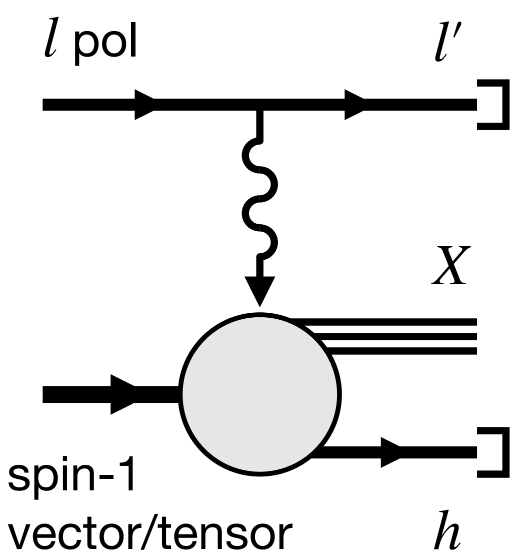
Semi-inclusive scattering from a polarized spin-1 target: Structures, observables, applications

C. Weiss (JLab) [weiss@jlab.org], Tensor SIDIS Workshop, JLab Lab, 3-5 June 2026 [Webpage]



Here: General kinematics, finite Q^2, W^2, current and target fragmentation regions

Based on: Cosyn, Weiss, arXiv:2603.23699 [INSPIRE]; see also 2603.23700 [INSPIRE]

See also: Zhao, Bacchetta, Kumano, Liu, Zhou JHEP 12, 067: Current fragmentation + TMDs

Kinematics

Basis vectors and collinear frames

Final-state hadron variables

Spin-1 target

Density matrix and polarization parameters

Preparation of vector/tensor polarization

Cross section and observables

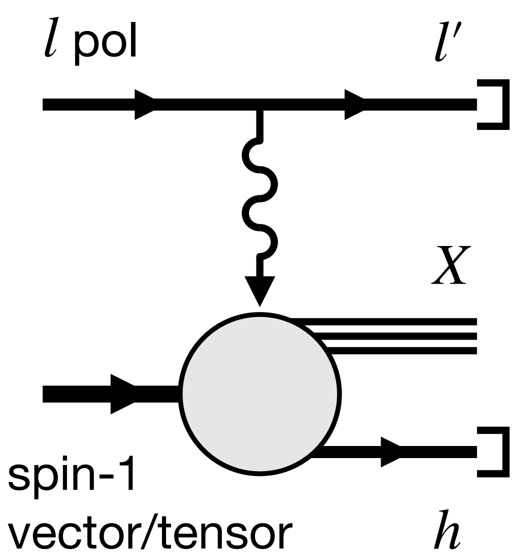
Semi-inclusive structure functions

Spin asymmetries vector/tensor

Extensions

Spin > 1 targets

Tensor polarization in $N \to \Delta$ transitions



DIS kinematics, current fragmentation region

TMD factorization: Tensor-polarized TMD distributions
Bacchetta, Mulders 2000; Boer et al 2016, Zhao et al 2025; Kumano, Kuroki 2026

Higher ϕ_h harmonics for spin-orbit studies

DIS kinematics, target fragmentation region

Fracture functions: Factorization theorem, leading-twist structures
Trentadue, Veneziano 1994; Collins 1998; Anselmino et al 2011

Polarized deuteron + spectator nucleon detected:
Fracture functions calculable from nuclear dynamics!
Frankfurt, Strikman 1983; Cosyn, Weiss 2020, 2026

Finite Q^2, W^2 kinematics, forward or backward hadron angles

Quasi-elastic nuclear breakup
Rekalo et al. 1987+; Dmitrasinovic, Gross 1989; Arenhövel et al 1993+;
Jeschonnek, Van Orden 2009+; Flores et al. 2023; Grassi et al 2023

Initial-state polarization and final-state interaction effects

Kinematics: Basis vectors

Relativistically covariant description

Longitudinal subspace $\{q, P\}$

Transverse subspace: Electron, produced hadron

Longitudinal basis vectors

$$e_q \equiv \frac{q}{\sqrt{-q^2}} \qquad e_L \equiv \frac{P + (e_q P) e_q}{\sqrt{P^2 + (e_q P)^2}}$$

set aligned with q

$$e_P \equiv \frac{P}{\sqrt{P^2}} \qquad e_{L*} \equiv \frac{q - (e_P q) e_P}{\sqrt{-q^2 + (e_P q)^2}}$$

alt. set, aligned with P

Transverse basis vectors

$$e_x \equiv \frac{l_T}{\sqrt{-l_T^2}} \qquad e_y \equiv \epsilon(e_L e_q e_x)$$

set aligned with lepton momentum l_T

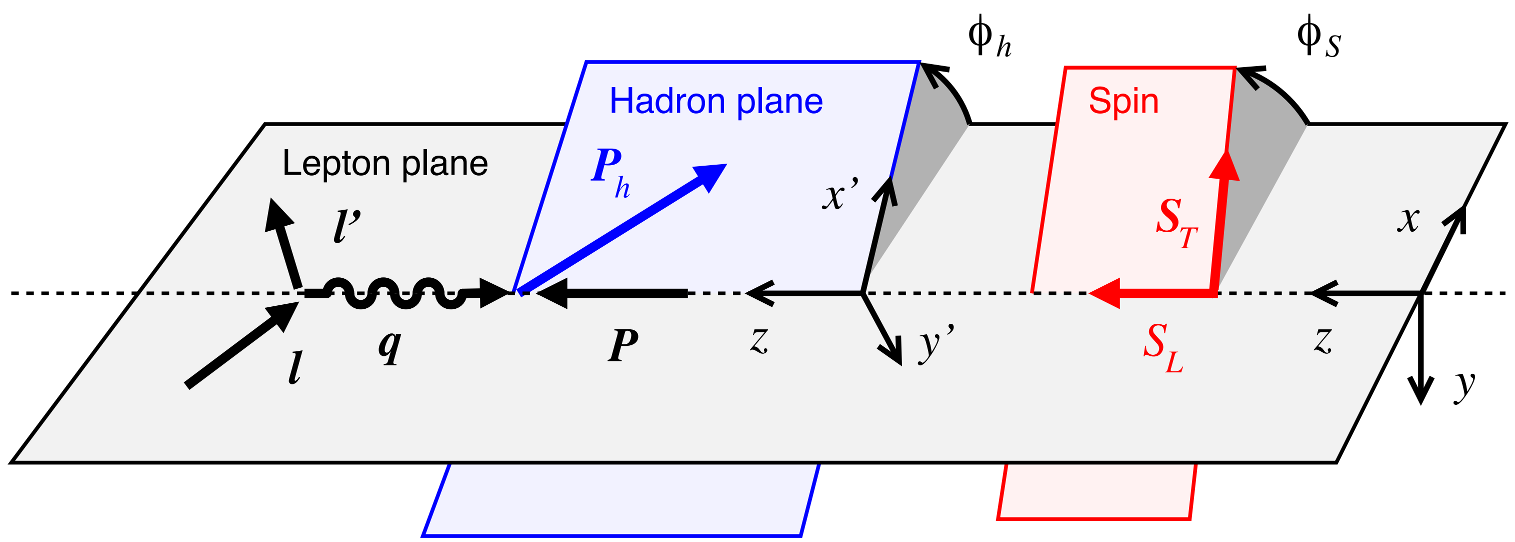
$$e_{x'} \equiv \frac{P_{hT}}{\sqrt{P_{hT}^2}} \qquad e_{y'} \equiv \epsilon(e_L e_q e_{x'})$$

alt.set, aligned with hadron momentum P_{hT}

Complete basis

$\{e_L, e_q, e_x, e_y\}$ alt. $\{e_P, e_{L*}, e_x, e_y\}$ or with $e_{x'}, e_{y'}$

Kinematics: Collinear frames



Frames where $\mathbf{q} \parallel \mathbf{P}$

Equivalence class of frames connected by boosts

Contains important frames:

Target rest frame $\mathbf{P} = 0$

Photon-target CM frame $\mathbf{q} + \mathbf{P} = 0$

Breit frame $q^0 = 0$

Coordinate system

Choose $q^z < 0$, convenient for dynamical calculations with target structure

Final state hadron momentum

$\mathbf{P}_h \rightarrow P_h^z, |\mathbf{P}_{hT}|, \phi_h$

$P_h^z \rightarrow$ invariant variable z_h (alt: light cone fractions ζ_h, η_h)

Angles ϕ_h, ϕ_S defined in Trento convention (even though coordinate system $q^z < 0$)

All components/angles can be expressed as scalar products with basis vectors - invariants

Spin-1 target: Density matrix

$$\rho(\lambda, \lambda'), \qquad \sum_{\lambda} \rho(\lambda, \lambda) = 1$$

Spin density matrix in rest frame, $\lambda = (+1,0,-1)$, describes target polarization state

$$\rho^{\alpha\beta} \equiv \sum_{\lambda,\lambda'} \rho(\lambda, \lambda')\, \epsilon^{\alpha}(\lambda) \epsilon^{\beta *}(\lambda')$$

Covariant form of density matrix (any P), $\epsilon^{\alpha}(\lambda)$ spin wave function, $P^{\alpha}\epsilon_{\alpha} = 0$

$$\rho^{\alpha\beta} = \frac{1}{3}\left(-g^{\alpha\beta} + \frac{P^{\alpha}P^{\beta}}{M^2} \right) + \frac{i}{2M}\epsilon^{\alpha\beta\gamma\delta} P_{\gamma} S_{\delta} \; - \; T^{\alpha\beta}$$

Parametrization of covariant density matrix

S^{α} axial 4-vector

$T^{\alpha\beta}$ symmetric traceless tensor

$$P^{\alpha} S_{\alpha} = 0$$

$$P^{\alpha} T_{\alpha\beta}, \, T^{\alpha\beta} P_{\beta} \; = \; 0$$

Polarization information represented by covariant parameters

Spin-1 target: Invariant polarization parameters

Contract polarization vector/tensor with basis vectors: Invariant polarization parameters

$$\{e_P,\, e_{L*},\, e_{x'},\, e_{y'}\}^{\alpha,\beta} \;\times\; S_\alpha,\, T_{\alpha\beta}$$

$$(e_{L*}S) \equiv S_L,$$

$$-(e_{x'}S) \equiv S_T \cos(\phi_S - \phi_h),$$

$$-(e_{y'}S) \equiv -\, S_T \sin(\phi_S - \phi_h),$$

3 vector polarization parameters
S_L, S_T, ϕ_S

$$(e_{L*}Te_{L*}) \equiv T_{LL},$$

$$-(e_{L*}Te_{x'}) \equiv T_{LT} \cos(\phi_{T_L} - \phi_h),$$

$$-(e_{L*}Te_{y'}) \equiv -\, T_{LT} \sin(\phi_{T_L} - \phi_h),$$

$$(e_{x'}Te_{x'}) - (e_{y'}Te_{y'}) \equiv T_{TT} \cos(2\phi_{T_T} - 2\phi_h),$$

$$(e_{x'}Te_{y'}) \equiv -\tfrac{1}{2}T_{TT} \sin(2\phi_{T_T} - 2\phi_h)$$

5 tensor polarization parameters
$T_{LL}, T_{LT}, T_{TT}, \phi_{T_L}, , \phi_{T_T}$

Also possible: Expand polarization vector/tensor covariantly in basis tensors

Expansion coefficients given by invariant polarization parameters

Connection with 3D spherical harmonic tensors in target rest frame

$N^\alpha, \qquad P^\alpha N_\alpha = 0, \qquad \Lambda = (+1,0,-1)$

Target in pure spin state with polarization Λ along axis N^α (described covariantly, specified in some frame)

$$S^\alpha = \Lambda N^\alpha$$

$$T^{\alpha\beta} = \frac{1}{6} W(\Lambda) \left(g^{\alpha\beta} - \frac{P^\alpha P^\beta}{P^2} + 3 N^\alpha N^\beta \right)$$

Vector and tensor polarization in pure state

$W(\Lambda) \equiv (1,-2,1)$ for $\Lambda = (+1,0,-1)$

Take sums/differences of Λ states

U: $\quad (\Lambda = +1) \; + \; (\Lambda = -1) \; + \; (\Lambda = 0)$

S: $\quad (\Lambda = +1) \; - \; (\Lambda = -1)$

T: $\quad (\Lambda = +1) \; + \; (\Lambda = -1) \; - \; 2\,(\Lambda = 0)$

Isolate unpolarized/vector/tensor terms

Sums/differences can be taken in measurements: Spin asymmetries, normalization conventional

Spin-1 target: Preparing target polarization

Target polarization preparation from pure states is formulated covariantly: N^α, Λ

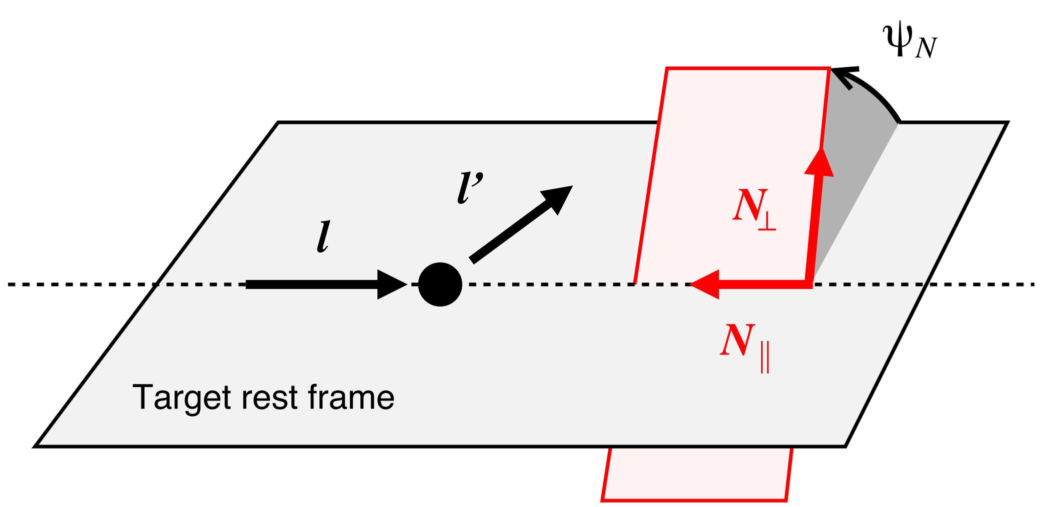
Can be specified in any frame where experimental information is available

Most relevant: Target polarization along lepton beam axis

Fixed-target: Lepton beam axis in target rest frame

Collider: Common axis of colliding beams (zero crossing angle)

Explicit expressions for vector/tensor polarization parameters $\{S_L, S_T, \phi_S\}$ and $\{T_{LL}, T_{LT}, T_{TT}, \phi_{T_L}, , \phi_{T_T}\}$ for target polarization along lepton beam axis available in CW26



Polarization directions denoted by || and ⊥

Polarization ⊥ described by angle ψ_N: Trento convention

Depolarization factors: How polarization along lepton beam axis converts to polarization in photon-target collinear frame

Cross section: Target spin dependence

$$d\sigma = \frac{1}{4I}\frac{4\pi e^2}{Q^4}\, L^{\mu\nu}\langle W_{\mu\nu}\rangle_\rho \, d\Gamma_{l'}\, d\Gamma_h$$

Differential cross section

$$W^{\mu\nu}(\lambda', \lambda) \equiv \frac{1}{4\pi}\sum_X (2\pi)^4 \delta^4(P + q - P_h - P_X) \times \langle A(\lambda') \,|\, J^{\dagger\mu} \,|\, X, h\rangle\langle X, h \,|\, J^\nu \,|\, A(\lambda)\rangle$$

Hadronic tensor for semi-inclusive scattering

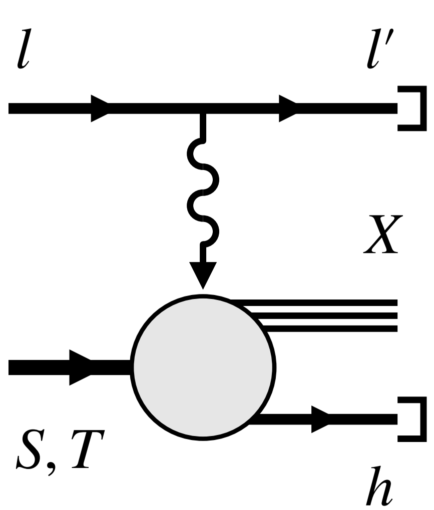
Matrix in target spin quantum numbers λ, λ'

$$\langle W^{\mu\nu}\rangle_\rho \equiv \sum_{\lambda'\lambda} \rho(\lambda, \lambda')\, W^{\mu\nu}(\lambda', \lambda)$$

Average with target spin density matrix

Gives rise to dependence on target polarization parameters S, T

Entangles target polarization with dependence on final-state hadron momentum: Spin-orbit effects



Decomposition of hadronic tensor

[kinematic tensor] $\times$ [polarization parameter] $\times$ [structure function]

formed from basis vectors

formed from S, T and basis vectors

contains dynamical information

Constrained by current conservation $q^\mu W_{\mu\nu}, W^{\mu\nu} q_\nu = 0$; parity, hermiticity

41 independent structures for spin-1 target. Number confirmed by helicity amplitude analysis

Contraction with leptonic tensor

Expressed through $\epsilon \equiv \dfrac{e_L^\mu e_L^\nu L_{\mu\nu}}{(e_{x'}^\mu e_{x'}^\nu + e_{y'}^\mu e_{y'}^\nu) L_{\mu\nu}} \leftrightarrow y$ L/T ratio of virtual photon flux

Gives rise to harmonic dependence on hadron angle $\cos(n\phi_h), \ \sin(n\phi_h)$

Dependence on lepton helicity $\lambda_e = \pm 1/2$

Cross section: Results

$$d\sigma = \frac{2\pi y^2 \alpha_{\text{em}}^2}{Q^4(1-\epsilon)}\, dx_A dQ^2 \frac{d\psi_{l'}}{2\pi} \times \left(\mathscr{F}_U + \mathscr{F}_S + \mathscr{F}_T\right)\, d\Gamma_{P_h}$$

flux factor
same as inclusive

contains structures
with U, S, T polarization

$$\begin{aligned}
\mathscr{F}_U &= F_{UU,T} + \epsilon F_{UU,L} \\
&+ \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} \\
&+ \epsilon\cos 2\phi_h F_{UU}^{\cos 2\phi_h} \\
&+ (2\lambda_e)\sqrt{2\epsilon(1-\epsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}
\end{aligned}$$

$$\begin{aligned}
\mathscr{F}_S &= S_L\left[\sqrt{2\epsilon(1+\epsilon)}\sin\phi_h F_{US_L}^{\sin\phi_h} + \epsilon\sin 2\phi_h F_{US_L}^{\sin 2\phi_h}\right] \\
&+ S_L(2\lambda_e)\left[\sqrt{1-\epsilon^2}F_{LS_L} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_h F_{LS_L}^{\cos\phi_h}\right] \\
&+ S_T\Big[\sin(\phi_h-\phi_S)\left(F_{US_T,T}^{\sin(\phi_h-\phi_S)} + \epsilon F_{US_T,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad + \epsilon\sin(\phi_h+\phi_S)F_{US_T}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \epsilon\sin(3\phi_h-\phi_S)F_{US_T}^{\sin(3\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)}\sin\phi_S F_{US_T}^{\sin\phi_S} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)}\sin(2\phi_h-\phi_S)F_{US_T}^{\sin(2\phi_h-\phi_S)}\Big] \\
&+ S_T(2\lambda_e)\Big[\sqrt{1-\epsilon^2}\cos(\phi_h-\phi_S)F_{LS_T}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LS_T}^{\cos\phi_S} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)}\cos(2\phi_h-\phi_S)F_{LS_T}^{\cos(2\phi_h-\phi_S)}\Big]\,. \quad (4.38)
\end{aligned}$$

Semi-inclusive structure functions:
$F(x, Q^2; \zeta_h, |\mathbf{P}_{hT}|)$

Notation: $F^{\text{[azimuthal]}}_{\text{[beam][target],[photon]}}$

U and S structures same as for spin-1/2 target

$$
\begin{aligned}
\mathscr{F}_T \; &= T_{LL} \Big[F_{UT_{LL},T} + \epsilon F_{UT_{LL},L} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)} \cos\phi_h F_{UT_{LL}}^{\cos\phi_h} \\
&\qquad + \epsilon \cos 2\phi_h F_{UT_{LL}}^{\cos 2\phi_h} \Big] \\
&+ T_{LL}(2\lambda_e) \sqrt{2\epsilon(1-\epsilon)} \sin\phi_h F_{LT_{LL}}^{\sin\phi_h} \\
&+ T_{LT} \Big[\cos(\phi_h - \phi_{T_L}) \Big(F_{UT_{LT},T}^{\cos(\phi_h-\phi_{T_L})} + \epsilon F_{UT_{LT},L}^{\cos(\phi_h-\phi_{T_L})} \Big) \\
&\qquad + \epsilon \cos(\phi_h + \phi_{T_L}) F_{UT_{LT}}^{\cos(\phi_h+\phi_{T_L})} \\
&\qquad + \epsilon \cos(3\phi_h - \phi_{T_L}) F_{UT_{LT}}^{\cos(3\phi_h-\phi_{T_L})} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)} \cos\phi_{T_L} F_{UT_{LT}}^{\cos\phi_{T_L}} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)} \cos(2\phi_h - \phi_{T_L}) F_{UT_{LT}}^{\cos(2\phi_h-\phi_{T_L})} \Big] \\
&+ T_{LT}(2\lambda_e) \Big[\sqrt{1-\epsilon^2} \sin(\phi_h - \phi_{T_L}) F_{LT_{LT}}^{\sin(\phi_h-\phi_{T_L})} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)} \sin\phi_{T_L} F_{LT_{LT}}^{\sin\phi_{T_L}} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)} \sin(2\phi_h - \phi_{T_L}) F_{LT_{LT}}^{\sin(2\phi_h-\phi_{T_L})} \Big] \\
&+ T_{TT} \Big[\cos(2\phi_h - 2\phi_{T_T}) \Big(F_{UT_{TT},T}^{\cos(2\phi_h-2\phi_{T_T})} + \epsilon F_{UT_{TT},L}^{\cos(2\phi_h-2\phi_{T_T})} \Big) \\
&\qquad + \epsilon \cos 2\phi_{T_T} F_{UT_{TT}}^{\cos 2\phi_{T_T}} \\
&\qquad + \epsilon \cos(4\phi_h - 2\phi_{T_T}) F_{UT_{TT}}^{\cos(4\phi_h-2\phi_{T_T})} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi_h - 2\phi_{T_T}) F_{UT_{TT}}^{\cos(\phi_h-2\phi_{T_T})} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)} \cos(3\phi_h - 2\phi_{T_T}) F_{UT_{TT}}^{\cos(3\phi_h-2\phi_{T_T})} \Big] \\
&+ T_{TT}(2\lambda_e) \Big[\sqrt{1-\epsilon^2} \sin(2\phi_h - 2\phi_{T_T}) F_{LT_{TT}}^{\sin(2\phi_h-2\phi_{T_T})} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)} \sin(\phi_h - 2\phi_{T_T}) F_{LT_{TT}}^{\sin(\phi_h-2\phi_{T_T})} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)} \sin(3\phi_h - 2\phi_{T_T}) F_{LT_{TT}}^{\sin(3\phi_h-2\phi_{T_T})} \Big] .
\end{aligned}
$$

T_{LL}: ϵ and ϕ_h dependence same as in U

T_{LT}: ϕ_h dependence same as S_T with replacement $\sin \to \cos$ for parity and $\phi_S \to \phi_{T_L}$

T_{TT}: ϕ_h dependence different from S_T

New harmonics $\cos(4\phi_h)$, $\sin(4\phi_h)$, unique to tensor polarization

ϕ_h-independent tensor-pol structure functions can be connected with conventional $b_1, .., b_4$

Observables: Azimuthal averages

$$\langle \mathscr{F} \rangle_w \equiv \int_0^{2\pi} \frac{d\phi_h}{2\pi}\, w(\phi_h)\mathscr{F}(\phi_h) \Big/ \int_0^{2\pi} \frac{d\phi_h}{2\pi}\, [w(\phi_h)]^2$$

Azimuthal average

$w(\phi_h) = 1,\ \cos(n\phi_h),\ \sin(n\phi_h), \quad n = 1 - 4$

$n = 0,1,2$ averages generally contain all S, T polarized structures

$n = 3$ averages

$$\begin{aligned}
&\langle \mathcal{F} \rangle_{\cos 3\phi_h} \\
&= -S_T \epsilon \sin\phi_S F_{US_T}^{\sin(3\phi_h - \phi_S)} \\
&+ T_{LT} \epsilon \cos(\phi_{T_L}) F_{UT_{LT}}^{\cos(3\phi_h - \phi_{T_L})} \\
&+ T_{TT} \sqrt{2\epsilon(1+\epsilon)} \cos(2\phi_{T_T}) F_{UT_{TT}}^{\cos(3\phi_h - 2\phi_{T_T})} \\
&- T_{TT} (2\lambda_e) \sqrt{2\epsilon(1-\epsilon)} \sin(2\phi_{T_T}) F_{LT_{TT}}^{\sin(3\phi_h - 2\phi_{T_T})},
\end{aligned}$$

special polarization angles →

$$\begin{aligned}
&\langle \mathcal{F} \rangle_{\cos 3\phi_h} (\phi_S = 0, \phi_{T_L} = \phi_{T_T} = 0) \\
&= T_{LT} \epsilon F_{UT_{LT}}^{\cos(3\phi_h - \phi_{T_L})} \\
&+ T_{TT} \sqrt{2\epsilon(1+\epsilon)} F_{UT_{TT}}^{\cos(3\phi_h - 2\phi_{T_T})},
\end{aligned}$$

$$\begin{aligned}
&\langle \mathcal{F} \rangle_{\sin 3\phi_h} \\
&= S_T \epsilon \cos\phi_S F_{US_T}^{\sin(3\phi_h - \phi_S)} \\
&+ T_{LT} \epsilon \sin(\phi_{T_L}) F_{UT_{LT}}^{\cos(3\phi_h - \phi_{T_L})} \\
&+ T_{TT} \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi_{T_T}) F_{UT_{TT}}^{\cos(3\phi_h - 2\phi_{T_T})} \\
&+ T_{TT} (2\lambda_e) \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi_{T_T}) F_{LT_{TT}}^{\sin(3\phi_h - 2\phi_{T_T})}.
\end{aligned}$$

→

$$\begin{aligned}
&\langle \mathcal{F} \rangle_{\sin 3\phi_h} (\phi_S = \pi/2, \phi_{T_L} = \phi_{T_T} = \pi/2) \\
&= T_{LT} \epsilon F_{UT_{LT}}^{\cos(3\phi_h - \phi_{T_L})} \\
&- T_{TT} (2\lambda_e) \sqrt{2\epsilon(1-\epsilon)} F_{LT_{TT}}^{\sin(3\phi_h - 2\phi_{T_T})},
\end{aligned}$$

Contain only T polarization!

Such angles can be achieved with || and ⊥ polarization relative to lepton beam axis

Observables: Azimuthal averages

$n = 4$ averages

$$\langle \mathcal{F} \rangle_{\cos 4\phi_h} = T_{TT} \epsilon \cos(2\phi_{T_T}) F_{UT_{TT}}^{\cos(4\phi_h - 2\phi_{T_T})},$$

$$\langle \mathcal{F} \rangle_{\sin 4\phi_h} = T_{TT} \epsilon \sin(2\phi_{T_T}) F_{UT_{TT}}^{\cos(4\phi_h - 2\phi_{T_T})}.$$

Contain only T_{TT} polarization

Depend on polarization angle as $2\phi_{T_T}$

$\sigma(\lambda_e = \pm\frac{1}{2}, \Lambda = \pm 1{,}0)$

Cross section for fixed final lepton x, Q^2; fixed final hadron ζ_h, P_{hT}, ϕ_h or integrated

$$A^V \equiv \frac{\sigma(+,+1) - \sigma(-,+1) - \sigma(+,-1) + \sigma(-,-1)}{\sigma(+,+1) + \sigma(-,+1) + \sigma(+,-1) + \sigma(-,-1)}$$

Double asymmetry in lepton helicity and target spin (|| or $\perp$ polarization)

Involves only ± 1 target spin states

$$A^V_{\parallel} \; = \; D_{\parallel[S_L]} \frac{F_{LS_L}}{\Sigma_{\parallel} F} \; + \; D_{\parallel[S_T]} \frac{F_{LS_T}^{\cos\phi_S}}{\Sigma_{\parallel} F}$$

Expansion in structure functions.
Kinematic factors D functions of y, x, Q^2

$$\Sigma_{\parallel} F \equiv F_{UU,T} \; + \; \epsilon F_{UU,L} \; + \; D_{\parallel[T_{LL}]} (F_{UT_{LL},T} + \epsilon F_{UT_{LL},L}) \\ + D_{\parallel[UT_{LT}]} F_{UT_{LT}}^{\cos\phi_{T_L}} \; + \; D_{\parallel[T_{TT}]} F_{UT_{TT}}^{\cos\phi_{T_T}}$$

Denominator involves tensor polarized SFs

Also version including $\Lambda = 0$ spin state in denominator, removing tensor polarization

Used to extract spin SFs

$$F_{LS_L} \sim g_1, \qquad F_{LS_T}^{\cos\phi_S} \sim \gamma(g_1 + g_2)$$

Similar form for $\perp$ polarization; different sensitivity to S_L and S_T structures

Observables: Spin asymmetries - tensor

$$\sigma(\Lambda) \equiv \frac{1}{2}\sum_{\lambda_e} \sigma(\lambda_e, \Lambda)$$

Cross section averaged over lepton helicity

$$A^T \equiv \frac{\sigma(+1) + \sigma(-1) - 2\sigma(0)}{\sigma(+1) + \sigma(-1) + \sigma(0)}$$

Tensor-polarized asymmetry in target spin
(|| or ⊥ polarization)

Involves ± 1 and 0 target spin states

$$A_{\parallel}^T = 2D_{\parallel[T_{LL}]} \frac{F_{UT_{LL},T} + \epsilon F_{UT_{LL},L}}{F_U} + 2D_{\parallel[UT_{LT}]} \frac{F_{UT_{LT}}^{\cos\phi_{T_L}}}{F_U} + 2D_{\parallel[T_{TT}]} \frac{F_{UT_{TT}}^{\cos 2\phi_{T_T}}}{F_U}$$

Expansion in structure functions.
Kinematic factors D functions of y, x, Q^2

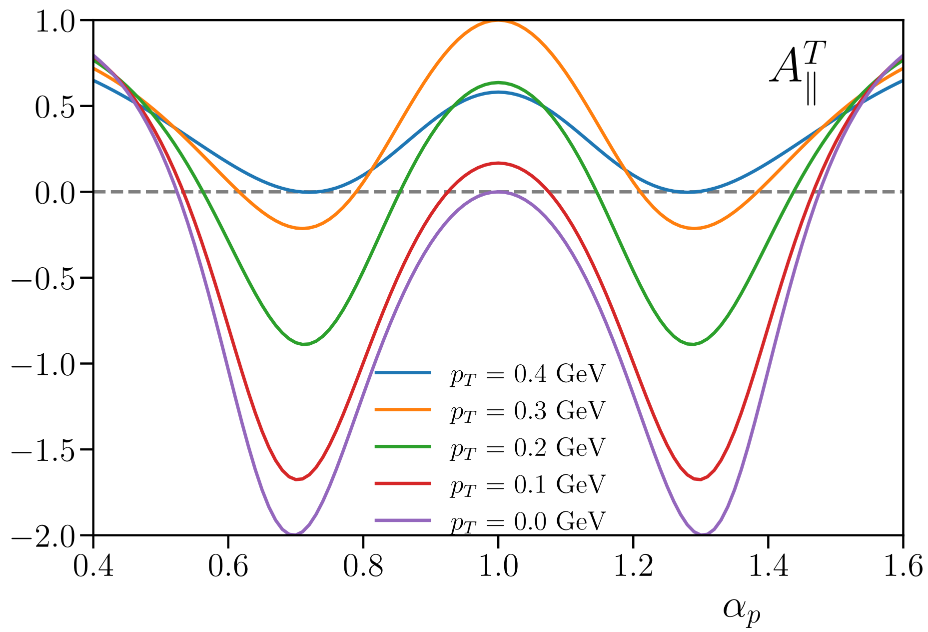
Denominator involves unpolarized SFs

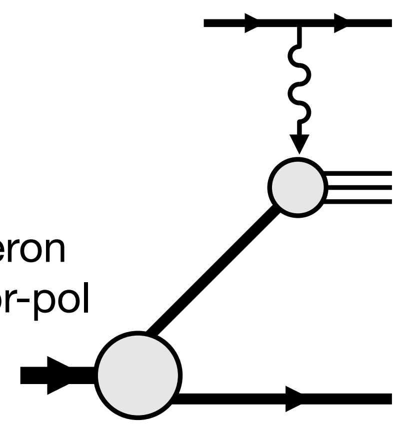
$$F_U \equiv F_{UU,T} + \epsilon F_{UU,L}$$

Can be used to extract tensor-polarized SFs

$$F_{UT_{LL},T}, F_{UT_{LL},L}, F_{UT_{LT},L}^{\cos\phi_{T_L}}, F_{UT_{TT},L}^{\cos 2\phi_{T_{TT}}}$$

Similar form for ⊥ polarization; different sensitivity to T_{LL}, T_{LT}, T_{TT} structures

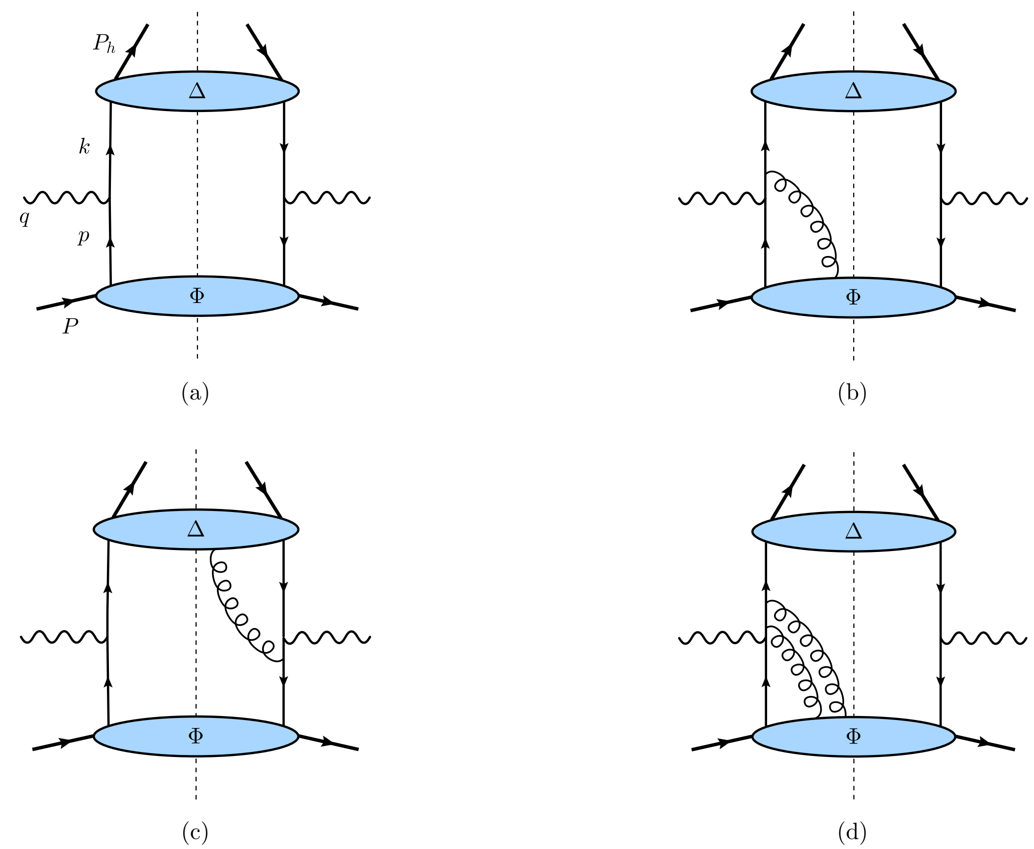




Target fragmentation: Polarized deuteron breakup due to nuclear dynamics ("spectator nucleon tagging")

Large tensor-pol asymmetries O(1) in D-wave dominated kinematics

→ Talk Cosyn



$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0)\mathcal{U}^c_{(0,\xi)}\psi_i(\xi)|P\rangle\Big|_{\xi^+=0}$$

Current fragmentation: TMD factorization of semi-inclusive structure functions

Quark/gluon correlation functions in polarized spin-1 target

→ Talks Sievert, Kumano, Zhao, Kuroki

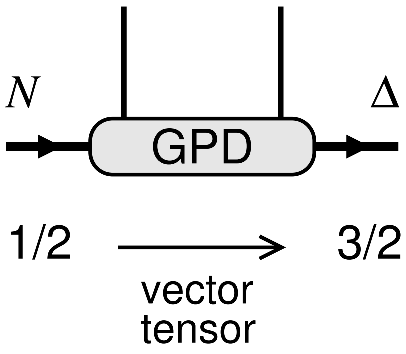
Semi-inclusive scattering on spin > 1 targets

Structural analysis of hadronic tensor can be generalized to spin > 1 (multipoles, helicity amplitudes)

Expected number of structures 5, 18, 41, 72, 113, ... for spin 0, 1/2, 1, 3/2, 2, ...

Practical relevance: Spin-3/2 nuclei 7Li, 11B (stable), spin-2 8Li (unstable)

Tensor polarization in $N \to \Delta$ transitions

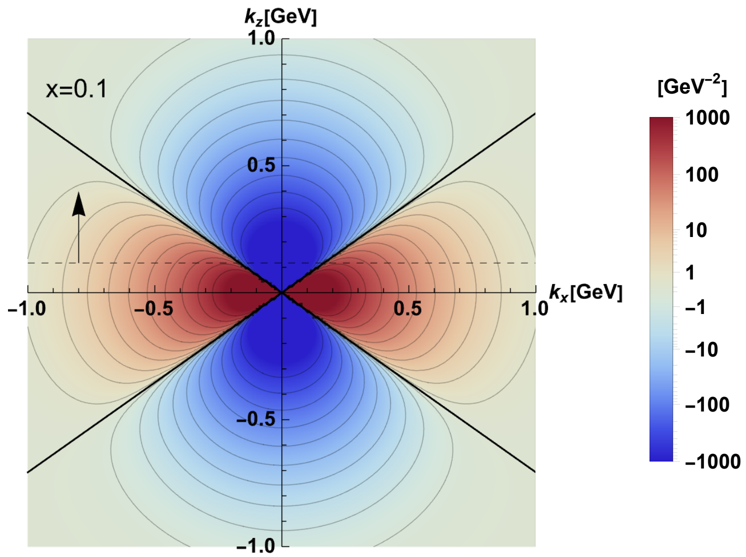


$1/2 \to 3/2$ spin transition supports transition vector and tensor

$N \to \Delta$ transition GPD contains "tensor-polarized" structure
J-Y. Kim et al. PRD 111, 114010 (2025) [INSPIRE]

Tensor-polarized transition PDF in forward limit $\Delta^+, \Delta_T = 0$
J-Y. Kim, C. Weiss, PLB 870 (2025) 139924 [INSPIRE]

Interesting connection with chiral symmetry breaking,
quark orbital angular momentum



Summary

Semi-inclusive spin-1 cross section has rich structure, resulting from "entanglement"
of measured hadron momentum with target polarization parameters

T_{LL}, T_{LT} structures in spin-1 cross section similar to U, S_T structures (present already in spin-1/2),
T_{TT} structures have genuinely new ϕ_h and polarization angle dependence

Tensor-polarized structures can be isolated through

- ϕ_h harmonics cos/sin$(3\phi_h)$ at special polarization angles, cos/sin$(4\phi_h)$
- Tensor-polarized asymmetries formed from ± 1 and 0 polarization states

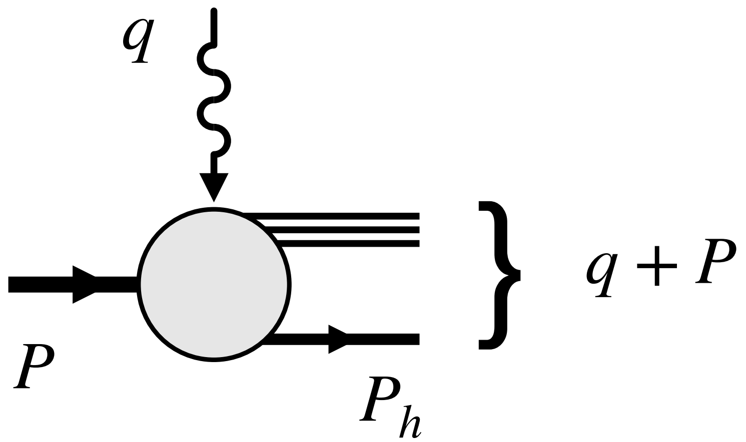
Large tensor-polarized asymmetries O(1) predicted in DIS on polarized deuteron with spectator tagging:
Special case of target fragmentation, structures calculable from nuclear dynamics

Useful techniques in kinematics (also for other applications):

- Natural basis vectors
- Covariant representation of spin density matrix and polarization preparation
- Light-front variables for current and target fragmentation

Supplemental material

Kinematics: Final-state hadron momentum



Various longitudinal momentum variables used

Energy fraction in target rest frame $z_h \equiv P_h \cdot P / q \cdot P$

Feynman variable $x_{\mathrm{F}} \equiv - P_h^z / P_{h,\mathrm{max}}^z$

Light-front variables in collinear frame

$P_h^{\pm} \equiv P_h^0 \pm P_h^z \qquad P_h^+ P_h^- = |\mathbf{P}_{hT}|^2 + M_h^2 \equiv M_{hT}^2$ LF momentum of observed hadron

$(q+P)^+, (q+P)^-$ LF momentum of total hadronic final state (given in terms of x, Q^2)

$\zeta_h \equiv \dfrac{P_h^+}{(q+P)^+} \qquad \eta_h \equiv \dfrac{P_h^-}{(q+P)^-}$ LF momentum fractions of observed hadron

$\zeta_h \eta_h = M_{hT}^2 / W^2$ Fractions connected by mass shell condition, use one or the other as variable!

Kinematics: Final-state hadron momentum

Current and target fragmentation regions in DIS limit $W^2 \gg$ masses

$\eta_h \sim 1, \qquad \zeta_h \sim M_{hT}^2/W^2$ current fragmentation region

$\zeta_h \sim 1, \qquad \eta_h \sim M_{hT}^2/W^2$ target fragmentation region

Invariant phase space element in hadron momentum

$$d\Gamma_h \equiv \frac{d^4 P_h}{(2\pi)^4}\,\delta(P_h^2 - M_h^2)\,\theta(P_h^0) \qquad = [2(2\pi)^3]^{-1}\,\frac{d\zeta_h}{\zeta_h}\,d^2 P_{hT}$$

(same form in η_h)

Advantages of light-front variables

Symmetric treatment of current and target fragmentation regions in DIS limit

Phase space element separates longitudinal and transverse momenta

Simple properties at finite W: Kinematic limits, phase space element