Model-Independent Determination of Proton Structure Functions at the Electron-Ion Collider

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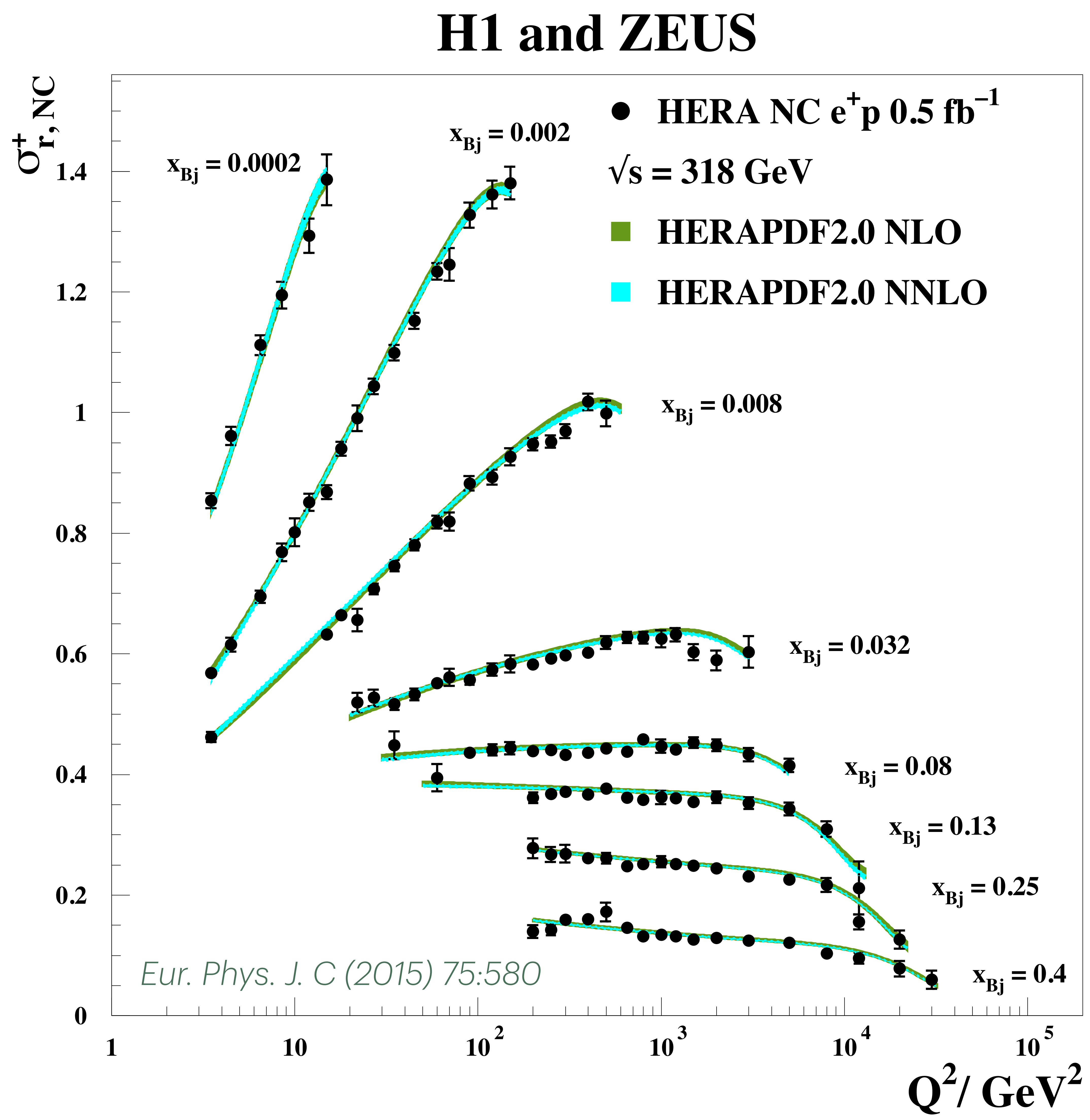
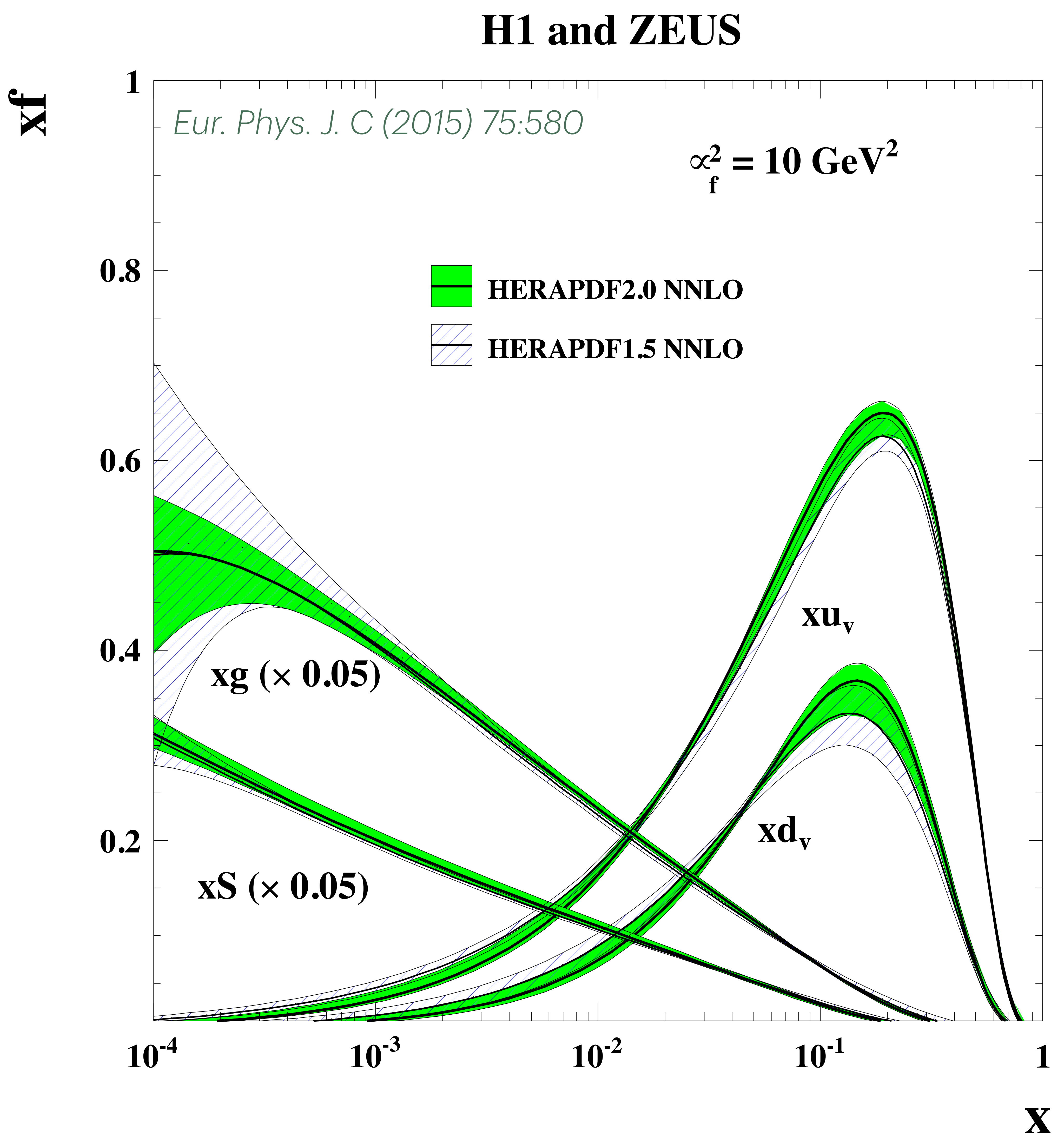
Why map the proton?

The proton is our main probe of nature, yet we can't calculate its insides

We have to measure them!

Why map the proton?



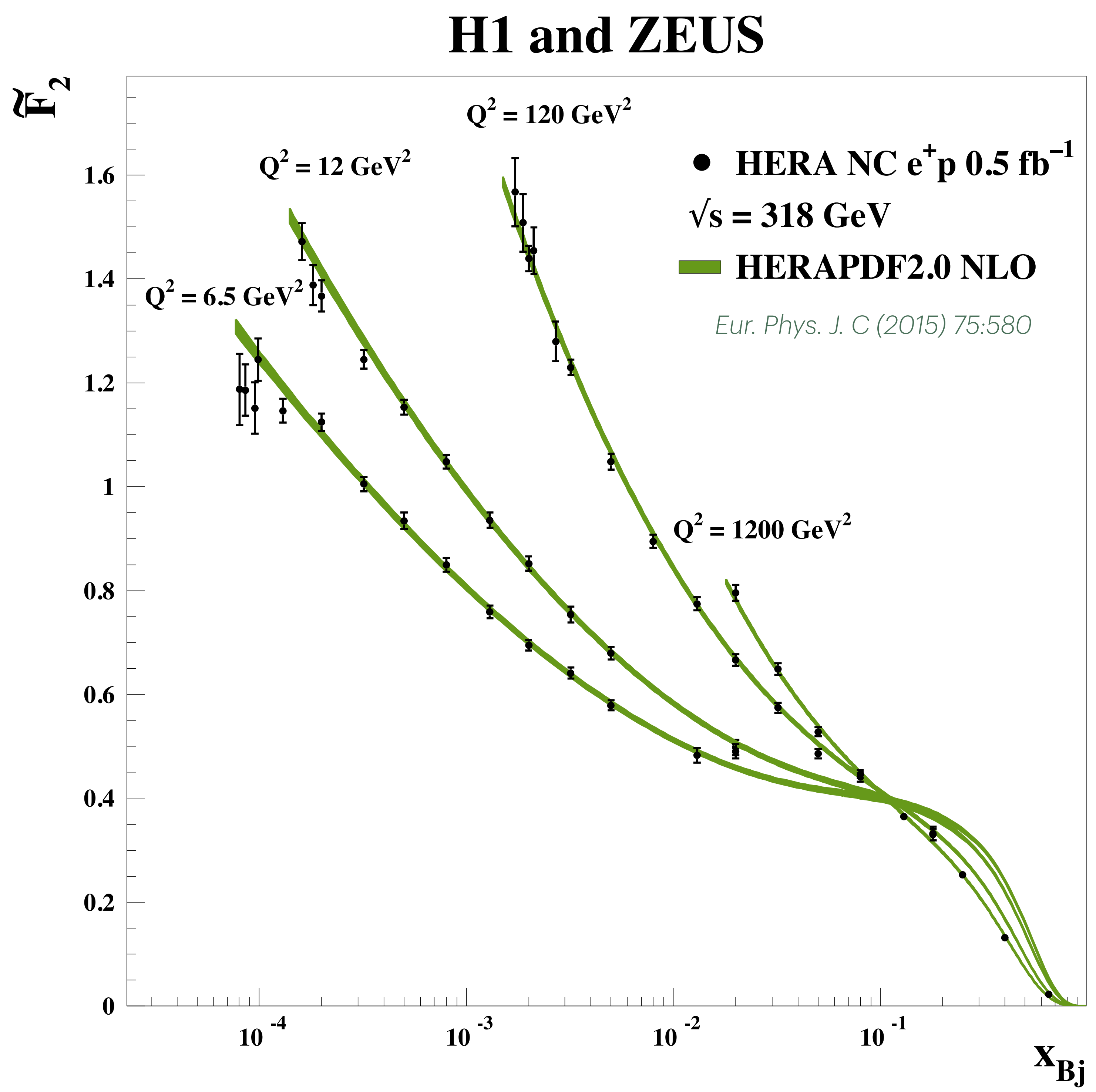


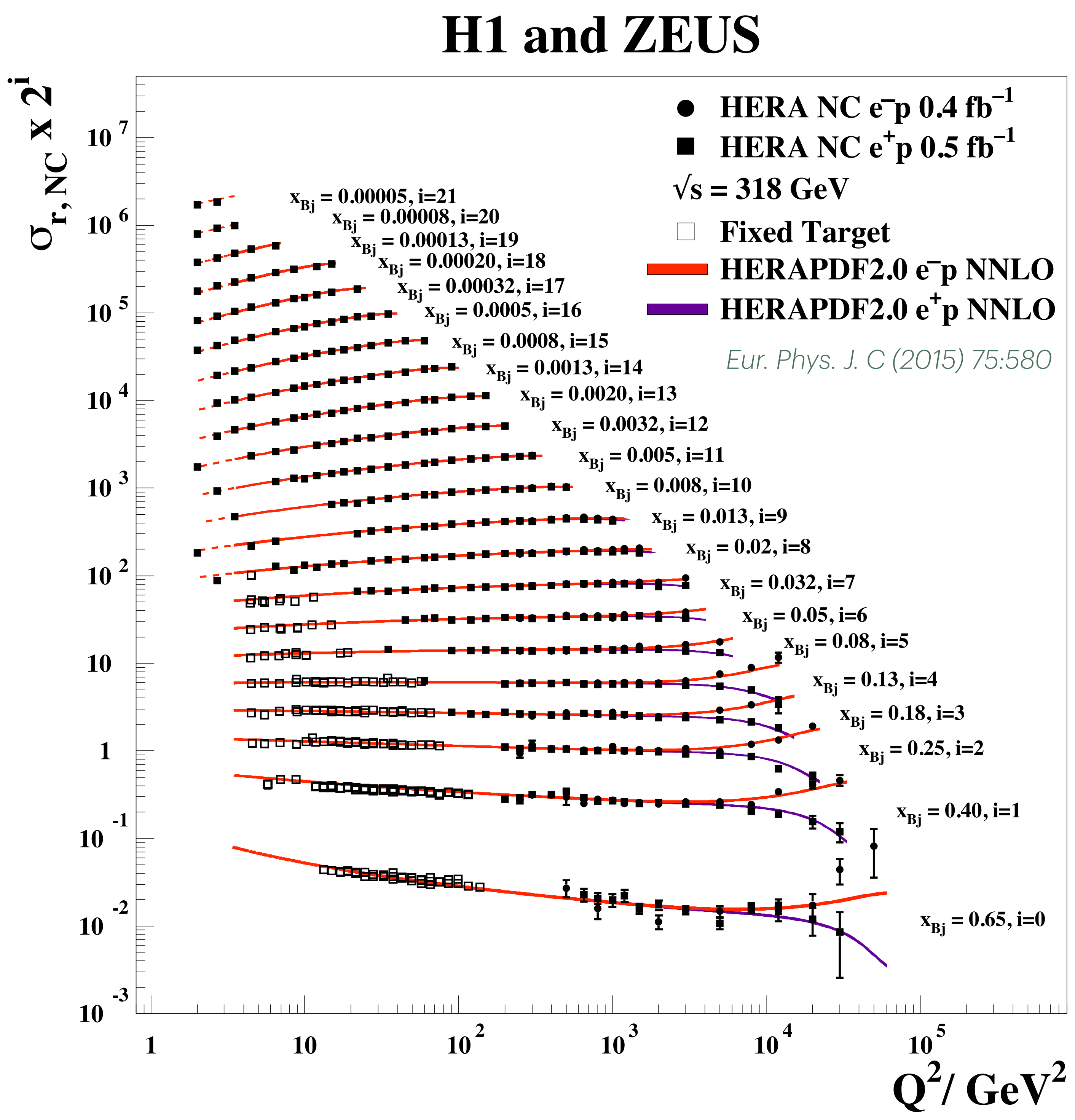
What are structure functions?

Structure functions give us information about the constituents of the proton

What do structure functions tell us?

- F_2: information about quarks.
- F_L: direct look at gluons.
- xF_3: matter-antimatter imbalance.



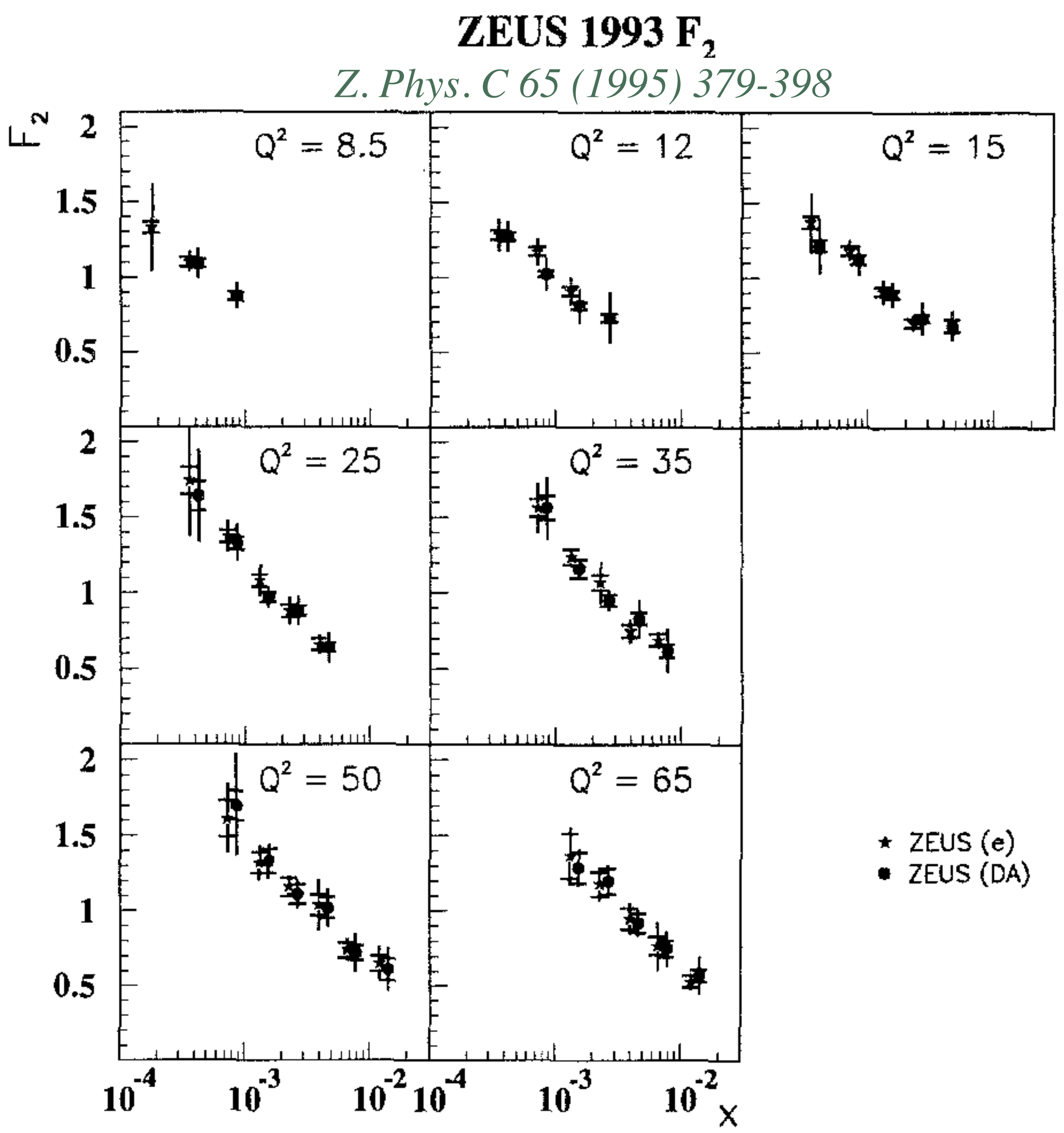


Altarelli-Martinelli relation (*Phys.Lett.B 76 (1978) 89-94*)

$$xg(x, Q^2) \approx 1.77 \frac{3\pi}{2\alpha_s(Q^2)} F_L(x, Q^2)$$

For a concise overview, see *Deep Inelastic Scattering* by Devenish and Cooper-Sarkar.

What can EIC do?



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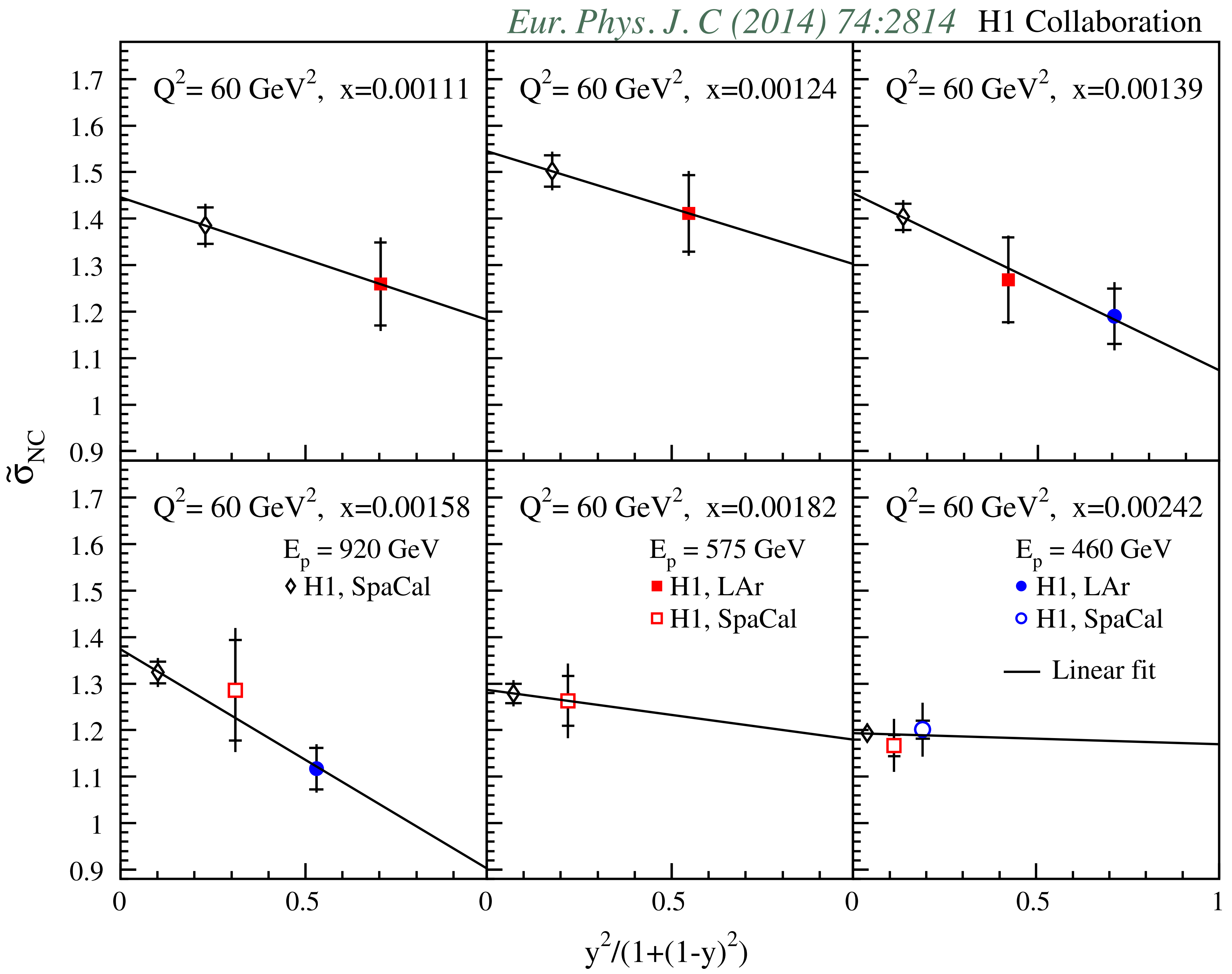
Include F_L and enough beam configurations.



Model dependent extractions:

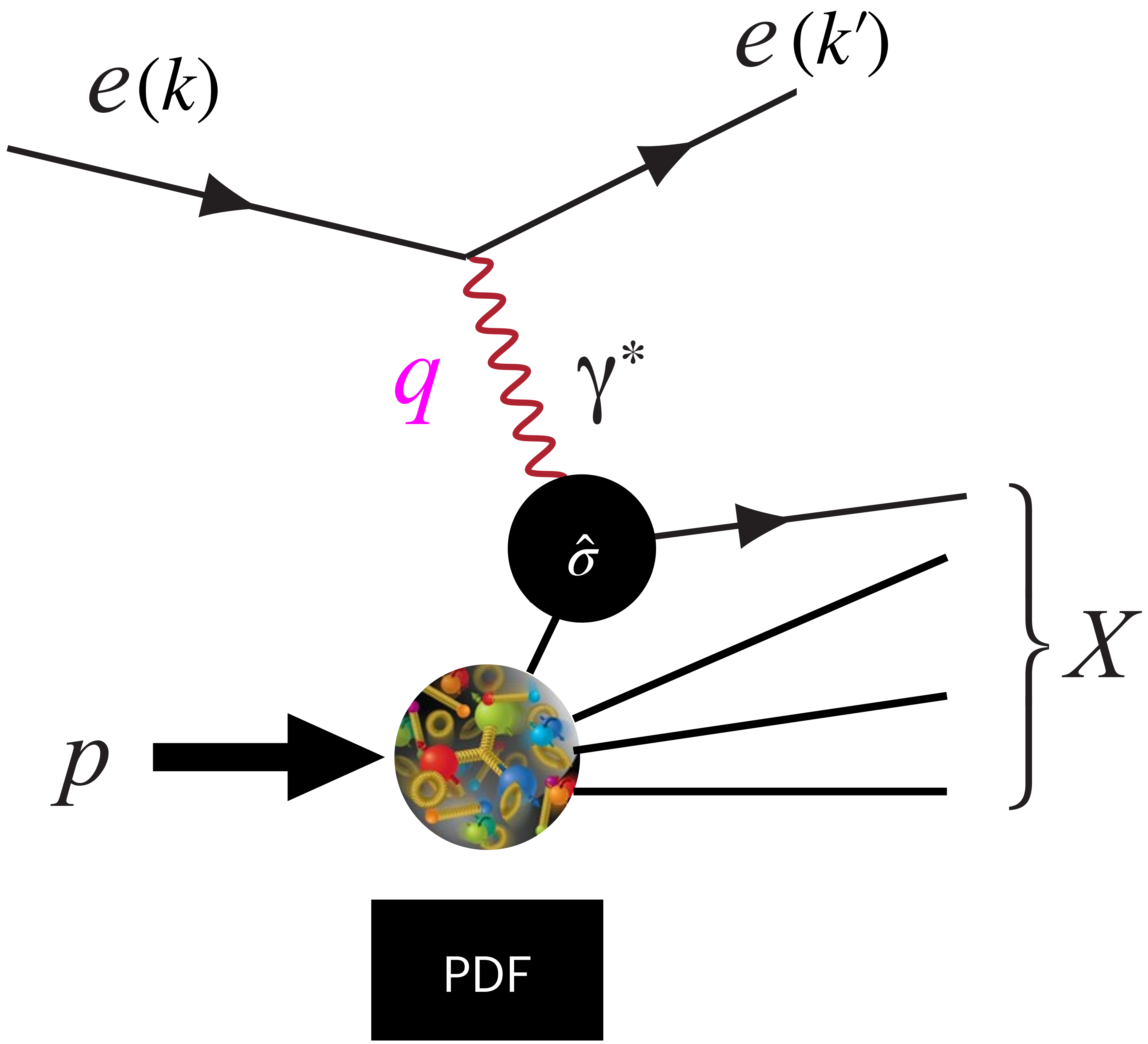
- Phys.Lett.B 282 (1992) 475-482
- ArXiv:hep-ph/9712539

At EIC we can extract the three of them without any assumptions.

Contents

- Deep Inelastic Scattering and structure functions.
- Overview of HERA results.
- Model independent extraction of F_2, F_L and xF_3 at EIC.

Overview of Deep Inelastic Scattering



The reduced cross-section for NC DIS is:

$$\sigma_r^{e^\pm p}(x, Q^2, y) = F_2\left(x, Q^2\right) \mp \frac{Y_-}{Y_+} xF_3\left(x, Q^2\right) - \frac{y^2}{Y_+} F_L\left(x, Q^2\right)$$

where x is the Bjorken scaling variable, $Q^2 = -q^2$ (virtuality), y the inelasticity and $Y_\pm = 1 \pm (1-y)^2$.

For $Q^2 \ll M_Z^2$:

$$\sigma_r^{e^\pm p}(x, Q^2, y) = F_2\left(x, Q^2\right) - \frac{y^2}{Y_+} F_L\left(x, Q^2\right)$$

x and Q^2 are known $\Longrightarrow$ F_2 and F_L from a linear fit.

This is known as the Rosenbluth-separation technique (*Phys. Rev. 79, 615*).

This method assumes $xF_3 = 0$, neglecting the exchange of Z bosons and the γZ interference.

Overview of Deep Inelastic Scattering

If xF_3 is not neglected, then:

$$\sigma_r^{e^\pm p}(x, Q^2, y) = F_2\left(x, Q^2\right) \mp \frac{Y_-}{Y_+} xF_3\left(x, Q^2\right) - \frac{y^2}{Y_+} F_L\left(x, Q^2\right)$$

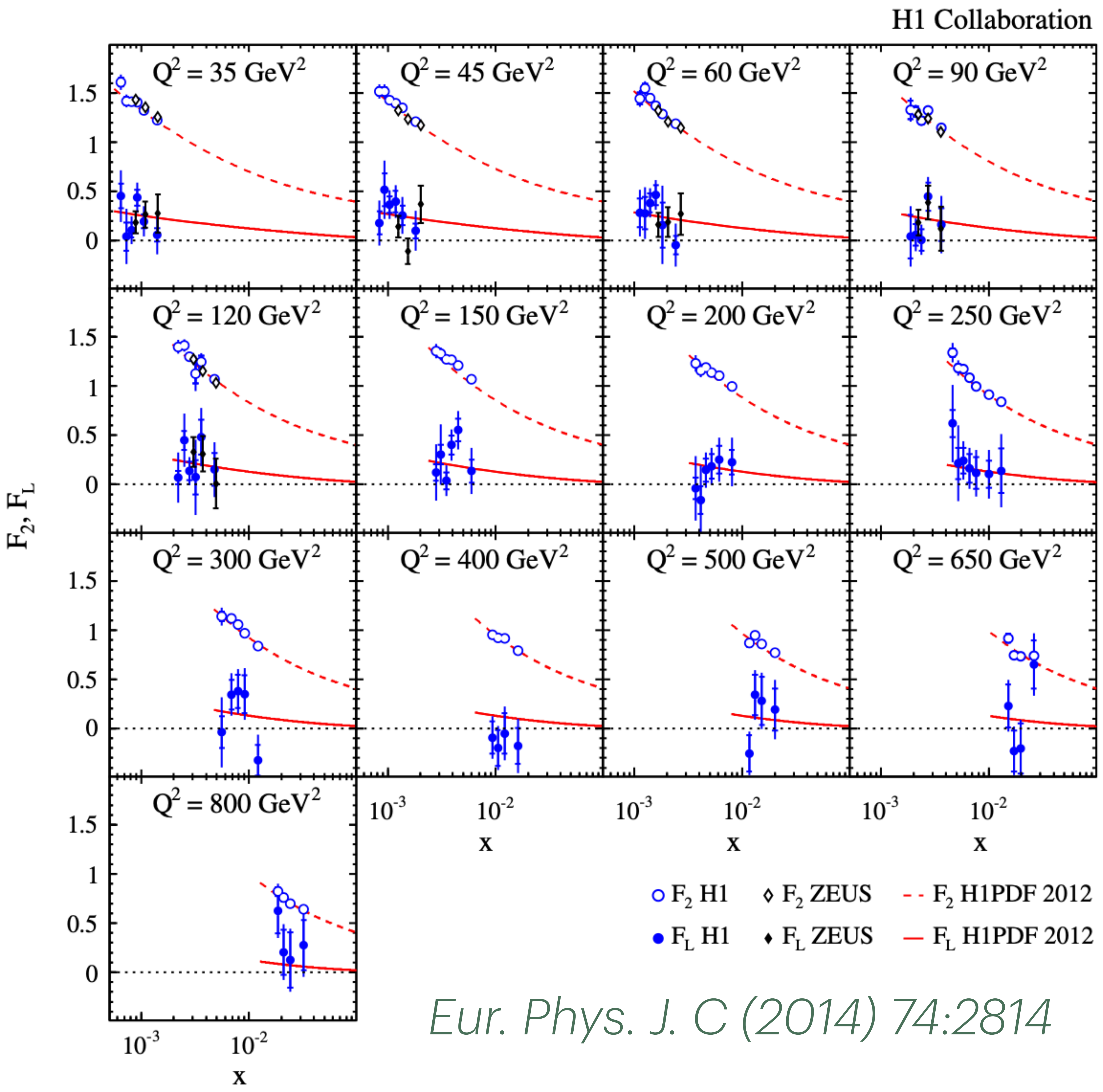
From this equation, it is possible to extract xF_3 using measurements from electron-proton and positron-proton interactions:

$$xF_3 = \frac{Y_+}{2Y_-}\left(\sigma_r^{e^-p} - \sigma_r^{e^+p}\right)$$

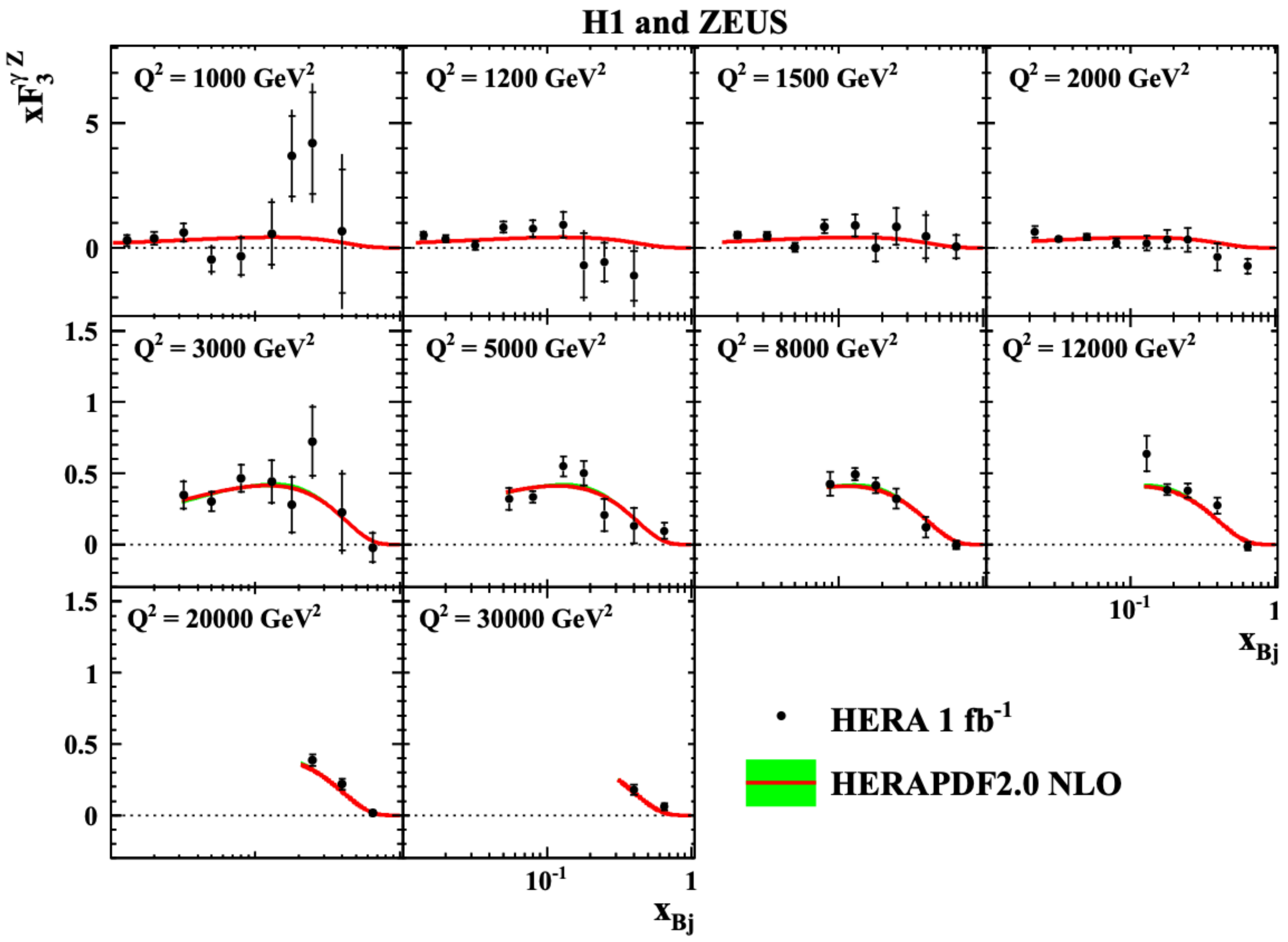
This was the only possible method that could be employed at HERA to extract measurements of xF_3.

However, there is a problem: one needs to have electrons and positrons available, but EIC will not have positrons and thus, this method can not be applied.

Results from HERA



Eur. Phys. J. C (2014) 74:2814



Eur. Phys. J. C (2015) 75:580

The simultaneous extraction of the 3 structure functions was not attempted at HERA.

Potential bias in the extraction of F_L

As the cross section is a measurable observable, it must satisfy:

$$\Delta = \sigma_r^{\text{full}} - \sigma_r^{\text{Rosenbluth}} = 0$$

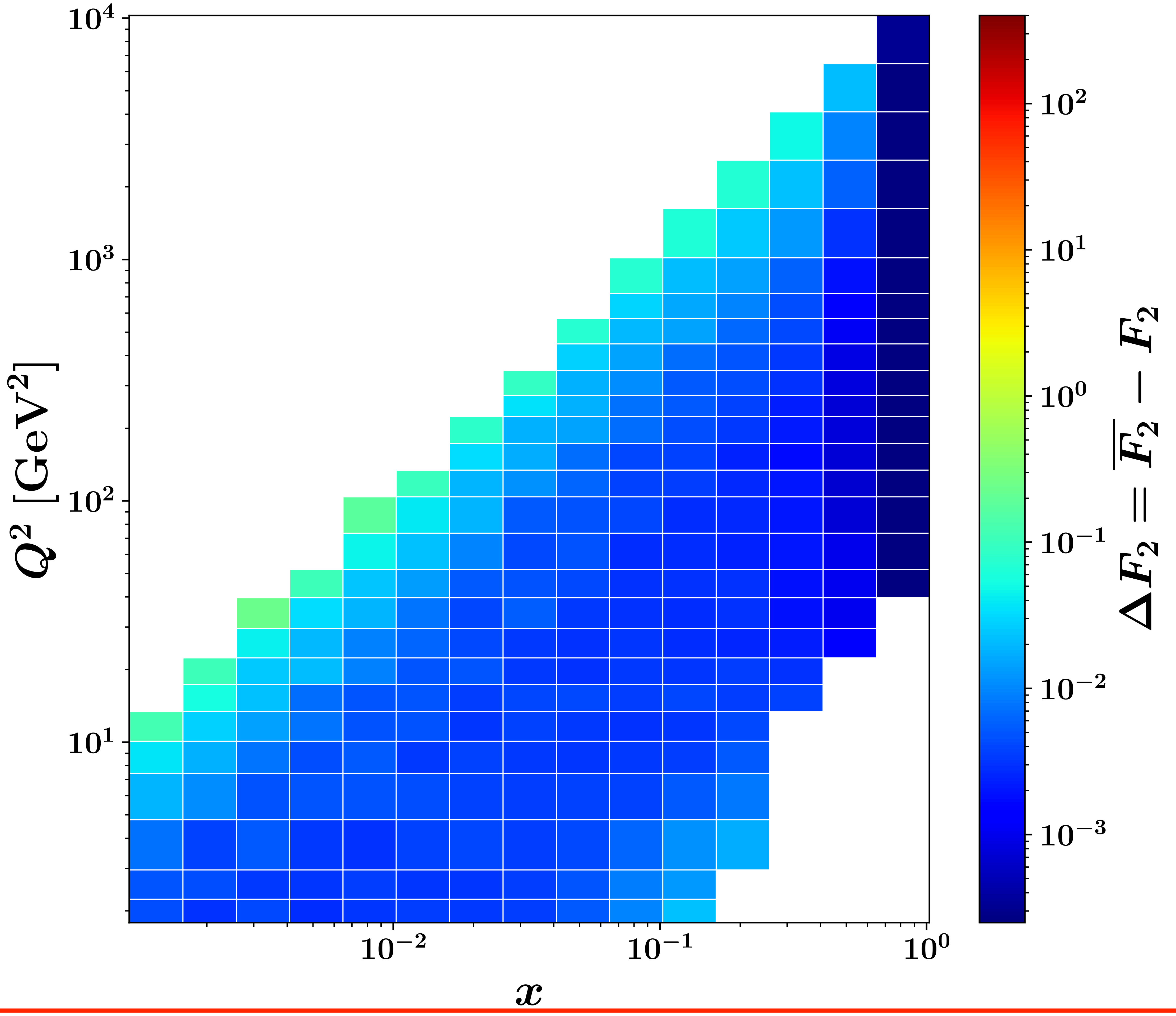
If we denote by $\overline{F}_2$ and $\overline{F}_L$ the structure functions obtained with the Rosenbluth method, one finds:

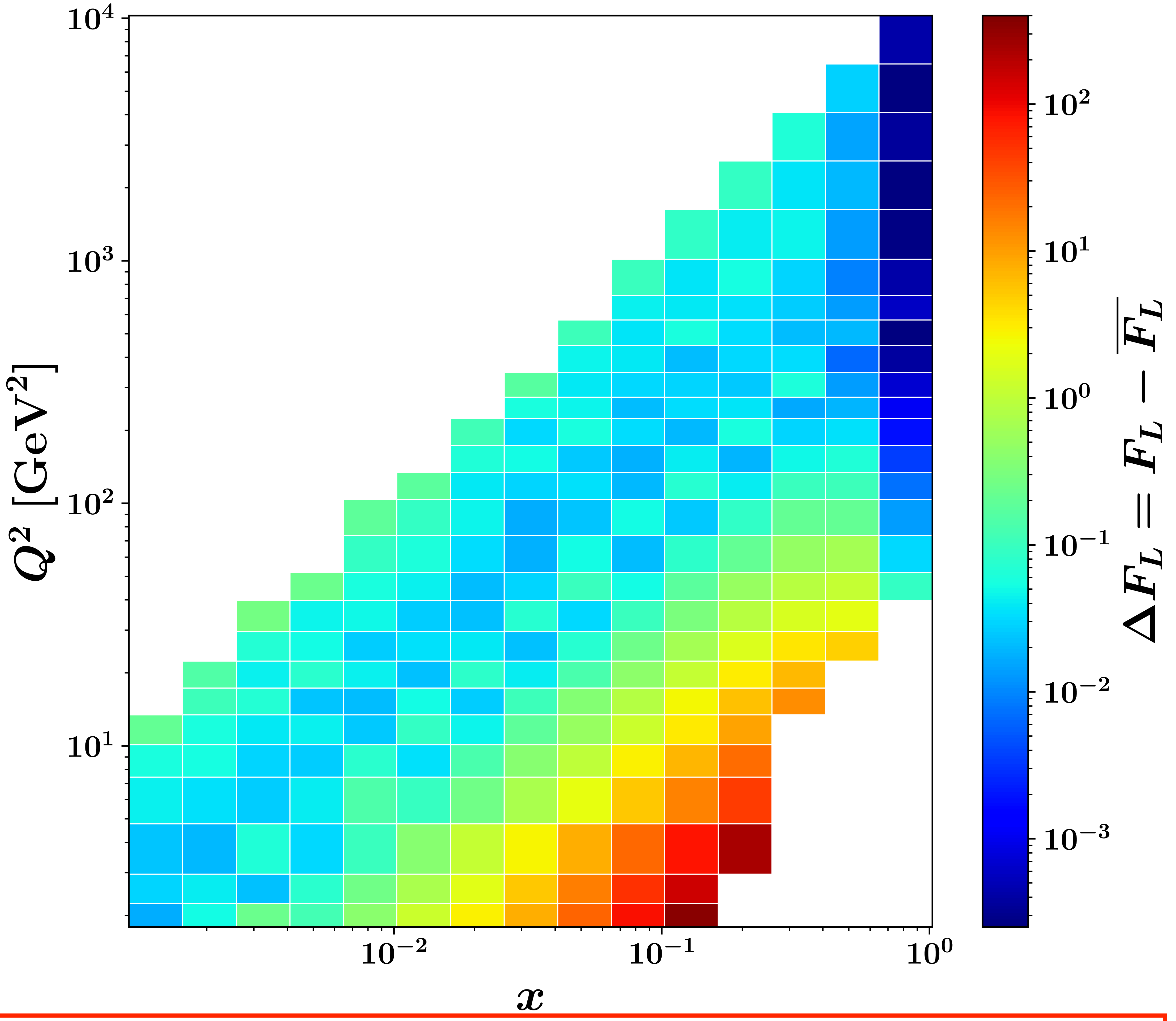
$$\Delta = F_2 - \overline{F}_2 + \frac{1}{Y_+}\left[(2y - y^2)xF_3 - y^2(F_L - \overline{F}_L)\right] = 0$$

As the difference $F_2 - \overline{F}_2$ can be neglected, the bias in F_L is:

$$\boxed{F_L - \overline{F}_L \sim xF_3}$$

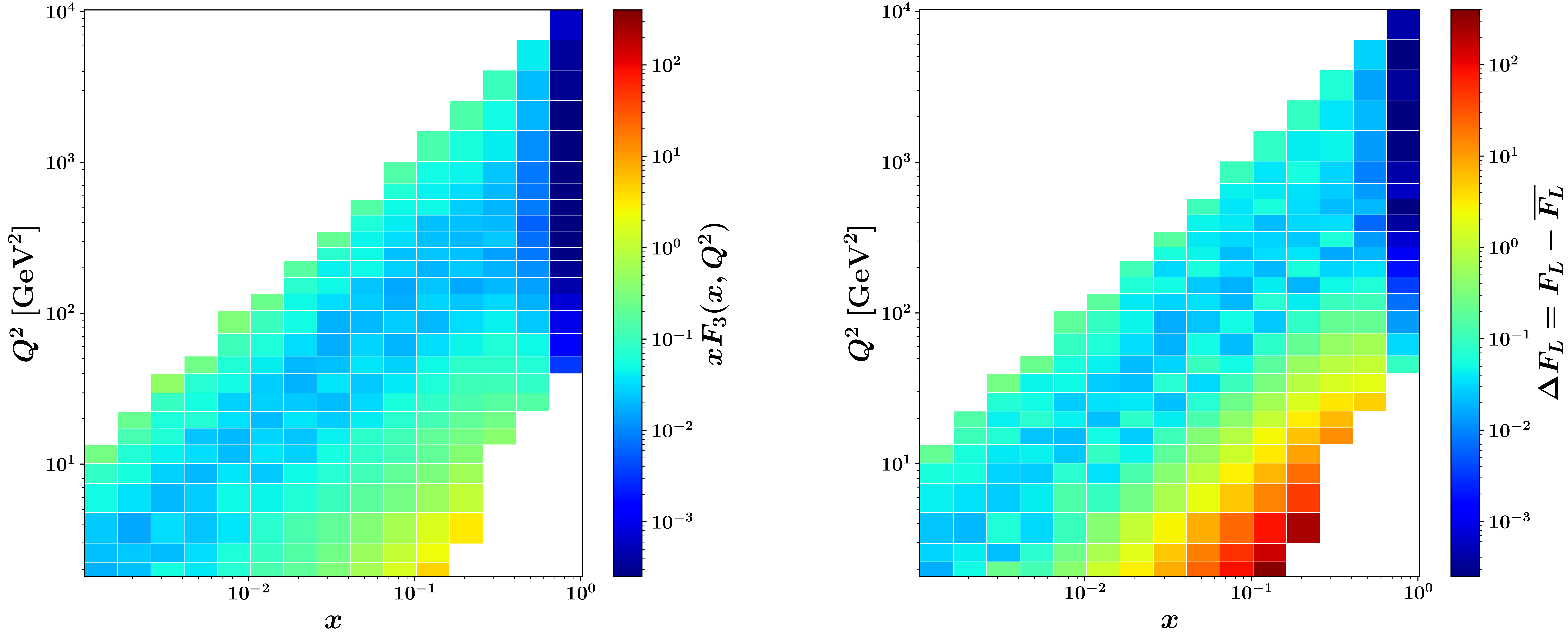
Potential bias in the extraction of F_L (S17 + optimistic)





The difference in F_2 due to the inclusion of xF_3 is negligible when compared to the difference in F_L

Potential bias in the extraction of F_L (S17 + optimistic)



Simultaneous extraction of F_2, F_L and xF_3

The extraction is done by fitting the structure functions according to the following relation with the reduced cross section:

$$\sigma_r^{e^\pm p}(x, Q^2, y) = F_2\left(x, Q^2\right) \mp \frac{Y_-}{Y_+} xF_3\left(x, Q^2\right) - \frac{y^2}{Y_+} F_L\left(x, Q^2\right)$$

As we are introducing an additional free parameter, we must be careful at the time of performing the fit. We propose the following fitting strategy:

- Obtain F_2 and F_L using the Rosenbluth method.

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As we are introducing an additional free parameter, we must be careful at the time of performing the fit. We propose the following fitting strategy:

- Obtain F_2 and F_L using the Rosenbluth method.
- Use those values as initial guesses of the full fit in order to improve numerical stability and convergence.

Additionally, we impose the condition $xF_3 \geq 0$ during the fitting process to ensure physically meaningful results.

Pseudo-data generation

S5

e-beam energy (GeV)	p-beam energy (GeV)	$\sqrt{s}$ (GeV)	Integrated lumi (fb^{-1})
18	275	141	15.4
10	275	105	100
10	100	63	79.0
5	100	45	61.0
5	41	29	4.4

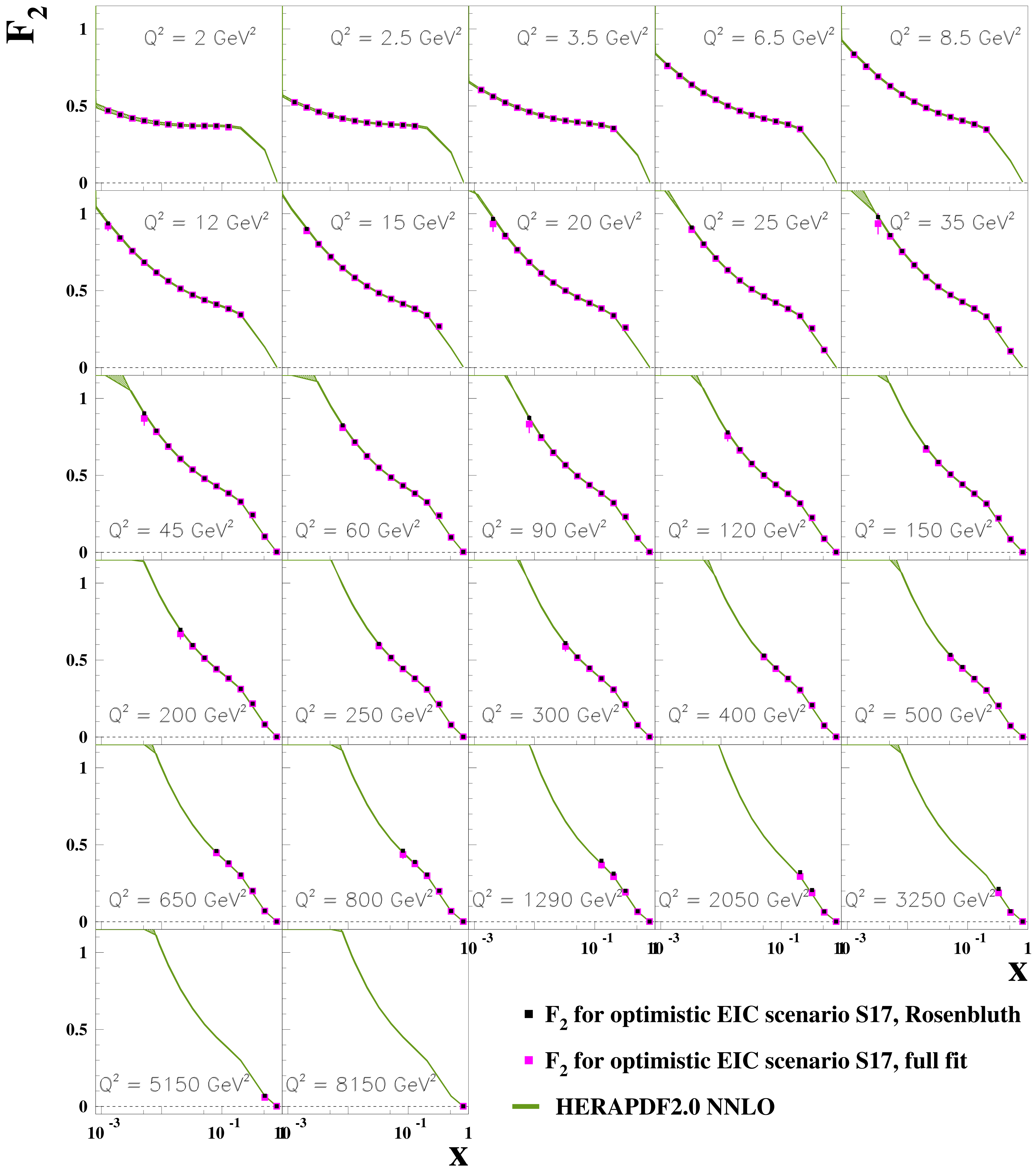
S17

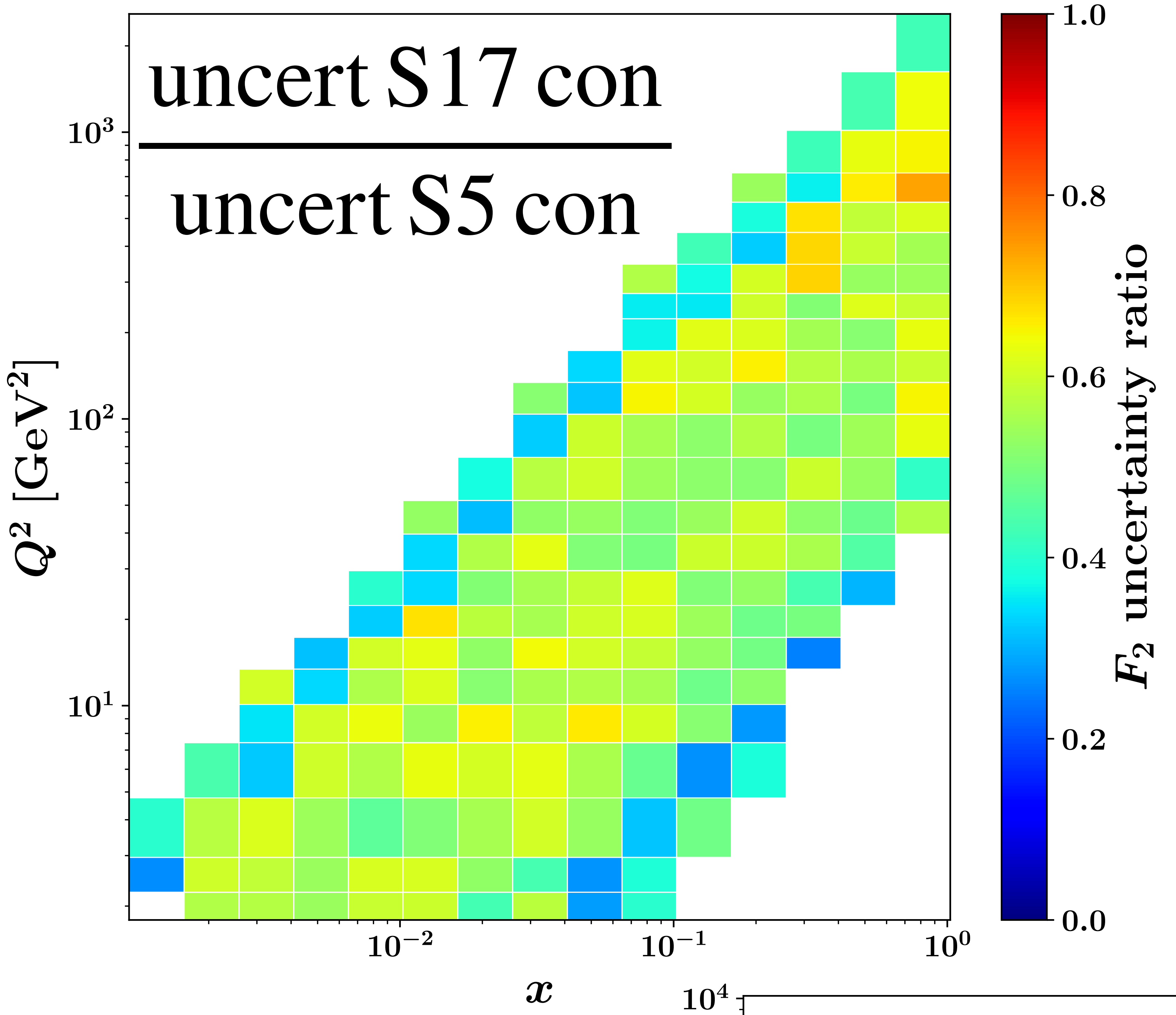
		E_p [GeV]					
		41	100	120	165	180	275
E_e [GeV]	5	29	45	49	57	60	74
	10	40	63	69	81	**85**	105
	18	54	85	93	109	114	141

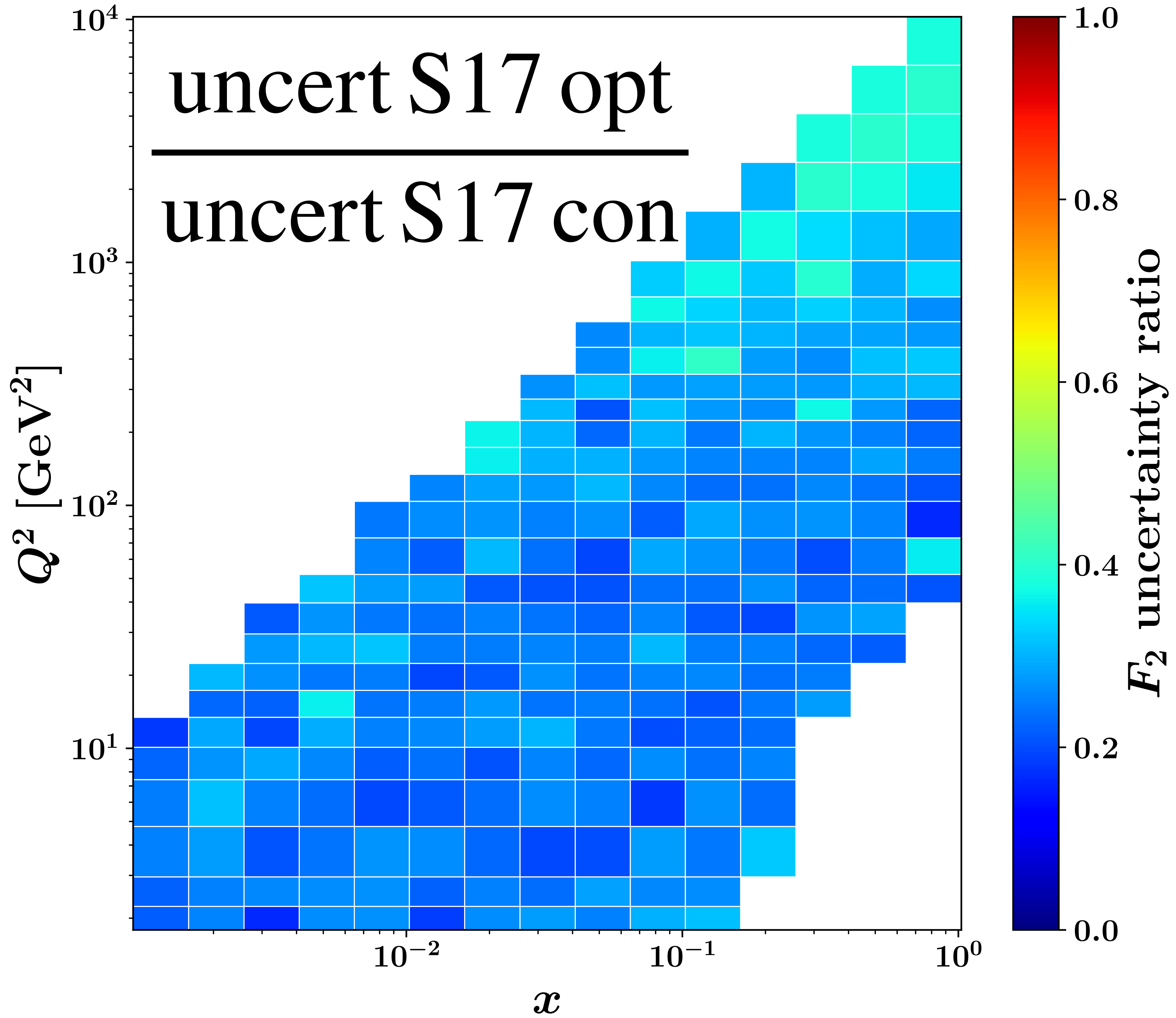
- Cross section generated with HERAPDF2.0 NNLO with xFitter according to the kinematic phase-space at EIC.
- Gaussian smearing according to two possible scenarios:
  - Conservative scenario: 1.9 % of correlated systematics and 3.4 % of uncorrelated systematics $\Longrightarrow$ total uncertainty of 3.9 %.
  - Optimistic scenario: total uncertainty of 1 %.

Note that the uncertainty due to the normalisation between different beam energies is not considered.

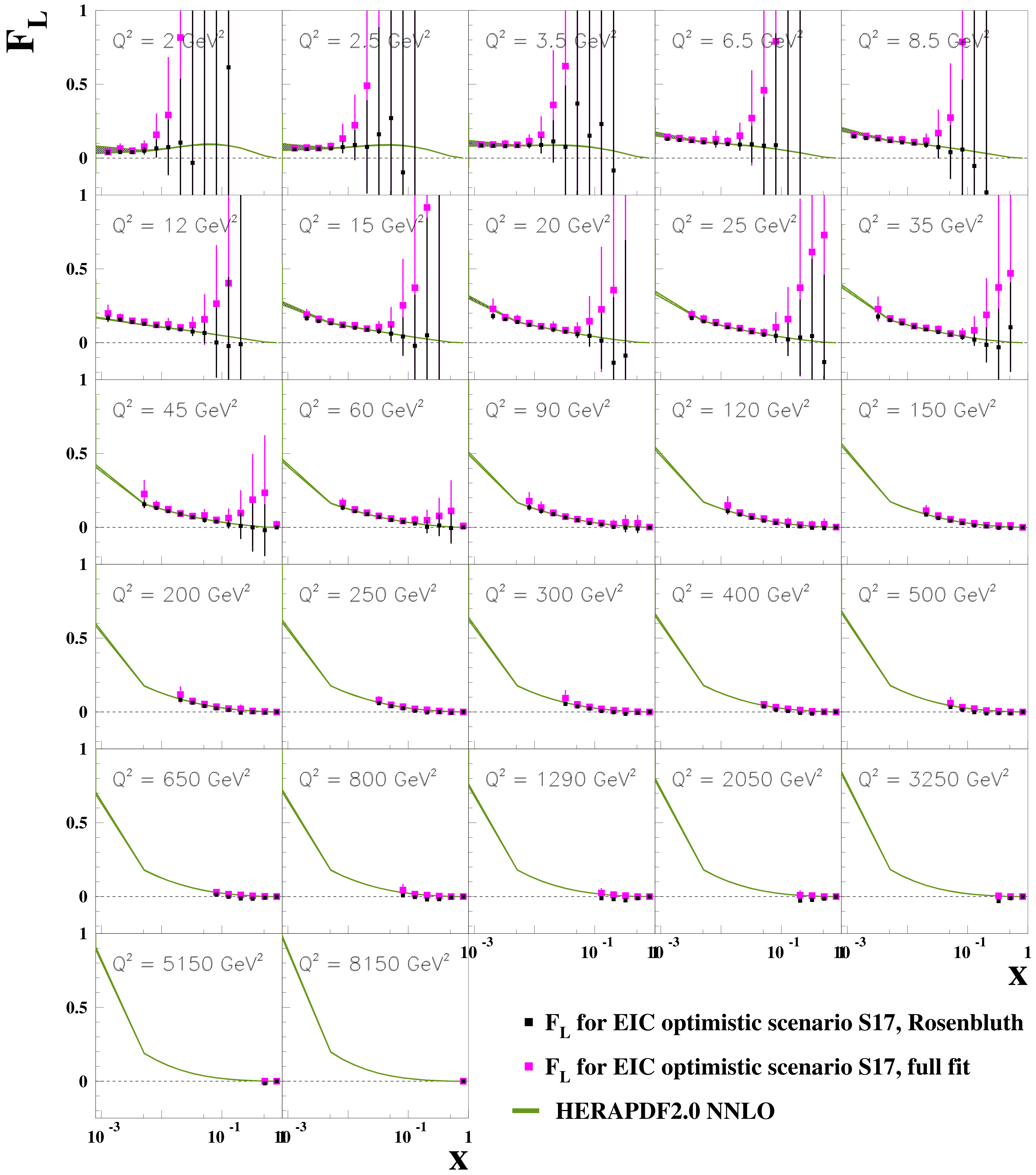
Results for F_2

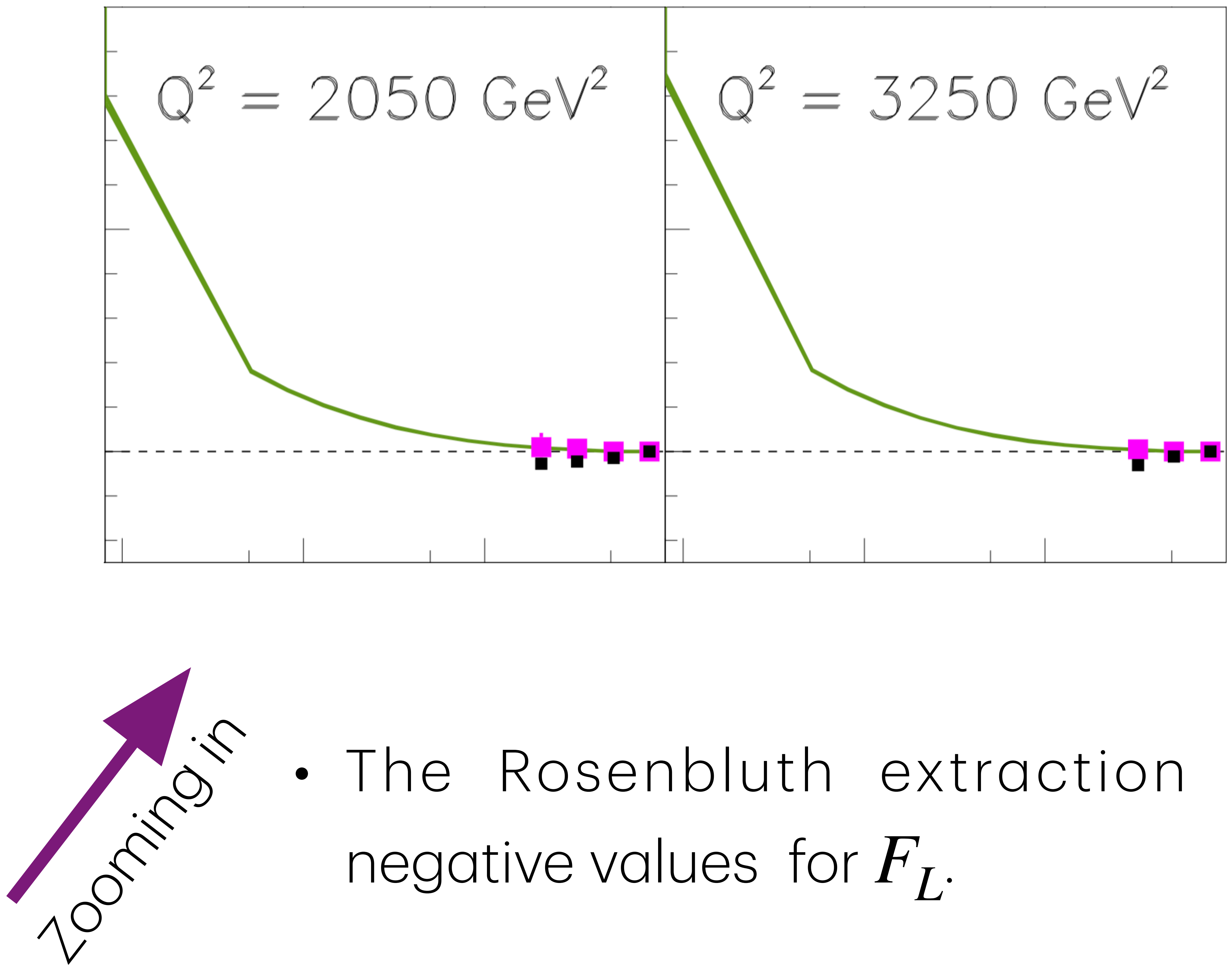






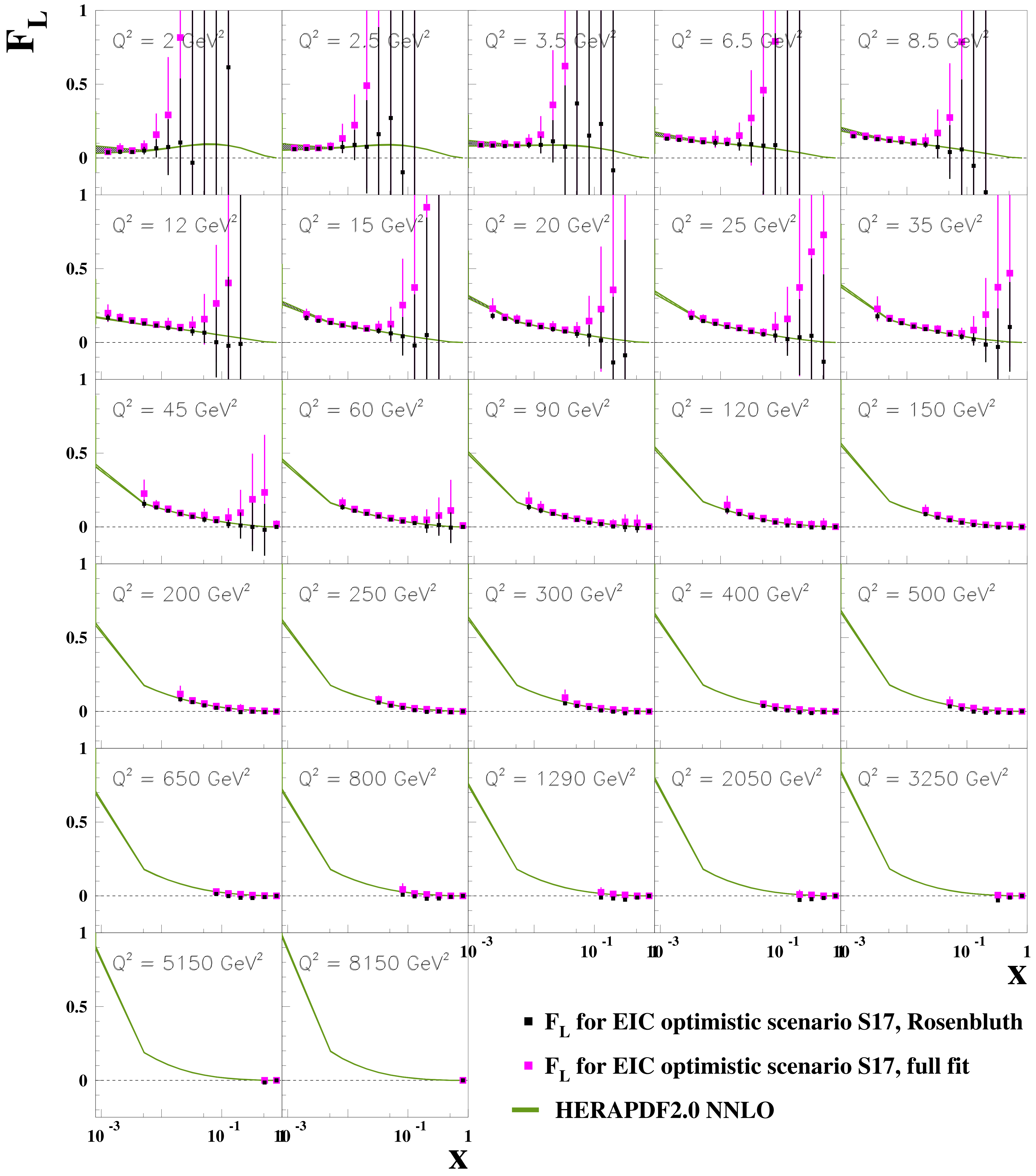
Results for F_L

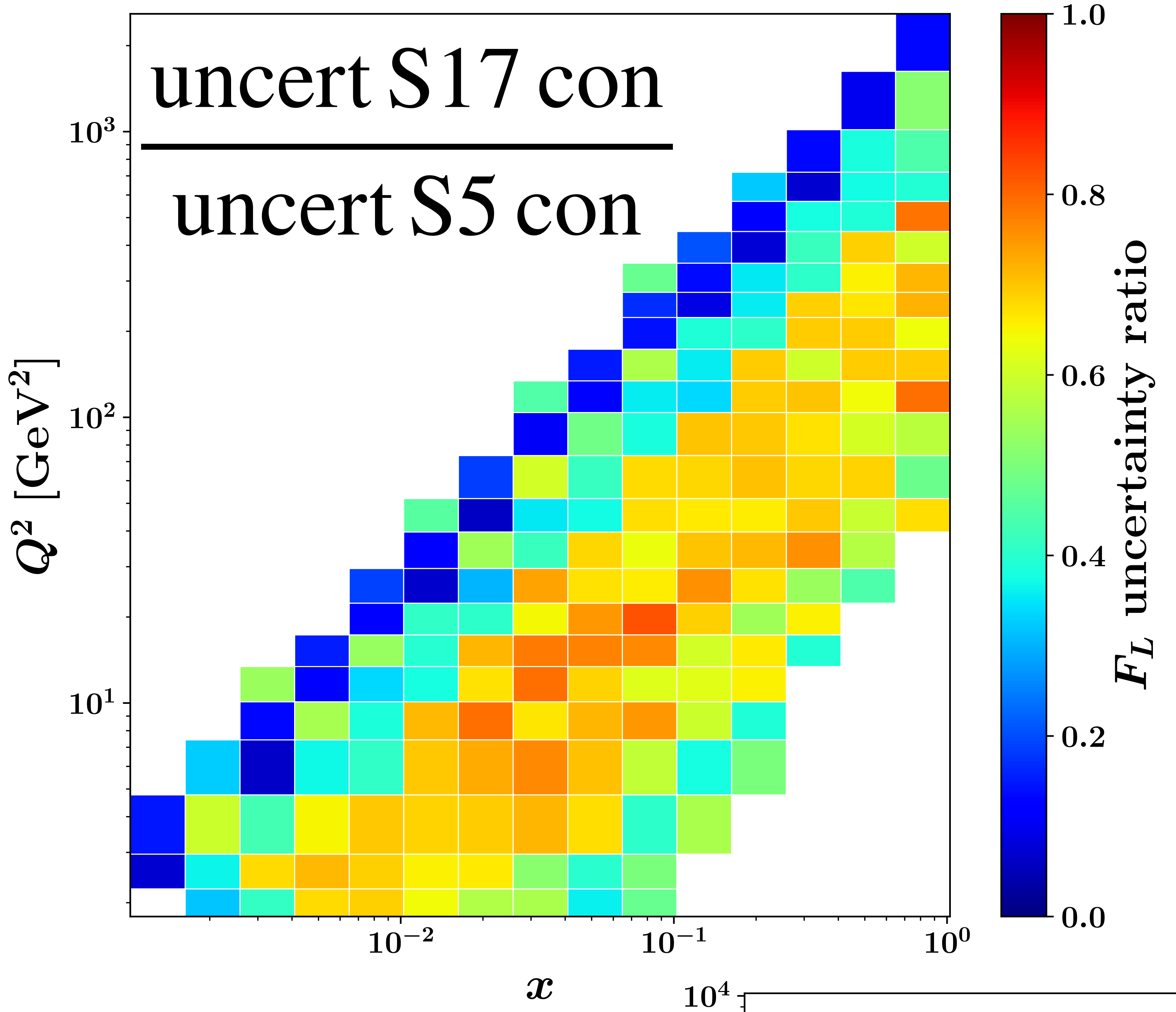


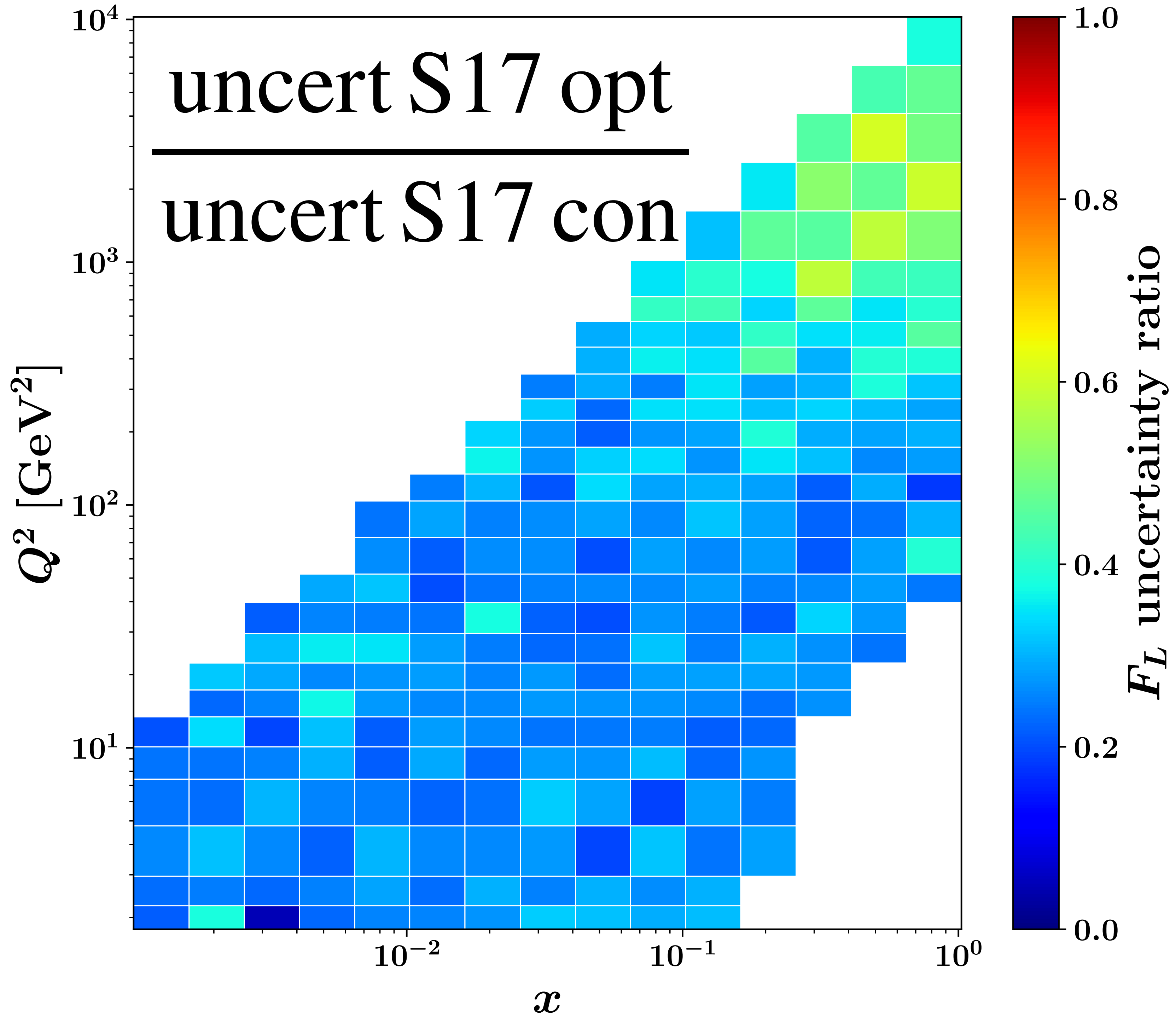


- The Rosenbluth extraction yields negative values for F_L.
- Corrected by the inclusion of xF_3.

Results for F_L

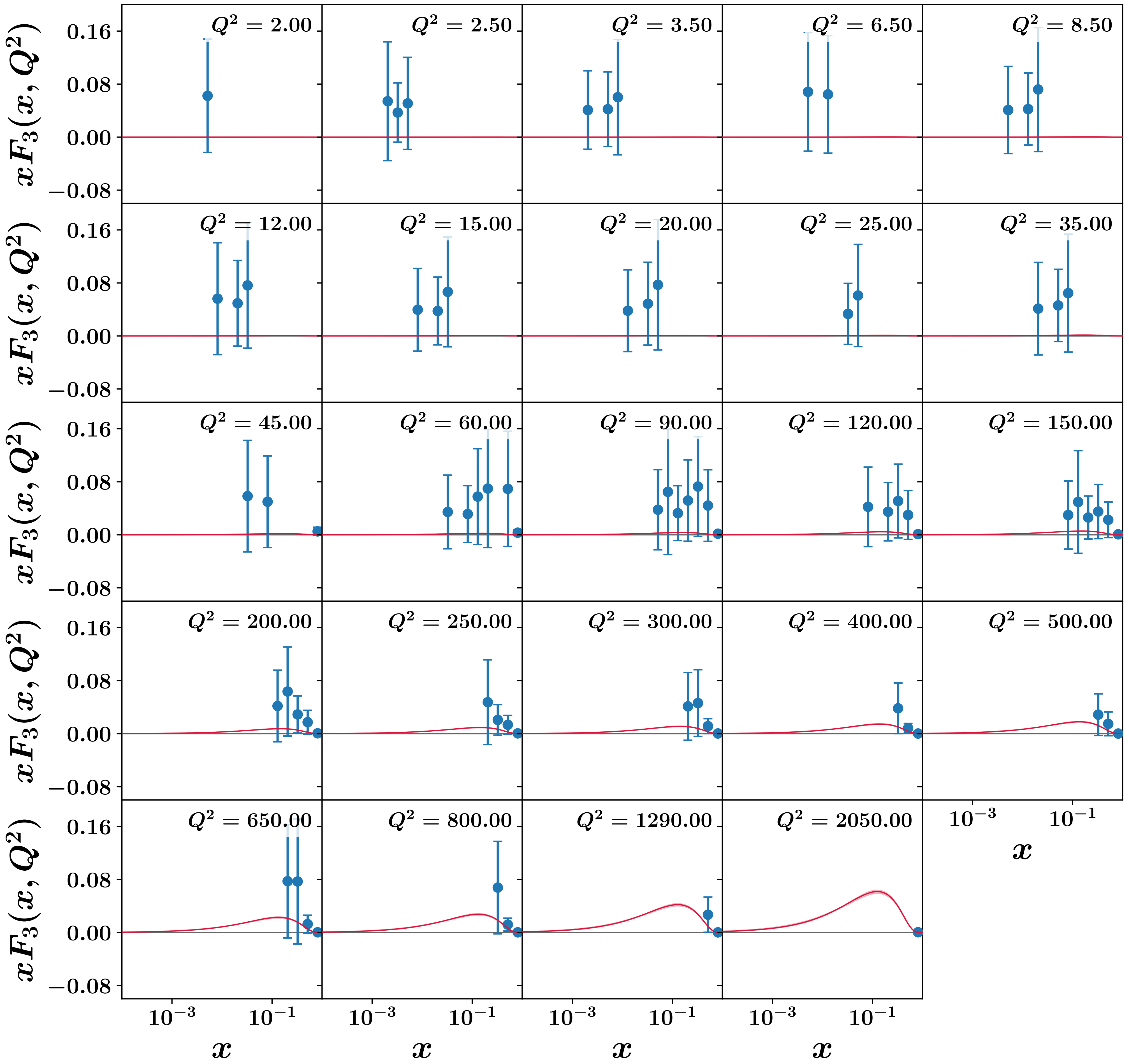




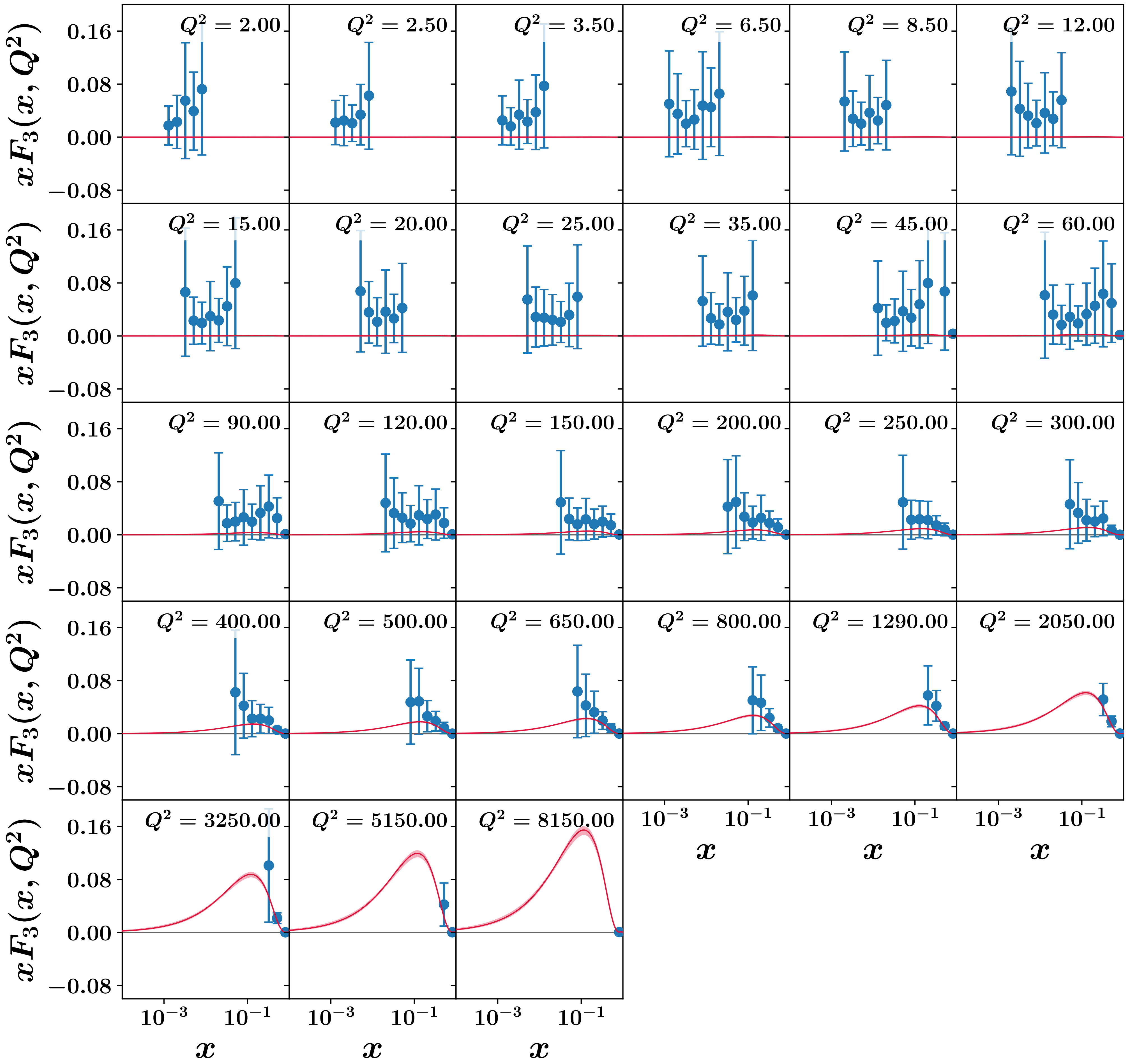


Results for xF_3 (conservative scenario)

S5



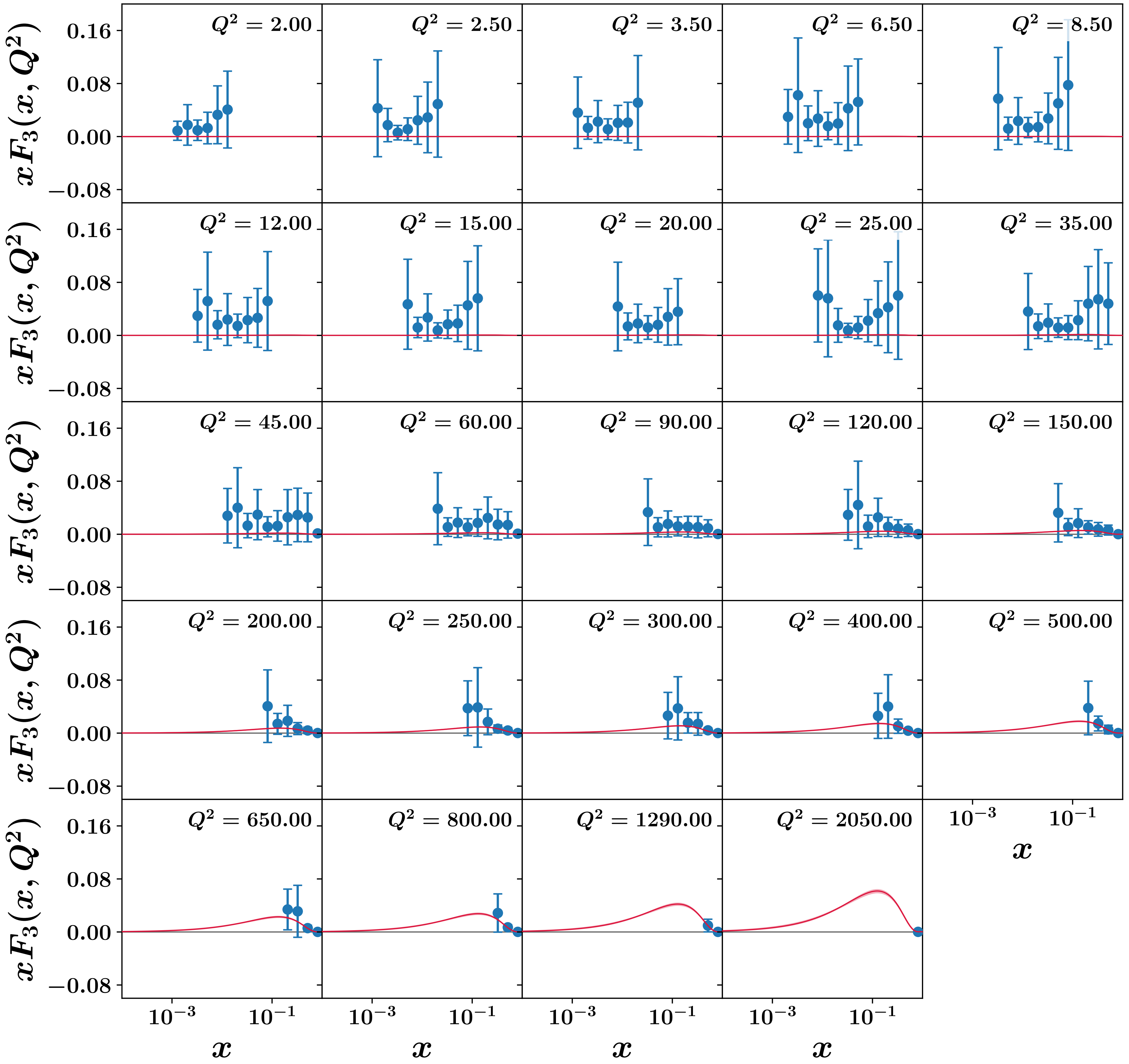
S17



Predictions to compare with are done with HERAPDF2.0 NNLO using YADISM (*Eur.Phys.J.C (2024) 84:697*) and points whose uncertainty is above 0.1 are not displayed for visual clarity.

Results for xF_3 (optimistic scenario)

S5



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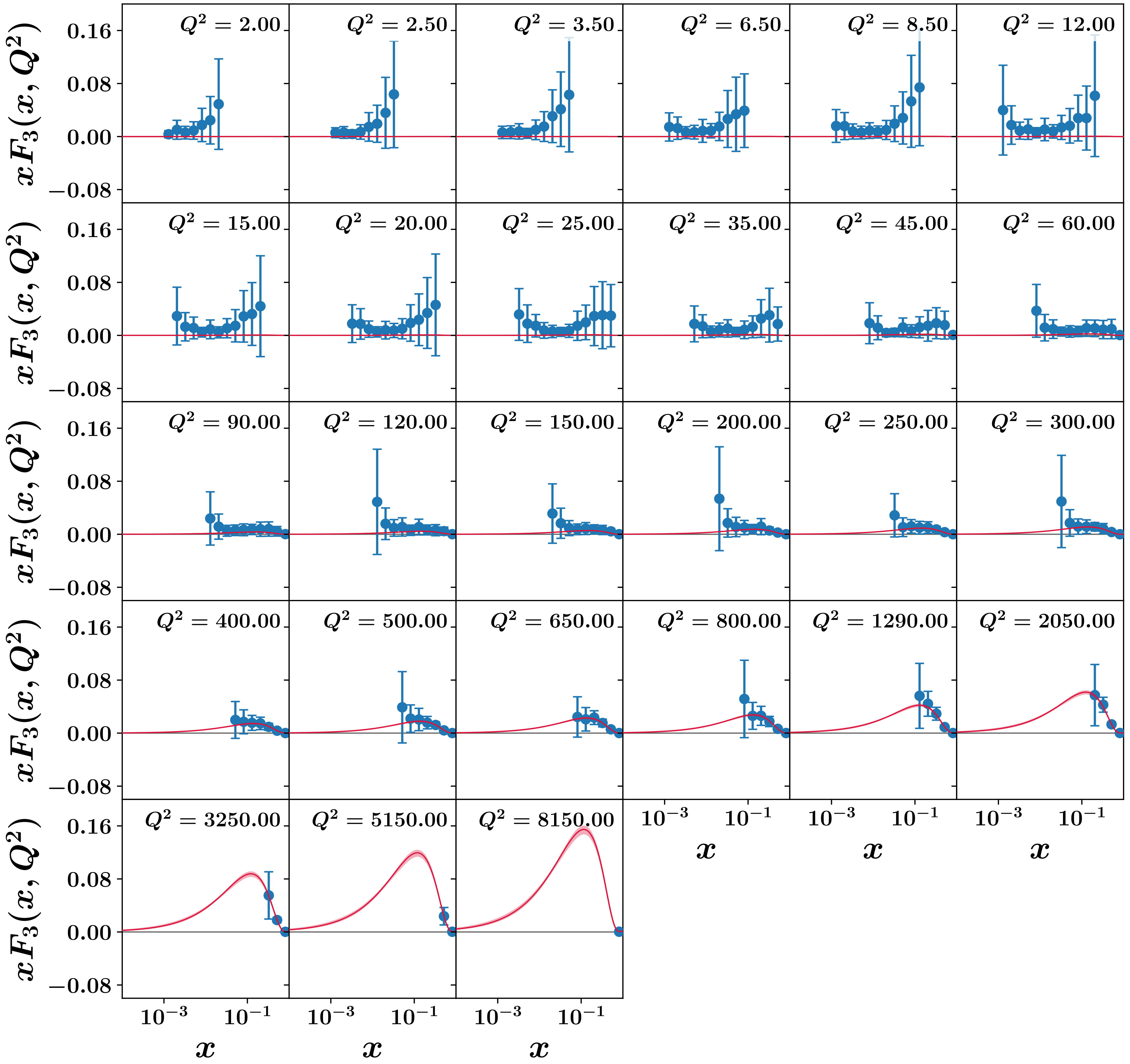

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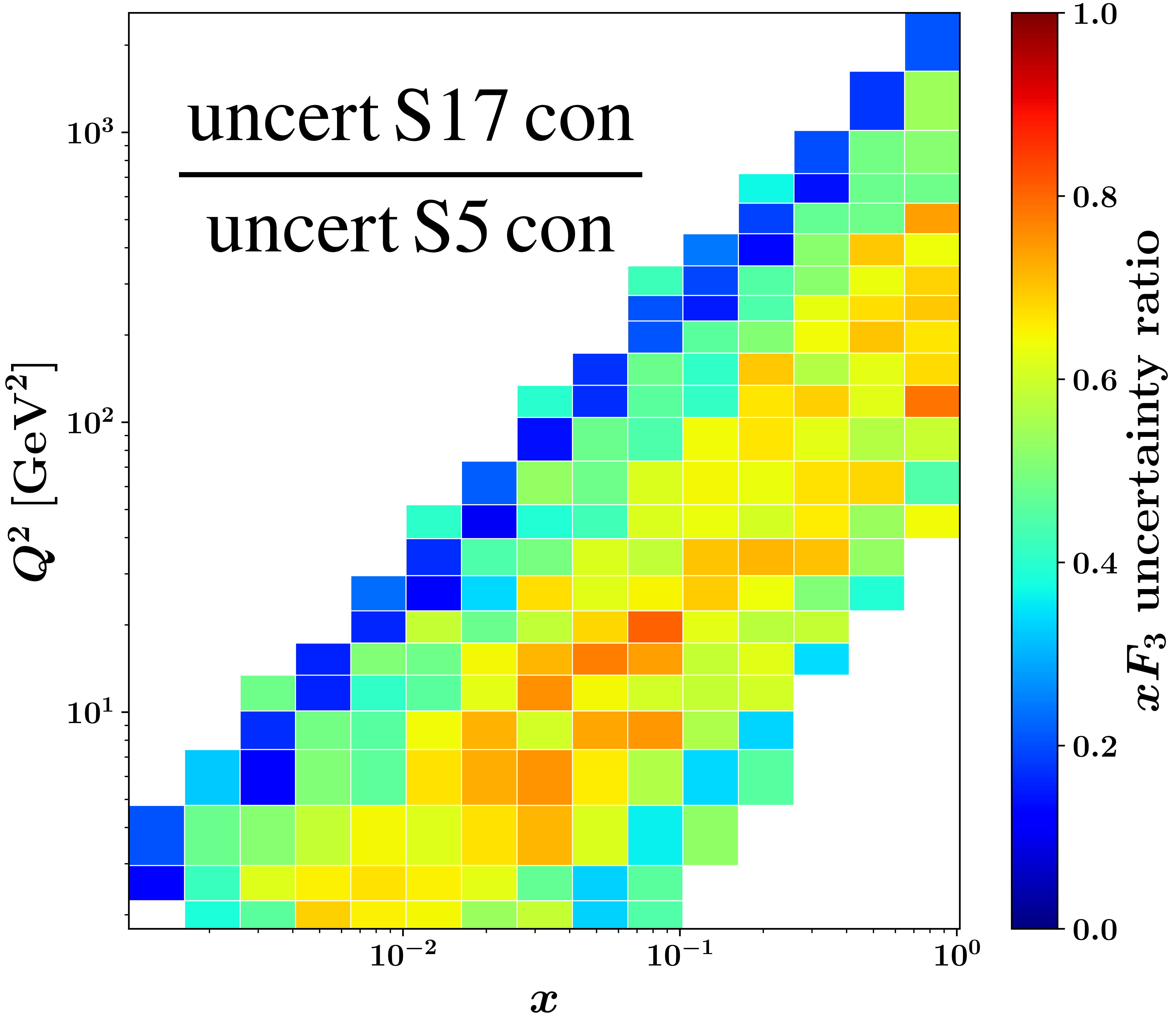
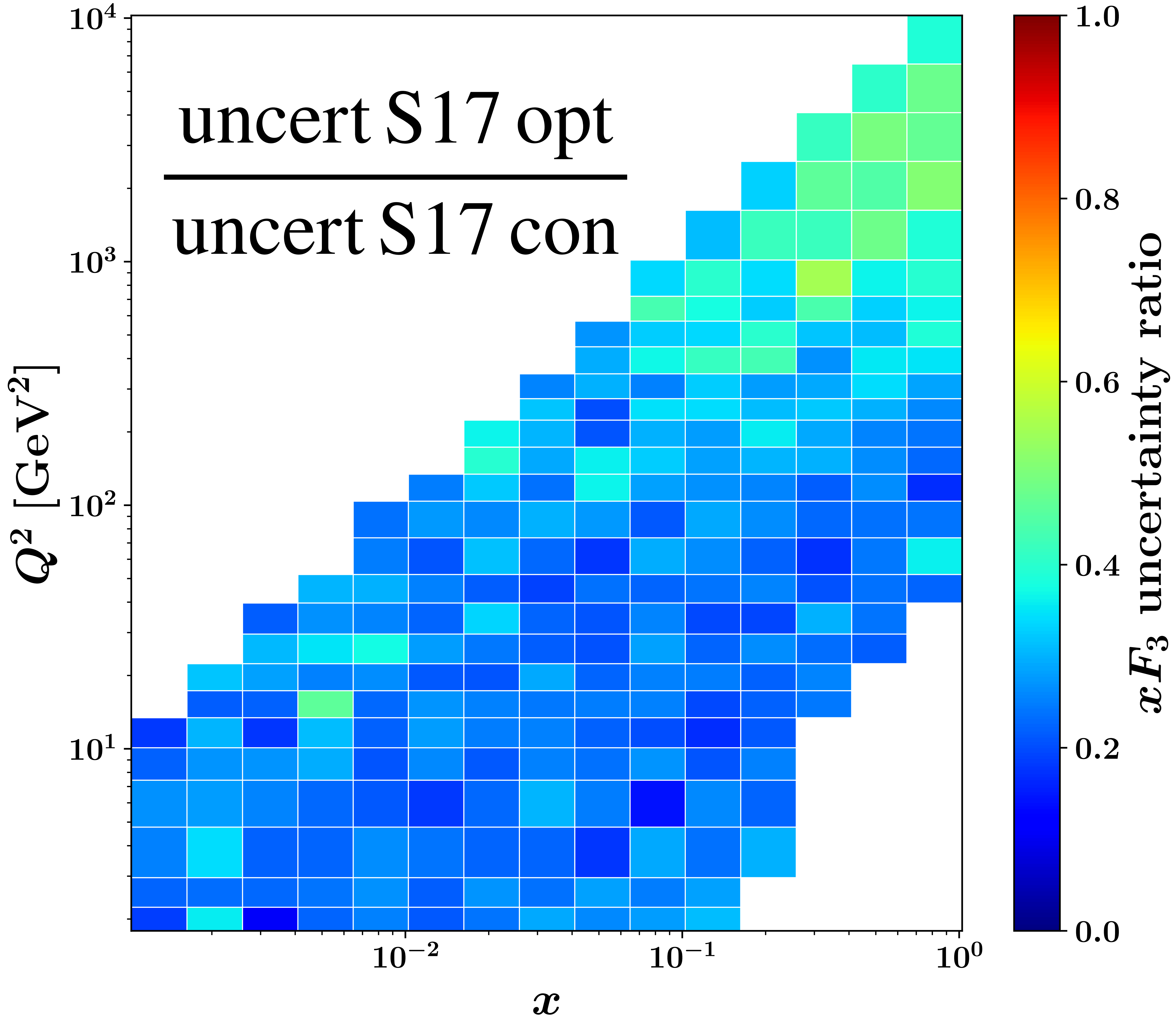
Results for xF_3

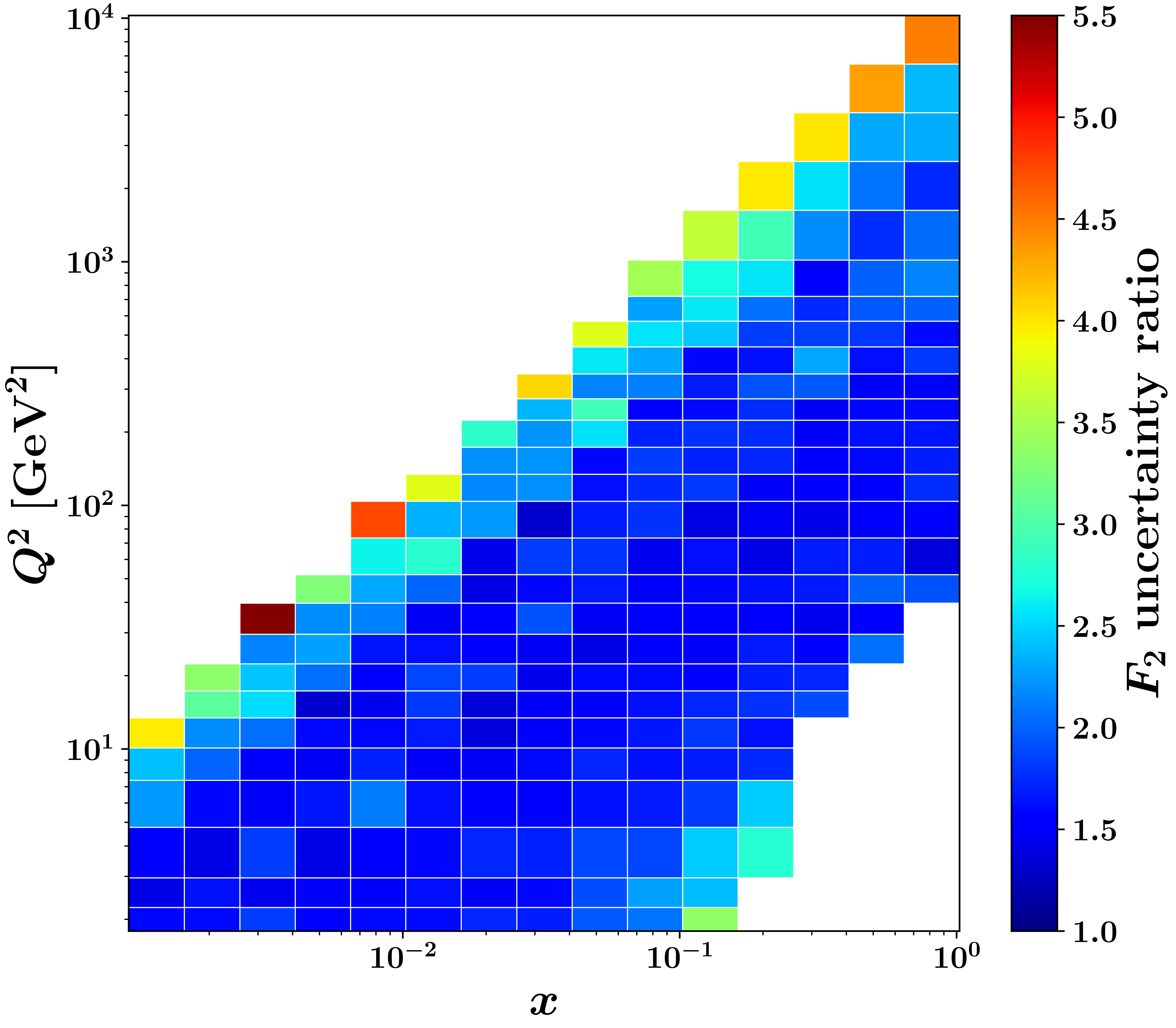


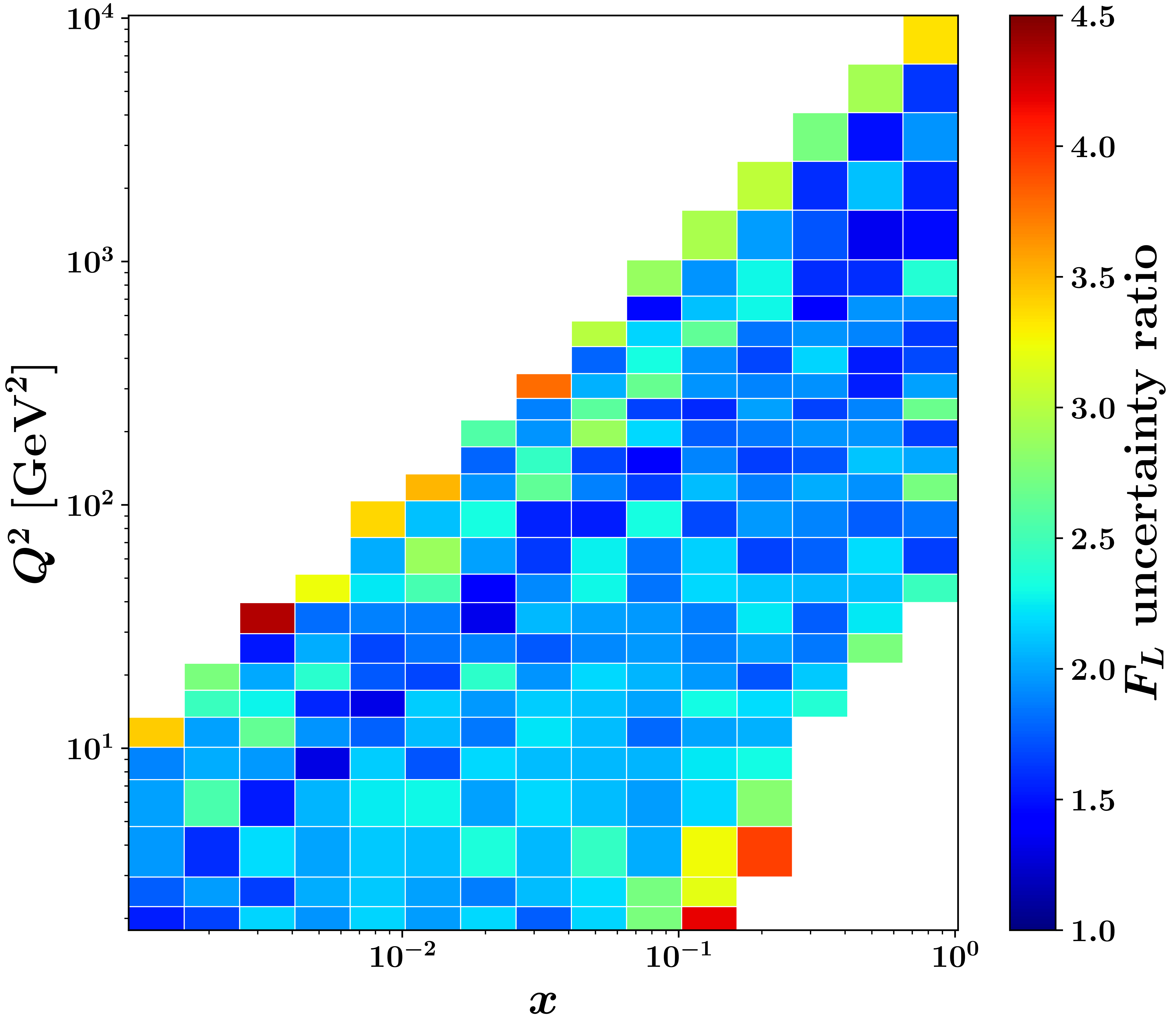


Uncertainties are considerably reduced in the optimistic scenario for the three structure functions.

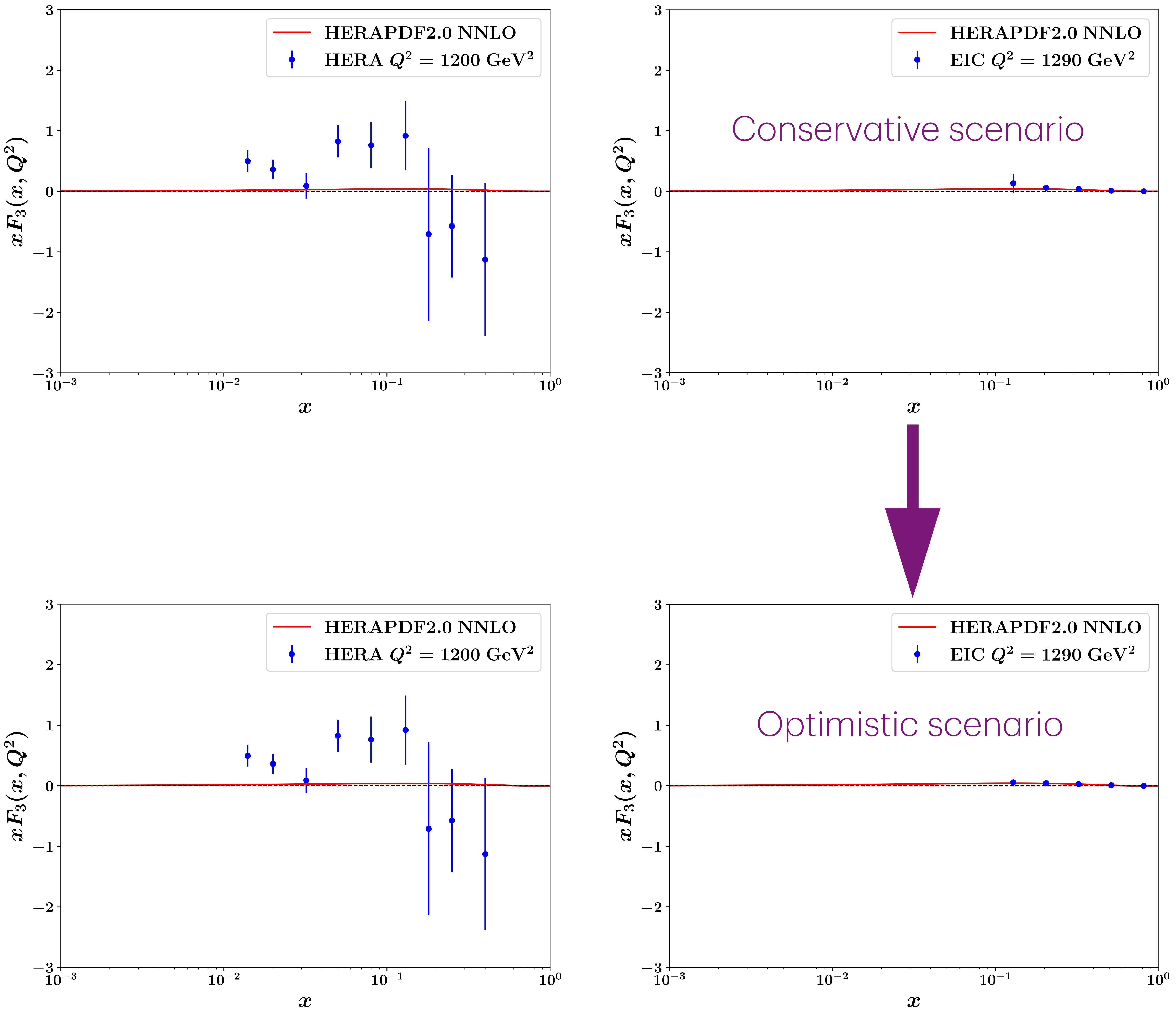
Comparison with Rosenbluth uncertainties (conservative scenario)

$$\frac{\text{uncert S17 full fit}}{\text{uncert S17 Rosenbluth}}$$





Comparison with HERA results



EIC will help to constrain xF_3 in an unexplored region of phase space

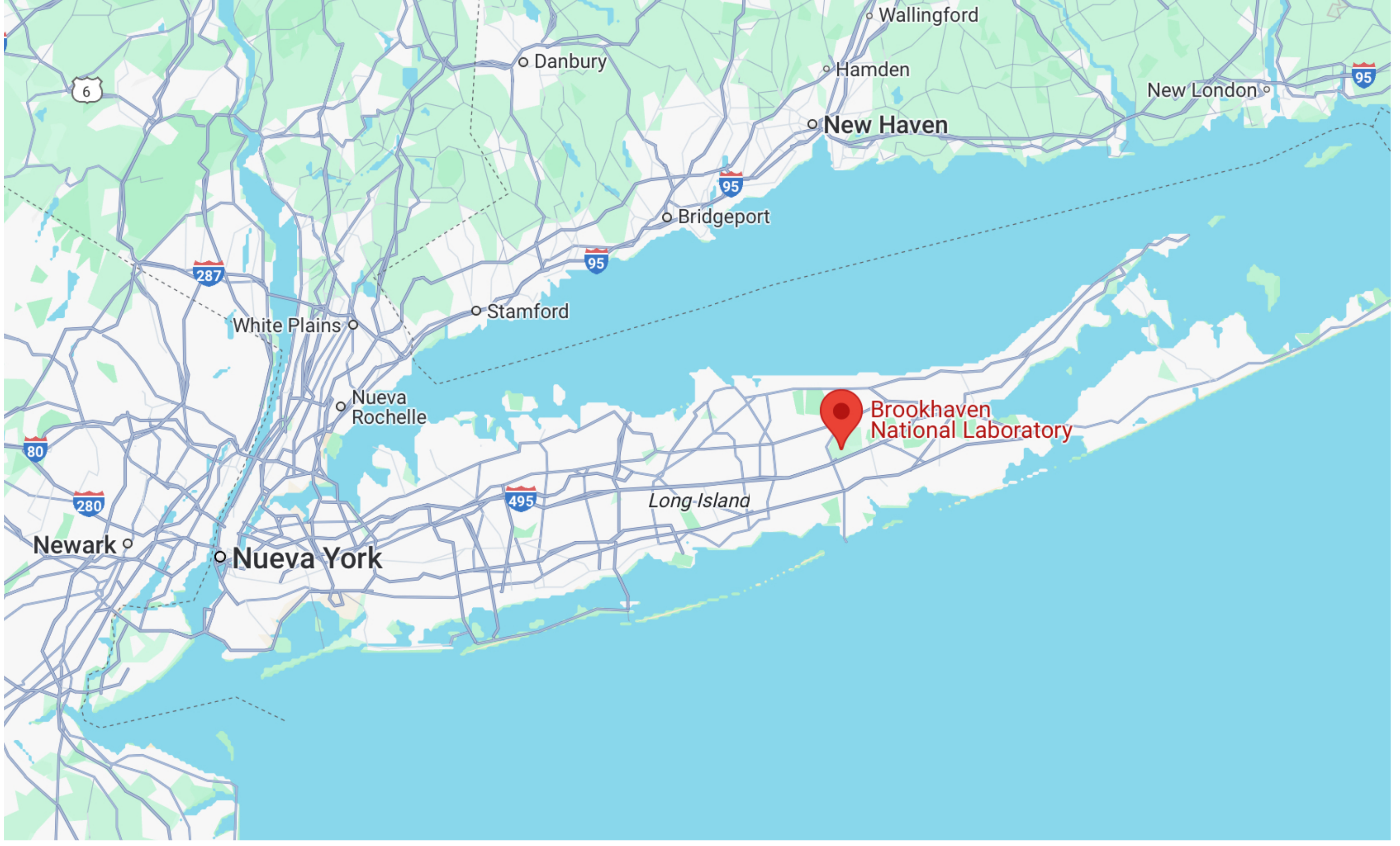
+

Unprecedented precision in the extraction of F_2 and F_L

Summary and conclusions

- Simultaneous extraction of the proton structure functions.
- Rosenbluth-type extraction introduces a potentially measurable bias in the longitudinal structure function, linked to $xF_3 \neq 0$.
- More beam configurations $\Longrightarrow$ better results.
- The extraction is systematics dominated $\Longrightarrow$ the optimistic scenario is key for the precise obtention of the proton structure functions.

Backup



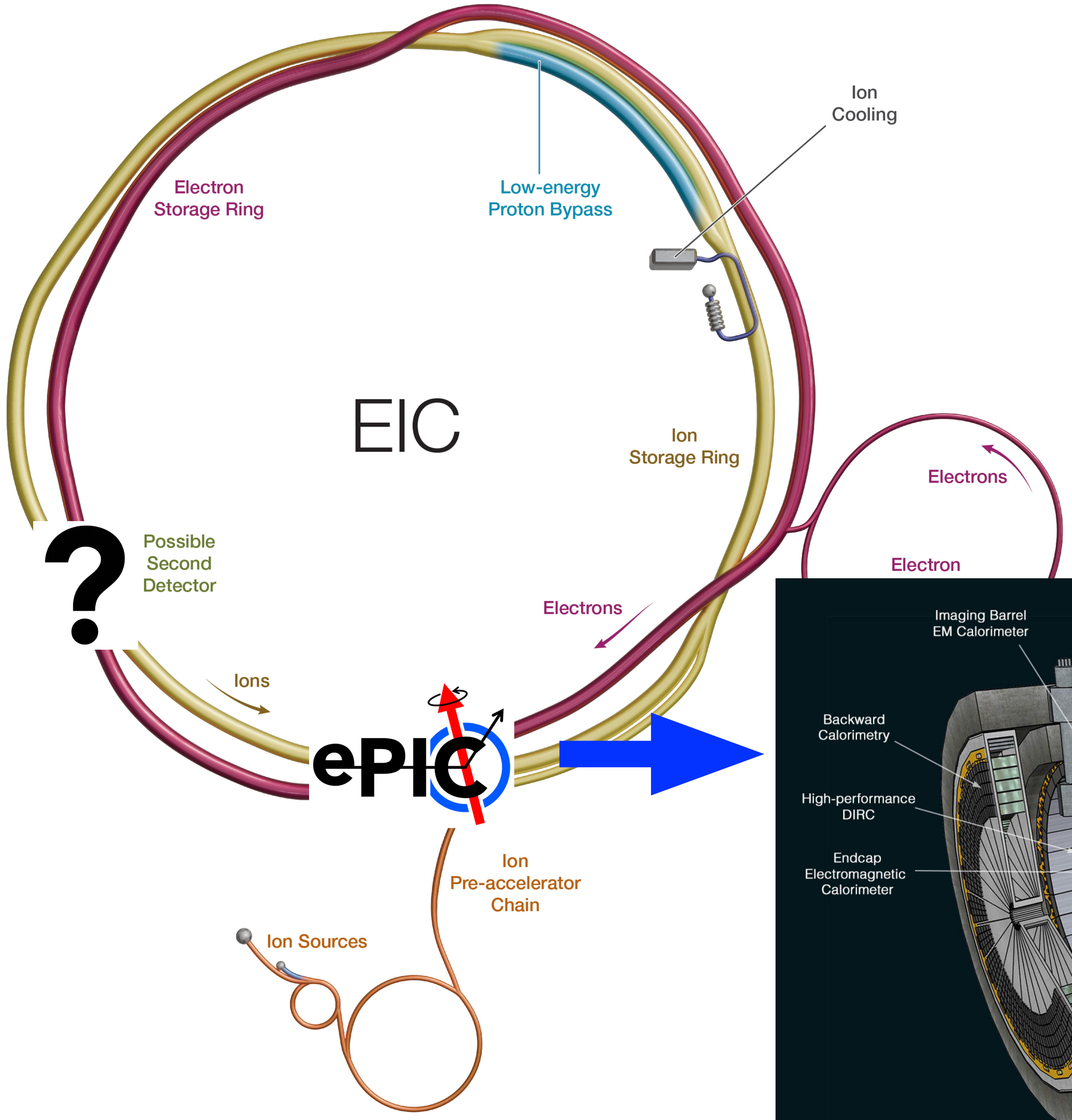
The Electron-Ion Collider

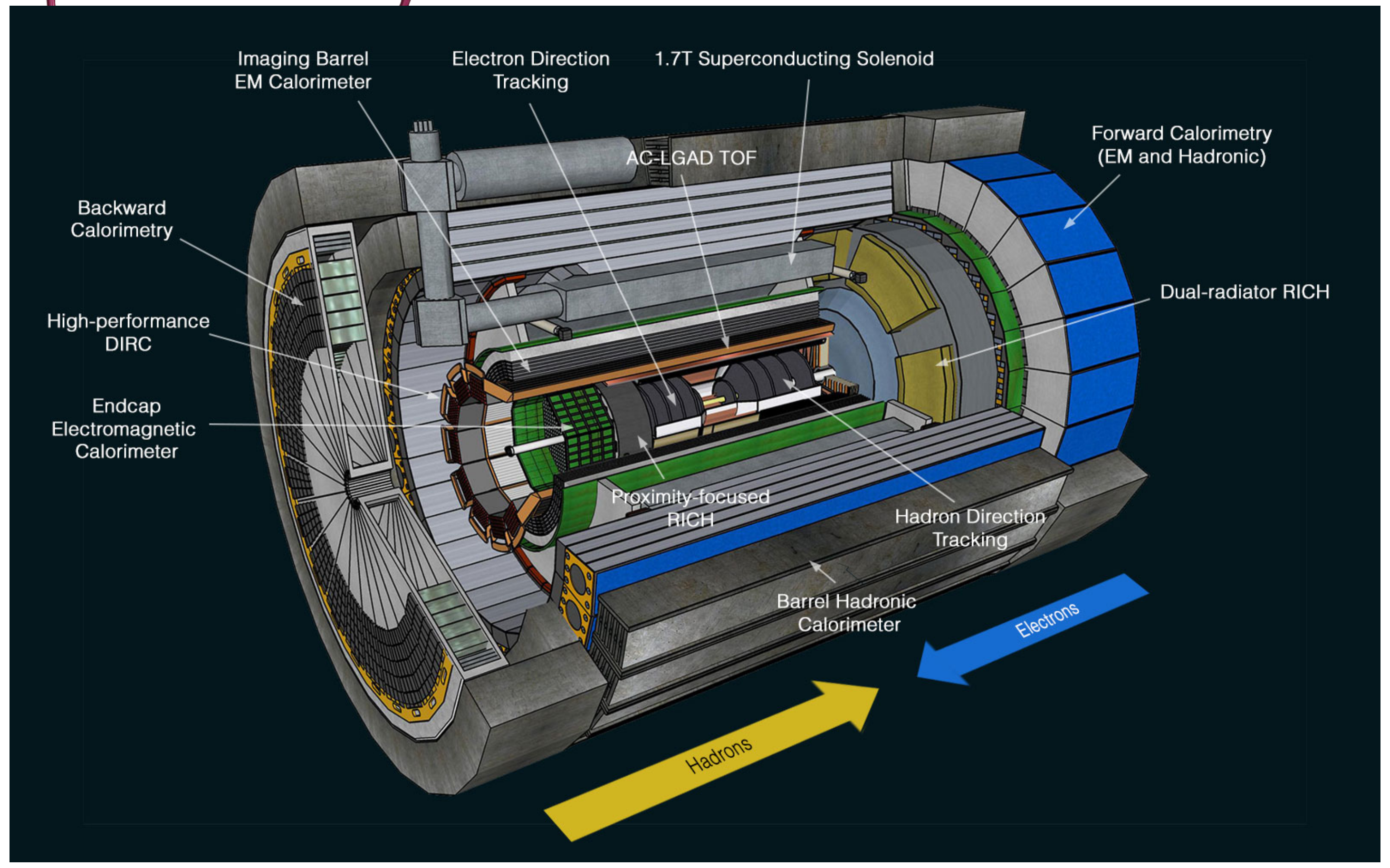
Only $e^{-}p$ collider since HERA

EIC will provide:

- Polarised beams $\Longrightarrow$ spin-dependent effects.
- Heavy ions for nuclear studies $\Longrightarrow$ nuclear PDFs.
- C.o.m energies ranging from 21 to 141 GeV.
- High luminosity $\left(10^{33} \text{ to } 10^{34}\ \text{cm}^{-2}\,\text{s}^{-1}\right)$.

This will be specially relevant for our analysis of the proton structure functions





ePIC will be the primary general-purpose detector at the future Electron-Ion Collider, designed to unravel the 3D structure and spin of the nucleon (*Nucl. Phys. A 1026 (2022) 122447*).

Overview of Deep Inelastic Scattering

If xF_3 is not neglected, the full cross section formula for NC DIS is:

$$\sigma_r^{e^\pm p}(x, Q^2, y) = F_2\left(x, Q^2\right) \mp \frac{Y_-}{Y_+} xF_3\left(x, Q^2\right) - \frac{y^2}{Y_+} F_L\left(x, Q^2\right)$$

where:

$$F_2 = F_2^{\gamma} - \kappa_z \nu_e F_2^{\gamma Z} + \kappa_z^2(\nu_e^2 + a_e^2) F_2^Z$$
$$F_L = F_L^{\gamma} - \kappa_z \nu_e F_L^{\gamma Z} + \kappa_z^2(\nu_e^2 + a_e^2) F_L^Z$$
$$xF_3 = -\kappa_z a_2 x F_3^{\gamma Z} + \kappa_z^2 2\nu_e a_e x F_3^Z$$

with

$$\kappa_z(Q^2) = \frac{Q^2}{(Q^2 + M_Z^2)(4\sin^2\theta_w \cos^2\theta_w)}$$

and ν_e and a_e are the vector and axial-vector weak couplings of the electron to the Z boson.

Electroweak effects are considerably suppressed for $Q^2 \ll M_Z^2$ but that does not mean we can fully neglect them!

MC replica method

Aiming to sample the distribution of possible outcomes for the values and uncertainties of the structure functions, 1000 replicas of each dataset are considered.

The averaging procedure, introduced in *PRD 105, 074006 (2022)*, is given by the following:

$$\bar{v} = \mathcal{S}_1 / N \qquad\qquad (\Delta v)^2 = \frac{\mathcal{S}_2 - \mathcal{S}_1^2 / N}{N - 1}$$

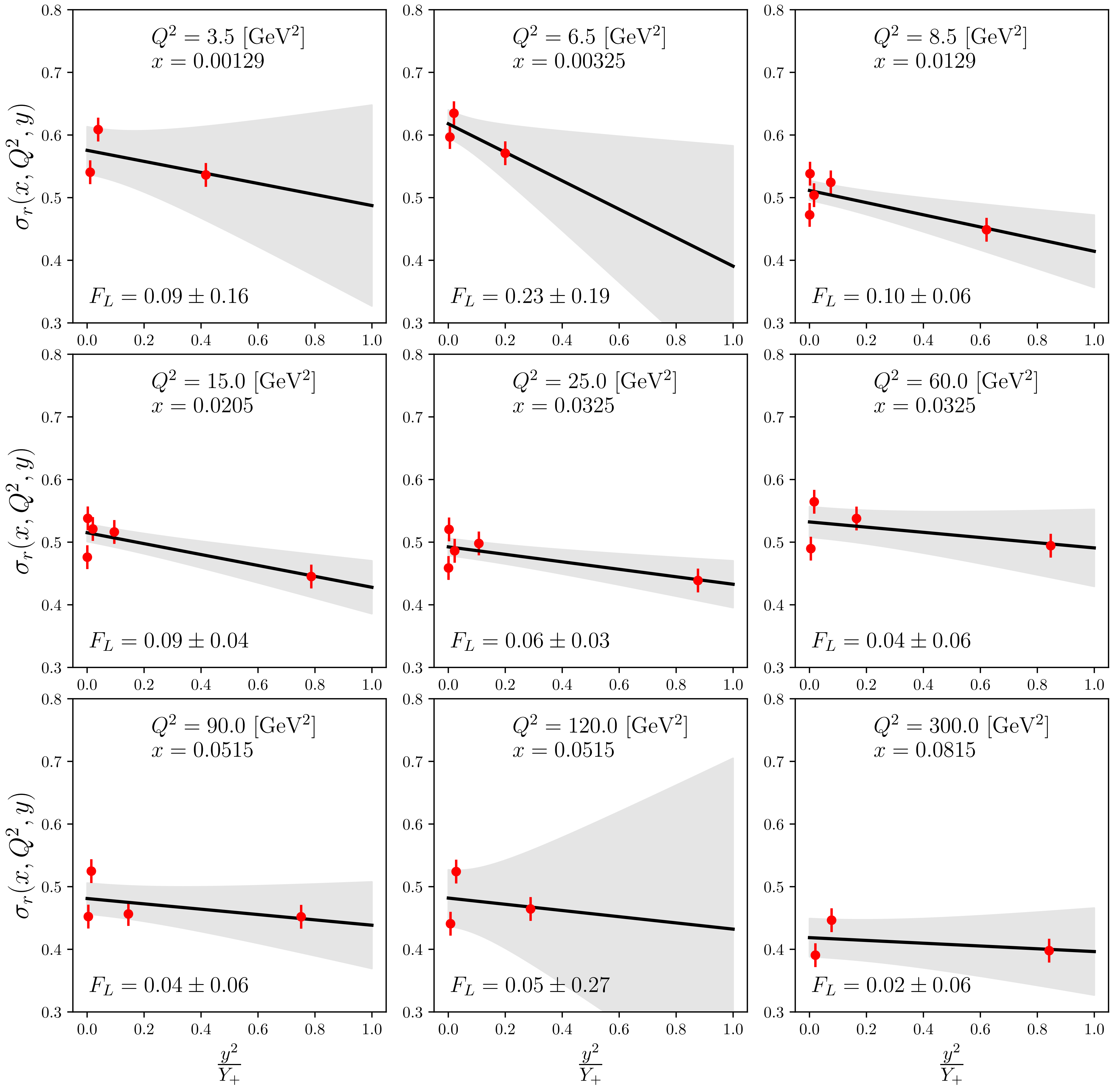
where:

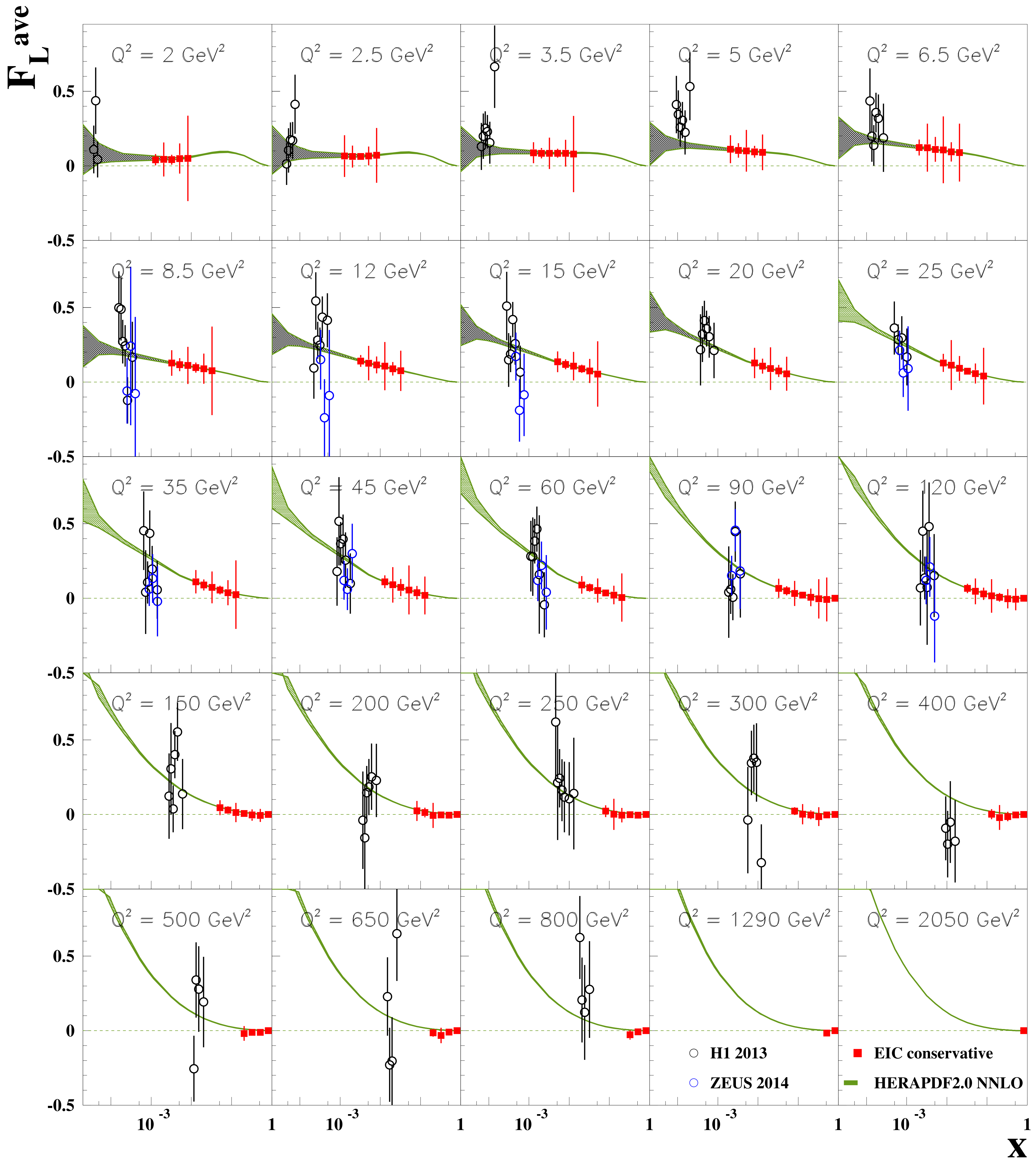
$$\mathcal{S}_n = \sum_{i=1}^{N} v_i^n$$

and v_i stands for extracted value of the structure function in the i-th MC replica.

F_L extraction with the Rosenbluth method

In *PRD 111, 056014 (2025)*, we obtained the prospects for the longitudinal structure function at EIC assuming $xF_3 = 0$.

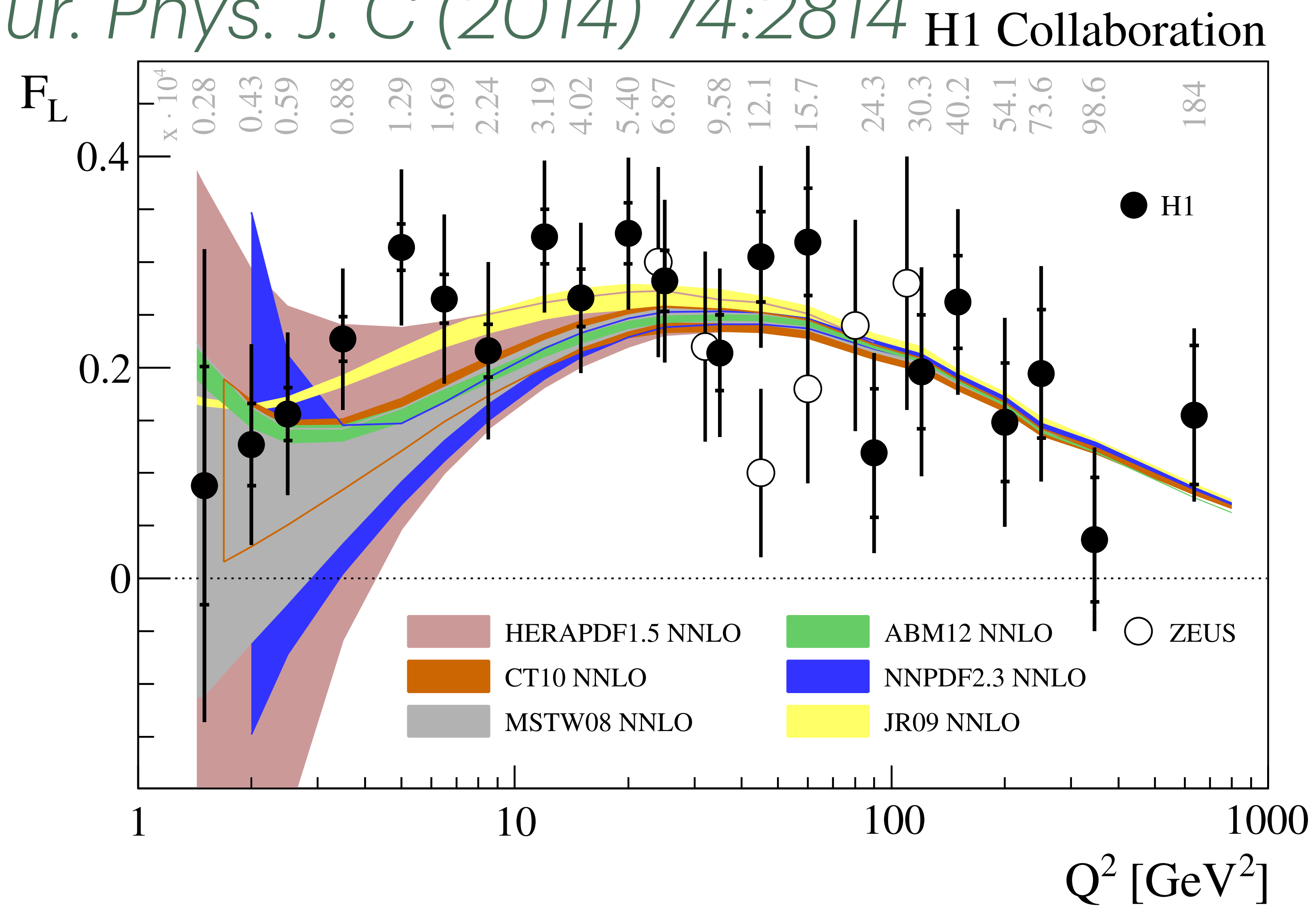




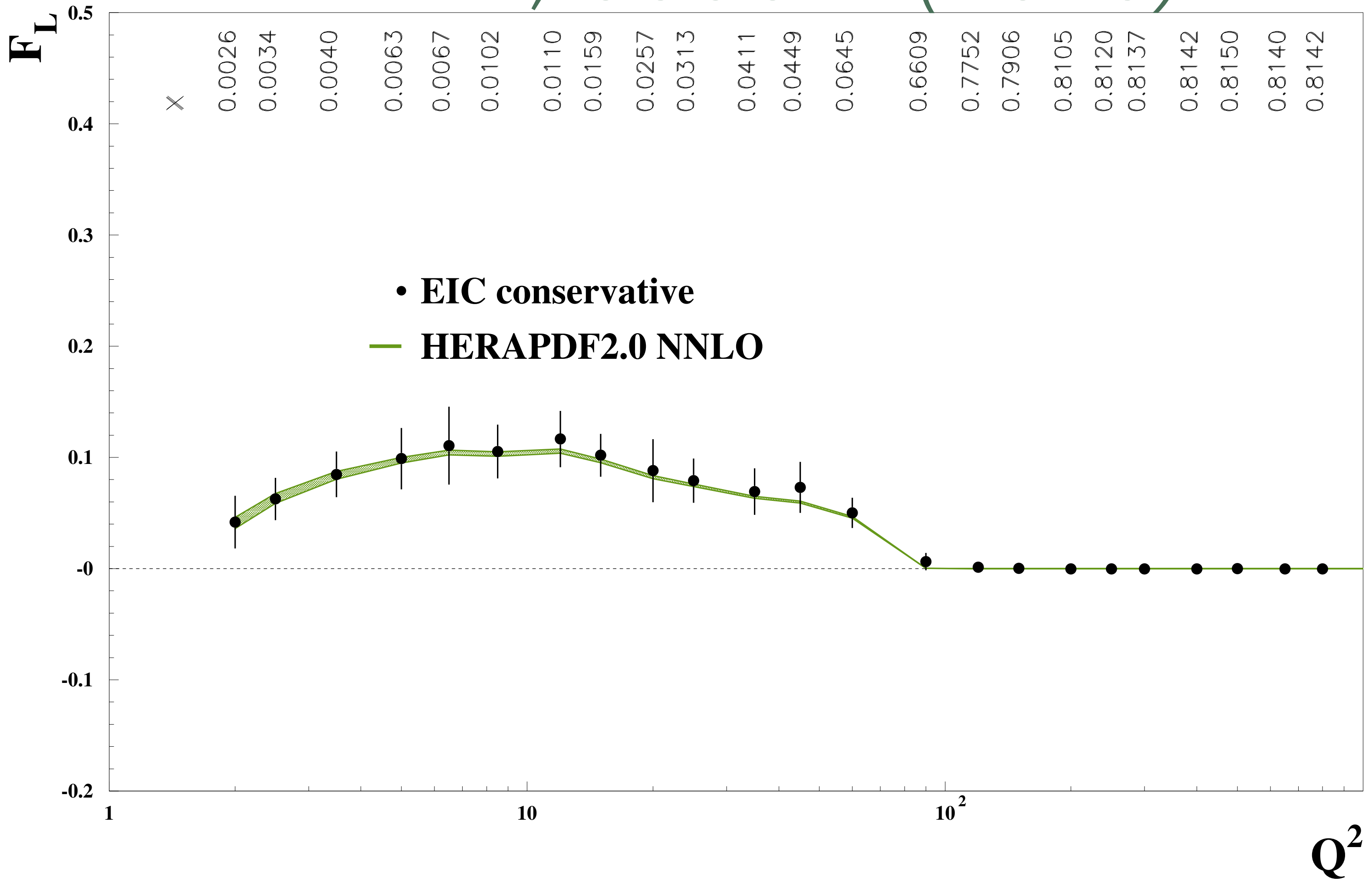
High y measurements are crucial!

F_L extraction with the Rosenbluth method

Eur. Phys. J. C (2014) 74:2814



PRD 111, 056014 (2025)



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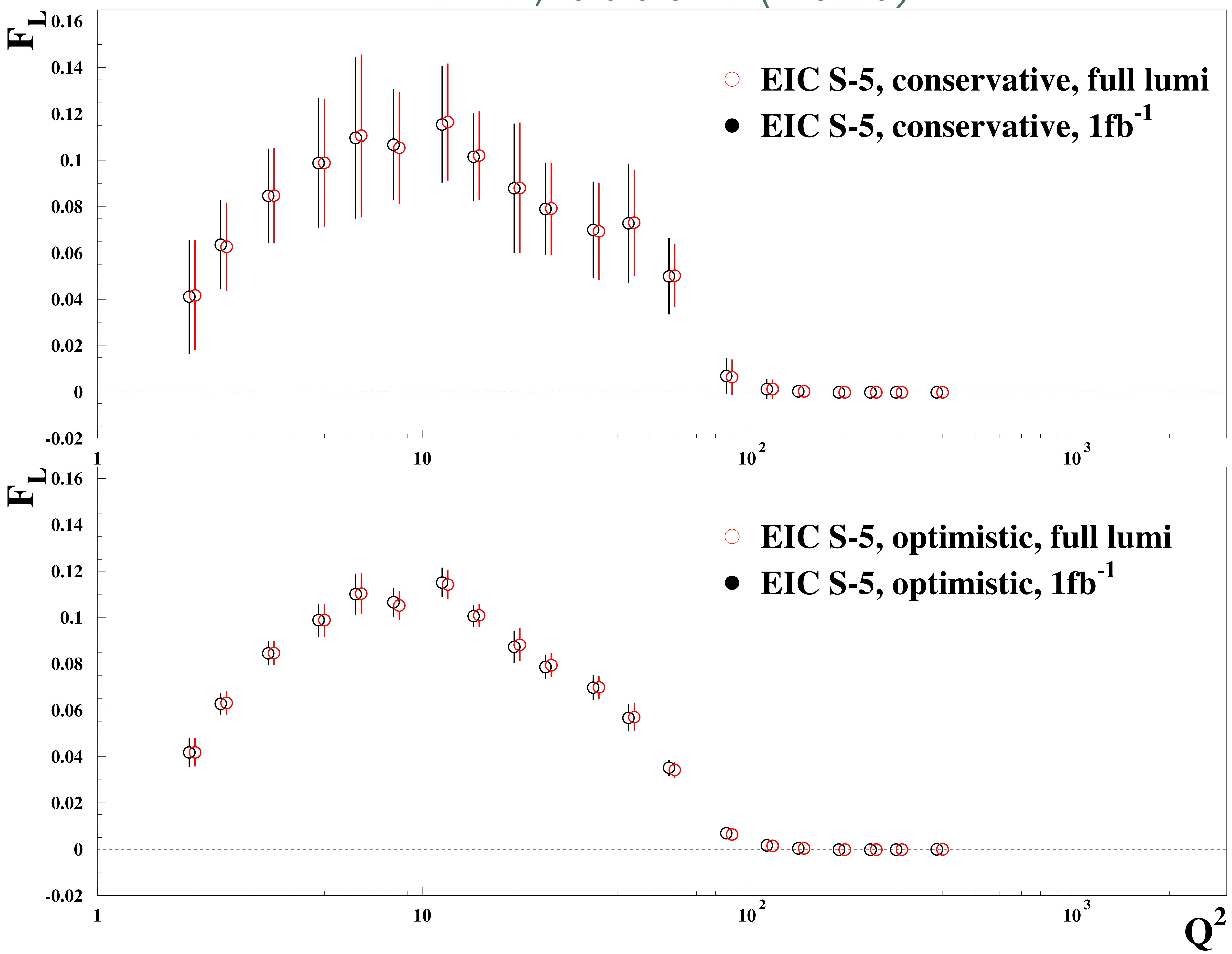
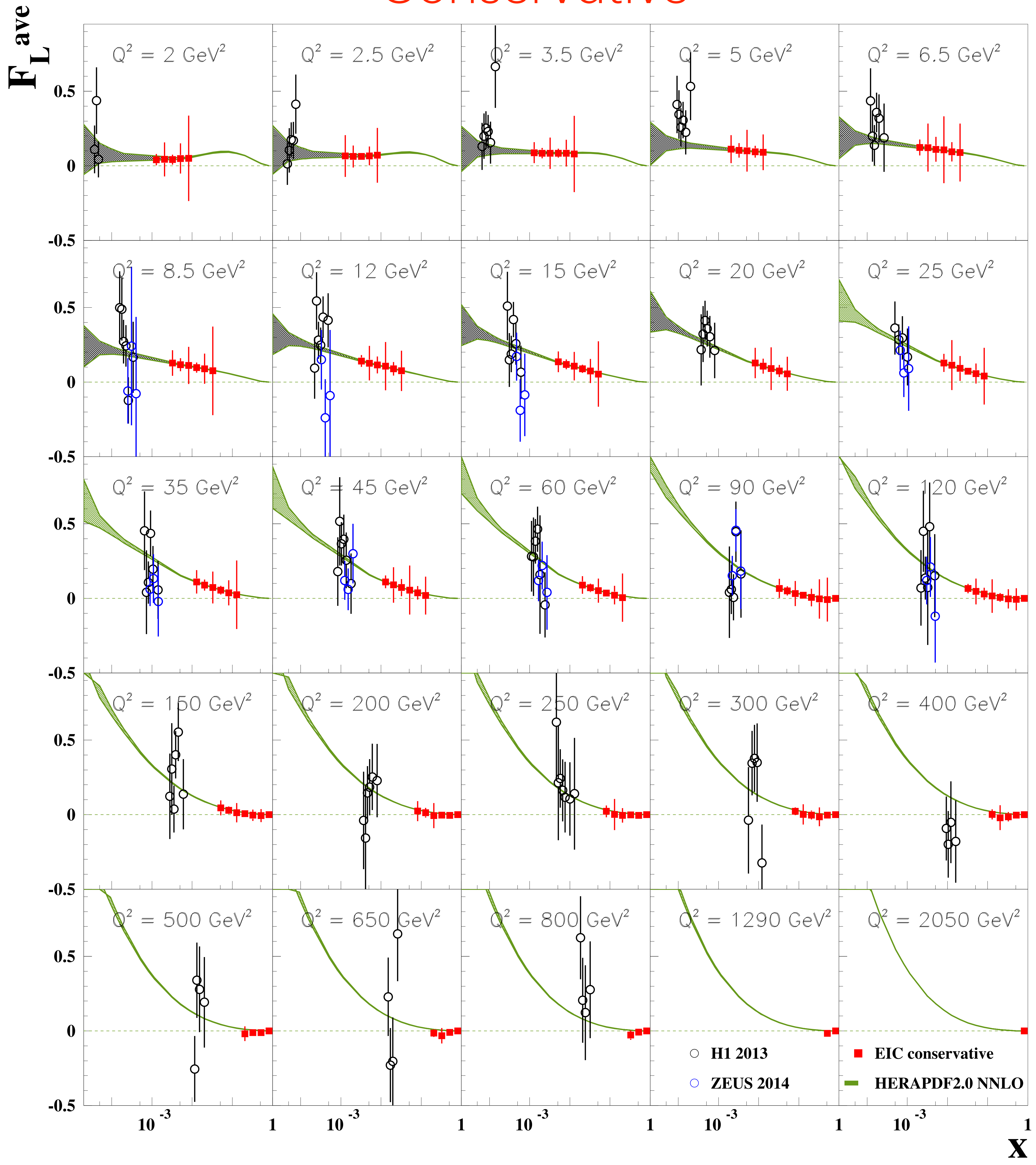


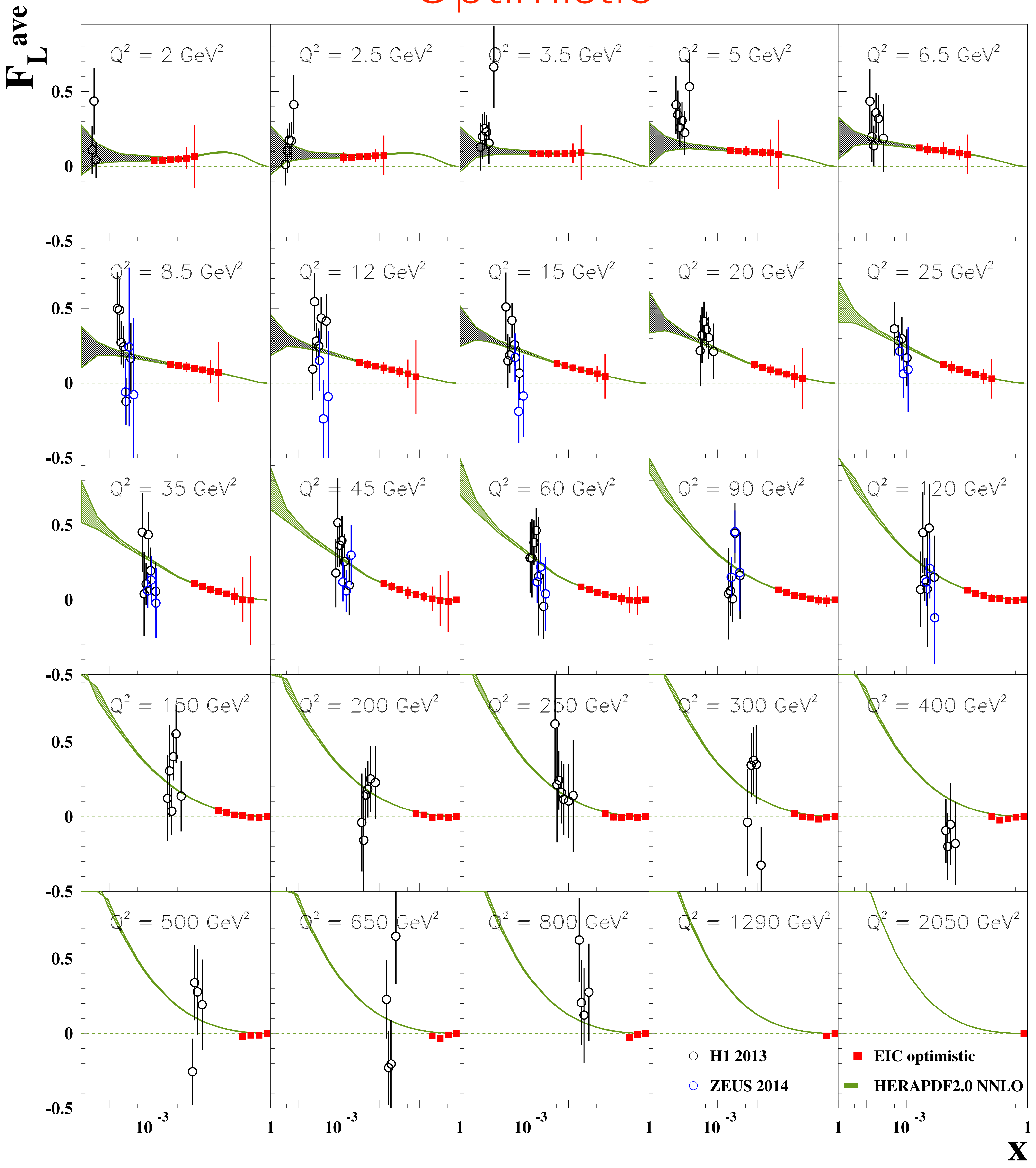
Modest luminosities already provide precise results

$F_L(x, Q^2)$ from Rosenbluth fit (conservative vs. optimistic)

Conservative

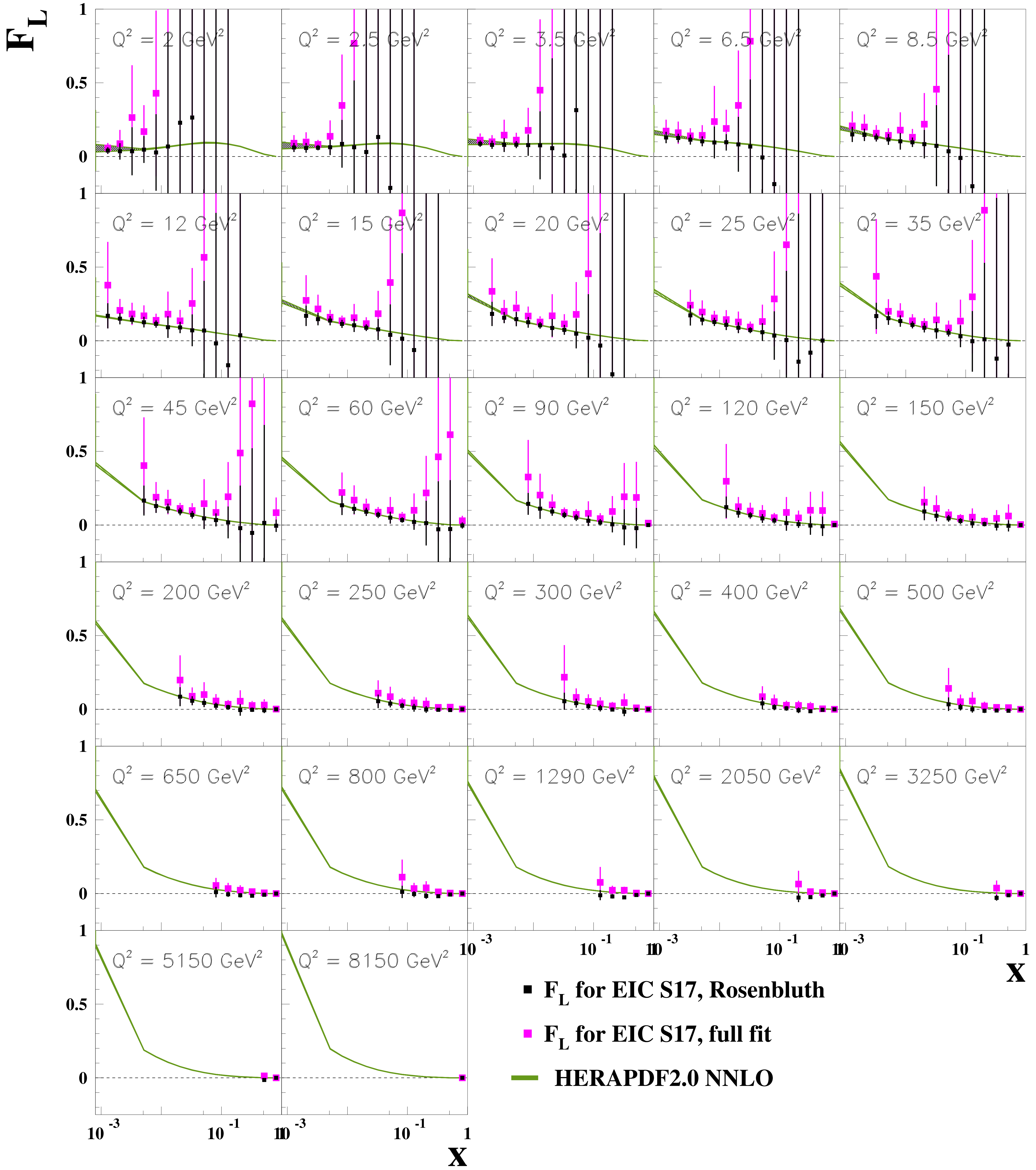


Optimistic



$F_L(x, Q^2)$ from full cross section fit (conservative vs. optimistic)

Conservative



Optimistic