L/T Separated Kaon Cross Sections from exclusive electroproduction at High Q^2 from Hall C at Jefferson Lab

Towards Improved Hadron Femtography with Hard Exclusive Reactions

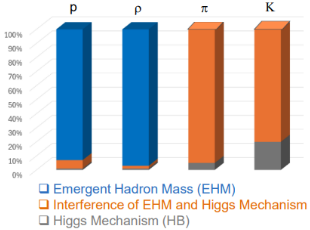
Chi-Kin Tam

On behalf of the KaonLT collaboration

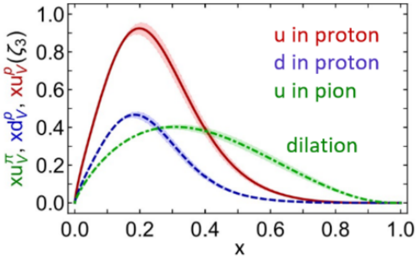
tamc@cua.edu / ckin@jlab.org

Understanding hadron structure through Mesons

THE CATHOLIC UNIVERSITY OF AMERICA



K. Raya, A. Bashir, D. Binosi, C.D. Roberts, J. Rodriguez-Quintero, Few Body Sust. **65** (**2024**) 2, 60
D. Binosi, C.D. Roberts, Z.-Q. Yao, PoS QCHCC24 (**2025**) 001



- proton reveals the large mass scale generated by QCD $\Rightarrow$ Emergent Hadron Mass (EHM).
- both π and K are massless in the chiral limit.
- Higgs mechanism leads to different mass generation for K and π due to the strange quark.

- Valence quark distribution of p, π are very different.
- direct information on the emergent hadron mass (EHM)

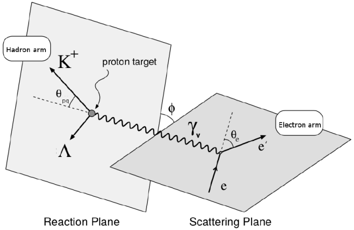
- π, K PDF difference measures how the internal momentum structure changes due to strange quark.
- Probing the PDFs of π and K mesons provides information on dynamic mass generation of hadrons and the role of the Higgs mechanism.

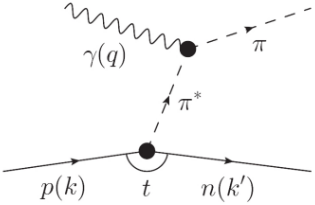
Experimental access to hadron structure through Meson Electroproduction

$Q^2 = -(p_e - p_{e'})^2$

$W = \sqrt{(p_{\text{tar}} + q)^2}$

$t = (p_K - q)^2$





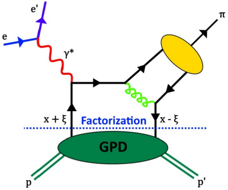
- Sullivan process : use the virtual meson cloud of the nucleon as the target
- virtual photon polarization $\epsilon = \left(1 + 2\frac{(E_e - E_{e'})^2 + Q^2}{Q^2}\tan^2\frac{\theta_{e'}}{2}\right)^{-1}$
- LT separate cross section by measuring at multiple ϵ settings

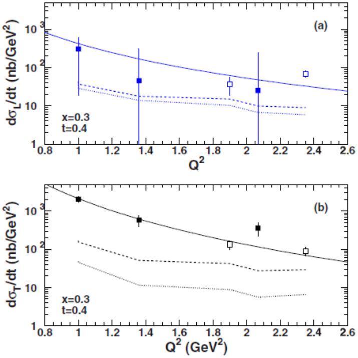
see G. Huber's talk on Wednesday.

$$2\pi\frac{d^2\sigma}{dt\,d\phi} = \frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)}\frac{d\sigma_{LT}}{dt}\cos\phi + \epsilon\frac{d\sigma_{TT}}{dt}\cos 2\phi$$

Test of QCD factorization theorem

THE CATHOLIC UNIVERSITY OF AMERICA





- Study the onset of QCD factorization through scaling test of $\sigma_L \sim Q^{-6}$ and $\sigma_T \sim Q^{-8}$ (higher threshold for K due to strange quark)
- Accessing nucleon GPDs through nucleons GPDS if σ_L is large
- Large σ_T allows the study of transversity GPDs

- Previous π data at $Q^2 > 1\,\mathrm{GeV}^2$ show σ_L-scaling but predicted higher $F_\pi(Q^2)$ than hard QCD.
- Difficult for kaon due to scarcity of data at high Q^2 and low $-t$. Previous results had to combine data from different experiments.
- Require measurements at higher Q^2 to study this transition region from hadronic to partonic degrees of freedom in exclusive processes.

Jlab data from *M. Carmignotto, et al., PRC 97, 025204*

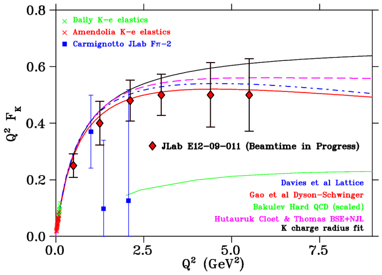
THE CATHOLIC UNIVERSITY OF AMERICA

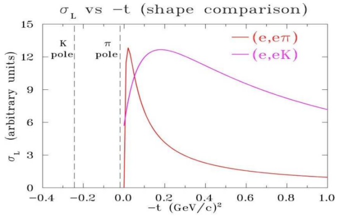
Kaon Form Factor $F_K(Q^2)$

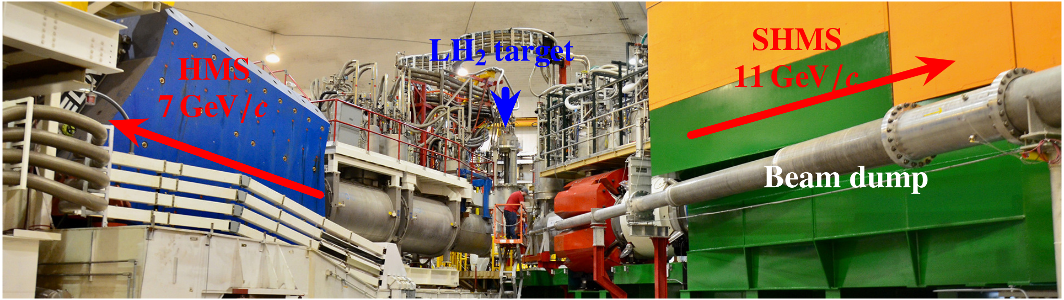
- Majority of kaon form factor $F_K(Q^2)$ data are only available at low $Q^2 < 0.13\,\mathrm{GeV}^2$ from elastic scattering experiments (*Amendolia, et al., PLB 178, (1986) 435*).
- Accessing $F_K(Q^2)$ at higher Q^2 is possible through Sullivan process where the kaon cloud within the nucleon is used $\Rightarrow$ Directly testing the validity for $F_K(Q^2)$ extraction.
- Is the kaon pole contribution dominant at low $-t$? Extract $F_K(Q^2)$ through the longitudinal cross section σ_L.
- As the K^+ pole is further away from the physical region than the π^+ pole, a test for the dominance of the kaon pole is required.

$$\frac{d\sigma_L}{dt} \sim \underbrace{\frac{-t}{(t-M_K^2)^2}}_{\text{kaon pole}} g^2_{K\Lambda N} Q^2 F_K^2(Q^2)$$

*$F_\pi(Q^2)$ are successfully extracted from σ_L at $-t$ close to $-t_{\min}$ in pion data.

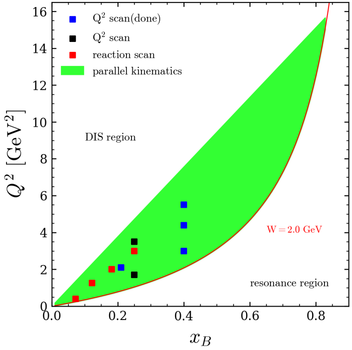






- E12-09-011 : First HMS-SHMS coincidence experiment in Hall C at 12 GeV era.
- High Momentum Spectrometer (HMS) and Super High Momentum Spectrometer (SHMS) for detecting scattered electrons and produced hadrons.
- SHMS allows small angle (5.5° – 40°) in the arms to achieve desirable kinematics.

Kinematics Summary



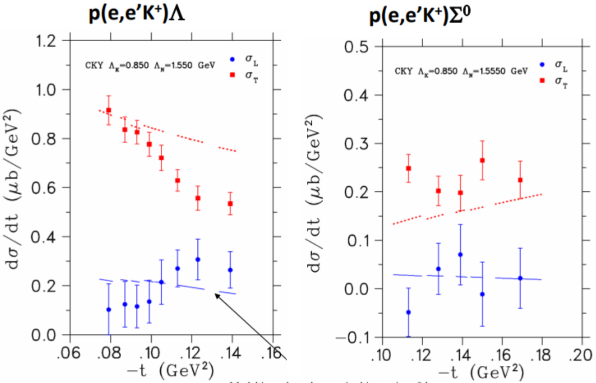
E [GeV]	Q^2 [GeV^2]	W [GeV]	$-t_{\rm min}$ [GeV^2]	x_B	ϵ
10.6 8.2	5.5	3.02	0.50	**0.40**	0.53 0.18
10.6 8.2	4.4	2.74	0.51	**0.40**	0.72 0.48
10.6 8.2	3.0	2.32	0.53	**0.40**	0.88 0.57
10.6 8.2	3.0	3.14	0.22	**0.25**	0.67 0.39
10.6 8.2	2.115	2.95	0.21	**0.21**	0.79 0.25
4.9 3.8	0.5	2.4	0.07	**0.09**	0.67 0.40

- Achieve high Q^2 and $W > 2\,\text{GeV}$ above resonance region for reliable K^+ measurements
- Measure cross sections at fixed $-t = 0.5\,\text{GeV}^2$, $x_B = 0.4$ allows scaling studies for QCD factorization.
- Form factor extraction from separated cross sections σ_L if pole dominance is observed at low $-t$.
- Reaction mechanism studies at low Q^2 through $K^+\Sigma^0/K^+\Lambda$ cross section ratio.
- Phase 2 runs in next few years will provide further evidence.

Preliminary KaonLT results at low Q^2

$$\frac{\sigma_L(\gamma^* p \rightarrow K^+\Sigma^0)}{\sigma_L(\gamma^* p \rightarrow K^+\Lambda^0)} \sim \frac{g^2_{K\Lambda N}}{g^2_{K\Sigma^0 N}}$$

- Kaon-pole dominance can be tested by measuring the ratio of the longitudinal cross sections for $K^+\Lambda$ and $K^+\Sigma^0$ production.
- model dependent extraction of form factor $F_K(Q^2)$ from σ_L.
- Preliminary results from the KaonLT experiment at $Q^2 = 0.5\,\text{GeV}^2$, $W = 2.40\,\text{GeV}$
- What can we learn from the higher Q^2 ?



Model is evaluated at precise kinematics of data.
Discontinuities indicate change in (Q^2,W) for each t-bin.

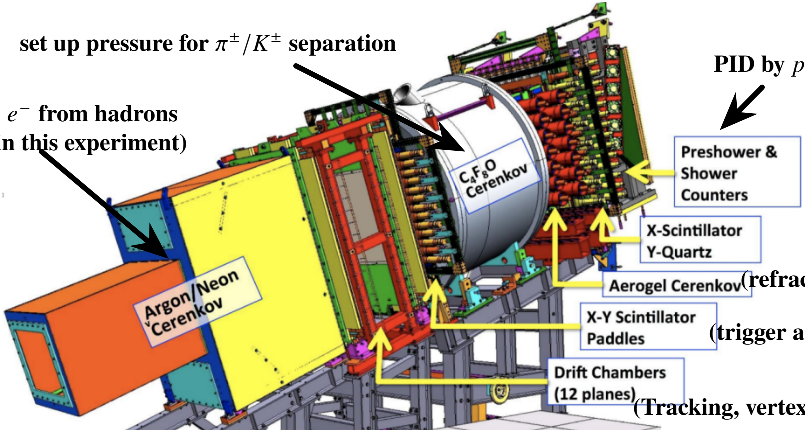
Results from A. Hamdi (U. Regina)
Publication in preparation

THE CATHOLIC UNIVERSITY OF AMERICA

set up pressure for $\pi^{\pm}/K^{\pm}$ separation

PID by p/E ratio

separates e^{-} from hadrons
(not used in this experiment)



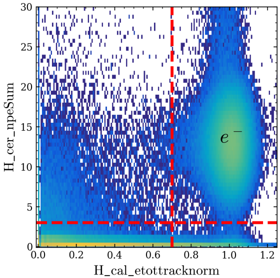
(refractive material)

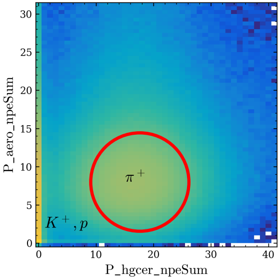
(trigger and TOF)

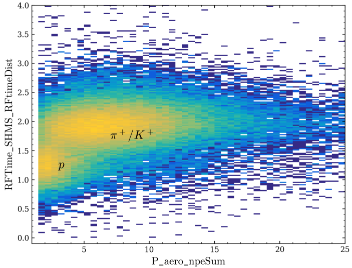
(Tracking, vertex reconstruction)

S. Ali, et al, 10.1016/j.nima.2025.171070

Particle Identification



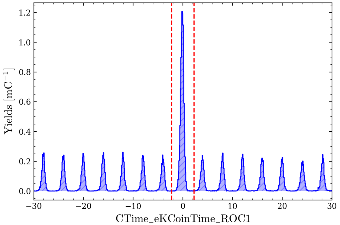


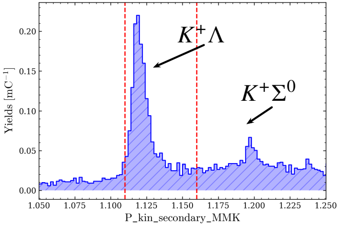


$Q^2 = 3.0\,\text{GeV}^2$, $W = 3.14\,\text{GeV}$
Center, Low $\epsilon \approx 0.3$

- HMS Aerogel Cherenkov n.p.e. > 1.5 + calorimeter $p/E > 0.7 \Rightarrow$ electron selection
- SHMS Aerogel Cherenkov n.p.e. > 2
- SHMS Heavy gas Cherenkov n.p.e. = 0 $\Rightarrow$ separates K^+ from π^+
- In some runs, RF time-of-flight (normalized to 1 bunch spacing) allows further separation of K^+ from protons

THE CATHOLIC UNIVERSITY OF AMERICA





$Q^2 = 3.0\,\mathrm{GeV}^2$, $W = 3.14\,\mathrm{GeV}$
Center, Low $\epsilon \approx 0.3$

Coincidence time

- $t_{\mathrm{coin}} = t_{\mathrm{SHMS}} - t_{\mathrm{HMS}} \in (-2.0, 2.0)$ ns $\Rightarrow$ 1 bunch spacing window selects true coincidence events
- Random coincidence events are selected from the sidebands with 3 bunch spacing windows each side.

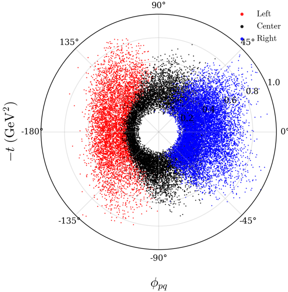
Missing mass cut

$$\mathrm{M}_X = \sqrt{(E_e + m_p - E_{e'} - E_{K^+})^2 - (\vec{p}_e - \vec{p}_{e'} - \vec{p}_{K^+})^2}$$

- $Q^2 = 3.0\,\mathrm{GeV}^2$, $W = 3.14\,\mathrm{GeV}$, Center, Low $\epsilon \approx 0.3$
- Nominal range of $K^+\Lambda$ is selected by varying the missing mass cut on both side until a stable separated cross section is obtained.
- Simultaneous measurement of Λ and Σ^0 hyperons is possible but only with abundant statistics in low Q^2.

Experimental coverage in $-t/\phi_{pq}$

Setting	$E = 8.2$ GeV, $\epsilon = 0.39$ HMS	$E = 8.2$ GeV, $\epsilon = 0.39$ SHMS	$E = 10.6$ GeV, $\epsilon = 0.67$ HMS	$E = 10.6$ GeV, $\epsilon = 0.67$ SHMS
Left	25.952°	9.973°	14.987°	12.523°
Center	25.952°	6.973°	14.487°	9.473°
Right	–	–	14.487°	6.558°



$Q^2 = 3.0\,\text{GeV}^2$, $W = 3.14\,\text{GeV}$
Center, High $\epsilon \approx 0.67$

$$2\pi \frac{d^2\sigma}{dt\,d\phi_{pq}} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT}}{dt} \cos\phi_{pq} + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi_{pq}$$

virtual photon polarization $\epsilon = \left(1 + 2\frac{(E_e - E_{e'})^2 + Q^2}{Q^2} \tan^2 \frac{\theta_{e'}}{2}\right)^{-1}$

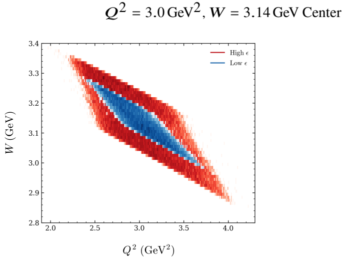
- Each (Q^2, W, ϵ) setting are subdivided into three SHMS angles to maximize ϕ_{pq} coverage.
- Complete ϕ_{pq} coverage is required for reliable Rosenbluth separation of σ_L and σ_T.
- Yields from overlapping coverage are taken as weighted averages to reduce systematic uncertainties.

Experimental coverage in $Q^2, W, -t$

THE CATHOLIC UNIVERSITY OF AMERICA

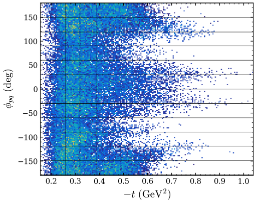
Common phase space in Q^2, W

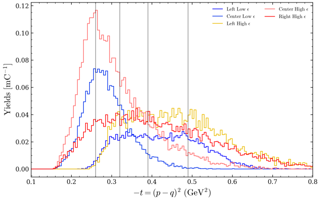
- LT separation requires common phase space in all ϵ settings.
- Boundaries in each setting are defined by kinematics and acceptance, and are estimated by a ”diamond” fit.
- Common phase space is defined by the overlapping region of the diamonds in all settings.



Optimal binning in $-t$

- $-t/\phi_{pq}$ bins are determined by requiring roughly equal statistics in each bin.
- Averaged kinematics are calculated for each bin, and used in cross section models.
- Uniform binning in ϕ_{pq}.



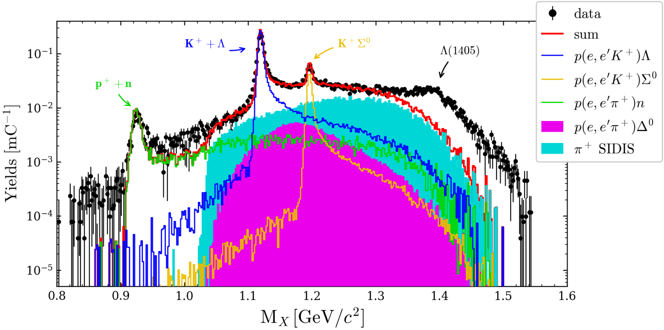


Background Contributions

THE CATHOLIC UNIVERSITY OF AMERICA

Physics background

- Pion contamination from SIDIS, $p(e, e'\pi)n$, Δ^0 are significant
- Kaon SIDIS mostly out of range
- diffractive ρ production not accounted for yet
- Visible excited states $\Lambda(1405)$ but do not contribute much under $p(e, e'K^+)\Lambda$



Random, dummy target

- Random coincidence ($\sim 5\%$)
- Aluminum frame of the target cell ($\sim 2\%$)

- Simultaneous fit to background process and isolate $K^+\Lambda$ yields
- Repeat for all $-t, \phi$ bins

$Q^2 = 3.0\,\text{GeV}^2$, $W = 3.0\,\text{GeV}$
Center Low $\epsilon = 0.39$

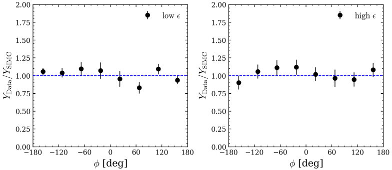
Rosenbluth Separation (Iterative approach)

- Prepare background subtracted yields for each $(-t, \phi)$ bin
- Propose functional form for structure functions $\sigma_{L,T,LT,TT}$
- Perform Self-consistent iterative method
  1. Compare data yields to SIMC
  2. Compute unseparated cross section $\sigma_{\mathrm{EXP}} = \dfrac{Y_{\mathrm{data}}}{Y_{\mathrm{simc}}} \cdot \sigma_{\mathrm{model}}$
  3. Extract separated cross sections through a simultaneous fit at each $-t$ bin
  4. Update the model paramters by fitting $\sigma_{L,T,LT,TT}(-t)$

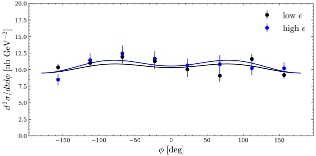
$$2\pi \frac{d^2\sigma}{dt\,d\phi} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT}}{dt} \cos\phi + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi$$

- Repeat until convergence is reached (e.g. $\Delta\sigma_{L,T,LT,TT} < 1\%$).

Yield Ratios ≈ 1



$Q^2 = 4.4\ \mathrm{GeV}^2$, $W = 2.74\ \mathrm{GeV}$
$-t_{\mathrm{center}} = 0.55\ \mathrm{GeV}^2$



SIMC Simulation

THE CATHOLIC UNIVERSITY OF AMERICA

- Simulate spectrometer acceptance, optics and reconstruct event vertex in the same way as data
- Radiation processes, multiple scattering, ionization energy loss and K^+ decay are taken into account
- Events are weighted by cross section model which depends on kinematic variables

$$\sigma_L = w \cdot p_0 \cdot \frac{t}{t^2 + M_K^2} \exp(-p_1 t)$$

$$\sigma_T = w \cdot p_2 \cdot \exp(-p_3 t)$$

$$\sigma_{LT} = w \cdot p_4 \cdot \exp(-p_5 t) \sin\theta^*$$

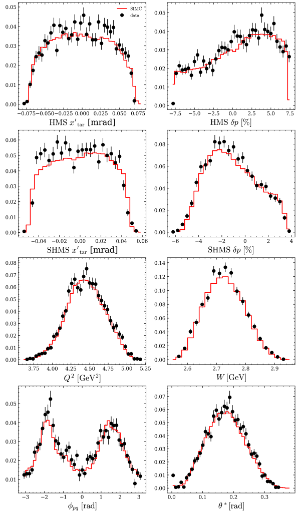
$$\sigma_{TT} = w \cdot p_6 \cdot \exp(-p_7 t) \sin^2\theta^*$$

$$w(W) = \frac{1}{(W^2 - M_p^2)^2}$$

- Agreement between data and SIMC in both HMS, SHMS acceptance and kinematics are required for reliable cross section extraction !

[link to SIMC code repository]

$Q^2 = 4.4\,\mathrm{GeV}^2$, $W = 2.74\,\mathrm{GeV}$
Center, Low $\epsilon \approx 0.5$



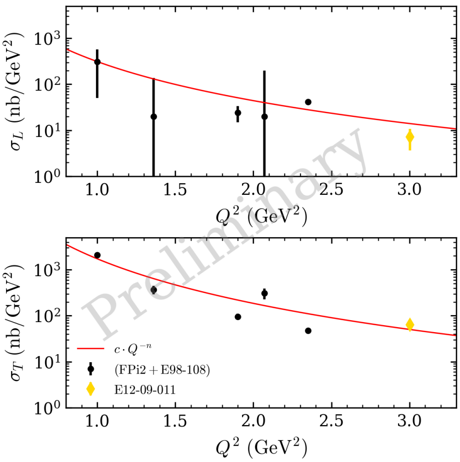
Q^{-n} scaling test at $x_B = 0.25, 0.3$

- L/T separated cross sections as a function of Q^2 for testing the onset of factorization theorem

E [GeV]	Q^2 [GeV^2]	W [GeV]	$-t_{min}$ [GeV^2]	x_B	ϵ
10.6 8.2	5.5	3.02	0.50	0.40	0.53 0.18
10.6 8.2	4.4	2.74	0.51	0.40	0.72 0.48
10.6 8.2	3.0	2.32	0.53	0.40	0.88 0.57
10.6 8.2	3.0	3.14	0.22	0.25	0.67 0.39
10.6 8.2	2.115	2.95	0.21	0.21	0.79 0.25

- L/T separated cross sections σ_L and σ_T at $x_B = 0.25$
- Scaling relation fitted with existing Jlab 6GeV data (`FPi2` and `E93-018`)
- Results show σ_L scales as Q^{-6} and σ_T scales as Q^{-8}
- More data points will be taken in Phase 2 Kaon LT experiment.

$p(e, e'K^+)\Lambda$

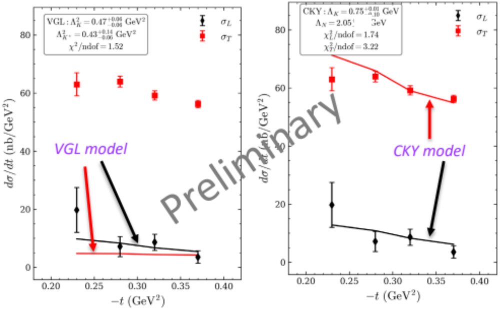


Investigating the $-t$-dependence : Form factor extraction

- L/T separated cross sections as a function of t up to the largest Q^2 to investigate the reaction mechanism towards kaon form factor extractions

E [GeV]	Q^2 [GeV^2]	W [GeV]	$-t_{min}$ [GeV^2]	x_B	ϵ
10.6 8.2	5.5	3.02	0.50	0.40	0.53 0.18
10.6 8.2	4.4	2.74	0.51	0.40	0.72 0.48
10.6 8.2	3.0	2.32	0.53	0.40	0.88 0.57
10.6 8.2	3.0	3.14	0.22	0.25	0.67 0.39
10.6 8.2	2.115	2.95	0.21	0.21	0.79 0.25

$\mathbf{p(e, e'K^+)\Lambda}$

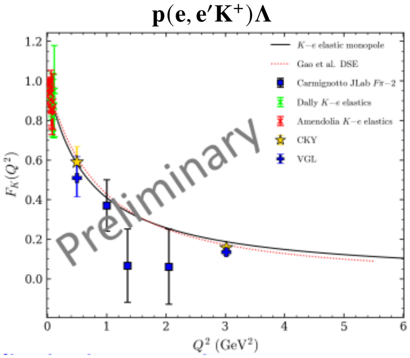


- VGL model *(Vanderhaeghen, Guidal, Laget, PRC 57(1998)1454)* : replace pole-like propagators with Regge propagators to describe the interaction with exchange of a family of particles with the same quantum numbers.
- CKY *(B-G Yu, T.K. Choi, K.J. Kong (private communication 2023))* : including nucleon pole in the s-channel with charge form factor
- σ_T is better described by the CKY model compared to the VGL model with prescription $\Lambda_K \approx \Lambda_{K^*}$.

- L/T separated cross sections as a function of t up to the largest Q^2 to investigate the reaction mechanism towards kaon form factor extractions

E [GeV]	Q^2 [GeV^2]	W [GeV]	$-t_{min}$ [GeV^2]	x_B	ϵ
10.6 8.2	5.5	3.02	0.50	0.40	0.53 0.18
10.6 8.2	4.4	2.74	0.51	0.40	0.72 0.48
10.6 8.2	3.0	2.32	0.53	0.40	0.88 0.57
10.6 8.2	3.0	3.14	0.22	0.25	0.67 0.39
10.6 8.2	2.115	2.95	0.21	0.21	0.79 0.25

$$F_K(Q^2) = \frac{1}{1 + Q^2/\Lambda_K^2}$$

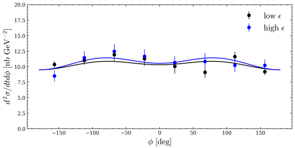


S.R. Amendolia et al., PLB 178 (1986) 435
E.B. Dally et al., PRL 45 (1980) 232.

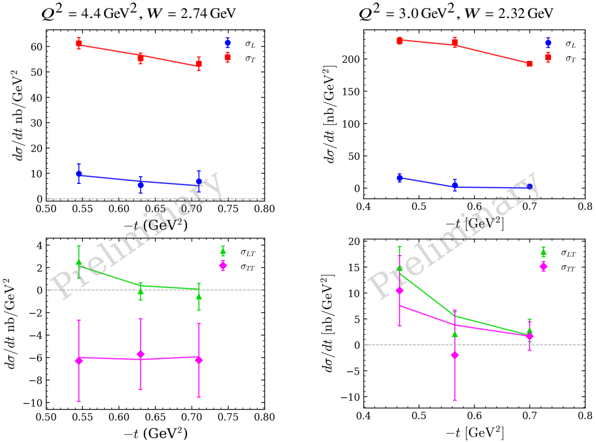
- Form Factor is extracted by comparing σ_L with model incorporating K^+ production and spectator nucleons.
- Preliminary results show that the extracted kaon form factor is consistent with the previous low Q^2 elastic data in Amendolia, et al.

”Surprisingly” large σ_T

- Separate cross sections as a function of t to test contribution of σ_L and σ_T to the cross section at $x_B = 0.4$.
- $\sigma_T \approx 60 \gg \sigma_L \approx 10\,\text{nb/GeV}^2$ at $Q^2 = 4.4\,\text{GeV}^2$.
- $\sigma_T \approx 200 \gg \sigma_L \approx 10\,\text{nb/GeV}^2$ at $Q^2 = 3.0\,\text{GeV}^2$.
- Interference terms are small but comparable to σ_L.



$Q^2 = 4.4\,\text{GeV}^2$, $W = 2.74\,\text{GeV}$
$-t_{\text{center}} = 0.55\,\text{GeV}^2$

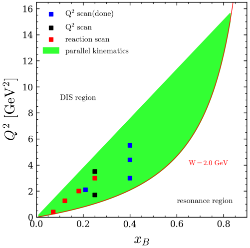


KaonLT Phase 1 Preliminary Results

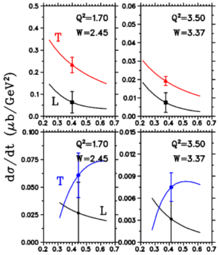
- [x] Extract Kaon cross sections $\sigma_{L,T,LT,TT}$ in high $Q^2 = 3.0, 4.4$ GeV^2 at $x_B = 0.25, 0.4$.
- [x] Preliminary results show that $\sigma_L \sim Q^{-6}$ and $\sigma_T \sim Q^{-8}$ in $x_B =\sim 0.3$.
- [x] Kaon Electromagnetic Form Factor F_K are extracted from σ_L using the VGL/CKY Regge models with the monopole prescription. The results are consistent with the charged radius line from Amenloia.
- [x] Significant σ_T contribution is observed in all settings, with $x_B = 0.4$ the most prominent.
- [ ] Systematic uncertainties from cuts, acceptance, and radiative corrections are under study.
- [ ] Remaining Phase 1 data at $Q^2 = 2.1, 5.5$ GeV^2 will be analyzed shortly.

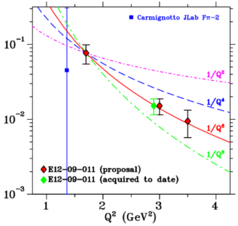
KaonLT Outlook

- KaonLT Phase 1 data: Complete higher Q^2 analysis.



projected data





- KaonLT Phase 2 experiment is expected to run as early as 2027.
  - Performing L/T separations for the Q^2 scan at $x_B = 0.25$ to validate the hard–soft reaction mechanism.
  - Improving the uncertainties of possible 12 GeV kaon form-factor extractions.
  - Establish a high-precision data base for $K^+\Lambda$ and $K^+\Sigma^0$ channels.

Our Team and Opportunities

Spokespersons (PionLT/KaonLT): Tanja Horn (CUA), Garth Huber (U. Regina), Dave Gaskell (JLab), Pete Markowitz (FIU)

Postdocs: Chi Kin Tam (CUA), Abdennacer Hamdi (U. Regina)
Former postdocs: Vladimir Berdnikov (now at JLab), Stephen Kay (now at U. of York)

PionLT and KaonLT collaborators from PAC proposals

Graduate Students:

KaonLT(E12-09-011)	PionLT (E12-19-006)
Richard Trotta (CUA, PhD 2024, now at UVA)	Nathan Heinrich (U. Regina)
Ali Usman (U. Regina, PhD 2024)	Muhammad Junaid (U. Regina)
Vijay Kumar (U. Regina, PhD 2025)	Kathleen Ramage (U. of Glasgow)
Alicia Postuma (U. Regina)	Sameer Jain (CUA)
	Nermin Sadoun (U. Regina)

Example of non-student driven Analyses: Gabriel Niculescu (JMU), Ioana Niculescu (JMU)

KaonLT	$p(e,e'K^+)\Sigma/p(e,e'K^+)\Lambda$ separated cross section ratios	Q^2=0.5, 2.115, 3.0, 4.4, 5.5	$g_{KN\Lambda}/g_{KN\Sigma}$ coupling constant ratio versus t. This is listed as one of our K^+-pole tests.	Requires extraction of Σ L/T/LT/TT cross sections from calibrated K^+ data.

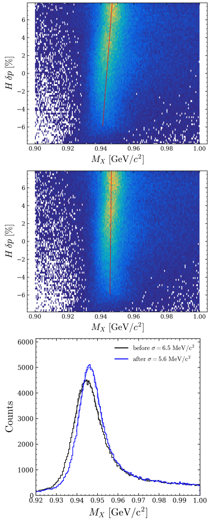
Thank you for your attention !

Link to full list of physics topics

Link to KaonLT experiment wiki

Backup

HMS Power Supply Saturation Issue

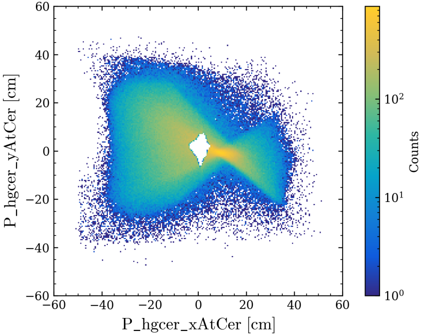


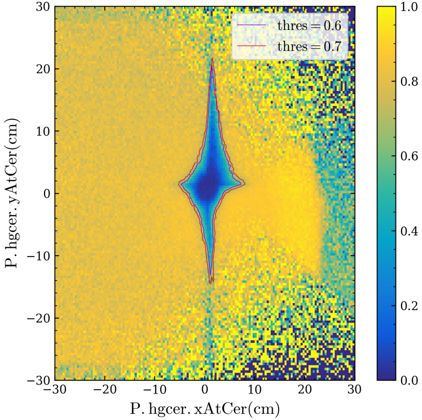
- HMS power supply saturation during experiment ⇒ Corrected by decorrelating reconstructed momentum from Missing Mass

$$\delta p^{\text{corr}}_{\text{HMS}} = \delta p^{\text{recon}}_{\text{HMS}} + \alpha \cdot x'_{\text{fp}}$$

- resolution improvement from $\sigma_{\text{MM}} \sim 0.7$ MeV to $\sigma_{\text{MM}} \sim 0.6$ MeV
- momentum-dependent correction applied to all runs, with α determined from the slope and resolution of $\pi^+ + n$ peak

THE CATHOLIC UNIVERSITY OF AMERICA





- Low efficiency region at the center of `HgCer` due to the mirror support structure
- relative efficiency $N_{\mathrm{did}}/\mathrm{N}_{\mathrm{should}}$ where denominator includes all detector cuts except the `HgCer` cut.
- Start at the center, radiates out and find the bin passing a certain threshold in efficiency.

- Each event in `SIMC` is weighted by a cross section model which depends on kinematic variables.
- Functional forms of the cross section model are not unique, what matters is that the model describes the data yields well in the kinematic region of interest.

Cross section model

$$\sigma_L = w \cdot p_0 \cdot \frac{t}{t^2 + M_K^2} \exp(-p_1 t)$$
$$\sigma_T = w \cdot p_2 \cdot \exp(-p_3 t)$$
$$\sigma_{LT} = w \cdot p_4 \cdot \exp(-p_5 t) \sin\theta^*$$
$$\sigma_{TT} = w \cdot p_6 \cdot \exp(-p_7 t) \sin^2\theta^*$$
$$w(W) = \frac{1}{(W^2 - M_p^2)^2}$$

- pole term embedded in σ_L
- $\sin\theta^*$ term in σ_{LT} and $\sin^2\theta^*$ term in σ_{TT} are required for vanishing of the interference terms at parallel kinematics.
- $w(W)$ dependence which describes existing data.

Linear Spline model

- Instead of encoding the t-dependence of the cross section model in a functional form, we can also use a **linear spline model** to describe the t-dependence of the cross section.
- Simple functional form is usually preferred, but in `KaonLT` it is difficult to determine the origin of inconsistencies between data and simulation
- Completely captures the t-dependence, or even overfitting the data $\Rightarrow$ If the complicated model cannot describe the data, there must be issues in data yields.

$$F(t) = \sum_{i=0}^{N-1} \beta_i B_i(t), \text{ where } \quad N = \text{number of knots}, \quad B_i(t) = \frac{t - t_i}{t_{i+1} - t_i}$$

$$\text{and} \quad t_i \le t < t_{i+1}, \quad i = 0, 1, \ldots, N-2$$

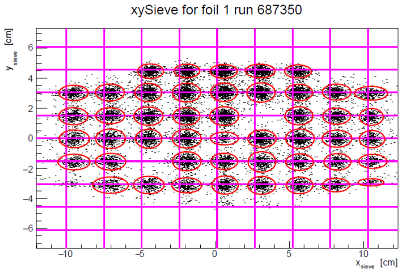
- Completely linearized the model speeds up the optimization !
- Instead of running iteration, simply solve the linear system of equations to find the optimal β_i coefficients.

$$x'_{\rm tar} = \sum_{i,j,k,l,m} X'_{ijklm} x^{i}_{\rm fp} x'^{j}_{\rm fp} y^{k}_{\rm fp} y'^{l}_{\rm fp} x^{m}_{\rm tar},$$

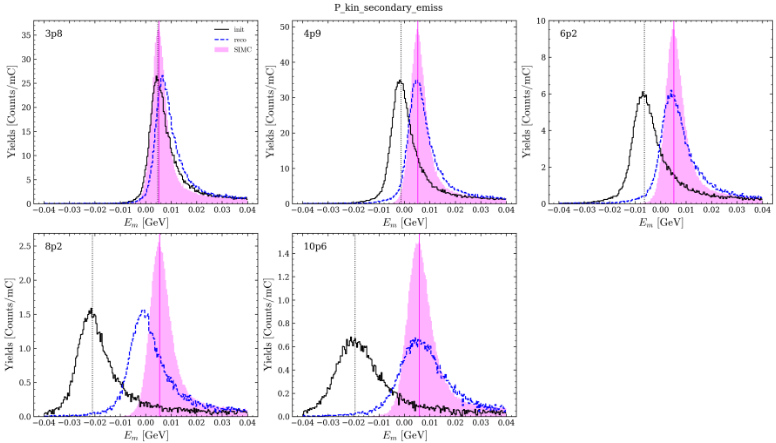
$$y_{\rm tar} = \sum_{i,j,k,l,m} Y_{ijklm} x^{i}_{\rm fp} x'^{j}_{\rm fp} y^{k}_{\rm fp} y'^{l}_{\rm fp} x^{m}_{\rm tar},$$

$$y'_{\rm tar} = \sum_{i,j,k,l,m} Y'_{ijklm} x^{i}_{\rm fp} x'^{j}_{\rm fp} y^{k}_{\rm fp} y'^{l}_{\rm fp} x^{m}_{\rm tar},$$

$$\delta = \sum_{i,j,k,l,m} D_{ijklm} x^{i}_{\rm fp} x'^{j}_{\rm fp} y^{k}_{\rm fp} y'^{l}_{\rm fp} x^{m}_{\rm tar}.$$

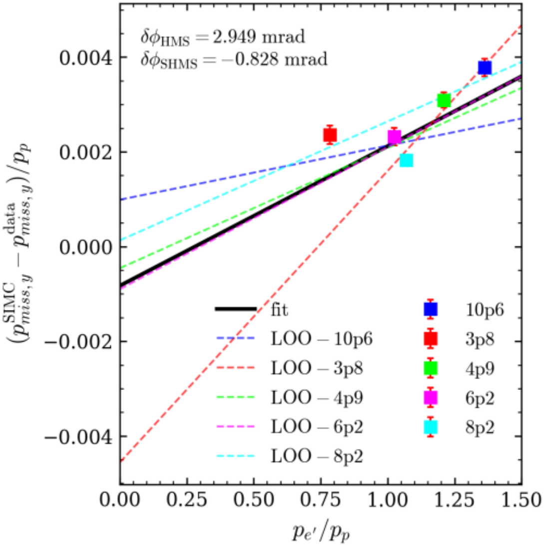


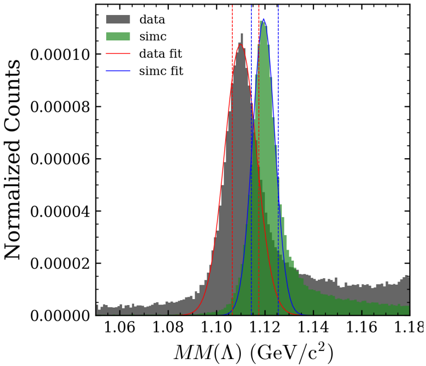
Heepcheck



- W, $P_{miss,x}$, $P_{miss,z}$ and E_{miss}.

Out-of-plane offsets





Problem

- Imperfect optic calibration leads to a shift in the missing mass peak $< 10\,\mathrm{MeV}$.
- Kinematics offsets found by HeepCheck is not perfect since it is an over-constrained problem.

Solution

- Apply a missing mass shift correction to each setting
- This correction is accompanied by a shift in $-t$ according to conservation laws while assuming the same Q^2 and W.

Systematics

THE CATHOLIC UNIVERSITY OF AMERICA

Source	pt-to-pt	t-correlated	Scale (earlier)	Scale (later)
Acceptance	0.4	0.4	2.0	1.0
PID	0.4	1.0	0.5	–
Coincidence Blocking	0.2	–	–	–
Tracking efficiency	0.1	0.1	1.5	1.5
Charge	0.2	0.5	0.5	–
Target thickness	0.2	0.8	0.8	–
Kinematics	0.4	1.0	–	–
Kaon Absorption	0.5	0.5	0.5	–
Kaon Decay	1.0	3.0	3.0	–
Radiative Corrections	0.1	0.4	2.0	2.0
Monte Carlo Model	0.2	1.0	0.5	0.5

Systematics : Missing mass range