

Fundamental tensor charge from pion electronic decays

with prospects for using machine learning

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22 July 2026



5th Workshop on Hadron Femtography
University of Virginia, Charlottesville, Virginia
20 – 24 July 2026

Topics discussed in this talk

Why we study the pion electronic decays (brief motivation)

The PiBeta/PEN experimental method and challenges

Pion radiative decays

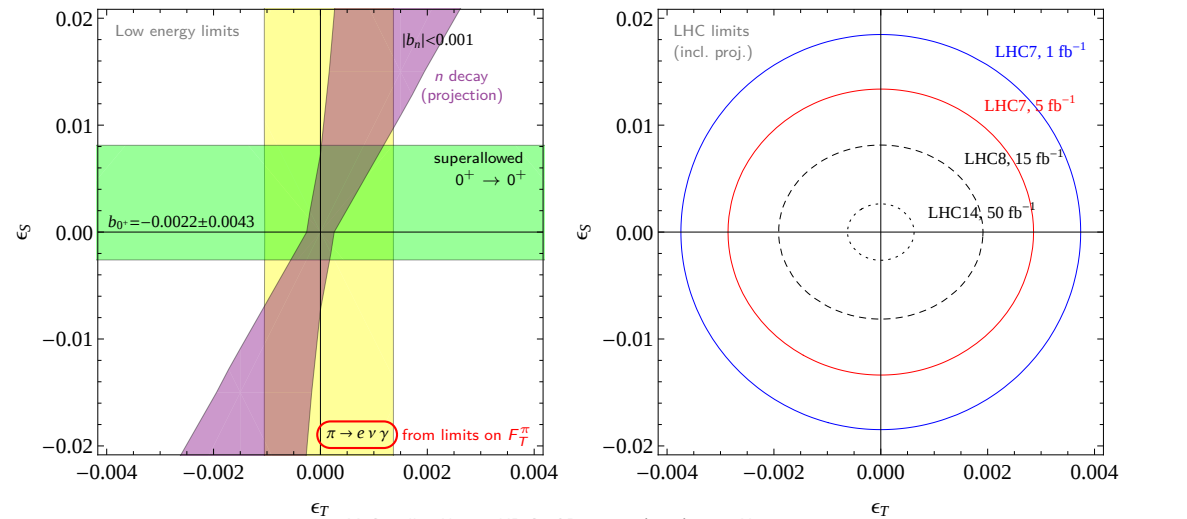
Improvements with machine learning (plans)

Near term plans



Why focus on the pion electronic (radiative) decay?

Select limits on fund. non-SM couplings (not including recent n /nucl. results discussed by Stefan B):



Known and measured pion and muon decays

decay	B.R.	physics interest
$\pi^+ \rightarrow \mu^+ \nu$	0.9998770 (4) ($\pi_{\mu 2}$)	
$\mu^+ \nu \gamma$	$2.00 (25) \times 10^{-4}$ ($\pi_{\mu 2 \gamma}$)	
$e^+ \nu$	$1.230 (4) \times 10^{-4}$ ($\pi_{e 2}$)	$\Leftarrow$ lepton universality, beyond SM terms ($T, \dots$)
$e^+ \nu \gamma$	$7.39(5) \times 10^{-7}$ ($\pi_{e 2 \gamma}$)	$\Leftarrow$ BSM terms ($T, \dots$), form fact's: $F_A^{(\pi)}, F_V^{(\pi)}, \dots$
$\pi^0 e^+ \nu$	$1.036 (6) \times 10^{-8}$ ($\pi_{e 3}$)	$\Leftarrow$ quark-lepton universality (V_{ud}), BSM loops
$e^+ \nu e^+ e^-$	$3.2 (5) \times 10^{-9}$ ($\pi_{e 2 ee}$)	
$\pi^0 \rightarrow \gamma \gamma$	0.98798 (32)	
$e^+ e^- \gamma$	$1.198 (32) \times 10^{-2}$ (Dalitz)	$\Leftarrow \chi$ anomaly, low energy chiral parameters
$e^+ e^- e^+ e^-$	$3.14 (30) \times 10^{-5}$	
$e^+ e^-$	$6.2 (5) \times 10^{-8}$	
$\mu^+ \rightarrow e^+ \nu \bar{\nu}$	~ 1.0 (Michel)	
$e^+ \nu \bar{\nu} \gamma$	0.014 (4) (RMD)	$\Leftarrow$ beyond SM weak interaction terms
$e^+ \nu \bar{\nu} e^+ e^-$	$3.4 (4) \times 10^{-5}$	

π_{e2} decay: SM calculations, lepton universality

- Early evidence for $V - A$ nature of weak interaction.

$$R_{e/\mu}^{\pi} = \frac{\Gamma(\pi \rightarrow e\bar{\nu}(\gamma))}{\Gamma(\pi \rightarrow \mu\bar{\nu}(\gamma))} = \frac{g_e^2}{g_{\mu}^2} \frac{m_e^2}{m_{\mu}^2} \frac{(1 - m_e^2/m_{\mu}^2)^2}{(1 - m_{\mu}^2/m_{\pi}^2)^2} (1 + \delta R_{e/\mu})$$

- Modern SM calc's.:

$$R_{e/\mu}^{\pi, \text{SM}} = \begin{cases} 1.2352(5) \times 10^{-4} & \text{Marciano and Sirlin, [PRL 71 (1993) 3629]} \\ 1.2354(2) \times 10^{-4} & \text{Finkemeier, [PL B 387 (1996) 391]} \\ 1.2352(1) \times 10^{-4} & \text{Cirigliano and Rosell, [PRL 99 (2007) 231801]} \end{cases}$$

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$\sim 2.5 \times 10^{-5}$

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- ▶ Experimental world average is $23\times$ less accurate than SM calculations! $[1.2327(23) \times 10^{-4}]$
(following publication of the PiENU result: A. Aguilar-Arevalo et al., PRL 115 (2015) 071801)

Mass reach of π_{e2} decay beyond the SM (New Physics)

$$\mathcal{L}_{\text{NP}} = \left[\pm \frac{\pi}{2\Lambda_V^2} \bar{u} \gamma_\alpha d \pm \frac{\pi}{2\Lambda_A^2} \bar{u} \gamma_\alpha \gamma_5 d \right] \bar{e} \gamma^\alpha (1 - \gamma_5) \nu + \left[\pm \frac{\pi}{2\Lambda_S^2} \bar{u} d \pm \frac{\pi}{2\Lambda_P^2} \bar{u} \gamma_5 d \right] \bar{e} (1 - \gamma_5) \nu,$$

(Λ_i ... scale of NP)

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At $\Delta R_{e/\mu}^\pi / R_{e/\mu}^\pi = 10^{-3}$,
 π_{e2} decay is sensitive to:

DIRECTLY	INDIRECTLY (LOOPS)
$\Lambda_P \leq 1000 \text{ TeV}, \quad \Lambda_A \leq 20 \text{ TeV}$	$\Lambda_S \leq 60 \text{ TeV} .$

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Further interesting limits on certain SUSY extensions, as well as in the neutrino sector, become accessible at $\Delta R_{e/\mu}^\pi / R_{e/\mu}^\pi < 10^{-3}$.

Summary of PiBeta and PEN objectives

Goals of the PiBeta experiment

	Decay	$\mathcal{O}(\text{B.R.})$	Goal $\delta R/R$	Attendant SM limits
$\pi_{e3}(\gamma) :$	$\pi^+ \rightarrow \pi^0 e^+ \nu_e(\gamma)$	$R_{e3}^\pi \sim 10^{-8}$	$\sim 5 \times 10^{-3}$	CKM V_{ud} & related
$\pi_{e2\gamma} :$	$\pi^+ \rightarrow e^+ \nu_e \gamma$	$R_{e2\gamma}^\pi \sim 10^{-7}$	$\leq 1 \times 10^{-2}$	$F_A^\pi, F_V^\pi, F_T^\pi$; χ^2 PT l.e.c.
RMD:	$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$	$R_{e2\gamma}^\pi \sim 10^{-3}$	$\leq 1 \times 10^{-2}$	Michel param.: $\bar{\eta}$

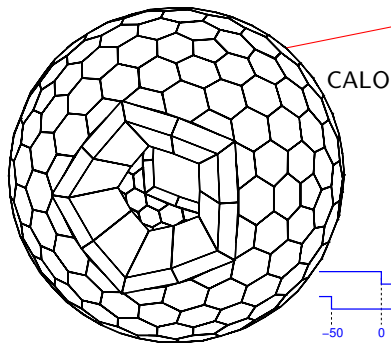
Goals of the PEN experiment:

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$\pi_{e2}(\gamma) :$	$\pi^+ \rightarrow e^+ \nu_e(\gamma)$	$R_{e2}^\pi \sim 10^{-4}$	$\sim 5 \times 10^{-4}$	lept. univ.; non- $V-A$, ...
$\pi_{e2\gamma} :$	$\pi^+ \rightarrow e^+ \nu_e \gamma$	$R_{e2\gamma}^\pi \sim 10^{-7}$	$\sim 1 \times 10^{-2}$	improve F_V^π & limit on F_T^π
RMD:	$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$	$R_{e2\gamma}^\pi \sim 10^{-6}$	$\sim 1 \times 10^{-2}$	improve $\bar{\eta}$



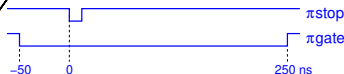
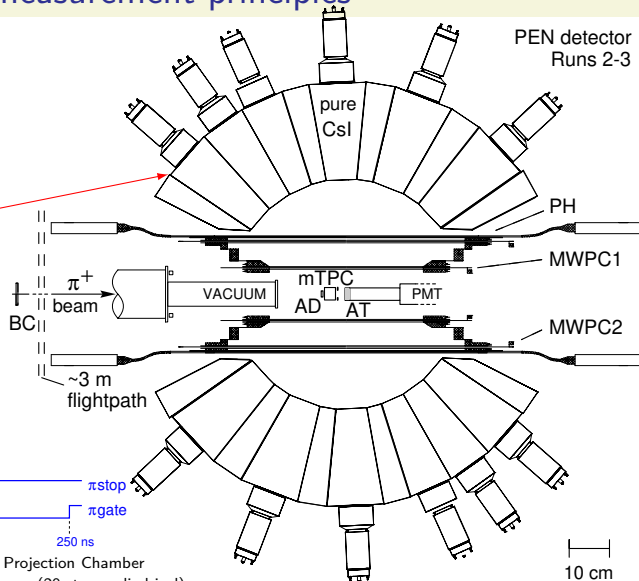
The PEN/PiBeta apparatus and measurement principles

- π E1 beam at PSI
- stopped π^+ beam
- beam tracking
- 240-elem. $12X_0$ spherical pure-CsI calo.
- tightly controlled temp/humidity/gains
- central tracking
- beam tracking
- fast-digitized wf's

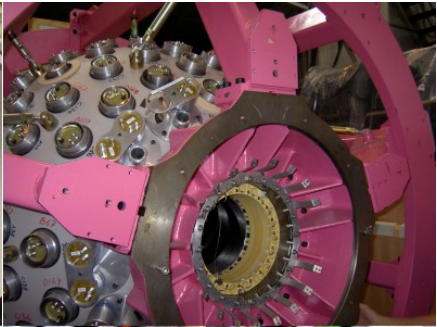
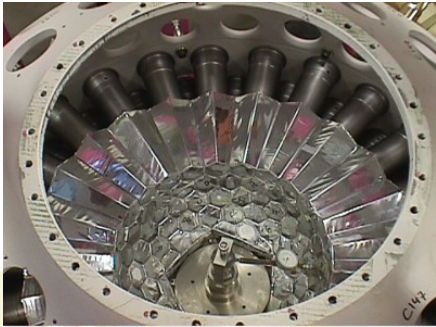


BC: Beam Counter
AD: Active Degradar
AT: Active Target

mTPC: mini-Time Projection Chamber
PH: Plastic Hodoscope (20 stave cylindrical)
MWPC: Multi-Wire Proportional Chamber (cylindrical)



A few photos of the
PiBeta/PEN
apparatus:



PEN measurement principles for $R_{e/\mu}^\pi$; key challenges

Basic principle: record pion decays at rest in a beam stopping target¹ and count each:

$$(a) \pi_{e2}(\gamma): \pi^+ \rightarrow e^+ \nu_e(\gamma), \quad \text{and} \quad (b) \pi_{\mu2}(\gamma): \pi^+ \rightarrow \mu^+ \nu_\mu(\gamma) \text{ decay event}$$

during an observation time window, and evaluate the yield ratio (a)/(b), applying corrections.

- ▶ As (a) and (b) cannot be fully identified in AT alone, use CALO and tracking detectors.
- ▶ Identify (b) through the subsequent decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu(\gamma)$.

¹A decay in flight measurement would present a wholly different set of challenges.

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Key challenges in achieving sub- 10^{-3} , or sub- 10^{-4} precision are of **systematic** nature:

- ▶ accurately **identify** processes (weak decay, hadronic interaction, etc.) for each event,
- ▶ accurately **count** and **sort** each type of decay event (without skipping/mislabeling any).

This requires **full tracking and detection** of beam and decay particles' interactions with matter.

Also required: **minimizing mass** (especially passive) in the particle path to AT/CALO, and **maximizing detection efficiency** and **resolution**: E , t and **spatial**.

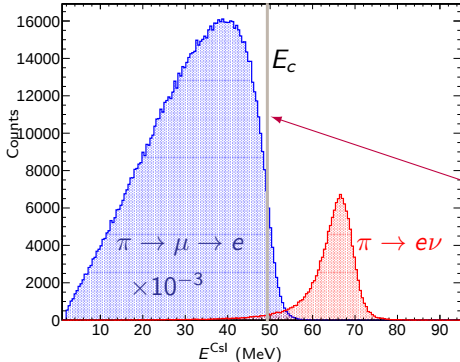
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Experimental branching ratio ($R_{e/\mu}^{\pi\text{-exp}}$)

Since: { timing gates affect number of π_{e2} and $\pi \rightarrow \mu \rightarrow e$ observations, and
 MWPC efficiency depends on energy,

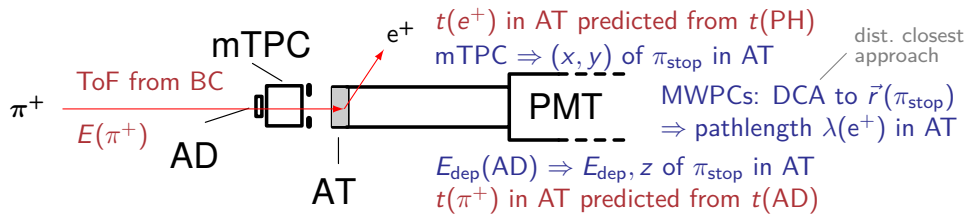
we have:

$$R_{e/\mu}^{\pi\text{-exp}} = \frac{N_{\pi \rightarrow e \nu}^{\text{peak}} (1 + \epsilon_{\text{tail}})}{N_{\pi \rightarrow \mu \nu}} \underbrace{\frac{f_{\pi \rightarrow \mu \rightarrow e}(T_e)}{f_{\pi \rightarrow e \nu}(T_e)}}_{r_f} \underbrace{\frac{\epsilon(E_{\mu \rightarrow e \nu \bar{\nu}})_{\text{MWPC}}}{\epsilon(E_{\pi \rightarrow e \nu})_{\text{MWPC}}}}_{r_{\text{PC}}} \underbrace{\frac{A_{\pi \rightarrow \mu \rightarrow e}}{A_{\pi \rightarrow e \nu}}}_{r_A} \underbrace{\frac{\epsilon_{\text{pileup}}}{\epsilon_{\delta\chi^2}}}_{r_{\text{cut}}}$$



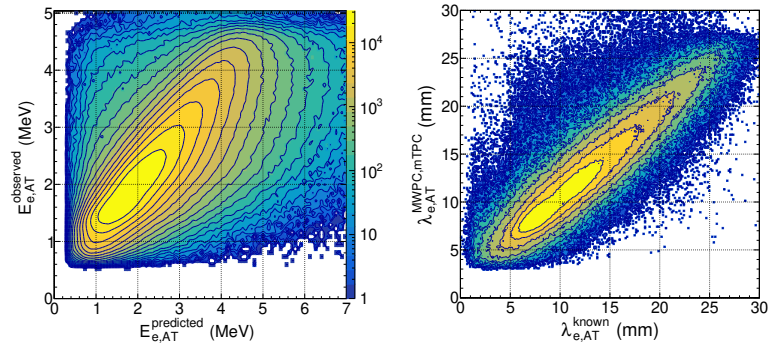
- E_c = cutoff energy
- N = number of events
- A = acceptance
- $\epsilon_{\text{tail}}(E_c)$ = tail to peak ratio
- $\epsilon(E)_{\text{MWPC}}$ = efficiency of MWPC
- $f(T_e)$ = probability from time

Discriminating π_{e2} and $\pi_{\mu2}$ in active target (AT)

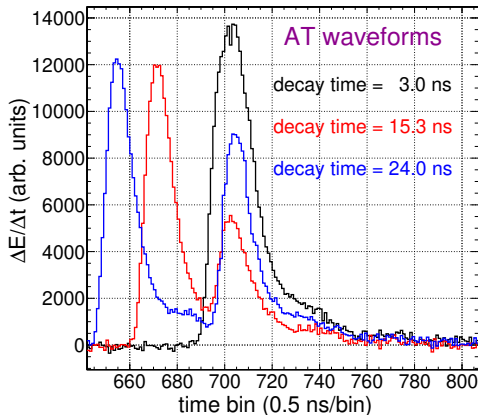
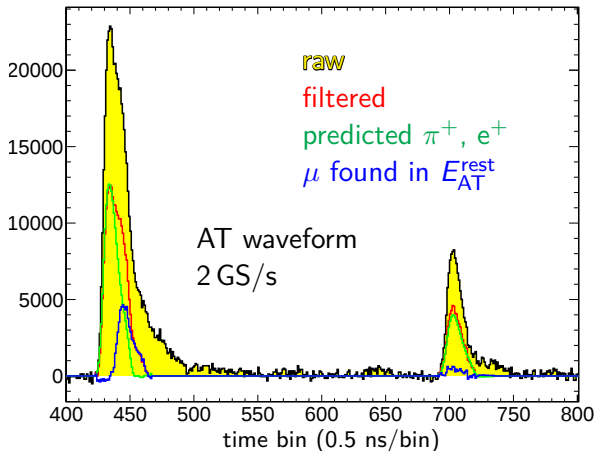


Predicted/known and reconstr.
 π^+ , e^+ energies/pathlengths in
AT agree VERY well:

$\Rightarrow E$ and t predictions are used
for $\pi_{e2}/\pi_{\mu2}$ discrimination.

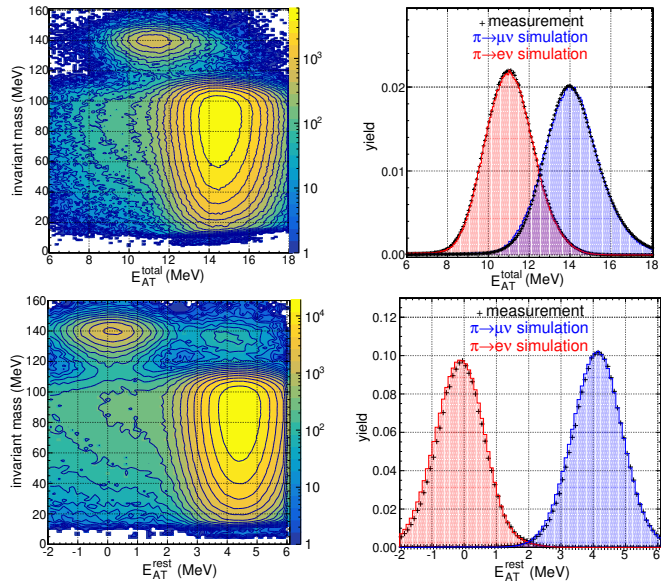


Decay type discrimination and target waveforms

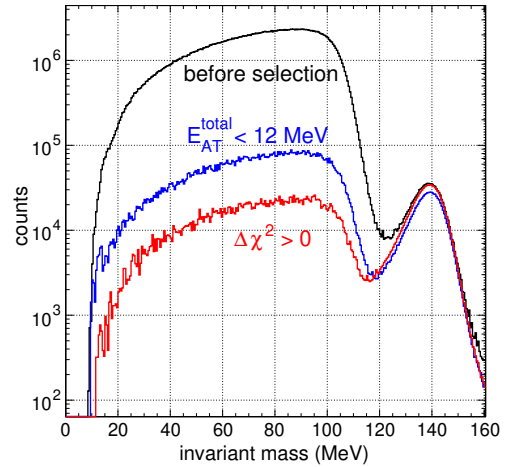


$E_{AT}^{\text{rest}} = E_{AT}^{\text{total}} - (E_{\pi,AT}^{\text{predicted}} + E_{e,AT}^{\text{predicted}})$ is evaluated binwise, and scanned to test for presence of μ peak:
 $\Delta\chi^2 \propto \chi_{3\text{-peak}}^2 - \chi_{2\text{-peak}}^2$ (normalized); calculated binwise for each event/waveform (no fit!).

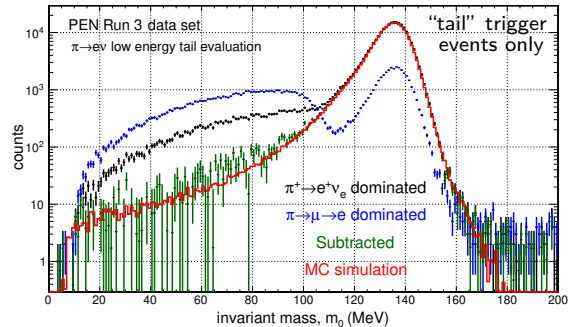
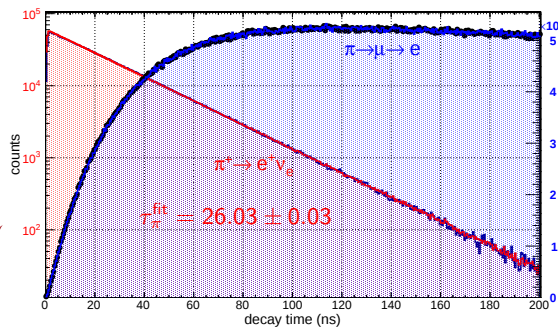
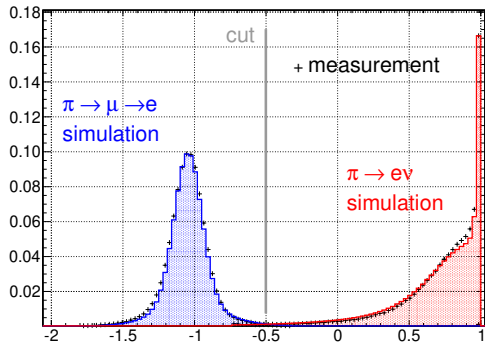
Select stages of suppressing $\pi_{\mu 2}$ decays in the π_{e2} “tail”



Recall: $E_{AT}^{rest} = E_{AT}^{total} - (E_{\pi,AT}^{predicted} + E_{e,AT}^{predicted})$
 $\Delta\chi^2$ sensitive to presence of μ peak



Results of $\Delta\chi^2 \pi_{e2}/\pi_{\mu2}$ sorting



Exp. subtracted "tail" not accurate enough!
 $\Rightarrow$ must rely on **simulations** for final $\times 4$ factor.



Radiative electronic ($\pi_{e2\gamma}$) decay:

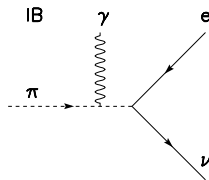
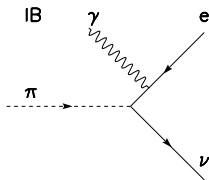
$$\pi^+ \rightarrow e^+ \nu_e \gamma$$

$$BR_{\text{non-IB}} \sim 10^{-7}$$

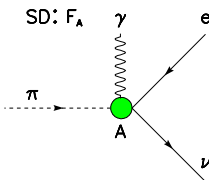
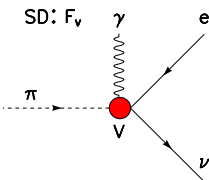
(unavoidable part of $\pi \rightarrow e\nu$ decay)

Physics of $\pi^+ \rightarrow e^+ \nu \gamma$ (RPD):

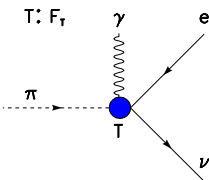
QED IB terms:



SM



and SD V , A terms:



A tensor interaction, too?

Exchange of $S=0$ leptoquarks

P Herczeg, PRD 49 (1994) 247



The $\pi \rightarrow e\nu\gamma$ amplitude and FF's

The IB amplitude (QED **uninteresting!**):

$$M_{\text{IB}} = -i \frac{e G_F V_{ud}}{\sqrt{2}} f_\pi m_e \epsilon^{\mu*} \bar{e} \left(\frac{k_\mu}{kq} - \frac{p_\mu}{pq} + \frac{\sigma_{\mu\nu} q^\nu}{2kq} \right) \times (1 - \gamma_5) \nu.$$

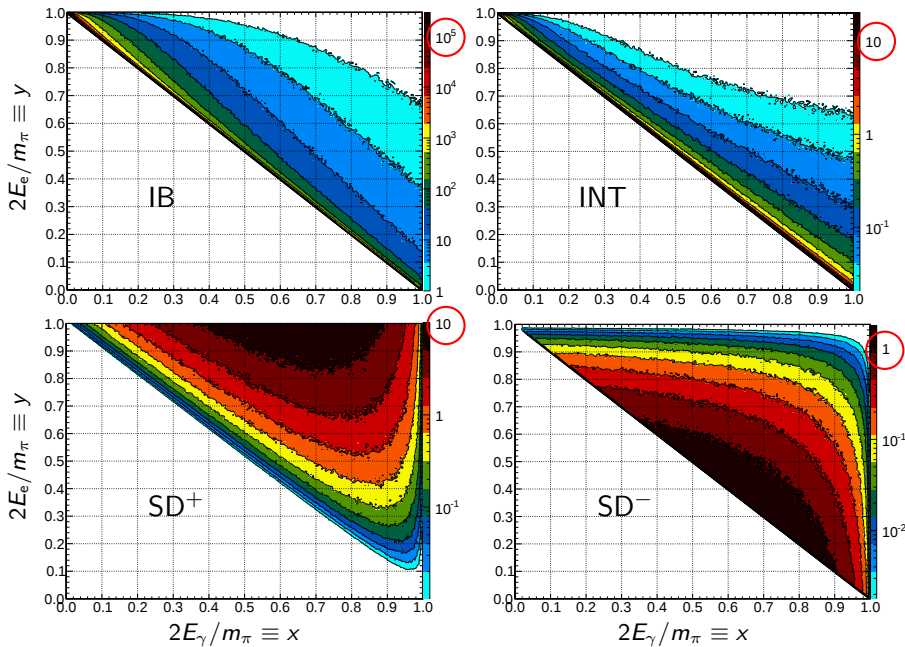
The structure-dependent amplitude (**interesting!**):

$$M_{\text{SD}} = \frac{e G_F V_{ud}}{m_\pi \sqrt{2}} \epsilon^{\nu*} \bar{e} \gamma^\mu (1 - \gamma_5) \nu \times [F_V \epsilon_{\mu\nu\sigma\tau} p^\sigma q^\tau + i F_A (g_{\mu\nu} pq - p_\nu q_\mu)].$$

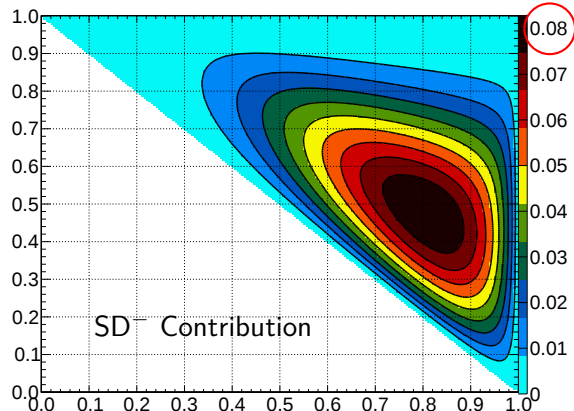
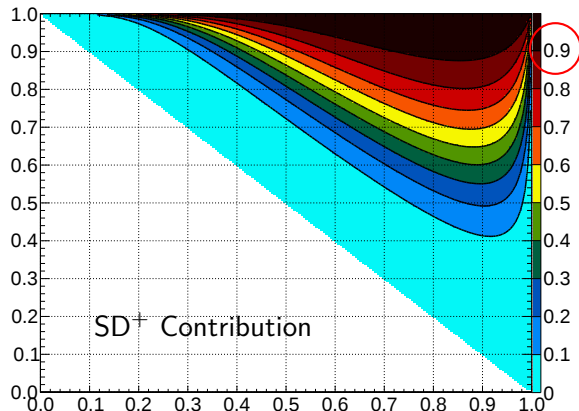
The SM branching ratio (with $x = 2E_\gamma/m_\pi$; $y = 2E_e/m_\pi$),

$$\begin{aligned} \frac{d\Gamma_{\pi e 2\gamma}}{dx dy} = & \frac{\alpha}{2\pi} \Gamma_{\pi e 2} \left\{ IB(x, y) + \left(\frac{m_\pi^2}{2f_\pi m_e} \right)^2 \right. \\ & \times [(F_V + F_A)^2 SD^+(x, y) + (F_V - F_A)^2 SD^-(x, y)] \\ & \left. + \frac{m_\pi}{f_\pi} [(F_V + F_A) S_{\text{int}}^+(x, y) + (F_V - F_A) S_{\text{int}}^-(x, y)] \right\}. \end{aligned}$$

Pion radiative decay regions



Best sensitivity for SD terms



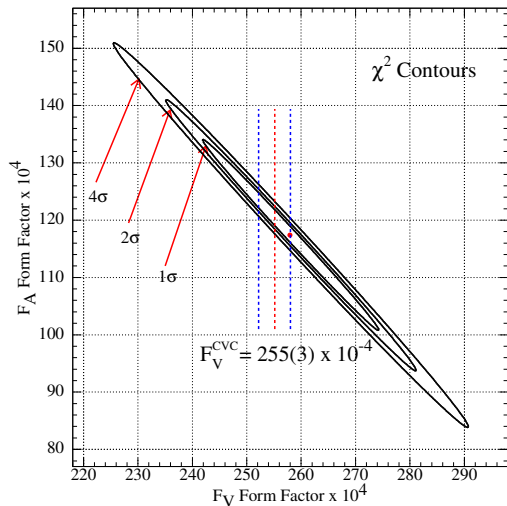
SD⁺ region favors high energy e⁺ and γ 's.

High energy track pairs occur with large opening angles.

Large solid angle coverage required $\Rightarrow$ good match to PEN!

SD⁻ is notoriously hard to measure directly.

Pion FF values and precision improvement factors (PIF) over previous work:

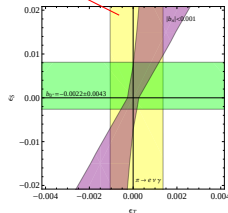


Observable	(PIF)
$F_V = 0.0258(17)$	(8 $\times$)
$F_A = 0.0119(1)_{(F_V^{CVC})}^{\text{exp}}$	(16 $\times$)
$a = 0.10(6)^*$	(∞)
$-5.2 < 10^4 \cdot F_T < 4.0$	90 % c.l.
$B_{\pi e 2\gamma} = 73.86(54) \times 10^{-8}^\dagger$	(17 $\times$)

* $a \dots q^2$ dependence of F_V

† for ($E_\gamma > 10$ MeV, and $\theta_{e\gamma} > 40^\circ$)

[Bychkov et al., PRL 103 (2009) 051802]



How can we do better?

⇒ make use of the PEN data.

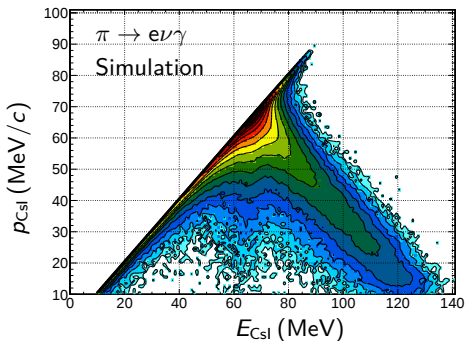
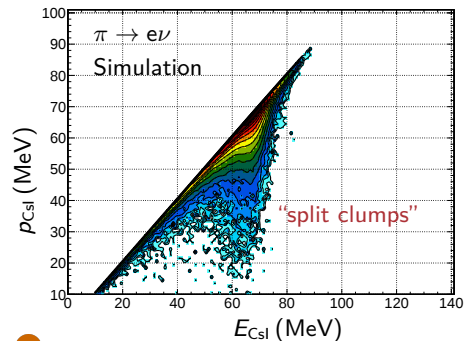
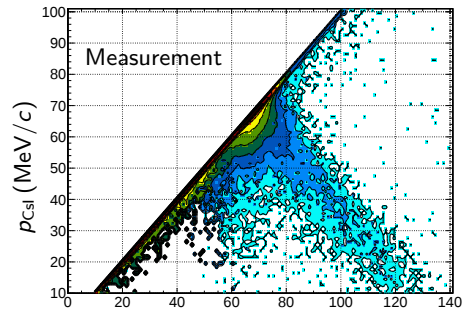
Identifying hard radiative decays in PEN

PEN indirectly measures p_ν in $\pi \rightarrow e\nu\gamma$

$$\vec{p}_e + \vec{p}_\gamma = -\vec{p}_\nu \equiv \vec{p}_{\text{obs}}; \text{ with:}$$

$$E_e + E_\gamma \equiv E_{\text{obs}}$$

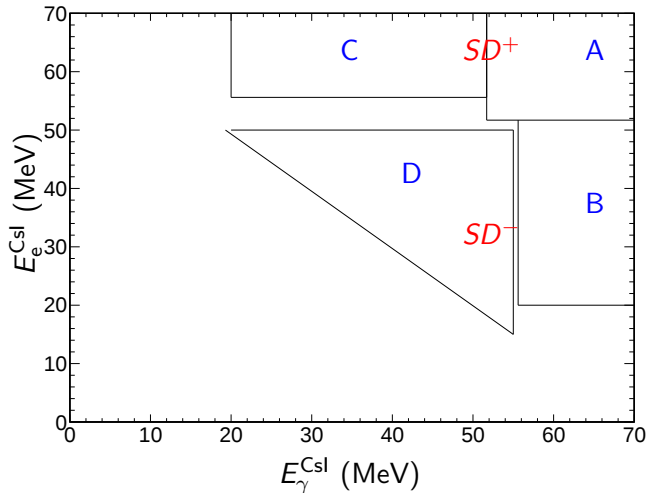
$$E_{\text{obs}} + p_{\text{obs}}c = m_\pi c^2$$



split clump: a photon separates and induces a detached shower

(analysis: C. Glaser)

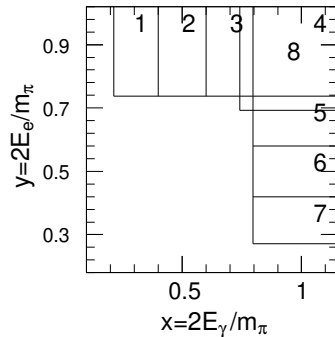
Data regions (in PiBeta and PEN)



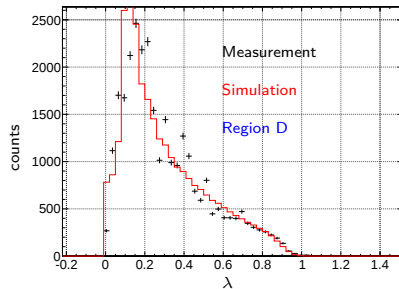
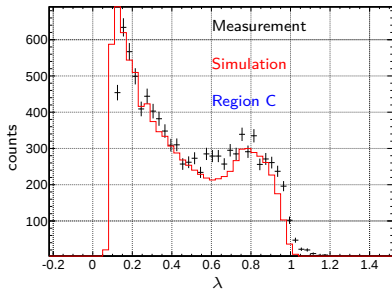
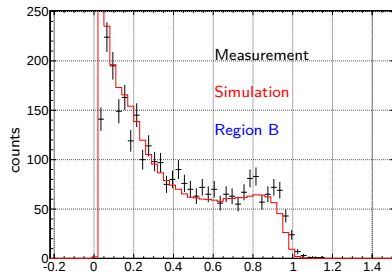
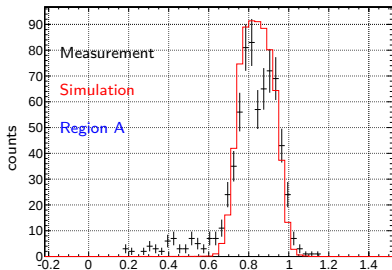
PiBeta (1999-01, 2004): regions A, B, and C.

[B was problematic in 1999-01 $\Rightarrow$ resolved with 2004 data set].

PiBeta final subdivision:



PEN Run 3 data

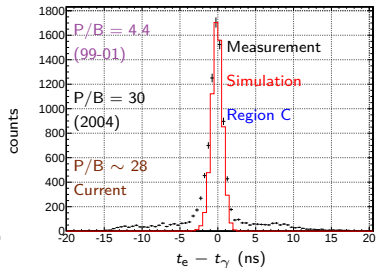
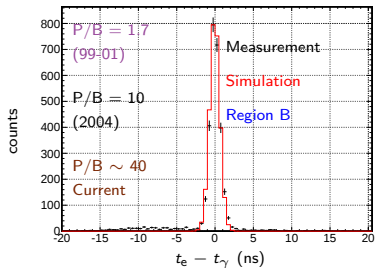
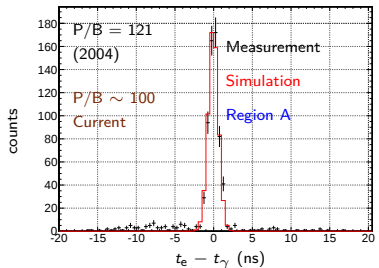
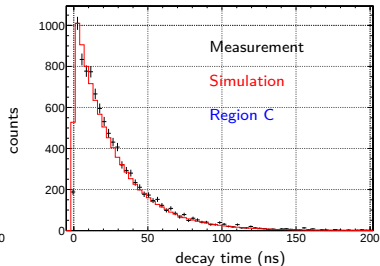
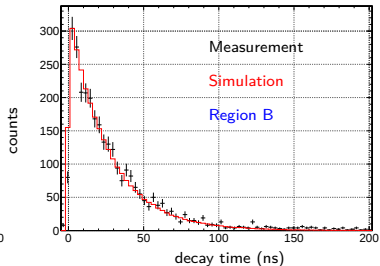
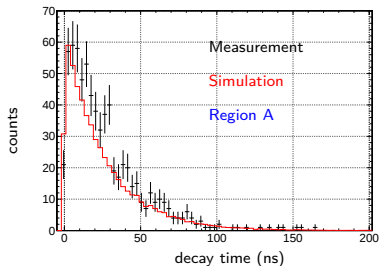


$$\lambda = 2E_e/m_\pi \sin^2(\theta_{e\gamma})$$

(C. Glaser)



Timing for radiative decays in PEN

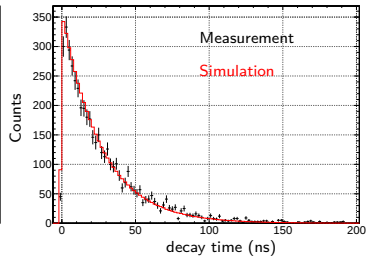
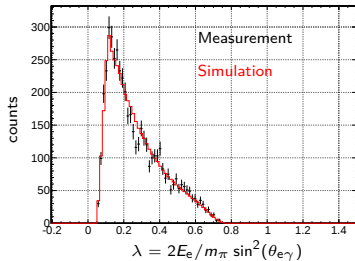
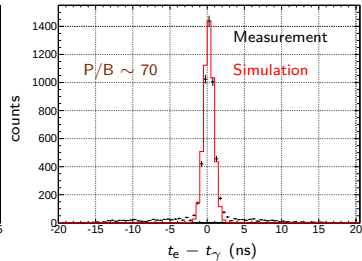
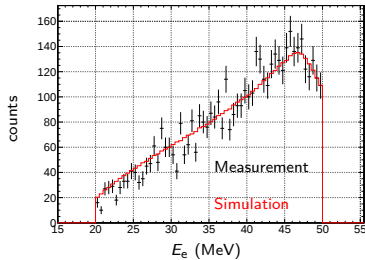
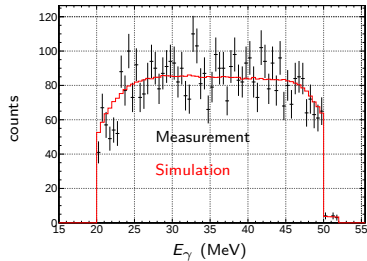


Measured spectra reflect instrumental response and event statistics.

(C. Glaser)



Region D: new in PEN

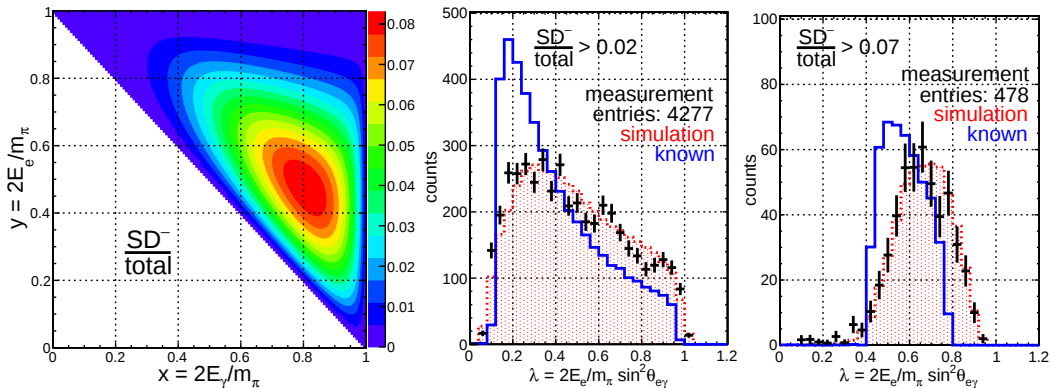


(C. Glaser)

Radiative decays: physics rich, accessible in PEN

1. $\pi^+ \rightarrow e^+ \nu_e \gamma$ and 2. $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$ (not discussed here).

New PEN data (subset below) offer first glimpse at elusive $SD^- \propto (F_V - F_A)^2$ term:



(C. Glaser)

Opportunities for improvement through machine learning

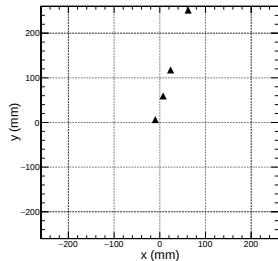
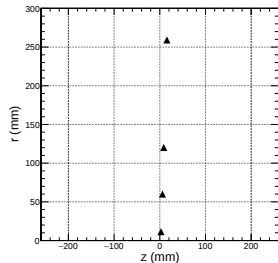
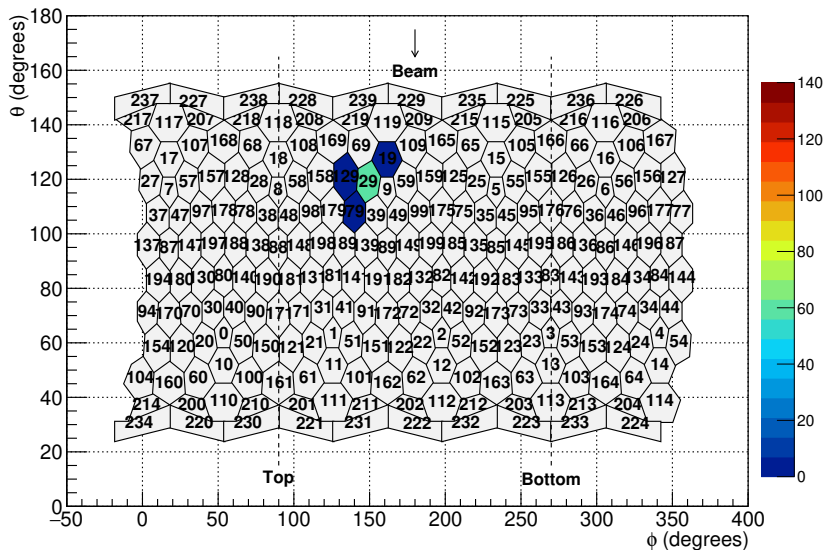
We plan to focus on:

- ▶ train and sort between π_{e2} “tail” signal and Michel (μ -decay) background using E_{AT}^{rest} in the overlap region;
- ▶ train and sort between $\pi \rightarrow e\nu$ and $\pi \rightarrow \mu\nu$ in the AT waveform (overlap region);
- ▶ combine the two above for a multidimensional smart cut;
- ▶ train and sort pattern recognition algorithms (graphical) to distinguish:
 - $\pi \rightarrow e\nu(\gamma)$ events without a detectable photon with a single well defined shower,
 - $\pi \rightarrow e\nu(\gamma)$ events without a detectable photon and a separate, “split clump” shower,
 - $\pi \rightarrow e\nu\gamma$ events with detected photon in a separate well defined shower,
 - $\pi \rightarrow e\nu\gamma$ events with detectable energy photon and small e - γ opening angle, resulting in a single, unresolved shower,
 - complex $\pi \rightarrow e\nu$ events with detectable energy photon, large e - γ opening angle, and one or more split clumps.

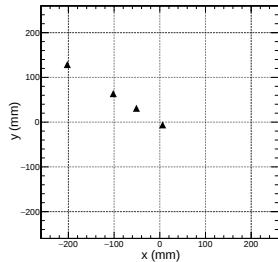
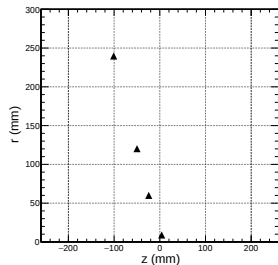
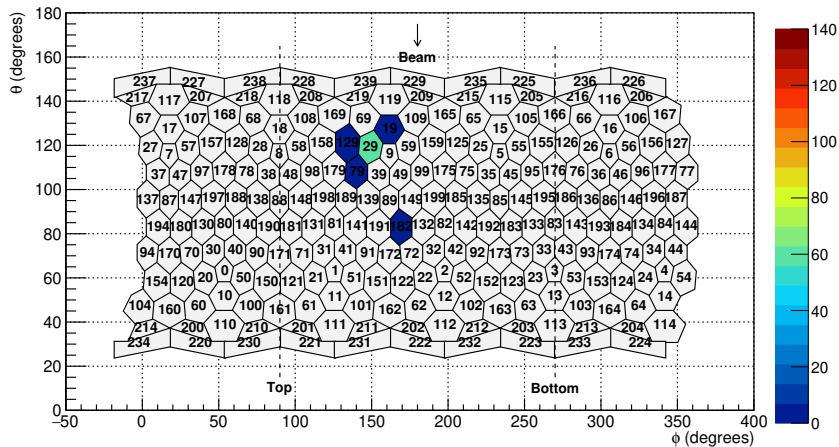
We review a few examples of **simulated events** below.

(C. Glaser, UVA)

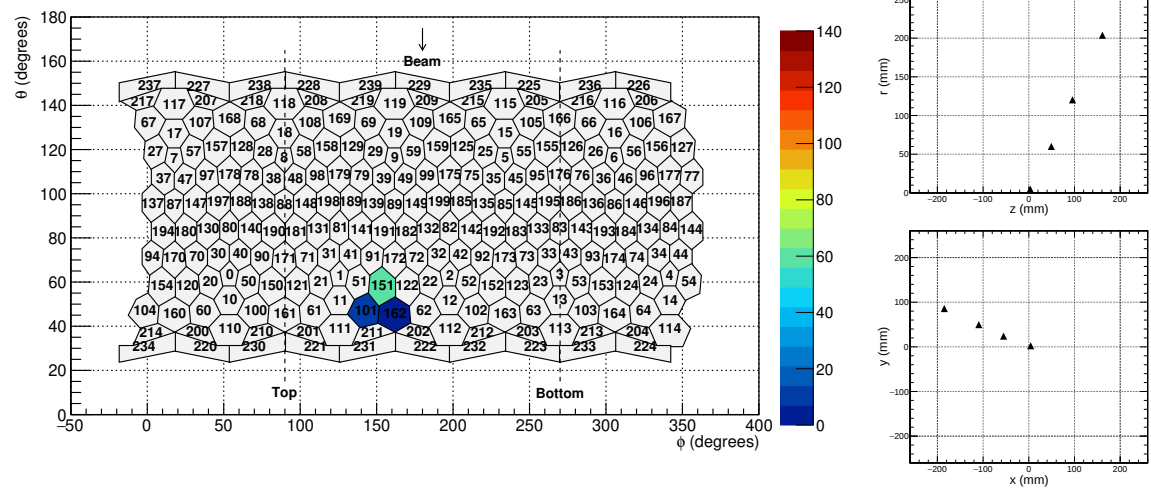
“Nonradiative” π_{e2} event – no split clump



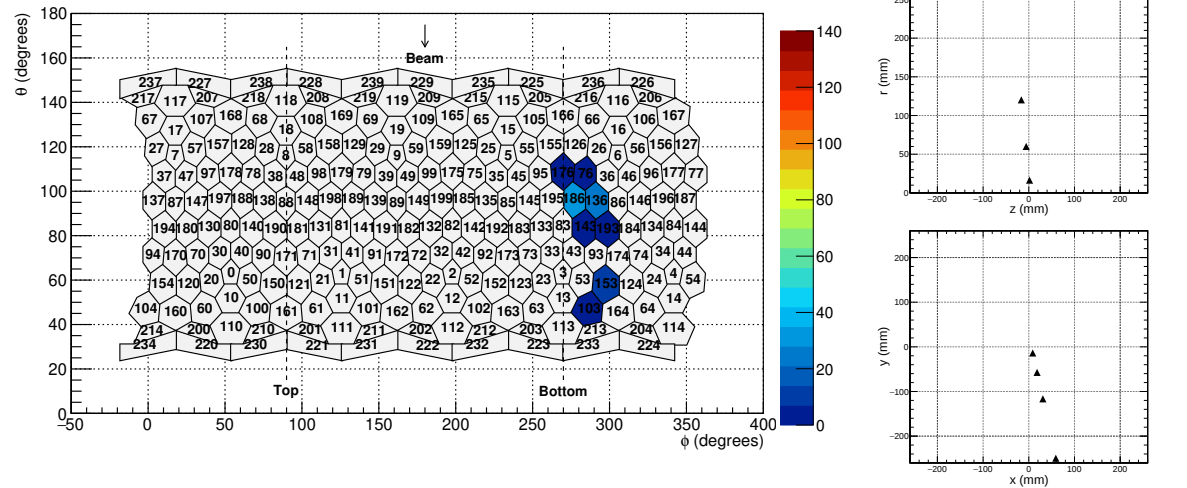
“Nonradiative” π_{e2} event – with split clump



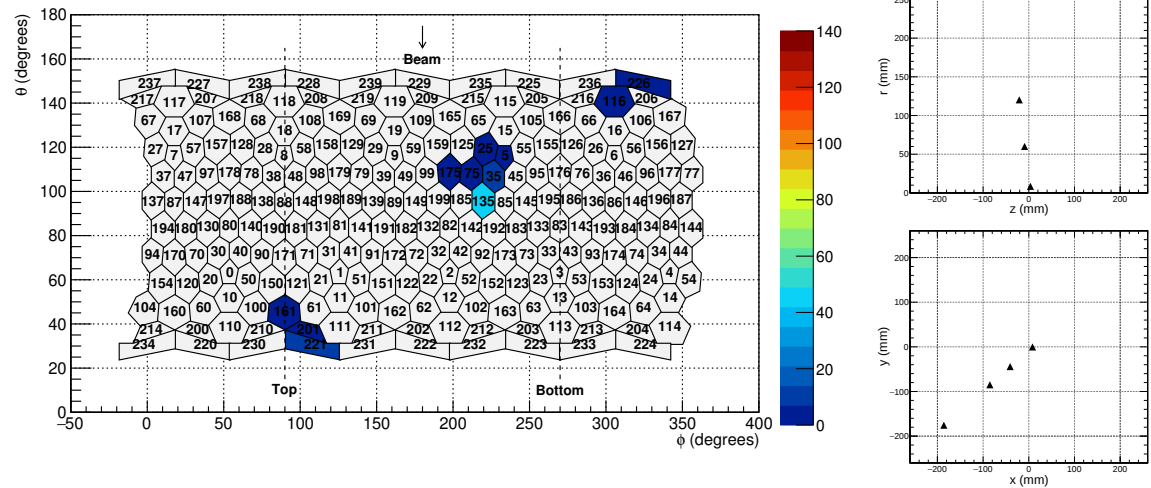
Radiative $\pi_{e2\gamma}$ event – one merged clump (small $\theta_{e\gamma}$)



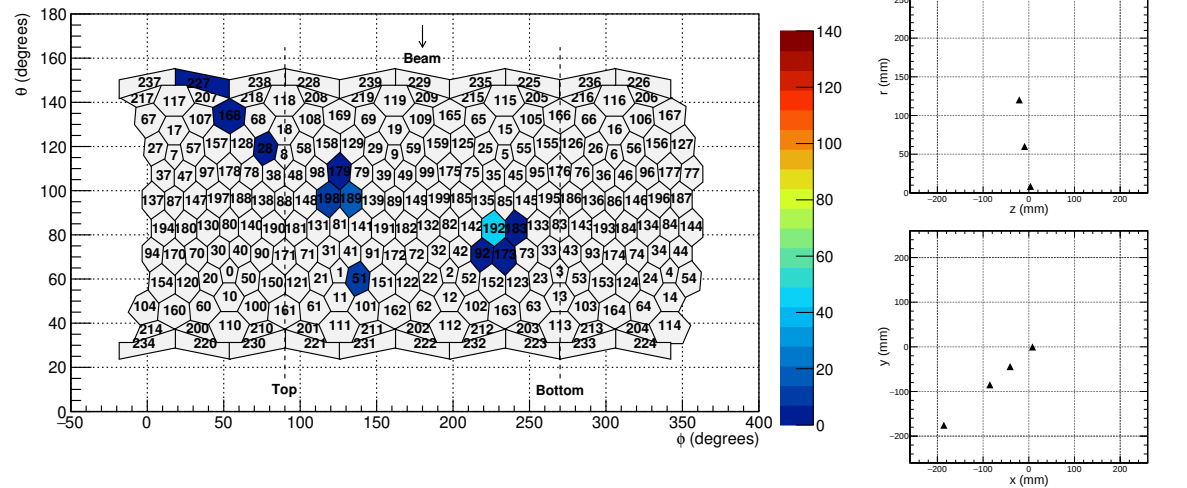
Radiative $\pi_{e2\gamma}$ event – two well separated clumps



Radiative $\pi_{e2\gamma}$ event – three separate clumps



Radiative $\pi_{e2\gamma}$ event – multiple clumps



Summary

- ▶ The PEN analysis is in its final phase.
- ▶ Prior to analysis unblinding, the PEN collaboration plans to publish technical details of the analysis and systematics.
- ▶ In addition to $R_{e/\mu}^\pi$, PEN collaboration plans to produce new physics results in radiative π and μ decays. This includes an updated analysis of $F_{A,V,T}^\pi \Rightarrow \epsilon_T$.
- ▶ We are actively pursuing avenues of analysis improvement through the application of machine learning techniques.
- ▶ The physics of PEN and PiBeta remains as compelling as ever. **PIONEER**, a new experiment with a large acceptance fast CALO, is under preparation at PSI.

Refs: [arXiv:2203.01981; arXiv:2203.05505]

Stay tuned!



Current and former PiBeta and PEN collaborators

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^dIRB, *Zagreb*, Croatia

^fIHEP, *Tbilisi*, Georgia

^h*Arizona* State Univ., USA

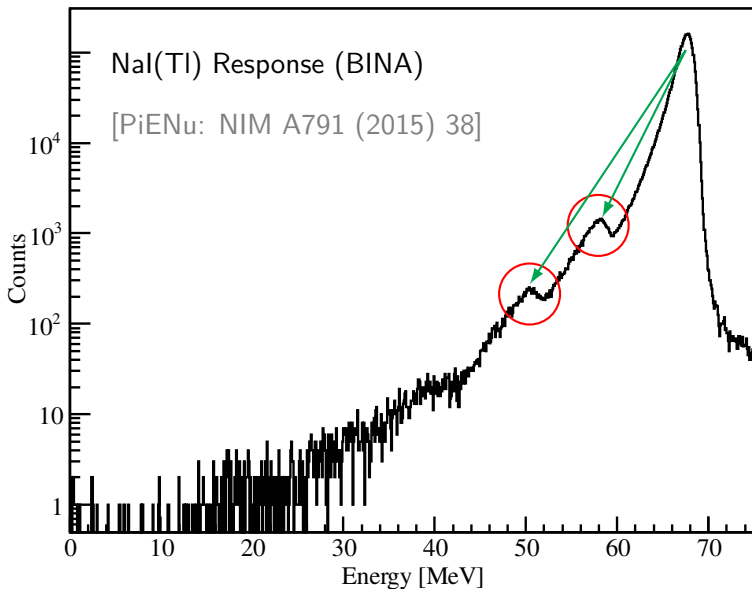
Home pages: <http://pibeta.phys.virginia.edu>
<http://pen.phys.virginia.edu>

Extra slides



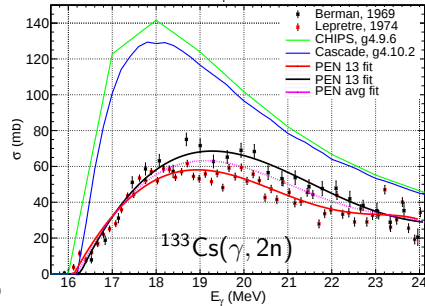
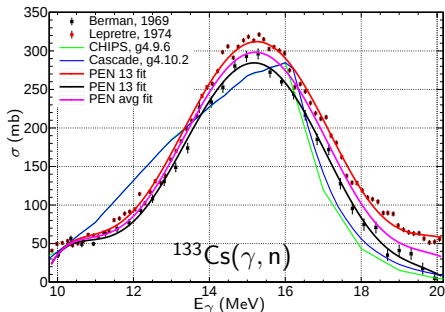
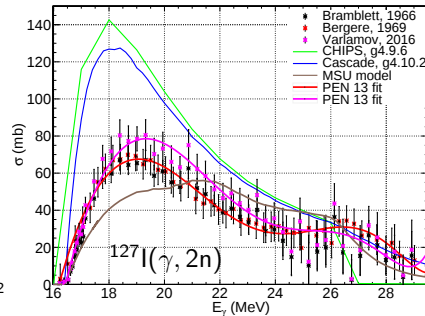
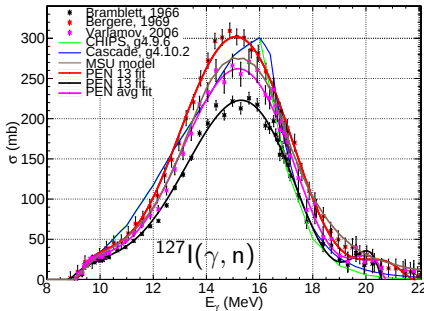
Tail fraction: photoneutron reactions

(γ, xn) reactions on calorimeter nuclei, **Cs** and **I**, shift counts from the main peak to the “tail” region if the neutrons are undetected.

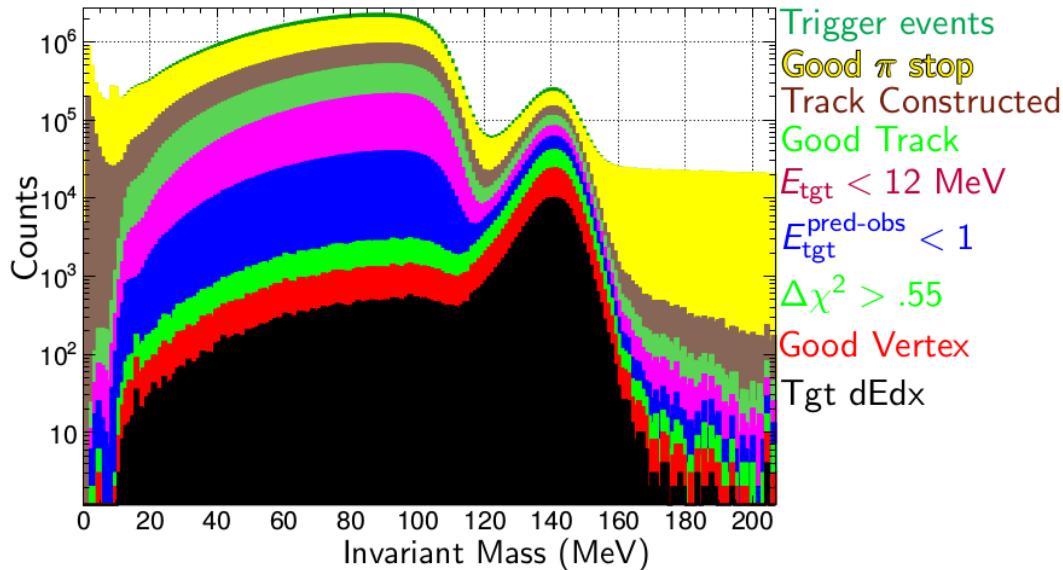


Photoneutron cross sections, $\sigma(\gamma, xn)$

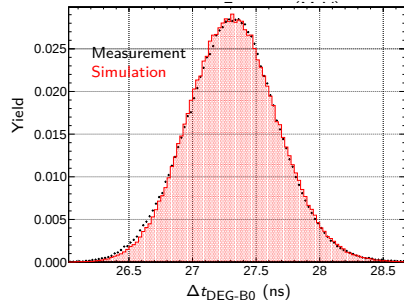
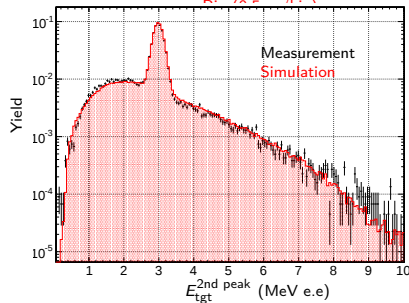
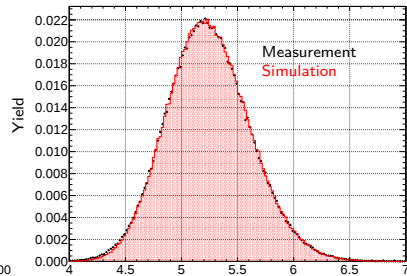
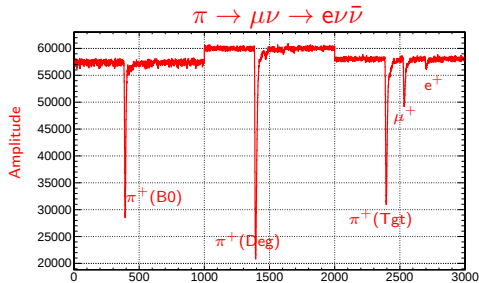
- ▶ Many inconsistencies among the data sets;
- ▶ Geant4 descriptions inadequate, often miss data by a wide margin.
- ▶ PEN was forced to implement its own parametrization in Geant4 (C. Glaser).
- ▶ This procedure works at the PEN goal precision, but would be inadequate at higher precision.



"Tail" trigger

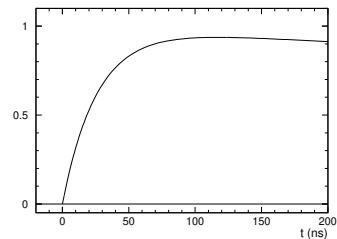


Realistic simulation of beam detectors vs. measurement



Choice of time interval, $f(T_e)$

$\pi \rightarrow \mu \rightarrow e$ ("Michel") timing selection: symmetric time window .



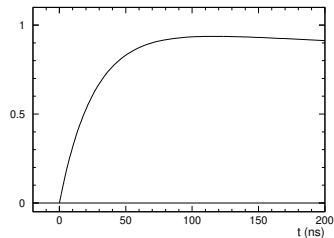
Choice of time interval, $f(T_e)$

$\pi \rightarrow \mu \rightarrow e$ ("Michel") timing selection: symmetric time window .

$$f_{\pi-\mu-e}(t) = \int_0^t f_{\pi \rightarrow \mu}(t-t') f_{\pi \rightarrow e}(t') dt' = \frac{1}{\tau_\mu - \tau_\pi} \left(e^{-t/\tau_\mu} - e^{-t/\tau_\pi} \right) .$$

$$|f_{\pi-\mu-e}|_{t-w}^{t+w} = \frac{2}{\tau_\mu - \tau_\pi} \left[\tau_\mu e^{-t/\tau_\mu} \sinh \frac{w}{\tau_\mu} - \tau_\pi e^{-t/\tau_\pi} \sinh \frac{w}{\tau_\pi} \right]$$

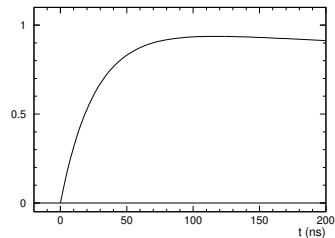
$$\delta f_{\pi-\mu-e} = \frac{2\delta t}{\tau_\mu - \tau_\pi} \left| e^{-t/\tau_\mu} \sinh \frac{w}{\tau_\mu} - e^{-t/\tau_\pi} \sinh \frac{w}{\tau_\pi} \right| \Rightarrow 0!!$$



Choice of time interval, $f(T_e)$

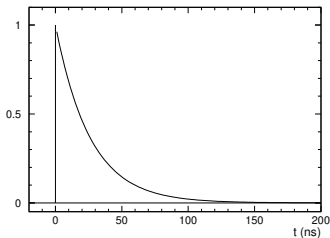
$\pi \rightarrow \mu \rightarrow e$ ("Michel") timing selection: symmetric time window .

$$f_{\pi-\mu-e}(t) = \int_0^t f_{\pi \rightarrow \mu}(t-t') f_{\pi \rightarrow e}(t') dt' = \frac{1}{\tau_\mu - \tau_\pi} \left(e^{-t/\tau_\mu} - e^{-t/\tau_\pi} \right) .$$



$$|f_{\pi-\mu-e}|_{t-w}^{t+w} = \frac{2}{\tau_\mu - \tau_\pi} \left[\tau_\mu e^{-t/\tau_\mu} \sinh \frac{w}{\tau_\mu} - \tau_\pi e^{-t/\tau_\pi} \sinh \frac{w}{\tau_\pi} \right]$$

$$\delta f_{\pi-\mu-e} = \frac{2\delta t}{\tau_\mu - \tau_\pi} \left| e^{-t/\tau_\mu} \sinh \frac{w}{\tau_\mu} - e^{-t/\tau_\pi} \sinh \frac{w}{\tau_\pi} \right| \Rightarrow 0!!$$

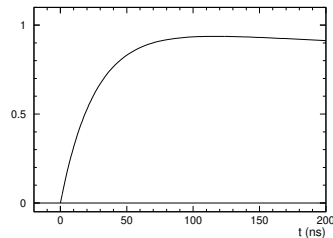


$\pi \rightarrow e\nu(\gamma)$ timing selection:

Choice of time interval, $f(T_e)$

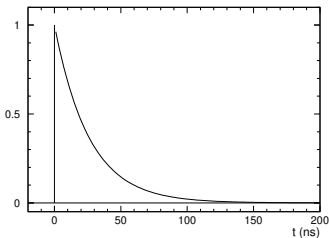
$\pi \rightarrow \mu \rightarrow e$ ("Michel") timing selection: symmetric time window .

$$f_{\pi-\mu-e}(t) = \int_0^t f_{\pi \rightarrow \mu}(t-t') f_{\pi \rightarrow e}(t') dt' = \frac{1}{\tau_\mu - \tau_\pi} \left(e^{-t/\tau_\mu} - e^{-t/\tau_\pi} \right) .$$



$$|f_{\pi-\mu-e}|_{t-w}^{t+w} = \frac{2}{\tau_\mu - \tau_\pi} \left[\tau_\mu e^{-t/\tau_\mu} \sinh \frac{w}{\tau_\mu} - \tau_\pi e^{-t/\tau_\pi} \sinh \frac{w}{\tau_\pi} \right]$$

$$\delta f_{\pi-\mu-e} = \frac{2\delta t}{\tau_\mu - \tau_\pi} \left| e^{-t/\tau_\mu} \sinh \frac{w}{\tau_\mu} - e^{-t/\tau_\pi} \sinh \frac{w}{\tau_\pi} \right| \Rightarrow 0!!$$

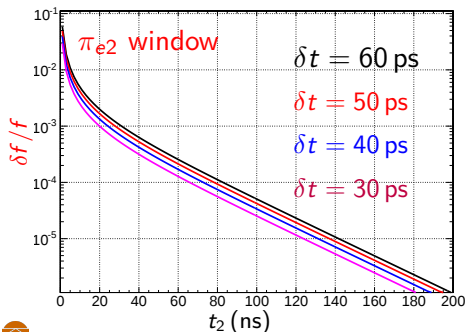
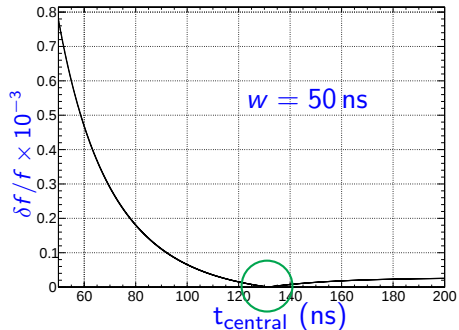
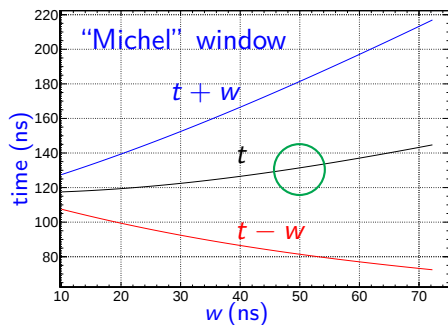


$\pi \rightarrow e\nu(\gamma)$ timing selection:

$$f_{\pi \rightarrow e\nu}(t_1, t_2) = \frac{1}{\tau_\pi} \int_{t_1}^{t_2} e^{-t/\tau_\pi} dt = e^{-t_1/\tau_\pi} - e^{-t_2/\tau_\pi}$$

Choose $t_1 < 0$: $\delta f_{\pi \rightarrow e\nu} = \frac{\delta t}{\tau_\pi} e^{-t_2/\tau_\pi}$.

Analysis requires: δt , w , t and t_2 .



r_f : decay time windows

$$\pi \rightarrow \mu \nu \rightarrow e \nu \bar{\nu}(\gamma)$$

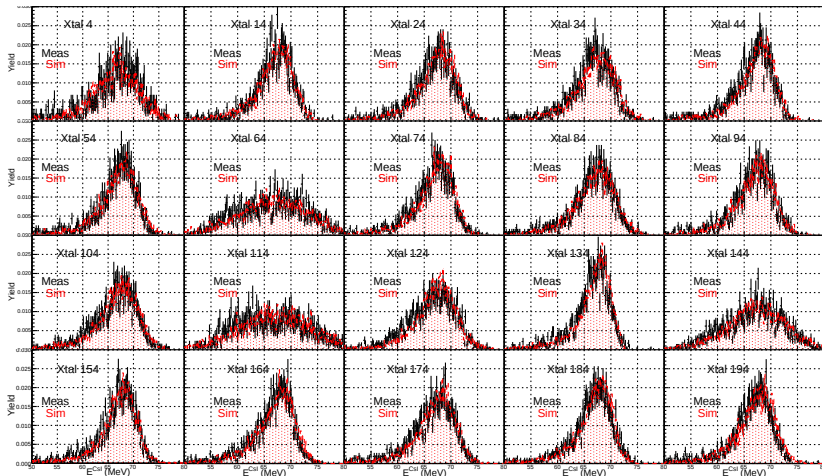
$\delta r_f / r_f$ negligible

$$\pi \rightarrow e \nu(\gamma)$$

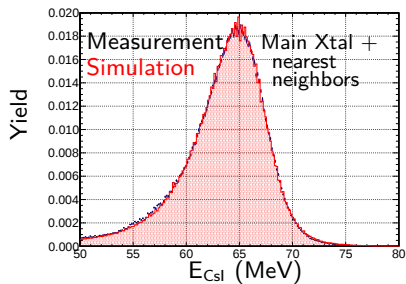
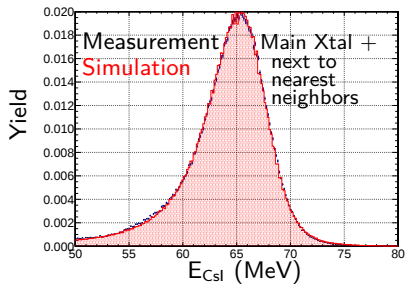
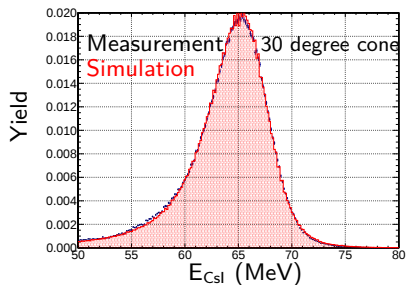
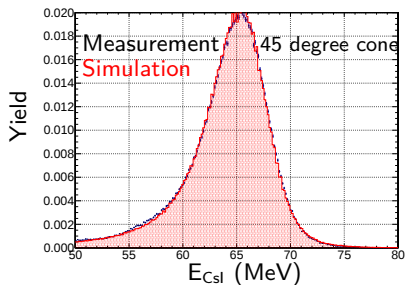
$\delta r_f / r_f$ negligible for $t_2 \gtrsim 90$ ns

CsI EM Calorimeter: realistic simulation

- ▶ different detector response, non-uniformities, $\Delta\Omega$ coverage;
- ▶ 240 PMTs, with slightly different properties, opt. couplings.



Csl simulation cont'd.



The PEN uncertainty budget

Type	Observable	Value	$\Delta R_{e/\mu}^\pi / R_{e/\mu}^\pi$
Systematic:	$\Delta\epsilon_{\text{tail}}$: low- E "tail" fraction*	$\simeq 0.038$	$\left\{ \begin{array}{l} \simeq 0.001^{\text{exp}} \\ 2 \times 10^{-4} _{\text{goal}}^{\text{MC}} \end{array} \right.$
	r_f : observed decay fractions	0.0441	$< 10^{-4}$
	r_{PC} : ratio of MWPC efficiencies	$\simeq .99$	$< 10^{-4}$
	r_A : acceptance ratio (blinded)	$\simeq 1$	$\leq 10^{-4}$
	r_{cut} : cut efficiency ratio	$\simeq 1.0153$	$\leq 4 \times 10^{-4}$
	$N_{\pi\text{DIF} \rightarrow e\nu} / N_{\pi \rightarrow e\nu} \mid^\dagger$	$< 2 \times 10^{-3}$	$10^{-6} - 10^{-5}$
	$N_{\pi\text{DIF} \rightarrow \mu\nu} / N_{\pi \rightarrow \mu\nu} \mid^\dagger$	2.3×10^{-3}	$10^{-6} - 10^{-5}$
Statistical:	$N_{\mu\text{DIF} \rightarrow e\nu\bar{\nu}} / N_{\mu \rightarrow \nu\bar{\nu}} \mid^\dagger$	1.4×10^{-4}	$< 10^{-5}$
	$\Delta N_{\pi \rightarrow e\nu} / N_{\pi \rightarrow e\nu}$		$\simeq 3 \times 10^{-4}$
Overall	goal		5×10^{-4}

* Depends on the invariant mass cutoff, here $m_0 = 117.5 \text{ MeV}$, which minimizes overall uncertainty.

† π_{DIF} : pion decay in flight; μ_{DIF} : muon decay in flight .

Lepton universality (and neutrinos)

From:

$$R_{e/\mu} = \frac{\Gamma(\pi \rightarrow e\bar{\nu}(\gamma))}{\Gamma(\pi \rightarrow \mu\bar{\nu}(\gamma))} = \frac{g_e^2}{g_\mu^2} \frac{m_e^2}{m_\mu^2} \frac{(1 - m_e^2/m_\mu^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} (1 + \delta R_{e/\mu})$$
$$R_{\tau/\pi} = \frac{\Gamma(\tau \rightarrow e\bar{\nu}(\gamma))}{\Gamma(\pi \rightarrow \mu\bar{\nu}(\gamma))} = \frac{g_\tau^2}{g_\mu^2} \frac{m_\tau^3}{2m_\mu^2 m_\pi} \frac{(1 - m_\pi^2/m_\tau^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} (1 + \delta R_{\tau/\pi})$$

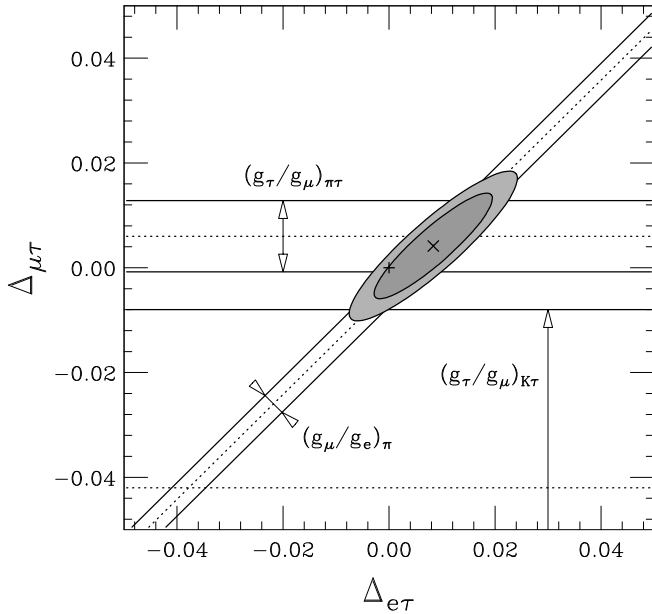
one can evaluate:

$$\left(\frac{g_e}{g_\mu}\right)_\pi = 0.9996 \pm 0.0012 \quad \text{and} \quad \left(\frac{g_\tau}{g_\mu}\right)_{\pi\tau} = 1.0030 \pm 0.0034.$$

For comparison,

$$\left(\frac{g_e}{g_\mu}\right)_W = 0.999 \pm 0.011 \quad \text{and} \quad \left(\frac{g_\tau}{g_e}\right)_W = 1.029 \pm 0.014.$$

- ▶ significant consequences in the **neutrino sector**;
- ▶ interesting limits on **MSSM extension observables**.



Loinaz et al., PRD **70** (2004) 113004

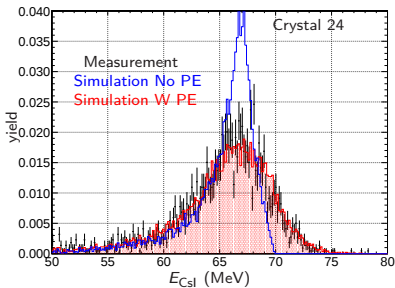
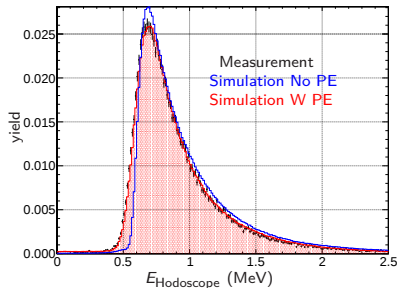
$$\Delta_{\ell\ell'} = 2 \left(\frac{g_\ell}{g_{\ell'}} - 1 \right)$$

Canonical Geant gives energies, timings, and positions

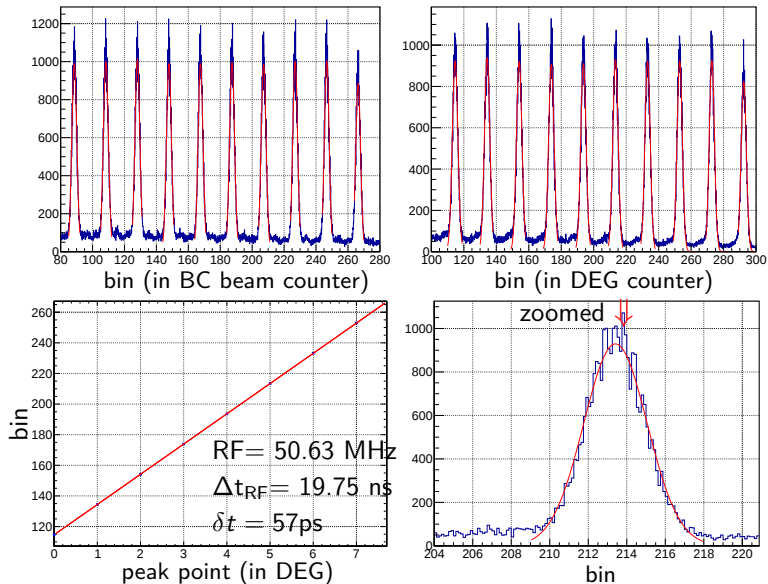
Requires additional physics input to simulate full detector response

In the Experiment:

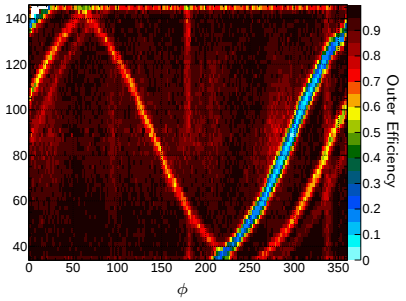
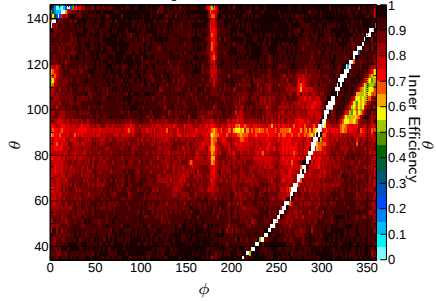
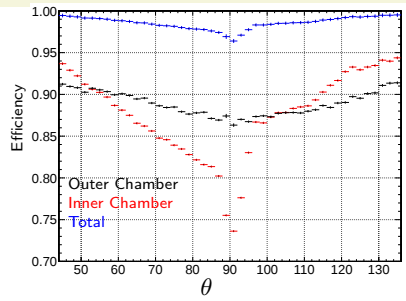
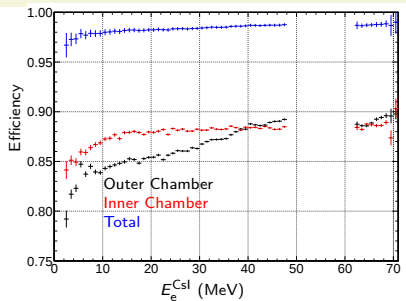
- ▶ digitized energies and timings of detector elements
- ▶ mTPC, beam counters, and target waveforms
- ▶ photoelectron statistics smear signal



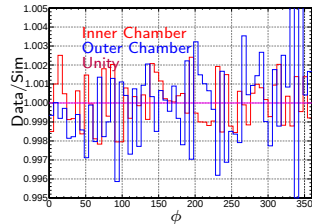
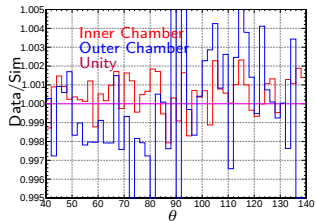
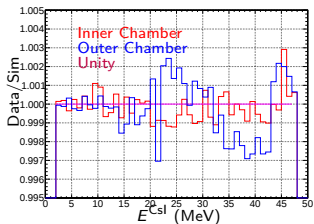
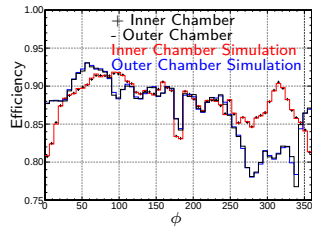
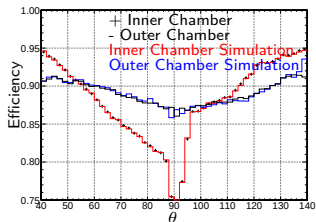
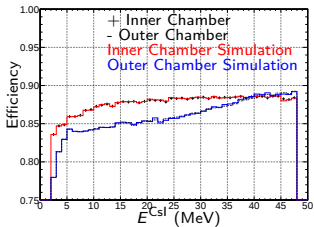
r_f : δt for beam particles



Chamber efficiencies



Chamber efficiencies: simulation



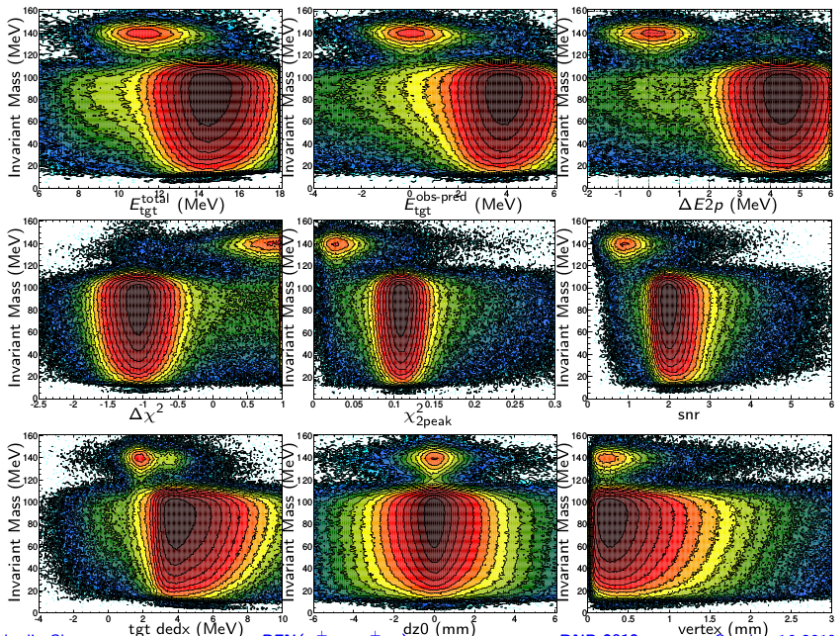
$dE/dx = g(E)$ in chamber gas $\pi \rightarrow e^+ \nu_e$ 70 MeV monoenergetic
 $\mu \rightarrow e \nu \bar{\nu}$ 0 – 52.5 MeV spectrum

Monte Carlo is weighted to simulate chamber efficiencies

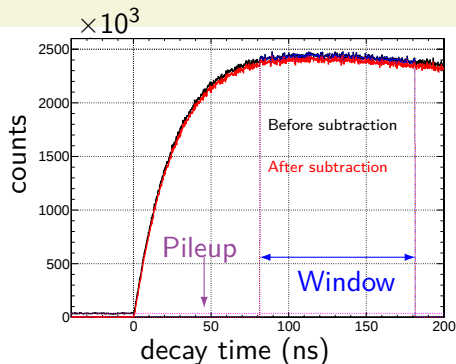
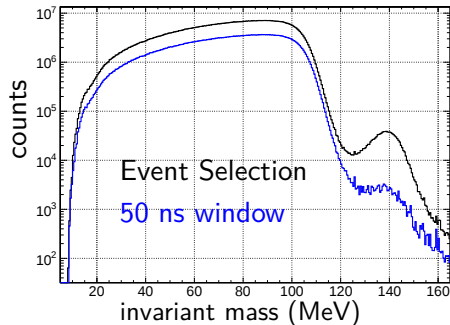
Absorbed into acceptances (blinded)



Observables to aid discrimination



r_N : number of $\pi \rightarrow \mu \rightarrow e$ events



$$N_{\pi-\mu-e, \text{ Run 2}} = (5203.57 \pm 0.32) \times 10^5$$

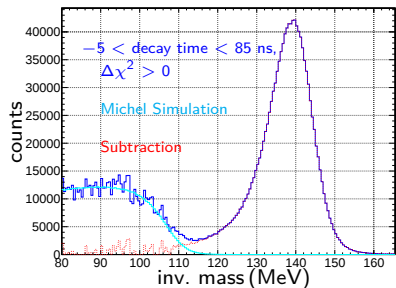
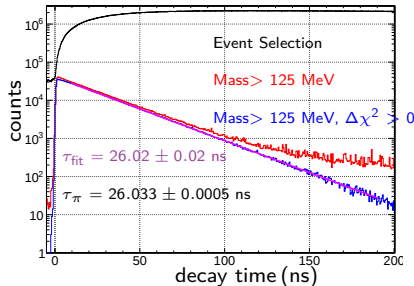
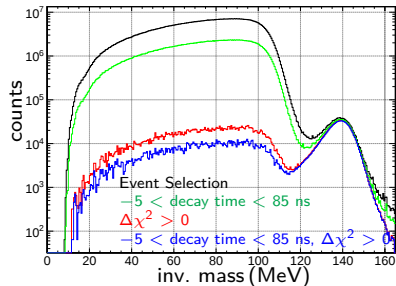
$$N_{\pi-\mu-e, \text{ Run 3}} = (9545.50 \pm 0.44) \times 10^5$$

$$\delta N_{\pi-\mu-e, \text{ Run 2}} / N_{\pi-\mu-e, \text{ Run 2}} = 6.2 \times 10^{-5}$$

$$\delta N_{\pi-\mu-e, \text{ Run 3}} / N_{\pi-\mu-e, \text{ Run 3}} = 4.6 \times 10^{-5}$$

contribution to $\Delta R_{e/\mu}^\pi / R_{e/\mu}^\pi \dots$ not significant

r_N : number of $\pi_{e2}(\gamma)$ events



Waveform cut is needed, $\epsilon \sim 97\%$

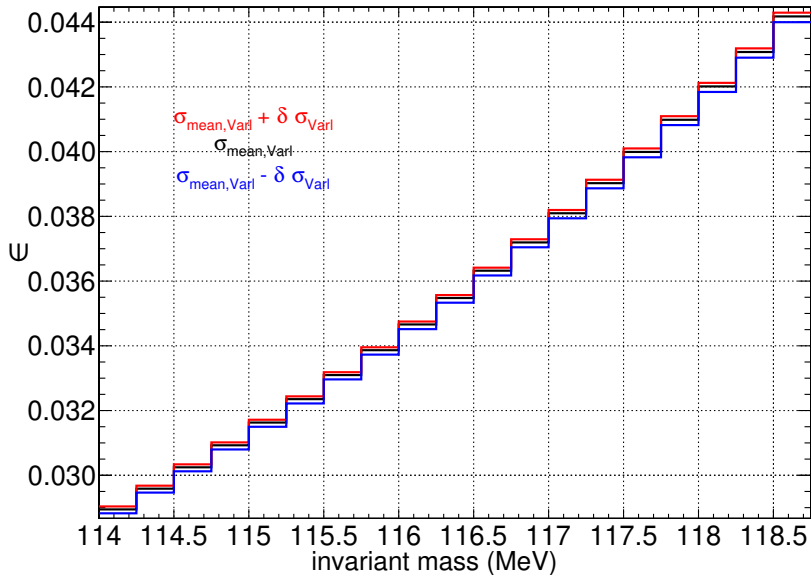
$$N_{\pi e2(\gamma), \text{Run 2}} = 1387431 \pm 1179.91$$

$$N_{\pi e2(\gamma), \text{Run 2}} = 2383805 \pm 1545.93$$

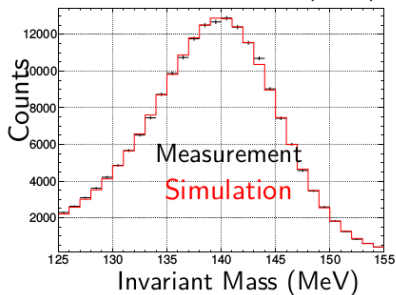
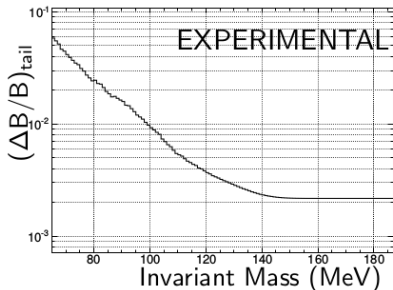
$$\frac{\delta N_{\pi e2(\gamma), \text{Run 2\&3}}}{N_{\pi e2(\gamma), \text{Run 2\&3}}} \simeq 5.1 \times 10^{-4}$$



π_{e2} tail including photoneutron corrections



MC simulated vs. experimental π_{e2} tail

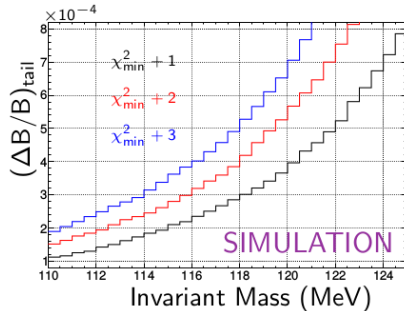


Simulation unavoidable!

Systematics from :

Gain Variation

Photo-nuclear physics



$$|F_V| \stackrel{\text{CVC}}{=} \frac{1}{\alpha} \sqrt{\frac{2\hbar}{\pi \tau_{\pi^0} m_\pi}} = 0.0255(3) .$$

$F_A \times 10^4$	reference
-------------------	-----------

106 ± 60	Bolotov et al. (1990)
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135 ± 16	Bay et al. (1986)
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60 ± 30	Piilonen et al. (1986)
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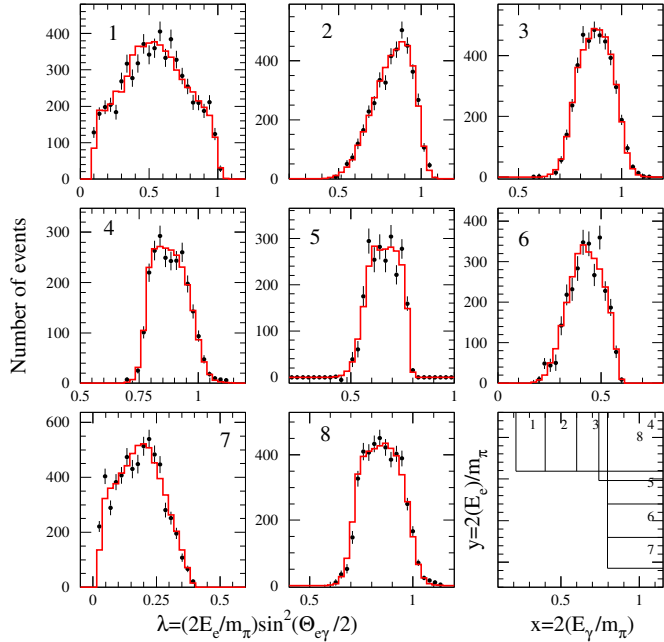
110 ± 30	Stetz et al. (1979)
-----------------	---------------------

116 ± 16	world average (PDG 2004)
-----------------	--------------------------

$$|F_V| \stackrel{\text{CVC}}{=} \frac{1}{\alpha} \sqrt{\frac{2\hbar}{\pi \tau_{\pi^0} m_\pi}} = 0.0255(3) .$$

$F_A \times 10^4$	reference	note
106 ± 60	Bolotov et al. (1990)	$(F_T = -56 \pm 17)$
135 ± 16	Bay et al. (1986)	
60 ± 30	Piilonen et al. (1986)	
110 ± 30	Stetz et al. (1979)	
116 ± 16	world average (PDG 2004)	

PiBeta $\pi_{e2\gamma}$ differential distributions:
 2009 analysis of 1999-01, 2004 data
 sets.



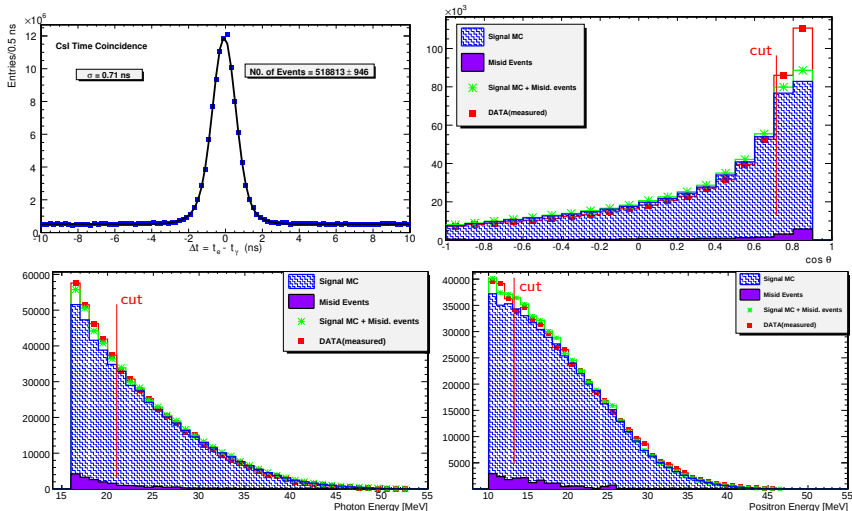
Radiative muon decay:

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$$

$BR \sim 10^{-3}$ for energetic γ 's

- ▶ Sensitive to admixtures beyond $V - A$
- ▶ Limiting factor in $\mu \rightarrow e\gamma$ LFV searches

RMD: $\mu^+ \rightarrow e^+ \nu \bar{\nu} \gamma$, [E. Munyangabe's analysis of 2004 PIBETA data]



"Split clumps" very well accounted for! ~ 30 -fold improvement in precision of the RMD BR .

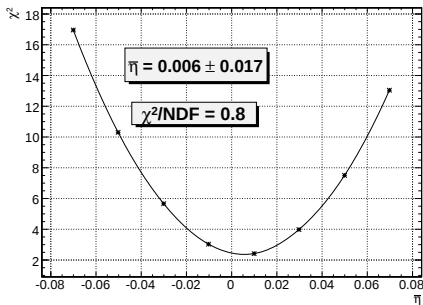
~ 4 -fold improvement over best previous limit on $\bar{\eta}$ Michel parameter.



RMD preliminary results, cont'd.

Preliminary result for RMD branching ratio (thesis E. Munyangabe):

$$B_{\text{exp}} = 4.365 (9)_{\text{stat.}} (42)_{\text{syst.}} \times 10^{-3}, \quad \boxed{29 \times}$$
$$B_{\text{SM}} = 4.342 (5)_{\text{stat-MC}} \times 10^{-3} \quad (\text{for } E_\gamma > 10 \text{ MeV}, \theta_{e\gamma} > 30^\circ)$$



Analysis of PS subset:

$13 \text{ MeV} < E_\gamma < 45 \text{ MeV}$, and

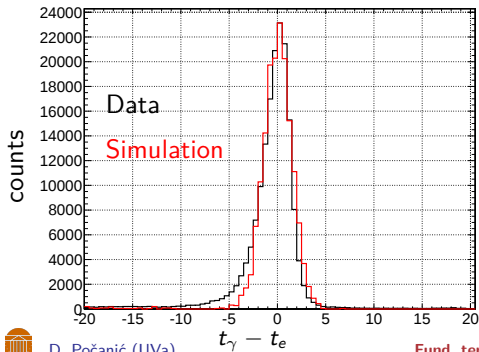
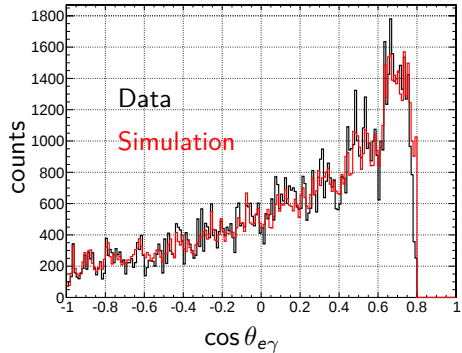
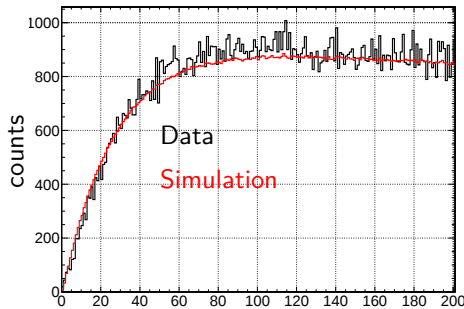
$10 \text{ MeV} < E_{e^+} < 43 \text{ MeV}$, yields

$$\bar{\eta} = 0.006 (17)_{\text{stat.}} (18)_{\text{syst.}}, \text{ or}$$

$$\bar{\eta} < 0.028 \quad (68\% \text{CL}).$$

$\sim 4 \times$ better than best previous experiment (Eichenberger et al, 84).

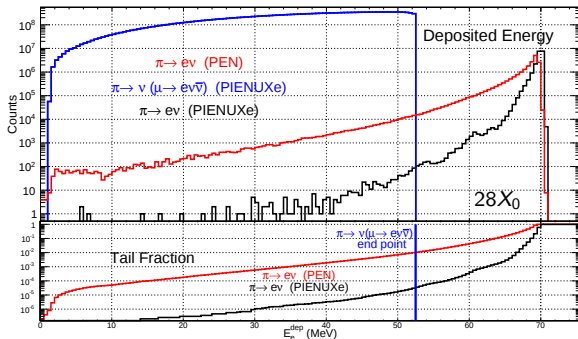
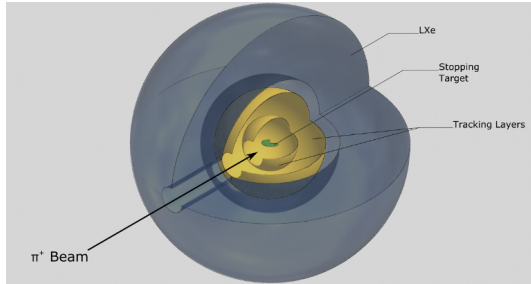
NB: preliminary results!



PEN RMD plots (Run 3 data set)

Simple track requirement,
Data selection using target cuts,
Adding coverage to PiBeta data set.

Ready for new BR and $\bar{\eta}$ evaluations.



Concept for an improved RarePi experiment

Goals:

1. $(\Delta R/R)_{e/\mu}^{\pi} < 10^{-4}$ (to match theory),
2. $(\Delta R/R)_{e3}^{\pi} \sim 2 - 3 \times 10^{-3}$ (per W. Marciano).

A possible setup:

- ▶ stopped π^+ in active target with full tracking,
- ▶ main detector: liquid Xe,
- ▶ $\langle r_{\text{stop}}^{\pi} \rangle \sim 1.5 - 2 \times 10^6 \text{ s}^{-1}$,
- ▶ with $T_{\text{run}}^{\text{live}} \sim 3 \times 10^7 \text{ s}$ (> 2 calendar years):
- ▶ $N_{e2(\gamma)}^{\pi} > 5 \times 10^9$: $(\Delta R_{e/\mu}^{\pi}/R)_{\text{stat}} < 2 \times 10^{-5}$;
- ▶ $N_{e3(\gamma)}^{\pi} \geq 4 \times 10^5$: $(\Delta R_{e3}^{\pi}/R)_{\text{stat}} < 2 \times 10^{-3}$.

Matching the theoretical precision in π_{e3} decay would require substantially higher beam stop rates and further attention to the target and central tracking detectors.