

ML-Assisted Determination of Hadronic Tensor from Lattice QCD for Neutrino-Nucleon Scattering

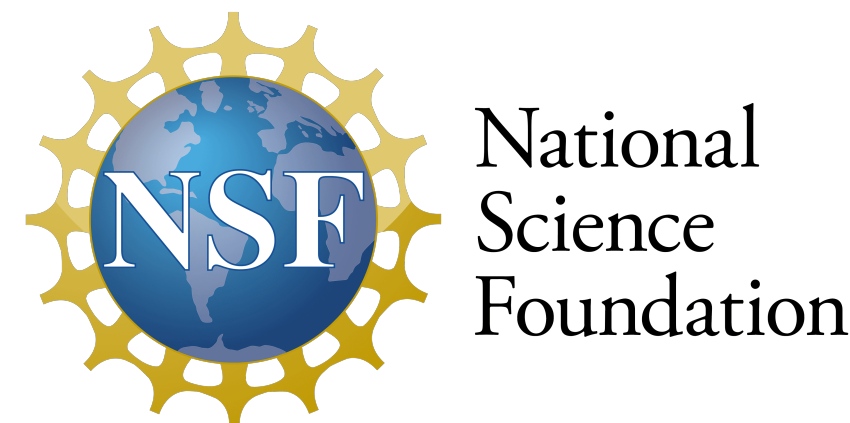
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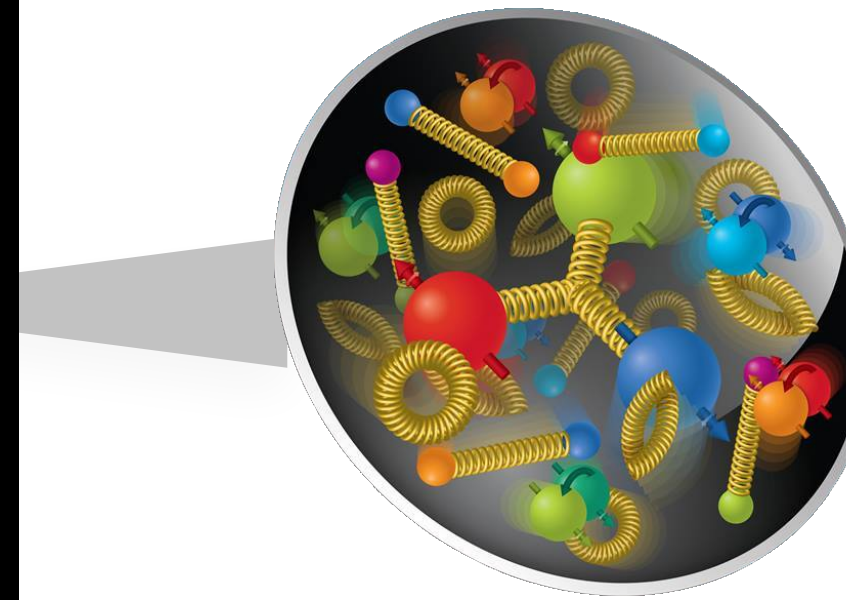
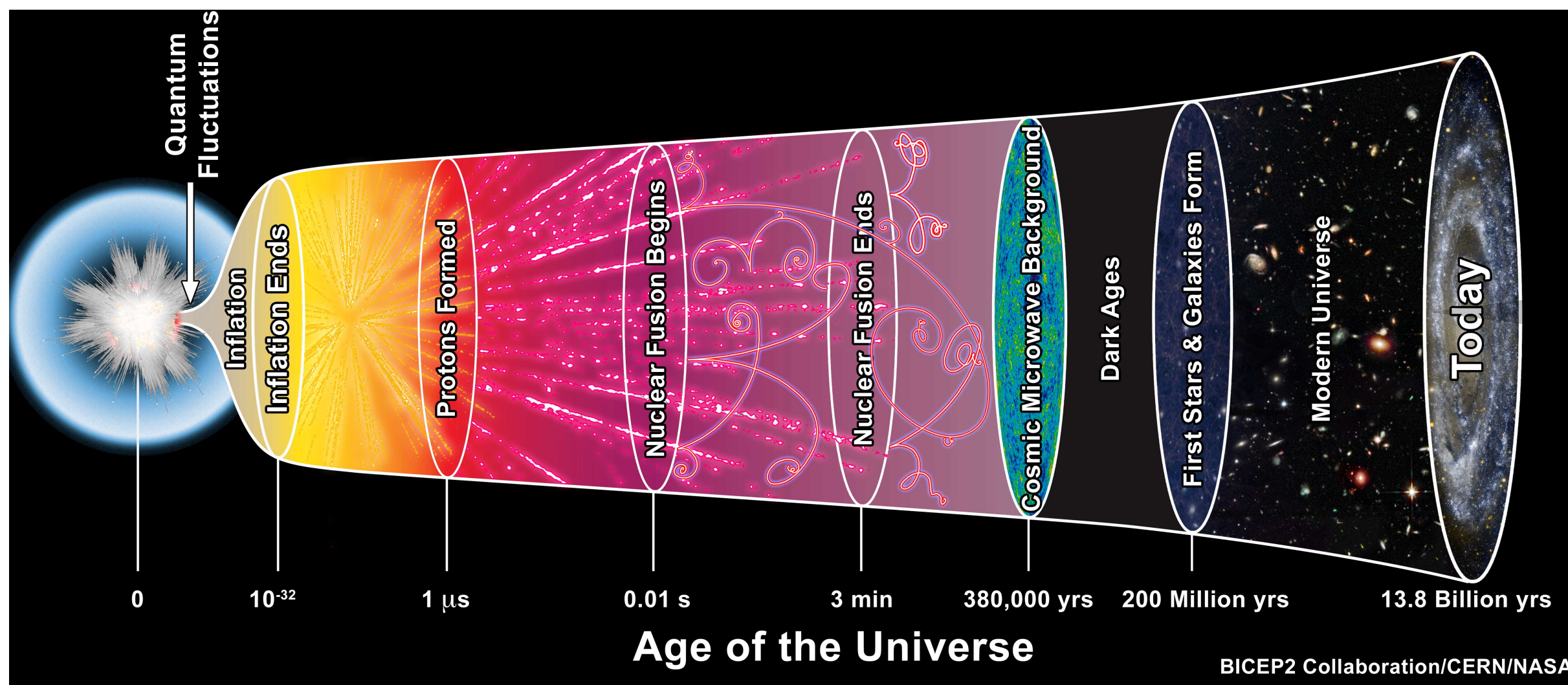
Towards Improved Hadron Femtography @ University of Virginia



Funding acknowledgment:



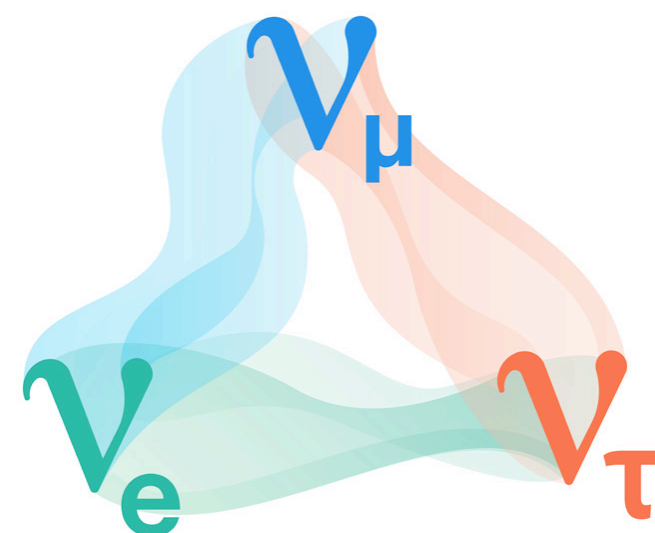
Symmetry is beautiful but asymmetry is why the universe exists



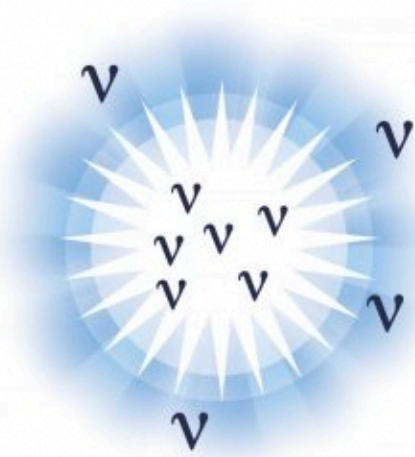
proton

Why the universe is the way it is?
Why only proton and no anti-proton?

- Why is the visible universe matter-dominated instead of being **EMPTY** filled with only radiation?
- Physics of neutrinos can potentially hold the key to answering this!

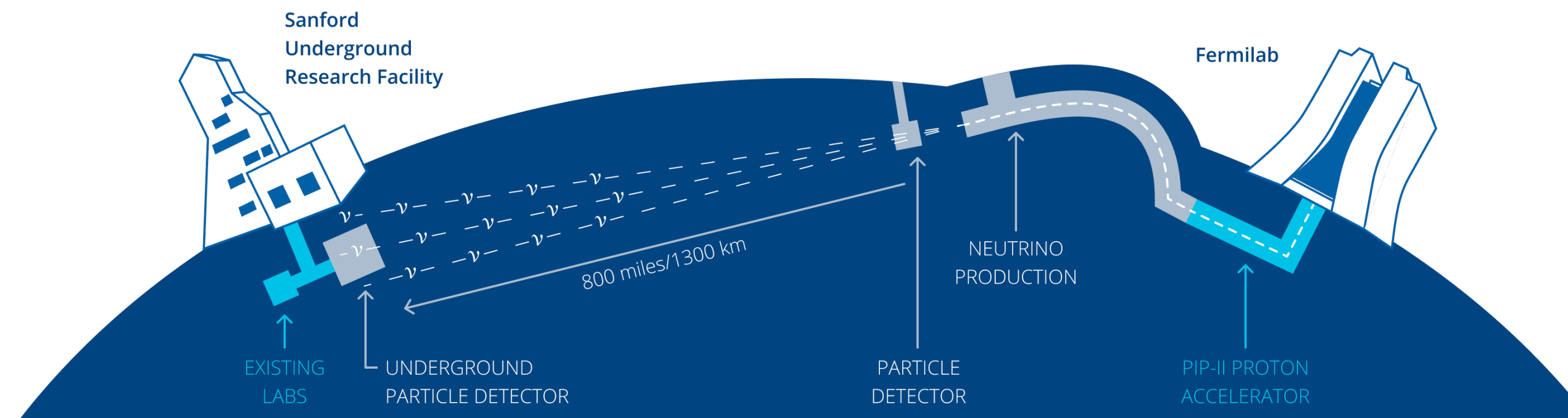


Aiming for groundbreaking discoveries

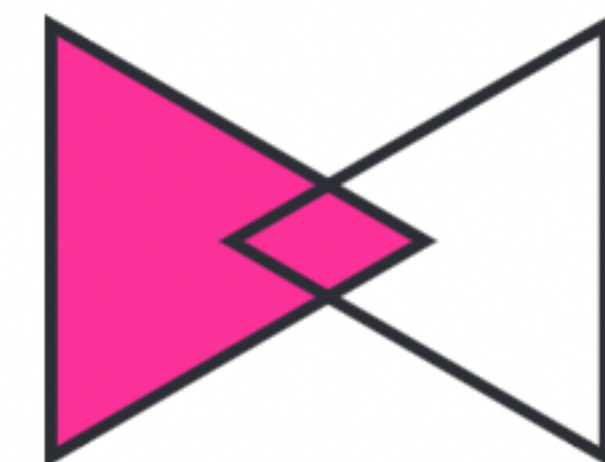


Origin of Matter

Could neutrinos be the reason that the universe is made of matter rather than antimatter? By exploring the phenomenon of neutrino oscillations, DUNE seeks to revolutionize our understanding of neutrinos and their role in the universe.



- Highest priority : Vision of the 2023 Particle Physics Project Prioritization Panel (P5)



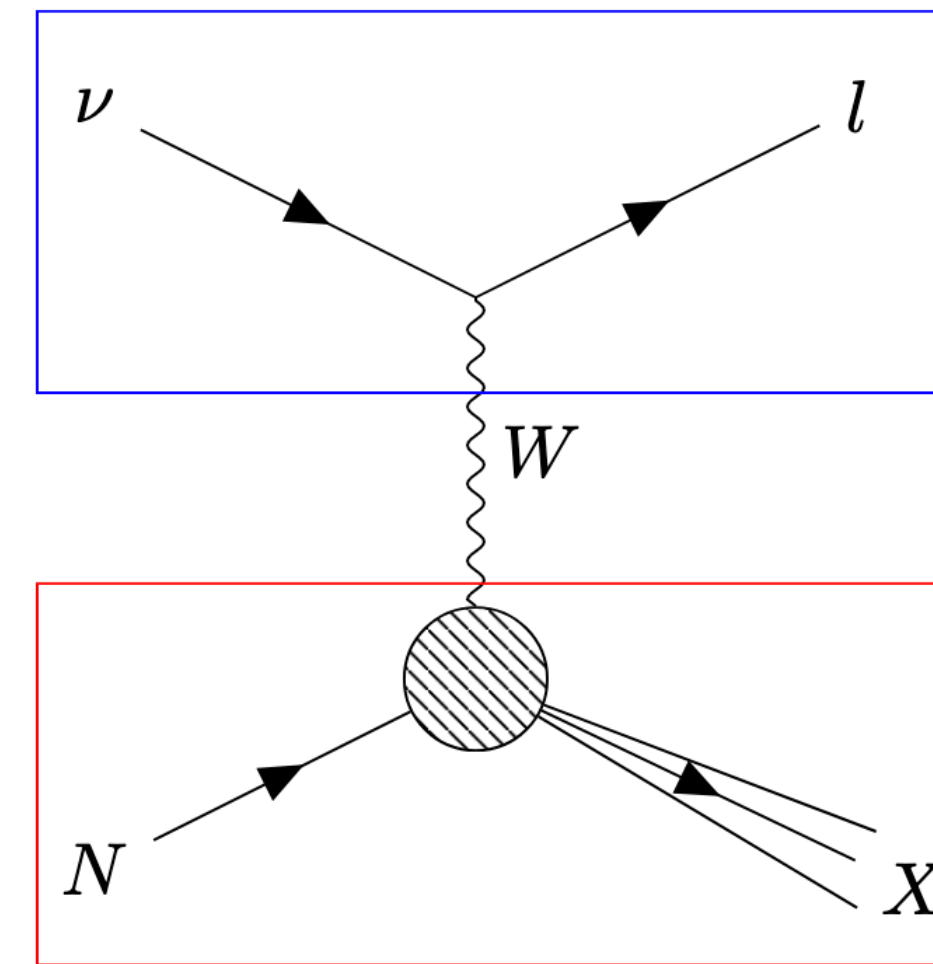
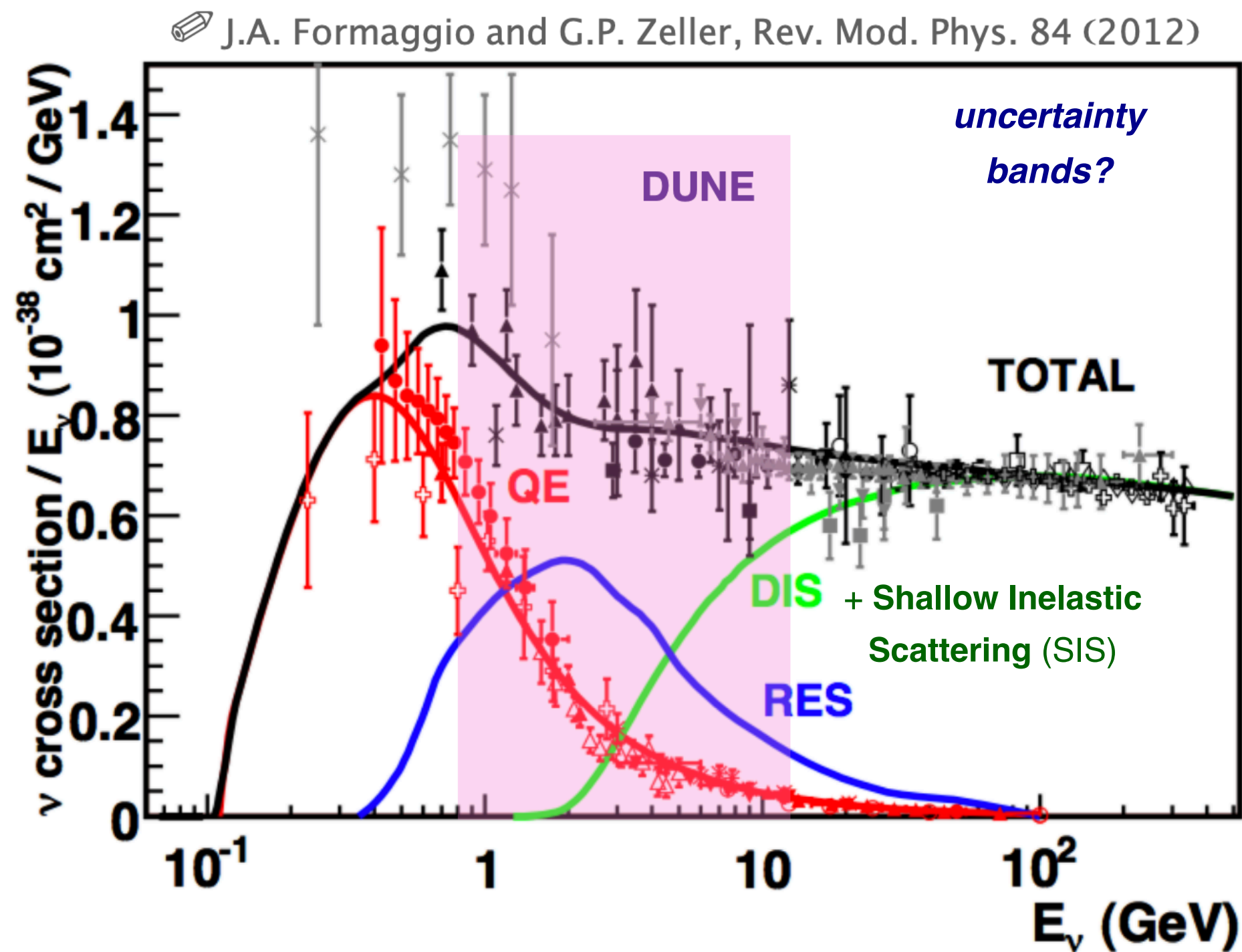
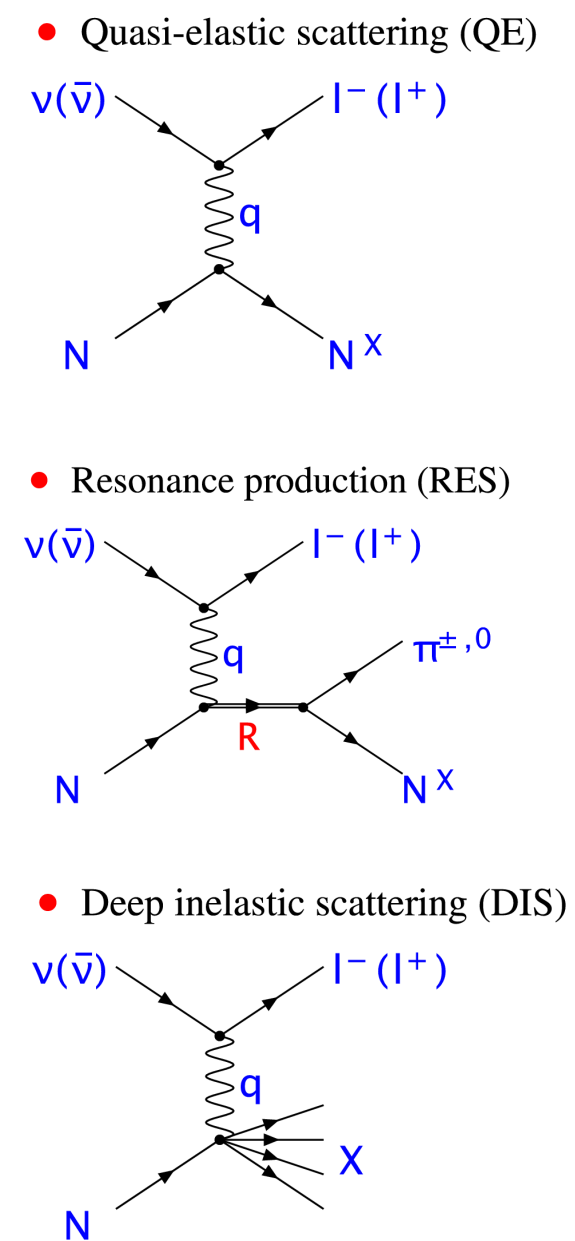
Decipher
the
Quantum
Realm

Elucidate the Mysteries
of Neutrinos

Challenges for the DUNE

Two major challenges:

- ▶ Neutrino beams are not mono-energetic (E_ν)
- ▶ Modeling of unknown nuclear effects (and even at the level of nucleons)



Leptonic tensor
(Easy)

Hadronic tensor
(Nonperturbative)
Calculable with
Lattice QCD

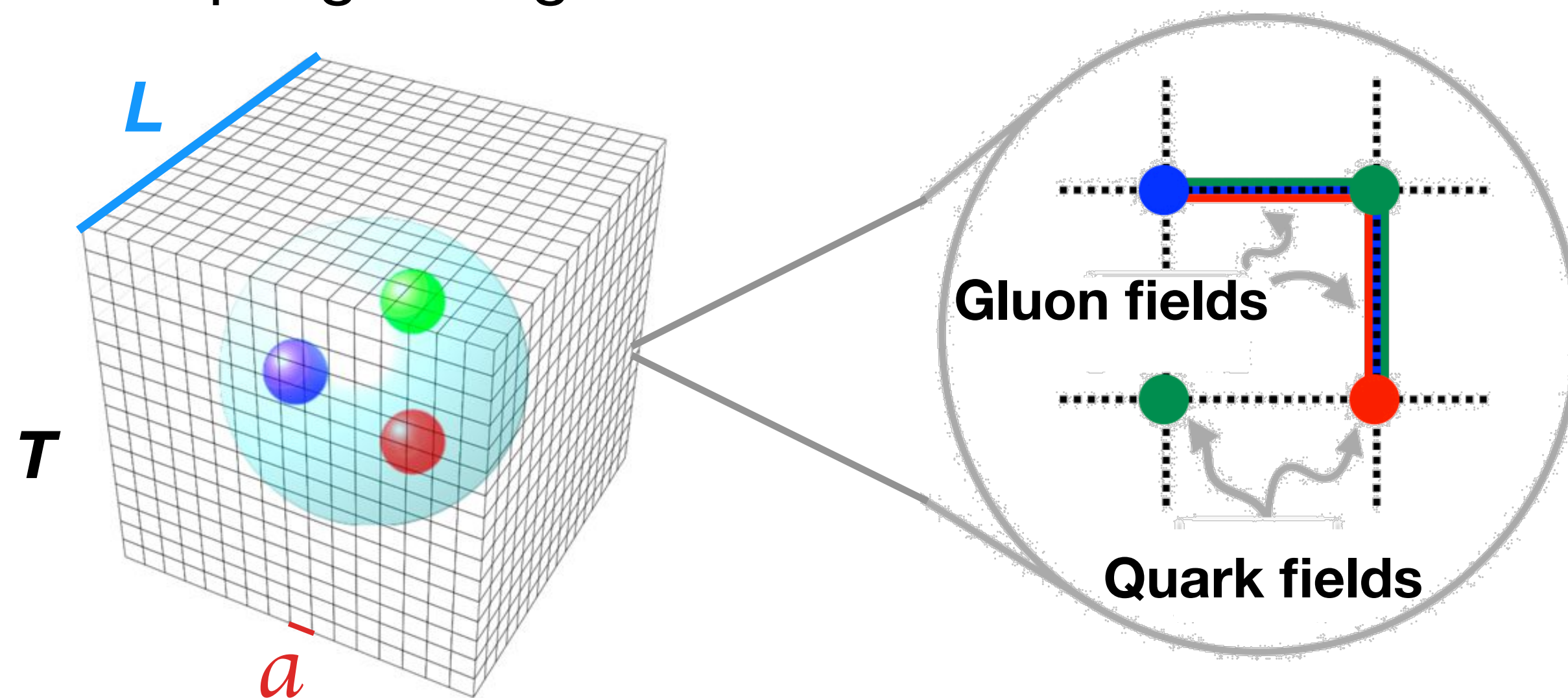
● **Research objective:** Hadronic tensor in QE, RES, SIS, and DIS regions using Lattice QCD [K. F. Liu, PRL 1994]

USQCD White Paper, Kronfeld, Richards, **RSS**, et al [EPJA 2019]
L. Ruso, et al [J.Phys.G 52 (2025) 4, 043001]

Field theoretical formalism for QCD: Lattice QCD

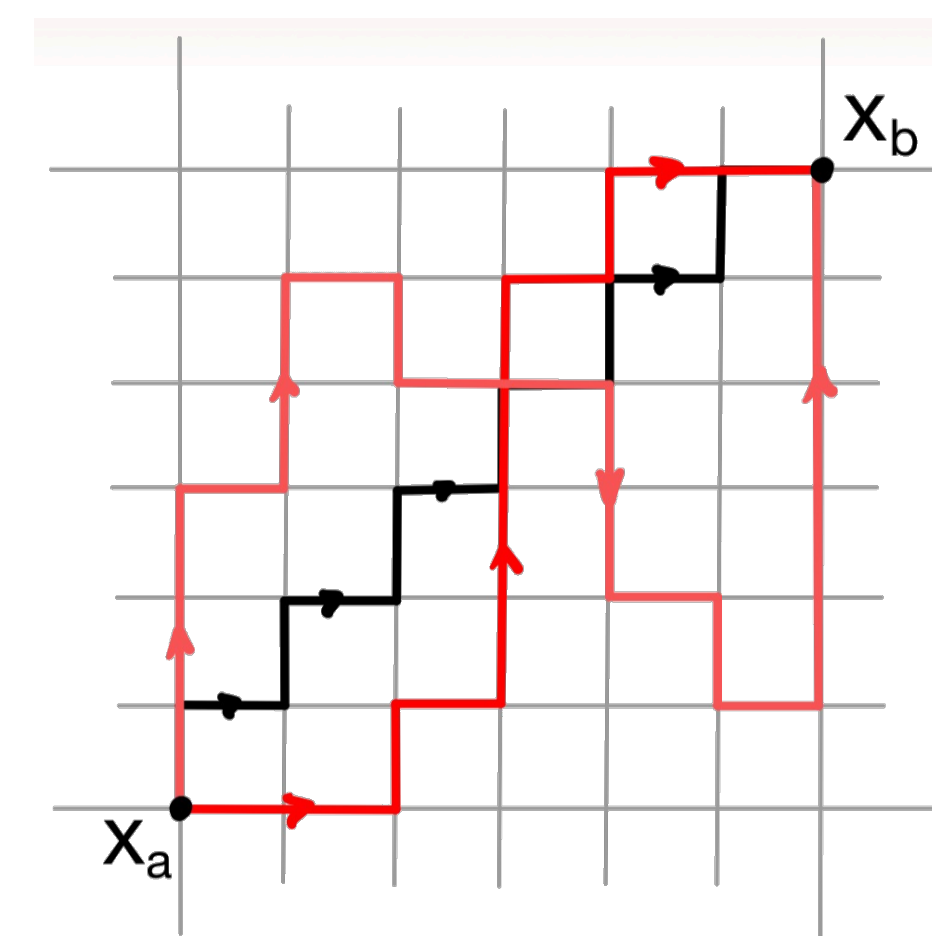
- LQCD: lattice regularization of QCD

► Quark mass and coupling strength



- First principle and nonperturbative approach to numerically study QCD
- Based on the path integral formalism of QFT
- RBC/UKQCD 32If DWF ensemble

► $L \times T = 32^3 \times 64$ $a = 0.063 \text{ fm}$ $m_\pi = 370 \text{ MeV}$



Hadronic tensor from lattice QCD

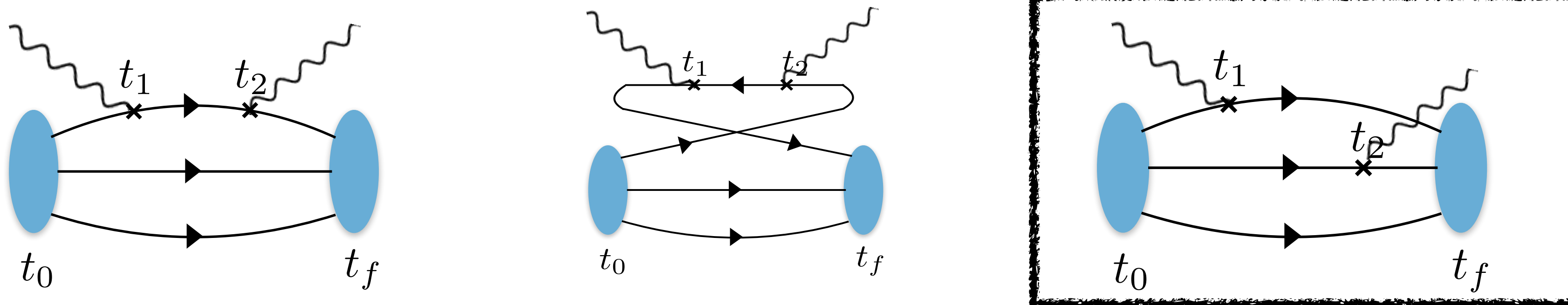
- Hadronic tensor is relevant in all energy regions

► USQCD white paper: Kronfeld, Richards, Liu, RSS, et al [2019] / A. Meyer, et al [Annu. Rev. 2022]

► Ability to investigate different channels with different current combinations on lattice

$$W_{\mu\nu}(q^2, \nu) = \frac{1}{\pi} \text{Im} T_{\mu\nu}(q^2, \nu) = \frac{1}{2} \sum_n \int \prod_{i=1}^n \left[\frac{d^3 p_i}{(2\pi)^3 2E p_i} \right] \langle N(p) | J_\mu(0) | n \rangle \langle n | J_\nu(0) | N(p) \rangle (2\pi)^3 \delta^4(p_n - p - q)$$

- All allowed intermediate states, different from excited-states from the nucleon source and sink



Cat's ear diagram

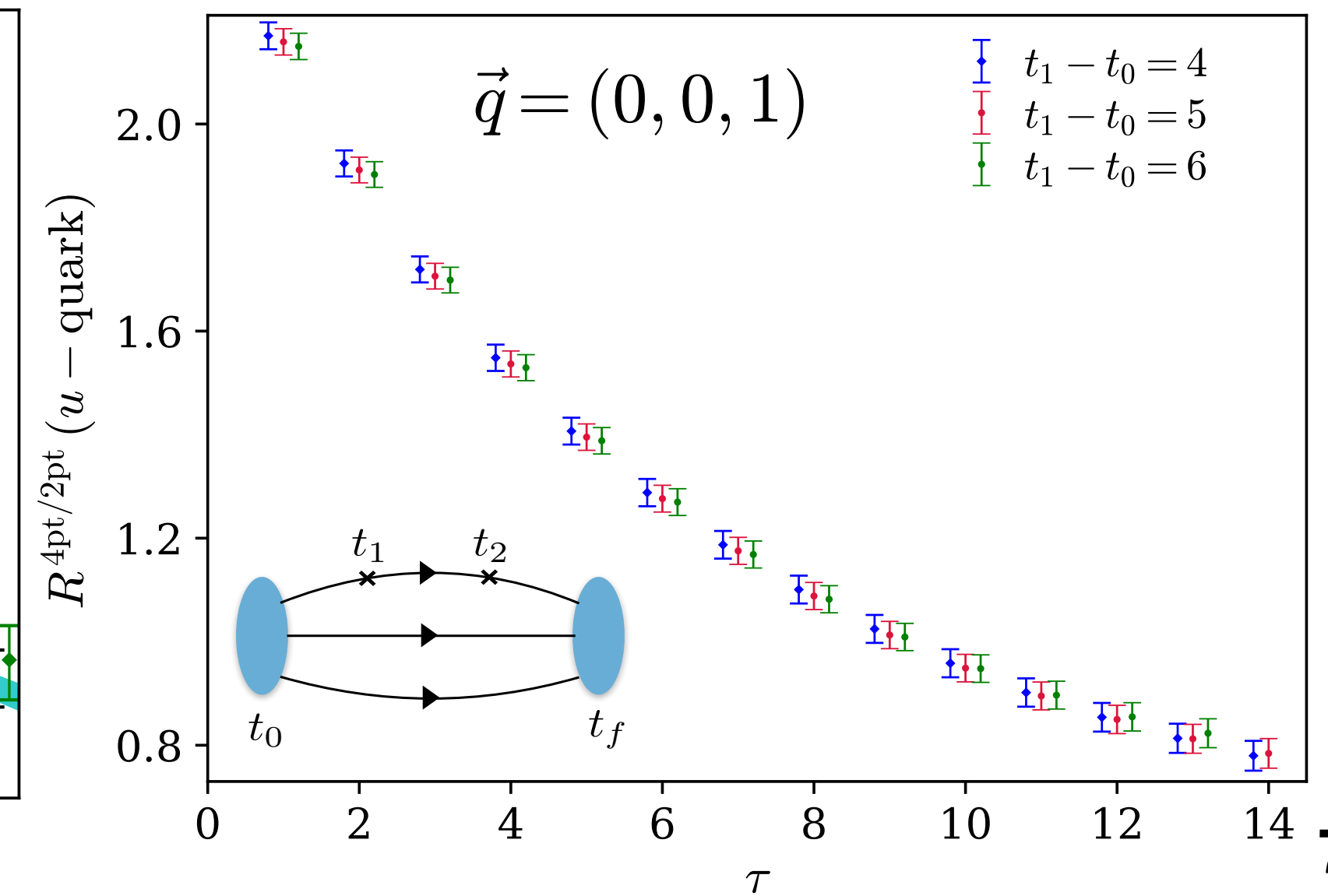
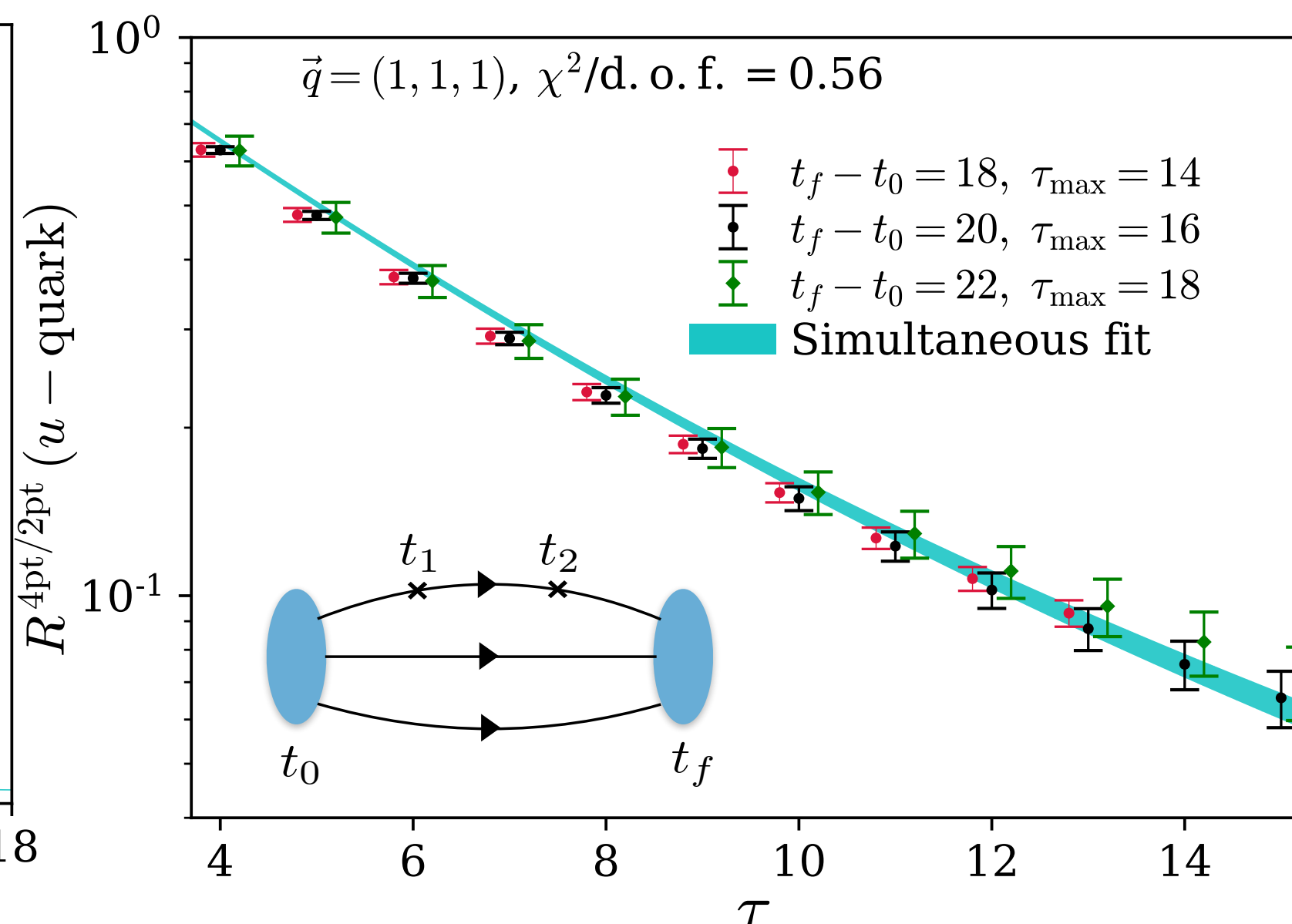
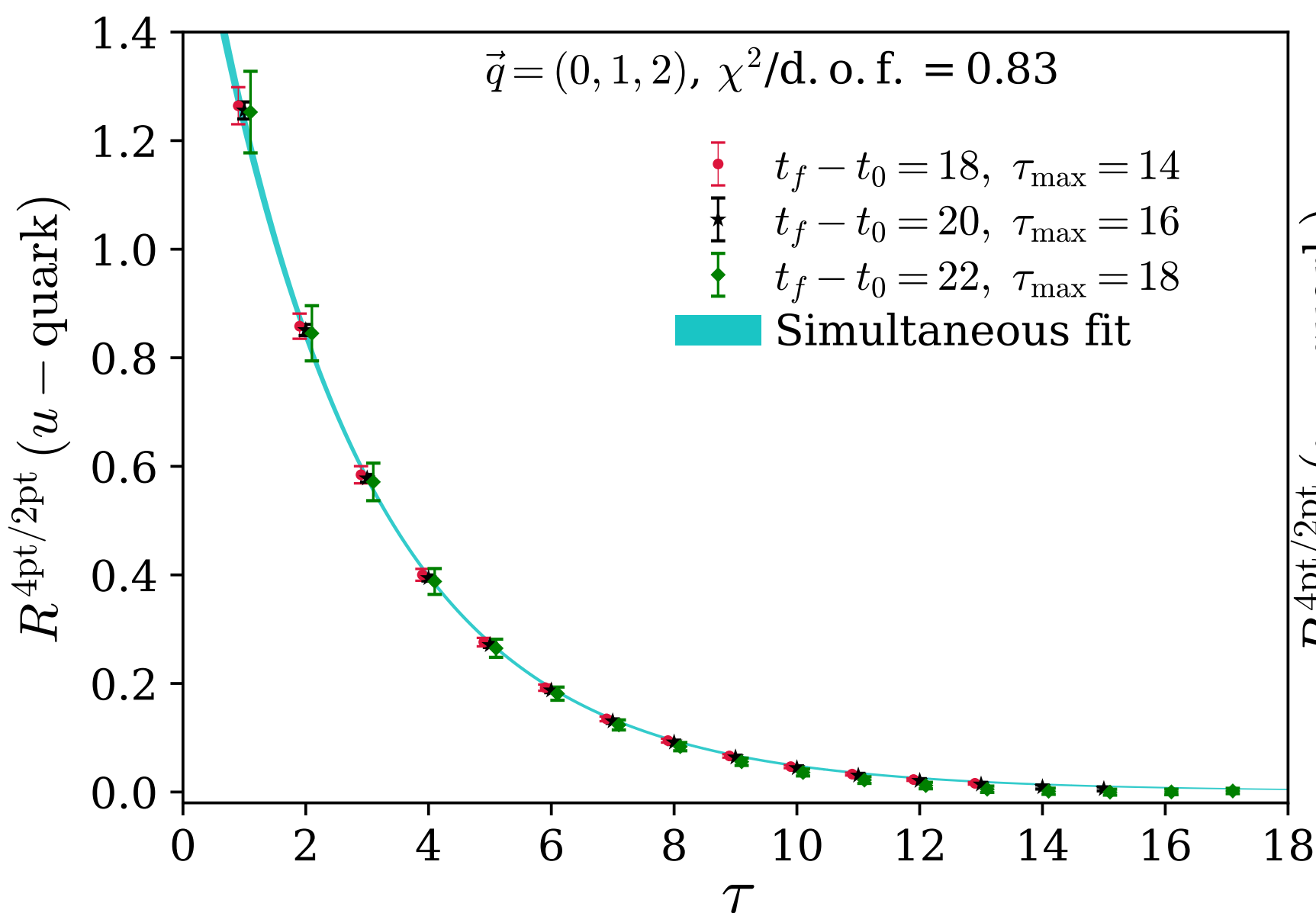
Lattice QCD calculation of hadronic tensor

- Fit the LQCD correlation function with:

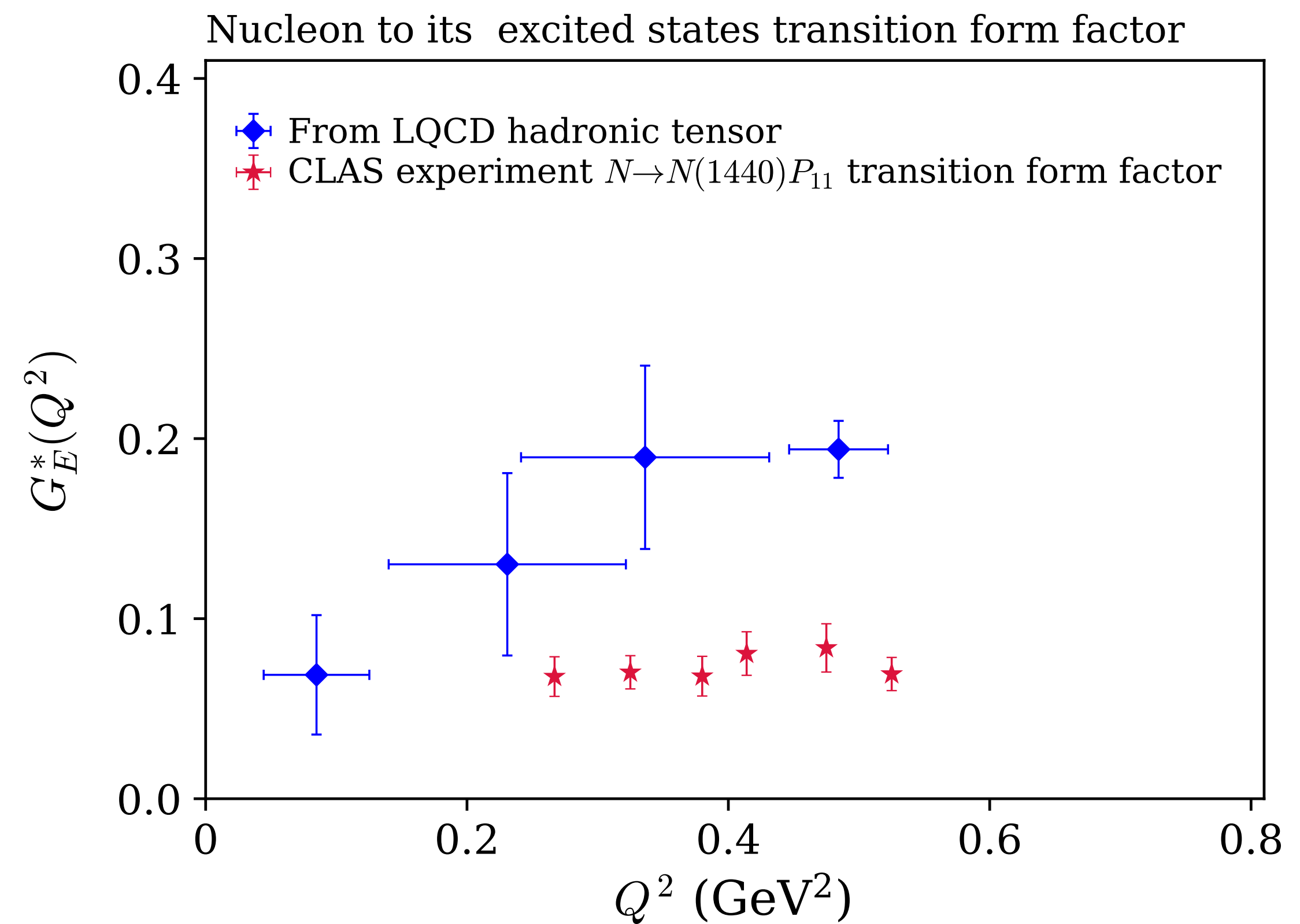
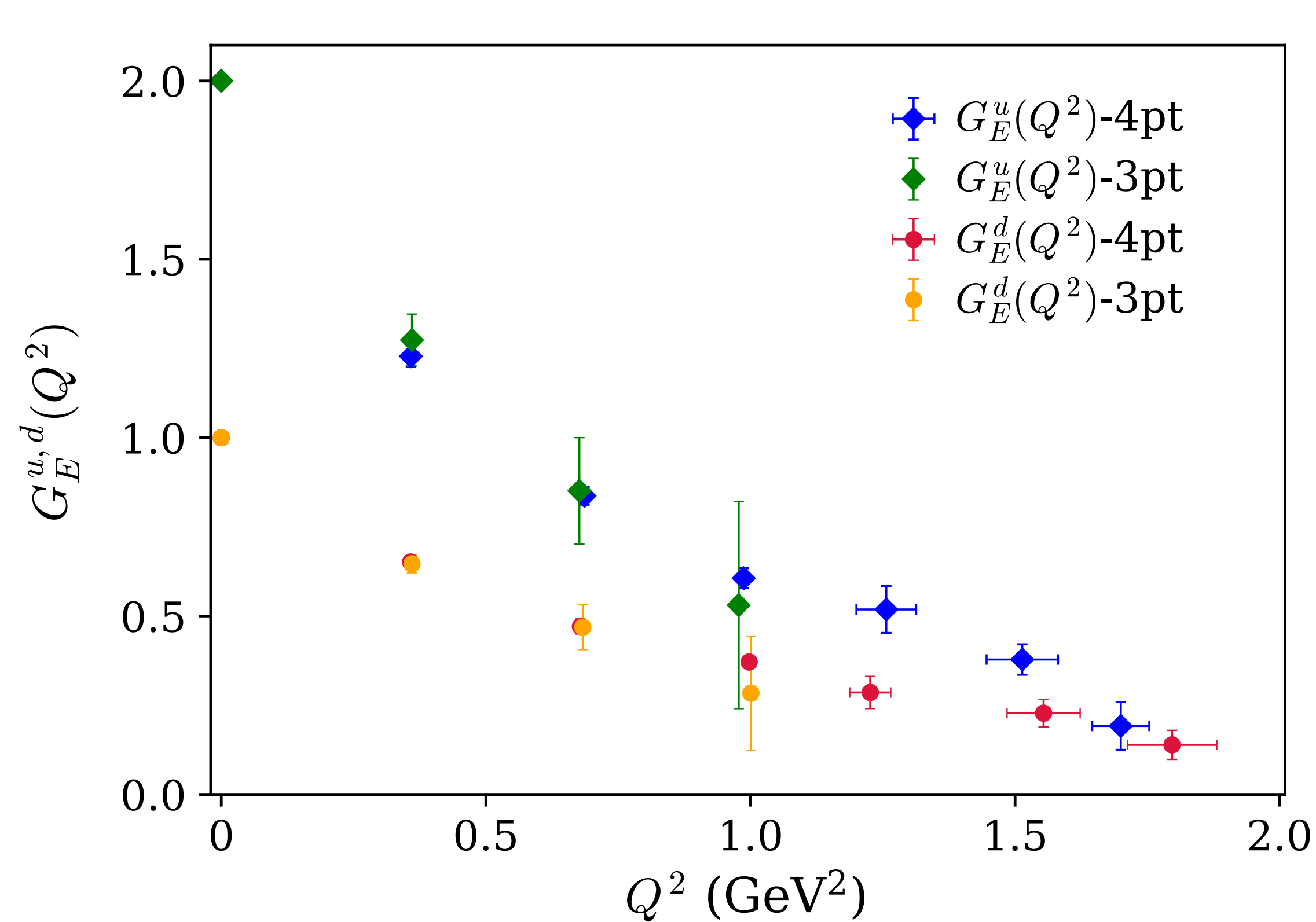
$$R^{4\text{pt}/2\text{pt}}(\vec{p}_f = \vec{0}, t_f, \vec{q}, t_2, t_1, \vec{p}_i = \vec{0}, t_0) \xrightarrow[t=n=N]{t \gg \tau \gg 0} \sum_n W_n(\vec{q}) e^{-(E_{n,\vec{q}} - E_{N,\vec{p}})\tau}$$

- Access to nucleon elastic and nucleon-to-first radially excited state in finite volume

$$R^{4\text{pt}/2\text{pt}}(\vec{p}_f = \vec{0}, t_f, \vec{q}, t_2, t_1, \vec{p}_i = \vec{0}, t_0) = \frac{1}{2} \left(\frac{E_{N,\vec{q}} + m_N}{E_{N,\vec{q}}} G_E^2 e^{-(E_{N,\vec{q}} - m_N)\tau} + \frac{E_R + m_R}{E_R} \left[\frac{(E_R - m_R)(m_N + m_R)}{q^2} \right]^2 (G_E^*)^2 e^{-(E_{R,\vec{q}} - m_N)\tau} \right) + \dots$$



Lattice QCD calculation of hadronic tensor



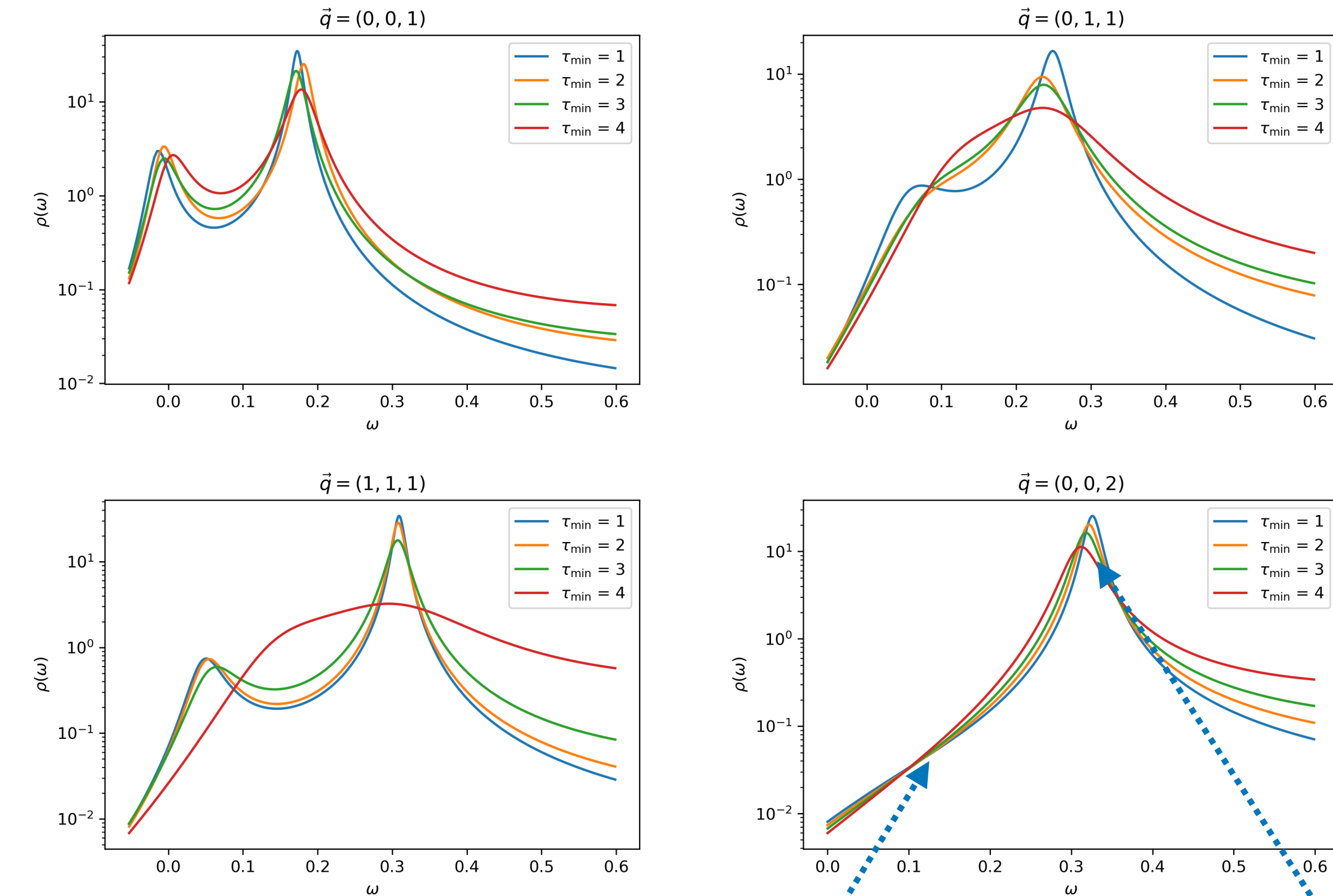
Liang, RSS, Wang, et al (XQCD Collaboration) [PRD 2026]

● First attempt towards studying resonance structure from hadronic tensor and encouraging result towards comparing with experimental data

Non-ML extraction of hadronic tensor & limitations

Bayesian reconstruction of spectral function: $R^{4\text{pt}/2\text{pt}}(\tau) \rightarrow \sum_n W_n e^{-(E_n - E_N)\tau} = \int d\omega \rho(\omega) e^{-\omega\tau}$

Burnier, Rothkopf
PRL 2013



$Q^2 \approx 1.6 \text{ GeV}^2$

$Q^2 \approx 0.5 \text{ GeV}^2$

$\vec{q} = (q_x, q_y, q_z)$	$m_1 = m_N \text{ (GeV)}$	$m_2^* \text{ (GeV)}$	$m_3^* \text{ (GeV)}$
(0,0,1)	1.25(20)	1.81(04)	1.81(04)
(0,1,1)	1.27(19)	1.89(04)	1.89(05)
(1,1,1)	1.38(19)	1.92(03)	1.98(03)
(0,0,2)	1.19(16)	1.96(14)	1.96(10)
(0,1,2)	1.14(40)	2.00(14)	2.03(10)
(1,1,2)	1.10(15)	1.98(06)	1.98(10)

Once again fails to isolate nearby intermediate states

Liang, RSS, Liu, et al (chiQCD collaboration)[Phys. Rev. D 2026]

Lattice QCD calculation of hadronic tensor

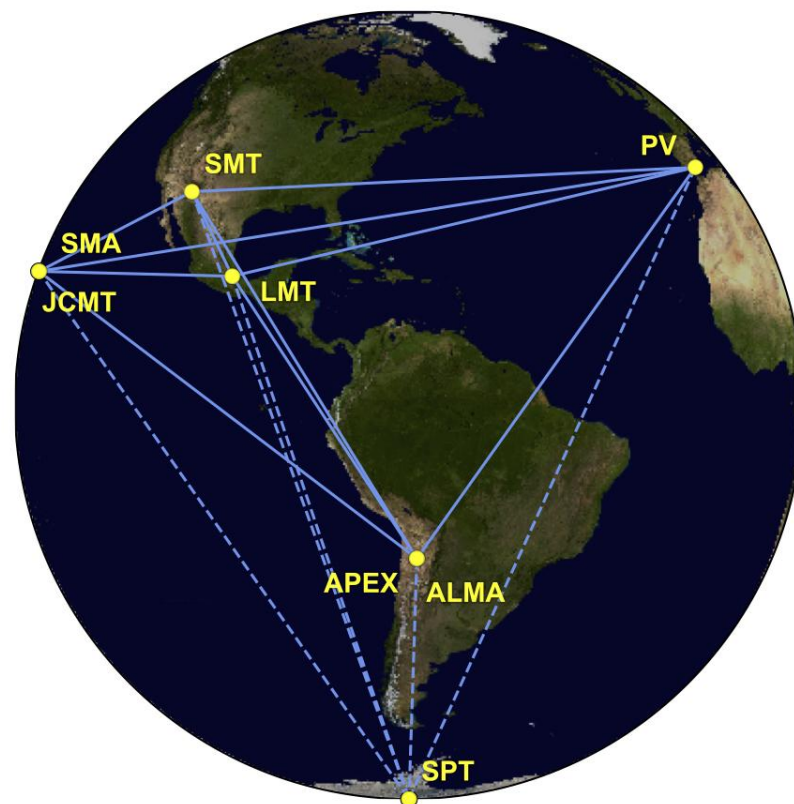
- LQCD calculation of the Euclidean hadronic tensor suffers from an inverse problem

$$W_{\mu\nu}^E(\vec{p}, \vec{q}, \tau) = \int_0^\infty \frac{d\nu}{2E_{N,\vec{p}}} e^{-\nu\tau} W_{\mu\nu}^M(\vec{p}, \vec{q}, \nu)$$

The inverse problem is the task of reconstructing the Minkowski space hadronic tensor from Euclidean lattice data, where limited and noisy information makes the reconstruction non-unique and numerically challenging.

- Inverse problem is everywhere

- ▶ Advantage of ML in the reconstruction of blackhole image



Event Horizon Telescope
2019a, ApJL, 875, L1



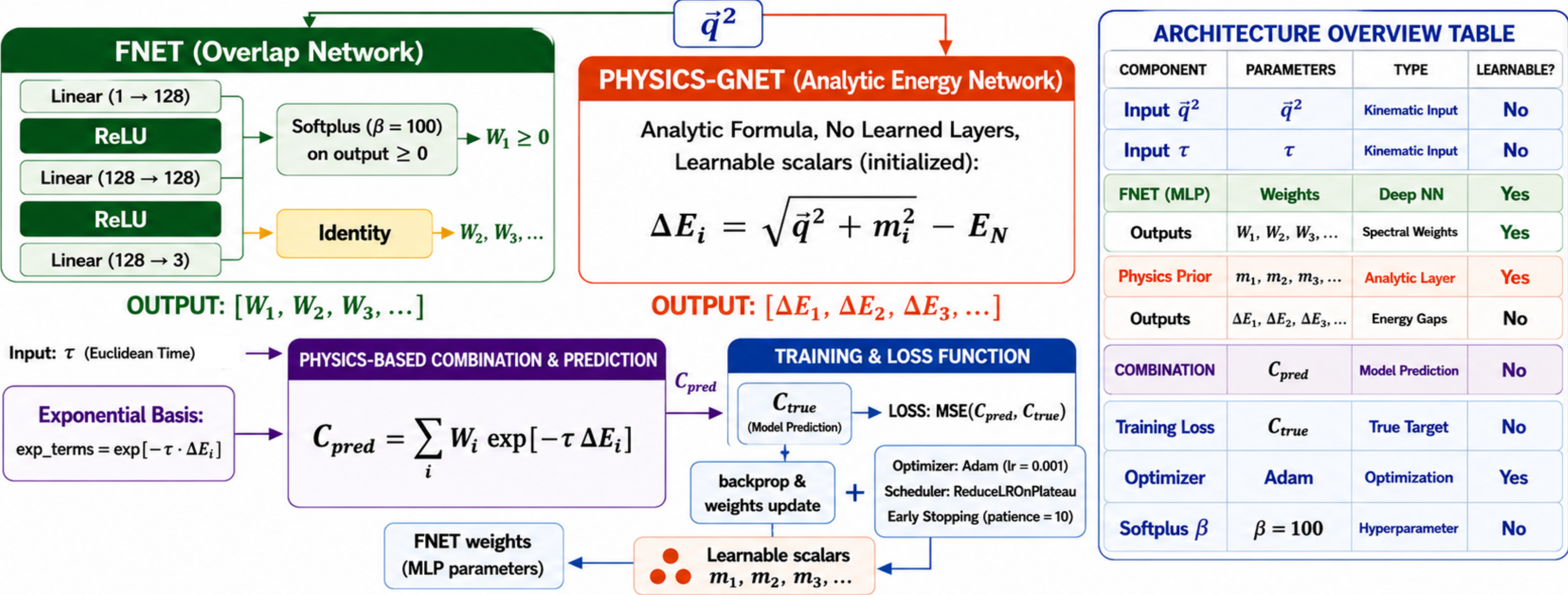
Impact of ML
2023, ApJL, 94, L7

ML-advantage in extracting the hadronic tensor

- Consider data at all momentum transfers and different quark-flavors together
 - ▶ Implement architecture of a neural network by decomposing the problem into two complementary subnetworks trained jointly by back-propagation through the physical model
 - ▶ The “**PhysicsGNet**” subnetwork holds the masses of the intermediate hadronic states obeying the dispersion relation for each momentum transfer as learnable scalar parameters and computes the energy gaps analytically, dramatically reducing the effective parameter space and eliminating an entire class of unphysical local minima
 - ▶ A two-hidden-layer (128 neurons in each layer) multi-layer perceptron (MLP) called “**FNet**” maps squared momentum transfer to overlapping spectral weights.
 - ▶ With $O(50)$ trained networks, the framework already identifies **3** distinct structures in the spectral function at high precision
 - ▶ Ongoing investigation for 4th structure

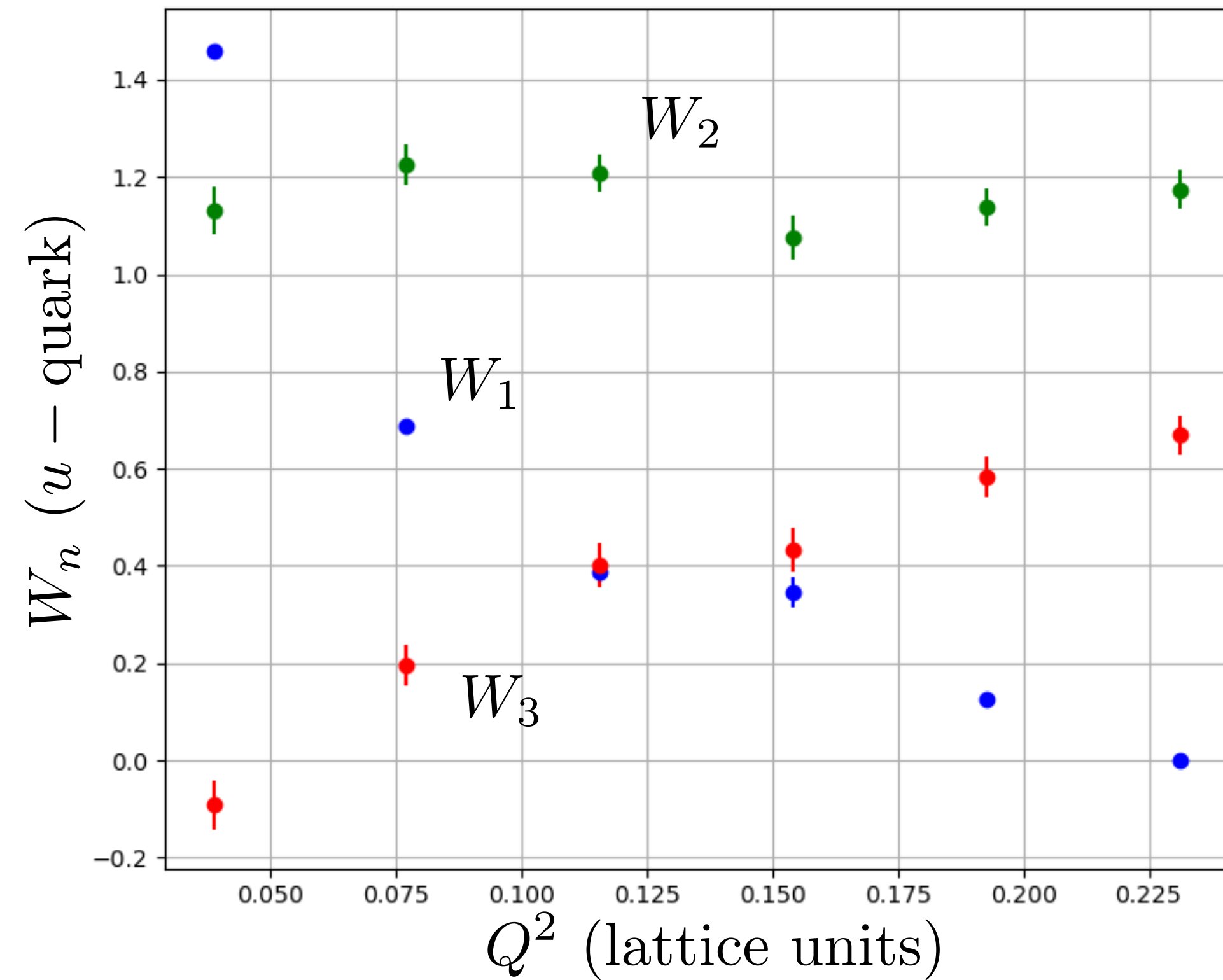
ML extraction of hadronic tensor

- Consider data at all momentum transfers and different quark-flavors together

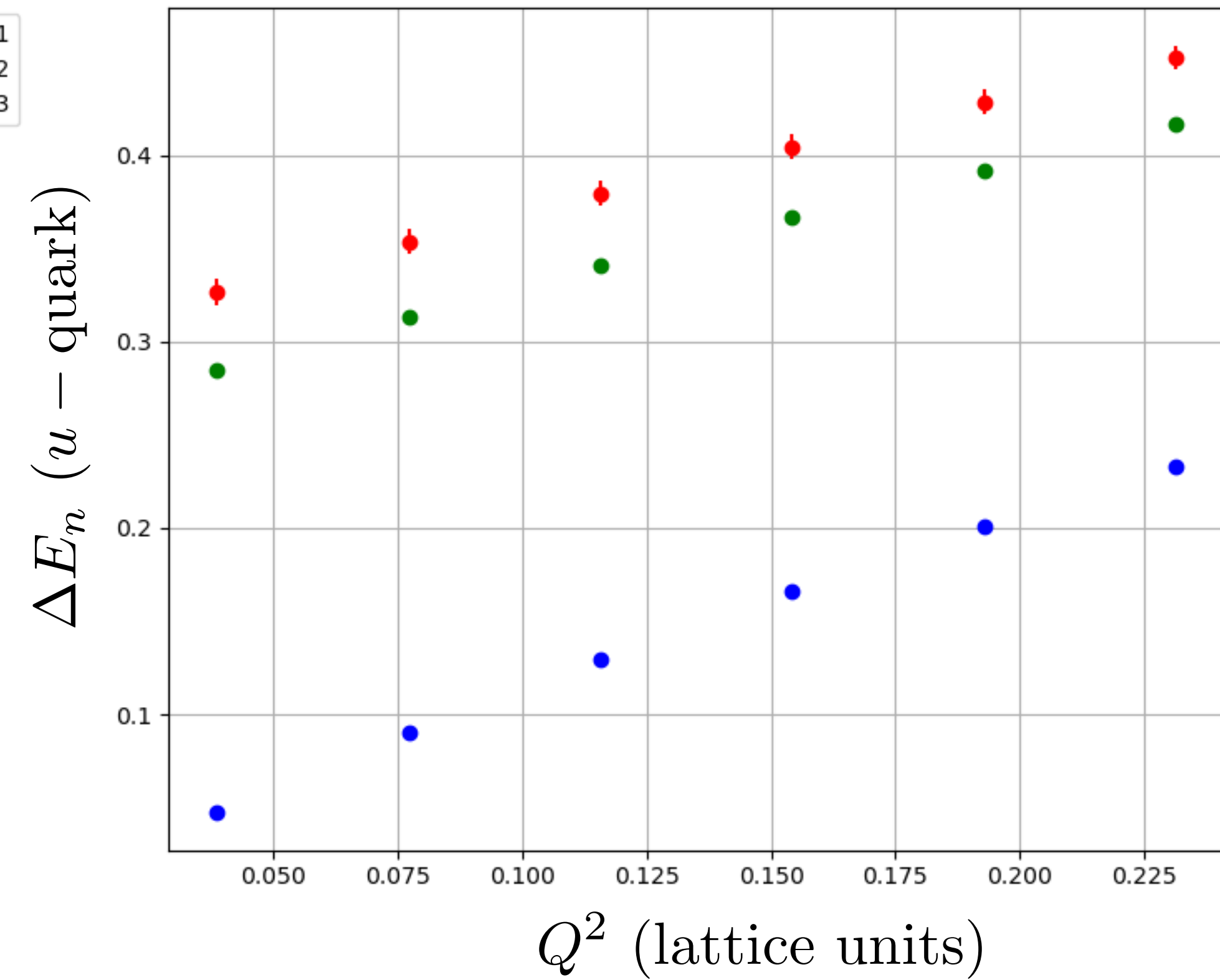


ML extraction of hadronic tensor

● PhysicsGNet and FNet extraction of form factors



Estimated f1
Estimated f2
Estimated f3



Estimated g1
Estimated g2
Estimated g3

In preparation
Kamruzzaman (Computer Science), RSS, et al

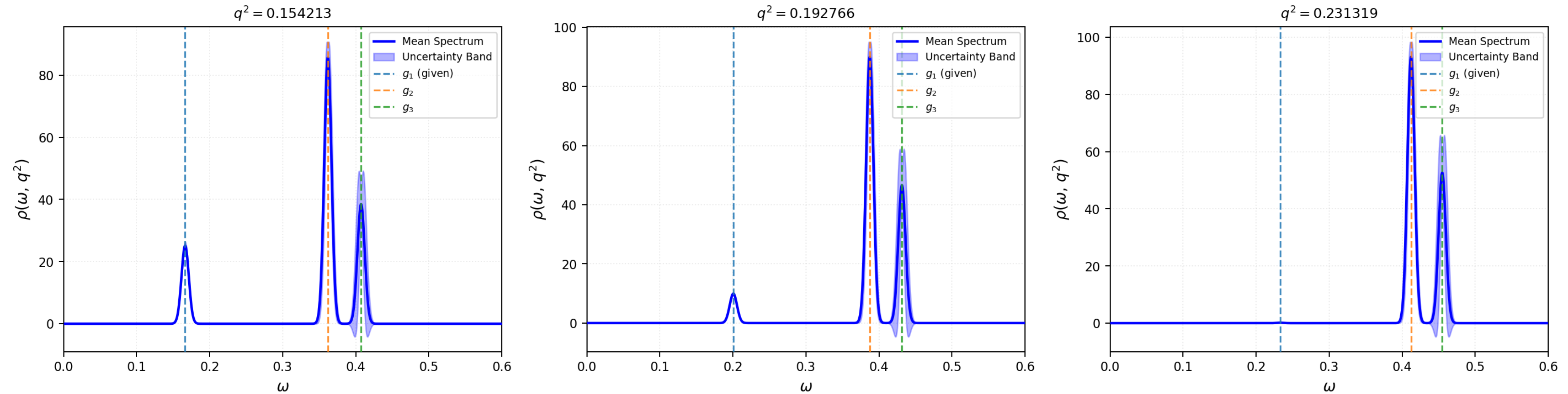
ML extraction of hadronic tensor

- Reconstruction of spectral functions: instead of learning several parameters such as peak positions and widths, the network may contain tens of thousands of parameters
 - ▶ Parametrizing the spectral function directly with a deep neural network $\boxed{\rho(\omega) = f_{\theta}(\omega)}$
 - ▶ Forward Laplace transform as the training objective
 - ▶ Second-derivative Tikhonov regulariser
 - ▶ Bayesian hyperparameter optimisation (Optuna / Tree-structured Parzen Estimator)

$$C(\tau, q^2) = \int_0^{\infty} \rho(\omega, q^2) e^{-\omega\tau} d\omega$$

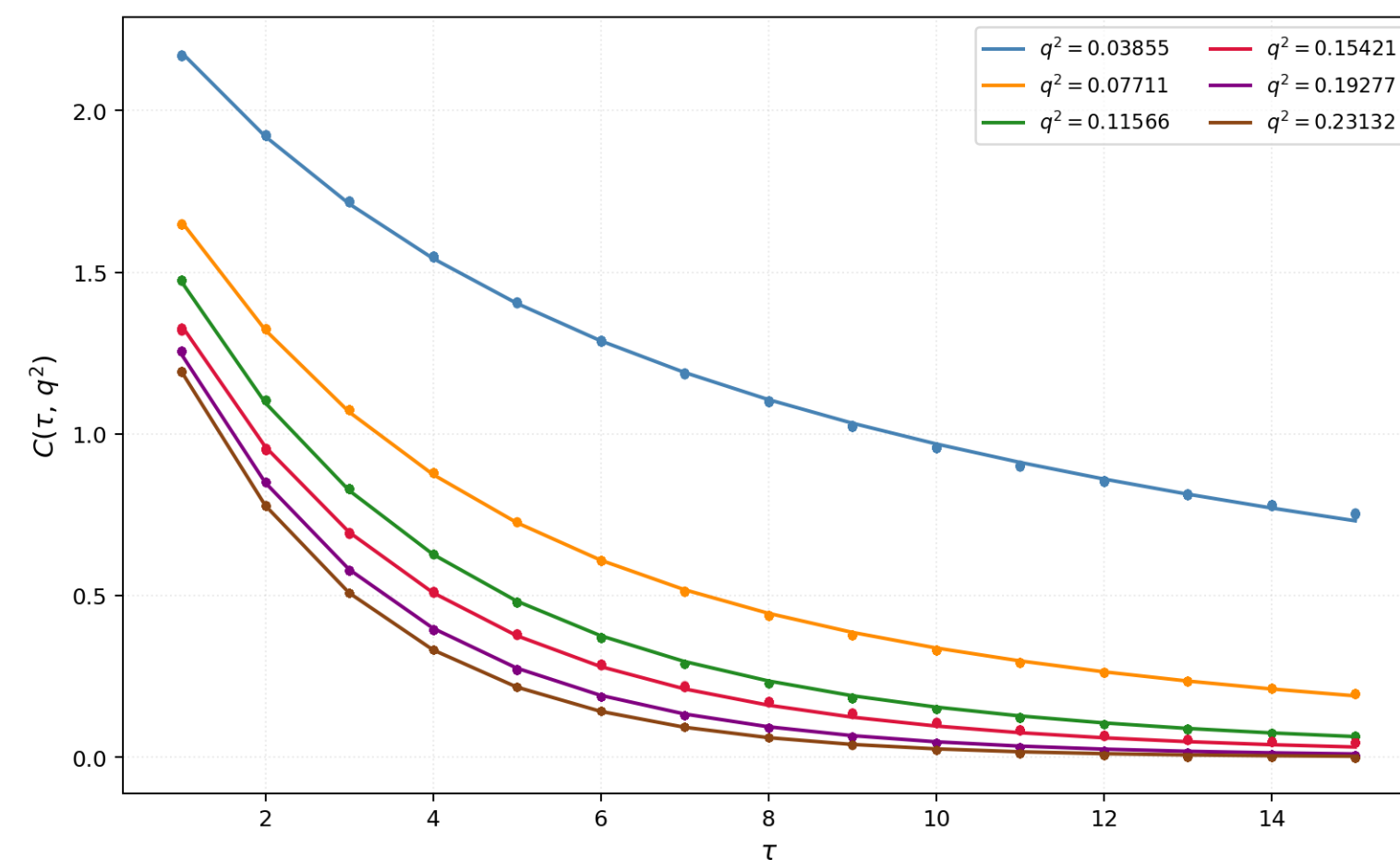
ML extraction of hadronic tensor

Reconstruction of spectral functions

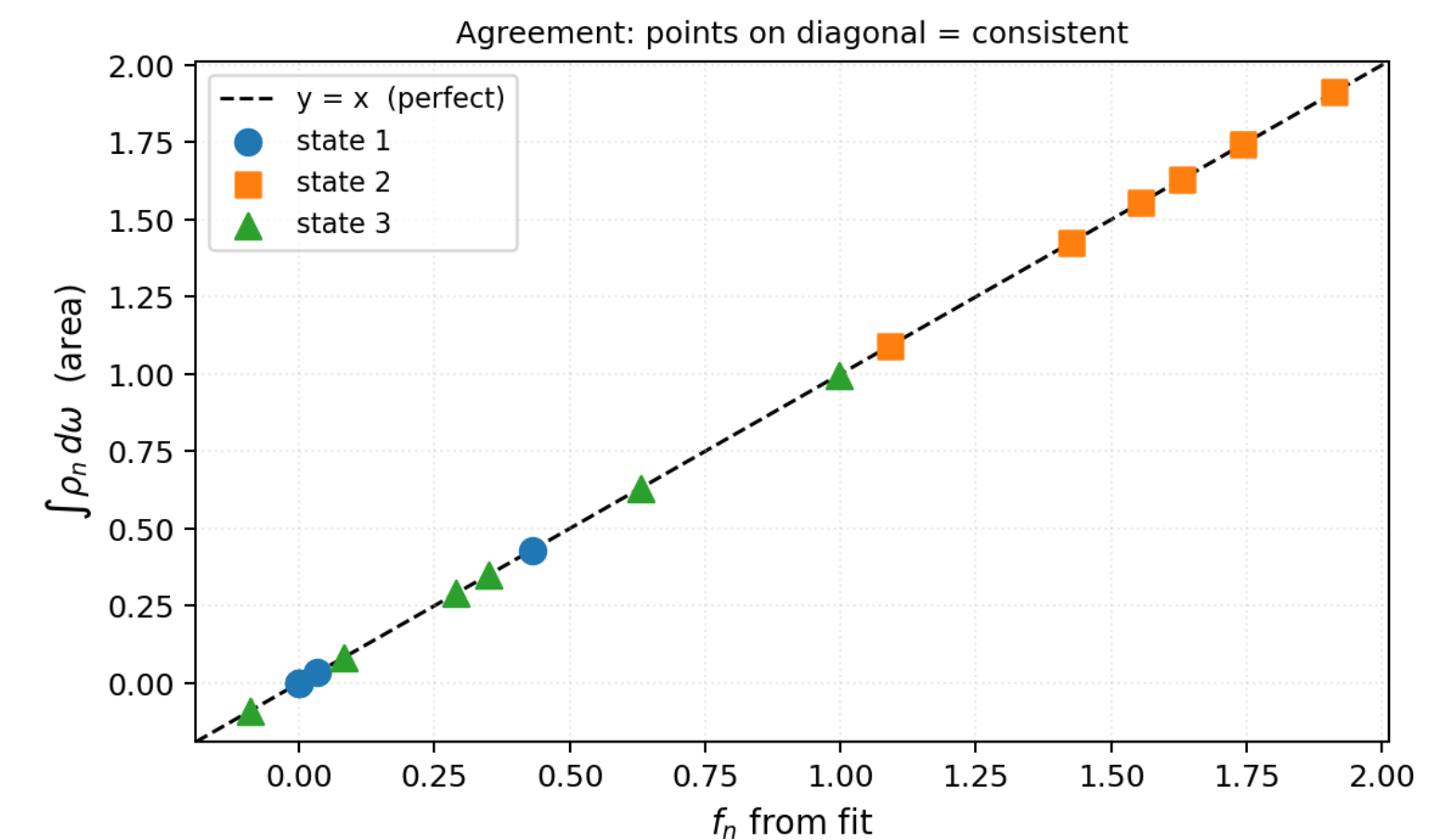


Verifications

Laplace Verification $C_{\text{rec}}(\tau) = \int_0^1 \rho e^{-\omega\tau} d\omega$ — Quark U

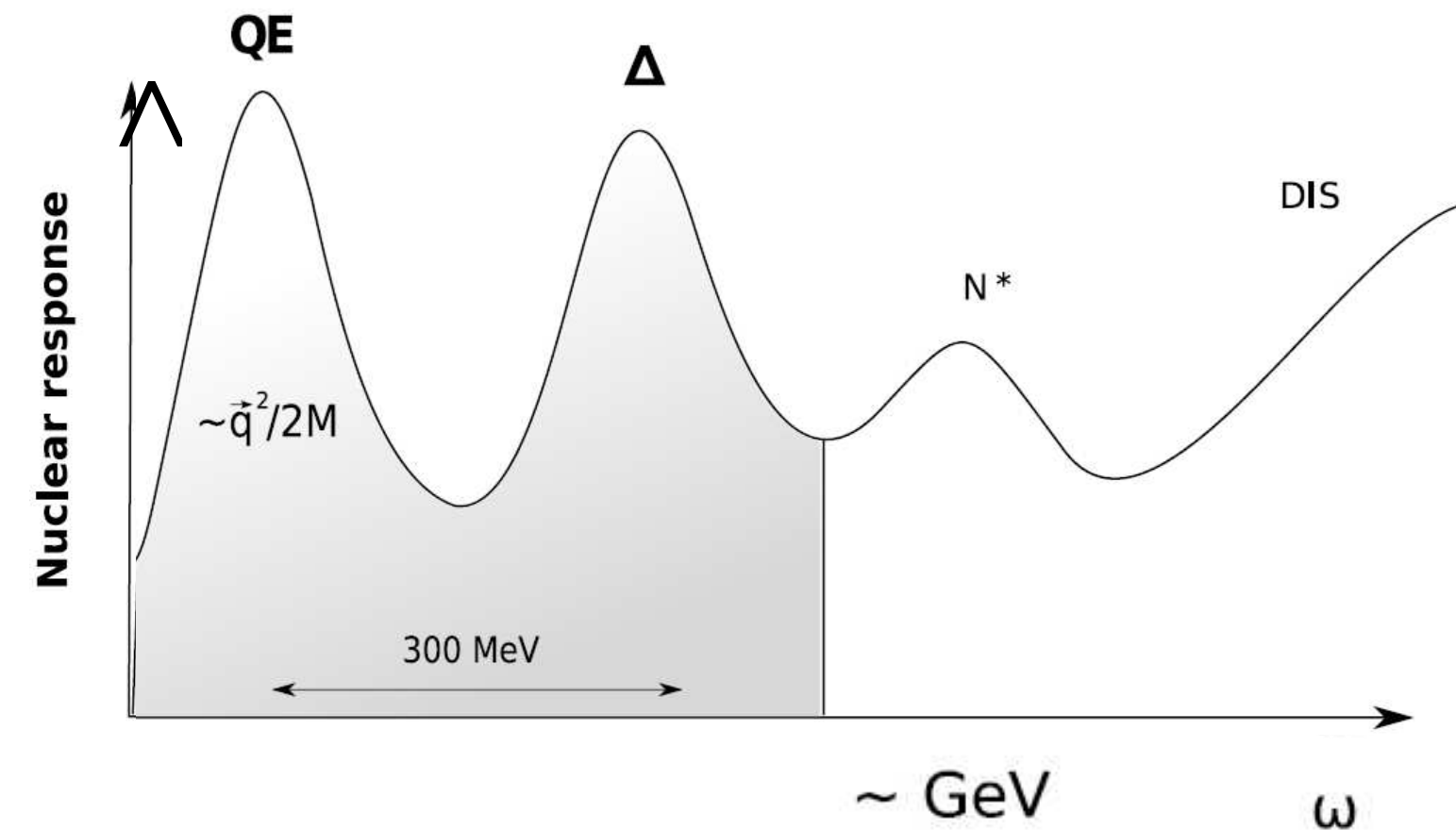


Peak-Area Verification — Quark U
(area under each Gaussian peak vs f_n from fit)



Impact of hadronic tensor calculation in lattice QCD

- First attempt towards studying resonance structure from hadronic tensor and encouraging result towards comparing with experimental data
- Next, nucleon-to-Delta transition (most dominant resonance structure for neutrino oscillation experiment)



- Calculation of DIS structure function using hadronic tensor (with Christian Zimmermann - UKY/LBL)
 - By integrating LQCD with AI-driven methodologies, we can **predict** neutrino-nucleon scattering cross sections across multiple channels with precision beyond conventional methods can achieve alone
- Direct impact on explaining and constraining various nuclear models and experimental data

Thank you!