Theory of spin-1 PDFs and TMDs

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Tensor SIDIS workshop and b1/Azz Collaboration meeting,
JLab, Newport News, Virginia, USA, June 3-5, 2026,
https://indico.jlab.org/event/1070/

Ref. S. Kumano, Euro. Phys. J. A 60 (2024) 205.

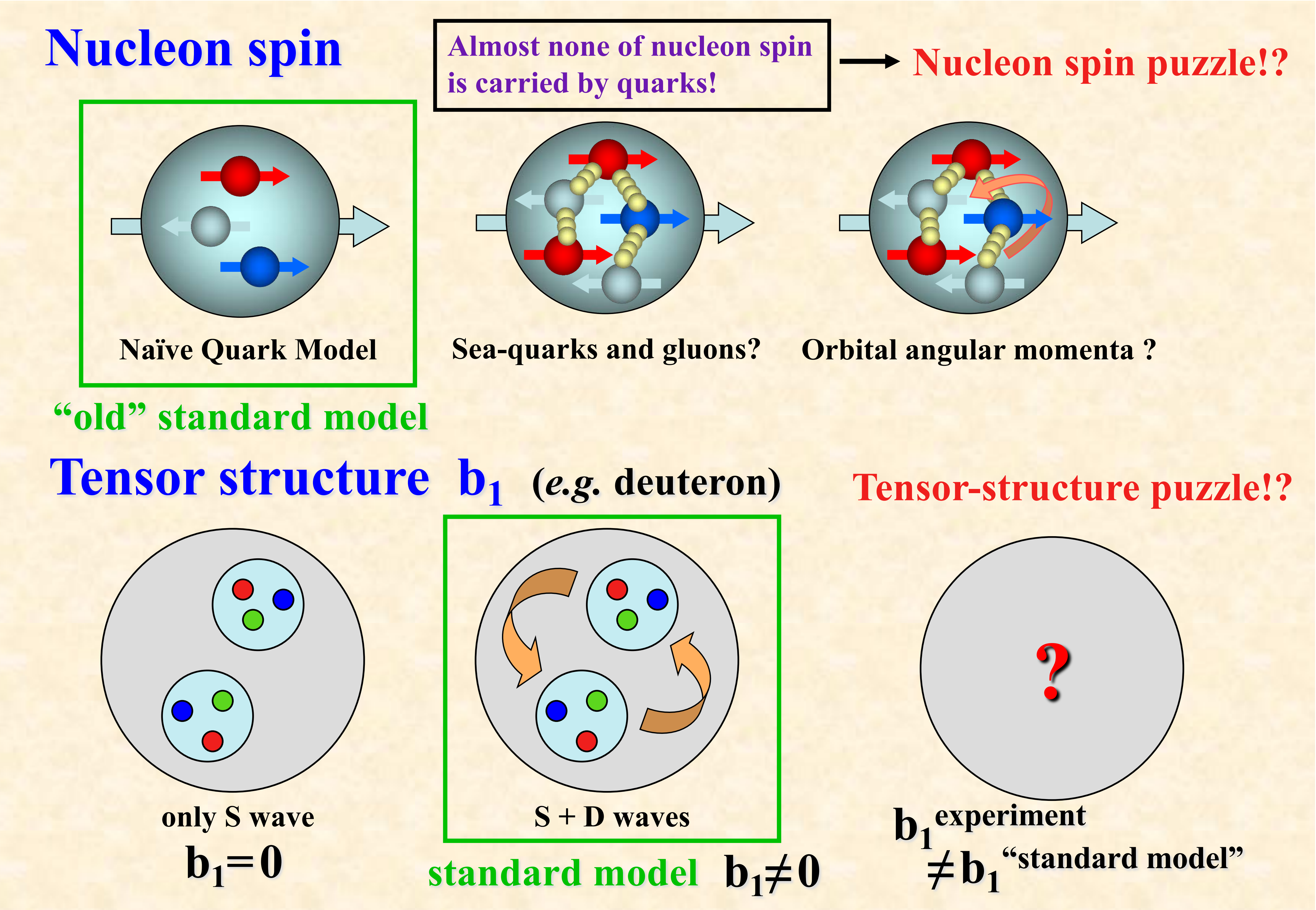
June 4, 2026

Contents

I may skip some slides.

I may skip.

Introduction


Nucleon spin
Almost none of nucleon spin is carried by quarks!
Nucleon spin puzzle!?
Naïve Quark Model
Sea-quarks and gluons?
Orbital angular momenta ?
"old" standard model
Tensor structure b_1 (*e.g.* deuteron)
Tensor-structure puzzle!?
?
only S wave
$b_1 = 0$
S + D waves
standard model $b_1 \neq 0$
$b_1^{\text{experiment}} \neq b_1^{\text{"standard model"}}$

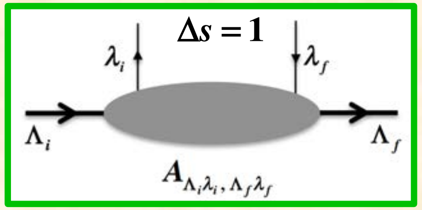
Gluon transversity $\Delta_T g$

Note on our notations:
Tensor-polarized gluon distribution: $\delta_T g$
Gluon transversity: $\Delta_T g$

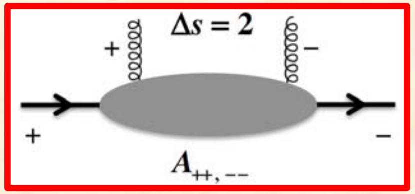
Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, conservation $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2}, +\frac{1}{2}+\frac{1}{2}\right) - A\left(+\frac{1}{2}-\frac{1}{2}, +\frac{1}{2}-\frac{1}{2}\right)$

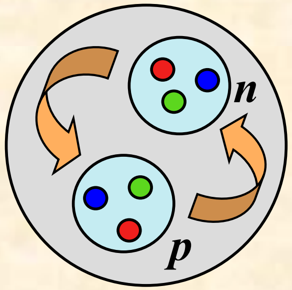
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2}, -\frac{1}{2}-\frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \to \lambda_f = -\frac{1}{2}$ quark spin flip ($\Delta s = 1$)



Gluon transversity in deuteron: $\Delta_T g(x) \sim A(+1+1, -1-1)$, ~~$A\left(+\frac{1}{2}+1, -\frac{1}{2}-1\right)$~~ not possible for nucleon



Note: Gluon transversity does not exist for spin-1/2 nucleons.



S + D waves

$b_1\ (\delta_T q,\ \delta_T g) \neq 0 \iff$ still $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?

Physics beyond "the standard model" in nuclear physics?

(Physics beyond the standard model in particle physics???)

JLab PAC-38 (2011) proposal, PR12-11-110

Full approval in 2023

2011

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-38.
(Update to LOI-11-003)

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2023

PR12-13-011

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-40
(Update to PR12-11-110)

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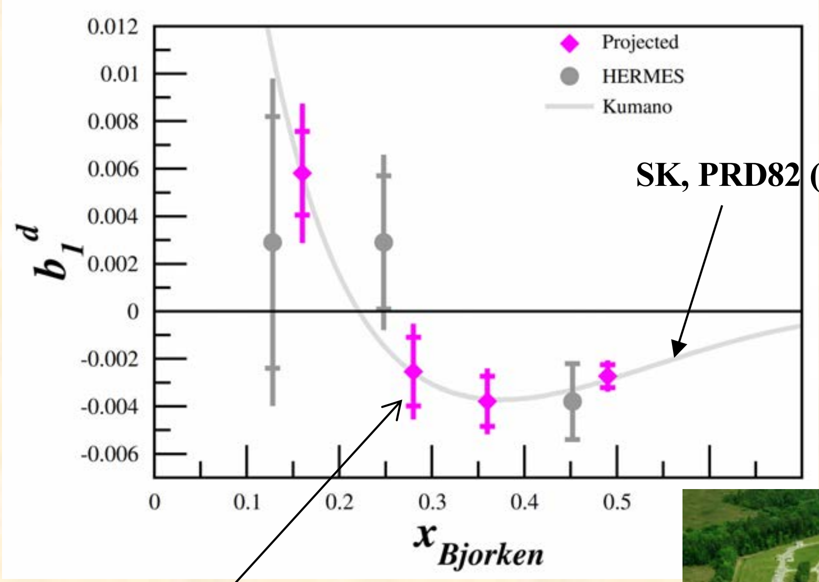
b_1 experiment



SK, PRD82 (2010) 017501

Expected errors by JLab

See J. Poudel, A. Bacchetta, J.-P. Chen, N. Santiesteban, EPJA 61 (2025) 81 for updated information.

Gluon transversity

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

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Eur. Phys. J. A (2025) 61:81
https://doi.org/10.1140/epja/s10050-025-01558-w

THE EUROPEAN PHYSICAL JOURNAL A

Review

Experimental study of tensor structure function of deuteron

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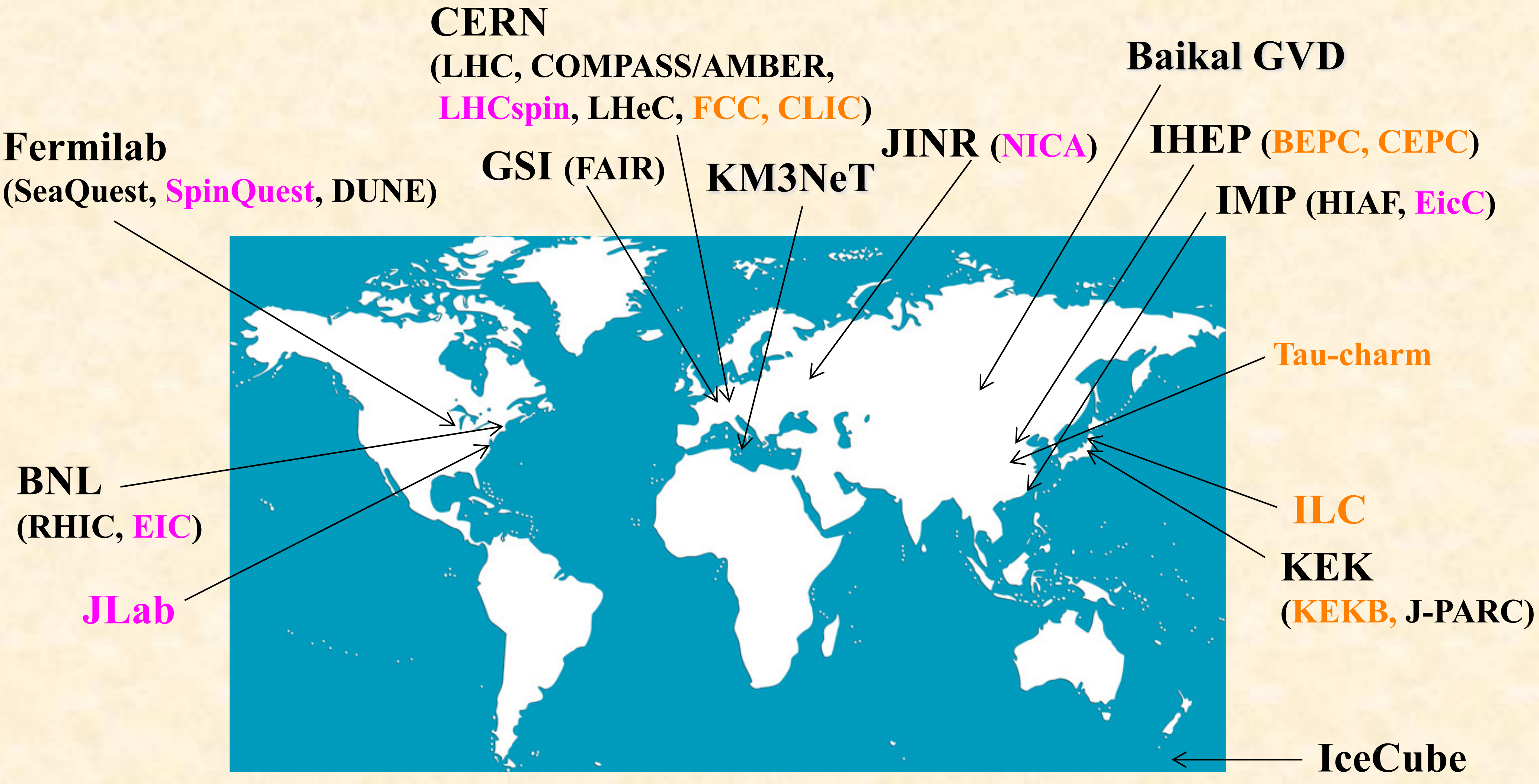
High-energy hadron physics experiments



Facilities on spin-1 hadron structure functions including future possibilities

(time-like processes).

Tensor-polarized structure function b_1

Electron scattering from a spin-1 hadron

P. Hoodbhoy, R. L. Jaffe, and A. Manohar, NP B312 (1989) 571.
[L. L. Frankfurt and M. I. Strikman, NP A405 (1983) 557.]

$$W_{\mu\nu} = \boxed{-F_1 g_{\mu\nu} + F_2 \frac{p_\mu p_\nu}{\nu} + g_1 \frac{i}{\nu} \varepsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + g_2 \frac{i}{\nu^2} \varepsilon_{\mu\nu\lambda\sigma} q^\lambda \left(p \cdot q s^\sigma - s \cdot q p^\sigma\right)}$$

spin-1/2, spin-1

$$\boxed{-b_1 r_{\mu\nu} + \frac{1}{6} b_2 \left(s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}\right) + \frac{1}{2} b_3 \left(s_{\mu\nu} - u_{\mu\nu}\right) + \frac{1}{2} b_4 \left(s_{\mu\nu} - t_{\mu\nu}\right)}$$

spin-1 only

Note: Obvious factors from $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$ are not explicitly written.

E^μ = polarization vector

$$\nu = p \cdot q, \quad \kappa = 1 + M^2 Q^2 / \nu^2, \quad E^2 = -M^2, \quad s^\sigma = -\frac{i}{M^2} \varepsilon^{\sigma\alpha\beta\tau} E^*_\alpha E_\beta p_\tau$$

$$r_{\mu\nu} = \frac{1}{\nu^2}\left(q \cdot E^* q \cdot E - \frac{1}{3}\nu^2 \kappa\right) g_{\mu\nu}, \quad s_{\mu\nu} = \frac{2}{\nu^2}\left(q \cdot E^* q \cdot E - \frac{1}{3}\nu^2 \kappa\right) \frac{p_\mu p_\nu}{\nu}$$

$$t_{\mu\nu} = \frac{1}{2\nu^2}\left(q \cdot E^* p_\mu E_\nu + q \cdot E^* p_\nu E_\mu + q \cdot E p_\mu E^*_\nu + q \cdot E p_\nu E^*_\mu - \frac{4}{3}\nu p_\mu p_\nu\right)$$

$$u_{\mu\nu} = \frac{1}{\nu}\left(E^*_\mu E_\nu + E^*_\nu E_\mu + \frac{2}{3} M^2 g_{\mu\nu} - \frac{2}{3} p_\mu p_\nu\right)$$

$b_1, \cdots, b_4$ tems are defined so that they vanish by spin average.

b_1, b_2 tems are defined to satisfy $2xb_1 = b_2$ in the Bjorken scaling limit.

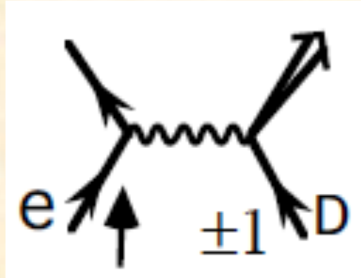
$2xb_1 = b_2$ in the scaling limit $\sim O(1)$

b_3, b_4 = twist-4 $\sim \frac{M^2}{Q^2}$

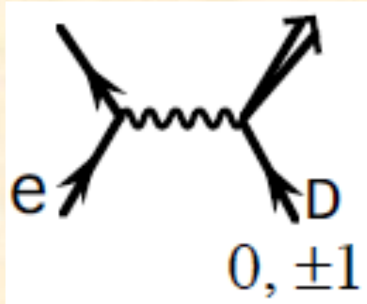
Structure Functions

$$F_1 \propto \langle d\sigma \rangle$$

$$g_1 \propto d\sigma(\uparrow,+1) - d\sigma(\uparrow,-1)$$



$$b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2}$$



note: $\sigma(0) - \frac{\sigma(+1)+\sigma(-1)}{2} = 3\langle\sigma\rangle - \frac{3}{2}\left[\sigma(+1)+\sigma(-1)\right]$

Parton Model

$$F_1 = \frac{1}{2}\sum_i e_i^2\left(q_i + \bar{q}_i\right) \qquad q_i = \frac{1}{3}\left(q_i^{+1} + q_i^{0} + q_i^{-1}\right)$$

$$g_1 = \frac{1}{2}\sum_i e_i^2\left(\Delta q_i + \Delta\bar{q}_i\right) \qquad \Delta q_i = q_{i\uparrow}^{+1} - q_{i\downarrow}^{+1}$$

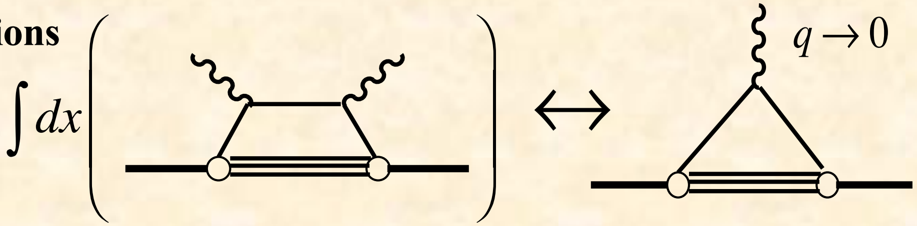
$$\left[q_{\uparrow}^{H}\left(x,Q^2\right)\right]$$

$$b_1 = \frac{1}{2}\sum_i e_i^2\left(\delta_T q_i + \delta_T \bar{q}_i\right) \qquad \delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2}$$

Sum rule for b_1

Constraint on valence-tensor polarization (sum rule)

Folow Feynman's book on Photon-Hadron Interactions



F.E.Close and SK, PRD42, 2377 (1990)

Intuitive derivation without calculation:

$$\int dx\, b_1(x) = \text{ dimensionless quantity}$$
$$= (\text{mass})^2 \cdot (\text{quadrupole moment})$$

$$\int dx\, b_1^D(x) = \frac{5}{36}\int dx\left[\delta_T u_v(x) + \delta_T d_v(x)\right] + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_{i,D}(x)$$

$$b_1 = \frac{1}{2}\sum_i e_i^2\left(\delta_T q_i + \delta_T \bar{q}_i\right)$$

$$\delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2}$$

$$\delta_T q_v \equiv \delta_T q - \delta_T \bar{q}$$

Elastic amplitude in a parton model

$$\Gamma_{H,H} = \langle p,H | J_0(0) | p,H \rangle = \sum_i e_i \int dx\left[q_i^H(x) - \bar{q}_i^H(x)\right]$$

$$\Gamma_{0,0} - \frac{\Gamma_{1,1} + \Gamma_{-1,-1}}{2} = \frac{1}{3}\int dx\left[\delta_T u_v(x) + \delta_T d_v(x)\right]$$

Macroscopically

$$\Gamma_{0,0} = \lim_{t\to 0}\left[F_c(t) - \frac{t}{3M^2}F_Q(t)\right], \quad \Gamma_{+1,+1} = \Gamma_{-1,-1} = \lim_{t\to 0}\left[F_c(t) + \frac{t}{6M^2}F_Q(t)\right]$$

$$\Gamma_{0,0} - \frac{1}{2}\left(\Gamma_{1,1} + \Gamma_{-1,-1}\right) = -\lim_{t\to 0}\frac{t}{2M^2}F_Q(t)$$

$$\int dx\, b_1^D(x) = \frac{5}{36}3\left[\Gamma_{0,0} - \frac{1}{2}\left(\Gamma_{1,1} + \Gamma_{-1,-1}\right)\right] + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_{i,D}(x)$$
$$= -\frac{5}{24}\lim_{t\to 0}\frac{t}{M^2}F_Q(t) + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_{i,D}(x)$$
$$= 0\ (\text{valence}) + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_{i,D}(x)$$

$$\int dx\, b_1^D(x) = -\frac{5}{24}\lim_{t\to 0}\frac{t}{M^2}F_Q(t) + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_{i,D}(x)$$

Constraint on tensor-polarized valence quarks: $\int dx\, \delta_T q_v(x) = 0$

Similarity to the Gottfried sum rule

SK, Phys. Rept. 303 (1998) 183.

may skip

$$S_G = \int_0^1 \frac{dx}{x}\left[F_2^{\mu p}(x) - F_2^{\mu n}(x)\right]$$
$$= \frac{1}{3} + \frac{2}{3}\int_0^1 dx\left[\bar{u}(x) - \bar{d}(x)\right]$$
$$= \frac{1}{3} \quad \text{if } \bar{u} = \bar{d}$$

(Gottfried sum rule)

$$F_2^{\mu p}(x)_{\rm LO} = x\left[\frac{4}{9}\{u(x)+\bar{u}(x)\} + \frac{1}{9}\{d(x)+\bar{d}(x)\} + \frac{1}{9}\{s(x)+\bar{s}(x)\}\right]$$
$$F_2^{\mu n}(x)_{\rm LO} = x\left[\frac{4}{9}\{u(x)+\bar{u}(x)\} + \frac{1}{9}\{d(x)+\bar{d}(x)\} + \frac{1}{9}\{s(x)+\bar{s}(x)\}\right]_n$$
$$= x\left[\frac{4}{9}\{d(x)+\bar{d}(x)\} + \frac{1}{9}\{u(x)+\bar{u}(x)\} + \frac{1}{9}\{s(x)+\bar{s}(x)\}\right]$$
$$\frac{1}{x}\left[F_2^{\mu p}(x)_{\rm LO} - F_2^{\mu n}(x)_{\rm LO}\right] = \frac{3}{9}\{u(x)+\bar{u}(x)\} - \frac{3}{9}\{d(x)+\bar{d}(x)\}$$
$$\int_0^1 \frac{dx}{x}\left[F_2^{\mu p}(x)_{\rm LO} - F_2^{\mu n}(x)_{\rm LO}\right] = \int_0^1 dx\left[\frac{1}{3}\{u_v(x)+2\bar{u}(x)\} - \frac{1}{3}\{d_v(x)+2\bar{d}(x)\}\right]$$
$$= \frac{2}{3} - \frac{1}{3} + \frac{2}{3}\int_0^1 dx\left[\bar{u}(x) - \bar{d}(x)\right]$$

NMC measurement (PRL 66 (1991) 2712; PRD 50 (1994) R1)

$$\int_{0.004}^{0.8} \frac{dx}{x}\left[F_2^{\mu p}(x) - F_2^{\mu n}(x)\right] = 0.221 \pm 0.008 \pm 0.019$$

Extrapolating the NMC data, they obtained

$$S_G = 0.235 \pm 0.026$$

30% is missing! $\Rightarrow$ $\bar{u} < \bar{d}$?

$$\int_0^1 \frac{dx}{x}\left[F_2^p(x) - F_2^n(x)\right] = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\left[\bar{u}(x) - \bar{d}(x)\right]$$
$$\int dx\, b_1^D(x) = \lim_{t\to 0} -\frac{5}{24}\frac{t}{M^2}F_Q(t) + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_i(x)$$

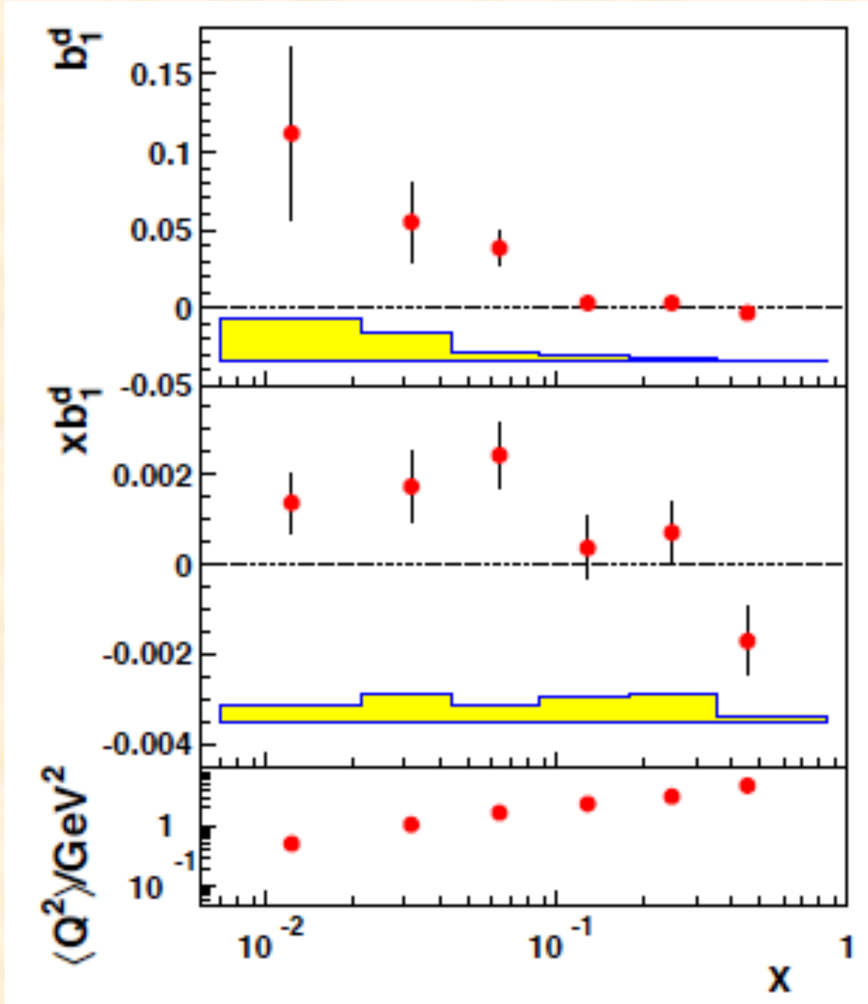
As the Gottfried-sum-rule violation indicated $\bar{u} < \bar{d}$, the b_1-sum-rule violation suggests a finite tensor polarization for antiquarks ($\delta_T \bar{u} \neq 0$).

HERMES results on b_1

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.

27.6 GeV/c $\rightleftarrows$, 0

positron → ● deuteron



b_1 measurement in the kinematical region

$0.01 < x < 0.45,\ \ 0.5\ \text{GeV}^2 < Q^2 < 5\ \text{GeV}^2$

b_1 sum in the restricted Q^2 range $Q^2 > 1\ \text{GeV}^2$

$$\int_{0.02}^{0.85} dx\, b_1(x) = [0.35 \pm 0.10(\text{stat}) \pm 0.18(\text{sys})] \times 10^{-2}$$

at $Q^2 = 5\ \text{GeV}^2$

A. Efremov and O. Teryaev (1982), $\int dx x b_1(x)$.

$$\int dx\, b_1^D(x) = \lim_{t\to 0} -\frac{5}{24}\frac{t}{M^2} F_Q(t) + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_i(x) = 0\ \ ?$$

$$\int \frac{dx}{x}\left[F_2^p(x) - F_2^n(x)\right] = \frac{1}{3}\int dx\left[u_v - d_v\right] + \frac{2}{3}\int dx\left[\bar{u} - \bar{d}\right] \neq 1/3$$

Drell-Yan experiments probe these antiquark distributions.

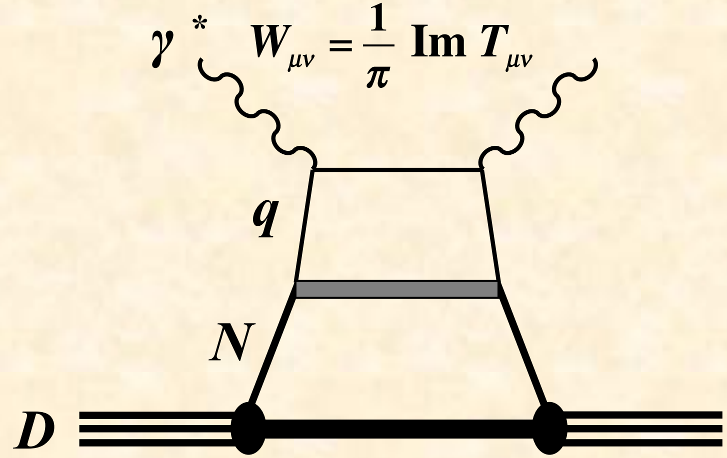
“Standard” deuteron model prediction for b_1

Theory 1: Basic convolution approach

Convolution model: $A_{hH,\,hH}(x,Q^2) = \int \frac{dy}{y} \sum_s f_s^H(y) \hat{A}_{hs,hs}(x/y,\, Q^2) \equiv \sum_s f_s^H(y) \otimes \hat{A}_{hs,hs}(y, Q^2)$

$$A_{hH,\,h'H'} = \varepsilon_{h'}^{*\mu} W_{\mu\nu}^{H'H} \varepsilon_h^{\nu}, \quad b_1 = A_{+0,+0} - \frac{A_{++,++} + A_{+-,+-}}{2}$$

$$\hat{A}_{+\uparrow,+\uparrow} = F_1 - g_1, \quad \hat{A}_{+\downarrow,+\downarrow} = F_1 + g_1$$



Momentum distribution: $f^H(y) = \int d^3p\, y\, |\phi^H(\vec{p})|^2 \delta\left(y - \frac{E - p_z}{M_N} \right)$

$$y = \frac{Mp \cdot q}{M_N P \cdot q} \simeq \frac{2p^-}{P^-}, \qquad f^H(y) \equiv f_\uparrow^H(y) + f_\downarrow^H(y)$$

D-state admixture: $\phi^H(\vec{p}) = \phi_{\ell=0}^H(\vec{p}) + \phi_{\ell=2}^H(\vec{p})$

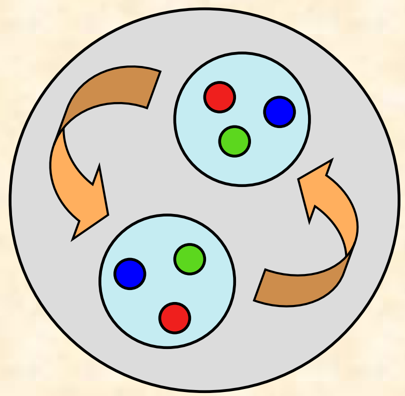
⇓

$$b_1(x) = \int \frac{dy}{y} \delta_T f(y) F_1^N(x/y,\, Q^2)$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y) + f^-(y)}{2}$$

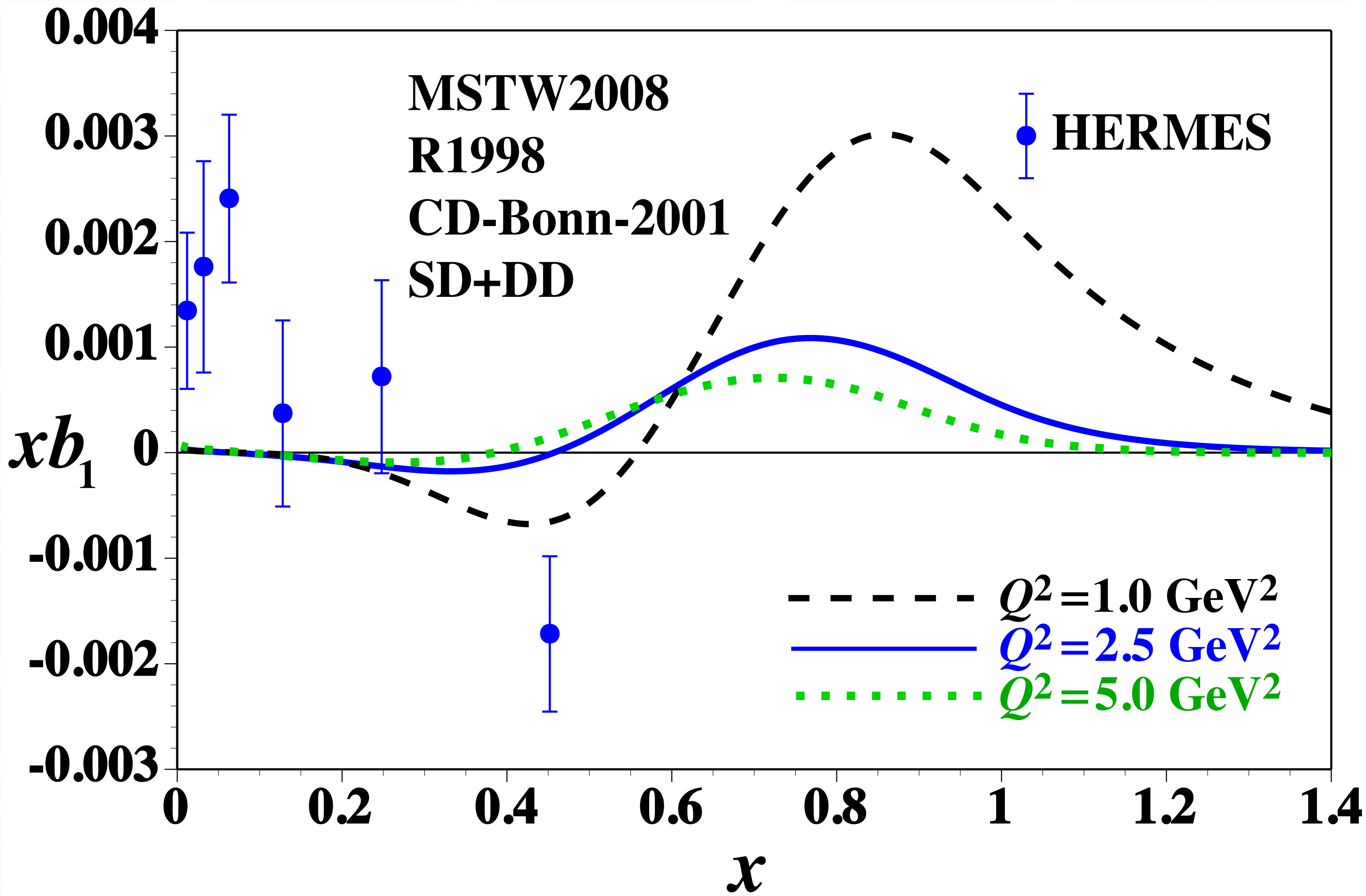
$$= \int d^3p\, y \left[-\frac{3}{4\sqrt{2}\pi} \phi_0(p)\phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] (3\cos^2\theta - 1) \delta\left(y - \frac{p \cdot q}{M_N \nu} \right)$$

Standard model of the deuteron



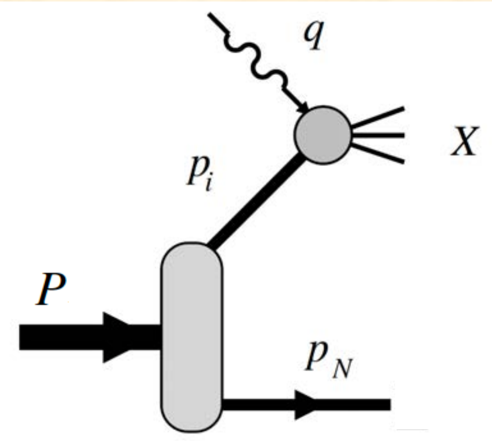
S + D waves

Comparison with HERMES measurements



Theory 2: Virtual nucleon approximation with higher-twist effects

L. L. Frankfurt and M. I. Strikman, Phys. Rep. 76, 215 (1981);
B. D. Keister and W. Polyzou, Adv. Nucl. Phys. 20, 225 (1991);
W. Cosyn and M. Sargsian, Phys. Rev. C 84, 014601 (2011);
W. Cosyn, W. Melnitchouk, and M. Sargsian, Phys. Rev. C 89, 014612 (2014).
W. Cosyn and C. Weiss, Phys. Rev. C 102 (2020) 065204.



Virtual nulceon approximation (VNA)

$$W_{\mu\nu}^{\lambda'\lambda}(P,q) = 4(2\pi)^3 \int d\Gamma_N \frac{\alpha_N}{\alpha_i} W_{\mu\nu}^{N}(p_i,q)\rho_D(\lambda',\lambda)$$

momentum-fractions for interacting (i) and spectator nucleons (N):

$$\alpha_i = \frac{2p_i^-}{P^-}, \;\; \alpha_N = \frac{2p_N^-}{P^-} = 2-\alpha_i, \;\; P = p_i + p_N$$

phase space: $d\Gamma_N = \dfrac{d^3 p_N}{2E_{p_N}(2\pi)^3}$

deuteron density: $\rho_D(\lambda',\lambda) = \displaystyle\sum_{\lambda_N,\lambda_N'} \frac{[\psi_{\lambda'}^{D}(\vec{k},\lambda_N',\lambda_N)]^\dagger \psi_{\lambda'}^{D}(\vec{k},\lambda_N',\lambda_N)}{\alpha_N \alpha_i}$

Results on b_1 in the convolution description

Very different from

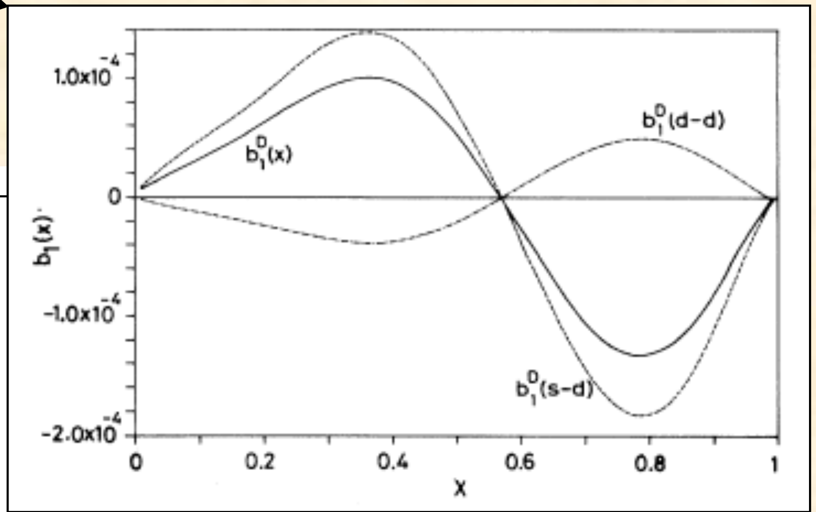
P. Hoodbhoy, R. L. Jaffe, and A. Manohar, NP B312 (1989) 571.

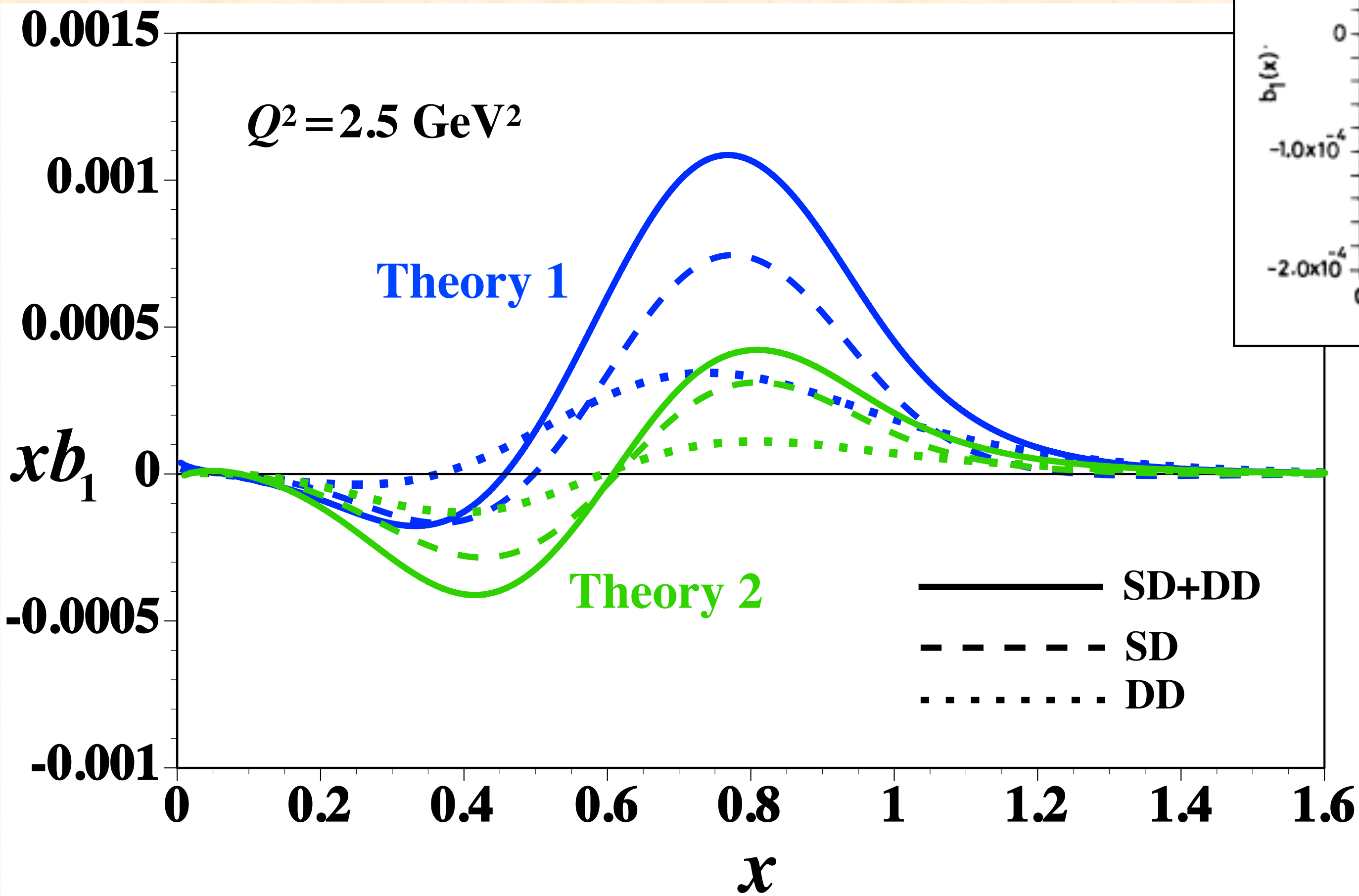
H. Khan and P. Hoodbhoy, PRC44 (1991) 1219;

(1) SD term is opposite,

(2) $b_1(x)$ exists even at $x > 1$,

(3) $|b_1(\text{CDKS})| = 10^{-3} \gg |b_1(\text{KH})| = 10^{-4}$.





"Standard-model" prediction for b_1 of deuteron

$$b_1(x) = \int \frac{dy}{y} \delta_T f(y) F_1^N(x/y, Q^2), \quad y = \frac{M p\cdot q}{M_N P\cdot q} \simeq \frac{2p^-}{P^-}$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y)+f^-(y)}{2}$$

$$= \int d^3p\, y \left[-\frac{3}{4\sqrt{2}\pi} \phi_0(p)\phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] (3\cos^2\theta - 1)\, \delta\left(y - \frac{p\cdot q}{M_N \nu} \right)$$

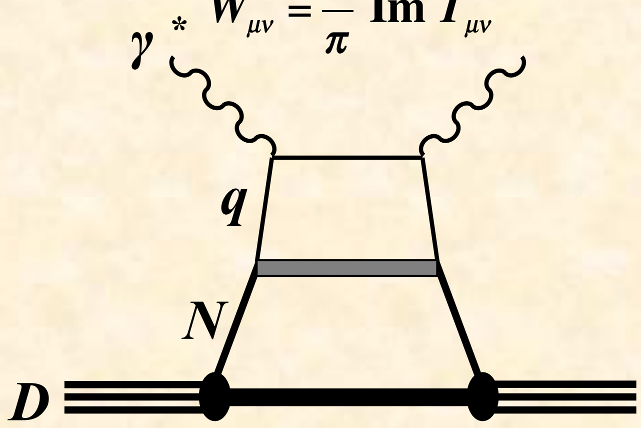
S-D term D-D term

Nucleon momentum distribution:

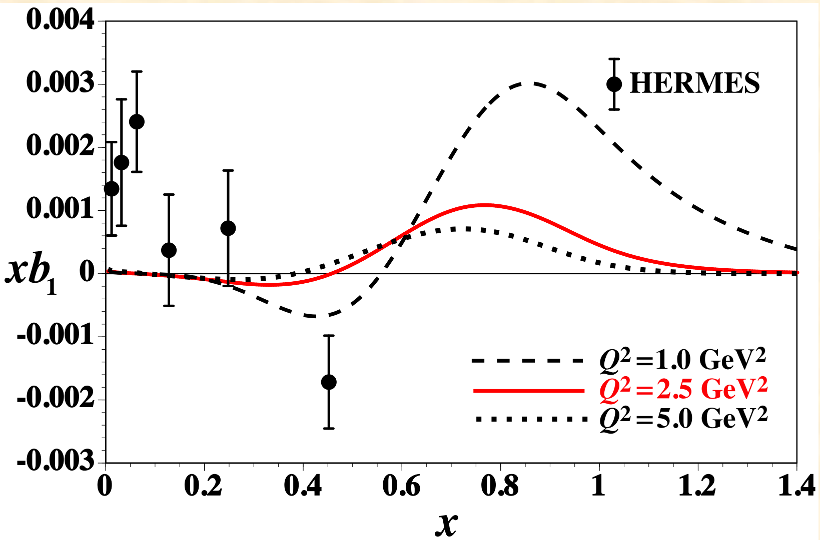
$$f^H(y) \equiv f^H_\uparrow(y) + f^H_\downarrow(y) = \int d^3p\, y\, |\phi^H(\vec{p})|^2 \delta\left(y - \frac{E - p_z}{M_N} \right)$$

D-state admixture: $\phi^H(\vec{p}) = \phi^H_{\ell=0}(\vec{p}) + \phi^H_{\ell=2}(\vec{p})$

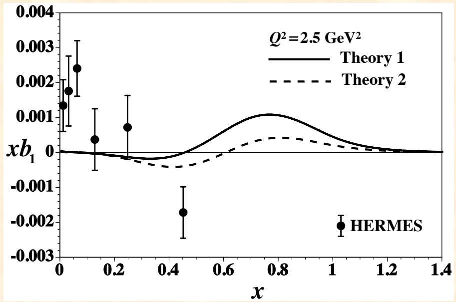
$$W_{\mu\nu} = \frac{1}{\pi} \operatorname{Im} T_{\mu\nu}$$



Standard model of the deuteron



W. Cosyn, Yu-Bing Dong, SK, M. Sargsian, Phys. Rev. D 95 (2017) 074036.



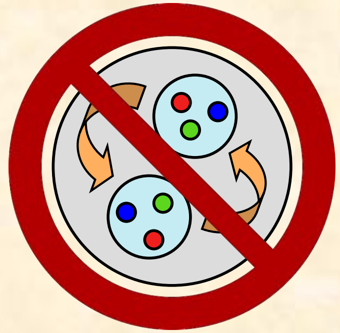
$|b_1(\text{theory})| \ll |b_1(\text{HERMES})|$
at $x < 0.5$

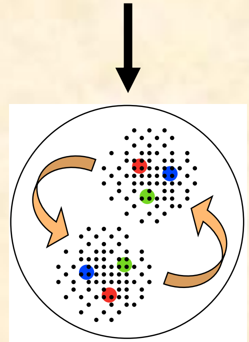
Standard convolution model does not work for the deuteron tensor structure!?

G. A. Miller, PRC 89 (2014) 045203,

Interesting suggestions:
hidden-color, 6-quark, · · ·

$$|6q\rangle = |NN\rangle + |\Delta\Delta\rangle + |CC\rangle + \cdots$$





?

Tensor-polarized PDFs at hadron accelerator facilities (*e.g.* Fermilab)

Spin asymmetries in the parton model

unpolarized: q_a, longitudinally polarized: Δq_a,
transversely polarized: $\Delta_T q_a$, tensor polarized: δq_a

Unpolarized cross section

$$\left\langle \frac{d\sigma}{dx_A dx_B d\Omega} \right\rangle = \frac{\alpha^2}{4Q^2}(1+\cos^2\theta)\frac{1}{3}\sum_a e_a^2\left[q_a(x_A)\bar{q}_a(x_B)+\bar{q}_a(x_A)q_a(x_B)\right]$$

Spin asymmetries

M. Hino and SK,
PRD 59 (1999) 094026;
60 (1999) 054018.

$$A_{LL} = \frac{\sum_a e_a^2\left[\Delta q_a(x_A)\Delta\bar{q}_a(x_B)+\Delta\bar{q}_a(x_A)\Delta q_a(x_B)\right]}{\sum_a e_a^2\left[q_a(x_A)\bar{q}_a(x_B)+\bar{q}_a(x_A)q_a(x_B)\right]}$$

$$A_{TT} = \frac{\sin^2\theta\cos(2\phi)}{1+\cos^2\theta}\frac{\sum_a e_a^2\left[\Delta_T q_a(x_A)\Delta_T\bar{q}_a(x_B)+\Delta_T\bar{q}_a(x_A)\Delta_T q_a(x_B)\right]}{\sum_a e_a^2\left[q_a(x_A)\bar{q}_a(x_B)+\bar{q}_a(x_A)q_a(x_B)\right]}$$

$$A_{UQ_0} = \frac{\sum_a e_a^2\left[q_a(x_A)\delta_T\bar{q}_a(x_B)+\bar{q}_a(x_A)\delta_T q_a(x_B)\right]}{2\sum_a e_a^2\left[q_a(x_A)\bar{q}_a(x_B)+\bar{q}_a(x_A)q_a(x_B)\right]}$$

$$A_{LT} = A_{TL} = A_{UT} = A_{TU} = A_{TQ_0} = A_{UQ_1}$$
$$= A_{LQ_1} = A_{TQ_1} = A_{UQ_2} = A_{LQ_2} = A_{TQ_2} = 0$$

Advantage of the hadron reaction ($\delta\bar{q}$ measurement)

$$A_{UQ_0}(\text{large } x_F) \approx \frac{\sum_a e_a^2 q_a(x_A)\delta_T\bar{q}_a(x_B)}{2\sum_a e_a^2 q_a(x_A)\bar{q}_a(x_B)}$$

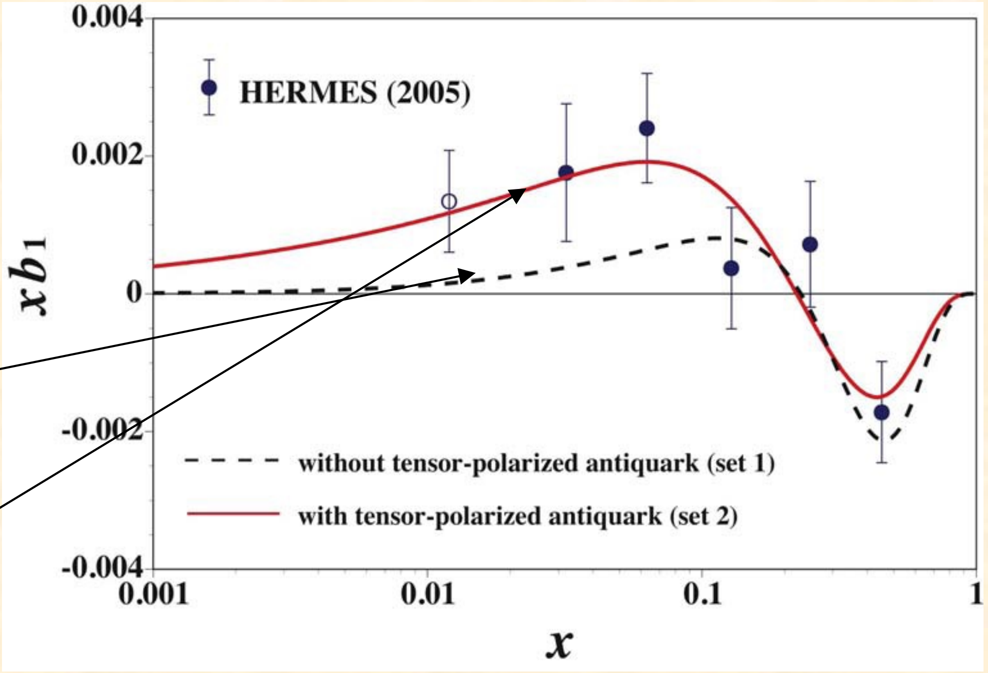
Note: $\delta \neq$ transversity in my notation

Tensor-polarized PDFs

SK, PRD 82 (2010) 017501.

Two-types of fit results:

- set-1 ($\delta_T \bar{q} = 0$): $\chi^2 / \text{d.o.f.} = 2.83$
  Without $\delta_T \bar{q}$, the fit is not good enough.
- set-2 ($\delta_T \bar{q} \neq 0$): $\chi^2 / \text{d.o.f.} = 1.57$
  With finite $\delta_T \bar{q}$, the fit is reasonably good.

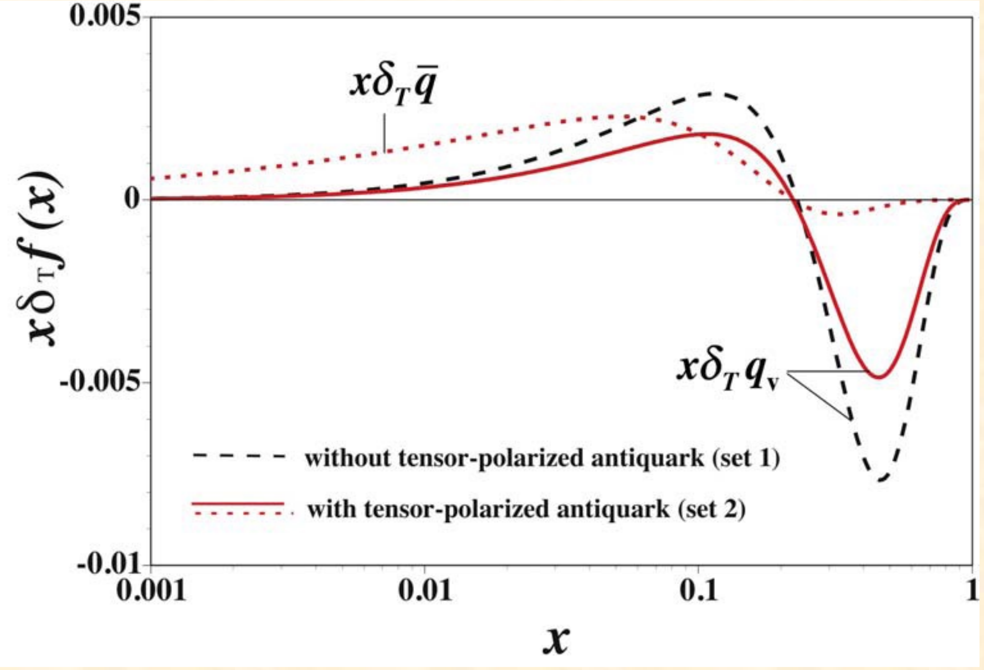


Obtained tensor-polarized distributions
$\delta_T q(x)$, $\delta_T \bar{q}(x)$ from the HERMES data.

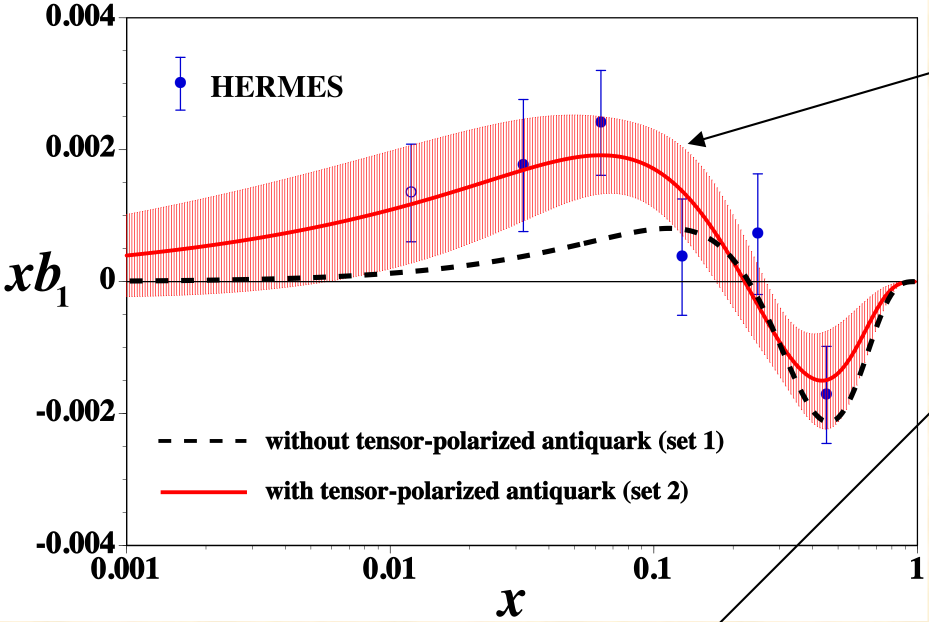
→ They could be used for
- experimental proposals,
- comparison with theoretical models.

Finite tensor polarization for antiquarks:

$$\int_0^1 dx b_1(x) = 0.058$$
$$= \frac{1}{9} \int_0^1 dx \left[4\delta_T \bar{u}(x) + \delta_T \bar{d}(x) + \delta_T \bar{s}(x) \right]$$

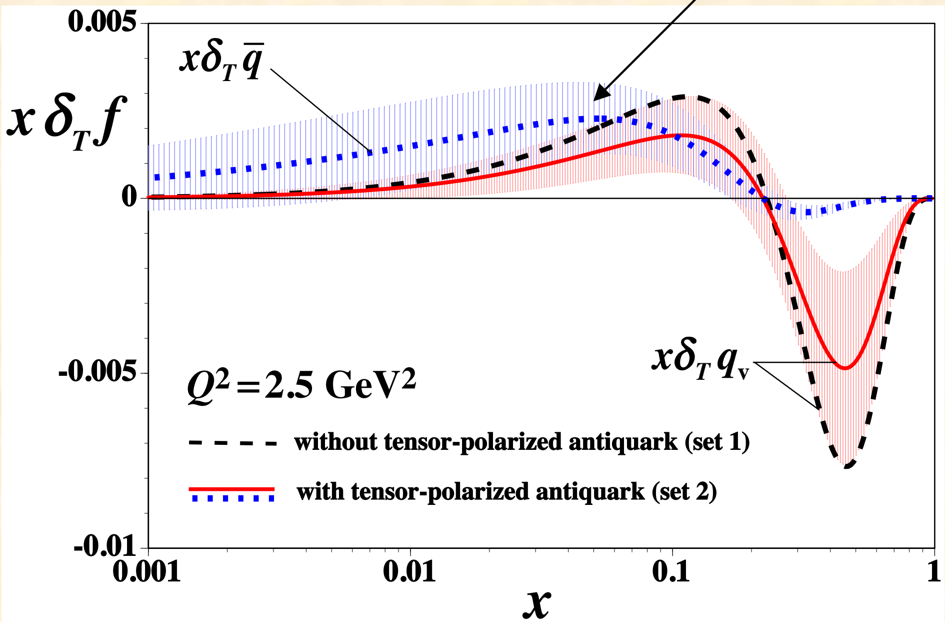


Tensor-polarized PDFs with errors



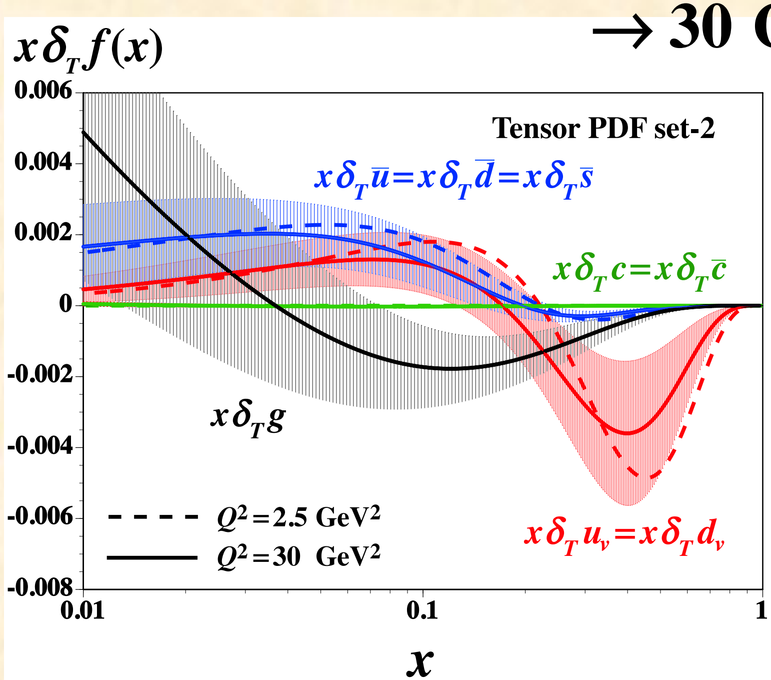
still large errors,
need experimental improvement
→ JLab, EIC, ...

experimental measurement
for antiquark distributions
→ Fermilab, ...



Q^2 evolution

$Q^2 = 2.5$ GeV^2
$\rightarrow 30$ GeV^2



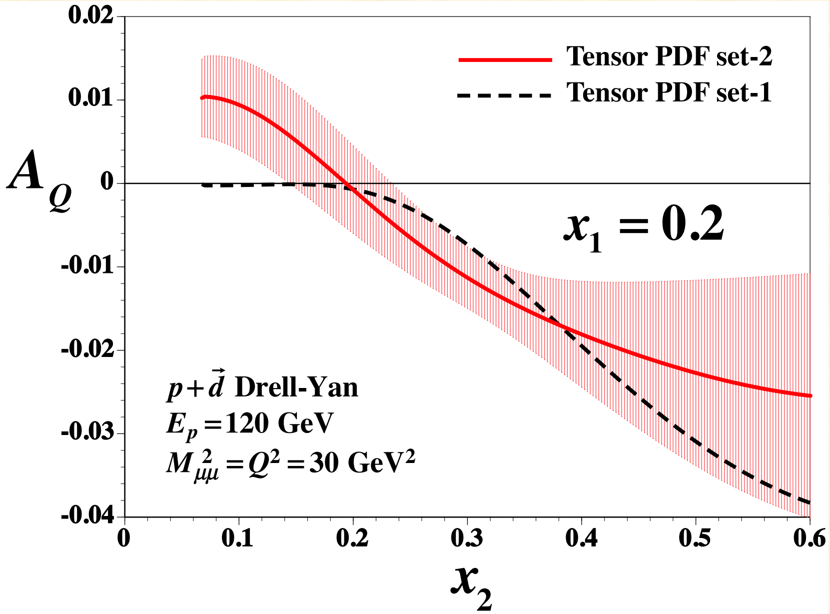
Tensor-polarized spin asymmetry at Fermilab

Spin asymmetry in proton-deuteron Drell-Yan process with tensor-polarized deuteron

$$A_{UQ_0} = \frac{\sum_a e_a^2 \left[q_a(x_A)\, \delta_T \bar{q}_a(x_B) + \bar{q}_a(x_A)\, \delta_T q_a(x_B) \right]}{2 \sum_a e_a^2 \left[q_a(x_A)\, \bar{q}_a(x_B) + \bar{q}_a(x_A)\, q_a(x_B) \right]}$$

Polarized fixed-target experiments at the Fermilab Main Injector

E1039-SpinQuest



SK and Qin-Tao Song,
PRD 94 (2016) 054022

Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

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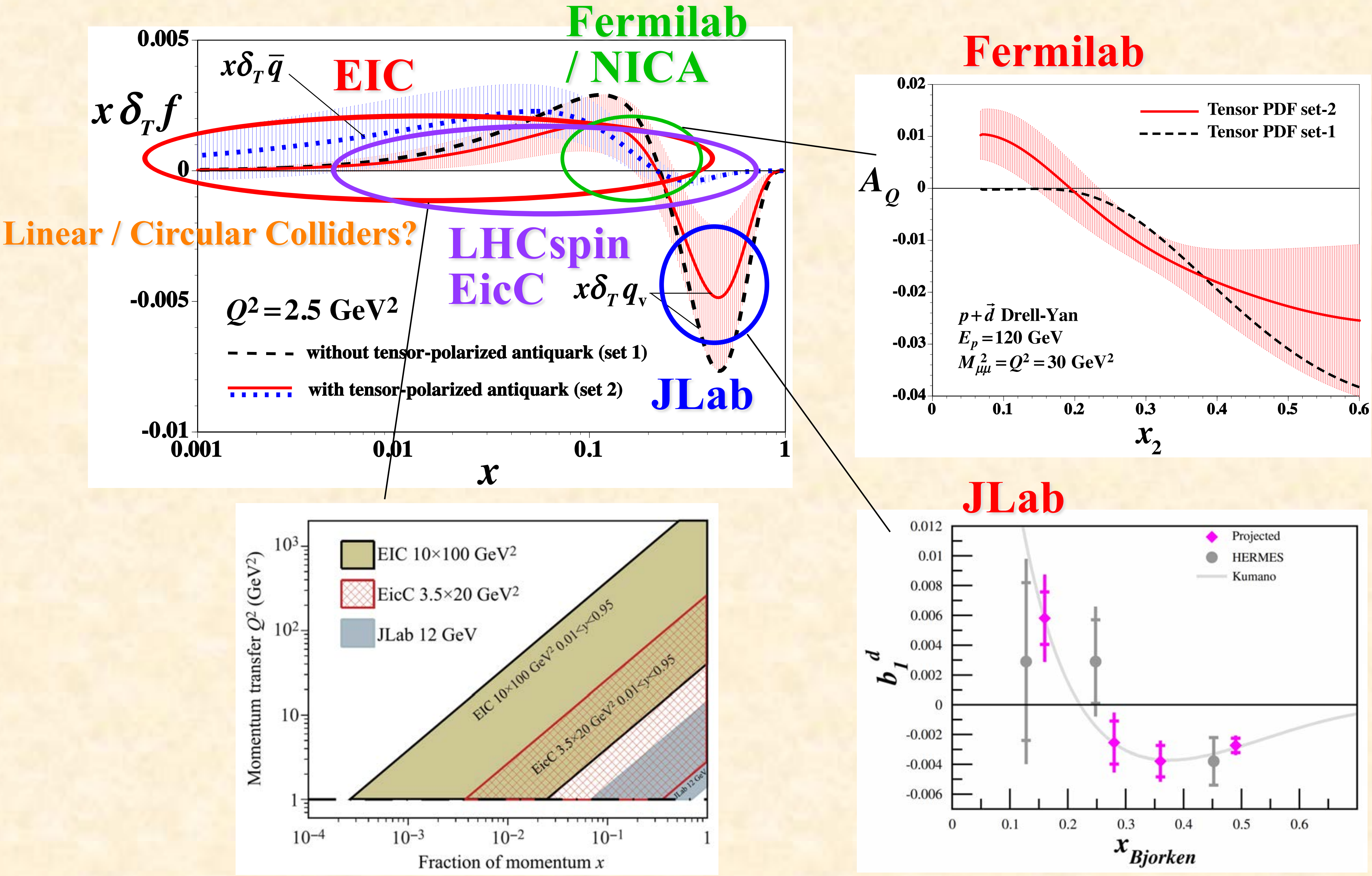
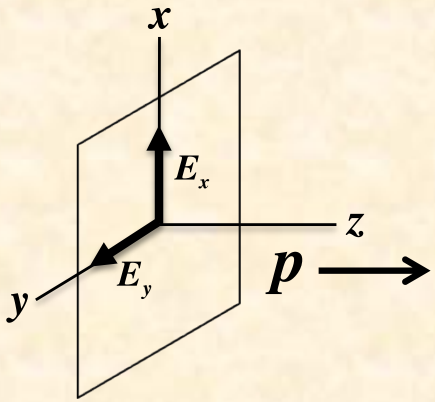
Tokyo Institute of Technology, Tokyo 152-8551, Japan
D. Crabb, D. Day, D. Keller, O. Rondon
University of Virginia, Charlottesville, VA 22904

x regions of b_1 in 2030's



Gluon transversity

Gluon transversity distribution in deuteron



Linear-polarization difference: $d\sigma(E_x - E_y) \propto \Delta_T g$

$$\Delta_T g(x) = \int \frac{d\xi^-}{2\pi} xp^+ e^{ixp^+\xi^-} \left\langle pE_x \middle| A^x(0)A^x(\xi) - A^y(0)A^y(\xi) \middle| pE_x \right\rangle_{\xi^+=\vec{\xi}_T=0}$$
$$= g_{\hat{x}/\hat{x}} - g_{\hat{y}/\hat{x}}$$

Confusing situation of gluon transversity

(no consensus even on its notation: publication # ≈ different notation #)

$$\Delta_2 G(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \quad [13,\ 44],$$
$$a(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \quad [23,\ 25],$$
$$\Delta_L g(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \quad [19],$$
$$\delta G(x) = -g_{\hat{x}/\hat{x}}(x) + g_{\hat{y}/\hat{x}}(x) \quad [26,\ 45],$$
$$h_{1TT,g}(x) = -g_{\hat{x}/\hat{x}}(x) + g_{\hat{y}/\hat{x}}(x) \quad [36,\ 38,\ 46],$$
$$\Delta_T g(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \quad [47],\ \text{this work},$$

SK and Qin-Tao Song,
PRD 101 (2020) 054011 & 094013.

→ One can imagine how premature this field is!

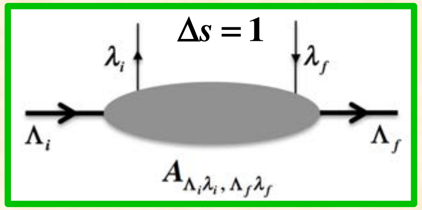
Gluon transversity $\Delta_T g$

Note on our notations:
Tensor-polarized gluon distribution: $\delta_T g$
Gluon transversity: $\Delta_T g$

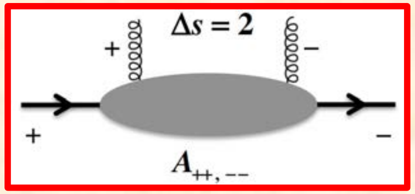
Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, **conservation** $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ +\frac{1}{2}+\frac{1}{2}\right) - A\left(+\frac{1}{2}-\frac{1}{2},\ +\frac{1}{2}-\frac{1}{2}\right)$

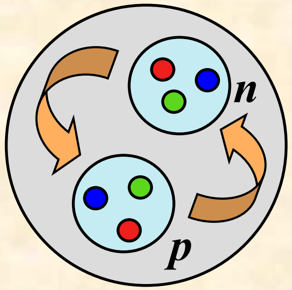
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ -\frac{1}{2}-\frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \to \lambda_f = -\frac{1}{2}$ **quark spin flip** ($\Delta s = 1$)



Gluon transversity in deuteron: $\Delta_T g(x) \sim A(+1+1,\ -1-1)$, ~~$A\left(+\frac{1}{2}+1,\ -\frac{1}{2}-1\right)$~~ **not possible for nucleon**



Note: Gluon transversity does not exist for spin-1/2 nucleons.



S + D waves

$b_1\ (\delta_T q,\ \delta_T g) \neq 0 \ \Leftrightarrow$ **still** $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$**?**

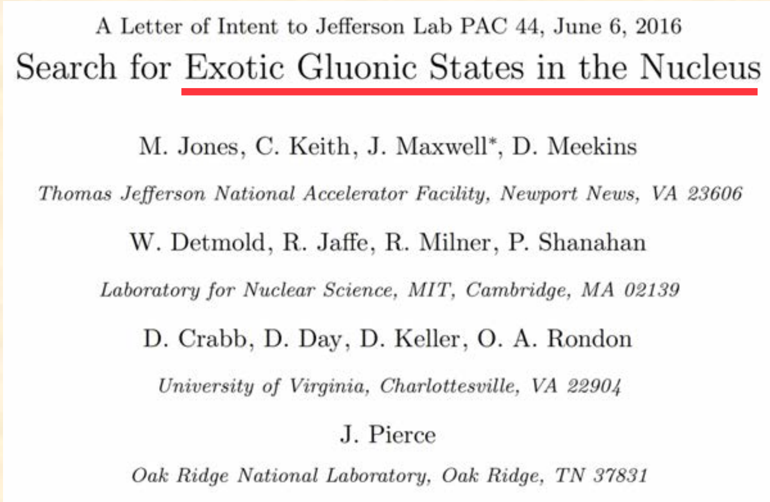
Physics beyond "the standard model" in nuclear physics?

(Physics beyond the standard model in particle physics???)

Letter of Intent at Jefferson Lab

Jefferson Lab,
Electron accelerator ~12 GeV

LoI, arXiv:1803.11206



A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan

Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon

University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831

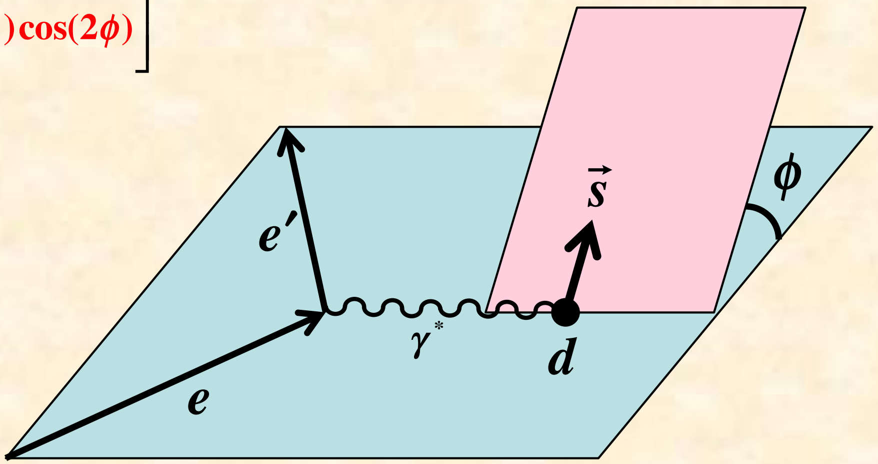
For development of polarized deuteron target,
see D. Keller, D. Crabb, D. Day
Nucl. Inst. Meth. Phys. Res. A981 (2020) 164504.

Electron scattering with polarized-deuteron target

$$\left.\frac{d\sigma}{dx\,dy\,d\phi}\right|_{Q^2 \gg M^2} = \frac{e^4 ME}{4\pi^2 Q^4}\left[xy^2 F_1(x,Q^2) + (1-y)F_2(x,Q^2) - \frac{1}{2}x(1-y)\Delta(x,Q^2)\cos(2\phi)\right]$$

$$\Delta(x,Q^2) = \frac{\alpha_s}{2\pi}\sum_q e_q^2 x^2 \int_x^1 \frac{dy}{y^3}\Delta_T g(y,Q^2)$$

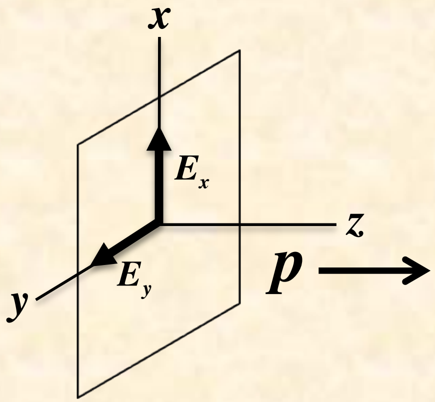
By looking at the deuteron-polarization angle ϕ,
the quark transversty $\Delta_T g$ can be measured.



Lattice QCD estimates:
W. Detmold and P. E. Shanahan,
PRD 94 (2016) 014507; 95 (2017) 079902.

Gluon transversity distribution in deuteron

may skip



Linear-polarization difference: $d\sigma(E_x - E_y) \propto \Delta_T g$

$$\Delta_T g(x) = \int \frac{d\xi^-}{2\pi} x p^+ e^{ixp^+\xi^-} \left\langle pE_x \left| A^x(0)A^x(\xi) - A^y(0)A^y(\xi) \right| pE_x \right\rangle_{\xi^+=\vec{\xi}_T=0}$$

$$= g_{\hat{x}/\hat{x}} - g_{\hat{y}/\hat{x}}$$

$g_{\hat{y}/\hat{x}}$ = gluon distribution with the gluon linear polarization ε_y in the deuteron linear polarization E_x

Polarization vectors $\vec{E}_x = \vec{\varepsilon}_x = (1,\ 0,\ 0)$, $\vec{E}_y = \vec{\varepsilon}_y = (0,\ 1,\ 0)$

Spin and tensor of the deuteron

$$S^\mu = \frac{1}{M}\varepsilon^{\mu\nu\alpha\beta} p_\nu \,\mathrm{Im}(E_\alpha^* E_\beta), \quad T^{\mu\nu} = -\frac{1}{3}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - \mathrm{Re}(E^{\mu *} E^\nu)$$

$$E^\mu = (0,\ \vec{E}), \quad \vec{E}_\pm = \frac{1}{\sqrt{2}}(\mp 1,\ -i,\ 0), \quad \vec{E}_0 = (0,\ 0,\ 1)$$

- $\vec{E}_+, \vec{E}_0, \vec{E}_-$: Spin states with z-components of spin $s_z = +1,\ 0,\ -1$
- $\vec{E}_x = (1,\ 0,\ 0)$, $\vec{E}_y = (0,\ 1,\ 0)$: Linear polarizations
  → to measure gluon transversity

(1) Prepare $s_x = 0$ [$\vec{E}_x = (1,\ 0,\ 0)$] by taking the quantization axis x
and $s_y = 0$ [$\vec{E}_y = (0,\ 1,\ 0)$] by taking the quantization axis y.

(2) Combination of transverse polarizations.

$$\boldsymbol{S} = (S_T^x,\ S_T^y,\ S_L),$$

$$\boldsymbol{T} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3}S_{LL} \end{pmatrix}$$

$$S_{TT}^{xy} = S_{LT}^x = S_{LT}^y = 0$$

Polarizations	$\vec{E}$	S_T^x	S_T^y	S_L	S_{LL}	S_{TT}^{xx}
Longitudinal $+z$	$\frac{1}{\sqrt{2}}(-1,\ -i,\ 0)$	0	0	+1	$+\frac{1}{2}$	0
Longitudinal $-z$	$\frac{1}{\sqrt{2}}(+1,\ -i,\ 0)$	0	0	−1	$+\frac{1}{2}$	0
Transverse $+x$	$\frac{1}{\sqrt{2}}(0,\ -1,\ -i)$	+1	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $-x$	$\frac{1}{\sqrt{2}}(0,\ +1,\ -i)$	−1	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $+y$	$\frac{1}{\sqrt{2}}(-i,\ 0,\ -1)$	0	+1	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Transverse $-y$	$\frac{1}{\sqrt{2}}(-i,\ 0,\ +1)$	0	−1	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Linear x	$(1,\ 0,\ 0)$	0	0	0	$+\frac{1}{2}$	−1
Linear y	$(0,\ 1,\ 0)$	0	0	0	$+\frac{1}{2}$	+1

Transverse polarization

Linear polarization

Proton-deuteron Drell-Yan cross section

SK and Qin-Tao Song,
PRD 101 (2020) 054011 & 094013.

Drell-Yan cross section

$$\frac{d\sigma_{pd\to\mu^+\mu^-X}(E_x-E_y)}{d\tau\, dq_T^2\, d\phi\, dy} = \frac{\alpha^2\alpha_s C_F q_T^2}{6\pi s^3}\cos(2\phi)\int_{\min(x_a)}^{1} dx_a \frac{1}{(x_a x_b)^2(x_a-x_1)(\tau-x_a x_2)^2}\sum_q e_q^2 x_a\left[q_A(x_a)+\bar{q}_A(x_a)\right]x_b\Delta_T g_B(x_b)$$

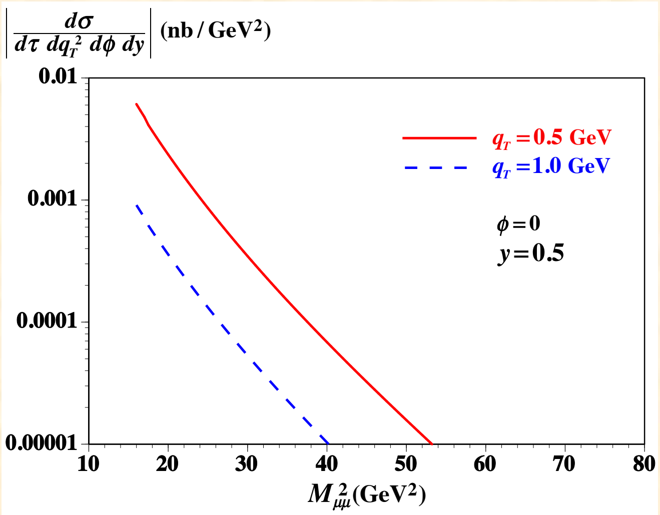
$$C_F=\frac{N_c^2-1}{2N_c},\quad \min(x_a)=\frac{x_1-\tau}{1-x_2},\quad x_b=\frac{x_a x_2-\tau}{x_a-\tau}$$

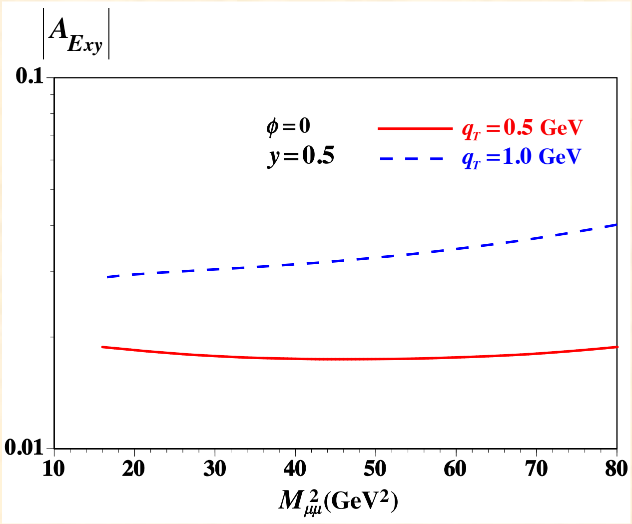
= (unpolarized PDFs of proton) * (gluon transversity distribution in the deuteron)

- Consider the Fermilab-E1039 experiment with the proton beam of $p = 120$ GeV
- No available $\Delta_T g$, so we may tentatively assume $\Delta_T g = \Delta g_p + \Delta g_n$ $\left(\text{or } \frac{\Delta g_p+\Delta g_n}{2},\ \frac{\Delta g_p+\Delta g_n}{4}\right)$
- CTEQ14 for $q(x)+\bar{q}(x)$, NNPDFpol1.1 for $\Delta g(x)$

Cross section: Dimuon mass squred ($M_{\mu\mu}^2 = Q^2$) dependence

Spin asymmetry: $$A_{E_{xy}} = \frac{\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_x) - \frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_y)}{\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_x) + \frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_y)}$$





Proposal at Fermilab-PAC (D. Keller)

Experimental possibility at Fermilab in 2030's

Polarized fixed-target experiments at the Main Injector, Proton beam = 120 GeV

Fermilab-E1039 (SpinQuest)

Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

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Fermilab experimentalists are interested in the gluon transversity by replacing the E1039 proton target for the deuteron one. (Spokesperson of E1039: D. Keller) However, there was no theoretical formalism until our work.

SK and Q.-T. Song, PRD 101 (2020) 054011 & 094013.

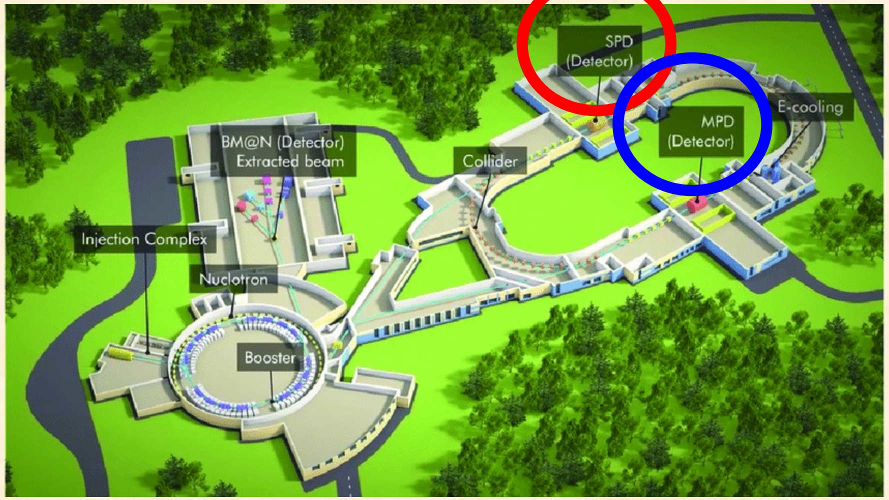
D. Keller. Technical Report, FERMILAB-PUB-22-381-743V, 2022, https://arxiv.org/abs/2205.01249.

The Transverse Structure of the Deuteron with Drell-Yan

The SpinQuest Collaboration[a]

Proposal for a Fermilab-PAC

Nuclotron-based Ion Collider fAcility (NICA)



SPD (Spin Physics Detector for physics with polarized beams)
MPD (MultiPurpose Detector for heavy ion physics)

$$\vec{p}+\vec{p}: \quad \sqrt{s_{pp}} = 12 \sim 27 \text{ GeV}$$

$$\vec{d}+\vec{d}: \quad \sqrt{s_{NN}} = 4 \sim 14 \text{ GeV}$$

$\vec{p}+\vec{d}$ is also possilbe.

On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbuzov *et al.* (NICA project), Nucl. Part. Phys. 119 (2021) 103858.


Progress in Particle and Nuclear Physics 119 (2021) 103858

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Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

A. Arbuzov [a], A. Bacchetta [b,c], M. Butenschoen [d], F.G. Celiberto [b,c,e,f], U. D'Alesio [g,h], M. Deka [a], I. Denisenko [a], M.G. Echevarria [i], A. Efremov [a], N.Ya. Ivanov [a,j], A. Guskov [a,k,*], A. Karpishkov [l,a], Ya. Klopot [a,m], B.A. Kniehl [d], A. Kotzinian [j,o], S. Kumano [p], J.P. Lansberg [q], Keh-Fei Liu [r], F. Murgia [h], M. Nefedov [l], B. Parsamyan [a,n,o], C. Pisano [g,h], M. Radici [c], A. Rymbekova [a], V. Saleev [l,a], A. Shipilova [l,a], Qin-Tao Song [s], O. Teryaev [a]

Unique opportunity in high-energy spin physics,
especially on the deuteron spin physics.

→ Theoretical formalisms need to be developed.

TMDs, PDFs, and fragmentation functions for spin-1 hadrons up to twist 4

TMD correlation functions for spin-1 hadrons

Correlation functions

$$\Phi_{ij}(k,P,T) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \left\langle P,T \left| \bar{\psi}_j(0) W(0,\xi) \psi_i(\xi) \right| P,T \right\rangle$$

$$W(0,\xi) = P\exp\left[-ig\int_0^\xi d\xi \cdot A(\xi)\right]$$

Spin vector: $S^\mu = S_L \frac{P^+}{M}\bar{n}^\mu - S_L \frac{M}{2P^+} n^\mu + S_T^\mu$

Tensor: $T^{\mu\nu} = \frac{1}{2}\left[\frac{4}{3}S_{LL}\frac{(P^+)^2}{M^2}\bar{n}^\mu\bar{n}^\nu + \frac{P^+}{M}\bar{n}^{\{\mu}S_{LT}^{\nu\}} - \frac{2}{3}S_{LL}\left(\bar{n}^{\{\mu}n^{\nu\}} - g_T^{\mu\nu}\right) + S_{TT}^{\mu\nu} - \frac{M}{2P^+}n^{\{\mu}S_{LT}^{\nu\}} + \frac{1}{3}S_{LL}\frac{M^2}{(P^+)^2}n^\mu n^\nu\right]$

Tensor part (twist-2): Bacchetta, Mulders, PRD 62 (2000) 114004

$$\Phi(k,P,T) = \left(\frac{A_{13}}{M}I + \frac{A_{14}}{M^2}\not{P} + \frac{A_{15}}{M^2}\not{k} + \frac{A_{16}}{M^3}\sigma_{\rho\sigma}P^\rho k^\sigma\right)k_\mu k_\nu T^{\mu\nu} + \left[A_{17}\gamma_\nu + \left(\frac{A_{18}}{M}P^\rho + \frac{A_{19}}{M}k^\rho\right)\sigma_{\nu\rho} + \frac{A_{20}}{M^2}\varepsilon_{\tau\nu\rho\sigma}P^\rho k^\sigma\gamma^\tau\gamma_5\right]k_\mu T^{\mu\nu}$$

Tensor part (twist-2, 3, 4): n^μ **dependent terms are added for up to twist 4.**

[For the spin-1/2 nucleon: Goeke, Metzand, Schlegel, PLB 618 (2005) ,90; Metz, Schweitzer, Teckentrup, PLB 680 (2009) 141.]

Kumano-Song-2021, for the details see PRD 103 (2021) 014025

$$\Phi(k,P,T\,|\,n) = \left(\frac{A_{13}}{M}I + \frac{A_{14}}{M^2}\not{P} + \frac{A_{15}}{M^2}\not{k} + \frac{A_{16}}{M^3}\sigma_{\rho\sigma}P^\rho k^\sigma\right)k_\mu k_\nu T^{\mu\nu} + \left[A_{17}\gamma_\nu + \left(\frac{A_{18}}{M}P^\rho + \frac{A_{19}}{M}k^\rho\right)\sigma_{\nu\rho} + \frac{A_{20}}{M^2}\varepsilon_{\tau\nu\rho\sigma}P^\rho k^\sigma\gamma^\tau\gamma_5\right]k_\mu T^{\mu\nu}$$

Bacchetta -Mulders (2000)

$$+\left(\frac{B_{21}M}{P\cdot n}k_\mu + \frac{B_{22}M^3}{(P\cdot n)^2}n_\mu\right)n_\nu T^{\mu\nu} + i\gamma_5\varepsilon_{\mu\tau\rho\sigma}P^\rho\left(\frac{B_{23}}{(P\cdot n)M}k^\tau n^\sigma k_\nu + \frac{B_{24}M}{(P\cdot n)^2}k^\tau n^\sigma n_\nu\right)T^{\mu\nu}$$

$$+\left[\frac{B_{25}}{P\cdot n}\not{n}k_\mu k_\nu + \left(\frac{B_{26}M^2}{(P\cdot n)^2}\not{n} + \frac{B_{28}}{P\cdot n}\not{P} + \frac{B_{30}}{P\cdot n}\not{k}\right)k_\mu n_\nu + \left(\frac{B_{27}M^4}{(P\cdot n)^3}\not{n} + \frac{B_{29}M^2}{(P\cdot n)^2}\not{P} + \frac{B_{31}M^2}{(P\cdot n)^2}\not{k}\right)n_\mu n_\nu + \frac{B_{32}M^2}{P\cdot n}\gamma_\mu n_\nu\right]T^{\mu\nu}$$

$$-\left[\varepsilon_{\mu\tau\rho\sigma}\gamma^\tau P^\rho\left(\frac{B_{34}}{P\cdot n}n^\sigma k_\nu + \frac{B_{33}}{P\cdot n}k^\sigma n_\nu + \frac{B_{35}M^2}{(P\cdot n)^2}n^\sigma n_\nu\right) + \varepsilon_{\lambda\tau\rho\sigma}k^\lambda\gamma^\tau P^\rho n^\sigma\left(\frac{B_{36}}{P\cdot nM^2}k_\mu k_\nu + \frac{B_{37}}{(P\cdot n)^2}k_\mu n_\nu + \frac{B_{38}M^2}{(P\cdot n)^3}n_\mu n_\nu\right)\right]\gamma_5 T^{\mu\nu}$$

$$+\varepsilon_{\mu\tau\rho\sigma}k^\tau P^\rho n^\sigma\left(\frac{B_{39}}{(P\cdot n)^2}k_\nu + \frac{B_{40}M^2}{(P\cdot n)^3}n_\nu\right)\not{n}\gamma_5 T^{\mu\nu}$$

$$+\sigma_{\rho\sigma}\left[P^\rho k^\sigma\left(\frac{B_{41}}{(P\cdot n)M}k_\mu n_\nu + \frac{B_{42}M}{(P\cdot n)^2}n_\mu n_\nu\right) + P^\rho n^\sigma\left(\frac{B_{43}}{(P\cdot n)M}k_\mu k_\nu + \frac{B_{44}M}{(P\cdot n)^2}k_\mu n_\nu + \frac{B_{45}M^3}{(P\cdot n)^3}n_\mu n_\nu\right)\right]T^{\mu\nu}$$

$$+\sigma_{\rho\sigma}\left[k^\rho n^\sigma\left(\frac{B_{46}}{(P\cdot n)M}k_\mu k_\nu + \frac{B_{47}M}{(P\cdot n)^2}k_\mu n_\nu + \frac{B_{48}M^3}{(P\cdot n)^3}n_\mu n_\nu\right)\right]T^{\mu\nu} + \sigma_{\mu\sigma}\left[n^\sigma\left(\frac{B_{49}M}{P\cdot n}k_\nu + \frac{B_{50}M^3}{(P\cdot n)^2}n_\nu\right) + \left(\frac{B_{51}M}{P\cdot n}P^\sigma + \frac{B_{52}M}{P\cdot n}k^\sigma\right)n_\nu\right]T^{\mu\nu}$$

New terms in our paper (2021)

From this correlation function, new tensor-polarized TMDs are defined in twist-3 and 4 in addition to twist-2 ones.

Terms associated with $n = \frac{1}{\sqrt{2}}(1,\ 0,\ 0,\ -1)$

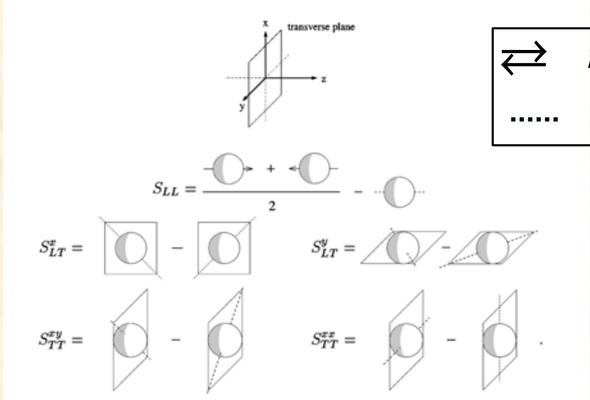
TMDs and their sum rules for spin-1 hadrons

see our PRD paper for the details

Twist-2 TMDs

Bacchetta-Mulders, PRD 62 (2000) 114004.

Quark \ Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1]$, $[h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}]$, $[h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}]$, $[h_{1TT}^\perp]$



⇄ $m_s = \pm 1$

...... $m_s = 0$

Time-reversal invariance in colliear corrlation functions (PDFs)

$$\int d^2k_T \, \Phi_{\text{T-odd}}(x, k_T^2) = 0$$

Sum rules for the TMDs of spin-1 hadrons

$$\int d^2k_T \, h_{1LT}(x, k_T^2) = 0, \qquad \int d^2k_T \, g_{LT}(x, k_T^2) = 0,$$

$$\int d^2k_T \, h_{LL}(x, k_T^2) = 0, \qquad \int d^2k_T \, h_{3LT}(x, k_T^2) = 0$$

Twist-3 TMDs

SK and Qin-Tao Song, PRD 103 (2021) 014025.

Quark \ Hadron	$\gamma^i, 1, i\gamma_5$ T-even	$\gamma^i, 1, i\gamma_5$ T-odd	$\gamma^+\gamma_5$ T-even	$\gamma^+\gamma_5$ T-odd	σ^{ij}, σ^{-+} T-even	σ^{ij}, σ^{-+} T-odd
U	$f^\perp$ $[e]$			$g^\perp$		$[h]$
L		$f_L^\perp$ $[e_L]$	$g_L^\perp$		$[h_L]$	
T		f_T, $f_T^\perp$ $[e_T, e_T^\perp]$	g_T, $g_T^\perp$		$[h_T]$, $[h_T^\perp]$	
LL	$f_{LL}^\perp$ $[e_{LL}]$			$g_{LL}^\perp$		$[h_{LL}]$
LT	f_{LT}, $f_{LT}^\perp$ $[e_{LT}, e_{LT}^\perp]$			g_{LT}, $g_{LT}^\perp$		$[h_{LT}]$, $[h_{LT}^\perp]$
TT	f_{TT}, $f_{TT}^\perp$ $[e_{TT}, e_{TT}^\perp]$			g_{TT}, $g_{TT}^\perp$		$[h_{TT}]$, $[h_{TT}^\perp]$

Twist-4 TMDs

Quark \ Hadron	γ^- T-even	γ^- T-odd	$\gamma^-\gamma_5$ T-even	$\gamma^-\gamma_5$ T-odd	σ^{i-} T-even	σ^{i-} T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}]$, $[h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}]$, $[h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}]$, $[h_{3TT}^\perp]$

PDFs for spin-1 hadrons

Twist-2 PDFs

Hadron \ Quark	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					
L			$g_{1L}\,(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}\,(b_1)$					
LT						*1
TT						

*1: $h_{1LT}(x)$, *2: $g_{LT}(x)$, *3: $h_{LL}(x)$, *4: $h_{3LT}(x)$

Because of the time-reversal invariance, the collinear PDF vanishes. However, since the time-reversal invariance cannot be imposed in the fragmentation functions, we should note that the corresponding fragmentation function should exist as a collinear fragmentation function.

[ ] = chiral odd

Twist-3 PDFs

Hadron \ Quark	$\gamma^i, 1, i\gamma_5$ T-even	$\gamma^i, 1, i\gamma_5$ T-odd	$\gamma^+\gamma_5$ T-even	$\gamma^+\gamma_5$ T-odd	σ^{ij}, σ^{-+} T-even	σ^{ij}, σ^{-+} T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					*3
LT	f_{LT}			*2		
TT						

Twist-4 PDFs

Hadron \ Quark	γ^- T-even	γ^- T-odd	$\gamma^-\gamma_5$ T-even	$\gamma^-\gamma_5$ T-odd	σ^{i-} T-even	σ^{i-} T-odd
U	f_3					
L			g_{3L}			
T					$[h_{3T}]$	
LL	f_{3LL}					
LT						*4
TT						

New collinear PDFs

SK and Qin-Tao Song, PRD 103 (2021) 014025.

New fragmentation functions (FFs) for spin-1 hadrons

see arXiv:2201.05397

Corresponding fragmentation functions exist for the spin-1 haddrons

simply by changing function names and kinematical variables.

Collinear FFs:
X. Ji, PRD 49, 114 (1994).

TMD distribution functions: $f,\ g,\ h,\ e\ ;\quad x,\ k_T,\ S,\ T,\ M,\ n,\ \gamma^+,\ \sigma^{i+}$

$\Downarrow$

TMD fragmentation functions: $D,\ G,\ H,\ E\ ;\quad z,\ k_T,\ S_h,\ T_h,\ M_h,\ \bar{n},\ \gamma^-,\ \sigma^{i-}$

Shandong group:
K.-B Chen, W.-H. Ynag,
S.-Y. Wei, Z.-T. Liang,
PRD 94 (2016) 034003.

Collinear FFs, twist 2

Hadron \ Quark	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	D_1					
L			G_{1L}			
T					$[H_1]$	
LL	D_{1LL}					
LT						$[H_{1LT}]$
TT						

[ ] = chiral odd

Collinear FFs, twist 3

Hadron \ Quark	$\gamma^i, 1, i\gamma_5$ T-even	$\gamma^i, 1, i\gamma_5$ T-odd	$\gamma^i\gamma_5$ T-even	$\gamma^i\gamma_5$ T-odd	σ^{ij}, σ^{-+} T-even	σ^{ij}, σ^{-+} T-odd
U	$[E]$					
L					$[H_L]$	
T			G_T			
LL	$[E_{LL}]$					$[H_{LL}]$
LT	D_{LT}			G_{LT}		
TT						

Collinear FFs, twist 4

Hadron \ Quark	γ^- T-even	γ^- T-odd	$\gamma^-\gamma_5$ T-even	$\gamma^-\gamma_5$ T-odd	σ^{i-} T-even	σ^{i-} T-odd
U	D_3					
L			G_{3L}			
T					$[H_{3T}]$	
LL	D_{3LL}					
LT						$[H_{3LT}]$
TT						

TMD FFs, twist 2

Hadron \ Quark	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	D_1					$[H_1^\perp]$
L			G_{1L}		$[H_{1L}^\perp]$	
T		$D_{1T}^\perp$	G_{1T}		$[H_1], [H_{1T}^\perp]$	
LL	D_{1LL}					$[H_{1LL}^\perp]$
LT	D_{1LT}			G_{1LT}		$[H_{1LT}], [H_{1LT}^\perp]$
TT	D_{1TT}			G_{1TT}		$[H_{1TT}], [H_{1TT}^\perp]$

TMD FFs, twist 3

Hadron \ Quark	$\gamma^i, 1, i\gamma_5$ T-even	$\gamma^i, 1, i\gamma_5$ T-odd	$\gamma^i\gamma_5$ T-even	$\gamma^i\gamma_5$ T-odd	σ^{ij}, σ^{-+} T-even	σ^{ij}, σ^{-+} T-odd
U	$D^\perp$ $[E]$			$G^\perp$		$[H]$
L		$D_L^\perp$ $[E_L]$	$G_L^\perp$		$[H_L]$	
T		$D_T, D_T^\perp$ $[E_T, E_T^\perp]$	$G_T, G_T^\perp$		$[H_T], [H_T^\perp]$	
LL	$D_{LL}^\perp$ $[E_{LL}]$			$G_{LL}^\perp$		$[H_{LL}]$
LT	$D_{LT}, D_{LT}^\perp$ $[E_{LT}, E_{LT}^\perp]$			$G_{LT}, G_{LT}^\perp$		$[H_{LT}], [H_{LT}^\perp]$
TT	$D_{TT}, D_{TT}^\perp$ $[E_{TT}, E_{TT}^\perp]$			$G_{TT}, G_{TT}^\perp$		$[H_{TT}], [H_{TT}^\perp]$

TMD FFs, twist 4

Hadron \ Quark	γ^- T-even	γ^- T-odd	$\gamma^-\gamma_5$ T-even	$\gamma^-\gamma_5$ T-odd	σ^{i-} T-even	σ^{i-} T-odd
U	D_3					$[H_3^\perp]$
L			G_{3L}		$[H_{3L}^\perp]$	
T		$D_{3T}^\perp$	G_{3T}		$[H_{3T}], [H_{3T}^\perp]$	
LL	D_{3LL}					$[H_{3LL}^\perp]$
LT	D_{3LT}			G_{3LT}		$[H_{3LT}], [H_{3LT}^\perp]$
TT	D_{3TT}			G_{3TT}		$[H_{3TT}], [H_{3TT}^\perp]$

New TMD FFs

Analogous relations to Wandzura-Wilczek relation and Burkhardt-Cottingham sum rule

may skip

Twist-2 PDFs

Quark \ Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					
L			$g_{1L}(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}(b_1)$					
LT						*1
TT						

Twist-3 PDFs

Quark \ Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					*3
LT	f_{LT}			*2		
TT						

[ ] = chiral odd

We derived analogous relations to Wandzura-Wilczek relation and Burkhardt-Cottingham sum rule for f_{LT} and f_{1LL}.

SK and Qin-Tao Song, JHEP 09 (2021) 141.

For spin-1/2 nucleons,

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) \text{ (Wandzura-Wilczek relation)}, \quad \int_0^1 dx\, g_2(x) = 0 \text{ (Burkhardt-Cottingham sum rule)}$$

For tensor-polarized spin-1 hadrons, we obtained

$$f_{2LT}^{+}(x) = -f_{1LL}^{+}(x) + \int_x^1 \frac{dy}{y} f_{1LL}^{+}(y), \quad \int_0^1 dx\ f_{2LT}^{+}(x) = 0, \quad f_{2LT}(x) \equiv \frac{2}{3} f_{LT}(x) - f_{1LL}(x)$$

$$\int_0^1 dx\ f_{LT}^{+}(x) = 0 \ \text{ if } \int_0^1 dx\ f_{1LL}^{+}(x) = \frac{2}{3}\int_0^1 dx\ b_1^{+}(x) = 0$$

Existence of multiparton distribution functions: $F_{G,LT}(x_1,x_2)$, $G_{G,LT}(x_1,x_2)$, $H_{G,LL}^{\perp}(x_1,x_2)$, $H_{G,TT}(x_1,x_2)$

Relations from equation of motion and Lorentz-invariance relation for spin-1 hadrons

SK and Qin-Tao Song, PLB 826 (2022) 136908.

may skip

- $x f_{LT}(x) - \int_{-1}^{+1} dy \left[F_{D,LT}(x,y) + G_{D,LT}(x,y) \right] = 0, \quad x f_{LT}(x) - f_{1LT}^{(1)}(x) - \mathcal{P}\int_{-1}^{+1} dy \frac{F_{G,LT}(x,y) + G_{G,LT}(x,y)}{x-y} = 0$
- $x e_{LL}(x) - 2\int_{-1}^{+1} dy H_{D,LL}^{\perp}(x,y) - \frac{m}{M} f_{1LL}(x) = 0, \quad x e_{LL}(x) - 2\mathcal{P}\int_{-1}^{+1} dy \frac{H_{G,LL}^{\perp}(x,y)}{x-y} - \frac{m}{M} f_{1LL}(x) = 0$

and the Lorentz-invariance relation

- $\frac{d f_{1LT}^{(1)}(x)}{dx} - f_{LT}(x) + \frac{3}{2} f_{1LL}(x) - 2\mathcal{P}\int_{-1}^{+1} dy \frac{F_{G,LT}(x,y)}{(x-y)^2} = 0$

Lorentz invariance
= frame independence of twist-3 observables

transverse-momemtum moment of TMD: $f^{(1)}(x) = \int d^2 k_T \frac{\vec{k}_T^2}{2M^2} f(x, k_T^2)$

Twist-2 PDFs

Quark / Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					
L			g_{1L} (g_1)			
T					[h_1]	
LL	f_{1LL} (b_1)					
LT						
TT						

Twist-3 PDFs

Quark / Hadron	$\gamma^i, 1, i\gamma_5$ T-even	$\gamma^i, 1, i\gamma_5$ T-odd	$\gamma^+\gamma_5$ T-even	$\gamma^+\gamma_5$ T-odd	σ^{ij}, σ^{-+} T-even	σ^{ij}, σ^{-+} T-odd
U	[e]					
L					[h_L]	
T			g_T			
LL	[e_{LL}]					
LT	f_{LT}					*1
TT						

Twist-3 TMDs

Quark / Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					[$h_1^\perp$]
L			g_{1L}		[$h_{1L}^\perp$]	
T		$f_{1T}^\perp$	g_{1T}		[h_1], [$h_{1T}^\perp$]	
LL	f_{1LL}					[$h_{1LL}^\perp$]
LT	f_{1LT}			g_{1LT}		[h_{1LT}], [$h_{1LT}^\perp$]
TT	f_{1TT}			g_{1TT}		[h_{1TT}], [$h_{1TT}^\perp$]

[ ] = chiral odd

Relations on fragmentation functions

Qin-Tao Song,
PRD 108 (2023) 094041.

- $E_{LL}(z)+iH_{LL}(z)-\frac{m_q}{M}zD_{1LL}(z)=2z\left[-iH_{1LL}^{\perp(1)}(z)+\mathcal{P}\int_z^\infty\frac{dz_1}{(z_1)^2}\frac{H_{G,LL}^{\perp}(z,z_1)}{1/z-1/z_1}\right]$
- $D_{LT}(z)+iG_{LT}(z)+i\frac{m_q}{M}zH_{1LT}(z)=-z\left[iG_{1LT}^{(1)}(z)-\int_z^\infty\frac{dz_1}{(z_1)^2}\frac{G_{G,LL}(z,z_1)}{1/z-1/z_1}\right]-z\left[D_{1LT}^{(1)}(z)+\int_z^\infty\frac{dz_1}{(z_1)^2}\frac{D_{G,LT}(z,z_1)}{1/z-1/z_1}\right]$
- $iH_{1TT}^{(1)}(z)+\int_z^\infty\frac{dz_1}{(z_1)^2}\frac{H_{G,TT}(z,z_1)}{1/z-1/z_1}=0$
- $\frac{3}{2}D_{1LL}(z)-D_{LT}(z)-z\left(1-z\frac{d}{dz}\right)D_{1LT}^{(1)}(z)=-2\int_z^\infty\frac{dz_1}{(z_1)^2}\frac{\mathrm{Re}\left[D_{G,LT}(z,z_1)\right]}{(1/z-1/z_1)^2}$
- $H_{LL}(z)+2H_{1LT}(z)+z\left(1-z\frac{d}{dz}\right)H_{1LL}^{\perp(1)}(z)=-2\int_z^\infty\frac{dz_1}{(z_1)^2}\frac{\mathrm{Im}\left[H_{G,LL}^{\perp}(z,z_1)\right]}{(1/z-1/z_1)^2}$
- $G_{LT}(z)+z\left(1-z\frac{d}{dz}\right)G_{1LT}^{(1)}(z)=-2\int_z^\infty\frac{dz_1}{(z_1)^2}\frac{\mathrm{Im}\left[H_{G,LT}(z,z_1)\right]}{(1/z-1/z_1)^2}$

Twist-2 TMD FFs

Twist-2 FFs

[ ] = chiral odd

Quark / Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5/\sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_1					
L			G_{1L}			
T					$[H_1]$	
LL	D_{1LL}					
LT						$[H_{1LT}]$
TT						

Twist-3 FFs

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^i\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[E]$					
L					$[H_L]$	
T			G_T			
LL	$[E_{LL}]$					$[H_{LL}]$
LT	D_{LT}			G_{LT}		
TT						

Twist-2 TMD FFs

Quark / Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5/\sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_1					$[H_1^{\perp}]$
L			G_{1L}		$[H_{1L}^{\perp}]$	
T		$D_{1T}^{\perp}$	G_{1T}		$[H_1]$, $[H_{1T}^{\perp}]$	
LL	D_{1LL}					$[H_{1LL}^{\perp}]$
LT	D_{1LT}			G_{1LT}		$[H_{1LT}]$ $[H_{1LT}^{\perp}]$
TT	D_{1TT}			G_{1TT}		$[H_{1TT}]$ $[H_{1TT}^{\perp}]$

Future prospects and summary

Recent progress

Twist-3 PDFs

Hadron \ Quark	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					
LT	f_{LT}					
TT						

Twist-2 TMDs

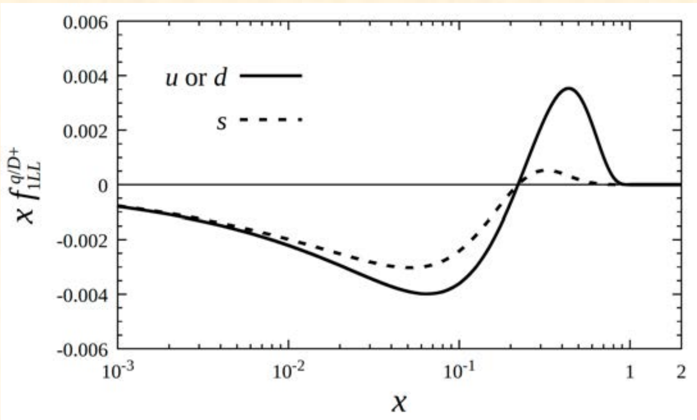
Hadron \ Quark	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

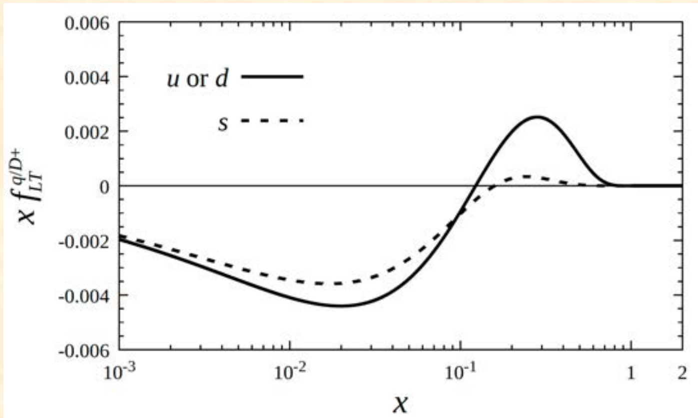
- **Drell-Yan: Si-Yi Qiao and Qin-Tao Song, PRD 111 (2025) 054026**
- **Semi-inclusive DIS: J. Zhao, A. Bacchetta, S. Kumano, T. Liu, and Y.-J. Zhou, JHEP 12 (2025) 067; W. Cosyn and C. Weiss, arXv:2603.23699, 2603.23700.**

- **f_{LT}(x) estimate: S. Kumano, K. Kuroki, PLB 875 (2026) 140348.**
- **OPE derivation of twist-2 relations, arXiv:2605.00430.**

Wandzura-Wilczek-like relation : $f_{LT}^{q^+}(x) = \frac{3}{2}\int_x^2 \frac{dy}{y} f_{1LL}^{q^+}(y)$

$$f_{1LL}^{q^+} = f_{1LL}^{q} + f_{1LL}^{\bar{q}} = -\frac{2}{3}\left(\delta_T q + \delta_T \bar{q}\right) = -\frac{2}{3}\left(b_1^q + b_1^{\bar{q}}\right)$$





Other works on spin-1

I may miss your papers.

GPDs

E.R. Berger, F. Cano, M. Diehl, B. Pire, Phys. Rev. Lett. 87, 142302 (2001);
W. Cosyn, B. Pire, Phys. Rev. D 98, 074020 (2018).

Lightcone models on ρ and D

B.D. Sun, Y.B. Dong, Phys. Rev. D 96, 036019 (2017); 99, 016023 (2019); 101, 096008 (2020);
N. Kumar, Phys. Rev. D 99, 014039 (2019);
S. Kaur, C. Mondal, X. Zhao, C.-R. Ji, arXiv:2507.09886.

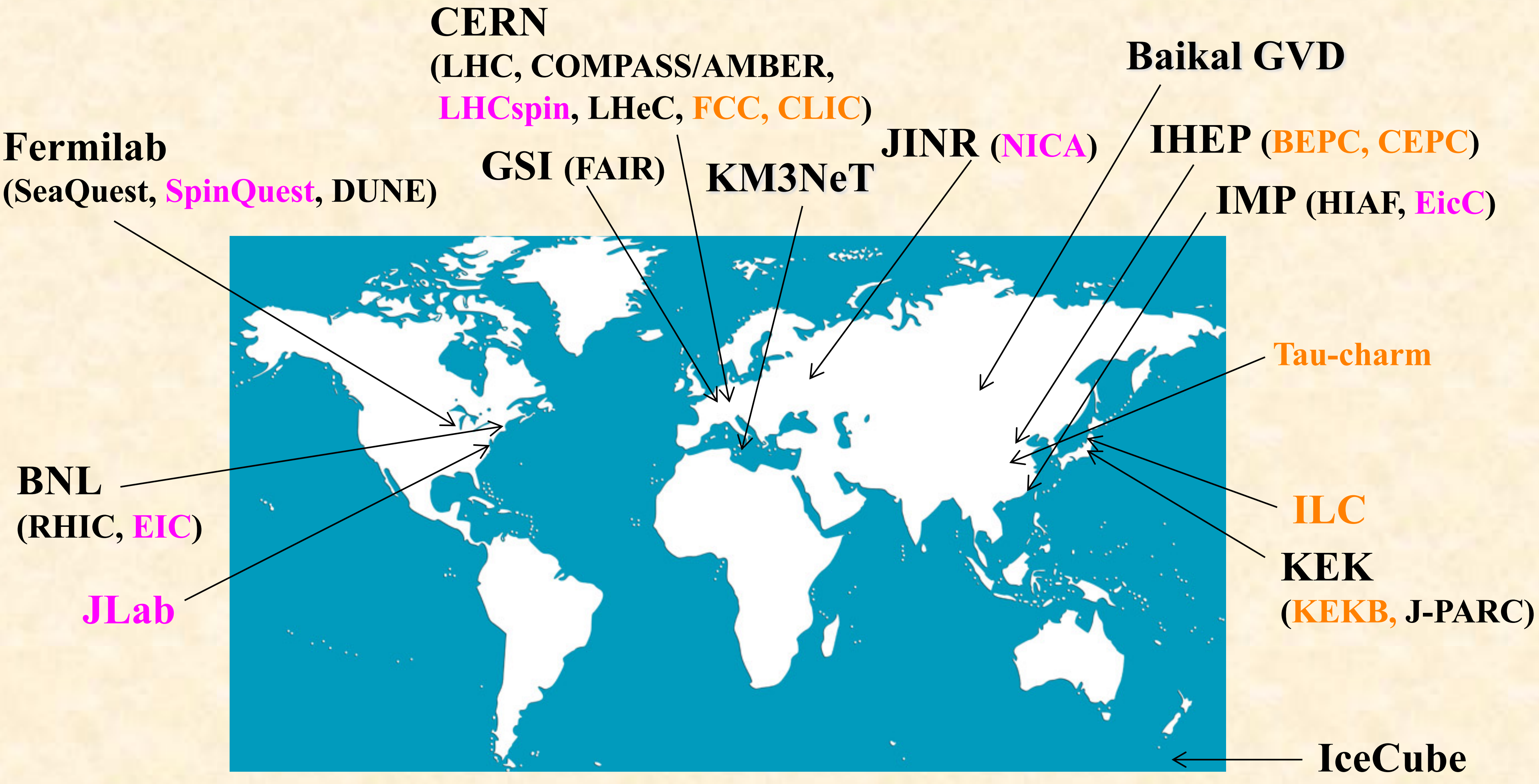
Lightcone models on TMDs

C. Shi, et al., Phys. Rev. D 106, 014026 (2022);
S. Kaur, et al., Phys. Lett. B 851, 138563 (2024).

Spin-3/2

J. Zhao, Z. Zhang, Z.-T. Liang, T. Liu, Y.-J Zhou, Phys. Rev. D 106 (2022) 094006; 109 (2024) 074017;
D. Fu, B.-D. Sun, Y.-B. Dong, Phys. Rev. D, 106 (2022) 116012; 10 (2023) 116021;
D.-Y. Fu, Y.-B. Dong, S. Kumano, Phys. Rev. D 109 (2024) 096006; 112 (2025) 096027;
D.-Y. Fu, Y.-B. Dong, S. Kumano, J.-J. Xie, arXiv:2602.11587 (PRD in press).

High-energy hadron physics experiments



Facilities on spin-1 hadron structure functions including future possibilities (time-like processes).

Spin-1 deuteron experiments from the middle of 2020's

JLab

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-38.
(Update to LOI-11-003)

J.-P. Chen (co-spokesperson), P. Solvignon (co-spokesperson), K. Allada, A. Camsonne, A. Deur, D. Gaskell, C. Keith, S. Wood, J. Zhang
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

N. Kalantarians (co-spokesperson), O. Rondon (co-spokesperson) Donal B. Day, Hovhannes Baghdasaryan, Charles Hanretty Richard Lindgren, Blaine Norum, Zhihong Ye
University of Virginia, Charlottesville, VA 22903

K. Slifer† (co-spokesperson), A. Atkins, T. Badman, J. Calarco, J. Maxwell, S. Phillips, R. Zielinski
University of New Hampshire, Durham, NH 03861

J. Dunne, D. Dutta
Mississippi State University, Mississippi State, MS 39762

G. Ron
Hebrew University of Jerusalem, Jerusalem

W. Bertozzi, S. Gilad, A. Kelleher, V. Sulkosky
Massachusetts Institute of Technology, Cambridge, MA 02139

K. Adhikari
Old Dominion University, Norfolk, VA 23529

R. Gilman
Rutgers, The State University of New Jersey, Piscataway, NJ 08854

Seonho Choi, Hoyoung Kang, Hyekoo Kang, Yoomin Oh
Seoul National University, Seoul 151-747 Korea

Proposal (approved),
Experiment: late 2020's

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan
Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon
University of Virginia, Charlottesville, VA 22904

J. Pierce
Oak Ridge National Laboratory, Oak Ridge, TN 37831

Fermilab

The Transverse Structure of the Deuteron with Drell-Yan

D. Keller[1]
[1] *University of Virginia, Charlottesville, VA 22904*

Proposal,
Fermilab-PAC: 2022
Experiment: 2020's

NICA

Progress in Particle and Nuclear Physics 119 (2021) 103858
Contents lists available at ScienceDirect
Progress in Particle and Nuclear Physics
journal homepage: www.elsevier.com/locate/ppnp
ELSEVIER

Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

A. Arbuzov[a], A. Bacchetta[b,c], M. Butenschoen[d], F.G. Celiberto[b,c,e,f], U. D'Alesio[g,h], M. Deka[a], I. Denisenko[a], M.G. Echevarria[i], A. Efremov[a], N.Ya. Ivanov[a,j], A. Guskov[a,k,*], A. Karpishkov[l,a], Ya. Klopot[a,m], B.A. Kniehl[d], A. Kotzinian[j,o], S. Kumano[p], J.P. Lansberg[q], Keh-Fei Liu[r], F. Murgia[h], M. Nefedov[l], B. Parsamyan[a,n,o], C. Pisano[g,h], M. Radici[c], A. Rymbekova[a], V. Saleev[l,a], A. Shipilova[l,a], Qin-Tao Song[s], O. Teryaev[a]

Prog. Nucl. Part. Phys.
119 (2021) 103858,
Experiment: middle of 2020's

LHCspin

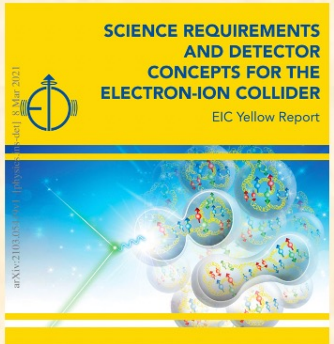
CERN-ESPP-Note-2018-111

The LHCSpin Project

C. A. Aidala[1], A. Bacchetta[2,3], M. Boglione[4,5], G. Bozzi[2,3], V. Carassiti[6,7], M. Chiosso[4,5], R. Cimino[8], G. Ciullo[6,7], M. Contalbrigo[6,7], U. D'Alesio[9,10], P. Di Nezza[8], R. Engels[11], K. Grigoryev[11], D. Keller[12], P. Lenisa[6,7], S. Liuti[12], A. Metz[13], P.J. Mulders[14,15], F. Murgia[10], A. Nass[11], D. Panzieri[5,16], L. L. Pappalardo[6,7], B. Pasquini[2,3], C. Pisano[9,10], M. Radici[3], F. Rathmann[11], D. Reggiani[17], M. Schlegel[18], S. Scopetta[19,20], E. Steffens[21], A. Vasilyev[22]

arXiv:1901.08002,
Experiment: ~2028

2030's EIC/EicC



R. Abdul Khalek *et al.*
Nucl. Phys. A 1026 (2022) 122447.

D. P. Anderle *et al.*,
Front. Phys. 16 (2021) 64701.

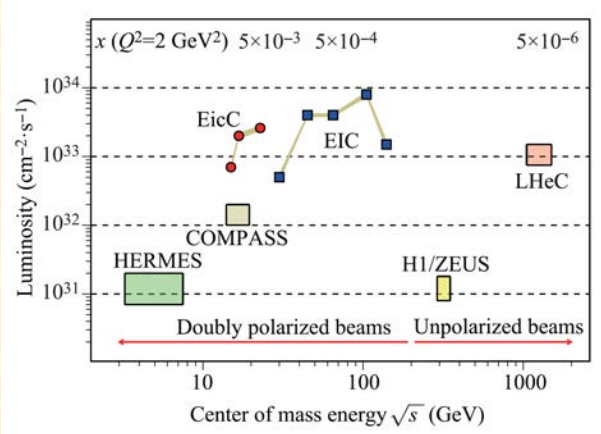
Frontiers of Physics
https://doi.org/10.1007/s11467-021-1062-0
Front. Phys. 16(6), 64701 (2021)

REVIEW ARTICLE

Electron-ion collider in China

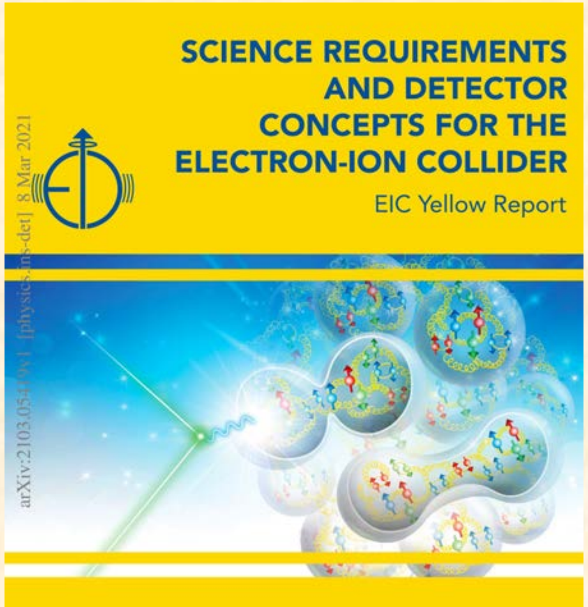
Daniele P. Anderle[1], Valerio Bertone[2], Xu Cao[3,4], Lei Chang[5], Ningbo Chang[6], Gu Chen[7], Xurong Chen[3,4], Zhuojun Chen[8], Zhufang Cui[9], Lingyun Dai[8], Weitian Deng[10], Minghui Ding[11], Xu Feng[12], Chang Gong[12], Longcheng Gui[13], Feng-Kun Guo[4,14], Chengdong Han[3,4], Jun He[15], Tie-Jiun Hou[16], Hongxia Huang[15], Yin Huang[17], KrešImir KumeričKi[18], L. P. Kaptari[3,19], Demin Li[20], Hengne Li[1], Minxiang Li[3,21], Xueqian Li[5], Yutie Liang[3,4], Zuotang Liang[22], Chen Liu[22], Chuan Liu[12], Guoming Liu[1], Jie Liu[3,4], Liuming Liu[3,4], Xiang Liu[21], Tianbo Liu[22], Xiaofeng Luo[23], Zhun Lyu[24], Boqiang Ma[12], Fu Ma[3,4], Jianping Ma[4,14], Yugang Ma[4,25,26], Lijun Mao[3,4], Cédric Mezrag[2], Hervé Moutarde[2], Jialun Ping[15], Sixue Qin[27], Hang Ren[3,4], Craig D. Roberts[9], Juan Rojo[28,29], Guodong Shen[3,4], Chao Shi[30], Qintao Song[20], Hao Sun[31], Paweł Sznajder[32], Enke Wang[1], Fan Wang[9], Qian Wang[1], Rong Wang[3,4], Ruiru Wang[3,4], Taofeng Wang[33], Wei Wang[34], Xiaoyu Wang[20], Xiaoyun Wang[35], Jiajun Wu[4], Xinggang Wu[27], Lei Xia[36], Bowen Xiao[23,37], Guoqing Xiao[3,4], Ju-Jun Xie[3,4], Yaping Xie[3,4], Hongxi Xing[1], Hushan Xu[3,4], Nu Xu[3,4,23], Shusheng Xu[38], Mengshi Yan[12], Wenbiao Yan[36], Wencheng Yan[20], Xinhu Yan[39], Jiancheng Yang[3,4], Yi-Bo Yang[4,14], Zhi Yang[40], Deliang Yao[8], Zhihong Ye[41], Peilin Yin[38], C.-P. Yuan[42], Wenlong Zhan[3,4], Jianhui Zhang[43], Jinlong Zhang[22], Pengming Zhang[44], Yifei Zhang[36], Chao-Hsi Chang[4,14], Zhenyu Zhang[45], Hongwei Zhao[3,4], Kuang-Ta Chao[12], Qiang Zhao[4,46], Yuxiang Zhao[3,4], Zhengguo Zhao[36], Liang Zheng[47], Jian Zhou[22], Xiang Zhou[45], Xiaorong Zhou[36], Bingsong Zou[4,14], Liping Zou[3,4]

Electron-ion collider projects in the world



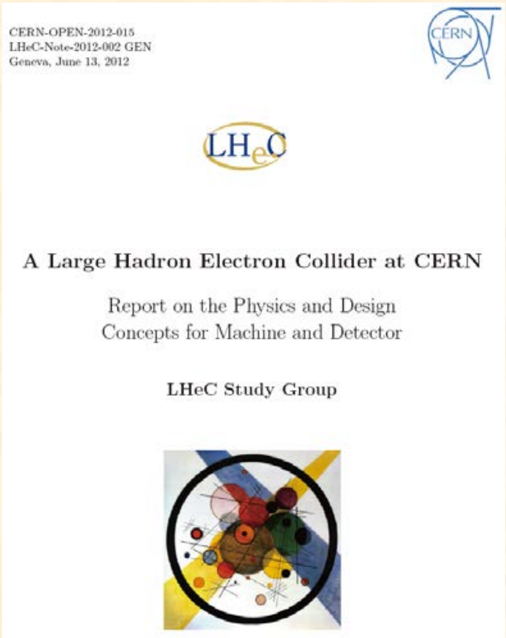
EIC-US

R. Abdul Khalek *et al.*, arXiv:2103.05419.



LHeC

J. L. Abelleira Fernandez *et al.*, J. Phys. G: Nucl. Part. Phys. 39 (2012) 075001.


CERN-OPEN-2012-015
LHeC-Note-2012-002 GEN
Geneva, June 13, 2012

LHeC

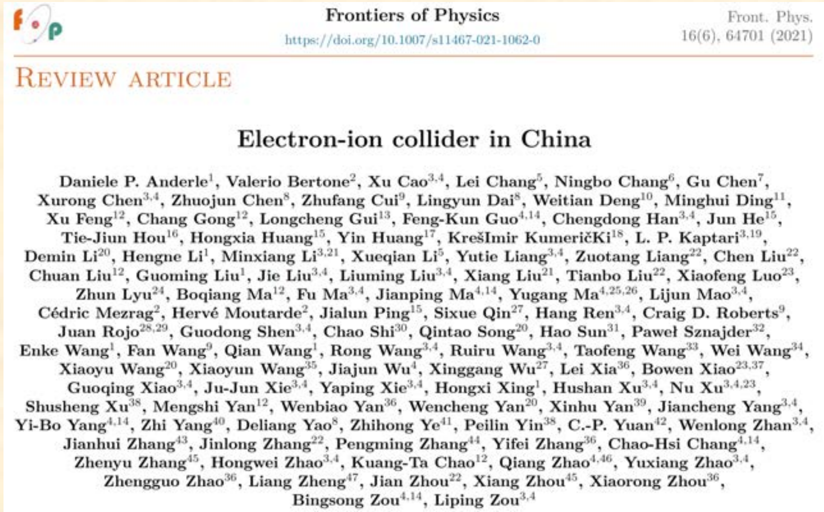
A Large Hadron Electron Collider at CERN

Report on the Physics and Design Concepts for Machine and Detector

LHeC Study Group

Institute of Modern Physics, High Intensity Heavy Ion Accelerator Facility (HIAF)
→ Electron-ion collider in China (EicC)

D. P. Anderle *et al.*, Front. Phys. 16 (2021) 64701.


Frontiers of Physics
https://doi.org/10.1007/s11467-021-1062-0
Front. Phys. 16(6), 64701 (2021)

REVIEW ARTICLE

Electron-ion collider in China

Daniele P. Anderle[1], Valerio Bertone[2], Xu Cao[3,4], Lei Chang[5], Ningbo Chang[6], Gu Chen[7], Xurong Chen[3,4], Zhuojun Chen[8], Zhufang Cui[9], Lingyun Dai[8], Weitian Deng[10], Minghui Ding[11], Xu Feng[12], Chang Gong[12], Longcheng Gui[13], Feng-Kun Guo[4,14], Chengdong Han[3,4], Jun He[15], Tie-Jiun Hou[16], Hongxia Huang[15], Yin Huang[17], KrešImir KumeričKi[18], L. P. Kaptari[3,19], Demin Li[20], Hengne Li[1], Minxiang Li[3,21], Xueqian Li[5], Yutie Liang[3,4], Zuotang Liang[22], Chen Liu[22], Chuan Liu[12], Guoming Liu[1], Jie Liu[3,4], Liuming Liu[3,4], Xiang Liu[21], Tianbo Liu[22], Xiaofeng Luo[23], Zhun Lyu[24], Boqiang Ma[12], Fu Ma[3,4], Jianping Ma[4,14], Yugang Ma[4,25,26], Lijun Mao[3,4], Cédric Mezrag[2], Hervé Moutarde[2], Jialun Ping[15], Sixue Qin[27], Hang Ren[3,4], Craig D. Roberts[9], Juan Rojo[28,29], Guodong Shen[3,4], Chao Shi[30], Qintao Song[20], Hao Sun[31], Paweł Sznajder[32], Enke Wang[1], Fan Wang[9], Qian Wang[1], Rong Wang[3,4], Ruiru Wang[3,4], Taofeng Wang[33], Wei Wang[34], Xiaoyu Wang[20], Xiaoyun Wang[35], Jiajun Wu[4], Xinggang Wu[27], Lei Xia[36], Bowen Xiao[23,37], Guoqing Xiao[3,4], Ju-Jun Xie[3,4], Yaping Xie[3,4], Hongxi Xing[1], Hushan Xu[3,4], Nu Xu[3,4,23], Shusheng Xu[38], Mengshi Yan[12], Wenbiao Yan[36], Wencheng Yan[20], Xinhu Yan[39], Jiancheng Yang[3,4], Yi-Bo Yang[4,14], Zhi Yang[40], Deliang Yao[8], Zhihong Ye[41], Peilin Yin[38], C.-P. Yuan[42], Wenlong Zhan[3,4], Jianhui Zhang[43], Jinlong Zhang[22], Pengming Zhang[44], Yifei Zhang[36], Chao-Hsi Chang[4,14], Zhenyu Zhang[45], Hongwei Zhao[3,4], Kuang-Ta Chao[12], Qiang Zhao[4,46], Yuxiang Zhao[3,4], Zhengguo Zhao[36], Liang Zheng[47], Jian Zhou[22], Xiang Zhou[45], Xiaorong Zhou[36], Bingsong Zou[4,14], Liping Zou[3,4]

Summary

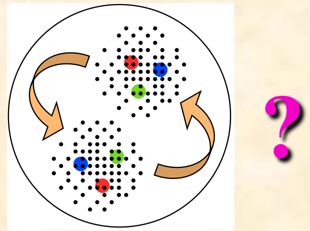
Spin-1 structure functions of the deuteron (additional spin structure to nucleon spin)

- Tensor structure in quark-gluon degrees of freedom
- Tensor-polarized structure function b_1 and PDFs, gluon transversity

Experiments at JLab, Fermilab, NICA, LHCspin/AMBER, EIC/EicC, • • •

- New signature beyond "standard" hadron physics?

(beyond the standard model in particle physics???)



- TMDs up to twist 4
- Higher-twist effects could be sizable at a few GeV^2 Q^2

There are various experimental projects on the polarized spin-1 deuteron in 2020's and 2030', and "exotic" hadron structure could be found by focusing on the spin-1 nature.

Comment: There is no nuclear effect in ρ and φ mesons, so that the gluon transversity, for example, could be sensitive to new physics?!

The End

The End