Tensor SIDIS workshop: short theory kickoff

Alessandro Bacchetta

Spin 1

see Bacchetta, Mulders, hep-ph/0007120

vector polarization

$$\boldsymbol{\rho} = \frac{1}{2}\left(\mathbf{1} + S^i \boldsymbol{\sigma}^i\right)$$

$$\boldsymbol{S} = (S_T^x, S_T^y, S_L)$$

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tensor polarization

$$\boldsymbol{\rho} = \frac{1}{3}\left(\mathbf{1} + \frac{3}{2} S^i \boldsymbol{\Sigma}^i + 3\, T^{ij} \boldsymbol{\Sigma}^{ij}\right)$$

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$$\boldsymbol{T} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}$$

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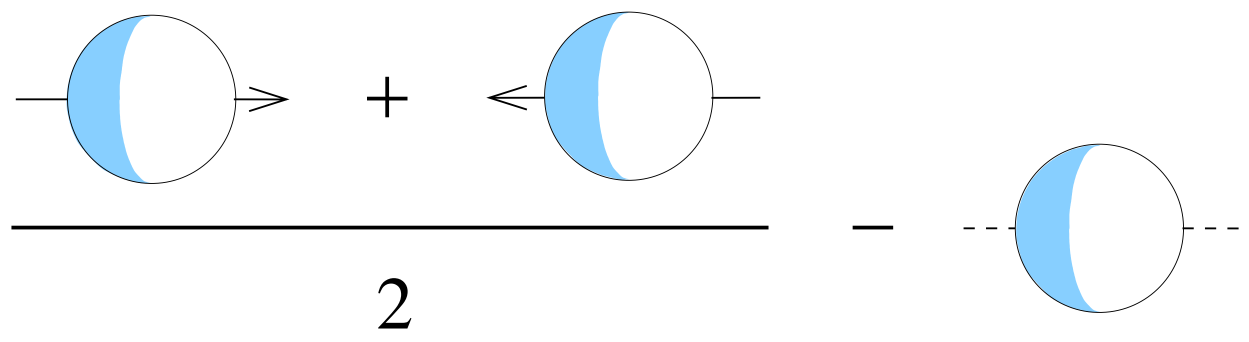
Beware that there are different conventions for the tensor

Spin-1 density matrix

$$\boldsymbol{\rho} = \begin{pmatrix} \frac{1}{3} + \frac{S_{LL}}{3} + \frac{S_L}{2} & \frac{S^x_{LT} - \mathrm{i}S^y_{LT}}{2\sqrt{2}} + \frac{S^x_T - \mathrm{i}S^y_T}{2\sqrt{2}} & \frac{S^{xx}_{TT} - \mathrm{i}S^{xy}_{TT}}{2} \\ \frac{S^x_{LT} + \mathrm{i}S^y_{LT}}{2\sqrt{2}} + \frac{S^x_T + \mathrm{i}S^y_T}{2\sqrt{2}} & \frac{1}{3} - \frac{2S_{LL}}{3} & \frac{-S^x_{LT} + \mathrm{i}S^y_{LT}}{2\sqrt{2}} + \frac{S^x_T - \mathrm{i}S^y_T}{2\sqrt{2}} \\ \frac{S^{xx}_{TT} + \mathrm{i}S^{xy}_{TT}}{2} & \frac{-S^x_{LT} - \mathrm{i}S^y_{LT}}{2\sqrt{2}} + \frac{S^x_T + \mathrm{i}S^y_T}{2\sqrt{2}} & \frac{1}{3} + \frac{S_{LL}}{3} - \frac{S_L}{2} \end{pmatrix}$$

Spin-1 density matrix

$$\boldsymbol{\rho} = \begin{pmatrix} \frac{1}{3} + \frac{S_{LL}}{3} + \frac{S_L}{2} & \frac{S_{LT}^{x} - \mathrm{i}S_{LT}^{y}}{2\sqrt{2}} + \frac{S_{T}^{x} - \mathrm{i}S_{T}^{y}}{2\sqrt{2}} & \frac{S_{TT}^{xx} - \mathrm{i}S_{TT}^{xy}}{2} \\ \frac{S_{LT}^{x} + \mathrm{i}S_{LT}^{y}}{2\sqrt{2}} + \frac{S_{T}^{x} + \mathrm{i}S_{T}^{y}}{2\sqrt{2}} & \frac{1}{3} - \frac{2S_{LL}}{3} & \frac{-S_{LT}^{x} + \mathrm{i}S_{LT}^{y}}{2\sqrt{2}} + \frac{S_{T}^{x} - \mathrm{i}S_{T}^{y}}{2\sqrt{2}} \\ \frac{S_{TT}^{xx} + \mathrm{i}S_{TT}^{xy}}{2} & \frac{-S_{LT}^{x} - \mathrm{i}S_{LT}^{y}}{2\sqrt{2}} + \frac{S_{T}^{x} + \mathrm{i}S_{T}^{y}}{2\sqrt{2}} & \frac{1}{3} + \frac{S_{LL}}{3} - \frac{S_L}{2} \end{pmatrix}$$

$S_{LL} =$ 

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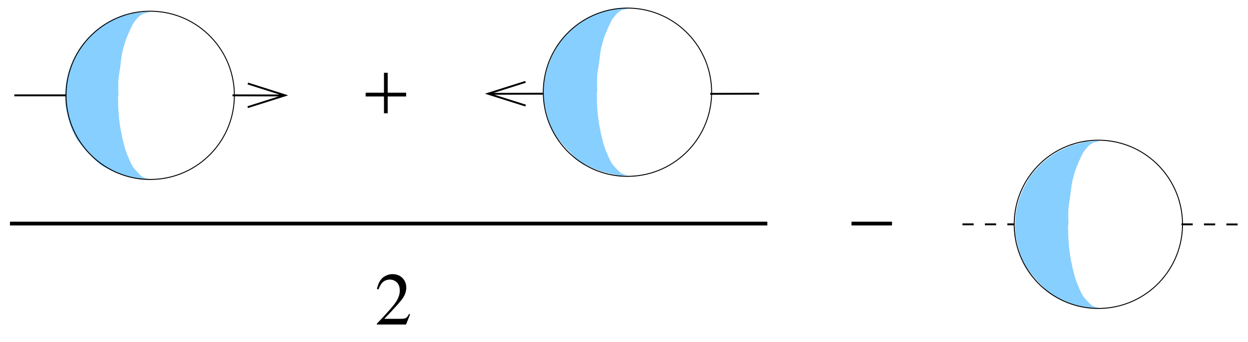
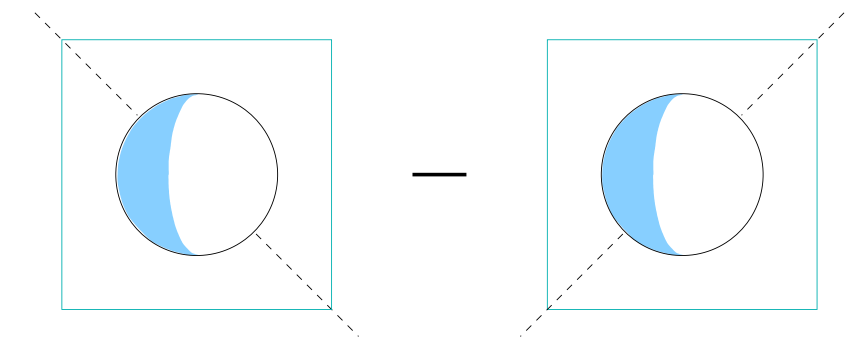
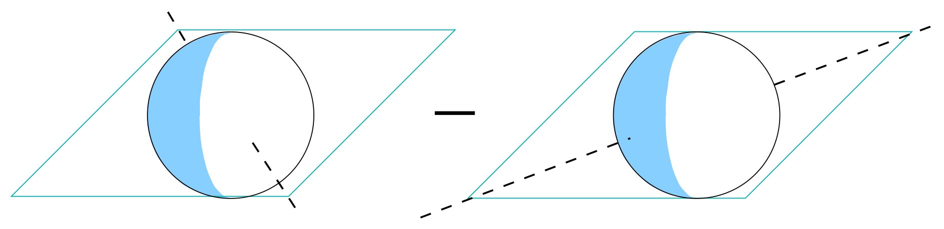
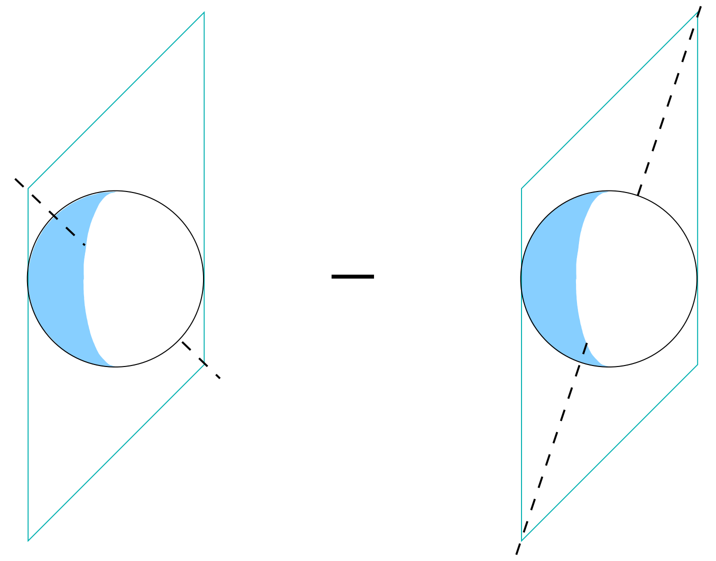
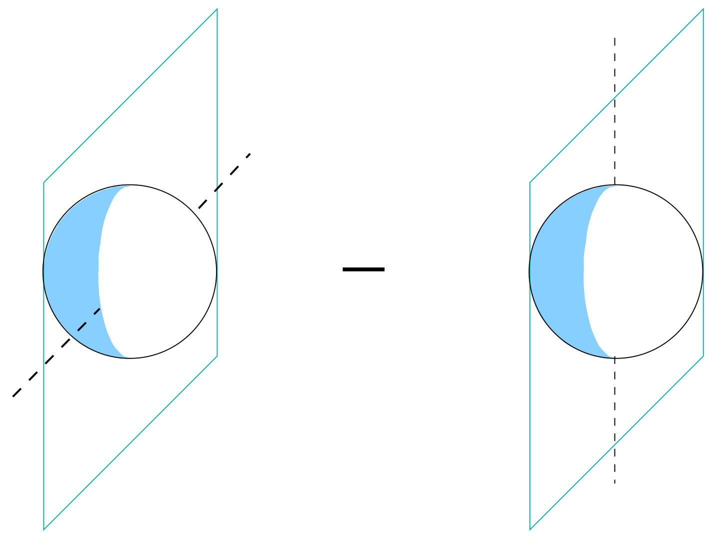
$S_{LL} =$ 

$S_{LT}^x =$ 

$S_{LT}^y =$ 

$S_{TT}^{xy} =$ 

$S_{TT}^{xx} =$ 

Observables

Inclusive DIS

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi} = \frac{2\alpha^2}{x_d y Q^2} \frac{y^2}{2(1-\varepsilon)} \Big\{ S_{LL} \Big(F_{U(LL),T} + \varepsilon F_{U(LL),L} \Big) + |S_{LT}| \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_{LT} F_{U(LT)}^{\cos\phi_{LT}} + |S_{TT}| \varepsilon \cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})} \Big\}$$

analogous to $b_1,\ b_2,\ b_3,\ b_4$

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analogous to b_1, b_2, b_3, b_4

Tensor-polarized PDF

$$F_{U(LL),T}(x_d) = x \sum_a e_a^2 f_{1LL}^a(x)$$

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analogous to b_1, b_2, b_3, b_4

Tensor-polarized PDF

$$F_{U(LL),T}(x_d) = x \sum_a e_a^2 f_{1LL}^a(x)$$

$$F_{U(LT)}^{\cos\phi_{LT}}(x_d) = -x \sum_a e_a^2 \frac{2M}{Q} f_{LT}^a(x)$$

Observables

Semi-inclusive DIS

see Zhao, Bacchetta, Kumano, Liu, Zhou, arxiv:2508.06134 and talk by Jing Zhao

18 structure functions for spin-½

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy dz d\phi_h d\psi dP_{h\perp}^2}$$

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18+23 structure functions for spin-1

$$\frac{d\sigma_{\mathrm{Tens}}}{dx_d dy dz d\phi_h d\psi dP_{h\perp}^2} = \frac{\alpha^2}{x_d y Q^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_d}\right)\Big\{S_{LL}\Big[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{U(LL)}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{U(LL)}^{\cos 2\phi_h} + \lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, F_{L(LL)}^{\sin\phi_h}\Big]$$

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$$F_{U(LL),T} = \mathcal{C}[f_{1LL} D_1]$$

Convolution in transverse momentum

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Tensor-polarized TMD PDF

$$F_{U(LL),T} = \mathcal{C}[f_{1LL} D_1]$$

Convolution in transverse momentum

Other examples

$$F_{U(LT),T}^{\cos(\phi_h-\phi_{LT})} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1LT} D_1\right]$$

leading twist, relatively simple convolution, requires LT tensor polarization

Other examples

$$F_{U(LT),T}^{\cos(\phi_h-\phi_{LT})} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1LT} D_1\right]$$

leading twist, relatively simple convolution, requires LT tensor polarization

$$F_{L(LL)}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe_{LL}H_1^{\perp} + \frac{M_h}{M} f_{1LL}\frac{\tilde{G}^{\perp}}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg_{LL}^{\perp}D_1 + \frac{M_h}{M} h_{1LL}^{\perp}\frac{\tilde{E}}{z}\right)\right]$$

subleading twist, similar to Cahn effect, requires LT tensor polarization

Other examples

$$F_{U(LT),T}^{\cos(\phi_h-\phi_{LT})} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1LT} D_1\right]$$

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subleading twist, similar to Cahn effect, requires LT tensor polarization

$$F_{U(TT)}^{\cos(2\phi_{TT})} = \mathcal{C}\left[\frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1TT} H_1^{\perp}\right]$$

leading twist, relatively simple angular dependence, requires TT tensor polarization

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Hopefully, we will start addressing these questions during the workshop!