

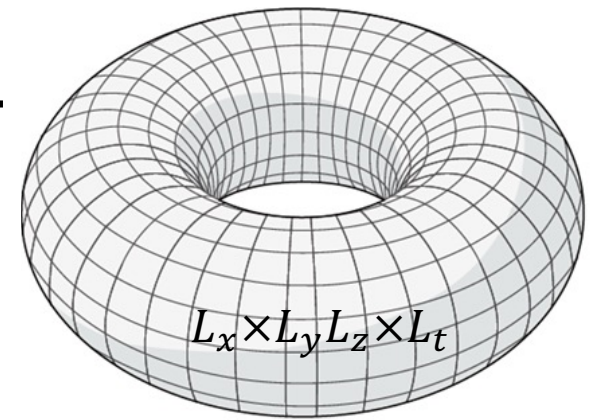
# Mellin moments from Lattice QCD: Gravitational Form Factors and Beyond

**Dimitra Anastasia Pefkou**

Vector Quarkonia and Pressure Gauges Workshop  
Jefferson Lab, 03/27/2026



# Lattice QCD: the first-principles tool



- Perturbative QCD fails at hadronic scales ( $\alpha_s \sim 1$ )
- **Lattice QCD formulates the path integral on a discretized Euclidean spacetime grid**
- Wick rotation:  $x_0 \rightarrow ix_0 \Rightarrow e^{iS} \rightarrow e^{-S}$  : statistical system in 4D
- $x_\mu \rightarrow a n_\mu, n_\mu \in \{1, 2, \dots, L_\mu\}$ ,  
 $a$  : lattice spacing, UV cutoff
- Generate configurations according to  $e^{-S^{\text{latt}}}$   
via importance sampling (e.g. Markov Chain MC)

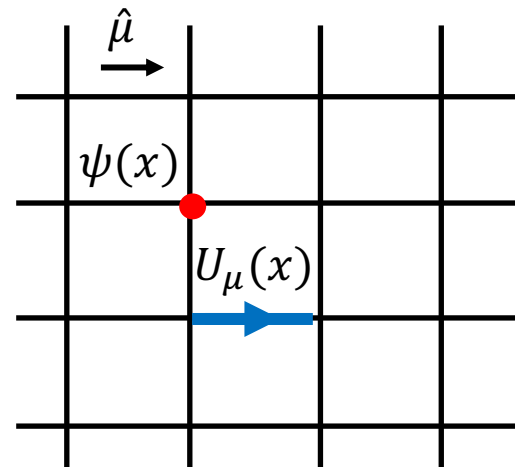


Perlmutter @ NERSC, LBL

Fields:

$$\psi(a n_\mu), \bar{\psi}(a n_\mu),$$

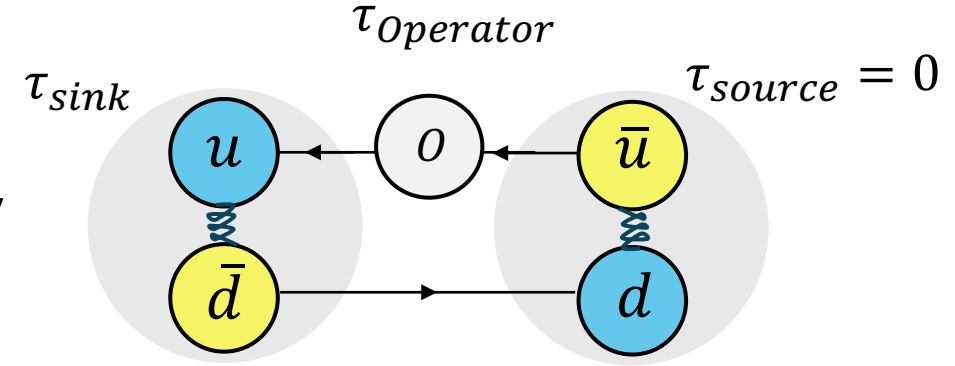
$$U_\mu(a n_\mu) = e^{-iaA_\mu(a(n_\mu + \frac{\hat{\mu}}{2}))}$$



JUWELS @ Jülich Supercomputing Center 2

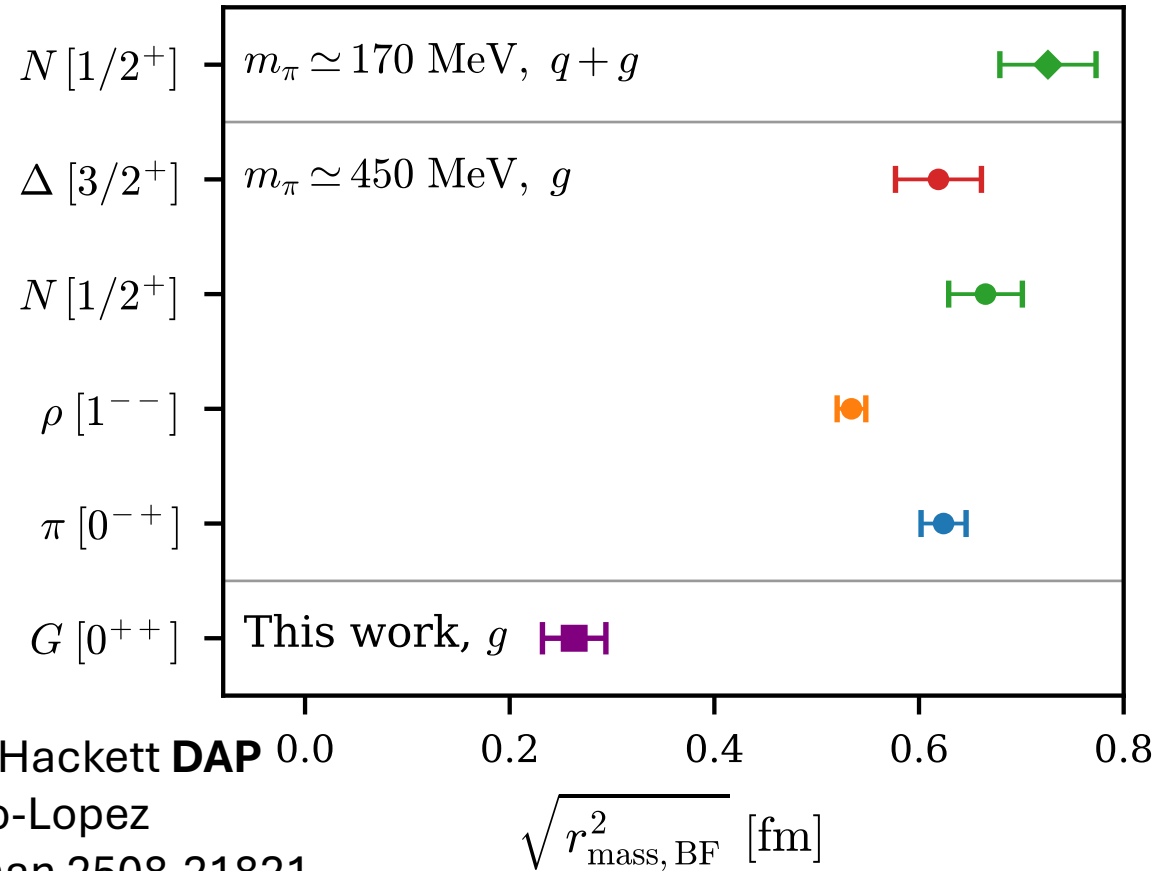
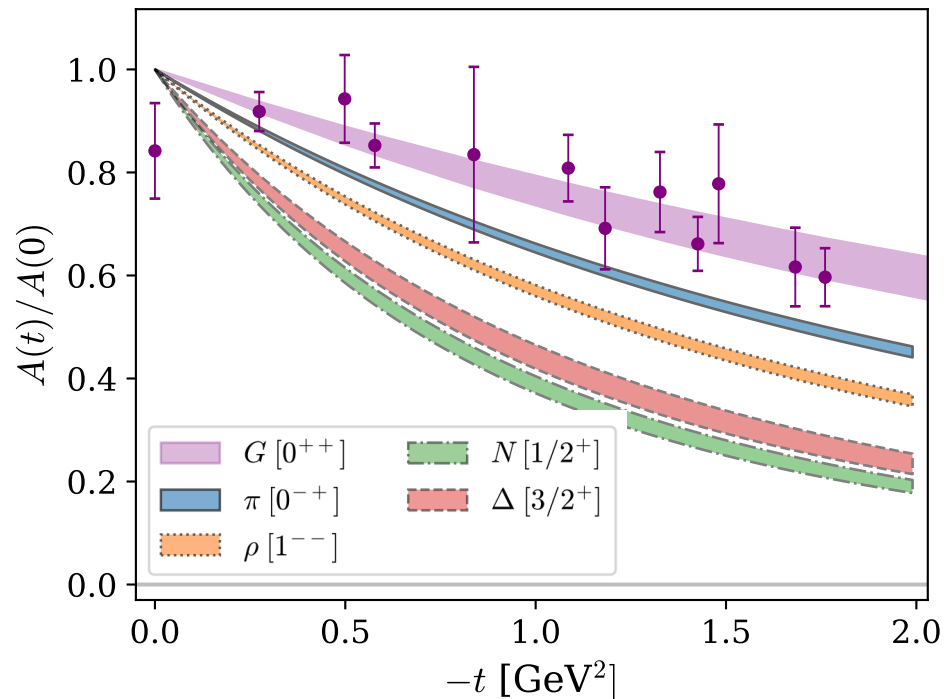
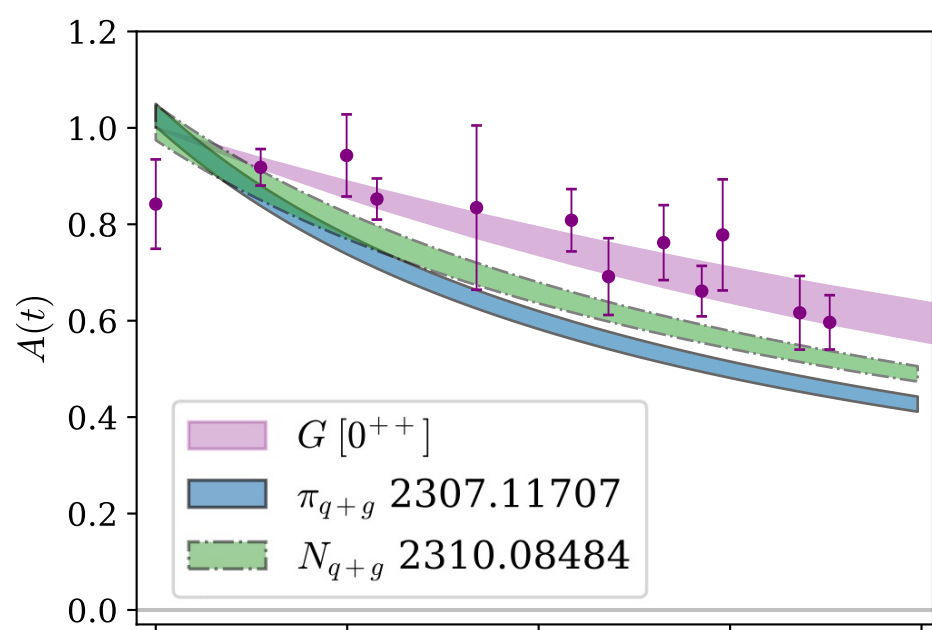
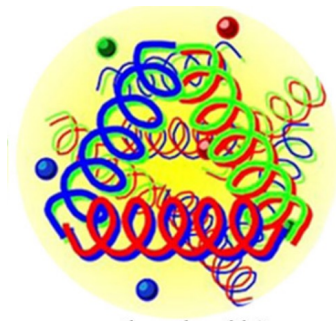
# Hadron structure from lattice QCD: 3-point correlation functions

- $\bar{h}(x)$  : operator chosen to create state that strongly overlaps with hadron ground state



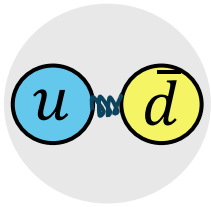
- $C^{3pt}(\mathbf{p}', \tau_s, \mathbf{\Delta}, \tau_o) = \sum_{\mathbf{x}, \mathbf{y}} e^{-i(\mathbf{p}' \cdot \mathbf{x} - \mathbf{\Delta} \cdot \mathbf{y})} \langle h(\mathbf{x}) O(\mathbf{y}) \bar{h}(0) \rangle$   
 $\xrightarrow[\text{use } A(t) = e^{Ht} A(0) e^{-Ht}]{\text{insert full set of states}} \sum_{n, m} Z_n^{\mathbf{p}'} Z_m^{\mathbf{p}} e^{-E_n^{\mathbf{p}'} \tau_o - E_m^{\mathbf{p}} (\tau_s - \tau_o)} \langle m(\mathbf{p}') | O(0) | n(\mathbf{p}) \rangle \quad \mathbf{p} = \mathbf{p}' - \mathbf{\Delta}$
- General idea: fit Euclidean time dependence to extract ground state ME  
Reality: much more complicated...
- Full systematic control: repeat at multiple lattice spacings  $a$ , volumes  $V$ , and pion masses  $m_\pi$  (physical point possible but often too expensive)

# GFFs in lattice Yang Mills theory: Hints of smaller size for scalar glueball



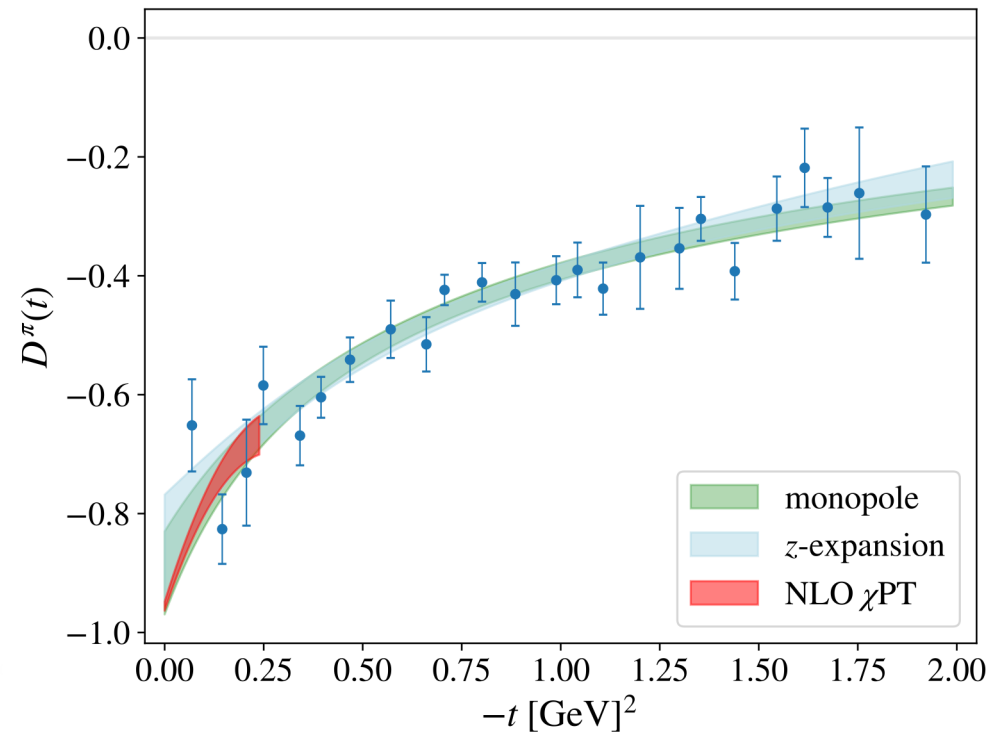
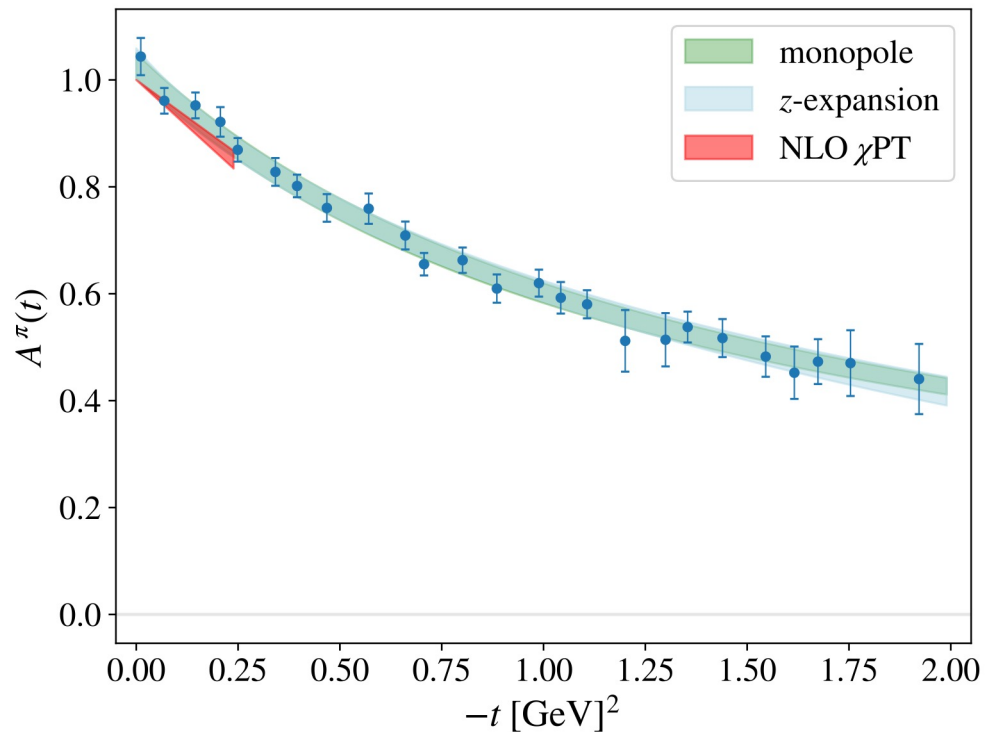
Abbott Hackett **DAP**  
 Romero-Lopez  
 Shanahan 2508.21821

# Pion GFFs from lattice QCD



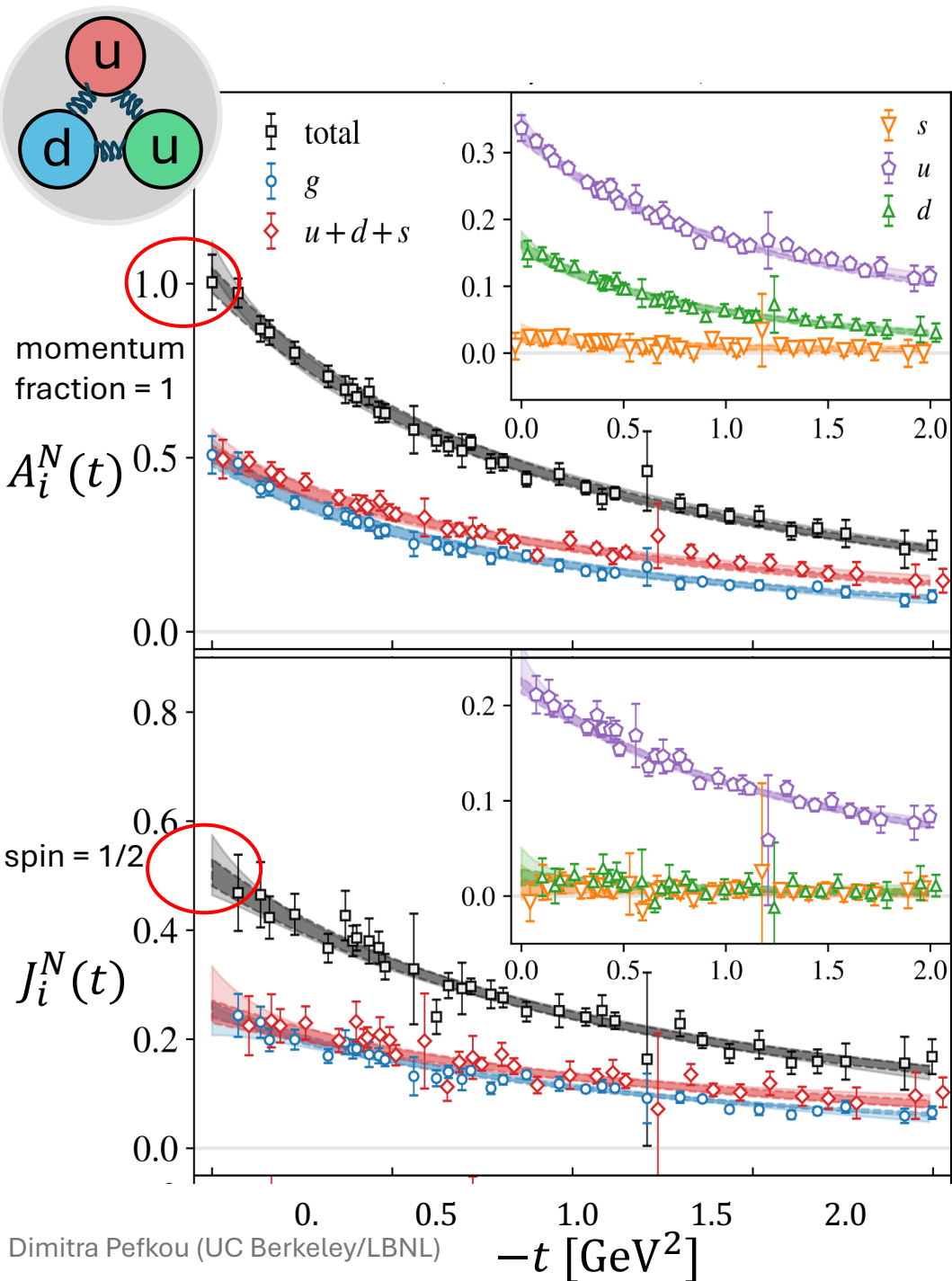
Hackett Oare **DAP**  
Shanahan 2307.11707

- Pros: almost physical pion mass
- Cons: single ensemble calculation, uncontrolled systematics



Total quark + gluon GFFs

- Momentum sum satisfied
- D-term consistent with chPT

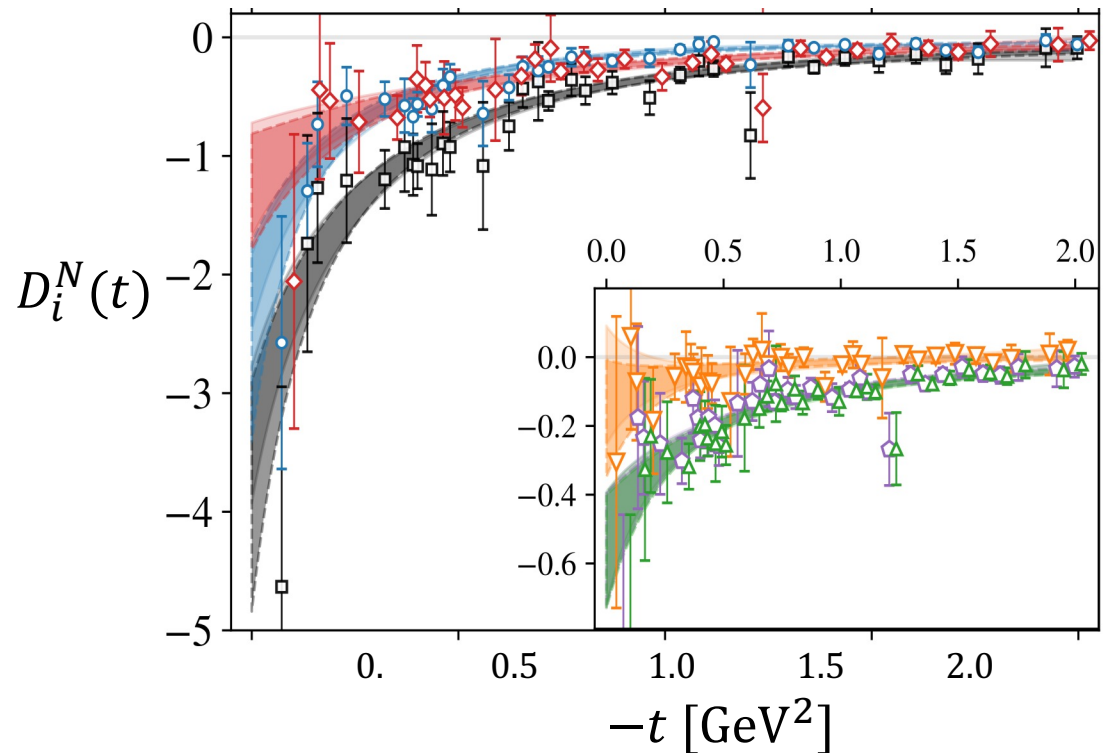


# Proton GFFs: full flavor decomposition from lattice QCD

$\overline{\text{MS}}, \mu = 2 \text{ GeV}$

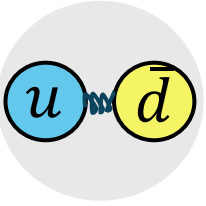
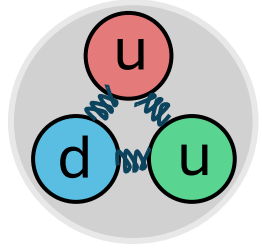
- Momentum and spin sum rules satisfied
- First determination of total D-term, but large uncertainties
- Single-ensemble calculation, systematics not fully controlled

Hackett **DAP**  
Shanahan  
2310.08484

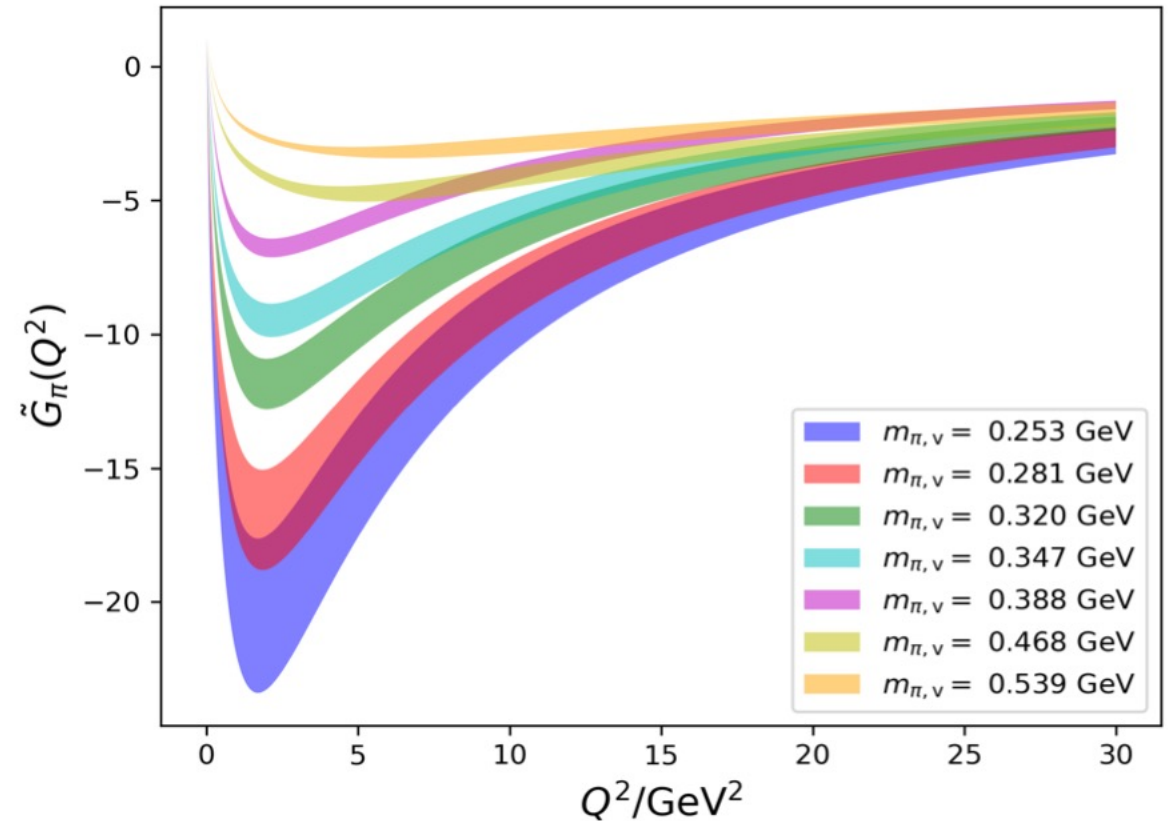
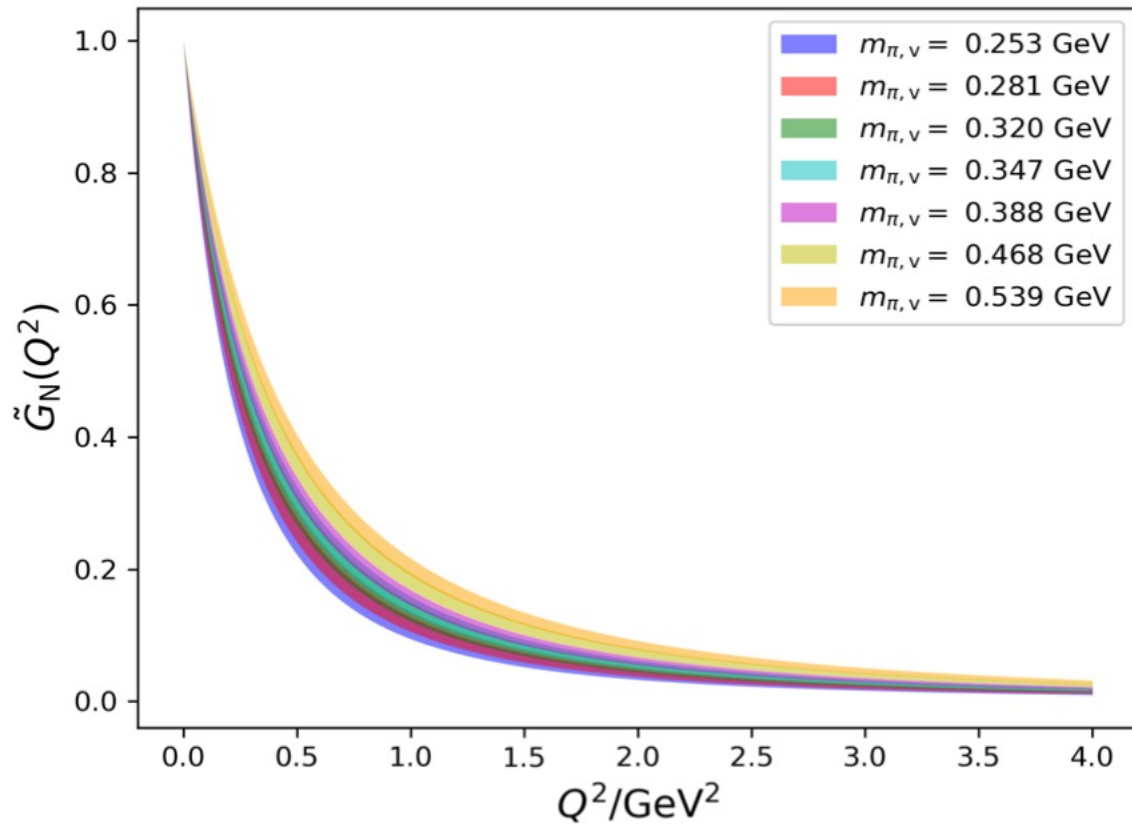


# Trace anomaly form factors from lattice QCD




Wang et al ( $\chi$ QCD) 2401.05496



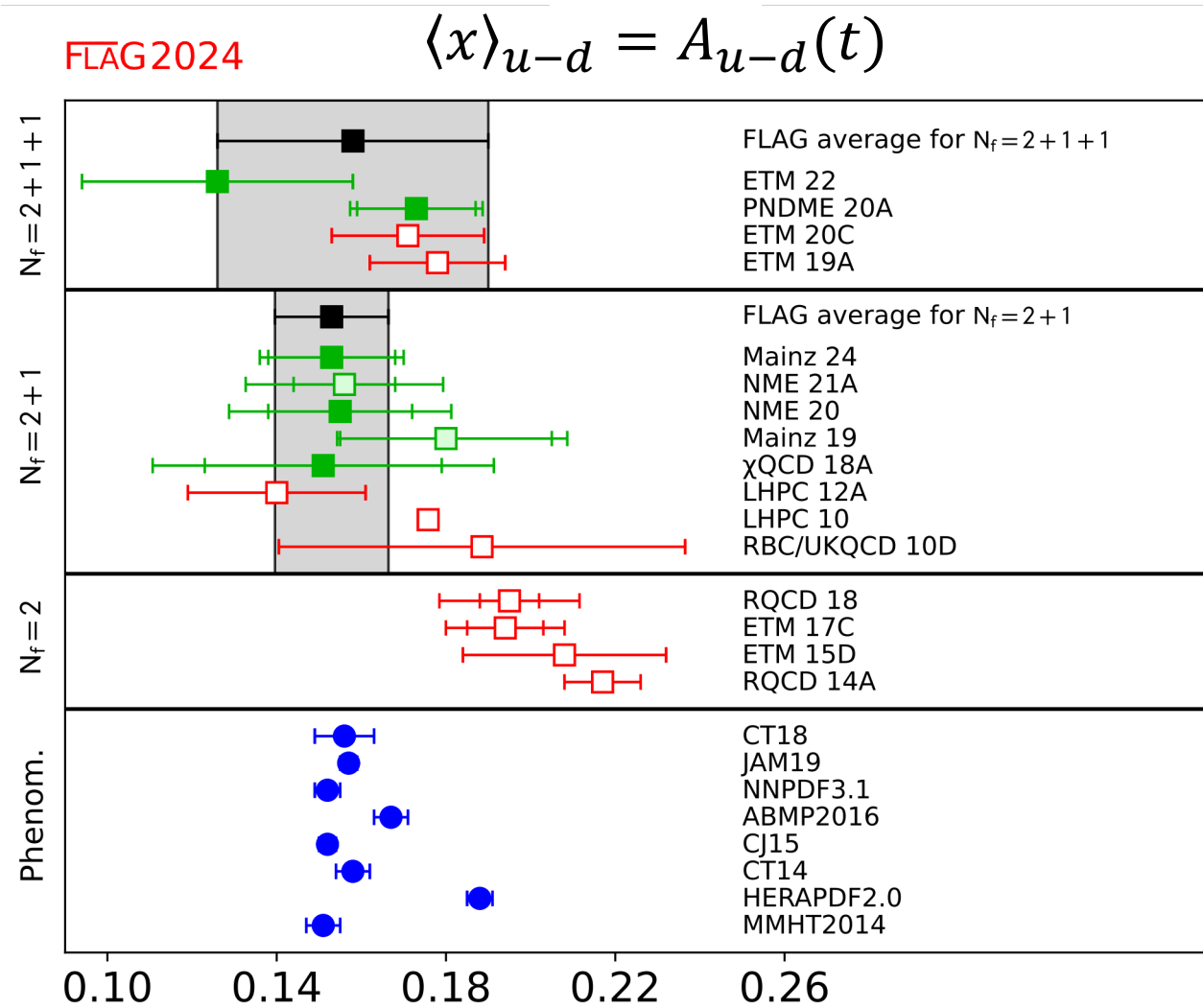
$$\tilde{G}_N = (A - B) \frac{Q^2}{4m^2} + 3D \frac{Q^2}{m^2} - F_\sigma$$



# Flavor Lattice Averaging Group (FLAG)

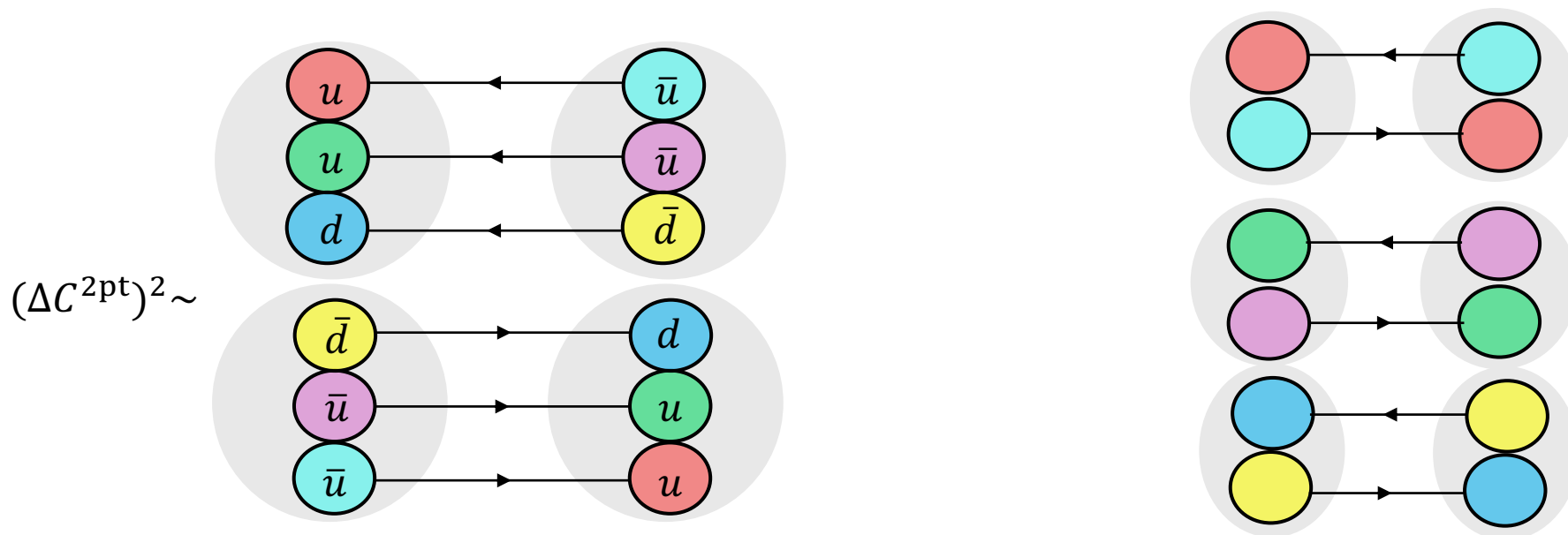
- Like PDG for lattice QCD results <http://flag.itp.unibe.ch/2024/>
- “We review lattice results related to pion, kaon, D-meson, B-meson, and nucleon physics with the aim of making them easily accessible to the nuclear and particle physics communities. [...] We consider nucleon matrix elements, and review the determinations of the axial, scalar and tensor bilinears, both isovector and flavor diagonal.”
- Rating of systematic effects with     
chiral extrapolation, continuum extrapolation, finite-volume effects, ...

# Only GFF info currently in FLAG



# Challenge: signal-to-noise (STN)

- STN of the nucleon  $C^{2pt}/\Delta C^{2pt} \sim e^{-(E_p - 3m_\pi/2)t_s}$  [Parisi Lepage]



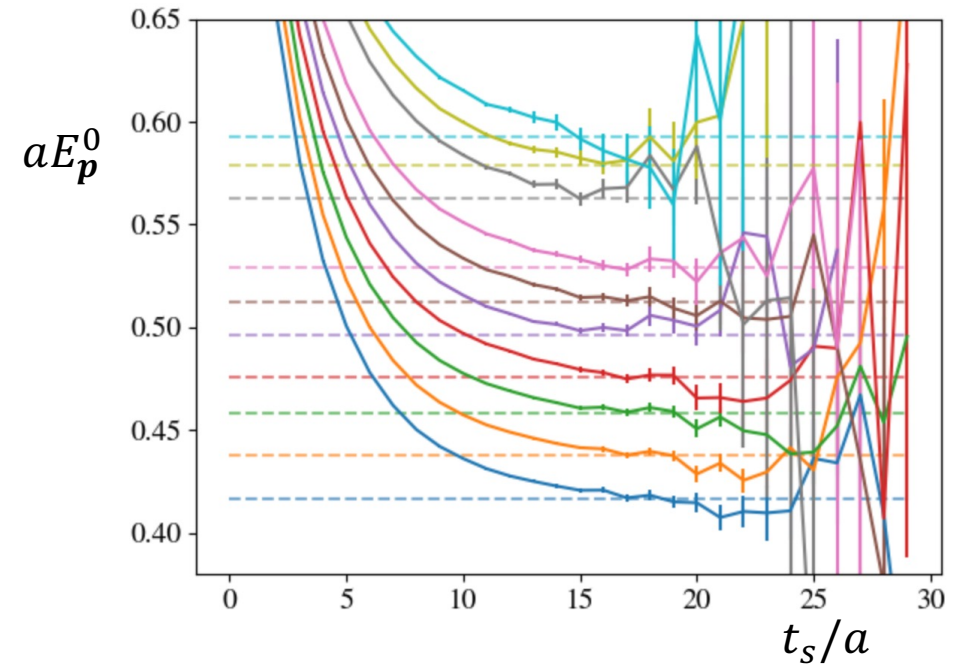
- Disconnected quark loops contain intractable all-to-all propagator  
 → hierarchical probing + low-mode deflation [Orginos Stathopoulos Gambir]  
 effective improvements to stochastic Hutchinson trace estimator
- Gluon contributions noisy due to strong UV fluctuations  
 → use smeared operator definition (ok in continuum limit)



# Challenge: excited state contamination

Small  $t_s$ : better STN, though contamination from higher states dominates  
 Large  $t_s$ : signal might degrade before true ground state is reached

- $C^{2\text{pt}}(\mathbf{p}, t_f) = |Z_{\mathbf{p}}^0|^2 e^{-E_{\mathbf{p}}^0 t_s} + |Z_{\mathbf{p}}^1|^2 e^{-E_{\mathbf{p}}^1 t_s} + \dots$
- $C^{3\text{pt}}(\mathbf{p}', t_f; \Delta, \tau) = Z_{\mathbf{p}}^{0*} Z_{\mathbf{p}'}^0 e^{-E_{\mathbf{p}'}^0 (t_s - \tau)} e^{-E_{\mathbf{p}}^0 t_s} \langle 0|O|0 \rangle$   
 $+ Z_{\mathbf{p}}^{0*} Z_{\mathbf{p}'}^1 e^{-E_{\mathbf{p}'}^1 (t_s - \tau)} e^{-E_{\mathbf{p}}^0 t_s} \langle 1|O|0 \rangle$   
 $+ Z_{\mathbf{p}}^{1*} Z_{\mathbf{p}'}^0 e^{-E_{\mathbf{p}'}^0 (t_s - \tau)} e^{-E_{\mathbf{p}}^1 t_s} \langle 0|O|1 \rangle$   
 $+ Z_{\mathbf{p}}^{1*} Z_{\mathbf{p}'}^1 e^{-E_{\mathbf{p}'}^1 (t_s - \tau)} e^{-E_{\mathbf{p}}^1 t_s} \langle 1|O|1 \rangle$   
 $+ \dots$



- Ground state matrix element  $\langle 0|O|0 \rangle$  needed to obtain form factors can be difficult to disentangle

# Lattice QCD $\rightarrow$ Physical world:

$m_\pi, m_K, \dots \rightarrow$  physical

- At heavier masses:
  - 1) quark propagators cheaper (Dirac matrix easier to invert)
  - 2) less excited state contamination (e.g  $N\pi$  heavier, further away from  $N$ )
  - 3) extrapolation to physical masses increases systematic uncertainty
- Thanks to algorithmic developments, more and more lattice QCD results directly at physical

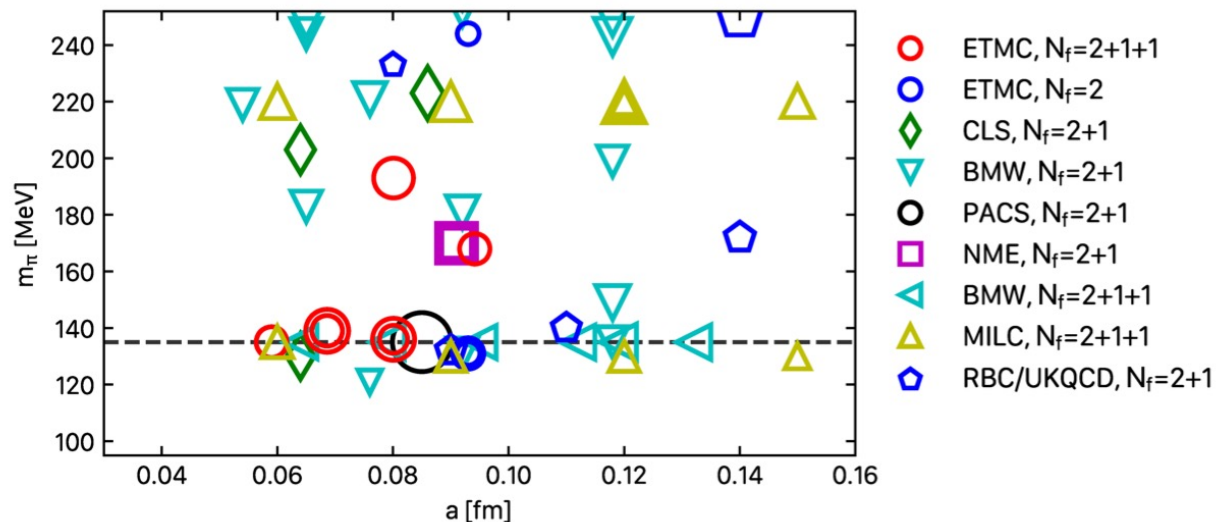


Figure from Alexandrou  
J.Phys.Conf.Ser. 2023

# Lattice QCD → Physical world:

## Challenge: Renormalization

$$H(4) \text{ irreps } \mathcal{R} \in \{\tau_1^{(3)}, \tau_3^{(6)}\}$$

- $\begin{pmatrix} T_q^{\overline{\text{MS}}} \\ T_g^{\overline{\text{MS}}} \end{pmatrix} = \begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\text{MS}}} & Z_{qg\mathcal{R}}^{\overline{\text{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\text{MS}}} & Z_{gg\mathcal{R}}^{\overline{\text{MS}}} \end{pmatrix} \begin{pmatrix} T_{q\mathcal{R}}^{\text{bare}} \\ T_{g\mathcal{R}}^{\text{bare}} \end{pmatrix}$  : quark isosinglet and gluon mix under renormalization  $u + d + s$

- $T_v^{\overline{\text{MS}}} = Z_{v\mathcal{R}}^{\overline{\text{MS}}} T_{v\mathcal{R}}^{\text{bare}}$  : non-singlet do not mix in the chiral limit  $u - d, u + d - 2s$

- Compute non-perturbatively via the RI-MOM scheme, convert to  $\overline{\text{MS}}$  scheme at  $\mu = 2 \text{ GeV}$

$$\begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\text{MS}}} & Z_{qg\mathcal{R}}^{\overline{\text{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\text{MS}}} & Z_{gg\mathcal{R}}^{\overline{\text{MS}}} \end{pmatrix}^{-1} (\mu^2) = \begin{pmatrix} R_{qq\mathcal{R}}^{\text{RI}} & R_{qg\mathcal{R}}^{\text{RI}} \\ R_{gq\mathcal{R}}^{\text{RI}} & R_{gg\mathcal{R}}^{\text{RI}} \end{pmatrix} (\mu_R^2) \\ \times \begin{pmatrix} C_{qq}^{\text{RI}/\overline{\text{MS}}} & C_{qg}^{\text{RI}/\overline{\text{MS}}} \\ C_{gq}^{\text{RI}/\overline{\text{MS}}} & C_{gg}^{\text{RI}/\overline{\text{MS}}} \end{pmatrix} (\mu^2, \mu_R^2)$$

# Lattice QCD → Physical world:

## Challenge: Renormalization

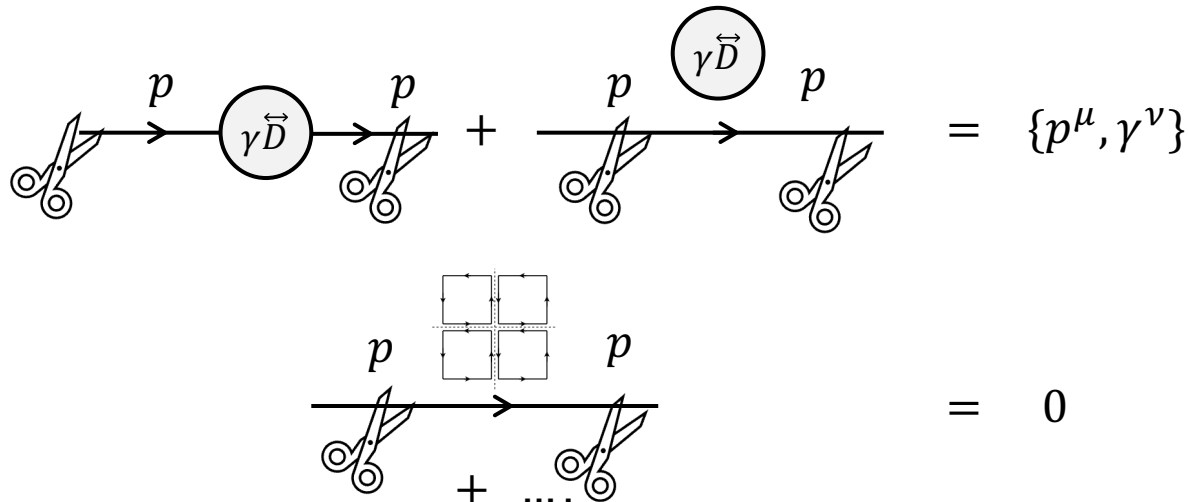
$$\begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\text{MS}}} & Z_{qg\mathcal{R}}^{\overline{\text{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\text{MS}}} & Z_{gg\mathcal{R}}^{\overline{\text{MS}}} \end{pmatrix}^{-1} (\mu^2) = \begin{pmatrix} R_{qq\mathcal{R}}^{\text{RI}} & R_{qg\mathcal{R}}^{\text{RI}} \\ R_{gq\mathcal{R}}^{\text{RI}} & R_{gg\mathcal{R}}^{\text{RI}} \end{pmatrix} (\mu_R^2) \times \begin{pmatrix} C_{qq}^{\text{RI}/\overline{\text{MS}}} & C_{qg}^{\text{RI}/\overline{\text{MS}}} \\ C_{gq}^{\text{RI}/\overline{\text{MS}}} & C_{gg}^{\text{RI}/\overline{\text{MS}}} \end{pmatrix} (\mu^2, \mu_R^2)$$

← RI-MOM  
← matching factors

### RI-MOM scheme:

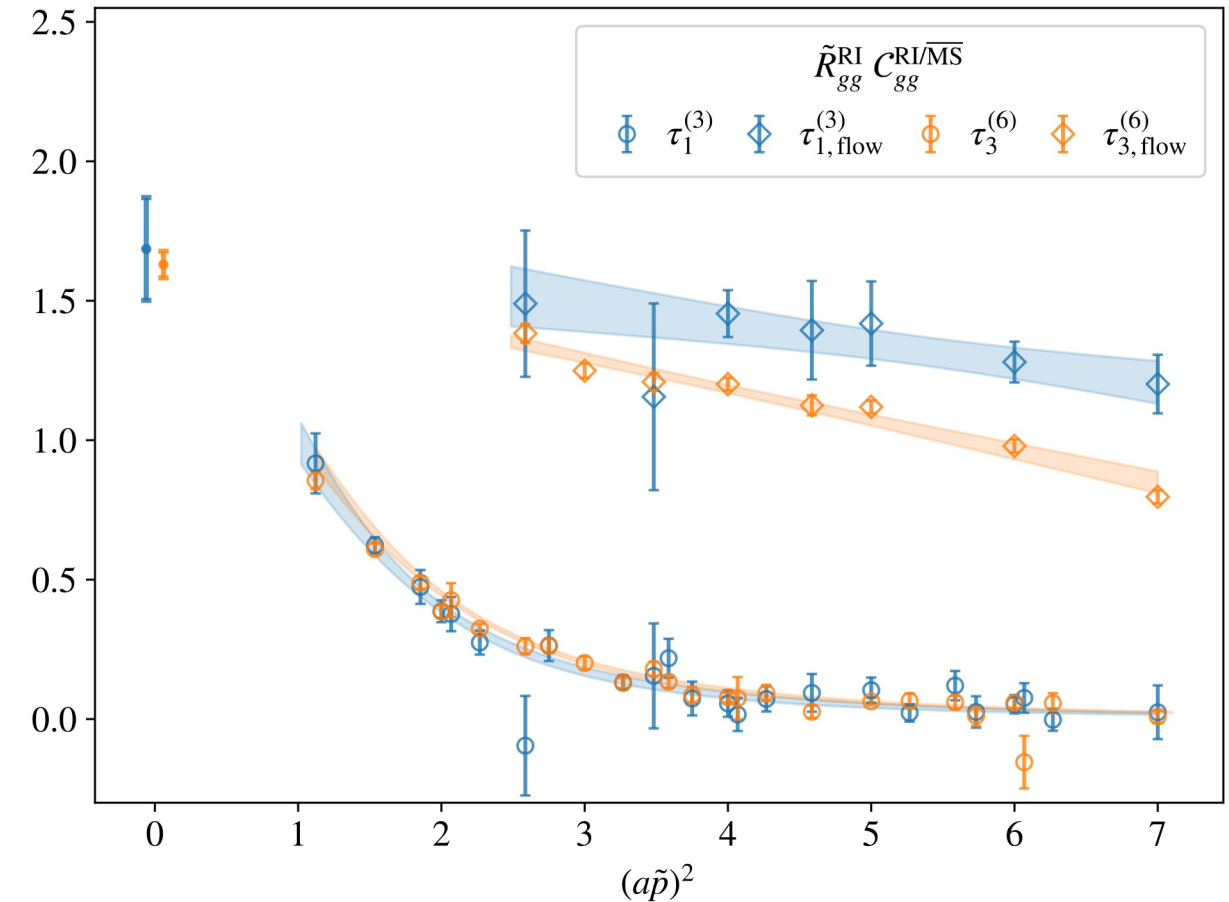
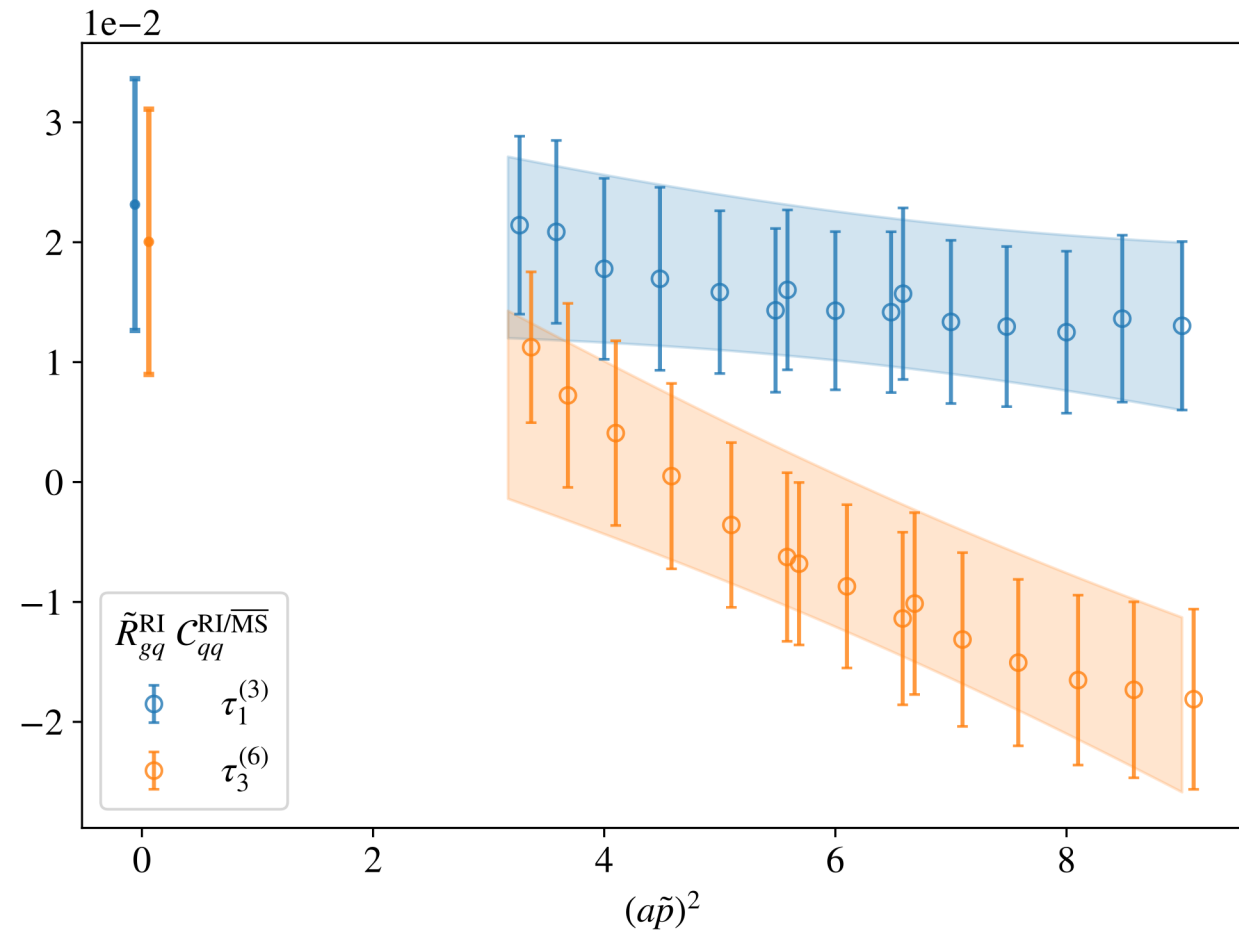
set amputated 3-point functions to their tree-level value

### Renormalization conditions:



- each  $R_{ij}^{\text{RI}} C_{jk}^{\text{RI}/\overline{\text{MS}}}$  has residual dependence on  $(a\tilde{p})^2$  due to lattice artifacts, non-perturbative effects, etc.
- model and fit this dependence to extract the renormalization coefficients
- For regular volume ensembles, gluon and disconnected have intractable noise → Use smaller volume ensemble to get renormalization factors

# Extraction of GFF renormalization coefficients: Challenging $(a\tilde{p})$ dependence

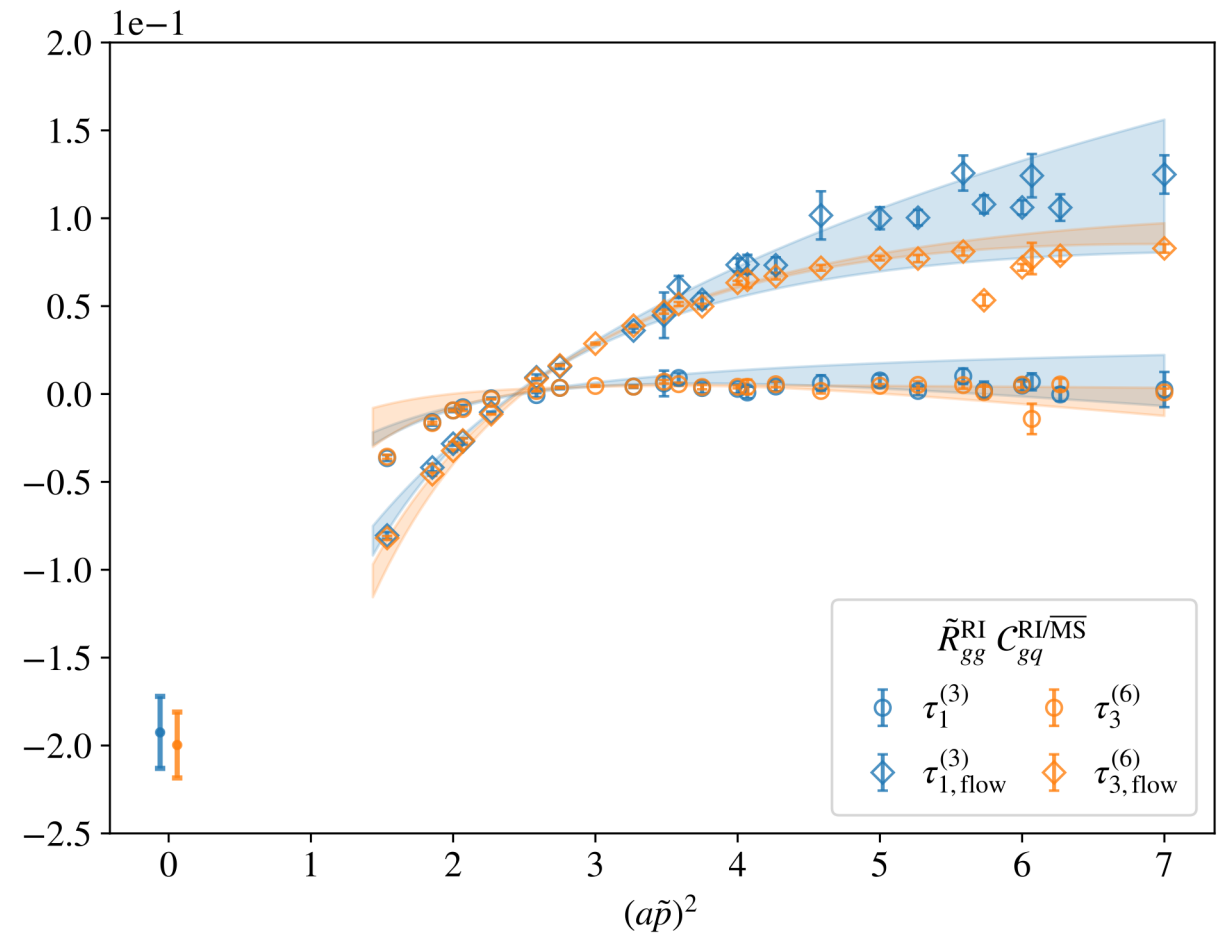
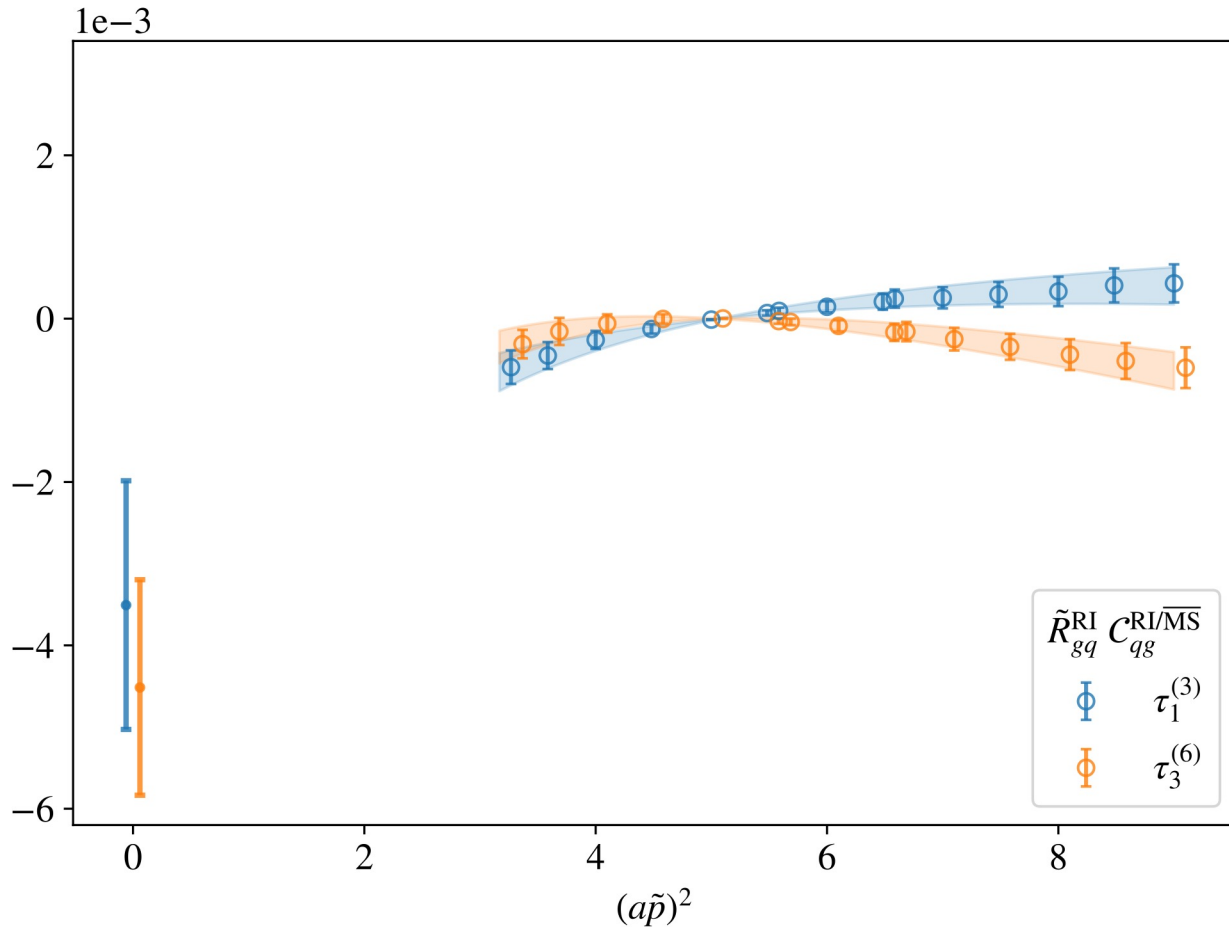


Fit  $(a\tilde{p})$  dependence due to discretization artifacts, non-perturbative effects, etc.

(inverse) polynomial

# Extraction of GFF renormalization coefficients:

## Challenging $(a\tilde{p})$ dependence

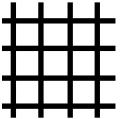
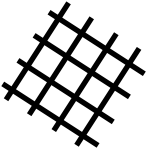


Fit  $(a\tilde{p})$  dependence due to discretization artifacts, non-perturbative effects, etc.

logarithmic

# Challenge: $a \rightarrow 0$ limit

- Ok for lowest moment (GFFs,  $\langle x \rangle$ , ...)

- Lattice:   $\neq$    $O(4) \rightarrow H(4)$

- Higher moments mix with lower-dimensional ones that transform under the same irreps of the hypercubic group
- This mixing comes proportional to  $1/a^k \rightarrow$  power-divergent as  $a \rightarrow 0$
- Same problem with higher twist, e.g. twist-4

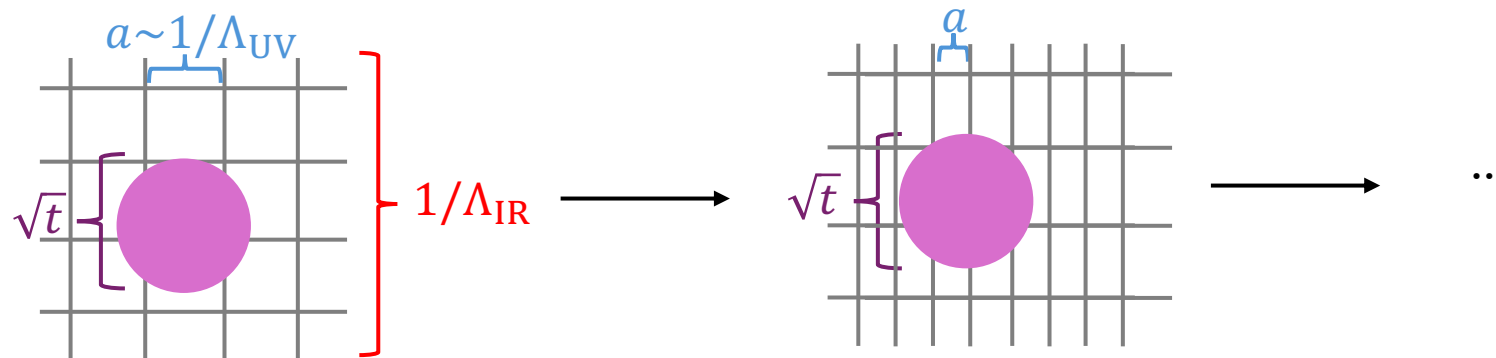
# Gradient flow: smearing the UV

Lüscher 1006.4518

Lüscher 1302.5246

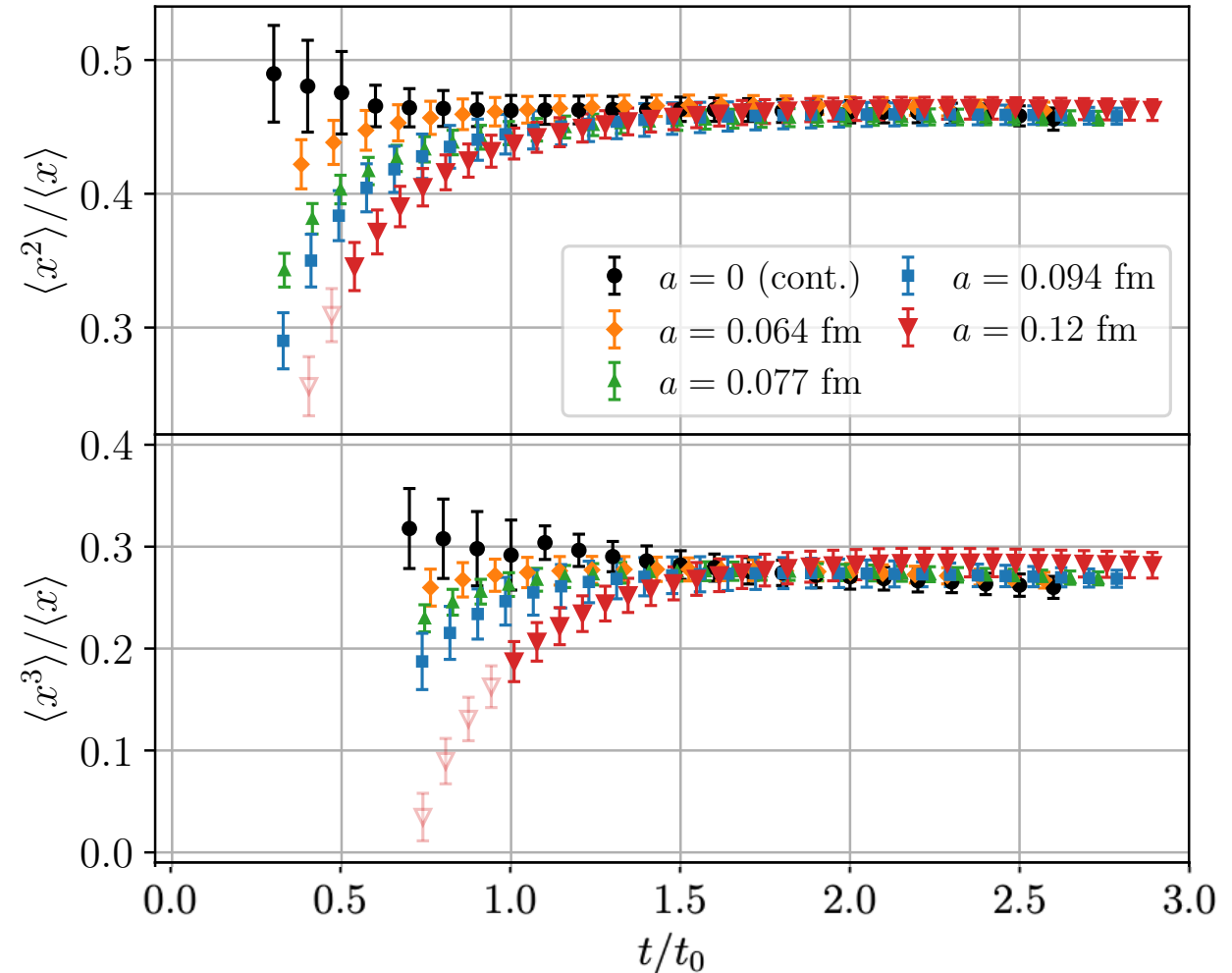
Lüscher Weisz 1101.0963

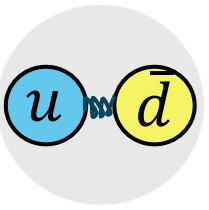
- **Gradient flow** (analogous to gauge sector Yang Mills flow) introduces a fictitious flow time  $t$ , diffusing gauge and fermion fields
- Physically, it acts as a continuous smearing radius  $\sqrt{8t}$ , exponentially suppressing UV fluctuations
- **Operators constructed from flowed fields require no renormalization for gauge, and only multiplicative for fermions**  
→ **bypasses power divergent mixing**
- Use gradient flow to obtain twist-2 Mellin moments of any order:  
Shindler 2311.18704



# Analyzing matrix elements of flowed operators

- We match the flowed matrix elements to the  $\overline{\text{MS}}$  scheme using perturbation theory (short flowtime expansion)
- We look for “window” where  $t$  is large enough to remove lattice artifacts, but small enough such that PT is valid
- Take ratios to cancel fermion multiplicative renormalization (for now)

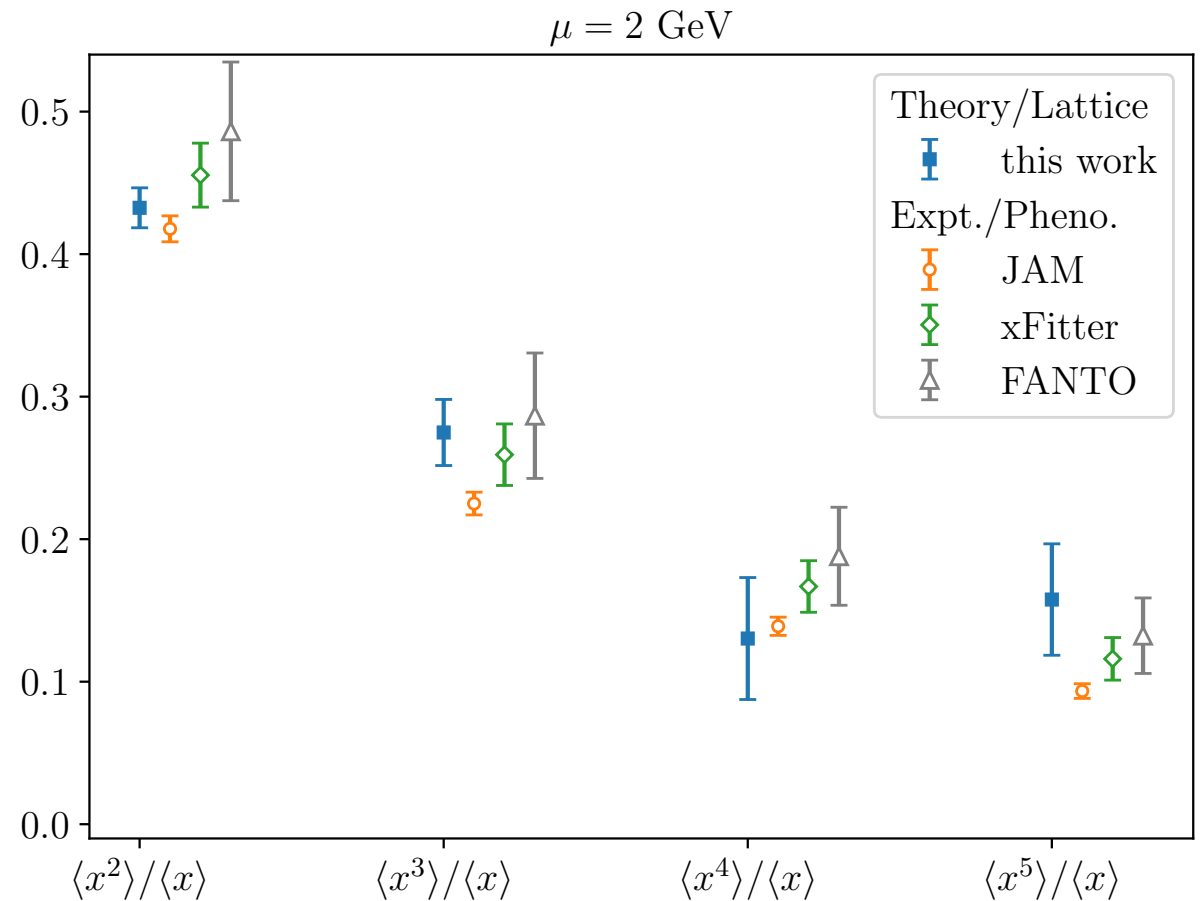


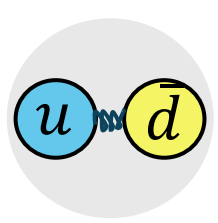


# First results: pion PDF moments up to $\langle x^5 \rangle$

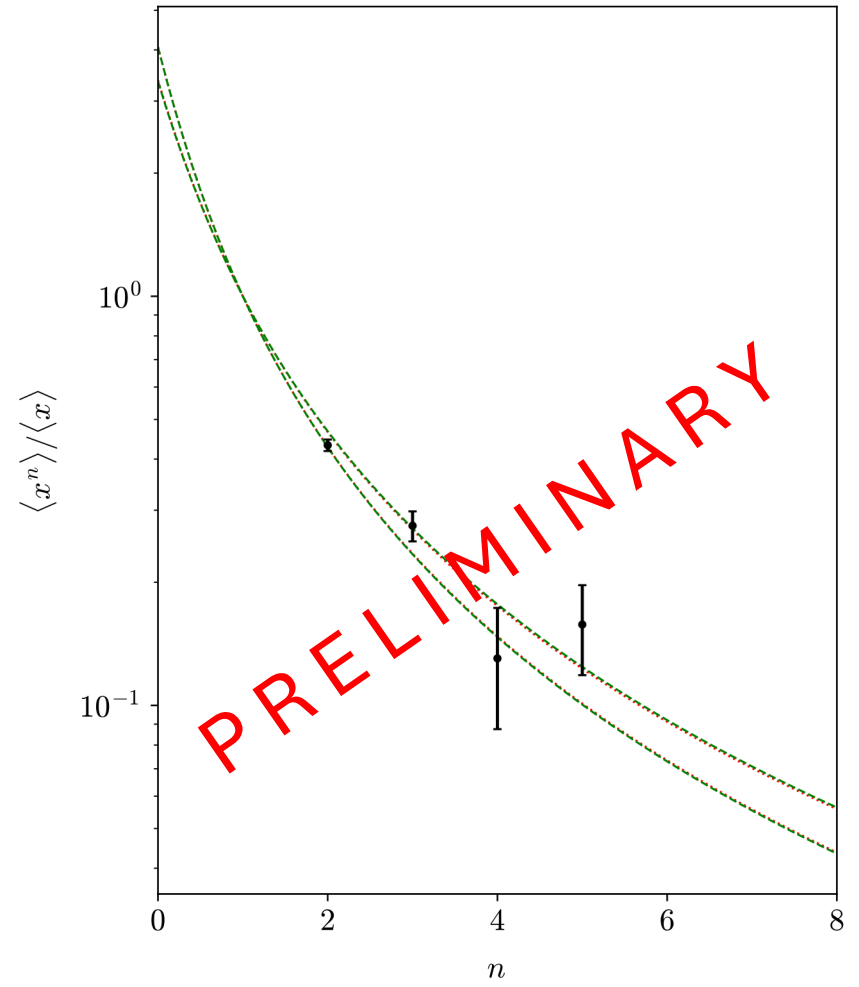
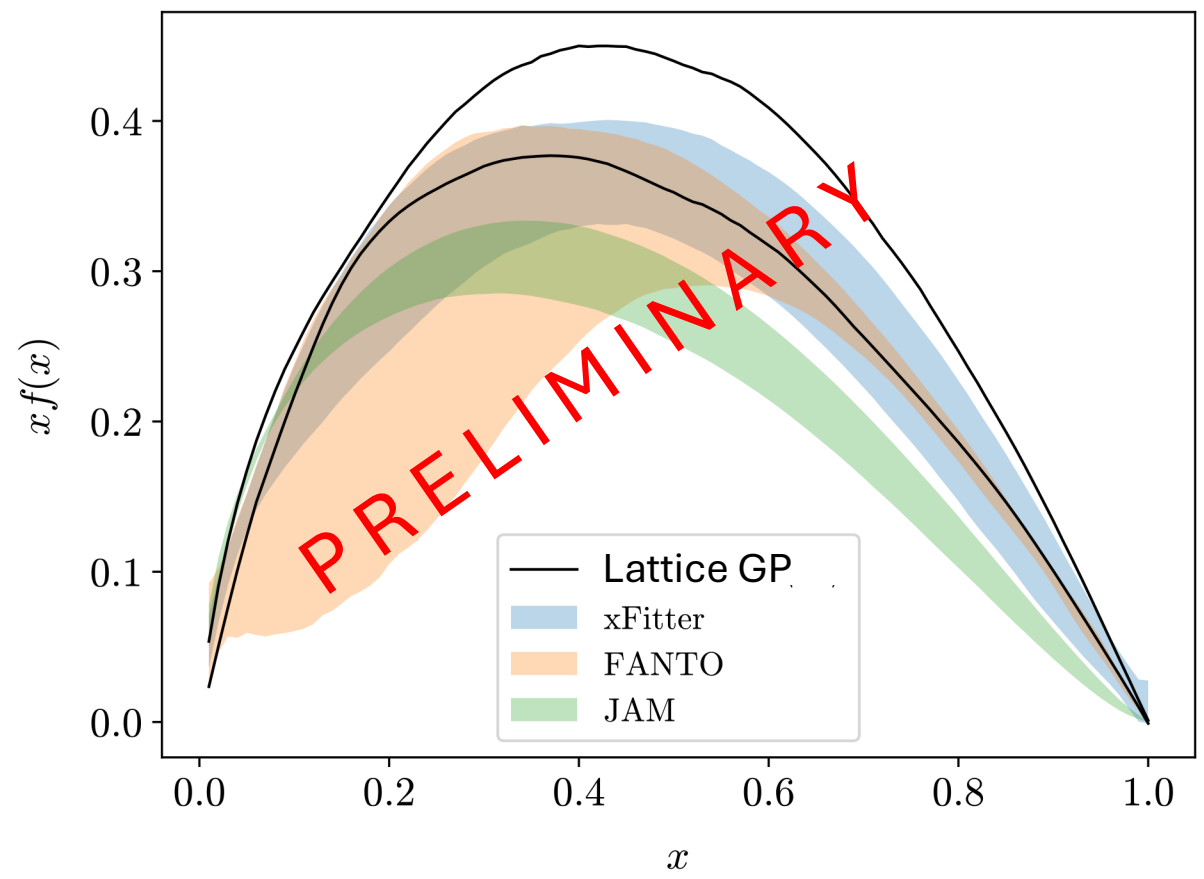
Francis, ..., **DAP** et al 2509.02427 } Just accepted  
Francis, ..., **DAP** et al 2510.26738 } to PRL + PRD

- Heavy (400 MeV) pions, but all other systematics under control
- These high-precision moments can be directly used e.g. in phenomenological global fits
- To do: physical point, singlet quark + gluon PDFs, **proton**



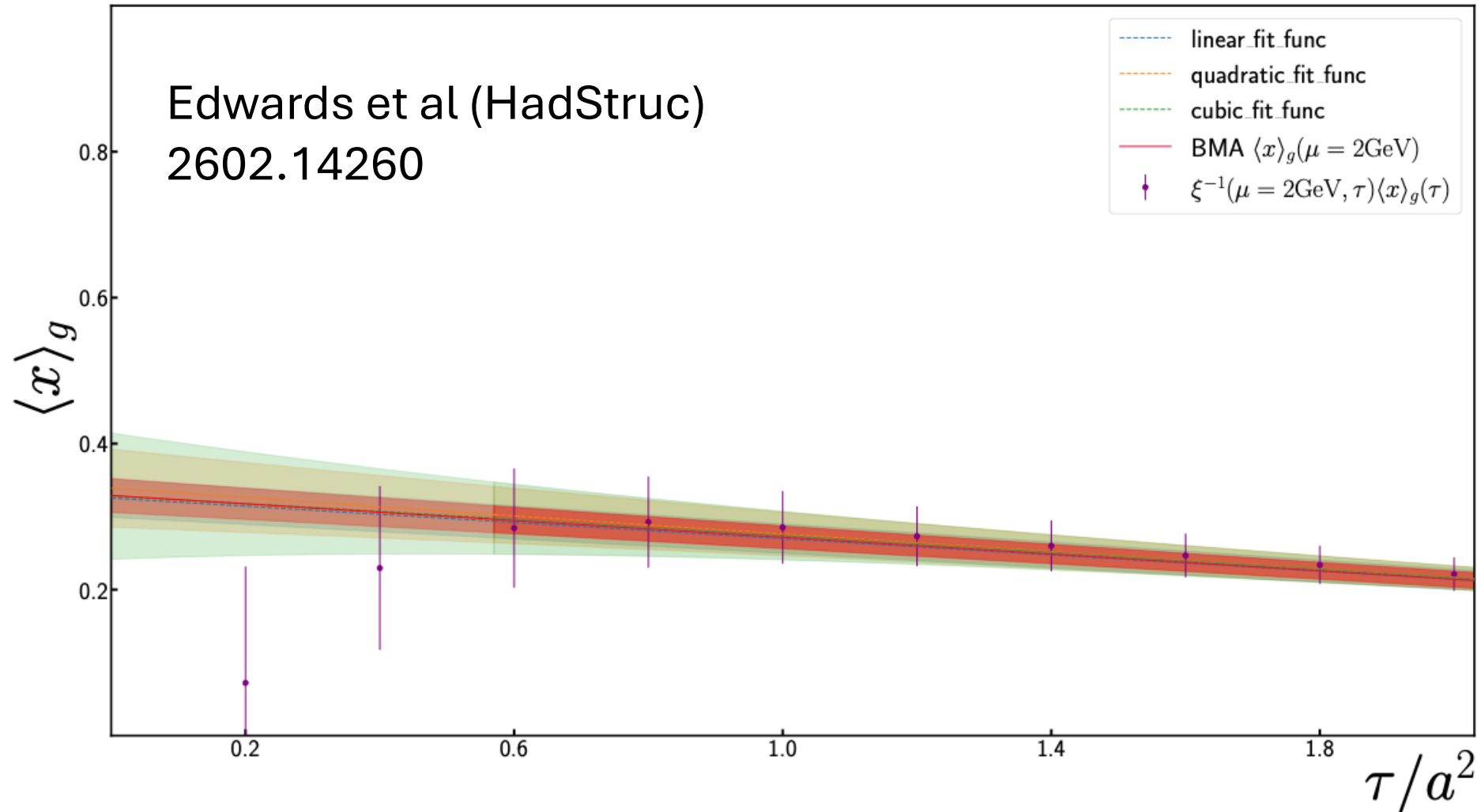


# Model-independent reconstruction of PDF from its moments using Gaussian Processes



ongoing by UC Berkeley PhD student Rohith Karur

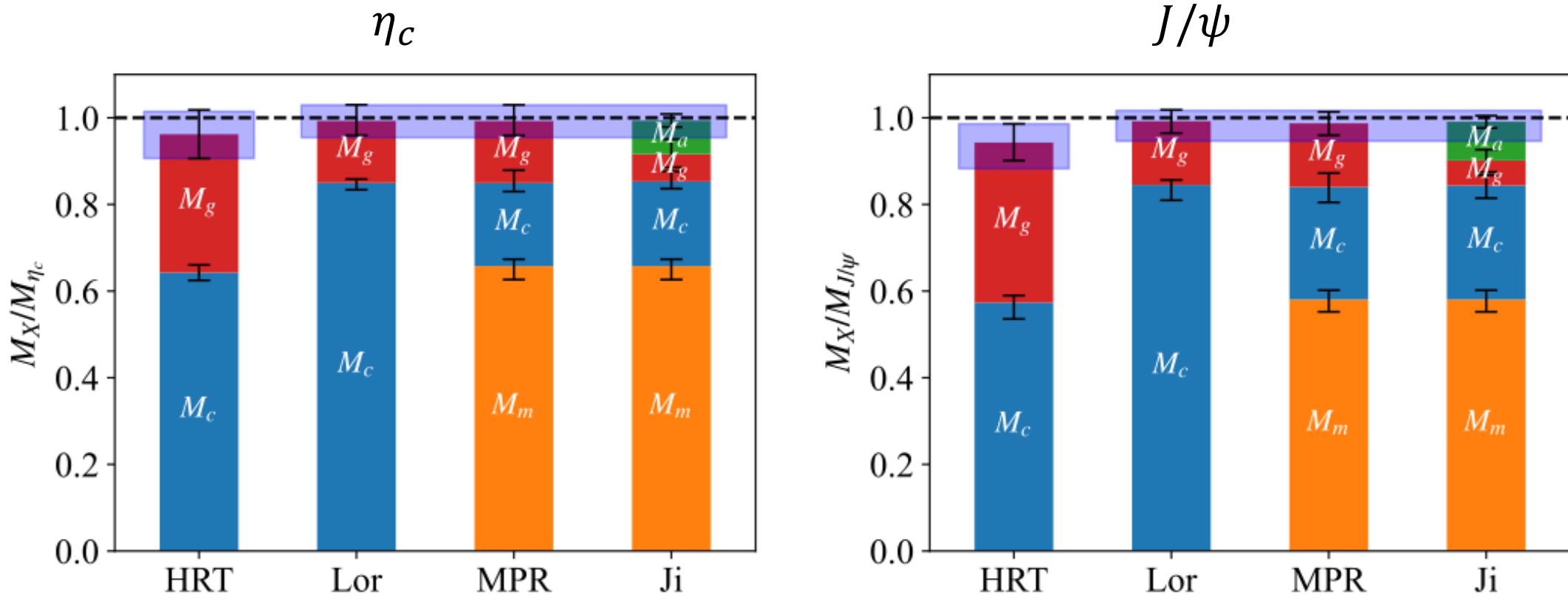
# Gradient flow as improved renormalization scheme for EMT



- Single-ensemble calculation
- $m_\pi \approx 358$  MeV
- No mixing with quark

# Gradient flow as improved renormalization scheme for EMT

Mass decomposition



Bollweg Ding Gao Luo Mukherjee 2601.13070

# Summary and outlook

- Lattice QCD provides direct access to matrix elements of local operators
- Mellin moment program crucial and complimentary to GPD/PDF calculations
- $\langle x \rangle$  calculations are maturing, but still no flavor decomposition with full systematic control
- First glimpses into gravitational form factors (single-ensemble)
- New tools (i.e., gradient flow) and a lot more work required to get better control, as well as to access higher moments and higher twist structure

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**THANK YOU**