A General Partial Wave Amplitude Formalism for Meson Electroprocution at the EIC

Dillon Leahy[1], Derek Glazier

University of Glasgow

EIC Early Career Workshop July 2026, Febuary 2026

[1] Email: d.leahy.1@research.gla.ac.uk
Linkedin: https://www.linkedin.com/in/dillonleahy/

Outline

1. Introduction
2. Formalism Overview
3. Results
4. Brief Extensions & Applications
5. Outlook

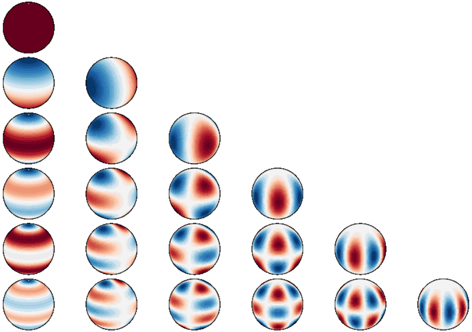


Figure 1: Example of some spherical harmonics. Red is +1 and blue is -1. Sourced from https://github.com/NVIDIA/torch-harmonics.

Introduction

What is the general idea?

- Analyse the independent angular contributions to our overall angular distribution $\mathrm{d}\sigma$
- Each "partial-wave" will contribute to an independent mode based on its quantum numbers
- Essentially a large scale Fourier transform to extract "moments"
- Followed by a non-linear minimisation to extract the underlying process amplitudes

Brief Results / Strengths:

- Amplitude extraction and longitudinal / transverse separation at a fixed beam energy
- "Moments" can encode interferences and magnitudes for many overlapping resonances
- Fully generalizable to polarised nucleon beams and recoils
- Further generalizable to higher spins and final products

Weaknesses:

- Specific cases result in ambiguous solutions and null results
- Scalability – currently use MINUIT to minimize non-linear system but this will not be practical for higher spins and/or polarized observables

Introduction: The Electron Ion Collider (EIC)

- The EIC will (hopefully) be the next electron–proton collider and the first since HERA
- High luminosities and clean $e^- p$ collisions will provide more than adequate stats to resolve such resonances
- Polarized beams provide a great way to take full advantage of the generality of this formalism
- Not just limited to standard meson spectroscopy at the EIC as this formalism is fully adaptable:
  - Possibility to extend to vector-vector production and the search for exotica
  - Possible connection to structure with the "moments" corresponding to distinct GPD related structures
  - The current formalism developed can work for any 2-pseudoscalar exclusive final state

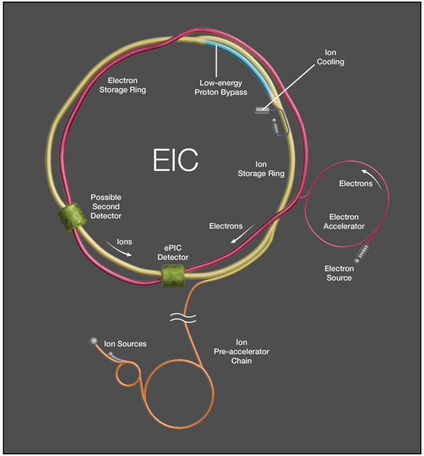


Figure 2: Schematic of the EIC. Taken from https://www.bnl.gov/eic/machine.php

Starting Point: CLAS 2009 $\pi^+\pi^-$ Moments Analysis

PRL **102,** 102001 (2009) PHYSICAL REVIEW LETTERS week ending 13 MARCH 2009

Measurement of Direct $f_0(980)$ Photoproduction on the Proton

M. Battaglieri,[1] R. De Vita,[1] A. P. Szczepaniak,[2] K. P. Adhikari,[34] M. Aghasyan,[19] M. J. Amaryan,[34] P. Ambrozewicz, M. Anghinolfi,[1] G. Asryan,[47] H. Avakian,[41] H. Bagdasaryan,[34] N. Baillie,[46] J. P. Ball,[4] N. A. Baltzell,[40] V. Batourine,[26]

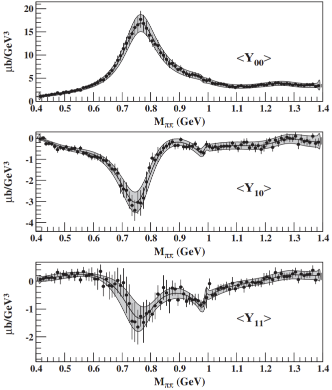


Fig A: *Extracted angular moments $\langle Y_{00}\rangle$, $\langle Y_{10}\rangle$, and $\langle Y_{11}\rangle$ mapping the clear S-P interference profiles.*

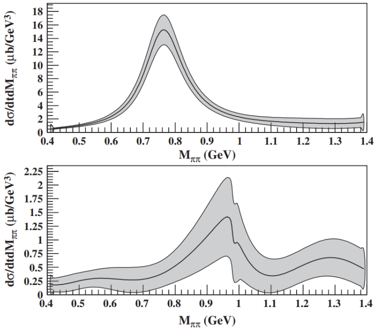


Fig B: *Resulting PWA cross sections cleanly splitting the small scalar S-wave from the dominant vector P-wave.*

Defining Angular Moments

$$\langle Y_{LM}\rangle = \sqrt{4\pi}\int d\Omega_\pi\, Y_{LM}\,\frac{d^3\sigma}{dt\, dM_{\pi\pi}\, d\Omega_\pi}$$

Disentangling S and P Waves

- **Overlap:** The $\pi^+\pi^-$ final state contains overlapping S and P wave amplitudes.
- **Scale Limit:** Direct scalar extraction is buried because the vector P-wave (ρ) cross section is $\sim 50\times$ larger.
- **Resolution:** Moments preserve the phase interference terms (e.g., $\langle Y_{10}\rangle$), allowing PWA to isolate the $f_0(980)$.
- model dependent due to unpolarised photon beam

- From standard field theory we can write out our 2-body decay intensity as

$$I(\Omega,\Phi) = \frac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}m_{ab}\,\mathrm{d}\Omega\,\mathrm{d}\Phi} = \kappa \sum_{\substack{\lambda,\lambda' \\ \lambda_1,\lambda_2}} A_{\lambda;\lambda_1\lambda_2}(\Omega)\,\rho^{\gamma^{(*)}}_{\lambda\lambda'}(\Phi)\,A^{*}_{\lambda';\lambda_1\lambda_2}(\Omega)\,.$$

- Can expand amplitudes out in a basis of partial-wave amplitudes $T^{l}_{\lambda m;\lambda_1\lambda_2}$:

$$A_{\lambda;\lambda_1\lambda_2}(\Omega) = \sum_{\ell m} T^{\ell}_{\lambda m;\lambda_1\lambda_2}\, Y^{m}_{\ell}(\Omega)$$

- Formalism then deconstructs the $I(\Omega,\Phi)$ in terms of each orthogonal Φ component

$$I(\Omega,\Phi) = \sum_{\alpha=0}^{8} I^{\alpha}(\Omega)\Pi^{\alpha}(\Phi)$$

- α collects our different polarizations: $0-3$ are purely transverse (TT) 4 is purely longitudinal (LL) and $5-8$ are interferences

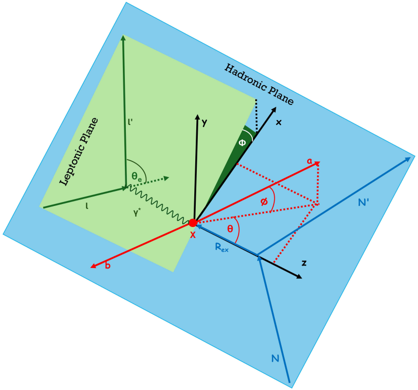


Figure 3: Gottfried-Jackson frame for the general decay.

Theory Overview (II) – Moments and Intensities

- We can then expand our polarized intensities in terms of their orthogonal Ω terms
- The amplitudes appear as billinears within the spin density matrix elements (SDMEs) $\rho^{\alpha,ll'}_{mm'}$
- put equations here

Definition

$$I^{\alpha}(\Omega) = \begin{cases} \sum_{LM} \left(\frac{2L+1}{4\pi}\right) H^{\alpha}_{LM} D^{L*}_{M0}(\Omega) & \text{if } \alpha = 0, \\ -\sum_{LM} \left(\frac{2L+1}{4\pi}\right) H^{\alpha}_{LM} D^{L*}_{M0}(\Omega), & \text{otherwise.} \end{cases}$$

$$H^{\alpha}(LM) = \begin{cases} \sum_{\substack{ll' \\ mm'}} \left(\frac{2l'+1}{2l+1}\right) C^{l0}_{l'0L0} C^{lm}_{l'm'LM} \rho^{\alpha,ll'}_{mm'} & \text{if } \alpha = 0, \\ -\sum_{\substack{ll' \\ mm'}} \left(\frac{2l'+1}{2l+1}\right) C^{l0}_{l'0L0} C^{lm}_{l'm'LM} \rho^{\alpha,ll'}_{mm'}, & \text{otherwise.} \end{cases}$$

- Can relate each term to standard concepts:
  - Angular distribution $I(\Omega, \Phi)$ (the signal)
  - Moments $H(LM)$ (Fourier coeffs)
  - Spherical harmonics $Y^M_L(\Omega)$ (sinusoids)
- For multiple resonances with different spins moments encode
  - Interference and phase between resonances,
  - Magnitudes of partial waves.

Theory Overview (III) – something

- The SDME above is directly related to the virutal photon SDME through the linear map

$$\rho^{\alpha,ll'}_{mm'} = \frac{1}{2}\sum_{\lambda} T^{l}_{\lambda m;\lambda_1\lambda_2}\Sigma^{\alpha}\left(T^{l'}_{\lambda' m';\lambda_1\lambda_2}\right)^{*}$$

- Σ^{α} are just some orthogonal base of basis matrices with corresponding coefficients Π^{α} that are model dependent
- Here we now focus on developing this formalism specifically for vector meson production, specifically the work of Schlling and Wolf [1]

$$\rho^{0,\ell\ell'}_{mm'} = \frac{1}{2}\sum_{\lambda=\pm1}\sum_{\lambda_1\lambda_2} T^{\ell}_{\lambda,m;\lambda_1\lambda_2} T^{\ell'*}_{\lambda,m';\lambda_1\lambda_2},$$

$$\rho^{1,\ell\ell'}_{mm'} = -\frac{1}{2}\sum_{\lambda=\pm1}\sum_{\lambda_1\lambda_2} T^{\ell}_{-\lambda,m;\lambda_1\lambda_2} T^{\ell'*}_{\lambda,m';\lambda_1\lambda_2},$$

$$\rho^{2,\ell\ell'}_{mm'} = \frac{\mathrm{i}}{2}\sum_{\lambda=\pm1}\sum_{\lambda_1\lambda_2} \lambda T^{\ell}_{-\lambda,m;\lambda_1\lambda_2} T^{\ell'*}_{\lambda,m';\lambda_1\lambda_2},$$

$$\rho^{3,\ell\ell'}_{mm'} = \frac{1}{2}\sum_{\lambda=\pm1}\sum_{\lambda_1\lambda_2} \lambda T^{\ell}_{\lambda,m;\lambda_1\lambda_2} T^{\ell'*}_{\lambda,m';\lambda_1\lambda_2},$$

$$\rho^{4,\ell\ell'}_{mm'} = \sum_{\lambda_1\lambda_2} T^{\ell}_{0,m;\lambda_1\lambda_2} T^{\ell'*}_{0,m';\lambda_1\lambda_2},$$

$$\rho^{5,\ell\ell'}_{mm'} = \frac{1}{2\sqrt{2}}\sum_{\lambda=\pm1}\sum_{\lambda_1\lambda_2} \lambda\left(T^{\ell}_{0,m;\lambda_1\lambda_2} T^{\ell'*}_{\lambda,m';\lambda_1\lambda_2} + T^{\ell}_{\lambda,m;\lambda_1\lambda_2} T^{\ell'*}_{0,m';\lambda_1\lambda_2}\right),$$

$$\rho^{6,\ell\ell'}_{mm'} = \frac{\mathrm{i}}{2\sqrt{2}}\sum_{\lambda=\pm1}\sum_{\lambda_1\lambda_2} \left(T^{\ell}_{0,m;\lambda_1\lambda_2} T^{\ell'*}_{\lambda,m';\lambda_1\lambda_2} - T^{\ell}_{\lambda,m;\lambda_1\lambda_2} T^{\ell'*}_{0,m';\lambda_1\lambda_2}\right),$$

$$\rho^{7,\ell\ell'}_{mm'} = \frac{1}{2\sqrt{2}}\sum_{\lambda=\pm1}\sum_{\lambda_1\lambda_2} \left(T^{\ell}_{0,m;\lambda_1\lambda_2} T^{\ell'*}_{\lambda,m';\lambda_1\lambda_2} + T^{\ell}_{\lambda,m;\lambda_1\lambda_2} T^{\ell'*}_{0,m';\lambda_1\lambda_2}\right),$$

$$\rho^{8,\ell\ell'}_{mm'} = \frac{\mathrm{i}}{2\sqrt{2}}\sum_{\lambda=\pm1}\sum_{\lambda_1\lambda_2} \lambda\left(T^{\ell}_{\lambda,m;\lambda_1\lambda_2} T^{\ell'*}_{0,m';\lambda_1\lambda_2} - T^{\ell}_{0,m;\lambda_1\lambda_2} T^{\ell'*}_{\lambda,m';\lambda_1\lambda_2}\right).$$

Theory Overview (IV) – The Reflectivity Basis

- The reflectivity basis is a transformation that is integral to this formalism
- Can use its symmetric quantities to reduce complexity and also relate the corresponding reflectivity eigenvalue $\epsilon = \pm 1$ to distinct exchanges [2]

Definitions:

$$^{\epsilon}T^{l}_{Tm;\lambda_1\lambda_2} = \frac{1}{2}\left(T^{l}_{+1m;\lambda_1\lambda_2} - (-1)^m T^{l}_{-1m;\lambda_1\lambda_2}\right) \text{ and } {}^{\epsilon}T^{l}_{Lm;\lambda_1\lambda_2} = \frac{1}{2}\left(T^{l}_{0m;\lambda_1\lambda_2} + (-1)^m T^{l}_{0m;\lambda_1\lambda_2}\right)$$

- New index L/T indicates whether we are dealing with a longitudinal or transverse
- Can also reduce the nucleon helicites$(\lambda_1\lambda_2)$ due to parity

$$[l]^{\epsilon}_{m;1} = {}^{\epsilon}T^{l}_{m;++} \quad \text{and} \quad [l]^{\epsilon}_{m;-1} = {}^{\epsilon}T^{l}_{m;+-}$$

- This **represents the spin-flip of the nucleon**
- $k = 1$ is non-spin flip ($++$ or - -) andn $k = -1$ is the spin flip state ($+$- or -$+$)

Interlude: Naturality

For the decay to two pseudoscalars our reflectivity is equivalent to the naturality of the exchange

Definition

- Naturality (natural parity) is defined as

$$\eta = P(-1)^J,$$

  where:
  - P is the parity and J is the total angular momentum of the meson.

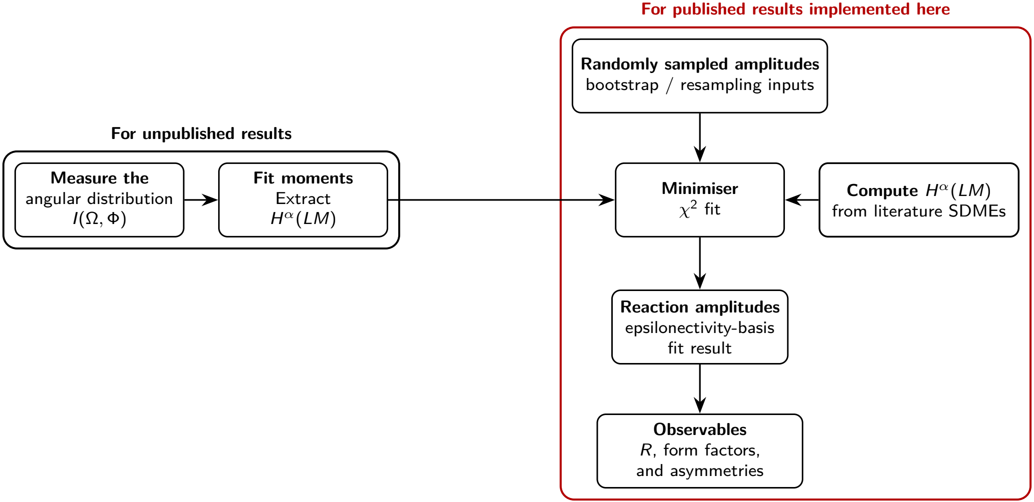
Examples

- **Natural** ($\eta = +1$) exchanges:
  - Pomerons, ρs, other vector mesons.
- **Unnatural** ($\eta = -1$) exchanges:
  - πs, other pseudoscalar mesons.

MINUIT Inversion Procedure

Evaluate the amplitudes through a non-linear minimisation procedure with bootstrap resampling for uncertainties

$$\chi^2 = \sum_{\alpha LM} \frac{H^{\alpha}_{obs}(LM) - H^{\alpha}(LM)}{\sigma_i^2(LM)}$$



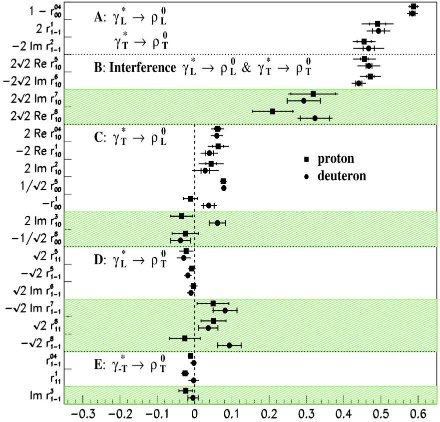
Application to existing unpolarised target data [3, 4, 5, 6]

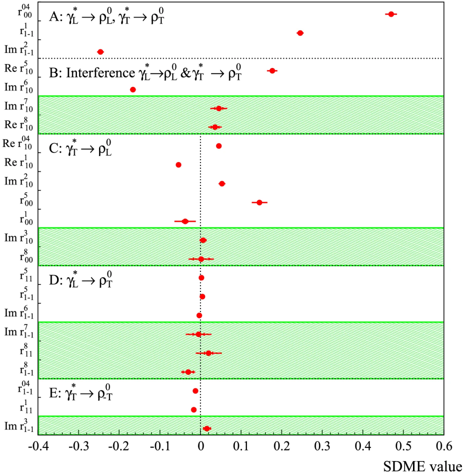
Spin density matrix elements in exclusive ρ^0 electroproduction on ^1H and ^2H targets at 27.5 GeV beam energy

Regular Article – Experimental Physics | Open access | Published: 17 July 2009
Volume 62, pages 659–695 (2009) Cite this article

Spin density matrix elements in exclusive ρ^0 meson muoproduction

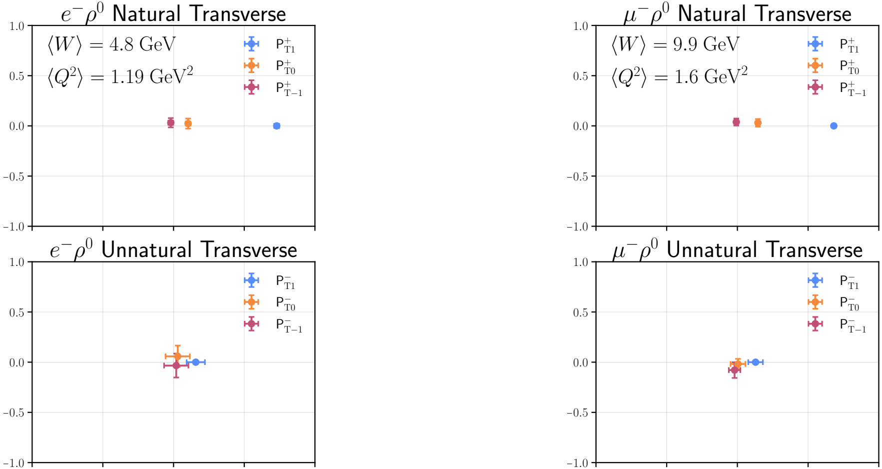
Regular Article – Experimental Physics | Open access | Published: 13 October 2023
Volume 83, article number 924 (2023) Cite this article





Real-World Data: ρ^0 Production (Transverse Sectors)

Applying the unconstrained reflectivity PWA numerical inversion to historical experimental data

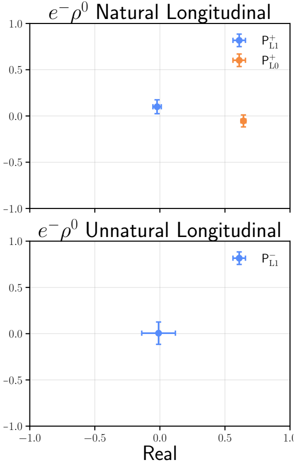


Electron Production: Extracted $e^-\rho^0$ transverse amplitudes . **Muon Production:** Extracted $\mu^-\rho^0$ transverse amplitudes.

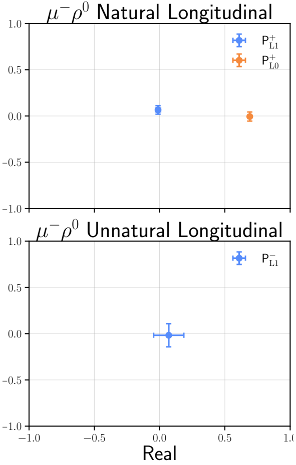
Observation: Both independent datasets display structural agreement, dominated by a highly prominent Natural Transverse $m = 1$ no-helicity-flip amplitude ($[\ell]^+_{T1}$) signifying expected Pomeron/Reggeon exchange trajectories.

Real-World Data: ρ^0 Production (Longitudinal Sectors)

The corresponding longitudinal sector extractions illustrate clean isolation of the primary diffractive structures alongside structural symmetry constraints :



Electron Production: Extracted $e^-\rho^0$ longitudinal amplitudes.

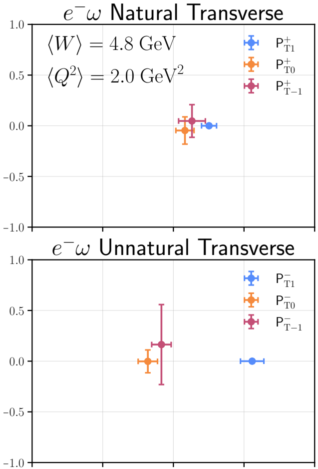


Muon Production: Extracted $\mu^-\rho^0$ longitudinal amplitudes.

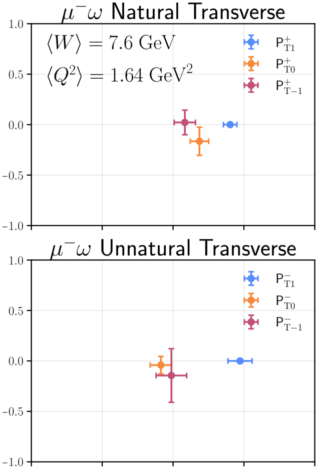
Key Observation: The longitudinal sector shows clear dominance of the Natural Longitudinal $m = 0$ non-helicity-flip wave($[\ell]^+_{l0}$) which relates to twist 2 GPDs.

Real-World Data: ω Production (Transverse Sectors)

On ω data the minimizer does not work as well...



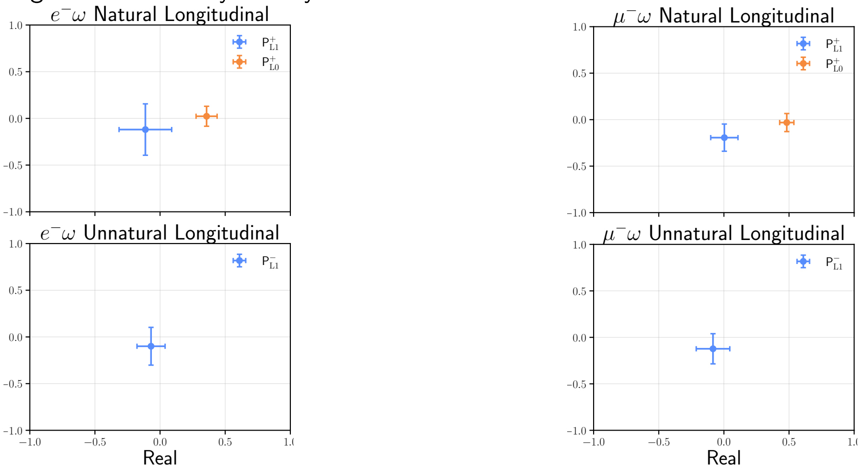
Electron Production: Extracted $e^-\omega$ transverse amplitudes .



Muon Production: Extracted $\mu^-\omega$ transverse amplitudes.

Observation: Minimizer solution is less certain than ρ^0, but can see a distinctly large Unnatural Transverse $m = 1$ corresponding to expected pion exchange.

The corresponding longitudinal sector extractions illustrate clean isolation of the primary diffractive structures alongside structural symmetry constraints :



Electron Production: Extracted $e^-\omega$ longitudinal amplitudes. **Muon Production:** Extracted $\mu^-\omega$ longitudinal amplitudes.

Key Observation: Similarly large Natural $m = 0$ component to the ρ^0.

Polarized Nucleons (I)

- One of the key aspects provided by the EIC will be polarised nucleon beams
- This will allow us to fully exploit this formalism and extract the nucleon helicity dependent amplitudes
- For polarised nucleons we sum over the 16 extra degrees of freedom given by each
- With our polarized electron this gives $9 \times 16 = 144$ different "polarization" mixtures

The Maths

- Now extend to include both nucleon density matrices

$$\rho^{\alpha,ll'}_{mm'} = \frac{1}{2}\sum_{\lambda} T^{l}_{\lambda m;\lambda_1\lambda_2} \Sigma^{\alpha}_{\lambda\lambda'} \sigma^{\beta}_{\lambda_1\lambda_1'} \sigma^{\delta}_{\lambda_2\lambda_2'} \left(T^{l'}_{\lambda' m';\lambda_1'\lambda_2'}\right)^*$$

- Things get complicated fast by summing over each index but we can summarise it quite neatly as all amplitudes end up as the form

$$\sum_{\epsilon\epsilon'} T^{\epsilon}_{Xm;\lambda_1\lambda_2} \sigma^{\beta}_{\lambda_1\lambda_1'} \sigma^{\delta}_{\lambda_2\lambda_2'} T^{\epsilon' *}_{Ym';\lambda_1'\lambda_2'} = 2\sum_{\epsilon\epsilon',kk'} C_{\beta\delta}(\epsilon,k)\, T^{\epsilon}_{Xm;k} T^{\epsilon' *}_{Ym';k'}$$

Polarized Nucleons (II)

(β, δ)	ϵ'	k'	$C_{\beta\delta}(\epsilon, k)$
$(0,0)$	ϵ	k	1
$(0,1)$	ϵ	$-k$	1
$(0,2)$	$-\epsilon$	$-k$	$-\mathrm{i}k$
$(0,3)$	$-\epsilon$	k	$-k$
$(1,0)$	ϵ	$-k$	ϵ
$(1,1)$	ϵ	k	ϵ
$(1,2)$	$-\epsilon$	k	$-\mathrm{i}k\epsilon$
$(1,3)$	$-\epsilon$	$-k$	$-k\epsilon$
$(2,0)$	$-\epsilon$	$-k$	$\mathrm{i}\epsilon$
$(2,1)$	$-\epsilon$	k	$\mathrm{i}\epsilon$
$(2,2)$	ϵ	k	$k\epsilon$
$(2,3)$	ϵ	$-k$	$-\mathrm{i}k\epsilon$
$(3,0)$	$-\epsilon$	k	1
$(3,1)$	$-\epsilon$	$-k$	1
$(3,2)$	ϵ	$-k$	$-\mathrm{i}k$
$(3,3)$	ϵ	k	$-k$

Polarized Nucleons (II) – Parameter Scaling

$\ell_{\max}$	N_{amps}	N_{real}	N_H
0	3	4	3
1	12	22	35
1 (no S-wave)	9	16	23
2	27	52	99
3	48	94	195
4	75	148	323
5	108	214	483

Table 2: Illustrative counting for the vector-meson-style electroproduction truncation using the inversion-counting method described in the text. Here $N_{\mathrm{amps}} = 3(\ell_{\max} + 1)^2$ counts the complex amplitudes, $N_{\mathrm{real}} = 2(N_{\mathrm{amps}} - 1)$ counts the real fit parameters after fixing two reference phases, and $N_H = 4(2\ell_{\max} + 1)^2 - 1$ counts the experimentally accessible moments. The $\ell_{\max} = 2$ row is the case used in the fixed-amplitude study. Also shown is the special P-wave-only case used when analysing vector-meson decay to two spin-0 particles.

Polarized Nucleons (III) – Parameter Scaling

$\ell_{\max}$	N_{amps}	N_{real}	N_H
0	6	11	56
1	24	47	568
1 (no S-wave)	18	35	376
2	54	107	1592
3	96	191	3128
4	150	299	5176
5	216	431	7736

Table 3: Illustrative counting for the case where both the target and recoil nucleons are polarized. The number of amplitudes is the same as in the singly polarized case, since the two k sectors have already been separated: $N_{\text{amps}} = 6(\ell_{\max}+1)^2$ and $N_{\text{real}} = 2N_{\text{amps}} - 1 = 12(\ell_{\max}+1)^2 - 1$. The additional spin observables increase the number of experimentally accessible moments to $N_H = 64(2\ell_{\max}+1)^2 - 8$.

Connection to Structure and GPDs (I)

- In the high-Q^2 deeply virtual limit, the production vertex probes partonic structure through GPDs.
- The advantage of the reflectivity PWA is that it first isolates amplitude sectors experimentally, before imposing a dynamical GPD model.
- For vector-meson production, the extracted P-wave amplitudes can be mapped onto helicity amplitudes and then to Compton form factors (CFFs) [7, 8].

Leading longitudinal sector

$$P^{+}_{0;1} \sim \mathcal{H}_V,$$
$$P^{+}_{0;-1} \sim \frac{\sqrt{-t'}}{2m_N}\,\mathcal{E}_V.$$

The non-flip amplitude is dominated by the vector CFF $\mathcal{H}_V$, while the flip amplitude accesses $\mathcal{E}_V$.

Why polarization matters

$$k = 1 \rightarrow \text{helicity conserving},$$
$$k = -1 \rightarrow \text{helicity flipping}.$$

Unpolarized data mainly gives diagonal sums over k. Transverse target or recoil polarization is needed to expose the interference that separates the suppressed E-type CFFs.

Connection to Structure and GPDs (II)

PWA label	Physics filter	Hard-exclusive limit	Structural information
$\epsilon = +1$	Natural / parity-matched exchange	Vector CFF sector	H, E-type GPD information
$\epsilon = -1$	Unnatural / parity-flipped exchange	Axial or transversity-sensitive sector	$\widetilde{H}, \widetilde{E}$ and chiral-odd sensitivity
$k = 1$	Nucleon non-spin flip	Helicity conserving	Dominant H-like contribution
$k = -1$	Nucleon spin flip	Helicity flipping	Access to E-like orbital-angular-momentum information

Interpretation

The extracted amplitudes provide a model-independent intermediate layer between angular data and GPD-based structure models. In the EIC regime, the same formalism can therefore serve both spectroscopy, through mass-dependent amplitudes, and nucleon tomography, through the Q^2, t, and polarization dependence of the CFFs [9, 10, 11].

Future Work

Physics interpretation

- Develop Regge- and GPD-based amplitude models for the extracted reflectivity waves.
- Use the Q^2 and t dependence to connect partial waves to transition form factors, CFFs, and spatial structure.

Applications to data

- Extend the present unpolarized-target studies to polarized target and recoil observables.
- Prepare the framework for high-statistics EIC measurements, where the larger response space should strongly constrain the amplitudes.

Computational development

- Replace brute-force MINUIT scans with scalable inference methods for high-dimensional amplitude extraction.
- Generalize from two spin-zero decay products to higher-spin and multi-body final states.

Summary & Questions

- Built a reflectivity-based partial-wave formalism for meson electroproduction.
- Connected measured angular moments to bilinear amplitude constraints, enabling a numerical inversion to reaction amplitudes.
- Demonstrated preliminary amplitude extractions for existing ρ^0 and ω electroproduction data.
- Extended the formalism to polarized target and recoil observables, where additional spin information unlocks otherwise inaccessible interference terms.
- Established the route from extracted amplitudes to transition form factors, CFFs, and GPD-sensitive structure observables.

Any Questions?

References I

[1] K. Schilling and G. Wolf. “How to analyse vector-meson production in inelastic lepton scattering”. In: *Nuclear Physics B* 61 (Sept. 1973), pp. 381–413. ISSN: 05503213. DOI: 10.1016/0550-3213(73)90371-4. URL: https://linkinghub.elsevier.com/retrieve/pii/0550321373903714 (visited on 01/06/2026).

[2] V. Mathieu et al. “Moments of angular distribution and beam asymmetries in $\eta\pi^0$ photoproduction at GlueX”. In: *Physical Review D* 100.5 (2019), p. 054017. DOI: 10.1103/PhysRevD.100.054017.

[3] A. Airapetian et al. “Spin density matrix elements in exclusive 0 electroproduction on1H and 2H targets at 27.5GeV beamenergy”. In: *The European Physical Journal C* 62.4 (Aug. 1, 2009), pp. 659–695. ISSN: 1434-6052. DOI: 10.1140/epjc/s10052-009-1082-3. URL: https://doi.org/10.1140/epjc/s10052-009-1082-3 (visited on 02/18/2026).

[4] G. D. Alexeev et al. “Spin density matrix elements in exclusive $$\rho ^0$$meson muoproduction”. In: *The European Physical Journal C* 83.10 (Oct. 13, 2023), p. 924. ISSN: 1434-6052. DOI: 10.1140/epjc/s10052-023-11359-4. URL: https://doi.org/10.1140/epjc/s10052-023-11359-4 (visited on 01/07/2026).

[5] A. Airapetian et al. “Spin density matrix elements in exclusive electroproduction on 1H and 2H targets at 27.5 GeV beam energy”. In: *European Physical Journal C* 74 (Nov. 15, 2014), p. 3110.

[6] G. D. Alexeev et al. “Spin density matrix elements in exclusive $$\omega $$meson muoproduction”. In: *The European Physical Journal C* 81.2 (Feb. 5, 2021), p. 126. ISSN: 1434-6052. DOI: 10.1140/epjc/s10052-020-08740-y. URL: https://doi.org/10.1140/epjc/s10052-020-08740-y (visited on 03/17/2026).

[7] M. Diehl. “Generalized parton distributions”. In: *Physics Reports* 388.2 (2003), pp. 41–277. ISSN: 0370-1573. DOI: https://doi.org/10.1016/j.physrep.2003.08.002. URL: https://www.sciencedirect.com/science/article/pii/S0370157303003338.

References III

[8] M. Diehl. "Kinematics and cross-section observables in exclusive rho meson electroproduction". In: *JHEP* 09 (2007), p. 064. DOI: 10.1088/1126-6708/2007/09/064. arXiv: 0704.1565 [hep-ph].

[9] S. V. Goloskokov and P. Kroll. "The Role of the quark and gluon GPDs in hard vector-meson electroproduction". In: *Eur. Phys. J. C* 53 (2008), pp. 367–384. DOI: 10.1140/epjc/s10052-007-0466-5. arXiv: 0708.3569 [hep-ph].

[10] S. V. Goloskokov and P. Kroll. "Transversity in exclusive vector-meson leptoproduction". In: *The European Physical Journal C* 74.2 (Feb. 2014), p. 2725. ISSN: 1434-6052. DOI: 10.1140/epjc/s10052-014-2725-6. URL: https://doi.org/10.1140/epjc/s10052-014-2725-6.

[11] S. V. Goloskokov and P. Kroll. "The pion pole in hard exclusive vector-meson leptoproduction". In: *The European Physical Journal A* 50.9 (Sept. 2014), p. 146. ISSN: 1434-601X. DOI: 10.1140/epja/i2014-14146-2. URL: https://doi.org/10.1140/epja/i2014-14146-2.

Backup Slides

- Maybe put a contents here for backup there is quite a lot

SDME Basis Matrices and Coefficient Vector

Full Spin Density Matrix

$$\begin{pmatrix} 1-\sqrt{1-\varepsilon^2}\,P\cos\alpha_2 & e^{-i\Phi}\left(\sqrt{\varepsilon(1+\varepsilon+2\delta)}-\sqrt{\varepsilon(1-\varepsilon)}\,P\cos\alpha_2\right) & -\varepsilon e^{-2i\Phi} \\ e^{i\Phi}\left(\sqrt{\varepsilon(1+\varepsilon+2\delta)}-\sqrt{\varepsilon(1-\varepsilon)}\,P\cos\alpha_2\right) & 2(\varepsilon+\delta) & -e^{-i\Phi}\left(\sqrt{\varepsilon(1+\varepsilon+2\delta)}+\sqrt{\varepsilon(1-\varepsilon)}\,P\cos\alpha_2\right) \\ -\varepsilon e^{2i\Phi} & -e^{i\Phi}\left(\sqrt{\varepsilon(1+\varepsilon+2\delta)}+\sqrt{\varepsilon(1-\varepsilon)}\,P\cos\alpha_2\right) & 1+\sqrt{1-\varepsilon^2}\,P\cos\alpha_2 \end{pmatrix}$$

Why can we decompose this?

- As our spin density matrix is a hermitian matrix it can always be decomposed into N^2 basis matrices (N is the dimension of the matrix)
- General complex matrices form a vector space over $\mathbb{R}$ of dimension $2N^2$
- Hermitian matrices are a subspace of this with half the d.o.f so they form a vector space of N^2

SDME Basis Matrices and Coefficient Vector

γ^* Vector of Coefficients

$$\Pi = \{\, 1,\ -\varepsilon\cos(2\Phi),\ -\varepsilon\sin(2\Phi),\ \tfrac{2m}{Q}(1-\varepsilon)P_0,\ \varepsilon+\delta,\ \sqrt{2\varepsilon(1+\varepsilon+2\delta)}\cos\Phi,\ \sqrt{2\varepsilon(1+\varepsilon+2\delta)}\sin\Phi,\ \tfrac{2m}{Q}(1-\varepsilon)(P_1\cos\Phi+P_2\sin\Phi),\ \tfrac{2m}{Q}(1-\varepsilon)(P_1\sin\Phi-P_2\cos\Phi)\,\}$$

Vector of Vector Meson SDME Coefficients

$$\Pi = \frac{1}{1+(\varepsilon+\delta)R}\{\, 1,\ -\varepsilon\cos(2\Phi),\ -\varepsilon\sin(2\Phi),\ \tfrac{2m}{Q}(1-\varepsilon)P_0,\ \varepsilon+\delta,\ \sqrt{2\varepsilon(1+\varepsilon+2\delta)}\cos\Phi,\ \sqrt{2\varepsilon(1+\varepsilon+2\delta)}\sin\Phi,\ \tfrac{2m}{Q}(1-\varepsilon)(P_1\cos\Phi+P_2\sin\Phi),\ \tfrac{2m}{Q}(1-\varepsilon)(P_1\sin\Phi-P_2\cos\Phi)\,\}$$

SDME Basis Matrices and Coefficient Vector

Basis Matrices

$$\Sigma^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \Sigma^1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \Sigma^2 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\Sigma^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \Sigma^4 = 2\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Sigma^5 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix},$$

$$\Sigma^6 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad \Sigma^7 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \Sigma^8 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

Parity Relations and the k-Basis

Proof

$$\hat{\mathcal{P}}\ {}^{(\epsilon)}T^{l}_{Lm;\lambda_1\lambda_2} = {}^{(\epsilon)}T_{L-m;-\lambda_1-\lambda_2} = \frac{1}{2}\left[T^{l}_{0-m;-\lambda_1-\lambda_2} - \epsilon(-1)^{m}T^{l}_{0m;-\lambda_1-\lambda_2}\right]$$
$$= \frac{1}{2}\left[(-1)^{m+\lambda_1-\lambda_2}T^{l}_{0m;\lambda_1\lambda_2} - \epsilon(-1)^{\lambda_1-\lambda_2}(-1)^{2m}T^{l}_{0-m;\lambda_1\lambda_2}\right]$$
$$\implies \text{under exchange m } \leftrightarrow -\text{m}$$
$${}^{(\epsilon)}T_{Lm;-\lambda_1-\lambda_2} = -\epsilon(-1)^{\lambda_1-\lambda_2}\ {}^{(\epsilon)}T_{Lm;\lambda_1\lambda_2}.$$

Casting L into k-basis

- Always have ${}^{(\epsilon)}T_{Lm;\lambda_1\lambda_2} = \pm\,{}^{(\epsilon)}T_{Lm;-\lambda_1-\lambda_2}$
- k-basis $[l]^{\epsilon}_{Lm;k}$ is valid also for the longitudinal part

Checking the Degrees of Freedom

Reflectivity Basis

- For our partial waves $[l]^{\epsilon}_{T m;k}$ and $[l]^{\epsilon}_{L m;k}$ the total d.o.f. is

$$\underbrace{2 \times 2(2l+1)}_{T} + \underbrace{2 \times \frac{2(2l+1)}{2}}_{L} = 6(2l+1),$$

where the longitudinal part is symmetric under $-m \Leftrightarrow m$: $[l]^{\epsilon}_{L m;k} = -\epsilon\,(-1)^m\,[l]^{\epsilon}_{L m;k}$.

- For $m = 0$, only an **unnatural contribution remains**.

Standard Basis

- In the standard basis $T^{l}_{\lambda m;\lambda_1 \lambda_2}$ (and considering parity):

$$\frac{3 \times 2^2(2l+1)}{2} = 6(2l+1),$$