

Rosenbluth separation of the elastic  
electron-proton scattering cross section at  
 $Q^2 = 9.5 \text{ (GeV/c)}^2$

PAC54 proposal PR12-26-007

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# Elastic electron–proton scattering L/T separation at high $Q^2$

Why is L/T important to measure?

How can Hall C do such a measurement?

Projected accuracy in 15.5-day run

# Elastic electron–proton scattering L/T separation at high $Q^2$

Why is L/T important to measure?

two-photon effects

How can Hall C do such a measurement?

next slides

Projected accuracy in 15.5-day run

# Electro-Magnetic Form Factors



One-photon approximation,  $\alpha_{em} = 1/137$ , hadron current

$$\mathcal{J}_{hadronic}^\mu = ie\bar{N}(p') \left[ \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_2(Q^2) \right] N(p)$$

At large  $Q^2$ , the study of  $G_E$  requires use of polarization observables => FFs at JLab

Rosenbluth (1950)

Akhiezer (1957)  
Arnold, Carlson and Gross (1981)

$1\gamma+2\gamma$  expression for  $\mathcal{M}$  has three complex functions,  $F_1, F_2, F_3$

$$\mathcal{M} = \frac{4\pi\alpha}{Q^2} \bar{u}' \gamma_\mu u \cdot \bar{N}' \left( \tilde{F}_1 \gamma^\mu - \tilde{F}_2 [\gamma^\mu, \gamma^\nu] \frac{q_\nu}{4M} + \tilde{F}_3 K_\nu \gamma^\nu \frac{P^\mu}{M^2} \right) N$$

$$\tilde{G}_M = \tilde{F}_1 + \tilde{F}_2 \quad \tilde{G}_E = \tilde{F}_1 - \tau \tilde{F}_2$$

$\tilde{F}_i$  are functions of  $(s - u)$  and  $t$

Guichon & Vanderhaeghen

$$d\sigma = d\sigma_{NS} \left\{ \epsilon (\tilde{G}_E + \frac{s-u}{4M^2} \tilde{F}_3)^2 + \tau (\tilde{G}_M + \epsilon \frac{s-u}{4M^2} \tilde{F}_3)^2 \right\}$$

$$\sigma_R = \epsilon G_E^2 + \tau G_M^2 + 2\tau G_M \text{Re} \left( \delta \tilde{G}_M + \epsilon \frac{s-u}{M^2} \tilde{F}_3 \right) + 2\epsilon G_E \text{Re} \left( \delta \tilde{G}_E + \frac{s-u}{M^2} \tilde{F}_3 \right)$$

Two-Photon Exchange

# Electro-Magnetic Form Factors



One-photon approximation,  $\alpha_{em} = 1/137$ , hadron current

$$\mathcal{J}_{hadronic}^\mu = ie\bar{N}(p') \left[ \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_2(Q^2) \right] N(p)$$

Rosenbluth (1950)

## SLAC results for the proton Form Factors

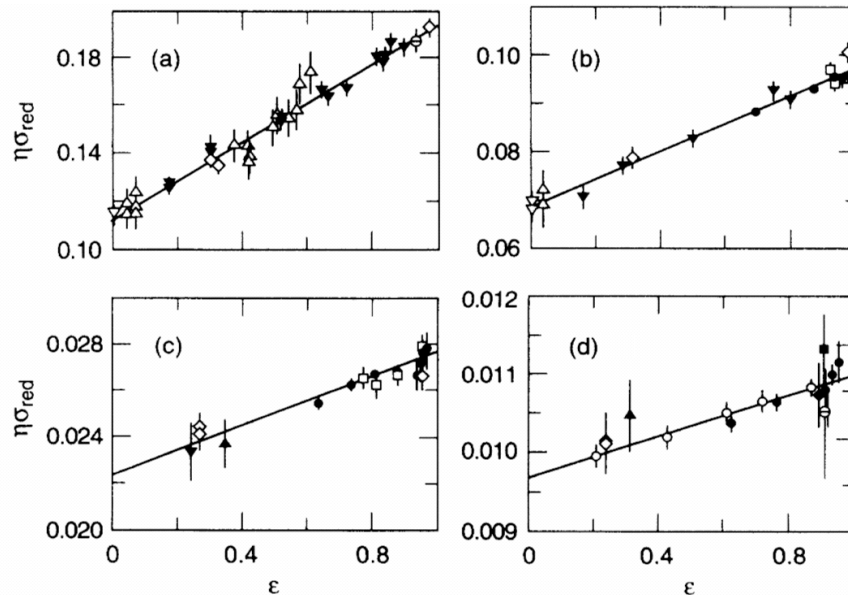
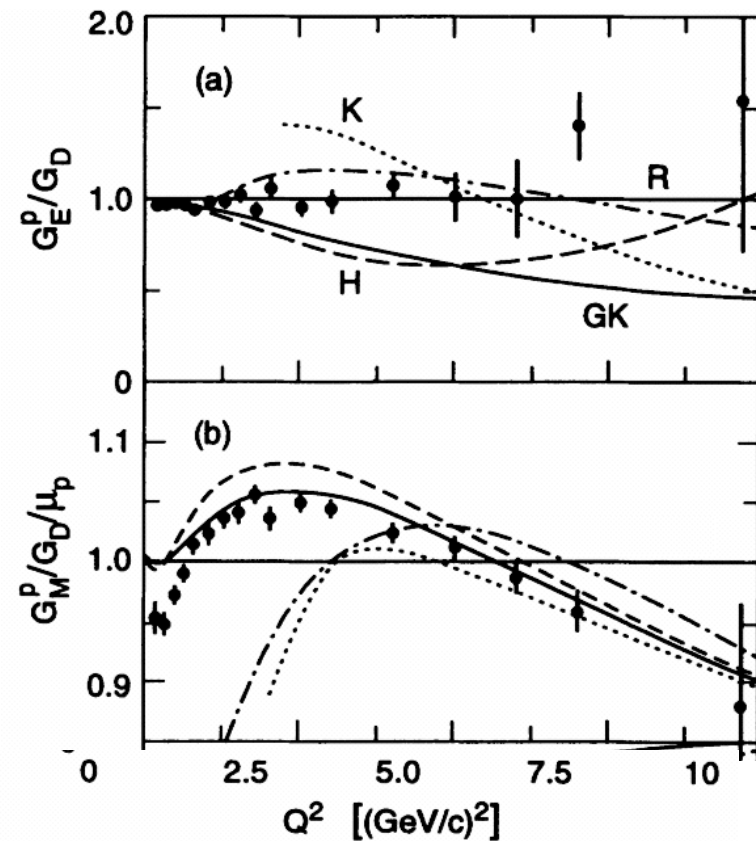
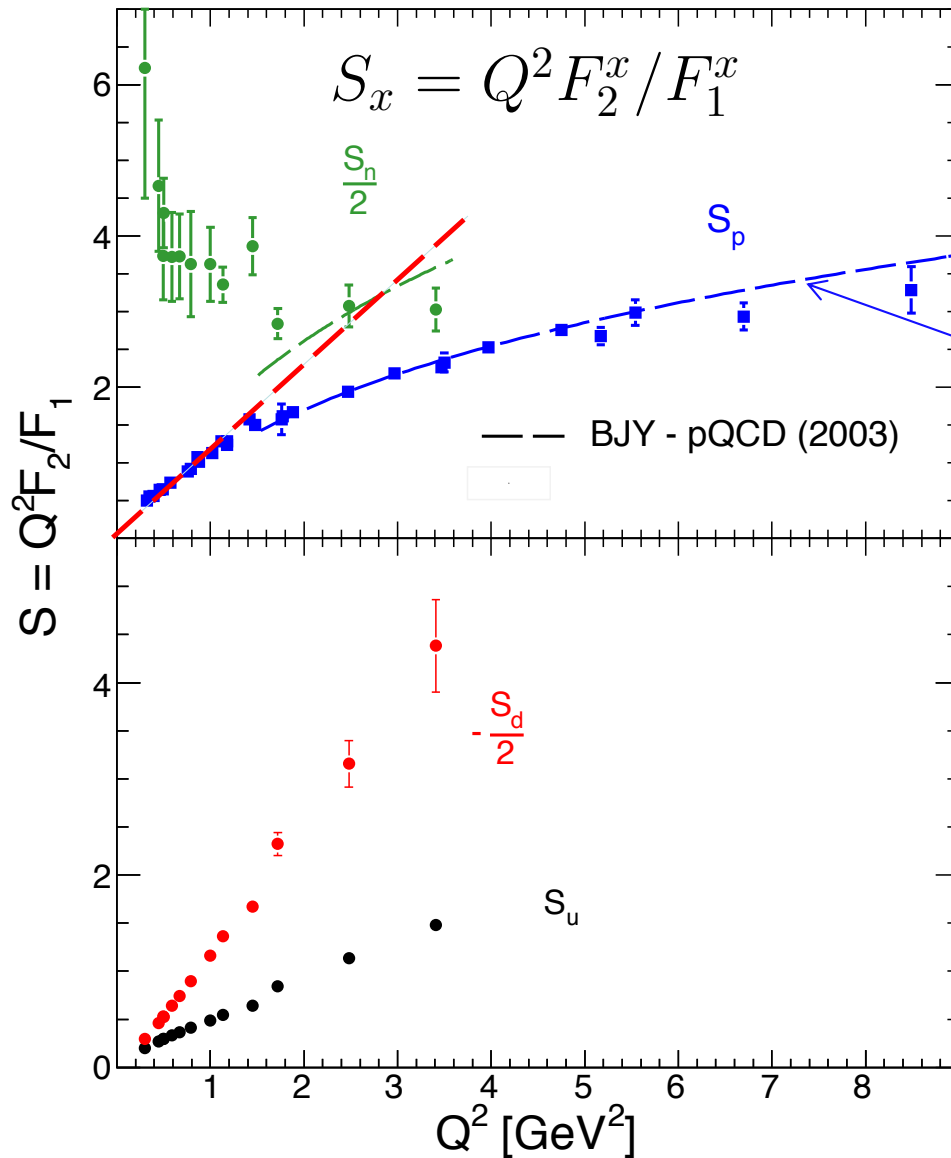


FIG. 9. Four typical Rosenbluth fits for the form factor extraction from the global data set at (a)  $Q^2 = 0.6$ , (b)  $Q^2 = 1.0$ , (c)  $Q^2 = 2.0$ , and (d)  $Q^2 = 3.0$   $(\text{GeV}/c)^2$ .



# The goal is understanding of the nucleon



pQCD prediction for large  $Q^2$ :  
 $S \rightarrow Q^2 F_2 / F_1 = \text{constant}$

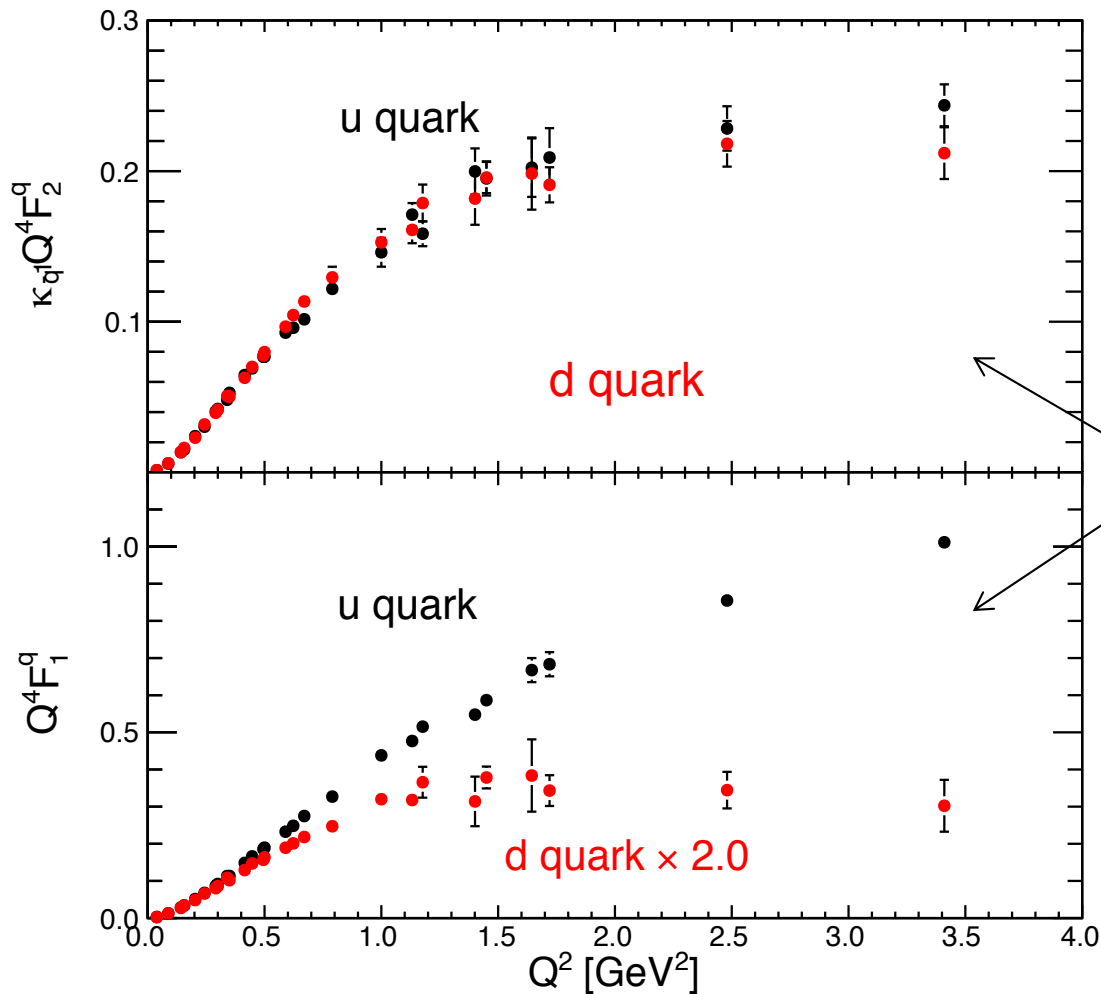
pQCD updated prediction:  
 $S \rightarrow [Q^2 / \ln^2(Q^2 / \Lambda^2)] F_2 / F_1$

Flavor separated contribution:  
 The log scaling for the proton  
 Form Factor ratio at a few  
 $\text{GeV}^2$  is likely "accidental".

The lines for individual flavor  
 are straight!

Cates, Jager, Riordan, BW  
 Physical Review Letters, 106, 252003 (2011)

# The flavor disparity in the nucleon



CJRW (u/d with new GEn data)  
Phys. Rev. Lett. 106 (2011)

Qattan, Arrington (2- $\gamma$  effects)  
Phys.Rev. C86 (2012) 065210

M.Diehl and P.Kroll (GPDs)  
Eur.Phys.J. C73 (2013) 2397

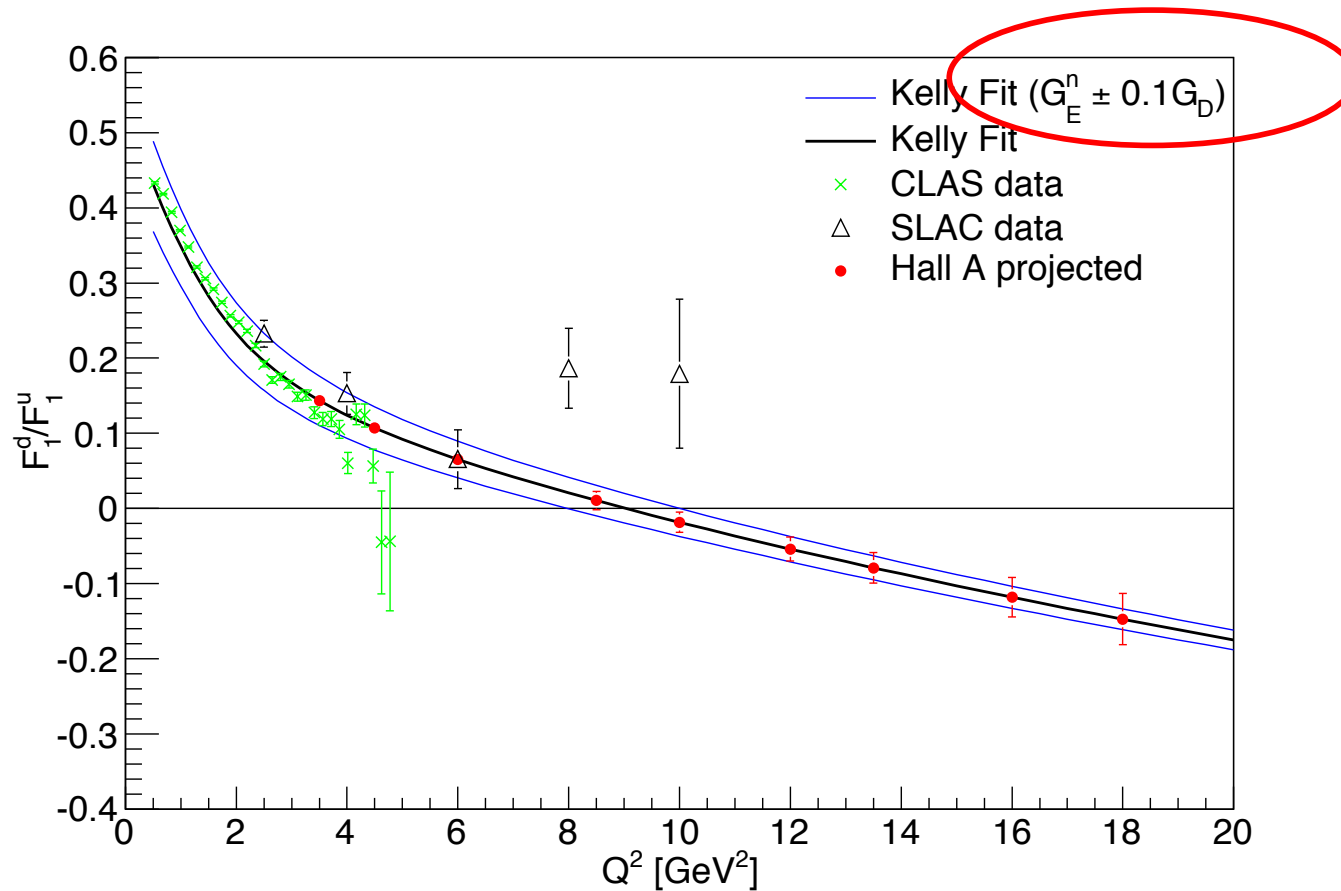
Using the D&K table of  $F^u$ ,  $F^d$

The down quark contribution  
to the  $F_1$  proton form factor is  
strongly suppressed at high  $Q^2$

When the virtual photon of 3 GeV<sup>2</sup> interacts with the down quark  
the proton more likely falls apart than in the case of the up quark

# $F_1$ decomposition at very large $Q^2$

$$F_1 = \frac{G_E + \tau G_M}{1 + \tau} \quad F_2 = -\frac{G_E - G_M}{1 + \tau}$$



# Electro-Magnetic Form Factors



One-photon approximation,  $\alpha_{em} = 1/137$ , hadron current

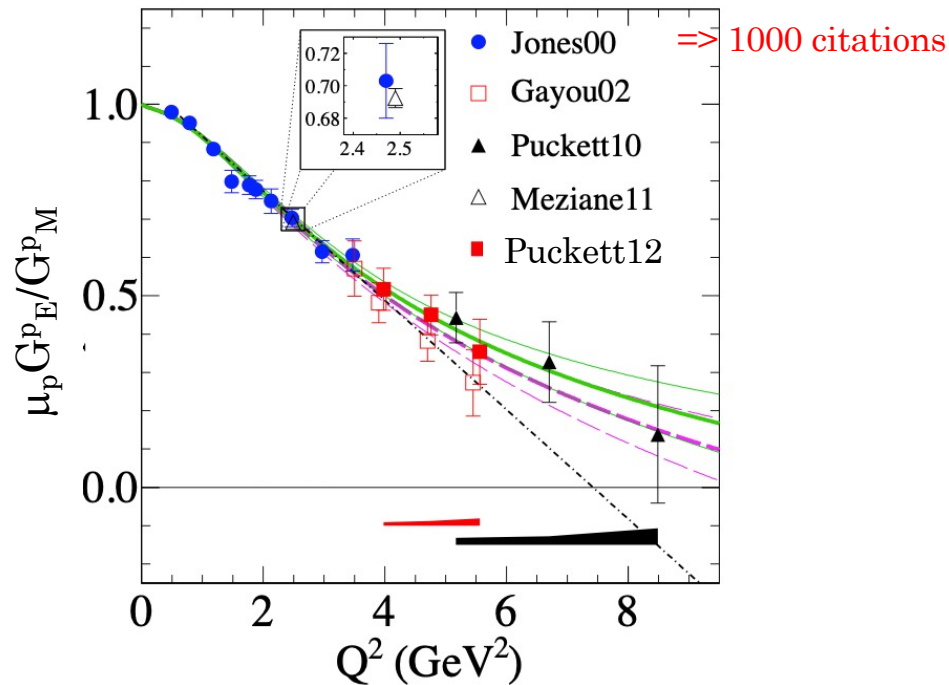
$$\mathcal{J}_{hadronic}^\mu = ie\bar{N}(p') \left[ \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_2(Q^2) \right] N(p)$$

At large  $Q^2$ , the study of  $G_E$  requires use of polarization observables  $\Rightarrow$  FFs at JLab

Rosenbluth (1950)

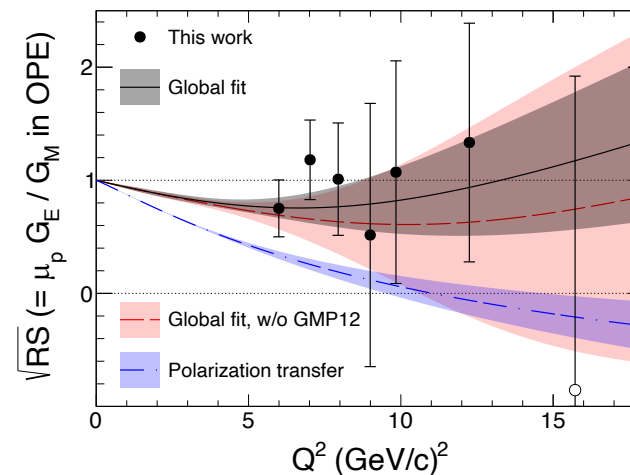
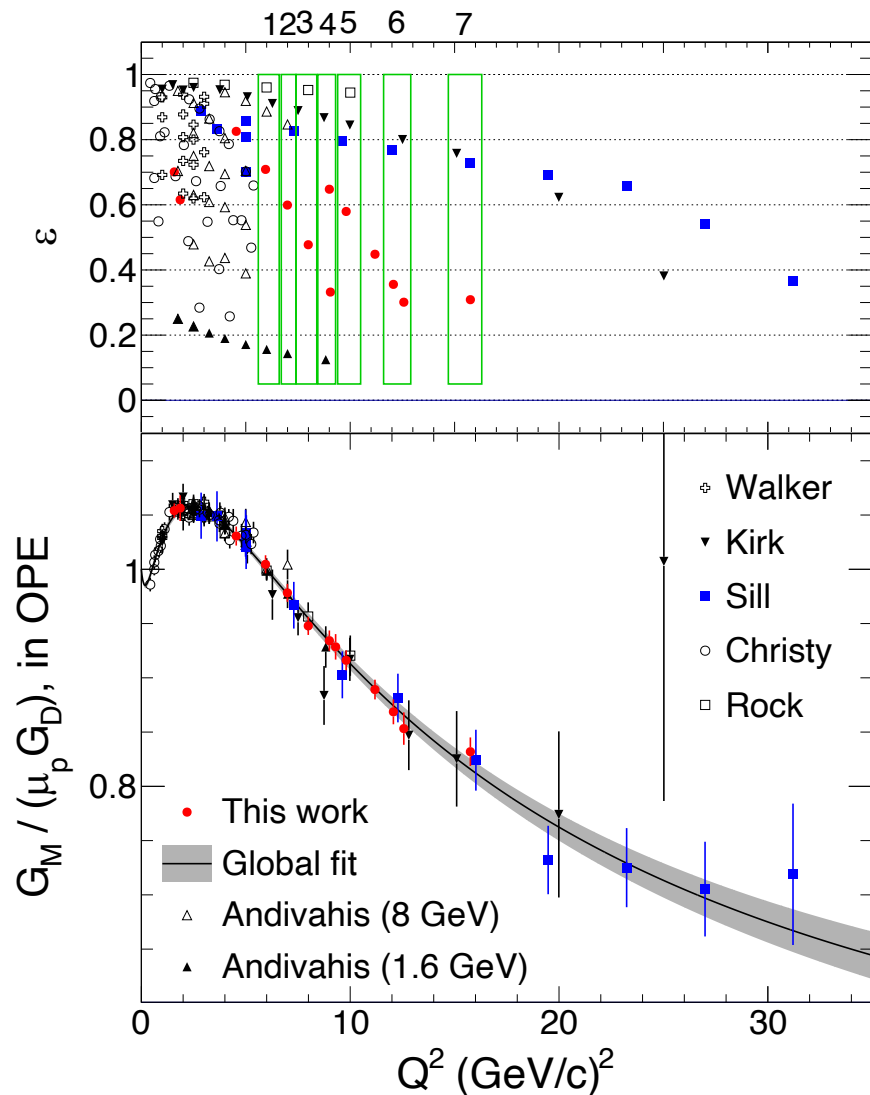
Akhiezer (1957)  
Arnold, Carlson  
and Gross (1981)

Perdrisat



# The GMp12 experiment (E12-07-108)

Phys.Rev.Lett. 128 (2022) 10, 102002



GMp12 fit:

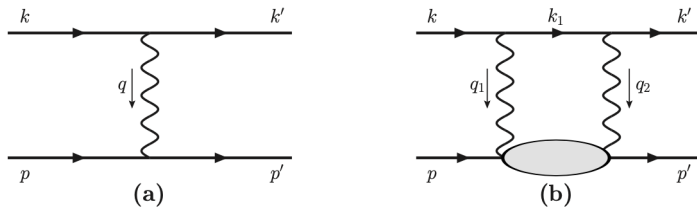
$$G_M = \mu_p (1 + a_1 \tau) / (1 + b_1 \tau + b_2 \tau^2 + b_3 \tau^3),$$

$$RS = 1 + c_1 \tau + c_2 \tau^2.$$

$a_1$	$b_1$	$b_2$	$b_3$	$c_1$	$c_2$
0.072(22)	10.73(11)	19.81(17)	4.75(65)	-0.46(12)	0.12(10)

courtesy of A. Gramolin and A. Puckett

# Proton E/M from cross section



$$\begin{aligned}\sigma_R &= \tau G_M^2(Q^2) + \varepsilon G_E^2(Q^2) = \sigma_T + \varepsilon \sigma_L \\ &= G_M^2(Q^2)(\tau + \varepsilon RS(Q^2)/\mu_p^2),\end{aligned}$$

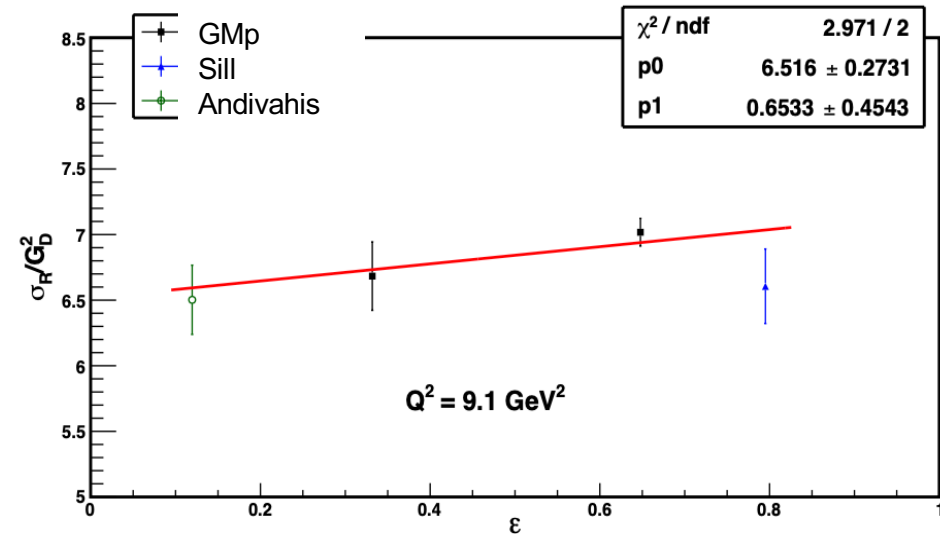
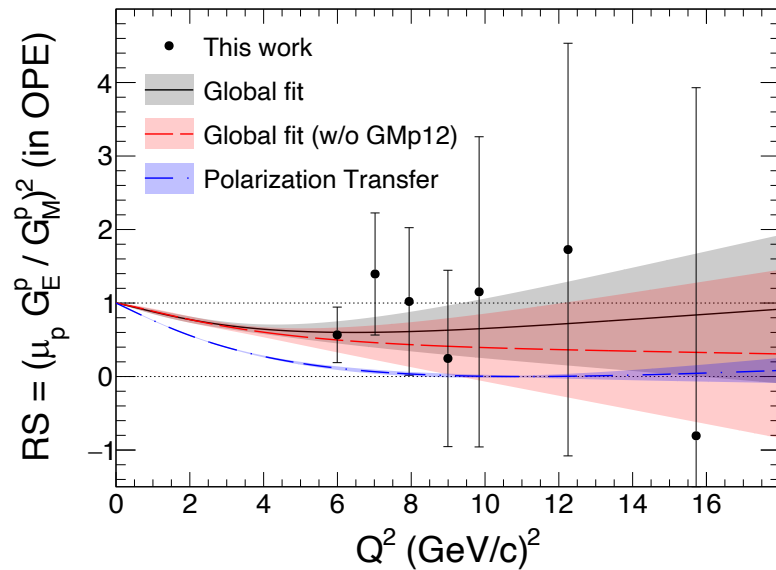
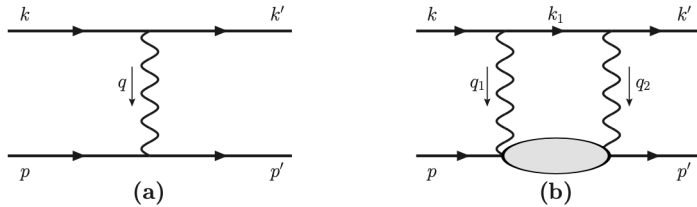


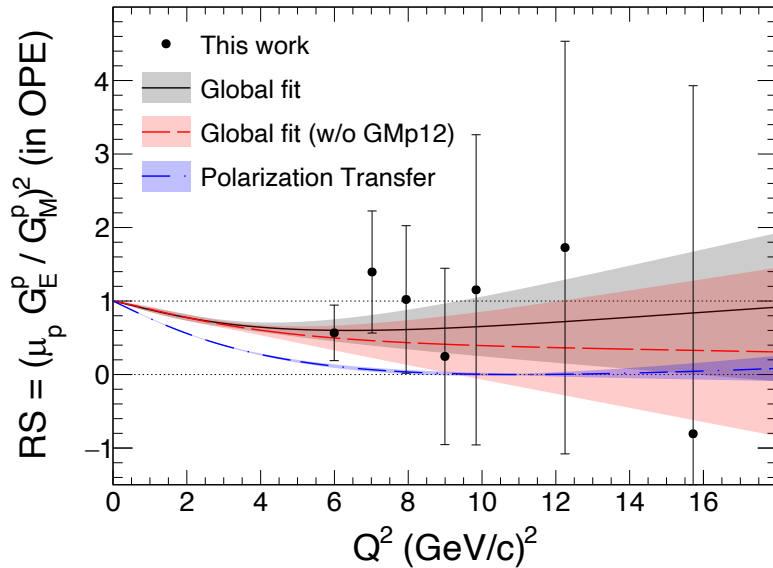
Figure 5.7. Reduced cross section normalized by  $G_D^2$  versus  $\varepsilon$  at  $Q^2 = 9.1 \text{ GeV}^2$ .

$$d\sigma/d\Omega \propto E_e^2/Q^4 \times 1/(Q^2)^4$$

# Proton E/M from cross section



$$\begin{aligned}\sigma_R &= \tau G_M^2(Q^2) + \varepsilon G_E^2(Q^2) = \sigma_T + \varepsilon \sigma_L \\ &= G_M^2(Q^2)(\tau + \varepsilon \text{RS}(Q^2)/\mu_p^2),\end{aligned}$$



$$d\sigma/d\Omega \propto E_e^2/Q^4 \times 1/(Q^2)^4$$

TABLE III. Rosenbluth separation results for the data groupings shown in the top panel of Fig. 1, after centering to the average  $Q_c^2$ . The quoted values of  $\sigma_L$  and  $\sigma_T$  as defined in Eq. (2), and  $G_M/(\mu_p G_D)$  and  $\mu_p G_E/G_M$  are obtained assuming validity of the OPE approximation. For the largest  $Q^2$ , where  $\sigma_L < 0$ , we quote  $-\sqrt{|\text{RS}|}$ .

$Q_c^2$ (GeV/c) <sup>2</sup>	$\sigma_T \times 10^5$	$\sigma_L \times 10^5$	$G_M/(\mu_p G_D)$ (OPE)	$\mu_p G_E/G_M$ (OPE)
5.994	167 ± 4	7.1 ± 4.6	1.000 ± 0.011	0.75 ± 0.25
7.020	104 ± 3	9.3 ± 5.3	0.967 ± 0.015	1.18 ± 0.35
7.943	71.0 ± 2.7	4.1 ± 3.9	0.943 ± 0.018	1.0 ± 0.5
8.994	49.8 ± 1.7	0.7 ± 3.0	0.934 ± 0.016	0.5 ± 1.2
9.840	36.9 ± 2.4	1.9 ± 3.5	0.909 ± 0.029	1.1 ± 1.0
12.249	18.0 ± 0.8	1.2 ± 1.8	0.858 ± 0.019	1.3 ± 1.1
15.721	8.6 ± 0.5	-0.2 ± 1.2	0.840 ± 0.025	(-0.9 ± 2.8)

# Electron-proton elastic cross section

$$\sigma_{Mott} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \quad \frac{d\sigma}{d\Omega} = \sigma_{Mott} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2G_M^2 \tan^2 \frac{\theta}{2} \right],$$

$$d\sigma/d\Omega \propto E_e^2/Q^4 \times 1/(Q^2)^4$$

In the L/T experiment with detection of **the scattered electron** we need to **measure** very accurately:

- Beam charge
- **Beam energy**
- Target thickness
- **Spectrometer momentum**
- **Spectrometer angle**
- Spectrometer solid angle
- Detector efficiency

$$\sigma_R = \tau G_M^2 + \epsilon G_E^2$$

$$\tau \equiv \frac{Q^2}{4M^2}$$

at  $Q^2 = 9.5 \text{ GeV}^2$

$$\tau \mu_p^2 \approx 21$$

# Electron-proton elastic cross section

$$\frac{\Delta\sigma(t)}{\Delta|t|} = \frac{\pi\alpha^2}{E_e^2 t^2} \cdot \left\{ G_E^2(t) \times \left[ \frac{(4ME_e + t)^2}{4M^2 - t} + t \right] - \frac{t}{4M^2} \cdot G_M^2(t) \times \left[ \frac{(4ME_e + t)^2}{4M^2 - t} - t \right] \right\}, \quad (3)$$

$$\partial\sigma/\partial E_e = -\frac{2 \cdot \sigma}{E_e} + \frac{2.34 \cdot \sigma}{11 \text{ GeV}} \approx \frac{0.34\sigma}{E_e}$$

In the L/T experiment with detection of **the scattered proton** we need to measure very accurately:

- Beam charge
- Target thickness **stability**
- Spectrometer solid angle
- Spectrometer **momentum stability**
- Detector **efficiency stability**

# Electron-proton elastic cross section

In the L/T experiment need to measure very accurately:

Demonstrated performance of the HMS and design specifications for the SHMS. Resolutions are quoted at 1 sigma.

<i>Parameter</i>	<i>HMS Performance</i>	<i>SHMS Specification</i>
Range of Central Momentum	0.4 to 7.4 GeV/c	2 to 11 GeV/c
Momentum Acceptance	$\pm 10\%$	-10% to +22%
Momentum Resolution	0.1% – 0.15%	0.03% – 0.08%
Scattering Angle Range	10.5° to 90°	5.5° to 40°
Target Length Accepted at 90°(HMS)/40° (SHMS)	10 cm	25 cm
Horizontal Angle Acceptance	$\pm 32$ mrad	$\pm 18$ mrad
Vertical Angle Acceptance	$\pm 85$ mrad	$\pm 45$ mrad
Solid Angle Acceptance	8.1 msr	4 msr
Horizontal Angle Resolution	0.8 mrad	0.5 – 1.2 mrad
Vertical Angle Resolution	1.0 mrad	0.3 – 1.1 mrad
Target resolution ( $y_{tar}$ )	0.3 cm	0.1 - 0.3 cm
Maximum Event Rate	4–5 kHz	4–5 kHz
Max. Flux within Acceptance	$\sim 5$ MHz	$\sim 5$ MHz
e/h Discrimination	>1000:1 at 98% efficiency	>1000:1 at 98% efficiency
$\pi$ /K Discrimination	100:1 at 95% efficiency	100:1 at 95% efficiency

# High accuracy L/T, 1999

**Letter –of-Intent:** LOI-99-003

**Title:** A Precision Measurement of  $G_p^E/G_p^M$  at  $Q^2 = 2.0$  and  $4.0 \text{ GeV}^2$

**Spokespersons:** B. Wojtsekhowski, W. Bertozzi, K. Fissum, D. Rowntree

Precision measurements of  $G_p^E$  are of great interest for providing information on the proton's

will allow improved acceptance determination as well as  $Q^2$  matching between the points at the level of the HRHh spectrometer resolution ( $\approx 10^{-4}$ ) since this spectrometer's magnetic field setting will be constant. While we expect the coincident measurement will provide the most accurate measurement of  $G_{E_p}/G_{M_p}$ , the singles measurement will provide a consistency

# High accuracy L/T, 1999

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**Spokespersons:** B. Wojtsekhowski, W. Bertozzi, K. Fissum, D. Rowntree

Precision measurements of  $G_p^E$  are of great interest for providing information on the proton's substructure. New precision measurements using recoil polarization from Hall A experiment 93-027 indicate a decreasing ratio of  $G_p^E/G_p^M$  with  $Q^2$ , contradicting some of the previous measurements using a Rosenbluth separation. This LOI discusses a new precision measurement in Hall A using the Rosenbluth technique. It would require control and understanding of systematic effects at the level of 1% in measurements of relative cross sections. The PAC acknowledges the interesting suggestions for limiting potential systematic errors presented in this letter but is not convinced that this 1% level can be achieved. In addition, as a cross-check on the recoil polarization technique a polarized beam-polarized target measurement would be more straightforward than the extremely challenging high-precision Rosenbluth separation discussed here. Also, while the PAC appreciates the importance of demonstrating the potential for doing high-precision Rosenbluth measurements with regard to future possible measurements (e.g. Coulomb Sum Rule), the PAC is not convinced that the physics motivation discussed in the present letter warrants the significant effort required to carry out such a difficult experiment.

# High accuracy L/T, 2001

**Proposal:** E-01-001

**Scientific Rating:** A

**Title:** New measurement of  $G_E/G_M$  for the proton.

**Spokesperson:** R. E. Segel and J. Arrington

**Motivation:** The disagreement between the Rosenbluth method and the polarization transfer method of existing determinations of  $G_E/G_M$  motivates this experiment to make a new Rosenbluth measurement with several improvements to the experimental method. It is of great importance to determine if there is a fundamental problem with either the Rosenbluth or polarization transfer methods, as they are also used for many other experiments.

**Measurement and Feasibility:** The new measurement will detect protons, which have fixed momentum at fixed  $Q^2$ , independent of epsilon. By simultaneously making measurements at very low  $Q^2$ , where there is no controversy, systematic errors are reduced compared to the previous Rosenbluth measurements, which detected electrons over a wide range of momentum at fixed  $Q^2$ , and did not have a simultaneous low  $Q^2$  measurement. Radiative corrections are also smaller using protons. The experiment uses standard equipment and methods, and appears to be straightforward to carry out.

**Issues:** The PAC believes it would be of higher scientific value to emphasize more precise measurements at the lower values of  $Q^2$ , where the Rosenbluth method and polarization transfer already have a significant difference. It will be very important to check the assumed linearity of the Rosenbluth separation with respect to epsilon at the optimal  $Q^2$  values by taking data at more epsilon points than proposed.

**Recommendation:** Approve for 10 days in Hall A.

# Experimental aspects

Stability / quality of the equipment, confirmed by experts

beam	x
target	x
magnets	x
optics + resolution (elastic peak shape)	x
pion rejection, proton PID	x
detector/DAQ efficiency	x

Specific considerations:

Max Q<sup>2</sup>, Peak shape calibration, Heavy gas PID

Approaches to the inelastic sources of the protons:

- target windows – z-cut with 20-cm long target
- pion related – take some data with a radiator

# Proton momentum spectra

PRL **94**, 142301 (2005)

PHYSICAL REVIEW LETTERS

week ending  
15 APRIL 2005

## Precision Rosenbluth Measurement of the Proton Elastic Form Factors

I. A. Qattan,<sup>1,2</sup> J. Arrington,<sup>2</sup> R. E. Segel,<sup>1</sup> X. Zheng,<sup>2</sup> K. Aniol,<sup>3</sup> O. K. Baker,<sup>4</sup> R. Beams,<sup>2</sup> E. J. Brash,<sup>5</sup> J. Calarco,<sup>6</sup>  
A. Camsonne,<sup>7</sup> J.-P. Chen,<sup>8</sup> M. E. Christy,<sup>4</sup> D. Dutta,<sup>9</sup> R. Ent,<sup>8</sup> S. Frullani,<sup>10</sup> D. Gaskell,<sup>11</sup> O. Gayou,<sup>12</sup> R. Gilman,<sup>13,8</sup>

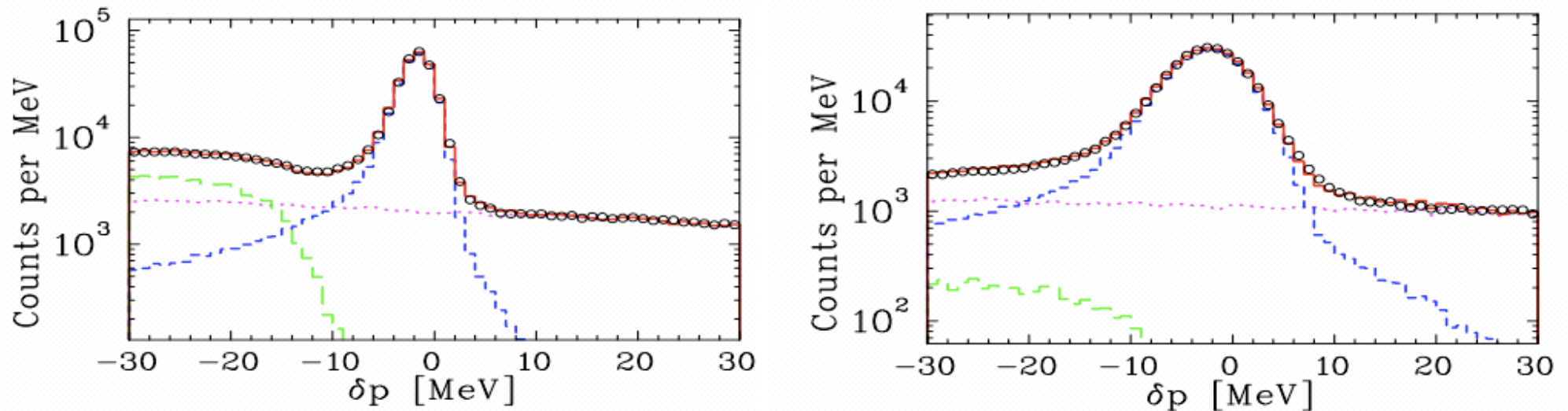
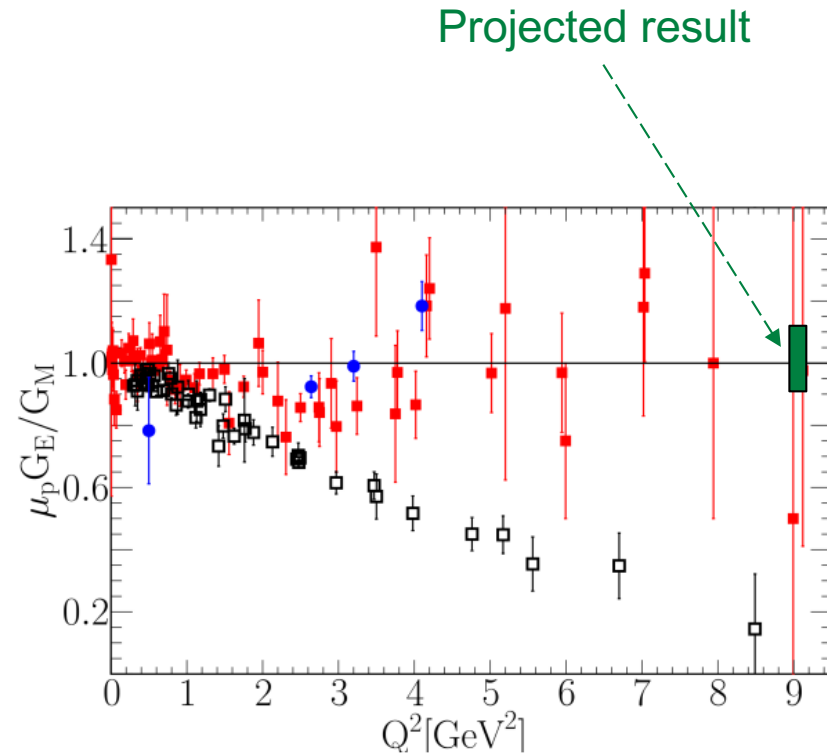
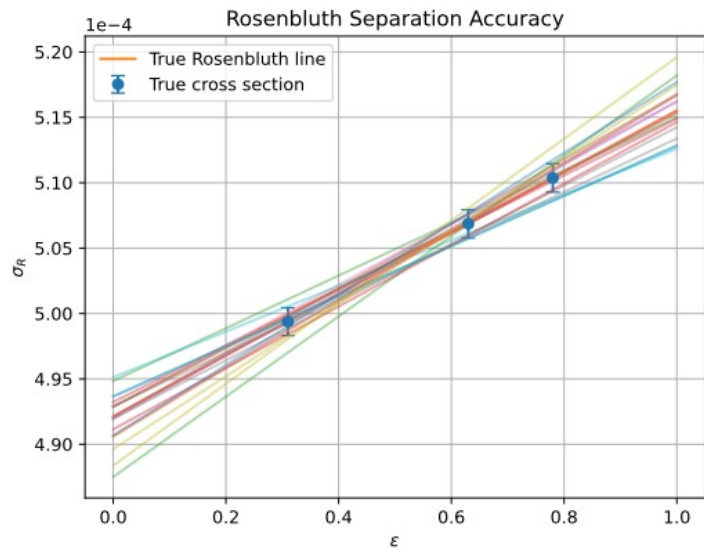


FIG. 1 (color online). The measured  $\delta p$  spectrum for the low (top) and high (bottom)  $\varepsilon$  points at  $Q^2 = 3.2 \text{ GeV}^2$  (circles). The dotted magenta line is the background from the target walls, the long-dash green line is the simulated background from  $\gamma p \rightarrow \pi^0 p$  and  $\gamma p \rightarrow \gamma p$  reactions, the short-dash blue line is the simulated elastic spectrum, and the solid red line (largely obscured by the data points) is the sum of the target wall, elastic, and background contributions.

# Plan of this experiment



Currently have 50-100% Rosenbluth separation uncertainty at  $Q^2 \sim 10 \text{ GeV}^2$

# Projected accuracy

The purpose of this experiment is to measure the  $e - p$  elastic scattering cross section at  $Q^2 = 9.5 \text{ (GeV}/c)^2$  at three values of  $\varepsilon$ .

Setting	Beam energy, GeV	Beam current, $\mu\text{A}$	Proton angle (deg)	Kinematical $\varepsilon$	$\sigma$ , $\text{cm}^2/\text{sr}_p$	Statistical $\delta\sigma/\sigma$ , %
1	6.6	50	12.7	0.307	$2.6 \times 10^{-36}$	0.075
2	8.8	50	19.1	0.635	$3.0 \times 10^{-36}$	0.075
3	11.0	50	22.0	0.782	$3.2 \times 10^{-36}$	0.075

$$\sigma \propto (\tau + \varepsilon \times R_{LT}^2) \quad \text{and} \quad \frac{\delta R_{LT}}{R_{LT}} \simeq \frac{\tau \mu_p^2}{2 \cdot \Delta\varepsilon} \times \frac{\delta\sigma}{\sigma}$$
$$\tau \mu_p^2 \approx 21$$

# Projected accuracy

Setting	Beam energy, GeV	Time days	Statistical $\delta\sigma/\sigma, \%$	Systematical point-to-point $\delta\sigma/\sigma, \%$
1	6.6	3.1	0.075	0.2
2	8.8	2.8	0.075	0.2
3	11.0	2.7	0.075	0.2

# Summary

- ❖ Accurate measurement of the L/T ratio at high  $Q^2$  will significantly boost understanding of the basic nucleon form factors.
- ❖ Accurate measurement of the L/T ratio at high  $Q^2$  is possible using a fixed momentum proton spectrometer.
- ❖ HMS and SHMS with 6-7 GeV/c allow us to do this experiment in 15.5 days for  $Q^2 = 9.5 \text{ GeV}^2$  to accuracy  $\delta(\mu G_E)/G_M \sim 10\%$
- ❖ L/T-10 will provide essential constraints on two-photon exchange contribution functions.