The Deuteron Tensor Structure Function b_1 from a Composite Structure Light-Front Wigner Formalism

Master correlator and Wigner decomposition

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Outline

- **Part 1**: Derivation of master formula for composite structure in a quark TMD correlator of a deuteron target
- **Part 2**: Spin tensorial decomposition of Wigner distribution in master correlator
- **Part 3**: Extracting $b_1(x)$ from the master correlator and prediction

Part 1: Derivation of master convolution formula

- Exact Fock-state decomposition of the deuteron TMD correlator
- Impulse approximation and QCD lensing
- Inclusive light-front wave function $\widetilde{\Psi}$
- Single-nucleon Wigner distribution W_{S_D,s_N}
- Master convolution formula

Starting point: leading-twist quark TMD correlator

The leading-twist TMD correlator for a spin-1 target with momentum P, spin S:

$$\Phi_{ij,D}(x, \boldsymbol{k}_\perp; P, S) = \int \frac{dr^- d^2 r_\perp}{(2\pi)^3}\, e^{i(k^+ r^- - \boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp)} \times {}_{\mathrm{in}}\langle P, S | \bar{\psi}_j(0)\, \mathcal{U}[0; r]\, \psi_i(r) | P, S \rangle_{\mathrm{in}}$$

- $x = k^+/P^+$: quark light-front momentum fraction
- $\mathcal{U}[0; r]$: gauge link (Wilson line)
- "in" states: Heisenberg picture

Steps: resolve the target matrix element via complete sets of Fock states.

Fock-state decomposition

Insert two complete sets of in-states:

$$\hat{\mathbf{1}} = \sum_n \frac{1}{S_n} \prod_{i=1}^{n} \int \frac{d^2 p_{i\perp}\, dp_i^+}{(2\pi)^3\, 2p_i^+} \,|p_1 \cdots p_n\rangle\langle p_1 \cdots p_n|$$

The target matrix element decomposes:

$$_{\rm in}\langle P, S|\, \bar{\psi}_j \mathcal{U} \psi_i \,|P, S\rangle_{\rm in} = \sum_{X,Y} {}_{\rm in}\langle P, S|X\rangle_{\rm in}\; {}_{\rm in}\langle X|\bar{\psi}_j \mathcal{U} \psi_i|Y\rangle_{\rm in} \times {}_{\rm in}\langle Y|P, S\rangle_{\rm in}$$

No QCD Lensing, should be included since it's a known effect. Just don't want complications for scope here.

Active–spectator decomposition

Partition each Fock state into **active** (struck by the quark bilinear) and **spectator** sectors:

$$|X\rangle = |\{P'_n\}\rangle \otimes |X'\rangle$$

Assumption 1: Impulse approximation

Quark bilinear acts only on active constituents; no quantum-number exchange with spectators.

$$\langle X|\bar{\psi}_j \mathcal{U} \psi_i|Y\rangle = \langle \{P'_n\}|\bar{\psi}_j \mathcal{U} \psi_i|\{P_n\}\rangle \ \times \ {}_{\rm in}\langle X'|Y'\rangle_{\rm in}$$

Assumption 2: Spectator gauge link

Wilson line does not scatter off spectators. Spectator interactions are kept in $\langle X'|Y'\rangle_{\rm in}$.

Hadronic amplitudes: light-front wave function

Overlap of the target state with a Fock state defines the LFWF:

$$
{}_{\rm in}\langle Y|P,S\rangle_{\rm in} = (2\pi)^3\, 2P^+\, \delta^{(2)}\Big(\sum_i \vec{P}_{Yi\perp} - \vec{P}_\perp\Big)\delta\Big(\sum_i P^+_{Yi} - P^+\Big)
$$
$$
\times\, \Psi_S(\{P^+_{Yi}/P^+\};\, \{\vec{P}_{Yi\perp}\})
$$

Spectator overlap (kept in full generality):

$$
{}_{\rm in}\langle X'|Y'\rangle_{\rm in} \;\propto\; \mathcal{S}^{\{s_{X'}\}}_{\{s_{Y'}\}}\big(\{P_{X'}\},\{P_{Y'}\}\big)
$$

- $\mathcal{S} = \mathbf{1}$ for free spectators
- Encodes all interactions among spectator
- **Not** restricted to free-particle final states

Inclusive light-front wave function

Defined by summing over all spectator configurations:

$$\widetilde{\Psi}_S^*(\{P_n'\}; P)\,\widetilde{\Psi}_S(\{P_n\}; P) \equiv \frac{1}{(2\pi)^3} \sum_{X',Y'} \int \Psi_S^*(X', \{P_n'\})\,\Psi_S(Y', \{P_n\})\,\mathcal{S}(X', Y')$$

$\otimes$ momentum-conservation deltas $\otimes$ phase-space factors

Interpretation

$|\widetilde{\Psi}|^2$ is an **inclusive** probability density: the probability to find the active constituent with momentum P_n inside the target, **summed over all spectator states**.

Crucial: the spectator overlap $\mathcal{S}$ is retained throughout. Final-state interactions among spectators are not neglected — they live inside $\mathcal{S}$.

Single active constituent: nucleon in deuteron

For the nucleonic Fock sector, the active constituent is one nucleon with momentum P_N, spin s_N:

$$
{}_{\rm in}\langle P,S|\bar{\psi}_j\mathcal{U}\psi_i|P,S\rangle_{\rm in} = \sum_{s_N}^{U,L,T}\int \frac{dP_N^+ d^2\boldsymbol{P}_{N\perp}}{2P_N^+}\underbrace{\frac{P^+}{P_N^+}\cdot\frac{P^+-P_N^+}{P^+}}_{=(1-\alpha)/\alpha} \times |\widetilde{\Psi}_S(P_N,s_N;P)|^2\cdot\langle P_N,s_N|\bar{\psi}_j\mathcal{U}\psi_i|P_N,s_N\rangle
$$

- Result of NO QCD Lensing
- $\alpha \equiv P_N^+/P^+$: light-front fraction of active nucleon
- $(1-\alpha)/\alpha$: kinematic terms

Wigner distribution: definition

Fourier transform the inclusive LFWF with respect to a momentum transfer Δ:

$$W_{S_D,s_N}(P_N, b; P) = \int \frac{d\Delta^+ d^2\Delta_\perp}{(2\pi)^3}\, e^{-i(b^-\Delta^+ - \boldsymbol{b}_\perp\cdot\Delta_\perp)} \times \frac{\widetilde{\Psi}^*_S(P_N + \frac{\Delta}{2}, s_N)}{\sqrt{2(P_N^+ + \Delta^+/2)}} \cdot \frac{\widetilde{\Psi}_S(P_N - \frac{\Delta}{2}, s_N)}{\sqrt{2(P_N^+ - \Delta^+/2)}}$$

Two indices:

- Target spin S: deuteron polarization
- Constituent spin s_N: nucleon spin projection

Projections recover familiar densities:

- $\int db^- d^2 b_\perp\, W \;=\; |\widetilde{\Psi}|^2/(2P_N^+)$ (momentum density)
- $\int dP_N^+ d^2\boldsymbol{P}_{N\perp}\, W \;=\;$ impact-parameter density

Embedded vs. intrinsic nucleon correlator

The quark momentum in the deuteron correlator decomposes relative to the active nucleon:

$$x' \equiv \frac{k^+}{P_N^+} = \frac{x}{\alpha}, \qquad \boldsymbol{k}_\perp = \boldsymbol{k}'_\perp + x' \, \boldsymbol{P}_{N\perp}$$

where $\boldsymbol{k}'_\perp$ is *intrinsic* to the active nucleon.

Embedded correlator

$$\Phi_{ij,N/D}(x', \boldsymbol{k}_\perp; P_N, s_N) = \int \frac{dr^- d^2 r_\perp}{(2\pi)^3} \, e^{i(x' P_N^+ r^- - \boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp)} \langle P_N, s_N | \bar{\psi}_j \mathcal{U} \psi_i | P_N, s_N \rangle$$

Intrinsic correlator

$$\Phi_{ij,N/D}(x', \boldsymbol{k}_\perp; P_N, s_N) \equiv \Phi_{ij,N}(x', \boldsymbol{k}'_\perp; P_N, s_N) \text{ wrt nucleon}$$

Master formula

Assembling all pieces:

$$\Phi_{ij,D}(x,\boldsymbol{k}_\perp;P,S_D) = \sum_{s_N} \int dP_N^+\, d^2\boldsymbol{P}_{N\perp}\, db^-\, d^2b_\perp \left(\frac{1-\alpha}{\alpha}\right) W_{S_D,s_N}(P_N,b;P) \times \Phi_{ij,N}\left(\frac{x}{\alpha}, \boldsymbol{k}_\perp - \frac{x}{\alpha}\boldsymbol{P}_{N\perp}; P_N, s_N\right)$$

The deuteron TMD correlator is a convolution of two pieces:

- **Nuclear physics:** W_{S_D,s_N} — Wigner distribution of nucleon in deuteron
- **Partonic physics:** $\Phi_{ij,N}$ — quark correlator in the nucleon

Two assumptions only: impulse approximation + spectator gauge link (removes extra function in convolution, QCD LENSING!).

Part 2: Tensorial decomposition and projection to polarization channels

- Spin structures of W_{s_D,s_N} in the rest frame
- $O(3)$ + P + T constraints
- 17 amplitudes: scalar, vector, tensor sectors
- Tensor sector: 9 structures, physical meaning

Wigner structure: vocabulary

In the deuteron rest frame, $W_{S_D,s_N}(\boldsymbol{p},\boldsymbol{b})$ is a scalar function of:

Phase-space vectors

- $\boldsymbol{p}$: nucleon momentum (in deuteron)
- $\boldsymbol{b}$: nucleon position (in deuteron)

Spin structure

- $\boldsymbol{\sigma}$: nucleon spin (constituent, spin-$\frac{1}{2}$)
- $\boldsymbol{S}$: deuteron vector polarization
- $Q_{ij} = S_i S_j - \frac{1}{3}\delta_{ij}\boldsymbol{S}^2$: tensor polarization

Symmetries enforced

- $O(3)$ rotational invariance (in rest frame)
- Parity: $\boldsymbol{p} \to -\boldsymbol{p}$, $\boldsymbol{b} \to -\boldsymbol{b}$; spins unchanged
- Time reversal: no QCD lensing $\Rightarrow$ T-even Wigner

Allowed scalars

$$\boldsymbol{p}^2, \quad \boldsymbol{b}^2, \quad \boldsymbol{p}\cdot\boldsymbol{b}$$

Allowed axial vector

$$\boldsymbol{p}\times\boldsymbol{b} \quad \text{(orbital direction)}$$

Spin decomposition: 17 independent amplitudes

NOTATION: F_i, G_i and H_i → coefficients

$$W(\boldsymbol{p},\boldsymbol{b}) = \underbrace{F_1 + F_2\,\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{b})}_{\text{scalar: } 2}$$

$$+\underbrace{G_1\,\boldsymbol{S}\cdot(\boldsymbol{p}\times\boldsymbol{b}) + G_2\,\boldsymbol{\sigma}\cdot\boldsymbol{S} + G_3\,(\boldsymbol{\sigma}\cdot\boldsymbol{p})(\boldsymbol{S}\cdot\boldsymbol{p}) + G_4\,(\boldsymbol{\sigma}\cdot\boldsymbol{b})(\boldsymbol{S}\cdot\boldsymbol{b})}_{\text{vector: } 4}$$

$$+\underbrace{G_5\,(\boldsymbol{p}\cdot\boldsymbol{b})(\boldsymbol{\sigma}\cdot\boldsymbol{p})(\boldsymbol{S}\cdot\boldsymbol{b}) + G_6\,(\boldsymbol{p}\cdot\boldsymbol{b})(\boldsymbol{\sigma}\cdot\boldsymbol{b})(\boldsymbol{S}\cdot\boldsymbol{p})}_{\text{vector: } 2}$$

$$+\underbrace{H_1\,Q_{ij}p_ip_j + H_2\,Q_{ij}b_ib_j + H_3\,(\boldsymbol{p}\cdot\boldsymbol{b})\,Q_{ij}(p_ib_j + p_jb_i)}_{\textbf{tensor spin-independent: 3}}$$

$$+\underbrace{H_4\,\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{b})\,Q_{ij}p_ip_j + H_5\,\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{b})\,Q_{ij}b_ib_j + H_6\,Q_{ij}\sigma_i(\boldsymbol{p}\times\boldsymbol{b})_j}_{\textbf{tensor spin-dependent: 3}}$$

$$+\underbrace{H_7\,(\boldsymbol{p}\cdot\boldsymbol{b})\,\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{b})\,Q_{ij}p_ib_j + H_8\,(\boldsymbol{p}\cdot\boldsymbol{b})\,Q_{ij}(\boldsymbol{p}\times\boldsymbol{\sigma})_ip_j + H_9\,(\boldsymbol{p}\cdot\boldsymbol{b})\,Q_{ij}(\boldsymbol{b}\times\boldsymbol{\sigma})_ib_j}_{\textbf{tensor spin-dependent: 3}}$$

Total: **2** scalar + **6** vector + **9** tensor = **17** structures.

Tensor sector: structural taxonomy

Nine tensor terms organize into patterns:

	Spin-independent (no $\boldsymbol{\sigma}$)	**Spin-orbit coupling** ($\times\,\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{b})$)	**Direct σ contraction** (σ_i into Q_{ij})
Momentum proj. $Q_{ij}p_ip_j$	H_1	H_4	—
Position proj. $Q_{ij}b_ib_j$	H_2	H_5	—
Mixed $Q_{ij}p_ib_j$, radial-weighted	H_3	H_7	H_8, H_9
Direct $Q_{ij}\sigma_i(\cdots)_j$	—	—	H_6

- Three **tensor contractions**: p_ip_j, b_ib_j, $p_ib_j + p_jb_i$
- Three ways the **nucleon spin** enters: absent, scalar spin-orbit factor, direct σ contraction
- Spin-orbit coupling $\leftrightarrow$ base structures via multiplication by $\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{b})$
- H_6, H_8, H_9 are **irreducible**: no spin-independent counterpart

Tensor sector I: spin-independent structures

Three terms with no $\boldsymbol{\sigma}$. The deuteron quadrupole probed by a spin-blind nucleon.

$H_1\, Q_{ij} p_i p_j$ — momentum quadrupole

Couples the deuteron's tensor polarization to the *nucleon momentum direction*.

Physical picture: nucleons in a tensor-polarized deuteron prefer to move along the alignment axis.

$H_2\, Q_{ij} b_i b_j$ — position quadrupole

Couples the deuteron's tensor polarization to the *nucleon spatial direction*.

$H_3\, (\boldsymbol{p} \cdot \boldsymbol{b})\, Q_{ij}(p_i b_j + p_j b_i)$ — mixed quadrupole

Off-diagonal contraction tying nucleon momentum and position, weighted by radial flow.

Tensor sector II: spin-orbit partners

Three terms with the scalar nucleon spin-orbit factor $\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{b}) = \boldsymbol{\sigma}_N\cdot\boldsymbol{L}_N$.

$H_4\,\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{b})\,Q_{ij}p_ip_j$

Nucleon spin-orbit coupling *modulated* by the deuteron quadrupole along $\boldsymbol{p}$.

$H_5\,\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{b})\,Q_{ij}b_ib_j$

Same spin-orbit, modulated by quadrupole along $\boldsymbol{b}$. Partner of H_4 under $\boldsymbol{p}\leftrightarrow\boldsymbol{b}$.

$H_7\,(\boldsymbol{p}\cdot\boldsymbol{b})\,\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{b})\,Q_{ij}p_ib_j$

Spin-orbital partner of H_3: radial flow + spin-orbit + mixed quadrupole, all in one.

Each of H_4, H_5, H_7 = (corresponding spin-indep. term) $\times\,\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{b})$.

Tensor sector III: irreducible spin couplings

Three terms where the nucleon spin enters by contracting *directly* into Q_{ij}.

$H_6 \, Q_{ij}\sigma_i(\boldsymbol{p}\times\boldsymbol{b})_j$

Quadrupole *links* nucleon spin to its orbital direction.

Spin-1 analog of $G_2\,\boldsymbol{\sigma}\cdot\boldsymbol{S}$ (vector polarization): irreducible spin-tensor coupling.

$H_8\,(\boldsymbol{p}\cdot\boldsymbol{b})\,Q_{ij}(\boldsymbol{p}\times\boldsymbol{\sigma})_i p_j$

Quadrupole links the nucleon's *spin-momentum* axial vector $\boldsymbol{p}\times\boldsymbol{\sigma}$ to its momentum.

$H_9\,(\boldsymbol{p}\cdot\boldsymbol{b})\,Q_{ij}(\boldsymbol{b}\times\boldsymbol{\sigma})_i b_j$

Position-space partner of H_8: quadrupole couples nucleon *spin-position* axial vector to its position.

No spin-independent counterpart exists for H_6, H_8, H_9.

Part 3: Extracting $b_1(x)$ from master convolution formula

- Unpolarized quark TMD in the deuteron
- Spin-1 tensor combination $\delta_T q_D^q$
- Constituent spin trace
- Spin-1 tensor projection $\Delta_T Q_{ij}$
- b_1^q master formula and prediction

Unpolarized quark TMD in the deuteron

Define the γ^+ projection in a fixed deuteron spin state S:

$$q_D^{q,S}(x,\boldsymbol{k}_\perp) \equiv f_{1,D}^{q}(x,\boldsymbol{k}_\perp;S) = \tfrac{1}{2}\mathrm{Tr}\big[\gamma^+\,\Phi_D^q(x,\boldsymbol{k}_\perp;P,S)\big]\,, \qquad S=+1,0,-1.$$

Inserting the master convolution:

$$q_D^{q,S}(x,\boldsymbol{k}_\perp) = \sum_{s_N,s_N'}\int d\alpha\, d^2\boldsymbol{P}_{N\perp}\left(\frac{1-\alpha}{\alpha}\right)\mathcal{I}_{S,s_N,s_N'}(\alpha,\boldsymbol{P}_{N\perp})\, q_N^{q,s_N,s_N'}\!\left(\frac{x}{\alpha},\boldsymbol{k}_\perp-\frac{x}{\alpha}\boldsymbol{P}_{N\perp}\right).$$

Nucleon input in the γ^+ channel

$$\Phi_N^{[\gamma^+]}(z,\ell_\perp;S_N) = f_1^{q/N}(z,\ell_\perp^2) - \frac{\epsilon_\perp^{ij}\,\ell_{\perp i}S_{N\perp j}}{M_N}\,f_{1T}^{\perp\,q/N}(z,\ell_\perp^2).$$

Unpolarized sector after $\ell_\perp$ integration: $\quad f_{1T}^{\perp\,q/N}=0, \qquad q_N^{q,s_N}\to q_N^q = f_1^{q/N}.$

Therefore, collinear PDF:

$$q_D^{q,S}(x) = \int d^2k_\perp\, q_D^{q,S}(x,\boldsymbol{k}_\perp) = \int d\alpha\, d^2\boldsymbol{P}_{N\perp}\left(\frac{1-\alpha}{\alpha}\right)\mathcal{I}_S(\alpha,\boldsymbol{P}_{N\perp})\, q_N^q\!\left(\frac{x}{\alpha}\right),$$

with $\mathcal{I}_S(\alpha,\boldsymbol{P}_{N\perp}) \equiv \mathcal{I}_{S,unp}(\alpha,\boldsymbol{P}_{N\perp}) \qquad \mathcal{I}_{S,s_N} = \int db^-\, d^2b_\perp\, W_{S_D,s_N}(P_N,b;P)$

Building the tensor combination b_1

Define the spin-projected deuteron quark PDFs:

$$q_D^{q,0}(x) \equiv q_D^{q,S=0}(x), \qquad q_D^{q,1}(x) \equiv \frac{1}{2}\left[q_D^{q,+1}(x) + q_D^{q,-1}(x)\right].$$

By parity,

$$q_D^{q,+1} = q_D^{q,-1}.$$

Tensor-polarized quark PDF:

$$\delta_T q_D^q(x) \equiv q_D^{q,0}(x) - q_D^{q,1}(x), \qquad b_1^q(x) = \frac{1}{2}\,\delta_T q_D^q(x).$$

The corresponding tensor-polarized light-front momentum density is

$$\Delta_T \mathcal{I}(\alpha, \boldsymbol{P}_{N\perp}) \equiv \mathcal{I}_0(\alpha, \boldsymbol{P}_{N\perp}) - \frac{1}{2}\left[\mathcal{I}_{+1}(\alpha, \boldsymbol{P}_{N\perp}) + \mathcal{I}_{-1}(\alpha, \boldsymbol{P}_{N\perp})\right].$$

Fixed-flavor tensor convolution

$$b_1^q(x) = \frac{1}{2}\int d\alpha\, d^2\boldsymbol{P}_{N\perp}\left(\frac{1-\alpha}{\alpha}\right)\Delta_T \mathcal{I}(\alpha, \boldsymbol{P}_{N\perp})\, q_N^q\!\left(\frac{x}{\alpha}\right)$$

Constituent spin trace

Since the $k_\perp$ integration selects the unpolarized sector, the corresponding sector must be projected in the Wigner distribution. The constituent spin sum becomes a Pauli trace:

$$\tfrac{1}{2}\,\mathrm{Tr}_\sigma[\sigma_i] = 0 \quad\Longrightarrow\quad \text{all terms linear in } \boldsymbol{\sigma} \text{ vanish}$$

The spin-traced Wigner density contains only constituent-spin-independent pieces:

$$\tfrac{1}{2}\,\mathrm{Tr}_\sigma\, W(\boldsymbol{p},\boldsymbol{b};S) = F_1 + H_1\, Q_{ij}\,p_i p_j + H_2\, Q_{ij}\,b_i b_j + H_3\,(\boldsymbol{p}\cdot\boldsymbol{b})\,Q_{ij}(p_i b_j + p_j b_i)$$

Six tensor amplitudes drop out at this step

$H_4, H_5, H_6, H_7, H_8, H_9$ all carry $\boldsymbol{\sigma}$ $\Rightarrow$ vanish after the trace.

Only H_1, H_2, H_3 survive into the unpolarized-quark projection.

The scalar F_1 contributes to the ordinary unpolarized deuteron PDF. The Q_{ij} pieces feed into the tensor-polarized combination $\delta_T q_D^q$.

Polarization channels that contribute to $b_1(x)$

	Spin-independent (no $\boldsymbol{\sigma}$)	Spin-orbit coupling ($\times\, \boldsymbol{\sigma}\cdot(\boldsymbol{p}\times \boldsymbol{b})$)	Direct σ contraction (σ_i into Q_{ij})
Momentum proj. $Q_{ij}p_ip_j$	$\boxed{H_1}$	~~H_4~~	—
Position proj. $Q_{ij}b_ib_j$	$\boxed{H_2}$	~~H_5~~	—
Mixed $Q_{ij}p_ib_j$, radial-weighted	$\boxed{H_3}$	~~H_7~~	~~H_8~~, ~~H_9~~
Direct $Q_{ij}\sigma_i(\cdots)_j$	—	—	~~H_6~~

- $\boxed{\cdot} \longrightarrow$ **accessible** with current data
- ~~X~~ $\longrightarrow$ **future** predictions

Tensor-polarized Wigner sector and the Δ_T operator

Factor the surviving tensor-polarized Wigner sector as

$$W_T(\boldsymbol{p},\boldsymbol{b}) \equiv Q_{ij}\, T_{ij}(\boldsymbol{p},\boldsymbol{b}), \qquad T_{ij} \equiv H_1\, p_i p_j + H_2\, b_i b_j + H_3\, (\boldsymbol{p}\cdot\boldsymbol{b})(p_i b_j + p_j b_i)$$

- Q_{ij}: deuteron spin-1 tensor (state-dependent)
- $T_{ij}(\boldsymbol{p},\boldsymbol{b})$: spatial Wigner dynamics (scalar coefficient functions)

Spin-1 tensor projection operator:

$$\Delta_T Q_{ij} \equiv \langle 0|Q_{ij}|0\rangle - \tfrac{1}{2}[\langle +1|Q_{ij}|+1\rangle + \langle -1|Q_{ij}|-1\rangle]$$

For quantization along $\hat{z}$:

$$\Delta_T Q_{ij} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}_{ij} \implies \Delta_T Q_{ij}\, p_i p_j = \tfrac{1}{2}\, p_\perp^2 - p_z^2$$

The combination $\frac{1}{2}p_\perp^2 - p_z^2$ is the *only* rank-two tensor surviving rotational covariance about $\hat{z}$.

b_1^q master formula

The spin-1 tensor combination reduces the nuclear weight to:

$$\delta_T \mathcal{I}(\boldsymbol{p}) = M_D \int d^3 b \, \Delta_T Q_{ij} \, T_{ij}(\boldsymbol{p}, \boldsymbol{b})$$

After the $\boldsymbol{b}$-integration, rotational covariance collapses H_1, H_2, H_3 into a *single* scalar function $C_T(p^2)$:

$$\delta_T \mathcal{I}(\boldsymbol{p}) = M_D \left(\tfrac{1}{2}\, p_\perp^2 - p_z^2\right) C_T(p^2) \longrightarrow p_z = \frac{\alpha^2 M_D^2 - m^2 - p_\perp^2}{2\alpha M_D}, \qquad \vec{p}^{\,2} = p_z^2 + p_\perp^2$$

Substituting into the fixed-flavor tensor convolution: (show integrated here instead bc we need to integrate to cancel channels)

$$\boxed{b_1^q(x) = \tfrac{1}{2}\, M_D \int d\alpha \, d^2 \boldsymbol{P}_{N\perp} \left(\frac{1-\alpha}{\alpha}\right) \left(\tfrac{1}{2}\, P_{N\perp}^2 - P_{Nz}^2\right) C_T(\boldsymbol{P}_N^2) \times \; q_N^q\!\left(\tfrac{x}{\alpha}\right)}$$

- Physical structure function: $b_1(x) = \sum_q e_q^2 \, b_1^q(x)$

Summary of the b_1 projection

17 Wigner amplitudes
2 scalar + 6 vector + 9 tensor

↓

γ^+ projection + $\int d^2k_\perp$ + spin-1 tensor combination δ_T
removes all scalar and vector amplitudes

↓

Equivalently, unpolarized intermediate state $\to \frac{1}{2}\mathrm{Tr}_{\sigma^0}$
removes $H_4, H_5, H_6, H_7, H_8, H_9$

↓

Surviving: H_1, H_2, H_3
ONLY H_1 and H_3 combine into $C_T(p^2)$ after $\boldsymbol{b}$-integration

- H_1 $(Q_{ij}p_ip_j)$, H_2 $(Q_{ij}b_ib_j)$, and H_3 $((\boldsymbol{p}\cdot\boldsymbol{b})Q_{ij}(p_ib_j + p_jb_i))$.

Preliminary: $b_1(x)$ with the Paris wave function

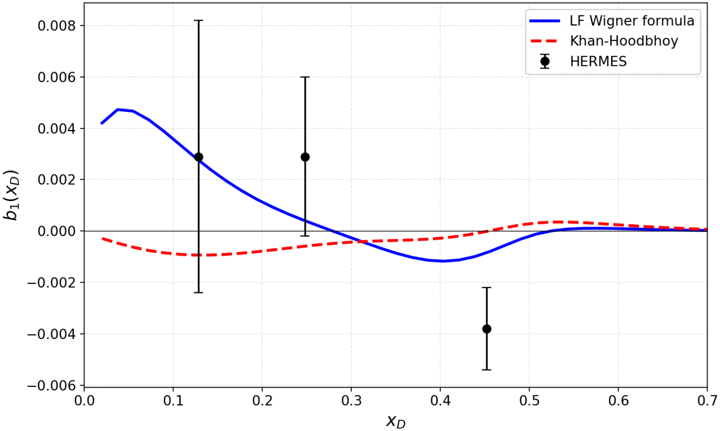

Figure: $b_1(x_D)$ from this formalism vs. Khan–Hoodbhoy, Paris WF and simple PDF

- **Preliminary work, to be reported in upcoming paper**

Summary

Master correlator

- Exact convolution of nuclear Wigner W with nucleon correlator $\Phi_{ij,N}$
- Two assumptions: impulse approximation + spectator gauge link (no QCD Lensing)

Wigner decomposition

- 17 amplitudes: 2 scalar + 6 vector + 9 tensor
- Tensor sector: 3×3 taxonomy
- 14 amplitudes are predictions for future programs

b_1 projection

- $\gamma^+ + \int d^2 k_\perp + \delta_T$ H_1, H_2, H_3
- Three amplitudes collapse into two functions $\to C_T(p^2)$
- b_1^q LF master convolution formula
- Numerical results with Paris WF (preliminary)

Thank you.