Spin-1 SIDIS & TMDs Theory

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Outline

- Introduction
- Kinematics and spin states
- Cross section in terms of structure functions
- Structure functions in terms of partonic functions
- Experimental feasibility and physical insights
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Introduction

- **Spin-1/2:** $\boldsymbol{\rho} : \mathbf{2} \times \mathbf{2}$ $\quad \boldsymbol{\rho} = \frac{1}{2}(\mathbf{1} + S^i \boldsymbol{\sigma}^i)$

  S^i : spin vector $\quad \boldsymbol{S} = (S_T^x, S_T^y, S_L)$ —3 independent components

- **Spin-1:** $\boldsymbol{\rho} : \mathbf{3} \times \mathbf{3}$ $\quad \boldsymbol{\rho} = \frac{1}{3}\left(\mathbf{1} + \frac{3}{2} S^i \boldsymbol{\Sigma}^i + 3T^{ij} \boldsymbol{\Sigma}^{ij}\right)$

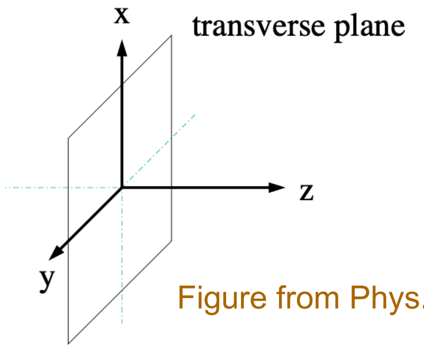
  T^{ij} : symmetric traceless spin tensor

$$T^{ij} = \frac{1}{2}\begin{pmatrix} S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & -2S_{LL} \end{pmatrix}$$

S_T^x, S_T^y, S_L **3**

$S_{LL}, S_{LT}^x, S_{LT}^y, S_{TT}^{xx}, S_{TT}^{xy}$ **5**

8 independent components



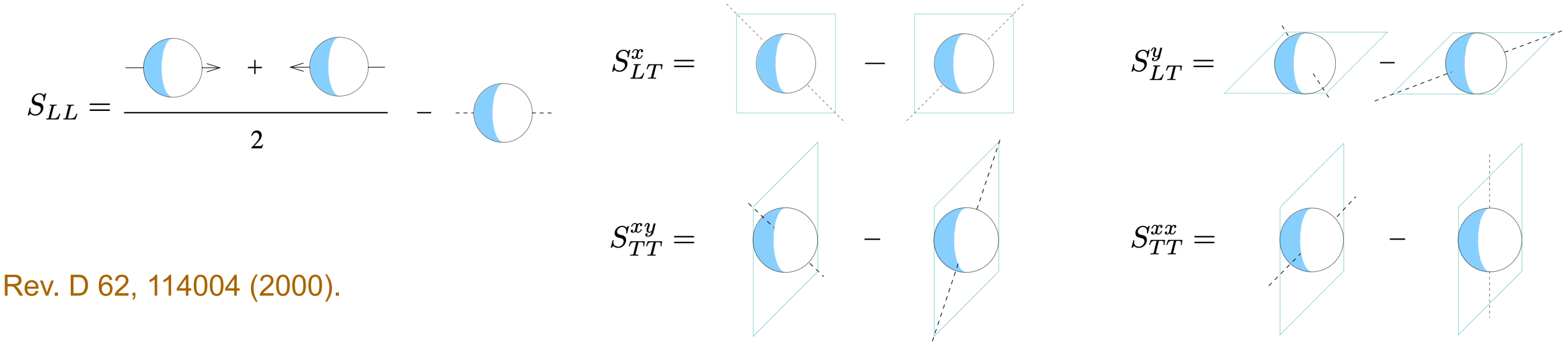


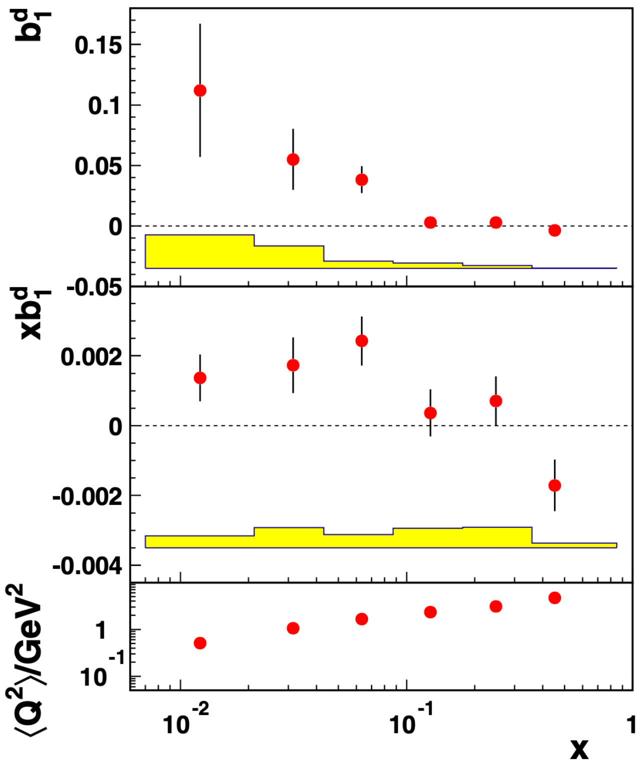
Figure from Phys. Rev. D 62, 114004 (2000).

Spin-1 hadrons offer unique access to explore novel effects beyond spin-1/2 targets.

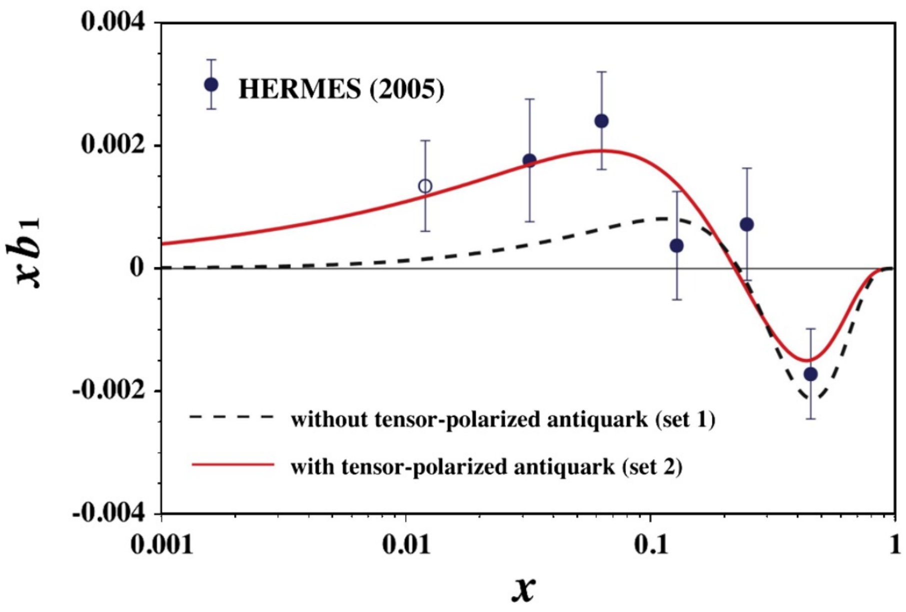
Introduction

■ Experiments with deuteron

- Deuteron is the simplest stable spin-1 nucleus (a weakly bound system of a proton and a neutron).
- HERMES experiment: First measurement of tensor polarized structure function b_1



HERMES collaboration, Phys. Rev. Lett. 95 242001 (2005).



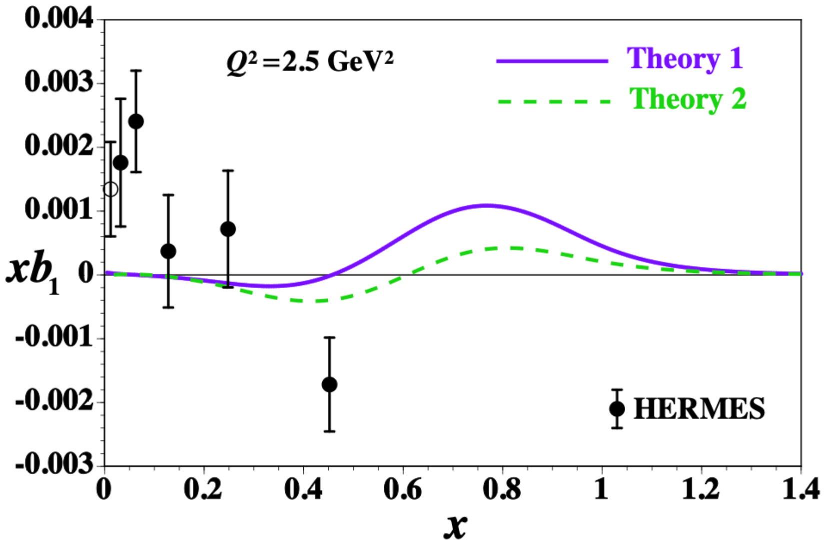
See S. Kumano's talk

S. Kumano, Phys. Rev. D 82 (2010) 017501

Large uncertainties require further experimental improvements.

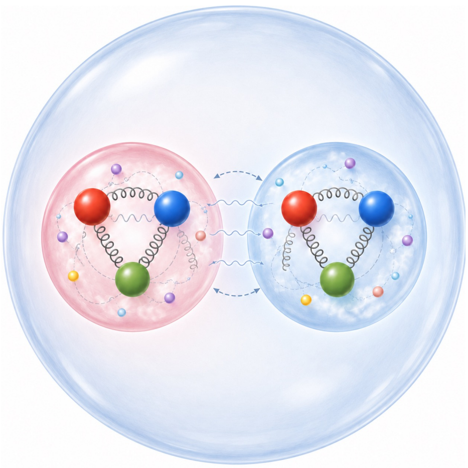
Introduction

- Model results compared with HERMES data



W. Cosyn, Y-B. Dong, S. Kumano, and, M. Sargsian, Phys. Rev. D 95, 074036 (2017)

See S. Kumano's talk



?

Possible mechanism:
Hidden color state, six quark system, …

G. A. Miller, Phys. Rev. C 89 (2014) 045203

Understanding the tensor structure of spin-1 deuteron is required for learning partonic structure of light nuclei.

Introduction

- **Upcoming experiments: JLab, Fermilab**

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-40

E12-13-011

Spin 1 Transverse Momentum Dependent Tensor Structure Functions in CLAS12

Jiwan Poudel, Alessandro Bacchetta, Jian-Ping Chen, Dustin Keller, Ishara Fernando et al. (Feb 27, 2025)

e-Print: 2502.20044 [hep-ph]

Experimental study of tensor structure function of deuteron

Jiwan Poudel (Jefferson Lab), Alessandro Bacchetta (INFN, Pavia), Jian-Ping Chen (Jefferson Lab), Nathaly Santiesteban (New Hampshire U.) (Apr 18, 2025)

Published in: *Eur.Phys.J.A* 61 (2025) 4, 81 • e-Print: 2506.04506 [nucl-ex]

Enhanced Tensor Polarization in Solid-State Targets

Dustin Keller, Don Crabb, Donal Day (Aug 21, 2020)

Published in: *Nucl.Instrum.Meth.A* 981 (2020) 164504 • e-Print: 2008.09515 [physics.ins-det]

The Transverse Structure of the Deuteron with Drell-Yan

SpinQuest Collaboration • D. Keller (Virginia U.) for the collaboration. (May 2, 2022)

e-Print: 2205.01249 [nucl-ex]

Inclusive DIS and SIDIS

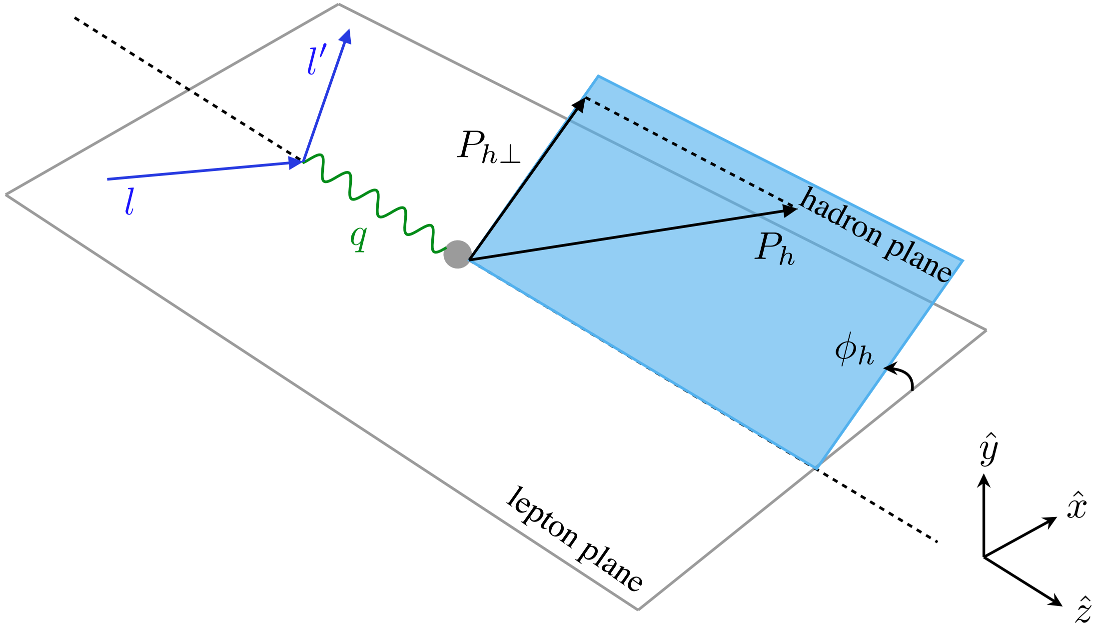
Proton-deuteron Drell-Yan

tensor polarized collinear PDFs and TMD PDFs

Corresponding theoretical study is required!

Kinematics and spin states

Trento conventions



Virtual photon-target frame

$$\ell(l) + d(P) \rightarrow \ell(l') + h(P_h) + X(P_X)$$

Spin-1 target, e.g. deuteron

Several commonly used dimensionless variables:

$$x_d = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad z = \frac{P\cdot P_h}{P\cdot q}, \quad \gamma = \frac{2Mx_d}{Q}$$

x_d is scaling variable for the deuteron. $x_B = 2x_d$

$$\cos\phi_h = -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \qquad \sin\phi_h = -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}$$

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P\cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{P\cdot q\sqrt{1+\gamma^2}}$$

Kinematics and spin states

Basis vectors: $\hat{t}^\mu = \frac{2x_d}{\sqrt{1+\gamma^2}Q}\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right), \quad \hat{x}^\mu = \frac{l_\perp^\mu}{|\boldsymbol{l}_\perp|}, \quad \hat{y}^\mu = \epsilon_\perp^{\mu\nu}\hat{x}_\nu \quad \hat{z}^\mu = -\frac{q^\mu}{Q}$

Spin vector:

$$S^\mu = S_L\frac{P^\mu - q^\mu M^2/(P\cdot q)}{M\sqrt{1+\gamma^2}} + S_T^\mu$$

$$S_L = \frac{S\cdot q}{P\cdot q}\frac{M}{\sqrt{1+\gamma^2}} \qquad S_T^\mu = g_\perp^{\mu\nu}S_\nu$$

Spin tensor:

$$T^{\mu\nu} = \frac{1}{2}\left\{\frac{2}{3}S_{LL}\left(\frac{2(\gamma^2 q^\mu - 2x_d P^\mu)(\gamma^2 q^\nu - 2x_d P^\nu)}{\gamma^2(\gamma^2+1)Q^2} + g_\perp^{\mu\nu}\right) + \frac{2x_d P^{\{\mu}S_{LT}^{\nu\}} - \gamma^2 q^{\{\mu}S_{LT}^{\nu\}}}{\gamma\sqrt{\gamma^2+1}Q} + S_{TT}^{\mu\nu}\right\}$$

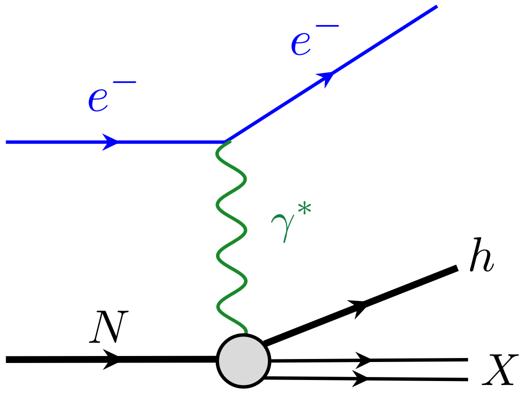
$$\begin{aligned} S_{LL} =& \frac{3}{2}T^{\mu\nu}\hat{L}_\mu\hat{L}_\nu, \\ S_{LT}^\mu =& -2T_{\alpha\beta}\hat{L}^\alpha g_\perp^{\mu\beta}, \\ S_{TT}^{\mu\nu} =& 2T_{\alpha\beta}g_\perp^{\alpha\mu}g_\perp^{\beta\nu} - T_{\alpha\beta}\hat{L}^\alpha\hat{L}^\beta g_\perp^{\mu\nu} \end{aligned}$$

$$\cos\phi_{LT} = -\frac{2T^{\mu\nu}\hat{L}_\mu\hat{x}_\nu}{|S_{LT}|}, \quad \sin\phi_{LT} = -\frac{2T^{\mu\nu}\hat{L}_\mu\hat{y}_\nu}{|S_{LT}|},$$

$$\cos 2\phi_{TT} = \frac{2T^{\mu\nu}\hat{x}_\mu\hat{x}_\nu + \frac{2}{3}T^{\mu\nu}\hat{L}_\mu\hat{L}_\nu}{|S_{TT}|}, \quad \sin 2\phi_{TT} = \frac{2T^{\mu\nu}\hat{x}_\mu\hat{y}_\nu}{|S_{TT}|}$$

Cross section in terms of structure functions

Decomposition of the hadronic tensor



$$\frac{d\sigma}{dx_d dy dz d\phi_h d\psi dP_{h\perp}^2} = \frac{\alpha^2 y}{8Q^4 z} L_{\mu\nu} W^{\mu\nu}$$

$$L_{\mu\nu} = 2\left(l_\mu l'_\nu + l_\nu l'_\mu - g_{\mu\nu} l\cdot l' + i\lambda_e \epsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma\right)$$

Polarized lepton beam

$$W^{\mu\nu}(q;P,S,T;P_h) = \sum_X \delta^4(P+q-P_h-P_X) \langle P,S,T|J^\mu(0)|P_X,P_h\rangle \langle P_X,P_h|J^\nu(0)|P,S,T\rangle$$

Hadronic tensor must satisfy:

Hermiticity: $W^{*\mu\nu}(q;P,S,T;P_h) = W^{\nu\mu}(q;P,S,T;P_h)$

Parity invariance : $W^{\mu\nu}(q;P,S,T;P_h) = W_{\mu\nu}\left(q,P,-\bar{S},\bar{T};P_h\right)$

Gauge invariance: $q_\mu W^{\mu\nu}(q;P,S,T;P_h) = W^{\mu\nu}(q;P,S,T;P_h)\, q_\nu = 0$

Cross section in terms of structure functions

Hadronic tensor $W^{\mu\nu}$ → **Basis tensors multiplied by scaler functions.**

9 basic Lorentz tensors:
$$h_U^{S\mu\nu} = \left\{ g_q^{\mu\nu}, P_q^\mu P_q^\nu, P_q^{\{\mu} P_{hq}^{\nu\}}, P_{hq}^\mu P_{hq}^\nu \right\}$$
$$\tilde{h}_U^{S\mu\nu} = \left\{ \epsilon^{\{\mu q P P_h} P_q^{\nu\}}, \epsilon^{\{\mu q P P_h} P_{hq}^{\nu\}} \right\}$$
$$h_U^{A\mu\nu} = \left\{ P_q^{[\mu} P_{hq}^{\nu]} \right\}$$
$$\tilde{h}_U^{A\mu\nu} = \left\{ \epsilon^{\mu\nu q P}, \epsilon^{\mu\nu q P_h} \right\}$$

$$P_q^\mu = P^\mu - \frac{P \cdot q}{q^2} q^\mu$$
$$P_{hq}^\mu = P_h^\mu - \frac{P_h \cdot q}{q^2} q^\mu$$
$$g_q^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$$

Polarized basis tensors = Polarization dependent (pseudo) scalars × *Basic Lorentz tensors*

Symmetric part: $h_T^{S\mu\nu} = \{T^{P_h P_h}, T^{P_h q}, T^{qq}\} h_U^{S\mu\nu}, \left\{\epsilon^{T^{P_h} P P_h q}, \epsilon^{T^q P P_h q}\right\} \tilde{h}_U^{S\mu\nu}$ **16**

Antisymmetric part: $h_T^{A\mu\nu} = \{T^{P_h P_h}, T^{P_h q}, T^{qq}\} h_U^{A\mu\nu}, \left\{\epsilon^{T^{P_h} P P_h q}, \epsilon^{T^q P P_h q}\right\} \tilde{h}_U^{A\mu\nu}$ **7**

23 basis tensors

$$W_T^{\mu\nu} = \sum_{i=1}^{16} V_{T,i}^S h_{T,i}^{S\mu\nu} + i \sum_{i=1}^{7} V_{T,i}^A h_{T,i}^{A\mu\nu}$$

subscript U, T: target polarization
superscript S, A: symmetric, antisymmetric
V_i: structure functions

K. b. Chen, W. h. Yang, S. y. Wei, and Z. t. Liang, Phys. Rev. D 94, 034003 (2016).

J. Zhao, Z. Zhang, Z-t. Liang, T. Liu, and Y-j. Zhou, Phys. Rev. D 109, 074017 (2024).

Cross section in terms of structure functions

General cross section

$$d\sigma = \begin{cases} d\sigma_U & \text{unpolarized} \\ + & \\ d\sigma_V & \text{vector polarized} \\ + & \\ d\sigma_T & \text{tensor polarized} \end{cases}$$

41

18

23

$F_{A(B),C}$: A: lepton beam polarization
B: target polarization
C: virtual photon polarization

$$\frac{d\sigma_{U+V}}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

Unpolarized

$$\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h}$$
$$+\varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}$$

Vector polarized

$$+S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\right]$$
$$+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right]$$
$$+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$
$$+\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$
$$+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$
$$+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$

Cross section in terms of structure functions

- **General cross section**

$$d\sigma = \begin{cases} d\sigma_U & \text{unpolarized} \\ + \\ d\sigma_V & \text{vector polarized} \\ + \\ d\sigma_T & \text{tensor polarized} \end{cases}$$

41

18

23

$F_{A(B),C}$: A: lepton beam polarization
B: target polarization
C: virtual photon polarization

Consistent with the result in
[W. Cosyn and C. Weiss, arXiv:2603.23699].

see C. Weiss's talk

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy dz d\phi_h d\psi dP_{h\perp}^2}$$

S_{LL}

$$= \frac{\alpha^2}{x_d y Q^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x_d}\right) \Big\{ S_{LL} \Big[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{U(LL)}^{\cos\phi_h}$$
$$+ \varepsilon \cos(2\phi_h)\, F_{U(LL)}^{\cos 2\phi_h} + \lambda_e \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, F_{L(LL)}^{\sin\phi_h} \Big]$$

S_{LT}

$$+ |S_{LT}| \Big[\cos(\phi_h - \phi_{LT}) \Big(F_{U(LT),T}^{\cos(\phi_h-\phi_{LT})} + \varepsilon F_{U(LT),L}^{\cos(\phi_h-\phi_{LT})} \Big)$$
$$+ \sqrt{2\varepsilon(1+\varepsilon)} \Big(\cos\phi_{LT} F_{U(LT)}^{\cos\phi_{LT}} + \cos(2\phi_h - \phi_{LT}) F_{U(LT)}^{\cos(2\phi_h-\phi_{LT})} \Big)$$
$$+ \varepsilon \Big(\cos(\phi_h + \phi_{LT}) F_{U(LT)}^{\cos(\phi_h+\phi_{LT})} + \cos(3\phi_h - \phi_{LT}) F_{U(LT)}^{\cos(3\phi_h-\phi_{LT})} \Big)$$
$$+ \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \Big(\sin\phi_{LT} F_{L(LT)}^{\sin\phi_{LT}} + \sin(2\phi_h - \phi_{LT}) F_{L(LT)}^{\sin(2\phi_h-\phi_{LT})} \Big)$$
$$+ \lambda_e \sqrt{1-\varepsilon^2}\, \sin(\phi_h - \phi_{LT}) F_{L(LT)}^{\sin(\phi_h-\phi_{LT})} \Big]$$

S_{TT}

$$+ |S_{TT}| \Big[\cos(2\phi_h - 2\phi_{TT}) \Big(F_{U(TT),T}^{\cos(2\phi_h-2\phi_{TT})} + \varepsilon F_{U(TT),L}^{\cos(2\phi_h-2\phi_{TT})} \Big)$$
$$+ \sqrt{2\varepsilon(1+\varepsilon)} \Big(\cos(\phi_h - 2\phi_{TT}) F_{U(TT)}^{\cos(\phi_h-2\phi_{TT})} + \cos(3\phi_h - 2\phi_{TT}) F_{U(TT)}^{\cos(3\phi_h-2\phi_{TT})} \Big)$$
$$+ \varepsilon \Big(\cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})} + \cos(4\phi_h - 2\phi_{TT}) F_{U(TT)}^{\cos(4\phi_h-2\phi_{TT})} \Big)$$
$$+ \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \Big(\sin(\phi_h - 2\phi_{TT}) F_{L(TT)}^{\sin(\phi_h-2\phi_{TT})} + \sin(3\phi_h - 2\phi_{TT}) F_{L(TT)}^{\sin(3\phi_h-2\phi_{TT})} \Big)$$
$$+ \lambda_e \sqrt{1-\varepsilon^2}\, \sin(2\phi_h - 2\phi_{TT}) F_{L(TT)}^{\sin(2\phi_h-2\phi_{TT})} \Big] \Big\}$$

Cross section in terms of structure functions

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy dz d\phi_h d\psi dP_{h\perp}^2} \xrightarrow{\text{Integrating over } P_{h\perp}} \frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi dz}$$ **5**

$P_{h\perp}$ integrated SIDIS:

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi dz} = \frac{2\alpha^2}{x_d y Q^2} \frac{y^2}{2(1-\varepsilon)} \left(1+\frac{\gamma^2}{2x_d}\right) \Big\{ S_{LL}\left(F_{U(LL),T} + \varepsilon F_{U(LL),L}\right) + |S_{LT}|\left(\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_{LT} F_{U(LT)}^{\cos\phi_{LT}} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_{LT} F_{L(LT)}^{\sin\phi_{LT}}\right) + |S_{TT}|\varepsilon\cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})} \Big\}$$

$$F_{U(LL),T}(x_d, z, Q^2) = \int d^2\boldsymbol{P}_{h\perp} F_{U(LL),T}(x_d, z, P_{h\perp}^2, Q^2)$$

$$\sum_h \int dz z \frac{d\sigma(\ell N \to \ell h X)}{dz dx_d dy d\psi} \longrightarrow \frac{\nu + M}{\nu} \frac{d\sigma(\ell N \to \ell X)}{dx_d dy d\psi}$$ **4**

Inclusive DIS:

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi} = \frac{2\alpha^2}{x_d y Q^2} \frac{y^2}{2(1-\varepsilon)} \Big\{ S_{LL}\left(F_{U(LL),T} + \varepsilon F_{U(LL),L}\right) + |S_{LT}|\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_{LT} F_{U(LT)}^{\cos\phi_{LT}} + |S_{TT}|\varepsilon\cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})} \Big\}$$

$$F_{U(LL),T}(x_d, Q^2) = \sum_h \int dz\, z\, F_{U(LL),T}(x_d, z, Q^2)$$

Cross section in terms of structure functions

The hadronic tensor of inclusive DIS for a spin-1 hadron is expressed as

$$W_{\mu\nu}^{\lambda'\lambda}(P,q) = \frac{1}{4\pi}\int d^4\xi e^{iq\cdot\xi}\langle P,\lambda'|[J_\mu^{em}(\xi),J_\nu^{em}(0)]|P,\lambda\rangle$$

$$= -F_1\hat{g}_{\mu\nu} + \frac{F_2}{M\nu}\hat{P}_\mu\hat{P}_\nu + \frac{ig_1}{\nu}\epsilon_{\mu\nu\lambda\sigma}q^\lambda S^\sigma + \frac{ig_2}{M\nu^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda(P\cdot qS^\sigma - S\cdot qP^\sigma)$$

Spin-1/2

$$- b_1 r_{\mu\nu} + \frac{1}{6}b_2(s_{\mu\nu}+t_{\mu\nu}+u_{\mu\nu}) + \frac{1}{2}b_3(s_{\mu\nu}-u_{\mu\nu}) + \frac{1}{2}b_4(s_{\mu\nu}-t_{\mu\nu}),$$

Tensor polarized

P. Hoodbhoy, R. L. Jaffe, and A. Manohar, Nucl. Phys. B 312, 571 (1989).

The relations between these two sets of structure functions:

$$F_{U(LL),T} = -\left[2(1+\gamma^2)b_1 - \frac{\gamma^2}{x_d}\left(\frac{1}{6}b_2 - \frac{1}{2}b_3\right)\right],$$

$$F_{U(LL),L} = \frac{1}{x_d}\left[2(1+\gamma^2)x_d b_1 - (1+\gamma^2)^2\left(\frac{1}{3}b_2 + b_3 + b_4\right) - (1+\gamma^2)\left(\frac{1}{3}b_2 - b_4\right) - \left(\frac{1}{3}b_2 - b_3\right)\right],$$

$$F_{U(LT)}^{\cos\phi_{LT}} = -\frac{\gamma}{2x_d}\left[(1+\gamma^2)\left(\frac{1}{3}b_2 - b_4\right) + \left(\frac{2}{3}b_2 - 2b_3\right)\right],$$

$$F_{U(TT)}^{\cos(2\phi_{TT})} = -\frac{\gamma^2}{2x_d}\left(\frac{1}{6}b_2 - \frac{1}{2}b_3\right).$$

W. Cosyn, B. R. Tomei, A. Sosa, and A. Zec, Eur. Phys. J. A 61, 83 (2025).

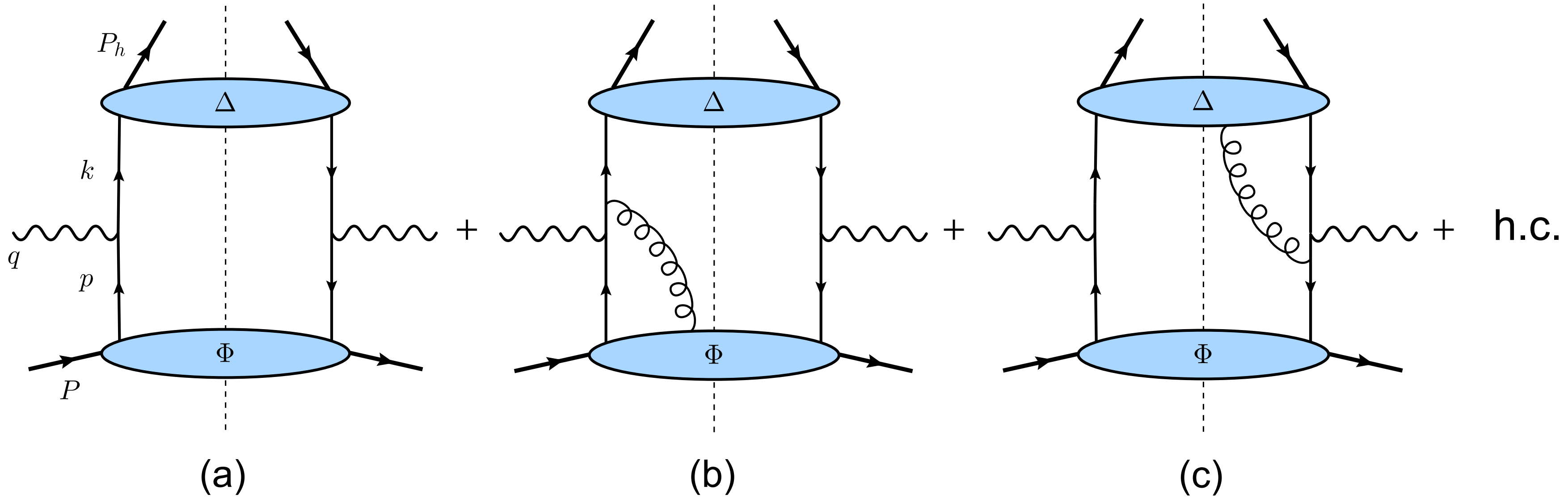
Structure functions in terms of partonic functions

■ Hadronic tensor up to subleading order in $1/Q$

$$W^{\mu\nu} = 2z\sum_a e_a^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T)\mathrm{Tr}\Big\{\Phi^a(x,p_T)\gamma^\mu\Delta^a(z,k_T)\gamma^\nu - \frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha \not{n}_+ \gamma^\nu \tilde{\Phi}^a_{A\alpha}(x,p_T)\gamma^\mu\Delta^a(z,k_T) + \gamma^\alpha \not{n}_- \gamma^\mu \tilde{\Delta}^a_{A\alpha}(z,k_T)\gamma^\nu\Phi^a(x,p_T) + \mathrm{h.c.}\Big]\Big\}$$

Φ, Δ : quark-quark correlators

$\tilde{\Phi}_{A\alpha}, \tilde{\Delta}_{A\alpha}$: quark-gluon-quark correlators



Structure functions in terms of partonic functions

Quark-quark correlators

quark-quark distribution correlator: $$\Phi_{ij}(x,p_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0)\mathcal{U}^c_{(0,\xi)}\psi_i(\xi)|P\rangle \Big|_{\xi^+=0}$$

$$\Phi(x,p_T;T) = \frac{1}{2}\Big\{ f_{1LL}S_{LL}\not{n}_+ - f_{1LT}\frac{S_{LT}\cdot p_T}{M}\not{n}_+ + f_{1TT}\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\not{n}_+ + \Big(g_{1LT}\frac{\epsilon_T^{\mu\nu}S_{LT\mu}p_{T\nu}}{M}\gamma_5\not{n}_+\Big) - \Big(g_{1TT}\frac{\epsilon_T^{\mu\nu}S_{TT\nu\rho}p_T^\rho p_{T\mu}}{M^2}\gamma_5\not{n}_+\Big) + \Big(ih_{1LL}^\perp S_{LL}\frac{[\not{p}_T,\not{n}_+]}{2M}\Big) + \Big(ih'_{1LT}\frac{[\not{S}_{LT},\not{n}_+]}{2}\Big) - \Big(ih_{1LT}^\perp\frac{S_{LT}\cdot p_T}{M}\frac{[\not{p}_T,\not{n}_+]}{2M}\Big) - \Big(h'_{1TT}\frac{\sigma^{\mu\nu}S_{TT\mu\rho}p_T^\rho n_{+\nu}}{M}\Big) + \Big(ih_{1TT}^\perp\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\frac{[\not{p}_T,\not{n}_+]}{2M}\Big)\Big\}$$

$$+ \frac{M}{2P^+}\Big\{ e_{LL}S_{LL} - e_{LT}^\perp\frac{S_{LT}\cdot p_T}{M} + e_{TT}^\perp\frac{p_T\cdot S_{TT}\cdot p_T}{M^2} + f_{LL}^\perp S_{LL}\frac{\not{p}_T}{M} + f'_{LT}\not{S}_{LT} - f_{LT}^\perp\frac{\not{p}_T S_{LT}\cdot p_T}{M^2} - f'_{TT}\frac{S_{TT}^{\mu\nu}\gamma_\mu p_{T\nu}}{M} + f_{TT}^\perp\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\frac{\not{p}_T}{M} - ie_{LT}\frac{S_{LT\mu}\epsilon_T^{\mu\nu}p_{T\nu}}{M}\gamma_5 + ie_{TT}\frac{S_{TT\mu\rho}p_T^\rho\epsilon_T^{\mu\nu}p_{T\nu}}{M^2}\gamma_5 - \Big(g_{LL}^\perp S_{LL}\gamma_5\frac{\varepsilon_T^{\mu\nu}\gamma_\mu p_{T\nu}}{M}\Big) - \Big(g'_{LT}\gamma_5\epsilon_T^{\mu\nu}\gamma_\mu S_{LT\nu}\Big) + \Big(g_{LT}^\perp\gamma_5\frac{\varepsilon_T^{\mu\nu}\gamma_\mu p_{T\nu}S_{LT}\cdot p_T}{M^2}\Big) + \Big(g'_{TT}\gamma_5\frac{\varepsilon_T^{\mu\nu}\gamma_\mu S_{TT\nu\alpha}p_T^\alpha}{M}\Big) - \Big(g_{TT}^\perp\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\gamma_5\frac{\varepsilon_T^{\mu\nu}\gamma_\mu p_{T\nu}}{M}\Big) - \Big(ih_{LL}S_{LL}\frac{[\not{n}_-,\not{n}_+]}{2}\Big) - \Big(ih_{LT}\frac{S_{LT}\cdot p_T}{M}\frac{[\not{n}_-,\not{n}_+]}{2}\Big) + \Big(ih_{LT}^\perp\frac{[\not{S}_{LT},\not{p}_T]}{2M}\Big) + \Big(ih_{TT}\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\frac{[\not{n}_-,\not{n}_+]}{2}\Big) - \Big(ih_{TT}^\perp\frac{[S_{TT}^{\mu\nu}\gamma_\mu p_{T\nu},\not{p}_T]}{2M^2}\Big)\Big\}$$

10 twist-2

A. Bacchetta and P. J. Mulders, Phys. Rev. D 62, 114004 (2000).

20 twist-3

S. Kumano and Q. T. Song, Phys. Rev. D 103, 014025 (2021).

Structure functions in terms of partonic functions

Spin-1 TMD PDFs

twist-2

Hadron Polarization	Quark Polarization: Unpolarized	Longitudinally Polarized	Transversely Polarized
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$
LL	f_{1LL}		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	$h_{1LT},\ h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	$h_{1TT},\ h_{1TT}^\perp$

A. Bacchetta and P. J. Mulders, Phys. Rev. D 62, 114004 (2000).

See S. Kumano's talk

twist-3

Hadron Polarization	Quark Polarization: Unpolarized	Longitudinally Polarized	Transversely Polarized
U	$f^\perp,\ e$	$g^\perp$	h
L	$f_L^\perp,\ e_L$	$g_L^\perp$	h_L
T	$f_T,\ f_T^\perp,\ e_T,\ e_T^\perp$	$g_T,\ g_T^\perp$	$h_T,\ h_T^\perp$
LL	$f_{LL}^\perp,\ e_{LL}$	$g_{LL}^\perp$	h_{LL}
LT	$f_{LT},\ f_{LT}^\perp,\ e_{LT},\ e_{LT}^\perp$	$g_{LT},\ g_{LT}^\perp$	$h_{LT},\ h_{LT}^\perp$
TT	$f_{TT},\ f_{TT}^\perp,\ e_{TT},\ e_{TT}^\perp$	$g_{TT},\ g_{TT}^\perp$	$h_{TT},\ h_{TT}^\perp$

twist-4

Hadron Polarization	Quark Polarization: Unpolarized	Longitudinally Polarized	Transversely Polarized
U	f_3		$h_3^\perp$
L		g_{3L}	$h_{3L}^\perp$
T	$f_{3T}^\perp$	g_{3T}	$h_{3T},\ h_{3T}^\perp$
LL	f_{3LL}		$h_{3LL}^\perp$
LT	f_{3LT}	g_{3LT}	$h_{3LT},\ h_{3LT}^\perp$
TT	f_{3TT}	g_{3TT}	$h_{3TT},\ h_{3TT}^\perp$

S. Kumano and Q. T. Song, Phys. Rev. D 103, 014025 (2021).

Structure functions in terms of partonic functions

quark-quark fragmentation correlator:

$$\Delta_{ij}(z,k_T) = \frac{1}{2z}\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3} e^{ik\cdot\xi} \langle 0|\mathcal{U}^{n_+}_{(+\infty,\xi)}\psi_i(\xi)|h,X\rangle\langle h,X|\bar{\psi}_j(0)\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\Big|_{\xi^+=0}$$

TMD PDFs → **TMD FFs**

$$n_+ \to n_-, \quad \epsilon_T \to -\epsilon_T, \quad P^+ \to P_h^-, \quad M \to M_h, \quad x \to 1/z$$

$$f \to D, \quad e \to E, \quad h \to H, \quad g \to G$$

The final-state hadron is unpolarized

$$\Delta(z,k_T) = \frac{1}{2}\left\{D_1 \not{n}_- + iH_1^\perp \frac{[\not{k}_T, \not{n}_-]}{2M_h}\right\}$$

twist-2 2

$$+\frac{M_h}{2P_h^-}\left\{E + D^\perp \frac{\not{k}_T}{2M_h} + iH\frac{[\not{n}_-, \not{n}_+]}{2} + G^\perp \gamma_5 \frac{\epsilon_T^{\rho\sigma}\gamma_\rho k_{T\sigma}}{M_h}\right\}$$

twist-3 4

Structure functions in terms of partonic functions

Quark-gluon-quark correlators

quark-gluon-quark correlator:

$$\left(\Phi_D^\mu\right)_{ij}(x,p_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P|\bar\psi_j(0)\mathcal{U}^{n_-}_{(0,\xi)} iD^\mu(\xi)\psi_i(\xi)|P\rangle\Big|_{\xi^+=0} \qquad iD^\mu(\xi) = i\partial^\mu + gA^\mu$$

plus-component: $\Phi_D^+(x,p_T) = xP^+\Phi(x,p_T)$

transverse component: $\tilde\Phi_A^\alpha(x,p_T) = \Phi_D^\alpha(x,p_T) - p_T^\alpha\Phi(x,p_T)$

Under the constraint of parity conservation

$$\begin{aligned}
\tilde\Phi_A^\alpha(x,p_T;T) = \frac{xM}{2}\Bigg\{\Bigg[&(\tilde f_{LL}^\perp - i\tilde g_{LL}^\perp)S_{LL}\frac{p_{T\rho}}{M} + (\tilde f'_{LT} - i\tilde g'_{LT})S_{LT\rho} - (\tilde f'_{TT} - i\tilde g'_{TT})S_{TT\rho\sigma}\frac{p_T^\sigma}{M} \\
&- (\tilde f_{LT}^\perp - i\,\tilde g_{LT}^\perp)\frac{S_{LT}\cdot p_T}{M}\frac{p_{T\rho}}{M} + (\tilde f_{TT}^\perp - i\,\tilde g_{TT}^\perp)\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\frac{p_{T\rho}}{M}\Bigg](g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\gamma_5) \\
&- \left[(\tilde h_{LT}^\perp + i\,\tilde e_{LT}^\perp)\frac{\epsilon_T^{\rho\sigma}S_{LT\rho}p_{T\sigma}}{M} + (\tilde h_{TT}^\perp + i\,\tilde e_{TT}^\perp)\frac{\epsilon_T^{\beta\rho}S_{TT\rho\sigma}p_{T\beta}p_T^\sigma}{M^2}\right]\gamma_T^\alpha\gamma_5 \\
&+ \left[(\tilde h_{LL} + i\tilde e_{LL})S_{LL} - (\tilde h_{LT} - i\tilde e_{LT})\frac{S_{LT}\cdot p_T}{M} - (\tilde h_{TT} - i\tilde e_{TT})\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\right]i\gamma_T^\alpha \\
&+ \ldots (g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\gamma_5)\Bigg\}\frac{\not n_+}{2}
\end{aligned}$$

twist-3

Structure functions in terms of partonic functions

QCD equation of motion: $[i\not{D}(\xi)-m]\psi(\xi)=\left[\gamma^+ iD^-(\xi)+\gamma^- iD^+(\xi)+\gamma_T^\alpha iD_\alpha(\xi)-m\right]\psi(\xi)=0$

$$\mathrm{Tr}\Gamma^+\left[x\gamma^-\Phi_3+\gamma_{T\rho}\tilde{\Phi}^\rho_{A,3}+\frac{\not{p}_T}{M}\Phi_2-\frac{m}{M}\Phi_2\right]=0 \qquad \Gamma^+=\{\gamma^+,\gamma^+\gamma_5,i\sigma^{\alpha+}\gamma_5\}$$

The relations between the twist-2 and twist-3 functions:

T-even

$$\begin{aligned}
xe_{LL}&=x\tilde{e}_{LL}+\frac{m}{M}f_{1LL},\\
xf^\perp_{LL}&=x\tilde{f}^\perp_{LL}+f_{1LL},\\
xe^\perp_{LT}&=x\tilde{e}_{LT}+\frac{m}{M}f_{1LT},\\
xe^\perp_{TT}&=x\tilde{e}_{TT}+\frac{m}{M}f_{1TT},\\
xe_{LT}&=x\tilde{e}^\perp_{LT},\\
xe_{TT}&=x\tilde{e}^\perp_{TT},\\
xf'_{LT}&=x\tilde{f}'_{LT},\\
xf_{LT}&=x\tilde{f}_{LT}-\frac{p_T^2}{2M^2}f_{1LT},\\
xf^\perp_{LT}&=x\tilde{f}^\perp_{LT}+f_{1LT},\\
xf'_{TT}&=x\tilde{f}'_{TT},\\
xf_{TT}&=x\tilde{f}_{TT}-\frac{p_T^2}{2M^2}f_{1TT},\\
xf^\perp_{TT}&=x\tilde{f}^\perp_{TT}+f_{1TT},
\end{aligned}$$

T-odd

$$\begin{aligned}
xg^\perp_{LL}&=x\tilde{g}^\perp_{LL}+\frac{m}{M}h^\perp_{1LL},\\
xg'_{LT}&=x\tilde{g}'_{LT}+\frac{p_T^2}{M^2}g_{1LT}+\frac{m}{M}h_{1LT}+\frac{m}{M}\frac{p_T^2}{2M^2}h^\perp_{1LT},\\
xg^\perp_{LT}&=x\tilde{g}^\perp_{LT}+g_{1LT}+\frac{m}{M}h^\perp_{1LT},\\
xg_{LT}&=x\tilde{g}_{LT}-\frac{p_T^2}{2M^2}g_{1LT}+\frac{m}{M}h_{1LT},\\
xg'_{TT}&=x\tilde{g}'_{TT}-\frac{p_T^2}{M^2}g_{1TT}+\frac{m}{M}h_{1TT}+\frac{m}{M}\frac{p_T^2}{2M^2}h^\perp_{1TT},\\
xg^\perp_{TT}&=x\tilde{g}^\perp_{TT}-g_{1TT}+\frac{m}{M}h^\perp_{1TT},\\
xg_{TT}&=x\tilde{g}_{TT}-\frac{p_T^2}{2M^2}g_{1TT}+\frac{m}{M}h_{1TT},\\
xh_{LL}&=x\tilde{h}_{LL}+\frac{p_T^2}{M^2}h^\perp_{1LL}.\\
xh_{LT}&=x\tilde{h}_{LT}+h_{1LT}-\frac{p_T^2}{2M^2}h^\perp_{1LT},\\
xh^\perp_{LT}&=x\tilde{h}^\perp_{LT}+h_{1LT}+\frac{p_T^2}{2M^2}h^\perp_{1LT}+\frac{m}{M}g_{1LT},\\
xh_{TT}&=x\tilde{h}_{TT}+h_{1TT}-\frac{p_T^2}{2M^2}h^\perp_{1TT},\\
xh^\perp_{TT}&=x\tilde{h}^\perp_{TT}+h_{1TT}+\frac{p_T^2}{2M^2}h^\perp_{1TT}-\frac{m}{M}g_{1TT},
\end{aligned}$$

Each of the TMD PDFs defined by quark-quark correlator have the corresponding relations.

Structure functions in terms of partonic functions

Results for structure functions

Substituting the correlators into hadronic tensor and using the relations from EoM relations,

$$W^{\mu\nu} = 2z\sum_a e_a^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T)\mathrm{Tr}\Big\{\Phi^a(x,p_T)\gamma^\mu\Delta^a(z,k_T)\gamma^\nu - \frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha \not{n}_+\gamma^\nu\tilde{\Phi}^a_{A\alpha}(x,p_T)\gamma^\mu\Delta^a(z,k_T) + \gamma^\alpha\not{n}_-\gamma^\mu\tilde{\Delta}^a_{A\alpha}(z,k_T)\gamma^\nu\Phi^a(x,p_T) + \mathrm{h.c.}\Big]\Big\}$$

Contracting it with the leptonic tensor, the structure functions are expressed as the convolution of TMD PDFs and TMD FFs.

For conciseness, we introduce the transverse momentum convolution notation

$$\mathcal{C}[wfD] = x\sum_a e_a^2 \int d^2\boldsymbol{k}_T d^2\boldsymbol{p}_T \delta^{(2)}\left(\boldsymbol{k}_T - \boldsymbol{p}_T - \boldsymbol{P}_{h\perp}/z\right) w\left(\boldsymbol{k}_T, \boldsymbol{p}_T\right) f^a\left(x, k_T^2\right) D^a\left(z, p_T^2\right)$$

Structure functions in terms of partonic functions

Up to twist-3, we have 21 nonzero structure functions:

$$F_{U(LL),L} = 0,$$

$$F_{U(LL)}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf_{LL}^{\perp}D_1 + \frac{M_h}{M}h_{1LL}^{\perp}\frac{\tilde{H}}{z}\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh_{LL}H_1^{\perp} + \frac{M_h}{M}f_{1LL}\frac{\tilde{D}^{\perp}}{z}\Big)\Big],$$

$$F_{U(LL)}^{\cos 2\phi_h} = \mathcal{C}\Big[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_{1LL}^{\perp}H_1^{\perp}\Big],$$

$$F_{L(LL)}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xe_{LL}H_1^{\perp} + \frac{M_h}{M}f_{1LL}\frac{\tilde{G}^{\perp}}{z}\Big) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xg_{LL}^{\perp}D_1 + \frac{M_h}{M}h_{1LL}^{\perp}\frac{\tilde{E}}{z}\Big)\Big],$$

$$F_{U(LT),T}^{\cos(\phi_h-\phi_{LT})} = \mathcal{C}\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}f_{1LT}D_1\Big],$$

$$F_{U(LT),L}^{\cos(\phi_h-\phi_{LT})} = 0,$$

$$F_{U(LT)}^{\cos\phi_{LT}} = \frac{2M}{Q}\mathcal{C}\Big\{ -\Big[\Big(xf_{LT}D_1 + \frac{M_h}{M}h_{1LT}\frac{\tilde{H}}{z}\Big)\Big] + \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xh_{LT}H_1^{\perp} + \frac{M_h}{M}g_{1LT}\frac{\tilde{G}^{\perp}}{z}\Big) + \Big(xh_{LT}^{\perp}H_1^{\perp} - \frac{M_h}{M}f_{1LT}\frac{\tilde{D}^{\perp}}{z}\Big)\Big]\Big\},$$

⋮

$\hat{\boldsymbol{h}} = \boldsymbol{P}_{h\perp}/|\boldsymbol{P}_{h\perp}|$: the direction of the transverse momentum

- S_{LL}-dependent structure functions have similar expressions to unpolarized structure functions
- **twist-2:** even number of $\phi_h, \phi_{LT}, \phi_{TT}$ **twist-3:** odd number of $\phi_h, \phi_{LT}, \phi_{TT}$
- Nonvanishing structure functions can be used to study the tensor-polarized TMD PDFs

Structure functions in terms of partonic functions

$P_{h\perp}$ integrated SIDIS:

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi dz} = \frac{2\alpha^2}{x_d y Q^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x_d}\right) \Big\{ S_{LL} \Big(F_{U(LL),T} + \varepsilon F_{U(LL),L} \Big) + |S_{LT}| \Big(\sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_{LT} F_{U(LT)}^{\cos\phi_{LT}} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_{LT} F_{L(LT)}^{\sin\phi_{LT}} \Big) + |S_{TT}| \varepsilon \cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})} \Big\}$$

$$F_{U(LL),T}(x_d, z) = x \sum_a e_a^2 f_{1LL}^a(x) D_1^a(z),$$

$$F_{U(LL),L}(x_d, z) = 0,$$

$$F_{U(LT)}^{\cos\phi_{LT}}(x_d, z) = -x \sum_a e_a^2 \frac{2M}{Q} f_{LT}^a(x) D_1^a(z),$$

$$F_{U(TT)}^{\cos(2\phi_{TT})}(x_d, z) = 0.$$

Inclusive DIS:

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi} = \frac{2\alpha^2}{x_d y Q^2} \frac{y^2}{2(1-\varepsilon)} \Big\{ S_{LL} \Big(F_{U(LL),T} + \varepsilon F_{U(LL),L} \Big) + |S_{LT}| \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_{LT} F_{U(LT)}^{\cos\phi_{LT}} + |S_{TT}| \varepsilon \cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})} \Big\}$$

$$F_{U(LL),T}(x_d) = x \sum_a e_a^2 f_{1LL}^a(x),$$ twist-2

$$F_{U(LL),L}(x_d) = 0,$$

$$F_{U(LT)}^{\cos\phi_{LT}}(x_d) = -x \sum_a e_a^2 \frac{2M}{Q} f_{LT}^a(x),$$ twist-3

$$F_{U(TT)}^{\cos(2\phi_{TT})}(x_d) = 0.$$

Extract tensor-polarized collinear PDFs

Experimental feasibility and physical insights

➢ Leading-twist observables are experimentally favorable.

- The leading-twist terms containing D_1 provide clearest access to tensor-polarized TMDs.

$$F_{U(LL),T}=\mathcal{C}[f_{1LL}D_1]$$

$$F_{U(LL)}^{\cos 2\phi_h}=\mathcal{C}\left[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_{1LL}^{\perp}H_1^{\perp}\right]$$

$$F_{U(LT),T}^{\cos(\phi_h-\phi_{LT})}=\mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}f_{1LT}D_1\right]$$

$$F_{U(LT)}^{\cos(\phi_h+\phi_{LT})}=\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}h_{1LT}H_1^{\perp}\right]$$

$$F_{U(LT)}^{\cos(3\phi_h-\phi_{LT})}=\mathcal{C}\left[-\frac{4(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\boldsymbol{p}_T^2-2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)}{2M^2M_h}h_{1LT}^{\perp}H_1^{\perp}\right]$$

$$F_{L(LT)}^{\sin(\phi_h-\phi_{LT})}=\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}g_{1LT}D_1\right]$$

$$F_{U(TT),T}^{\cos(2\phi_h-2\phi_{LT})}=\mathcal{C}\left[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2}{M^2}f_{1TT}D_1\right]$$

$$F_{U(TT)}^{\cos(2\phi_{TT})}=\mathcal{C}\left[\frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_{1TT}H_1^{\perp}\right]$$

$$F_{U(TT)}^{\cos(4\phi_h-2\phi_{TT})}=\mathcal{C}\left[-\left(\frac{4(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\boldsymbol{p}_T^2-8(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^3}{2M^3M_h}+\frac{4(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)\boldsymbol{p}_T^2}{2M^3M_h}\right)h_{1TT}^{\perp}H_1^{\perp}\right]$$

$$F_{L(TT)}^{\sin(2\phi_h-2\phi_{TT})}=\mathcal{C}\left[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2}{M^2}g_{1TT}D_1\right]$$

Experimental feasibility and physical insights

- **The S_{LL} sector is the most experimentally accessible.**

- In inclusive DIS: b_1 is a key measurement of the proposed experiment at JLab.
- In SIDIS： $F_{U(LL),T}$， $F_{U(LL)}^{\cos 2\phi_h}$ can be used to extract the S_{LL}-polarized TMD PDFs f_{1LL} and h_{1LL}, respectively.

$$F_{U(LL),T} = \mathcal{C}[f_{1LL} D_1] \qquad F_{U(LL)}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T)(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T) - \boldsymbol{k}_T \cdot \boldsymbol{p}_T}{M M_h} h_{1LL}^{\perp} H_1^{\perp} \right]$$

- **For the S_{LT} sector, the twist-3 modulation term $F_{U(LT)}^{\cos \phi_{LT}}$ is noteworthy.**

$F_{U(LT)}^{\cos \phi_{LT}}$ contribute in $P_{h\perp}$ integrated SIDIS and inclusive DIS.

$$F_{U(LT)}^{\cos \phi_{LT}}(x_d, z) = -x \sum_a e_a^2 \frac{2M}{Q} f_{LT}^a(x) D_1^a(z) \qquad F_{U(LT)}^{\cos \phi_{LT}}(x_d) = -x \sum_a e_a^2 \frac{2M}{Q} f_{LT}^a(x)$$

Twist-3 distribution function f_{LT} ⟶ See K. Kuroki's talk

- **Model calculations can provide quantitative guidance for future experiments.**

- Convolution model, light-front wave functions, spectator model, ……

Summary

- We express the tensor-polarized part of the SIDIS cross section in terms of 23 structure functions.
- In the parton model, we compute the structure functions up to the subleading power in $1/Q$ (twist-3).

  The 21 structure functions have nontrivial expressions in SIDIS.

  Only two structure functions contribute to the cross section in inclusive DIS.
- Upcoming experiments, such as those at JLab using tensor-polarized deuteron targets, will facilitate measurements, enhancing our understanding of nucleon spin dynamics and potential novel nuclear phenomena.

Thank you!

Back up

Structure functions in terms of partonic functions

QCD equation of motion

$$[i\not{D}(\xi) - m]\psi(\xi) = \left[\gamma^+ iD^-(\xi) + \gamma^- iD^+(\xi) + \gamma_T^\alpha iD_\alpha(\xi) - m\right]\psi(\xi) = 0$$

$$\Phi = \Phi_2 + \frac{M}{P^+}\Phi_3 + \left(\frac{M}{P^+}\right)^2 \Phi_4 \qquad \frac{1}{M}\tilde{\Phi}_A^\alpha = \tilde{\Phi}_{A,3}^\alpha + \frac{M}{P^+}\tilde{\Phi}_{A,4}^\alpha + \left(\frac{M}{P^+}\right)^2 \tilde{\Phi}_{A,5}^\alpha$$

$$\mathcal{P}_+\left[xM\gamma^-\Phi_3 + M\gamma_{T\rho}\,\tilde{\Phi}_{A,3}^\rho + \not{p}_T\Phi_2 - m\Phi_2\right] + \mathcal{P}_+\left[xM\gamma^-\Phi_4 + M\gamma_{T\rho}\tilde{\Phi}_{A,4}^\rho + \not{p}_T\Phi_3 - m\Phi_3\right]\frac{M}{P^+} = 0,$$

where we use the relations $\mathcal{P}_+\Phi_4 = \mathcal{P}_-\Phi_2 = 0$ $\quad \mathcal{P}_+\tilde{\Phi}_{A,5}^\alpha = \mathcal{P}_-\tilde{\Phi}_{A,3}^\alpha = 0$

$$\mathrm{Tr}\Gamma^+\left[x\gamma^-\Phi_3 + \gamma_{T\rho}\tilde{\Phi}_{A,3}^\rho + \frac{\not{p}_T}{M}\Phi_2 - \frac{m}{M}\Phi_2\right] = 0 \qquad \Gamma^+ = \{\gamma^+, \gamma^+\gamma_5, i\sigma^{\alpha+}\gamma_5\}$$

Structure functions in terms of partonic functions

$$\left(\Delta_D^\mu\right)_{ij}(z,k_T) =$$
$$\frac{1}{2z}\sum_X \int \frac{d\xi^+}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle 0|\,\mathcal{U}^{n_+}_{(+\infty,\xi)}\, iD^\mu(\xi)\;\psi_i(\xi)|h,X\rangle\,\langle h,X|\bar{\psi}_j\,(0)\,\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\Big|_{\xi^-=0}$$

$$\tilde{\Delta}_A^\alpha(z,k_T) = \Delta_D^\alpha(z,k_T) - k_T^\alpha \Delta(z,k_T)$$

$$\tilde{\Delta}_A^\alpha(z,k_T) = \frac{M_h}{2z}\left\{\left(\tilde{D}^\perp - i\tilde{G}^\perp\right)\frac{k_{T\rho}}{M_h}\left(g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\gamma_5\right) + \left(\tilde{H} + i\tilde{E}\right)i\gamma_T^\alpha + \ldots\left(g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\gamma_5\right)\right\}\frac{\not{n}_-}{2}.$$

$$\frac{E}{z} = \frac{\tilde{E}}{z} + \frac{m}{M_h}D_1,$$
$$\frac{D^\perp}{z} = \frac{\tilde{D}^\perp}{z} + D_1,$$
$$\frac{G^\perp}{z} = \frac{\tilde{G}^\perp}{z} + \frac{m}{M_h}H_1^\perp,$$
$$\frac{H}{z} = \frac{\tilde{H}}{z} + \frac{k_T^2}{M_h^2}H_1^\perp.$$

Structure functions in terms of partonic functions

21 nonzero structure functions:

$$F_{U(LL),L} = 0,$$

$$F_{U(LL)}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf_{LL}^{\perp}D_1 + \frac{M_h}{M}h_{1LL}^{\perp}\frac{\tilde{H}}{z}\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh_{LL}H_1^{\perp} + \frac{M_h}{M}f_{1LL}\frac{\tilde{D}^{\perp}}{z}\Big)\Big],$$

$$F_{U(LL)}^{\cos 2\phi_h} = \mathcal{C}\Big[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_{1LL}^{\perp}H_1^{\perp}\Big],$$

$$F_{L(LL)}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xe_{LL}H_1^{\perp} + \frac{M_h}{M}f_{1LL}\frac{\tilde{G}^{\perp}}{z}\Big) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xg_{LL}^{\perp}D_1 + \frac{M_h}{M}h_{1LL}^{\perp}\frac{\tilde{E}}{z}\Big)\Big],$$

$$F_{U(LT),T}^{\cos(\phi_h-\phi_{LT})} = \mathcal{C}\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}f_{1LT}D_1\Big],$$

$$F_{U(LT),L}^{\cos(\phi_h-\phi_{LT})} = 0,$$

$$F_{U(LT)}^{\cos\phi_{LT}} = \frac{2M}{Q}\mathcal{C}\Big\{-\Big[\Big(xf_{LT}D_1 + \frac{M_h}{M}h_{1LT}\frac{\tilde{H}}{z}\Big)\Big] + \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xh_{LT}H_1^{\perp} + \frac{M_h}{M}g_{1LT}\frac{\tilde{G}^{\perp}}{z}\Big) + \Big(xh_{LT}^{\perp}H_1^{\perp} - \frac{M_h}{M}f_{1LT}\frac{\tilde{D}^{\perp}}{z}\Big)\Big]\Big\},$$

Structure functions in terms of partonic functions

$$F_{U(LT)}^{\cos(\phi_h+\phi_{LT})} = \mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}h_{1LT}H_1^\perp\Big],$$

$$F_{U(LT)}^{\cos(3\phi_h-\phi_{LT})} = \mathcal{C}\Big[-\frac{4(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\boldsymbol{p}_T^2-2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)}{2M^2M_h}h_{1LT}^\perp H_1^\perp\Big],$$

$$F_{L(LT)}^{\sin\phi_{LT}} = \frac{2M}{Q}\mathcal{C}\Big\{\Big(xg_{LT}D_1+\frac{M_h}{M}h_{1LT}\frac{\tilde{E}}{z}\Big)+\frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xe_{LT}H_1^\perp-\frac{M_h}{M}g_{1LT}\frac{\tilde{D}^\perp}{z}\Big)-\Big(xe_{LT}^\perp H_1^\perp+\frac{M_h}{M}f_{1LT}\frac{\tilde{G}^\perp}{z}\Big)\Big]\Big\},$$

$$F_{L(LT)}^{\sin(2\phi_h-\phi_{LT})} = \frac{2M}{Q}\mathcal{C}\Big\{-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xe_{LT}H_1^\perp+\frac{M_h}{M}f_{1LT}\frac{\tilde{G}^\perp}{z}\Big)+\Big(xe_{LT}^\perp H_1^\perp-\frac{M_h}{M}g_{1LT}\frac{\tilde{D}^\perp}{z}\Big)\Big]\Big\},$$

$$F_{L(LT)}^{\sin(\phi_h-\phi_{LT})} = \mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}g_{1LT}D_1\Big],$$

$$F_{U(TT),T}^{\cos(2\phi_h-2\phi_{LT})} = \mathcal{C}\Big[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2}{M^2}f_{1TT}D_1\Big],$$

$$F_{U(TT),L}^{\cos(2\phi_h-2\phi_{LT})} = 0,$$

$$F_{U(TT)}^{\cos(\phi_h-2\phi_{TT})} = \frac{2M}{Q}\mathcal{C}\Big\{\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf_{TT}D_1-\frac{M_h}{M}h_{1TT}\frac{\tilde{H}}{z}\Big)+\frac{(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\boldsymbol{p}_T^2-2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)}{2M^2M_h}\Big[\Big(xh_{TT}H_1^\perp-\frac{M_h}{M}g_{1TT}\frac{\tilde{G}^\perp}{z}\Big)+\Big(xh_{TT}^\perp H_1^\perp-\frac{M_h}{M}f_{1TT}\frac{\tilde{D}^\perp}{z}\Big)\Big]\Big\},$$

$$F_{U(TT)}^{\cos(3\phi_h-2\phi_{TT})} = \frac{2M}{Q}\mathcal{C}\Big\{\frac{3(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\big(2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2\big)}{2M^3}\Big(xf_{TT}^\perp D_1+\frac{M_h}{M}h_{1TT}^\perp\frac{\tilde{H}^\perp}{z}\Big)$$
$$+\frac{4(\boldsymbol{h}\cdot\boldsymbol{k}_T)(\boldsymbol{h}\cdot\boldsymbol{p}_T)^2-2(\boldsymbol{h}\cdot\boldsymbol{p}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)-(\boldsymbol{h}\cdot\boldsymbol{k}_T)\boldsymbol{p}_T^2}{2M^2M_h}\times\Big[\Big(xh_{TT}^\perp H_1^\perp+\frac{M_h}{M}f_{1TT}\frac{\tilde{D}^\perp}{z}\Big)-\Big(xh_{TT}H_1^\perp+\frac{M_h}{M}g_{1TT}\frac{\tilde{G}^\perp}{z}\Big)\Big]\Big\},$$

Structure functions in terms of partonic functions

$$F_{U(TT)}^{\cos(2\phi_{TT})} = \mathcal{C}\left[\frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_{1TT}H_1^{\perp}\right],$$

$$F_{U(TT)}^{\cos(4\phi_h-2\phi_{TT})} = \mathcal{C}\left[-\left(\frac{4(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\boldsymbol{p}_T^2-8(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^3}{2M^3M_h}+\frac{4(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)\boldsymbol{p}_T^2}{2M^3M_h}\right)h_{1TT}^{\perp}H_1^{\perp}\right],$$

$$F_{L(TT)}^{\sin(\phi_h-2\phi_{TT})} = \frac{2M}{Q}\mathcal{C}\left\{\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg_{TT}D_1+\frac{M_h}{M}h_{1TT}\frac{\tilde{E}}{z}\right)-\frac{(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\boldsymbol{p}_T^2-2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)}{2M^2M_h}\left[\left(xe_{TT}H_1^{\perp}-\frac{M_h}{M}f_{1TT}\frac{\tilde{G}^{\perp}}{z}\right)-\left(xe_{TT}^{\perp}H_1^{\perp}-\frac{M_h}{M}g_{1TT}\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

$$F_{L(TT)}^{\sin(3\phi_h-2\phi_{TT})} = \frac{2M}{Q}\mathcal{C}\left\{-\frac{3(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\big(2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2\big)}{2M^3}\left(xg_{TT}^{\perp}D_1+\frac{M_h}{M}h_{1TT}^{\perp}\frac{\tilde{E}}{z}\right)\right.$$
$$\left.+\frac{4(\boldsymbol{h}\cdot\boldsymbol{k}_T)(\boldsymbol{h}\cdot\boldsymbol{p}_T)^2-2(\boldsymbol{h}\cdot\boldsymbol{p}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)-(\boldsymbol{h}\cdot\boldsymbol{k}_T)\boldsymbol{p}_T^2}{2M^2M_h}\times\left[\left(xe_{TT}H_1^{\perp}+\frac{M_h}{M}f_{1TT}\frac{\tilde{G}^{\perp}}{z}\right)+\left(xe_{TT}^{\perp}H_1^{\perp}+\frac{M_h}{M}g_{1TT}\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

$$F_{L(TT)}^{\sin(2\phi_h-2\phi_{TT})} = \mathcal{C}\left[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2}{M^2}g_{1TT}D_1\right]$$