Twist-3 PDF of Spin-1 Deuteron

S. Kumano and K. Kuroki, Phys. Lett. B 875 (2026) 140348
S. Kumano and K. Kuroki, arXiv:2605.00430

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Introduction

- Structure Functions $b_{1,\ldots,4}$
- Deuteron b_1
- Tensor Structure Puzzle

Structure Functions (SFs) $b_{1,\dots,4}$

Charged-lepton DIS, P. Hoodbhoy, R. L. Jaffe, A. Manohar, NPB 312 (1989) 571

$$W_{\mu\nu} = -F_1 g_{\mu\nu} + F_2 \frac{p_\mu p_\nu}{\nu} + g_1 \frac{i}{\nu} \varepsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + g_2 \frac{i}{\nu^2} \varepsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma)$$
$$-b_1 r_{\mu\nu} + \frac{1}{6} b_2 (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) + \frac{1}{2} b_3 (s_{\mu\nu} - u_{\mu\nu}) + \frac{1}{2} b_4 (s_{\mu\nu} - t_{\mu\nu})$$

Tensor-polarized Structure Functions

b_1, b_2, b_3, b_4

which vanish under spin average

- b_1, b_2: leading-twist
  $2x b_1(x) = b_2(x)$ in Bjorken limit
- b_3, b_4: higher-twist

Symmetric under $\mu \leftrightarrow \nu$
→ Unpolarized lepton beam ✓

$$r_{\mu\nu} = \frac{1}{\nu^2}\left(q \cdot E^* q \cdot E - \frac{1}{3}\nu^2 \kappa\right) g_{\mu\nu}$$
$$s_{\mu\nu} = \frac{2}{\nu^2}\left(q \cdot E^* q \cdot E - \frac{1}{3}\nu^2 \kappa\right) \frac{p^\mu p^\nu}{\nu}$$
$$t_{\mu\nu} = \frac{1}{2\nu^2}\left(q \cdot E^* p_{\{\mu} E_{\nu\}} + q \cdot E p_{\{\mu} E^*_{\nu\}} - \frac{4}{3}\nu p_\mu p_\nu\right)$$
$$u_{\mu\nu} = \frac{1}{\nu}\left(E^*_{\{\mu} E_{\nu\}}\right) + \frac{2}{3} M^2 g_{\mu\nu} - \frac{2}{3} p_\mu p_\nu$$

$\nu = p \cdot q$
$\kappa = 1 + M^2 Q^2/\nu^2$
E^μ: Polarization vector
$p \cdot E = 0,\ \ E^2 = -M^2$
$\sum_\lambda E^\mu(\lambda) E^{*\nu}(\lambda) = -g^{\mu\nu} M^2 + p^\mu p^\nu$
$s^\sigma = -\frac{i}{M^2} \varepsilon^{\sigma\alpha\beta\tau} E^*_\alpha E_\beta p_\tau$
$a^{\{\mu} b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$

b_1 sum rule F.E. Close, S. Kumano, PRD 42 (1990) 2377

$$\int dx\, b_1(x) = -\frac{5}{24} \lim_{t \to 0} \frac{t}{M^2} F_Q(t) + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_i(x) \quad \text{in parton model}$$

Useful to investigate $\delta_T \bar{q}_i$

Deuteron b_1

HERMES (2005)

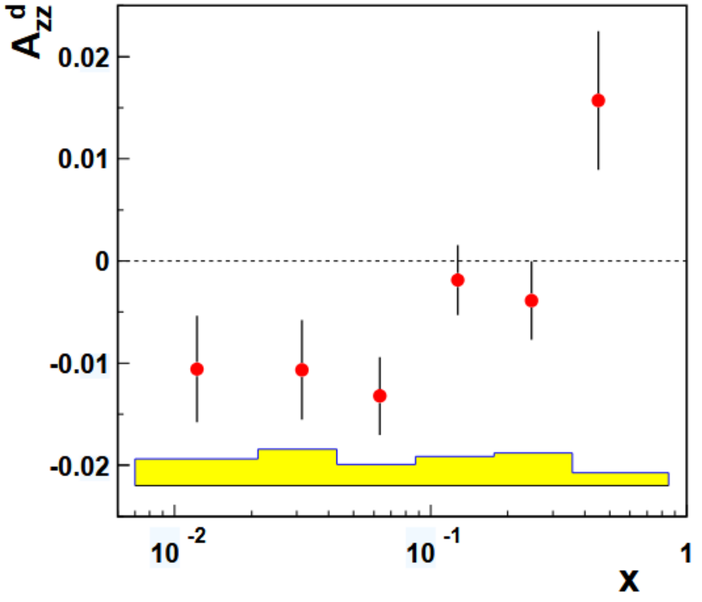
A. Airapetian et al. (HERMES), PRL 95 (2005) 242001

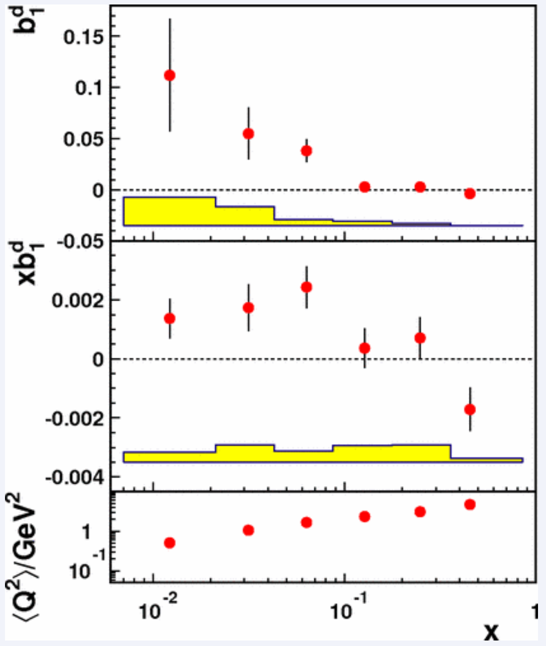
$e^+ \xrightarrow{27.6\ \text{GeV}} D \ \rightleftarrows, 0$

$0.5 < Q^2 < 5\ \text{GeV}^2,\ 0.01 < x < 0.45$

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{d^2\sigma^U}{dx\,dQ^2}\left(1 + \frac{1}{2} P_{ZZ} A_{ZZ}^D\right)$$

Tensor asymmetry: $A_{ZZ}^D \xrightarrow[\text{Bjorken limit}]{} A_{ZZ}^D = -\frac{2}{3}\frac{b_1^D}{F_1^D}$





- Finite at 2σ level
  Positive at low-x
  Negative at high-x

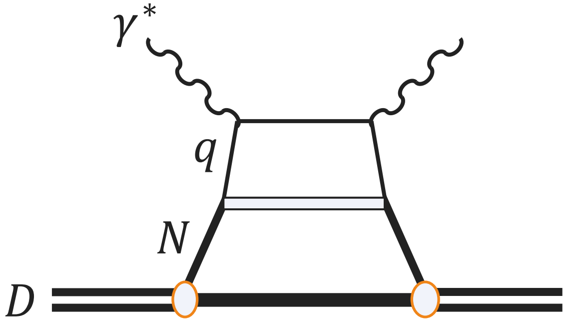
- **Finite Sum**

$$\int_{0.02}^{0.85} dx\, b_1^D(x) = \begin{bmatrix} 0.0035 \\ \pm 0.0010\ (\text{stat}) \\ \pm 0.0018\ (\text{sys}) \end{bmatrix}$$

$\delta_T \bar{q}_i \neq 0$?

Tensor Structure Puzzle

Convolution model
W. Cosyn, Y.-B. Dong, S. Kumano, M. Sargsian, PRD 95 (2017) 074036



$$b_1(x,Q^2) = \int \frac{dy}{y}\, \delta_T f(y)\, F_1^N\left(\frac{x}{y},Q^2\right)$$

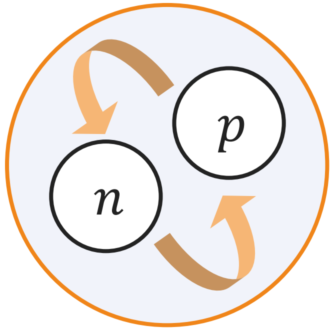
$$y = \frac{M p\cdot q}{M_N P\cdot q} \simeq \frac{2p^-}{P^-}$$

$$f^H(y) = \int d^3p\, y\, \left|\phi^H(p)\right|^2 \delta\left(y - \frac{E-p_z}{M_N}\right)$$

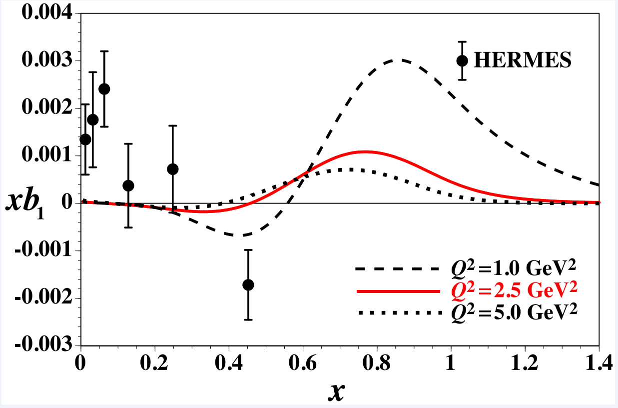
$$\phi^H = \phi^H_{l=0} + \phi^H_{l=2}$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y)+f^-(y)}{2}$$

$$= \int d^3p\, y\left[-\frac{3}{4\sqrt{2}\pi}\phi_0(p)\phi_2(p) + \frac{3}{16\pi}|\phi_2(p)|^2\right](3\cos^2\theta - 1)\,\delta\left(y - \frac{p\cdot q}{M_N \nu}\right)$$



S + D waves



indicate that these standard theoretical predications are much different from the HERMES measurements. Furthermore, significantly large distributions are predicted, at large x ($x > 0.8$) and even at extremely large x ($x > 1$). Since our results are very different from the HERMES measurement, new hadronic mechanisms could be needed for interpreting the data although there is still some room to improve the differences due to the higher-twist effects and the experimental extraction of b_1 from A_{zz}. The HERMES data have large uncertainties; however, upcoming JLab experimental measurements will improve on the size of the errors. In addition, there are

Purpose of this study

A quantitative estimate of twist-3 tensor-polarization

Theoretical Framework

- Collinear PDFs
- Twist-2 Relations

Collinear PDFs

S. Kumano, Q.-T. Song, PRD 103 (2021) 014025

Collinear Correlation Function (CF)

$$\Phi_{ij}(x,P,T) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \left\langle P,T \left| \overline{\psi}_j(0)\psi_i(\xi) \right| P,T \right\rangle_{\xi^+=0,\vec{\xi}_T=0}$$

$0 < x < 1$, $P^\mu = P^+ \bar{n}^\mu + \frac{M^2}{2P^+} n^\mu$, Gauge link is not explicitly written

$$n^\mu = \frac{1}{\sqrt{2}}(1,0,0,-1)$$
$$\bar{n}^\mu = \frac{1}{\sqrt{2}}(1,0,0,1)$$
$$g_T^{\mu\nu} = g^{\mu\nu} - \bar{n}^{\{\mu} n^{\nu\}}$$
$$a^{\{\mu} b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$$

Spin tensor

A. Bacchetta, P.J. Mulders, PRD 62 (2000) 114004

$$T^{\mu\nu} = \frac{1}{2}\left[\frac{4}{3} S_{LL} \frac{(P^+)^2}{M_D^2} \bar{n}^\mu \bar{n}^\nu - \frac{2}{3} S_{LL} \left(\bar{n}^{\{\mu} n^{\nu\}} - g_T^{\mu\nu}\right) + \frac{1}{3} S_{LL} \frac{M_D^2}{(P^+)^2} n^\mu n^\nu + \frac{P^+}{M_D} \bar{n}^{\{\mu} S_{LT}^{\nu\}} - \frac{M_D}{2P^+} n^{\{\mu} S_{LT}^{\nu\}} + S_{TT}^{\mu\nu}\right]$$

Tensor-polarization parameters: $S_{LL}, S_{LT}^\mu, S_{TT}^{\mu\nu}$

$n \cdot S_{LT} = \bar{n} \cdot S_{LT} = 0$, $n_\mu S_{TT}^{\mu\nu} = \bar{n}_\mu S_{TT}^{\mu\nu} = 0$, $S_{TT}^{\mu\nu} = S_{TT}^{\nu\mu}$, $g_{T\,\mu\nu} S_{TT}^{\mu\nu} = 0$

Collinear PDFs

$$\Phi(x,P,T) = \frac{1}{2}\left[S_{LL}(\bar{n}\cdot\gamma) f_{1LL}(x) + \frac{M_D}{P^+} S_{LL} 1 e_{LL}(x) + \frac{M_D}{P^+}(S_{LT}\cdot\gamma) f_{LT}(x) + \frac{M_D^2}{(P^+)^2} S_{LL}(n\cdot\gamma) f_{3LL}(x)\right]$$

Twist-2: f_{1LL} **Twist-3:** e_{LL}, f_{LT} **Twist-4:** f_{3LL}

e_{LL} is chiral-odd

Twist-2 Relations

Twist-3 functions = Twist-2 contributions + Pure twist-3 functions

e.g., Vector-polarized SF

$$g_2 = g_2^{\mathrm{WW}} + g_2^{(\mathrm{HT})}$$

Wandzura-Wilczek (WW) relation

S. Wandzura, F. Wilczek, PLB 72 (1977) 195

$$g_2^{\mathrm{WW}}(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$$

WW approximation: $g_2 = g_2^{\mathrm{WW}} + g_2^{(\mathrm{HT})} \approx g_2^{\mathrm{WW}}$

Burkhardt-Cottingham (BC) sum rule

H. Burkhardt, W.N. Cottingham, Annals. Phys. 56 (1970) 453

$$\int_0^1 dx\, g_2(x) = 0$$

WW approximation ⇒ BC Sum rule ✓

$$\int_0^1 dx\, g_2^{\mathrm{WW}}(x) = 0$$

Similar relations exist for tensor-polarized PDFs f_{1LL} and f_{LT}

Twist-2 Relations

S. Kumano, K. Kuroki, arXiv:2605.00430

The n-th moment of a non-local matrix element

$$\int_{-1}^{1} dx x^{n-1} \int \frac{d(P\cdot\xi)}{4\pi} e^{ixP\cdot\xi} \langle P, T | \bar{\psi}(0)\gamma^{\sigma}\psi(\xi) | P, T\rangle_{\substack{\xi^+=0\\ \vec{\xi}_T=0}} \cong S_{LL}\bar{n}^{\sigma} \int_{-1}^{1} dx x^{n-1} f_{1LL}(x) + \frac{M_D}{P\cdot n} S_{LT}^{\sigma} \int_{-1}^{1} dx x^{n-1} f_{LT}(x)$$

LHS can also be written as

$$\int_{-1}^{1} dx x^{n-1} \int \frac{d(P\cdot\xi)}{4\pi} e^{ixP\cdot\xi} \langle P, T | \bar{\psi}(0)\gamma^{\sigma}\psi(\xi) | P, T\rangle_{\substack{\xi^+=0\\ \vec{\xi}_T=0}} = \cdots = \frac{n_{\mu_1}\cdots n_{\mu_{n-1}}}{2(P\cdot n)^n} \langle P, T | R^{\sigma\{\mu_1\cdots\mu_{n-1}\}} | P, T\rangle$$

$$\psi(\xi) = \sum_{m=0}^{\infty} \frac{1}{m!} (\xi\cdot\partial)^m \psi(\xi)_{\xi=0}$$

$$\partial^{\mu_1}\cdots\partial^{\mu_{n-1}} \sum_{m=0}^{\infty} \frac{1}{m!} (\xi\cdot\partial)^m = \partial^{\{\mu_1}\cdots\partial^{\mu_{n-1}\}} \left(1 + \sum_{m=1}^{\infty} \frac{1}{m!} (\xi\cdot\partial)^m\right)$$

$$a^{\{\mu_1}\cdots a^{\mu_{n-1}\}} = \frac{1}{n!}(a^{\mu_1}\cdots a^{\mu_{n-1}} + \text{permutations})$$

Local operator

$$R^{\sigma\{\mu_1\cdots\mu_{n-1}\}} = i^{n-1}\bar{\psi}(0)\gamma^{\sigma} D^{\{\mu_1}\cdots D^{\mu_{n-1}\}}\psi(0) - \text{traces}$$
$$= R^{\{\sigma\mu_1\cdots\mu_{n-1}\}} + R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}$$

$R^{\{\sigma\mu_1\cdots\mu_{n-1}\}}$: twist-2

$R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}$: twist-3

traces : twist ≥ 4

Twist-2 Relations

S. Kumano, K. Kuroki, arXiv:2605.00430

Expand matrix elements in terms of momentum P^μ and spin tensor $T^{\mu\nu}$

$$\langle P,T|R^{\{\sigma\mu_1\cdots\mu_{n-1}\}}|P,T\rangle = \frac{2}{n}a_n M^2\left[\sum_{i=1}^{n-1}(T^{\sigma\mu_i}+T^{\mu_i\sigma})\prod_{j(\neq i)=1}^{n-1}P^{\mu_j}+\sum_{i=1}^{n-1}\sum_{j(\neq i)=1}^{n-1}T^{\mu_i\mu_j}P^\sigma\prod_{k(\neq i,j)=1}^{n-1}P^{\mu_k}\right]$$

$$\langle P,T|R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}|P,T\rangle = \frac{2}{n}d_n M^2\left[\sum_{i=1}^{n-1}\sum_{j(\neq i)=1}^{n-1}(P^\sigma T^{\mu_i\mu_j}-P^{\mu_i}T^{\sigma\mu_j})\prod_{k(\neq i,j)=1}^{n-1}P^{\mu_k}\right]$$

a_n and d_n are overall coefficients

LHS becomes

$$\frac{n_{\mu_1}\cdots n_{\mu_{n-1}}}{2(P\cdot n)^n}\langle P,T|R^{\sigma\{\mu_1\cdots\mu_{n-1}\}}|P,T\rangle \cong S_{LL}\bar{n}^\sigma\left[\frac{2(n-1)}{3}a_n\right]+\frac{M_D}{P\cdot n}S_{LT}^\sigma\left[\frac{n-1}{n}a_n-\frac{(n-1)(n-2)}{2n}d_n\right]$$

Moments of PDFs

$$\int_{-1}^{1}dx\,x^{n-1}f_{1LL}(x)=\int_0^1 dx\,x^{n-1}f_{1LL}^{\pm}(x)=\frac{2(n-1)}{3}a_n$$

$$\int_{-1}^{1}dx\,x^{n-1}f_{LT}(x)=\int_0^1 dx\,x^{n-1}f_{LT}^{\pm}(x)=\frac{n-1}{n}a_n-\frac{(n-1)(n-2)}{2n}d_n$$

$f^\pm(x)=f(x)\pm\bar{f}(x)$

$+$ $(-)$ for

n = even (odd) integer

Twist-3 f_{LT} contains twist-2 contribution a_n

Twist-2 Relations

S. Kumano, K. Kuroki, arXiv:2605.00430

$$f_{LT}^{\pm}(x) = f_{LT}^{\pm\,\text{twist-2}}(x) + f_{LT}^{\pm\,(\text{HT})}(x)$$

Twist-2 WW part — Pure twist-3 part

$$\int_0^1 dx\, x^{n-1}\, f_{LT}^{\pm(\text{HT})}(x) = -\frac{(n-1)(n-2)}{2n}\, d_n$$

WW-like twist-2 relation

S. Kumano, Q.-T. Song, JHEP 09 (2021) 141

$$f_{LT}^{\pm\,\text{twist-2}}(x) = \frac{3}{2}\int_x^1 \frac{dy}{y}\, f_{1LL}^{\pm}(y)$$

By defining $f_{2LT} \coloneqq \frac{2}{3} f_{LT} - f_{1LL}$,

$$f_{2LT}^{\pm\,\text{twist-2}}(x) = -f_{1LL}^{\pm}(x) + \int_x^1 \frac{dy}{y}\, f_{1LL}^{\pm}(y)$$

Multiparton Collinear CF

$$(\Phi_G^{\alpha})_{ij}(x_1, x_2) = \int \frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi}\, e^{ix_1 P^+ \xi_1^-} e^{i(x_1-x_2)P^+\xi_2^-} \times \left\langle P, T \left| \overline{\psi}_j(0)\, g G^{+\alpha}(\xi_2^-)\, \psi_i(\xi_1^-) \right| P, T \right\rangle$$

$$\Phi_G^{\alpha}(x_1, x_2) = \frac{M}{2}\Big[i S_{LT}^{\alpha} F_{G,LT}(x_1, x_2) - \epsilon_T^{\alpha\mu} S_{LT\,\mu}\gamma_5 G_{G,LT}(x_1, x_2) + i S_{LL}\gamma_T^{\alpha} H_{G,LL}^{\perp}(x_1, x_2) + i S_{TT}^{\alpha\mu}\gamma_\mu H_{G,TT}(x_1, x_2) \Big]\bar{n}\cdot\gamma$$

$$f_{LT}^{(\text{HT})}(x) = -\mathcal{P}\int_{-1}^{1} dy\, \frac{1}{x-y}\Big[\frac{\partial}{\partial x}\{F_{G,LT}(x,y) + G_{G,LT}(x,y)\} + \frac{\partial}{\partial x}\{F_{G,LT}(y,x) + G_{G,LT}(y,x)\}\Big]$$

Twist-2 Relations

S. Kumano, K. Kuroki, arXiv:2605.00430

By integrating WW-like relation over x,

BC-like sum rule

S. Kumano, Q.-T. Song, JHEP 09 (2021) 141

$$\int_0^1 dx\, f_{2LT}^{\pm\,\text{twist}-2}(x) = 0$$

$$\int_0^1 dx\, f_{2LT}^{\pm}(x) = \int_0^1 dx\, f_{2LT}^{\pm\,\text{twist}-2}(x) + \int_0^1 dx\, f_{2LT}^{(\text{HT})}(x) = \frac{3}{2}\int_0^1 dx\, f_{LT}^{(\text{HT})}(x)$$

Experimental sum verification

↓

Pure twist-3 $f_{LT}^{(\text{HT})}$

Numerical Results

- Phenomenological Twist-2 Distribution
- Twist-2 PDF f_{1LL}
- Twist-3 PDF f_{LT}
- Twist-3 Effects on SFs

Phenomenological Twist-2 Distribution

Parton model

$$b_1^D = \frac{1}{2}\sum_q e_q^2[\delta_T q^D + \delta_T \bar{q}^D]$$

$$\delta_T q^D = q^{D0} - \frac{q^{D+} + q^{D-}}{2}$$

$q^{D\lambda}$: Unpolarized quark distribution inside the spin-λ deuteron

Parametrization S. Kumano, PRD 82 (2010) 017501

Assumption: $q = u, d, s$, flavor-independent polarization

$$\delta_T q_v^D(x) = \delta_T w(x)\, q_v^D(x)$$
$$\delta_T \bar{q}^D(x) = \alpha_{\bar{q}}\, \delta_T w(x)\, \bar{q}^D(x)$$
$$\delta_T w(x) = a x^b (1-x)^c (x_0 - x)$$

$q_v^D, \bar{q}^D$: Unpolarized distribution in nucleon

A.D. Martin et al., EPJ C 63 (2009) 189

Parameters

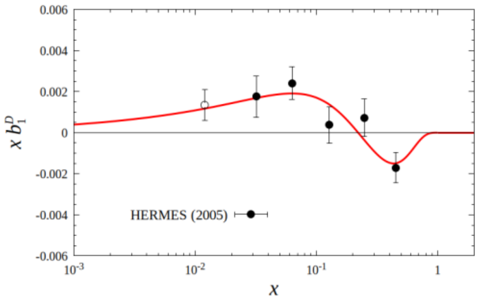
a, b

$c = 1$ (fixed)

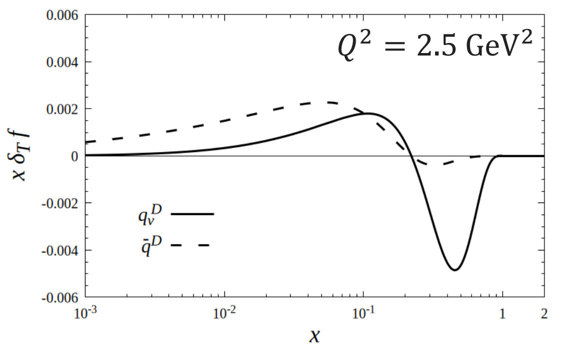
$x_0, \alpha_{\bar{q}}$

HERMES b_1

$\chi^2/\mathrm{d.o.f.} = 1.57$



$a = 0.221$
$b = 0.648$
$x_0 = 0.221$
$\alpha_{\bar{q}} = 3.20$

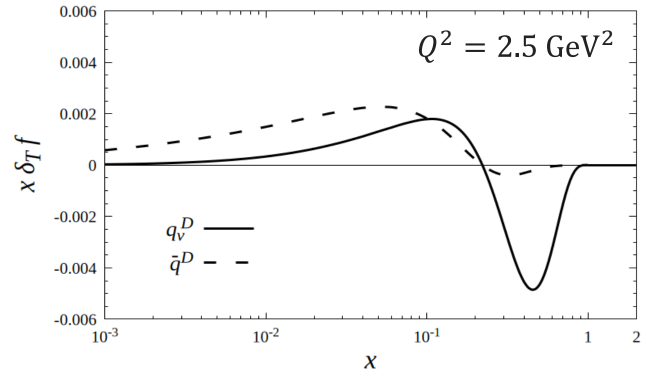


- Positive at $x \lesssim 0.22$
  Negative at $x \gtrsim 0.22$
- Finite $\delta_T \bar{q}^D$

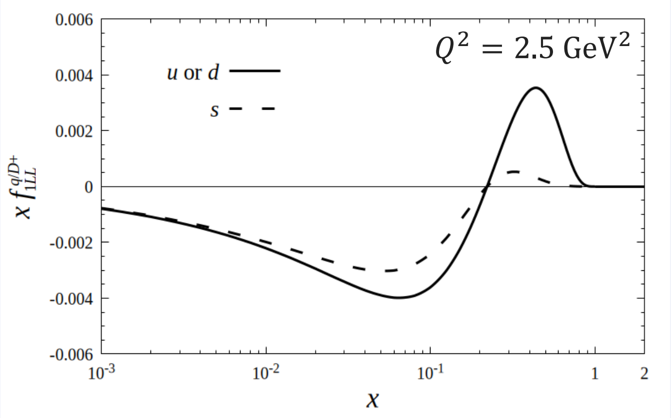
Twist-2 PDF f_{1LL}

S. Kumano, K. Kuroki, PLB 875 (2026) 140348

$$f_{1LL}^{q/D+}(x) = f_{1LL}^{q/D}(x) + f_{1LL}^{\bar{q}/D+}(x)$$
$$= -\frac{2}{3}[\delta_T q^D(x) + \delta_T \bar{q}^D(x)]$$
$$= -\frac{2}{3}[\delta_T q_v^D(x) + 2\delta_T \bar{q}^D(x)]$$



Twist-2 f_{1LL} of deuteron



- Negative at $x \lesssim 0.22$
  Positive at $x \gtrsim 0.22$
- Magnitude $\sim \mathcal{O}(10^{-3})$

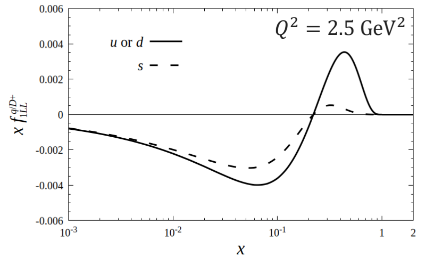
WW-like twist-2 relation to calculate twist-3 PDF f_{LT}

Twist-3 PDF f_{LT}

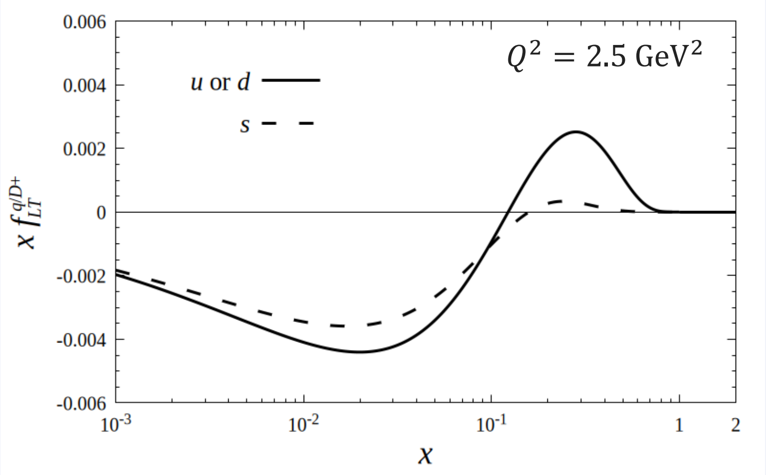
S. Kumano, K. Kuroki, PLB 875 (2026) 140348

Twist-2 relation (WW approximation)

$$f_{LT}^{q/D+}(x) \cong f_{LT}^{q/D+\,\text{twist}-2}(x) = \frac{3}{2}\int_x^2 \frac{dy}{y} f_{1LL}^{q/D+}(y)$$



Twist-3 f_{LT} of deuteron

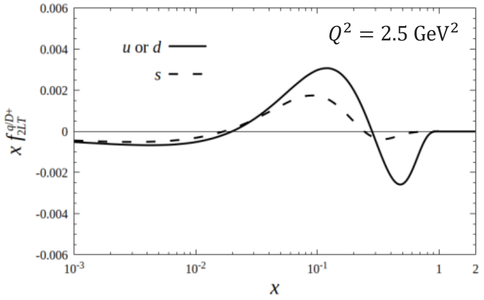


- Negative at $x \lesssim 0.1$
  Positive at $x \gtrsim 0.1$
- Magnitude $\sim \mathcal{O}(10^{-3})$
  About the same as twist-2 f_{1LL}

Sum rule

$$f_{2LT}^{q/D+}(x) = \frac{2}{3} f_{LT}^{q/D+}(x) - f_{1LL}^{q/D+}(x)$$

$$\int dx\, f_{2LT}^{q/D+}(x) = 0$$



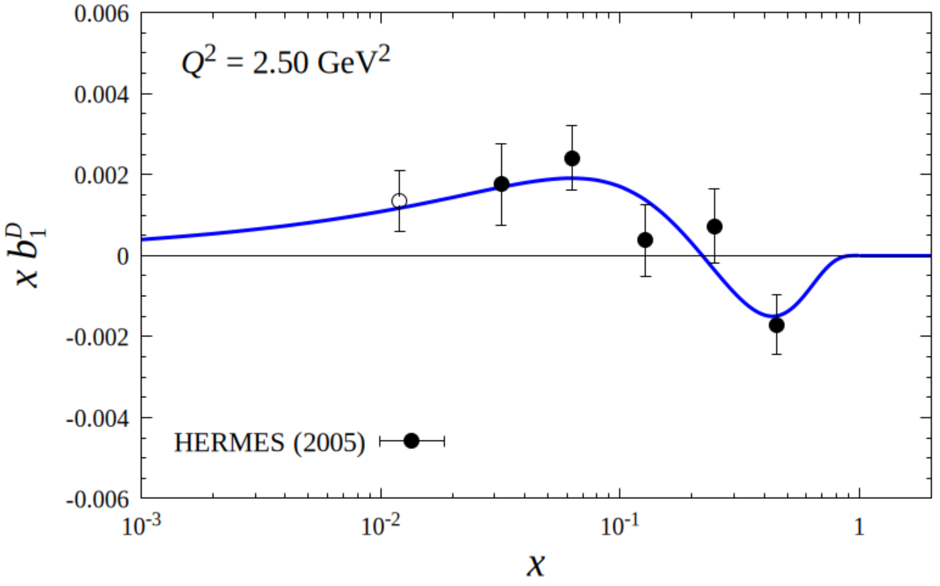
Twist-3 Effects on SFs

SFs in terms of PDFs up to twist-3

J. Zhao, A. Bacchetta, S. Kumano, T. Liu, Y.-J. Zhou, JHEP 12 (2025) 067

$$b_1 = -\frac{3}{4}\frac{1}{\gamma^2+1}\sum_q e_q^2 f_{1LL}^q \qquad \gamma = \frac{2M_D x}{Q}$$

$$b_2 = -\frac{3}{2}\frac{x}{(\gamma^2+1)^2}\sum_q e_q^2 f_{1LL}^q + 8\frac{\gamma x}{(\gamma^2+1)^2}\frac{M_D}{Q}\sum_q e_q^2 f_{LT}^q$$

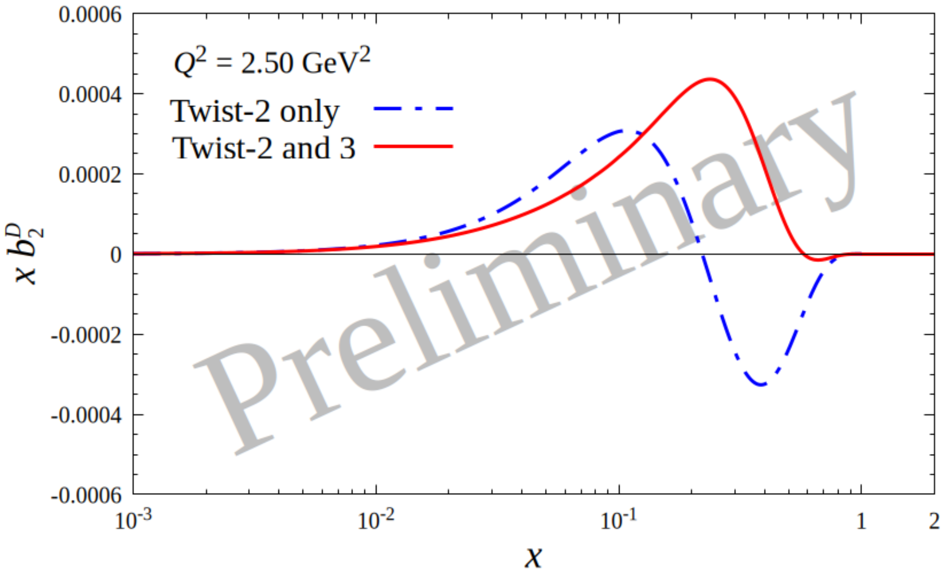


- Magnitudes: $b_1 \gg b_2$

  Callan-Gross-like relation in Bjorken limit

  $$2xb_1(x) = b_2(x)$$

- **Significant twist-3 contribution to b_2**



SFs of Other Convention

W. Cosyn, C. Weiss, arXiv:2603.23699

$$F^{\text{azimuthal}}_{\text{beam(target),photon}}$$

Inclusive SFs

$$F_{U(LL),T},\, F_{U(LL),L},\, F^{\cos\phi_{LT}}_{U(LT)},\, F^{\cos 2\phi_{TT}}_{U(TT)}$$

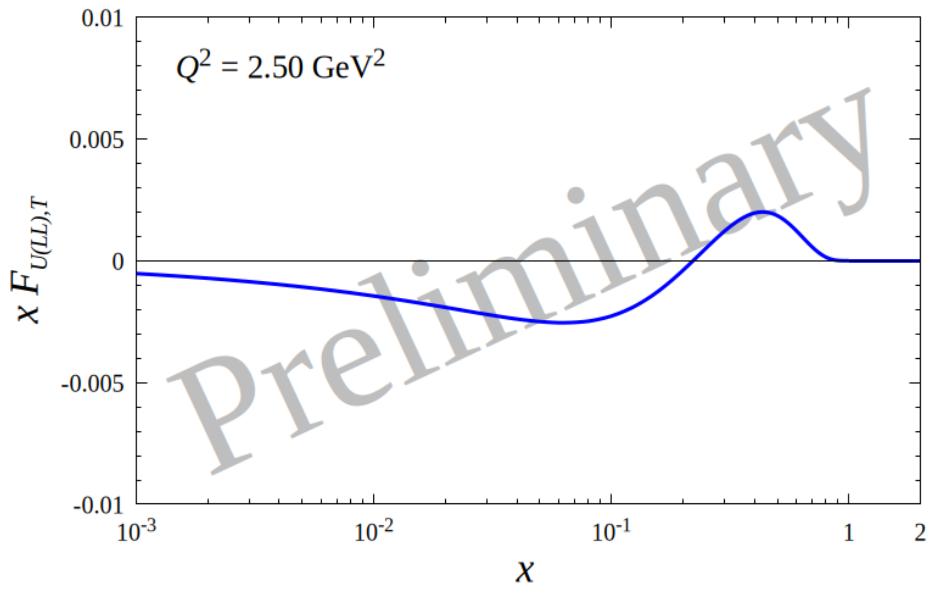
SFs in terms of PDFs up to twist-3

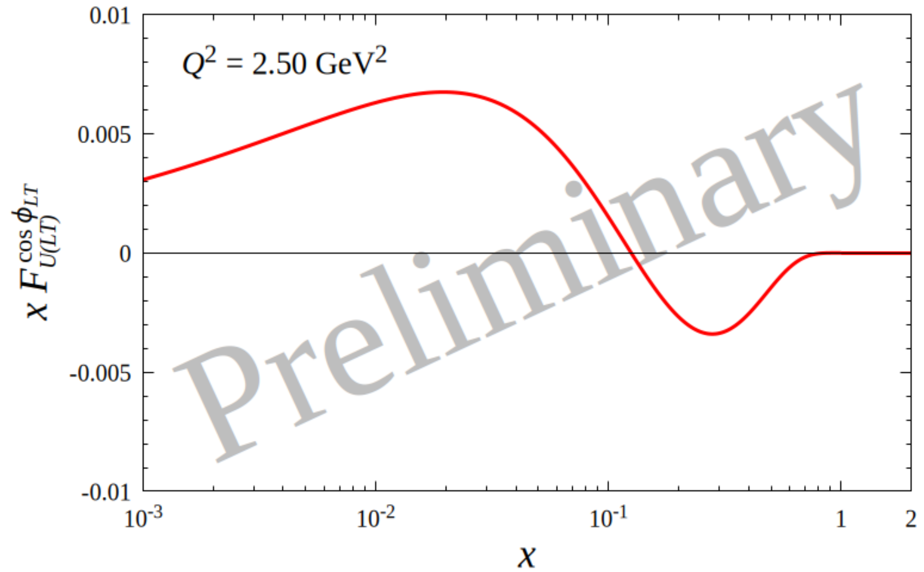
J. Zhao, A. Bacchetta, S. Kumano, T. Liu, Y.-J. Zhou, JHEP 12 (2025) 067

$$F_{U(LL),T} = \sum_q e_q^2 f_{1LL}^q \qquad F_{U(LL),L} = 0$$

$$F^{\cos\phi_{LT}}_{U(LT)} = \frac{2M_D}{Q} \sum_q e_q^2 f_{LT}^q \qquad F^{\cos 2\phi_{TT}}_{U(TT)} = 0$$

■ Magnitudes: $F_{U(LL),T} \approx F^{\cos\phi_{LT}}_{U(LT)}$





Summary

Numerical estimate of twist-3 tensor-polarization

Phenomenological twist-2 f_{1LL} ← HERMES b_1 data

WW-like twist-2 relation

Twist-3 f_{LT}

Magnitude of twist-3 f_{LT} is the same as twist-2 f_{1LL}

→ Significant contribution to SF b_2

Also accessible through

- SIDIS J. Zhao, A. Bacchetta, S. Kumano, T. Liu, Y.-J. Zhou, JHEP 12 (2025) 067
- Drell-Yan S.-Y. Qiao, Q.-T. Song, PRD 111 (2025) 054026

Backup

Twist-3 Contribution in SIDIS

J. Zhao, A. Bacchetta, S. Kumano, T. Liu, Y.-J. Zhou, JHEP 12 (2025) 067

$P_{h\perp}$-integrated SIDIS

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi dz} = \frac{2\alpha^2}{x_d y Q^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x_d}\right) \Big\{ S_{LL} \left(F_{U(LL),T} + \varepsilon F_{U(LL),L} \right)$$
$$+ |S_{LT}| \left(\sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_{LT} F_{U(LT)}^{\cos\phi_{LT}} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_{LT} F_{L(LT)}^{\sin\phi_{LT}} \right) + |S_{TT}| \varepsilon \cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})} \Big\}$$

$$F_{U(LL),T}(x_d, z) = x \sum_a e_a^2 f_{1LL}^a(x) D_1^a(z),$$

$$F_{U(LL),L}(x_d, z) = 0,$$

$$F_{U(LT)}^{\cos\phi_{LT}}(x_d, z) = -x \sum_a e_a^2 \frac{2M}{Q} f_{LT}^a(x) D_1^a(z),$$

$$F_{U(TT)}^{\cos(2\phi_{TT})}(x_d, z) = 0,$$

Twist-3 Contribution in Drell-Yan

S.-Y. Qiao, Q.-T. Song, PRD 111 (2025) 054026

$$\frac{d\hat{\sigma}}{dQ^2 d\Omega} = \sum_q \frac{\alpha^2 e_q^2}{4N_c Q^2} \int dx dy \delta(xyS - Q^2) \Big\{ S_{LL} [f_{1LL}^q(x) f_1^{\bar{q}}(y) + (q \leftrightarrow \bar{q})](1 + \cos^2\theta)$$
$$+ |S_{LT}| \frac{M}{Q} [(2x f_{LT}^q(x) - f_{1LT}^{(1)q}(x)) f_1^{\bar{q}}(y) + (q \leftrightarrow \bar{q})] \sin(2\theta) \cos\hat{\phi} \Big\},$$

$$l_1^\mu = \frac{Q}{2}(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$S_{LT}^\mu = |S_{LT}|(0, \cos\phi_s, \sin\phi_s, 0)$$

$$\hat{\phi} = \phi - \phi_s$$

$$\int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, T | \bar{\psi}_j(0) i\partial^\alpha \psi_i(\xi^-) | P, T \rangle$$
$$= \frac{M}{2} \{ i h_1^{\perp(1)}(x) \gamma_T^\alpha \not{n} + [f_{1LT}^{(1)}(x) S_{LT}^\alpha \not{n} + g_{1LT}^{(1)}(x) \tilde{S}_{LT}^\alpha \gamma_5 \not{n} - h_{1LL}^{\perp(1)}(x) S_{LL} \sigma^{\alpha\mu} \bar{n}_\mu + h_{1TT}^{\prime(1)}(x) S_{TT}^{\alpha\beta} \sigma_{\beta\mu} \bar{n}^\mu]\}$$

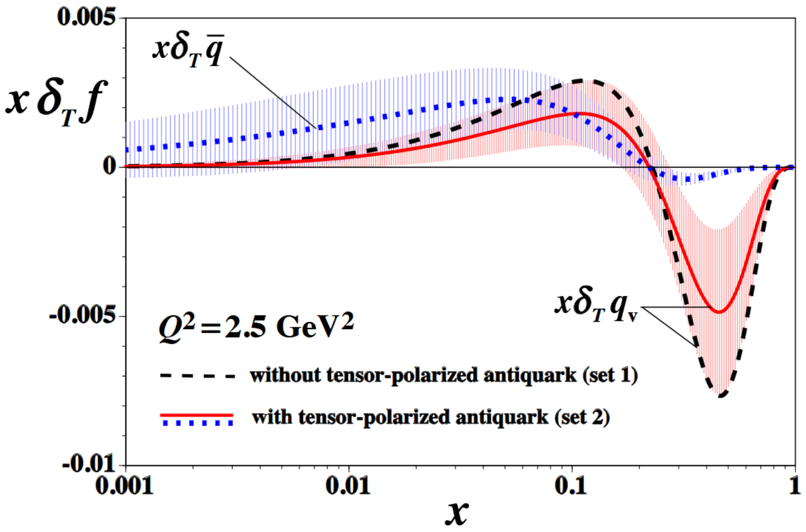
$$f^{(1)}(x) = \int d^2 k_T \frac{\vec{k}_T^2}{2M^2} f(x, k_T^2)$$

QCD Evolution

P. Hoodbhoy, R. L. Jaffe, A. Manohar, NPB 312 (1989) 571

the same as for electroproduction off nucleons. In particular, the anomalous dimensions of the operators $O_{V,A}$ and the coefficient functions $C_n^{(1,2,3)}$ are the standard ones. Thus the moments of F_1 and F_2 and g_1 have the same anomalous dimensions as for a spin-1/2 target. Since b_1, b_2 are obtained from the same operator tower as F_1, F_2, they obey the same scaling equations as F_1 and F_2. The anomalous dimensions of the moments of g_2 are determined by axial vector operators of twist-3, and of those of b_3 and b_4 by vector operators of twist-4.

S. Kumano, Q.-T. Song, PRD 94 (2016) 054022



DGLAP evolution