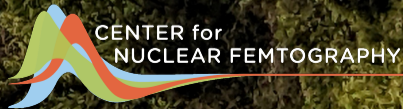


The background of the slide features a close-up of a tree trunk covered in vibrant green moss. Several light pink cherry blossoms are in various stages of bloom, some fully open and others as buds. The background is softly blurred, showing more of the blossoms and the sky.

*Vector Quarkonia as  
Pressure Gauges*

Adam Freese  
Center for Nuclear Femtography  
March 26-27, 2026



# Proton pressure

☛ Pressure & stress in the proton is a hot topic

✦ Nature papers from JLab!

✦ Burkert, Elouadrhiri & Girod, *Nature* (2018)

✦ Duran & al., *Nature* (2023)

✦ Popular science articles

✦ Science Daily (2018)

✦ Quanta magazine (2024)

✦ Dedicated research center

✦ **Center for Nuclear Fentography (CNF)**

✦ Controversies about interpretation

☛ **Vector Quarkonia as Pressure Gauges**

is a CNF + Theory Center mini-workshop dedicated to empirical extraction of proton pressure.

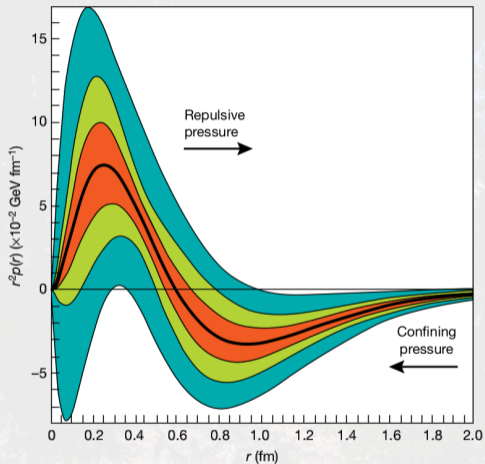


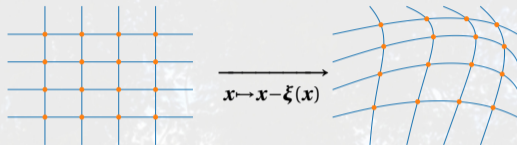
Figure: Burkert, Elouadrhiri & Girod, *Nature* (2018)

# The stress tensor

✦ The **stress tensor** is a  $3 \times 3$  sub-matrix of the energy-momentum tensor.

$$\begin{bmatrix} T^{00}(x) & T^{01}(x) & T^{02}(x) & T^{03}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix}$$

↑  
**Stress tensor**



✦ Work needed to deform a system:

$$W[\mathbf{x} \mapsto \mathbf{x} - \boldsymbol{\xi}(\mathbf{x})] = - \int d^3x \overbrace{T_{\text{QCD}}^{ij}(x)}^{\text{stress tensor}} \left( \overbrace{\frac{1}{2}(\partial_i \xi^j(x) + \partial_j \xi^i(x))}^{\text{strain}} + \overbrace{\frac{1}{2}(\partial_i \xi^j(x) - \partial_j \xi^i(x))}^{\text{torsion}} \right)$$

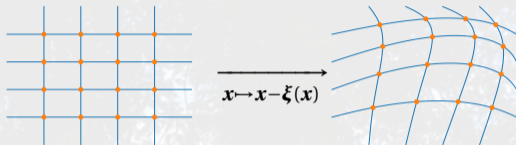
✦ **Stress:** internal forces resisting spatial deformation

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↑  
**Stress tensor**



Pressures (normal stresses)

- Work needed to deform a system:

$$W[\mathbf{x} \mapsto \mathbf{x} - \boldsymbol{\xi}(\mathbf{x})] = - \int d^3x \overbrace{T_{\text{QCD}}^{ij}(\mathbf{x})}^{\text{stress tensor}} \left( \overbrace{\frac{1}{2}(\partial_i \xi^j(\mathbf{x}) + \partial_j \xi^i(\mathbf{x}))}^{\text{strain}} + \overbrace{\frac{1}{2}(\partial_i \xi^j(\mathbf{x}) - \partial_j \xi^i(\mathbf{x}))}^{\text{torsion}} \right)$$

- ✦ **Stress:** internal forces resisting spatial deformation

# Stress & momentum flux density

- Continuity equation (integral form):

$$\frac{d}{dt} \left[ \underbrace{\int_V d^3x T^{0j}(\mathbf{x}, t)}_{\text{Momentum in region}} \right] = - \underbrace{\oint_{\partial V} dS \hat{n}_i T^{ij}(\mathbf{x}, t)}_{\text{Flux out of region}}$$

$$T^{\mu\nu}(\mathbf{x}) = \begin{bmatrix} T^{00}(\mathbf{x}) & T^{01}(\mathbf{x}) & T^{02}(\mathbf{x}) & T^{03}(\mathbf{x}) \\ T^{10}(\mathbf{x}) & T^{11}(\mathbf{x}) & T^{12}(\mathbf{x}) & T^{13}(\mathbf{x}) \\ T^{20}(\mathbf{x}) & T^{21}(\mathbf{x}) & T^{22}(\mathbf{x}) & T^{23}(\mathbf{x}) \\ T^{30}(\mathbf{x}) & T^{31}(\mathbf{x}) & T^{32}(\mathbf{x}) & T^{33}(\mathbf{x}) \end{bmatrix}$$

- Stresses are **momentum flux densities**

- ✦ Include **particle fluxes & forces**



**Particle flux**



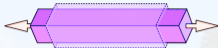
**Force**

- Kinetic pressure is pressure**

- ✦ It's the microscopic origin of gas & fluid pressure
- ✦ Photon flux is the origin of radiation pressure

# Tension and compression

Tension



Compression



Image: MikeRun ([Wikimedia](#)), very modified

## Hanging monkeys

- 🍃 **Negative stress** / negative “pressure”
- 🍃 **Tension** / pulling / stretching
- 🍃 Highest monkey under most tension

$$\nabla_i T^{ij} = -\rho g \hat{z}^j = f_{\text{grav}}^j$$

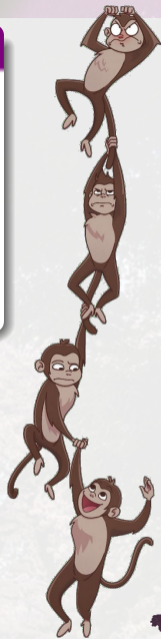
Monkey image by Angie Freese ([Instagram](#))

## Stacked Turtles

- 🍃 **Positive stress** / positive “pressure”
- 🍃 **Compression** / pushing / squishing
- 🍃 More compression at bottom of pile

$$\nabla_i T^{ij} = -\rho g \hat{z}^j = f_{\text{grav}}^j$$

Turtle image by FoxKids1302 ([DeviantArt](#))



# Tension and compression

## Cauchy momentum equation

Schrödinger, *Ann. Phys.* 387 (1927) 265

Nielsen & Martin, *PRB* 32 (1985) 3780

Polyakov & Son, *JHEP* 09 (2018) 156

AF, *PRD* 111 (2025) 034047

## Hanging monkeys

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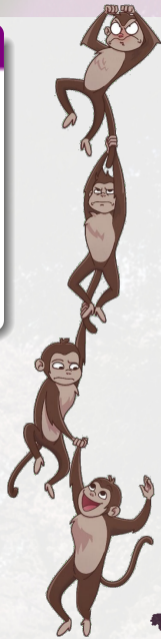
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## Stacked Turtles

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Turtle image by FoxKids1302 ([DeviantArt](#))



# Mechanical form factors

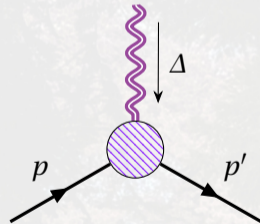
- Energy-momentum tensor parametrized using **mechanical form factors**

$$\langle \mathbf{p}', s' | \hat{T}_{q,g}^{\mu\nu}(0) | \mathbf{p}, s \rangle = \bar{u}(\mathbf{p}', s') \left\{ \frac{P^\mu P^\nu}{m} A_{q,g}(\Delta^2) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} D_{q,g}(\Delta^2) + m g^{\mu\nu} \bar{c}_{q,g}(\Delta^2) \right. \\ \left. + \frac{i P^{[\mu} \sigma^{\nu]\Delta}}{2m} J_{q,g}(\Delta^2) - \frac{i P^{[\mu} \sigma^{\nu]\Delta}}{2m} S_{q,g}(\Delta^2) \right\} u(\mathbf{p}, s)$$

- General form from Lorentz covariance
- Also known as **gravitational form factors**

$$P^\mu = \frac{1}{2}(p + p')^\mu$$

$$\Delta^\mu = (p' - p)^\mu$$



Kobzarev & Okun, *Sov Phys JETP* 16 (1963) 1343

H. Pagels, *Phys Rev* 144 (1966) 1250

X. Ji, *PRL* 78 (1997) 610

# Mechanical form factors and generalized parton distributions

☛ Mechanical form factors related to **generalized parton distributions** through Mellin moments:

$$\int_{-1}^1 dx x^p H_a(x, \xi, t) = A_a(t) + \xi^2 D_a(t) \quad p = \begin{cases} 1 & : a = q \\ 0 & : a = g \end{cases}$$

✧ Relationship ultimately due to operator product expansion

☛ Processes sensitive to GPDs can also give MFFs!

✧ Includes DVCS, **vector quarkonium production**, SDHEPs, etc.

☛ **Difficulty**: GPDs aren't measured directly—inverse problem!

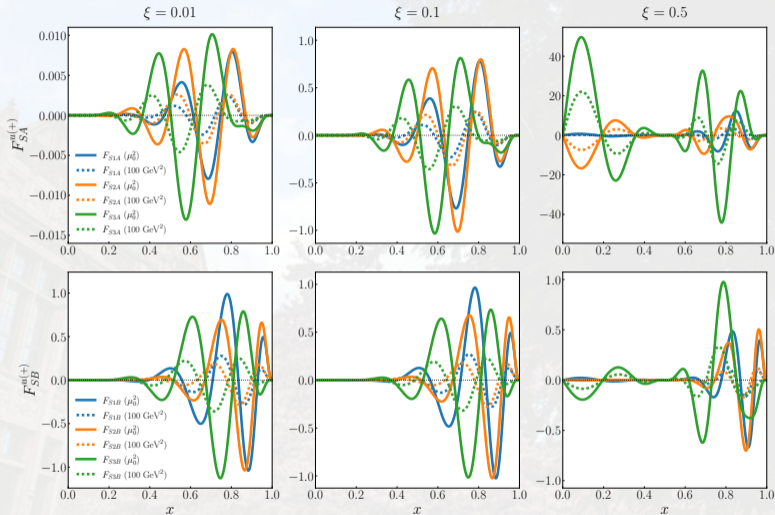
what is measurable

$$\int_{-1}^1 \frac{dx}{x} \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x, \xi, t)$$

what we want

$$\int_{-1}^1 dx H_g(x, \xi, t) = A_g(t) + \xi^2 D_g(t)$$

# Deconvolution problem in DVCS



These GPDs give the same DVCS amplitude.

Dubbed **shadow GPDs**

Hard to get MFFs from DVCS

Discovery by Bertone *et al.*, *Phys Rev D* 103 (2021) 114019

Figure from Moffat *et al.*, *Phys Rev D* 108 (2023) 036027

# Stepping outside the shadows

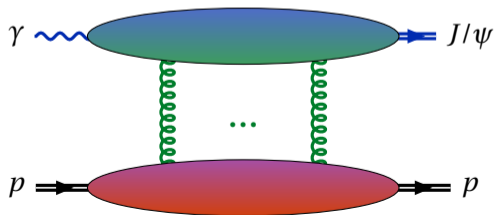
## Dispersion relations

☛ Dispersion relations allow direct access to D-term!

$$\text{Re } \mathcal{H}_q(\xi, t) \approx D_q(t) + \mathcal{P} \int dx \left( \frac{1}{\xi - x} - \frac{1}{\xi + x} \right) \text{Im } \mathcal{H}_q(x, t)$$

Burkert, Elouadrhiri & Girod, [Nature 557 \(2018\) 396](#)

## Quarkonium production



☛ Near-threshold approximation:

$$\mathcal{H}_q(\xi, t) \approx \frac{5}{2\xi^2} \left( A_q(t) + \xi^2 D_q(t) \right)$$

Hatta & Strikman, [Phys Lett B \(2021\) 136295](#)

Guo, Ji & Yuan [Phys Rev D \(2024\) 014014](#)

# Large- $\xi$ (electro-/photo-)production of vector quarkonia

## Threshold approximation

YH, Strikman 2102.12631 (Mellin moment)  
Guo, Ji, Liu 2103.11506 (Mellin moment)  
Guo, Ji, Yuan 2308.13006 (conformal moment)

what is measurable

what we want

$$\int_{-1}^1 dx \frac{1}{\xi - x - i\epsilon} \left\{ \begin{array}{l} \frac{1}{2} H^{q(+)}(x, \xi, t, \mu^2) \\ \frac{1}{x} H^g(x, \xi, t, \mu^2) \end{array} \right. \approx \frac{2}{\xi^2} \frac{5}{4} (A^a(t, \mu^2) + \xi^2 D^a(t, \mu^2))$$

Keep only the first term in the conformal partial wave expansion

Very good approximation when  $\xi = \mathcal{O}(1)$  and for **gluon** and **strangeness** GPDs  
(but not for light-quark GPDs)

Recently extended to NLO Guo, Yuan, Zhao, 2501.10532 → talk by Feng  
YH, Klest, Passek-K, Schoenleber, 2501.12343  
YH, Schoenleber 2502.12061

# Why a mini-workshop?

## Create community consensus

- ✦ Experiments & analyses to be done
  - ✦ Facility requirements
- ✦ Required tools for phenomenology
  - ✦ Global analysis workflows
  - ✦ Uncertainty quantification
- ✦ Any theoretical gaps
  - ✦ Clarify model dependence of frameworks
  - ✦ Established required kinematics
  - ✦ Factorization proofs?

## Workshop goals

- ✦ Discuss & converge on needs
- ✦ **Create white paper**
  - ✦ Communicate conclusions of workshop
  - ✦ Short document

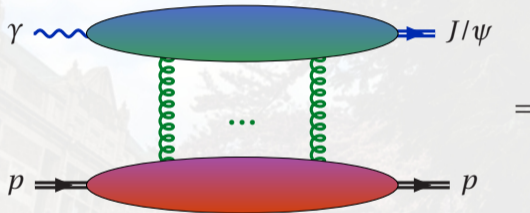
## Thursday

- Overview of experiments
  - Volker Burkert
  - Lubomir Pentchev
  - Rafo Paremuzyan
- Amplitude analysis (JPAC)
  - Arkaitz Rodas
- CNF Seminar: Yoshitaka Hatta**
- Transition GPD theory
  - Christian Weiss
- Ongoing analyses
  - Mariana Khachatryan
  - Bo Yu

## Friday

- Ongoing and future experiments
  - Richard Tyson
  - Mariana Tenorio Pita
  - Bhawani Singh
  - Henry Klest
- Global analysis
  - Melany Higuera Angulo
  - Yuxun Guo
- MFFs & GPDs from lattice QCD
  - Joe Karpie
  - Dimitra Pefkou
- Theoretical overview
  - Kiminad Mamo

# Vector Quarkonia as Pressure Gauges



**Thank you for your participation!**