,8}, R. Sassot¹¹, G. Schaef²⁰, P. Schweitzer¹², E. Sichterman^{17,4}, M. Stratmann¹⁹, M. Strikman²¹, M. Sullivan²¹, S. Taneja^{10,25}, T. Toll², D. Trbojević², T. Ulbrich³, R. Venugopalan¹, S.V. W. Wobisch^{29,4}, C. Weiss²³, B.-W. Xiao³, F. Yuan^{17,3}, Y.-H. Zhang³, and L. Zhang³

the spatial distribution of gluons in the nucleus. Exclusive vector meson production in diffractive $e + A$ collisions is the cleanest such process, due to the low number of particles in the final state. This would not only provide us with further insight into saturation physics but also constitute a highly important contribution to heavy-ion physics by providing a quantitative understanding of the initial conditions of a heavy-ion collision as described in sect. 3.4.2.

It might even shed some light on the role of glue and thus QCD in the nuclear structure of light nuclei (see sect. 3.3).

M. Brooks, A. Klein (CoSpokesperson), D. Kleinjan, K. Liu, M. Liu
M. McCumber, P. McGaughy, C. Da Silva
Los Alamos National Laboratory, Los Alamos, NM 87545



Progress in Particle and Nuclear Physics

Volume 119, July 2021, 103858



Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

A. Arbutov^a, A. Bacchetta^{b,c}, M. Butenschoen^d, F.G. Celiberto^{b,c,e,f}, U. D'Alesio^{g,h}, M. Deka^g, I. Denisenko^g, M.G. Echevarria^g, A. Efremov^g, N.Yu. Ivanov^g, A. Guskov^{g,h}, 张, A. Karpishkov^h, Y. Klöpfer^{g,m}, B.A. Kniehl^g, A. Kotzinian^{l,o}, S. Kumano^g, J.P. Lansberg^g, Keh-Fei Liu^f, F. Murgia^h, M. Nefedov^h, O. Teryaev^g



Frontiers of Physics

<https://doi.org/10.1007/s11467-021-1062-0>

Front. Phys.

16(6), 64701 (2021)

REVIEW ARTICLE

Electron-ion collider in China

Danielle P. Anderle¹, Valerio Bertone², Xu Cao^{3,4}, Lei Chang⁵, Ningbo Chang⁶, Gu Chen⁷, Xurong Chen^{3,4}, Zhuojun Chen³, Zhufang Cui⁹, Lingyun Dai³, Weitian Deng¹⁰, Minghui Ding¹¹, Xu Feng¹², Chang Gong¹², Longcheng Gui¹³, Feng-Kun Guo¹⁴, Chengdong Han^{3,4}, Jun He¹⁵, Tie-Jun Hou¹⁶, Hongxia Huang¹⁷, Yin Huang¹⁷, Kresimir Kumericki¹⁸, L. P. Kaptari^{19,20}, Dennis Li²¹, Hengze Li²², Mingxiang Li²³, Xiaojian Li²⁴, Yutian Liang²⁵, Zuotang Liang²⁶, Chen Liu²⁷, Chuan Liu²⁸, Guoming Liu¹, Jie Liu⁴, Liuning Liu^{3,4}, Xiang Liu^{3,4}, Tianbo Liu²⁹, Xiaofeng Luo³⁰, Zhun Lyu³¹, Boqiang Ma¹², Fu Ma^{3,4}, Jianping Ma^{31,4}, Yugang Ma^{3,26,36}, Lijun Mao³¹, Cédric Mezrag³², Hervé Moutarde³³, Jianjun Ping³⁴, Sibux Qin³⁷, Hang Ren³⁴, Craig D. Roberts³, Juan Rojas^{35,20}, Guodong Shen³¹, Chao Shi³⁸, Qintao Song³², Hao Sun³¹, Pavel Suvajdzic³⁹, Enke Wang⁴⁰, Fan Wang⁴¹, Qiao Wang⁴², Rong Wang⁴³, Huirui Wang⁴⁴, Tsoufeng Wang⁴⁵, Wei Wang⁴⁴, Xiaoyu Wang²², Xiaoyun Wang²², Jiahui Wu⁴⁶, Xianzeng Wu⁴⁷, Lei Xia²², Bowen Xiao^{23,27}.

Home > The European Physical Journal A > Article

Experimental study of tensor structure function of deuteron

Review | Published: 18 April 2025

Volume 61, article number 81, (2025) [Cite this article](#)

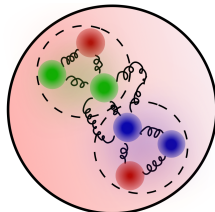
Jiwan Poudel , Alessandro Bacchetta, Jian-Ping Chen & Nathaly Santiesteban

Basis Light-Front Quantization (BLFQ)

Non-perturbative approach based on the Hamiltonian formalism :

$$P^+ P^- |\Psi\rangle = M^2 |\Psi\rangle$$

- ▶ To solve relativistic many-body bound state problems.
- ▶ facilitates with mass spectra and LFWFs.
- ▶ Successfully implemented to investigate the structures of various baryons and mesons.
- ▶ Motivation: To extend the approach to investigate light nuclei.



Fock state expansion of the deuteron state

$$|\Psi\rangle_D = \psi_{6q} |qqq qq\rangle + \psi_{6q+1g} |qqq qqg\rangle + \psi_{6q+q\bar{q}} |qqq qq q\bar{q}\rangle + \dots$$

- ▶ ψ_{\dots} : LFWFs associated with the Fock components $|\dots\rangle$.

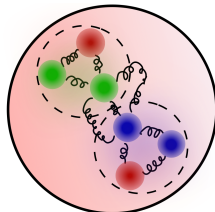
¹J.P.Vary, H. Honkanen, J. Li, P. Maris, S.J.Brodsky, A. Harindranath, G.F. de Teramond, PRC 81, 035205 (2010).

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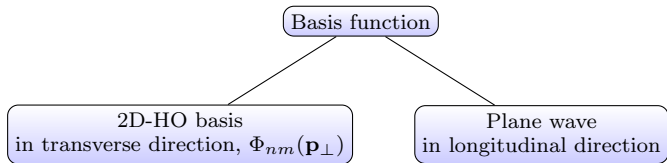


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¹J.P.Vary, H. Honkanen, J. Li, P. Maris, S.J.Brodsky, A. Harindranath, G.F. de Teramond, PRC 81, 035205 (2010).



- ▶ Parton's basis state is identified by $|\alpha_i\rangle = |k_i, n_i, m_i, \lambda_i\rangle$
- ▶ Many-body basis states are identified as the direct product of the Fock-particle basis states $|\alpha\rangle = \otimes |\alpha_i\rangle$.

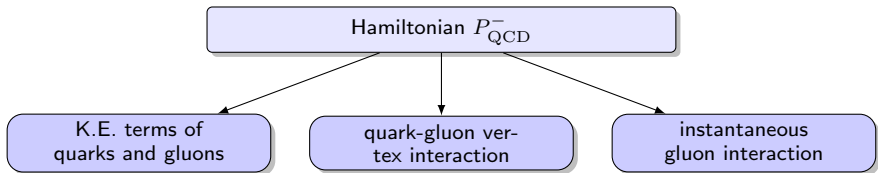
Fock space truncation:

$$|\Psi\rangle_D = \psi_{6q} |qqq qqq\rangle + \psi_{6q1g} |qqq qqq g\rangle$$

Basis truncation:

$$\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$$

$$\sum_i k_i = K, \quad x = \frac{k_i}{K}$$



Light-front QCD Hamiltonian

[Brodsky et al, 1998]

$$P_{-,LFQCD} = \frac{1}{2} \int d^3x \bar{\psi} \gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+} \psi - \frac{1}{2} \int d^3x A_a^i (i\partial^\perp)^2 A_a^i$$

$$+ g \int d^3x \bar{\psi} \gamma_\mu A^\mu \psi$$

$$+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma_\mu A^\mu \frac{\gamma^+}{i\partial^+} \gamma_\nu A^\nu \psi$$

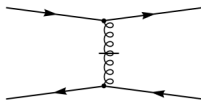
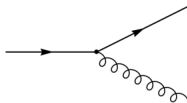
$$- i g^2 \int d^3x f^{abc} \bar{\psi} \gamma^+ T^c \psi \frac{1}{(i\partial^+)^2} (i\partial^+ A_a^\mu A_{\mu b})$$

$$+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi$$

$$+ i g \int d^3x f^{abc} i\partial^\mu A^\nu A_\mu^b A_\nu^c$$

$$- \frac{1}{2} g^2 \int d^3x f^{abc} f^{ade} i\partial^+ A_b^\mu A_{\mu c} \frac{1}{(i\partial^+)^2} (i\partial^+ A_d^+ A_{ve})$$

$$+ \frac{1}{4} g^2 \int d^3x f^{abc} f^{ade} A_b^\mu A_c^\nu A_{\mu d} A_{ve}$$



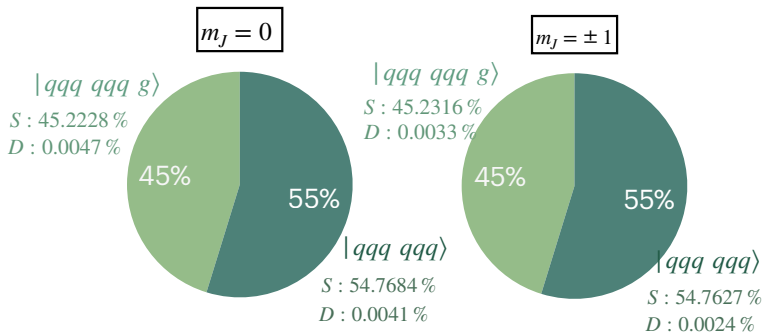
6

¹S.J. Brodsky, H.C. Pauli, S.S. Pinsky, Phys. Rep. 301, 299-486 (1998)

Parameters and Decomposition of Spin States

m_u	m_d	b	b_{inst}	g	m_f
1 GeV	0.95 GeV	0.30 GeV	5 GeV	1.90	42.56

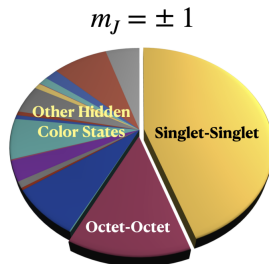
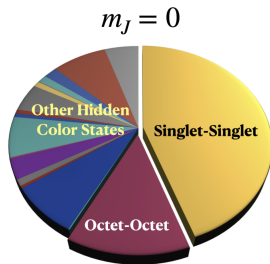
$N_{max} = 8; K = 9$



- ▶ Number of Color singlet states in $|6q\rangle$: 5
- ▶ Number of Color singlet states in $|6q + 1g\rangle$: 16

Decomposition of Color States

	Color Configurations	Total Color Singlet States	Probability (%) (Preliminary)	
			$m_J = 0$	$m_J = 1$
$ qqq\ qqq\rangle$	Singlet-Singlet	1	44.56	44.52
	Octet-Octet	4	12.98	13.02
$ qqq\ qqq\ g\rangle$	Decuplet-Octet-Octet	2	42.46	42.46
	Octet-Decuplet-Octet	2		
	Octet-Octet-Octet	8		
	Octet-Singlet-Octet	2		
	Singlet-Octet-Octet	2		



Electromagnetic Form Factors

SLAC-PUB-2318
May 1979
(T)

Phys.Rev.C 21, 1426 (1980)

SLAC-PUB-5763
March 1992
T/E

Phys.Rev.D 46, 2141 (1992)

Universal Properties of the Electromagnetic
Interactions of Spin-One Systems*

ELASTIC ELECTRON-DEUTERON SCATTERING AT HIGH ENERGY*

$$\begin{aligned} \langle V(P', \Lambda') | J^\mu | V(P, \Lambda) \rangle = & -\epsilon_{\Lambda'}^* \cdot \epsilon_\Lambda (P + P')^\mu F_1(Q^2) + (\epsilon_\Lambda^\mu q \cdot \epsilon_{\Lambda'}^* - \epsilon_{\Lambda'}^{*\mu} q \cdot \epsilon_\Lambda) F_2(Q^2) \\ & + \frac{(\epsilon_{\Lambda'}^* \cdot q)(\epsilon_\Lambda \cdot q)}{2M_V^2} (P + P')^\mu F_3(Q^2) \end{aligned}$$

PRD 70, 053015 (2004)

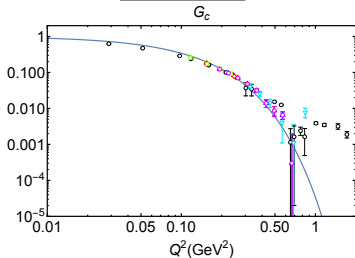
Overlap form of LFWFs

$$\begin{aligned} G_C = F_1 + \frac{2}{3}\eta G_Q &= \frac{1}{2P^+} \left[\frac{3-2\eta}{3} I_{++}^+ + \frac{4\eta}{3} \frac{I_{+0}^+}{\sqrt{2\eta}} + \frac{I_{+-}^+}{3} \right], \\ G_M = -F_2 &= \frac{2}{2P^+} \left[I_{++}^+ - \frac{I_{+0}^+}{\sqrt{2\eta}} \right], \\ G_Q = F_1 + F_2 + (1+\eta)F_3 &= \frac{1}{2P^+} \left[-I_{++}^+ + 2 \frac{I_{+0}^+}{\sqrt{2\eta}} - \frac{I_{+-}^+}{\eta} \right], \end{aligned}$$

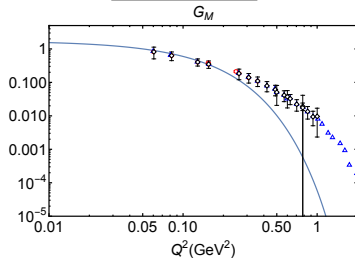
$$\begin{aligned} I_{\Lambda', \Lambda}^+(Q^2) &\triangleq \langle V(P', \Lambda') | \frac{J^+(0)}{2P^+} | V(P, \Lambda) \rangle \\ &= \sum_{\hbar, \bar{\hbar}} \int_0^1 \int_0^\infty \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \Psi_{\hbar\bar{\hbar}}^{\Lambda'^*}(x, \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp) \Psi_{\hbar\bar{\hbar}}^\Lambda(x, \mathbf{k}_\perp), \end{aligned}$$

EM Form Factors : Preliminary results from BLFQ

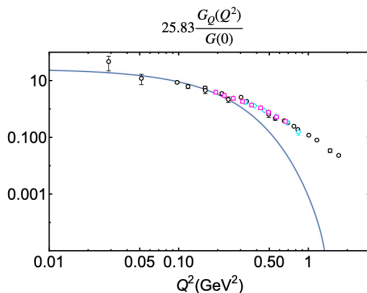
Charge FF



Magnetic FF



Quadrupole FF



m_J	M_D
0	1.927 GeV
± 1	1.874 GeV

$$\Delta M_D = 0.053 \text{ GeV}$$

**Rotational symmetry
is not severely broken**

$$\mu_D = 0.86 \text{ (Exp. } 0.857)$$

$$\mathcal{Q}_D = 0.20 \text{ GeV}^{-2} \text{ (} 7.34 \text{ GeV}^{-2})$$

Richness of Deuteron's Spin Structure

Quark \ Hadron	U (γ^+)		L ($\gamma^+\gamma_s$)		T ($i\sigma^{i+}\gamma_s / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		f_{1T}^\perp	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

- Spin-0: 2 TMDs
- Spin-1/2: 8 TMDs
- Spin-1: 18 TMDs

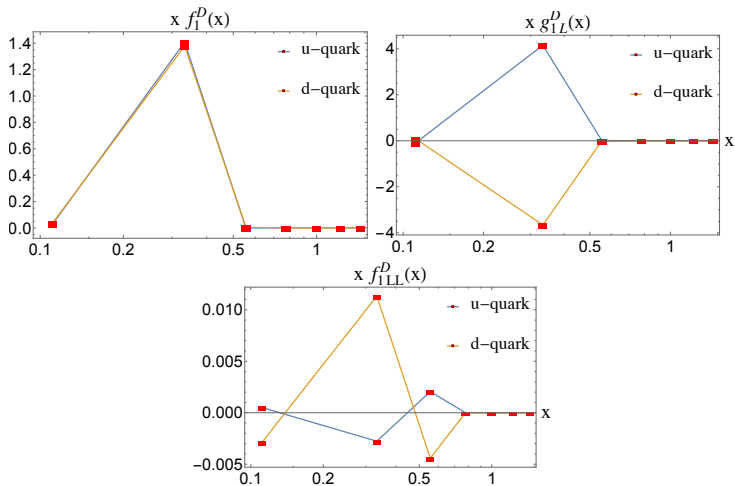
Twist-2 TMDs.

Kumano et al. 2406.01180

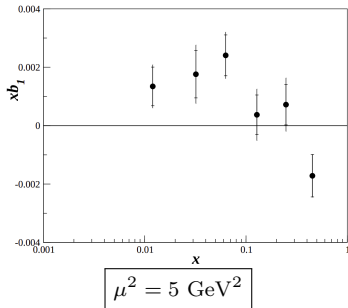
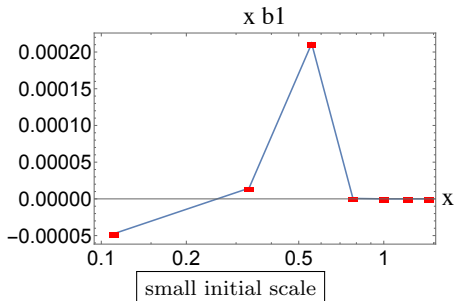
Quark \ Hadron	U (γ^+)		L ($\gamma^+\gamma_s$)		T ($i\sigma^{i+}\gamma_s / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					
L			$g_{1L}(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}(b_1)$					
LT						*1 $[h_{1LT}]$
TT						

Twist-2 PDFs.

PDFs : Preliminary results from BLFQ



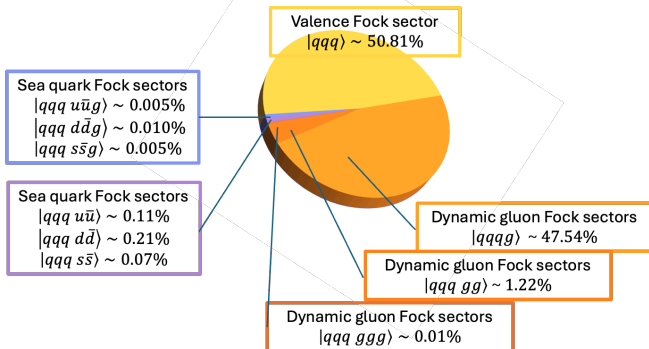
Tensor-polarized structure function b_1 : Preliminary results from BLFQ



- ▶ Qualitatively consistent with HERMES results.
- ▶ Small magnitude at small scale: negligible D -wave and small basis size
- ▶ Capability to achieve the gluon transversity.

Nucleon: Fock Sector Contributions

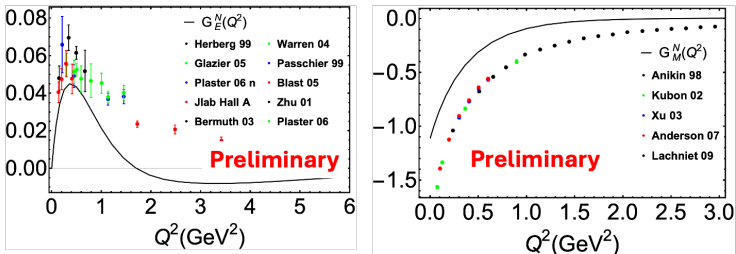
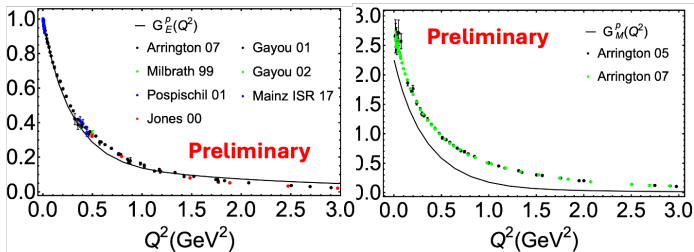
$$|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle \\ + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$



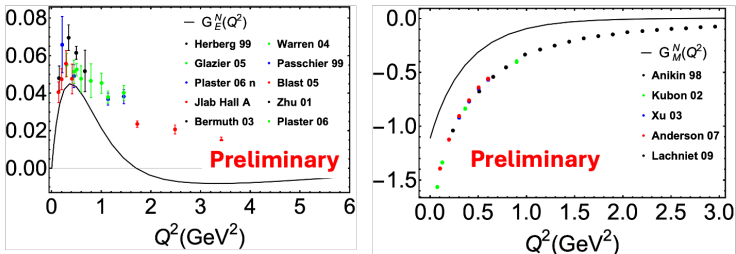
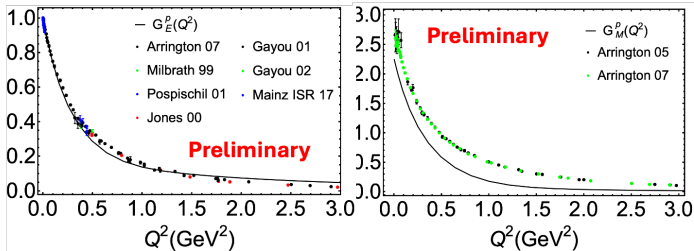
Parameters:

m_u	m_d	m_s	m_f	g	b
0.3 GeV	0.299 GeV	0.5 GeV	0.9 GeV	2.2	0.8 GeV

Nucleon: Electromagnetic Form Factors



Nucleon: Electromagnetic Form Factors



Future Research Program for Deuteron

Current Work

- ▶ $|qqq\ qqq\rangle$
- ▶ $|qqq\ qqq\ g\rangle$
- ▶ Hidden-color dynamics

Next 3 Years

- ▶ Improved QCD Hamiltonian by including all the QCD interactions
- ▶ Larger basis spaces
- ▶ Fock sectors up to 9 particles

Physics Goals

- ▶ EIC observables
- ▶ Tensor structure
- ▶ Gluon Transversity
- ▶ ^3He and ^4He

Conclusions

- ▶ Implemented QCD effects within the deuteron using the Hamiltonian-based approach: BLFQ.
- ▶ BLFQ (**Preliminary results**): obtained qualitative agreement with the available data of deuteron properties.

Thank you!

Basis Dimensions

