Skewness-dependent moments of the pion GPD from nonlocal quark-bilinear correlators

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Towards Improved Hadron Femtography | University of Virginia | Jul. 23, 2026

Based on Phys.Rev.D 113 (2026) 1, 014505

Introduction

Aspect	Pion	Proton
Spin	0	$1/2$
Unpolarized GPDs	$H^\pi(x,\xi,t)$	$H(x,\xi,t),\ E(x,\xi,t)$
Nonlocal decomposition	built from P^μ, Δ^μ, z^μ only	involves P^μ, Δ^μ, z^μ plus Dirac structures
Typical structure	$M^\mu_\pi \sim P^\mu A_1^\pi + \Delta^\mu A_2^\pi + m^2 z^\mu A_3^\pi$	$M^\mu_N \sim \bar{u}[\gamma^\mu,\ P^\mu,\ \Delta^\mu,\ z^\mu,\ i\sigma^{\mu\nu}\Delta_\nu, \ldots]u$
Lattice extraction	no spinor / polarization projectors	multiple projectors needed to isolate H and E
Experimental status	limited, mostly indirect	much richer constraints from DVCS and exclusive processes

Pion GPDs offer a clean, complementary benchmark for validating lattice GPD methodology

Theoretical definition of pion GPDs

- Pion Generalized Parton Distributions (GPDs)

$$H^{\pi}(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle \pi(p_f) \middle| \bar{\psi}\left(-\frac{z^-}{2}\right) \gamma^+ \mathscr{L}\left(-\frac{z^-}{2}, \frac{z^-}{2}\right) \psi\left(\frac{z^-}{2}\right) \middle| \pi(p_i) \right\rangle$$

x	$\Delta^\mu = (p_f - p_i)^\mu$	$-t = \Delta^2$	$\xi = \frac{(p_i^+ - p_f^+)}{(p_i^+ + p_f^+)}$	$P^\mu = \frac{(p_i^\mu + p_f^\mu)}{2}$
momentum fraction	momentum transfer	momentum transfer squared	skewness	average hadron momentum

$$f_{\mathrm{GPD}}(x, \mathbf{b_T}) \xrightarrow[\text{FT}]{b_\perp \leftrightarrow \Delta_\perp} H(x,0,t) \xleftarrow{\xi = 0} H(x,\xi,t)$$

Impact-Parameter Distributions

$H(x,0,t) \xrightarrow{\int dx}$ Form Factor $A_{1,0}$ (FF)

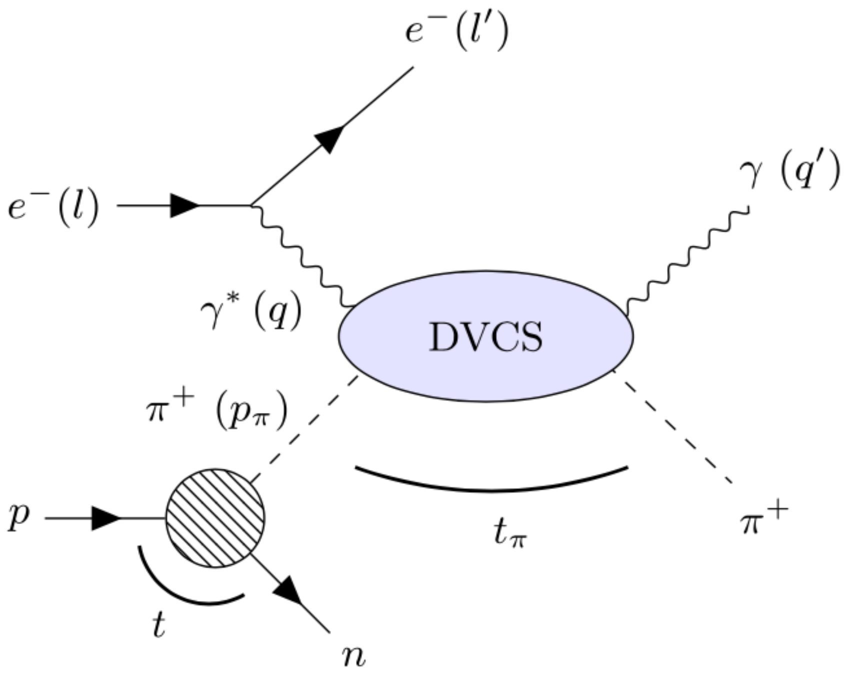
$H(x,\xi,t) \xrightarrow{\int_{-1}^{1} dx\, x^{n-1}}$ Generalized form factors $A_{n,k}$ (GFFs)

n-th Mellin moments

Experimental access to pion GPDs

- No fixed pion target → no direct $e\pi$ scattering
- Pion GPDs are accessible only **indirectly** (via Sullivan process/pion pole DVCS)

See Garth's talk.



Chávez et al., PRL 128 (2022) 202501

✓ Pion form factor from pion-pole production:

⇒ first moment only G. M. Huber et al., PRC 78, 045203 (2008); T. Horn et al., PRC 78, 058201 (2008); …

✓No direct experimental extraction of pion GPDs is available yet.

✓Future prospects: JLab 12 GeV, EIC…

From first principles: lattice QCD

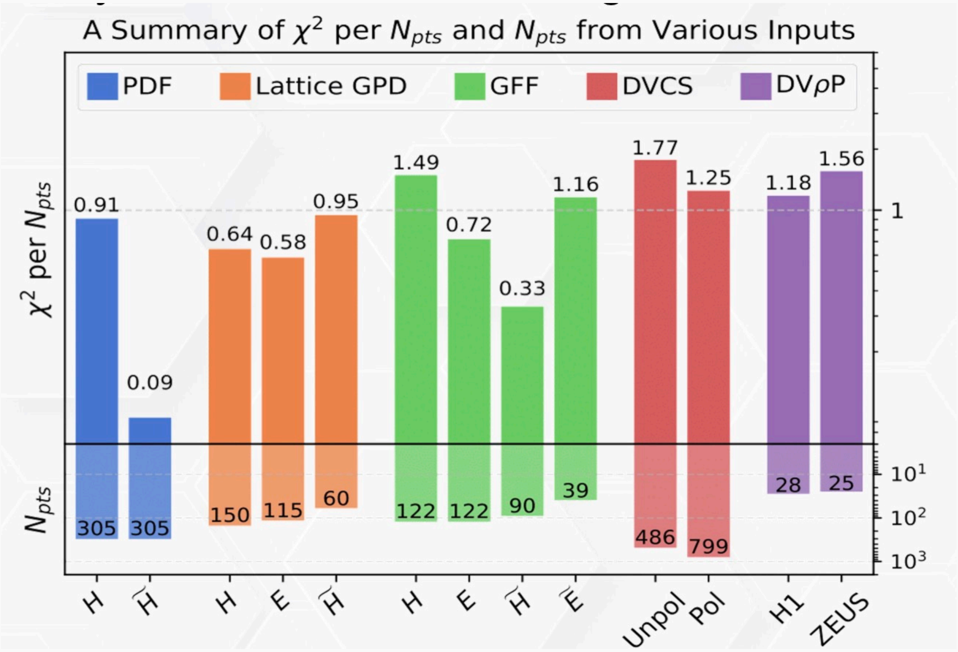
Experimental access to pion GPDs

- No fixed pion target → no direct $e\pi$ scattering
- Pion GPDs are accessible only **indirectly** (via Sullivan process/pion pole DVCS)

$e^-(l)$

$p \longrightarrow$

In addition to lattice x-dependent GPDs, lattice moments / GFFs provide complementary constraints on global fits.



More details in Yuxun's presentation (nucleon).

n:

RC 78, 045203 (2008);

3, 058201 (2008);

of

C…

CD

Pion GPDs from lattice QCD

- Traditionally, Mellin moments of pion GPD from **local operators**

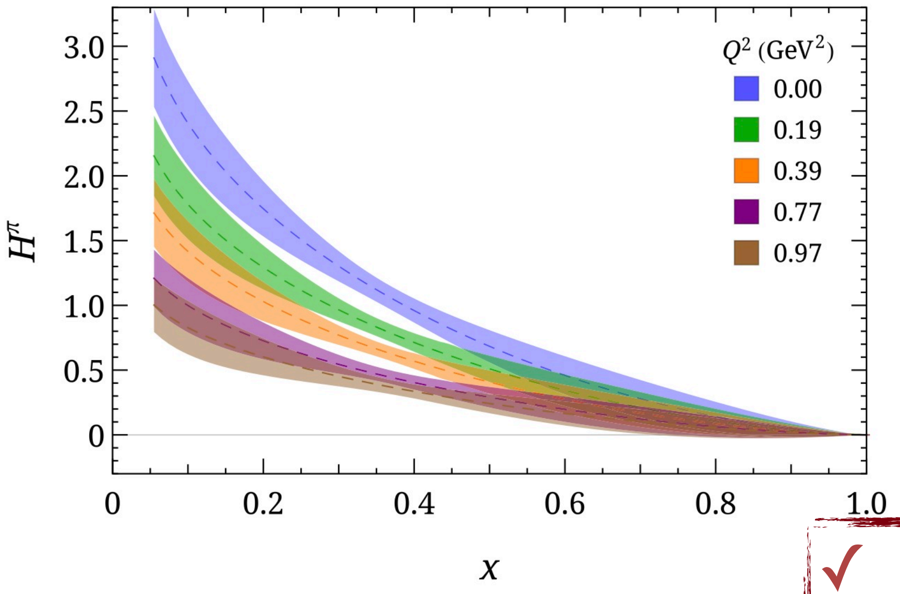
[1] QCDSF/UKQCD Collab., *PoS* LAT2005 (2006);
[2] QCDSF/UKQCD Collab., EPJC 51 (2007).
[3] G. Bali et al., PoS LATTICE2013, 447, arXiv:1311.7639
…

$$\langle \pi^+(p') \,|\, \mathcal{O}^{\mu\mu_1\mu_2\cdots\mu_n} \,|\, \pi^+(p) \rangle = \langle \pi^+(p') \,|\, \bar{u}(0)\,\gamma^{\{\mu}\, i\overleftrightarrow{D}^{\mu_1} i\overleftrightarrow{D}^{\mu_2} \cdots i\overleftrightarrow{D}^{\mu_n}\, u(0) \,|\, \pi^+(p) \rangle$$

$$= 2\,P^{\{\mu}P^{\mu_1}\cdots P^{\mu_n\}}A_{n+1,0}(\Delta^2) + 2\sum_{i=1,odd}^{n} \Delta^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}P^{\mu_{i+1}}\cdots P^{\mu_n\}}A_{n+1,i+1}(\Delta^2)$$

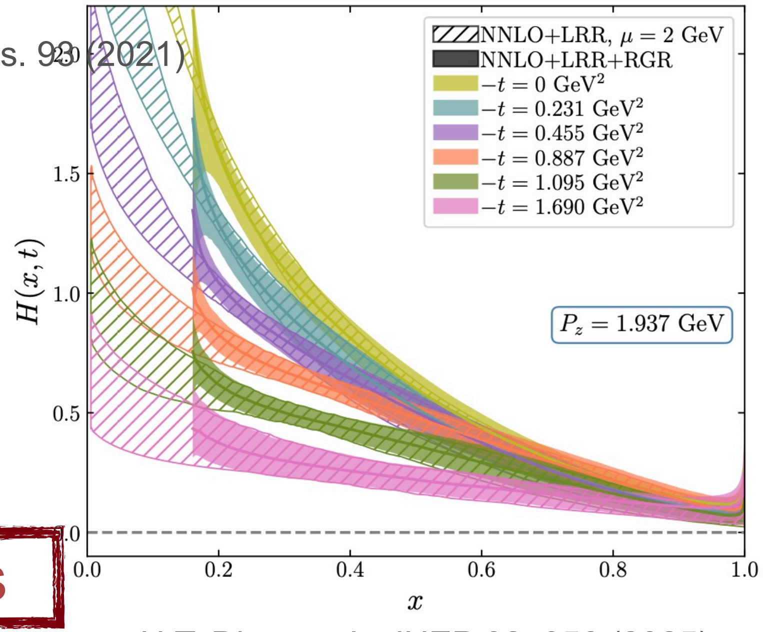
- Large momentum effective theory (LaMET) approach to x-dependent pion GPDs.

X.D. Ji, PRL 110 (2013);
X.D. Ji et. al., Rev.Mod.Phys. 93 (2021)



H.-W. Lin, Phys. Lett. B 846, 138181 (2023)

✓ Zero skewness



H.T. Ding et al., JHEP 02, 056 (2025)

Moments from nonlocal correlators

T. Izubuchi et al., PRD 98, 056004 (2018); Y.-S. Liu et al., PRD 100, 034006 (2019).

- Operator product expansion (OPE) of nonlocal operators:

☑ Pion and nucleon **PDFs** X. Gao et al., *PRD* 109 (2024); X. Gao et al., *PRD* 107 (2023); B. Joo et al., PRD 100, 114512 (2019) ; J. Karpie et al., JHEP 11, 178 (2018); …

$$\tilde{Q}(\lambda, \mu^2 z^2) = \sum_n C_n(\mu^2 z^2) \frac{(-i\lambda)^n}{n!} a_{n+1}; \quad a_{n+1} = \int_{-1}^{1} dx\, x^n q(x, \mu) \qquad \text{Ioffe time} : \lambda = zP_z$$

☑ Pion and nucleon **GPDs** (zero skewness) S. Bhattacharya et al., JHEP 01, 146 (2025); X. Gao et al., PRD 109, 094506 (2024); X. Gao et al., PRD 104 (2021) ; …

$$\tilde{H}(z, P_z, t) = \sum_n C_n(\mu^2 z^2) \frac{(-i\lambda)^n}{n!} A_{n,0}(t); \quad A_{n,0}(t) = \int_{-1}^{1} dx\, x^{n-1} H(x, t, \mu)$$

☑ Only nucleon **GPDs (nonzero** skewness) HadStruc Collab., JHEP 08, 162 (2024)

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^{k} H_n(\xi, t, \mu)\, \xi^{k-n} c_{k,k-n}(z^2\mu^2); \quad H_n(\xi, t, \mu) = \int_{-1}^{1} dx\, x^{n-1} H(x, \xi, t, \mu)$$

✓ $H_3 = A_{3,0} + \xi^2 A_{3,2}$ **becomes larger as ξ increases.**

✓ Only LL-resummed results were considered.

Opposite trend for pion case!!

Value at t = 0

GPD H^{u-d}		GPD E^{u-d}	
$A_{1,0}$: 0.974^{+12}_{-5}		$B_{1,0}$: 3.40^{+7}_{-1}	
$A_{2,0}$: 0.206^{+2}_{-6}		$B_{2,0}$: 0.370^{+9}_{-24}	
$A_{3,0}$: 0.064^{+2}_{-6}	$A_{3,2}$: 0.39^{+11}_{-3}	$B_{3,0}$: 0.063^{+24}_{-8}	$B_{3,2}$: 1.1^{+4}_{-8}
$A_{4,0}$: 0.065^{+5}_{-19}	$A_{4,2}$: 0.5^{+3}_{-3}	$B_{4,0}$: 0.06^{+16}_{-2}	$B_{4,2}$: > 1.1

HadStruc Collab., JHEP 08, 162 (2024)

Theoretical framework

- OPE of nonlocal operators:

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^{k} H_n(\xi, t, \mu)\, \xi^{k-n}\, c_{k,k-n}(z^2\mu^2) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2 z^2\right)$$

✓ Renormalized matrix elements

✓ n-th moment of pion GPDs.

✓ Perturbative matching matrix (lower-triangular)

Valence-quark pion matrix element and its LI decomposition

$$\widetilde{H}^R(z,P_z,\xi,t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^{k} H_n(\xi,t,\mu)\,\xi^{k-n}\, c_{k,k-n}(z^2\mu^2) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2 z^2\right)$$

✓ Renormalized matrix elements

Ratio scheme:

$$\widetilde{H}^R(z,P_z,\xi,t) = \frac{\widetilde{H}(z,P_z,\xi,t)}{\widetilde{H}(z,0,0,0)}$$

A. Radyushkin, PRD 98, 014019 (2018)

- Valence-quark matrix element:

$$M^\mu(z,P,\Delta) = \left\langle \pi(p_f) \middle| \bar{\psi}(-z/2)\gamma^\mu L(-z/2,z/2)\psi(z/2) \middle| \pi(p_i) \right\rangle^{u-d}$$

- Lorentz-invariant (LI) amplitudes $A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$:

$$M^\mu(z,P,\Delta) = P^\mu A_1 + z^\mu m^2 A_2 + \Delta^\mu A_3$$

- Traditional quasi-GPD defined with γ^0 is **frame dependent**:

$$\widetilde{H}(z,P_z,\xi,t) = \frac{1}{P^0} M^0(z,P,\Delta) = A_1 + \frac{\Delta^0}{P^0} A_3$$

S. Bhattacharya et al., PRD 106 (2022)

- LI quasi-GPD: combining $M^0\,(\gamma^0)$ and $M^x\,(\gamma^x)$

$$\widetilde{H}_{\mathrm{LI}}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_1 + \frac{z\cdot\Delta}{z\cdot P} A_3$$

Consistency with the γ^0 definition!

Polynomiality of GPD moments

$$\widetilde{H}^R(z,P_z,\xi,t) = \sum_{k=1}^{\infty}\frac{(-izP_z)^{k-1}}{(k-1)!}\sum_{n=1}^{k} H_n(\xi,t,\mu)\,\xi^{k-n}\,c_{k,k-n}(z^2\mu^2)+\mathcal{O}\left(\Lambda_{\rm QCD}^2 z^2\right)$$

✓ n-th moment of pion GPDs.

Polynomiality of GPD moments (ξ-dependent):

$$H_n(\xi,t) = \sum_{\substack{k=0\\ \text{even}}}^{n-1} \underbrace{\boxed{A_{n,k}(t)}}_{\text{GFFs}}(2\xi)^k \pm \underbrace{\text{mod}(n-1,2)\,(2\xi)^n\,C_n(t)}_{\text{D-term contribution (for even n)}}$$

Ji, PRL 78, 610 (1997); H. B. O'Connell et al., PLB 354 (1995)

GFFs

H. B. O'Connell et al., PLB 354 (1995)

1. monopole-like parameterization

$$A_{n,k}(t) = \frac{A_{n,k}(0)}{1-t/M_{n,k}^2}$$

G. Lee et al., PRD 92 (2015)

2. z-expansion (model-independent)

$$A_{n,k}(t) = \sum_{l=0}^{2} a_{n,k,l}\,\mathbf{z}^l, \quad \mathbf{z}(t,t_{\rm cut},t_0) = \frac{\sqrt{t_{\rm cut}-t}-\sqrt{t_{\rm cut}-t_0}}{\sqrt{t_{\rm cut}-t}+\sqrt{t_{\rm cut}-t_0}}$$

$\mathbf{z}$(t) is introduced to improve the convergence of the series expansion.

Matching and renormalization-group (RG) resummation

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^{k} H_n(\xi, t, \mu)\, \xi^{k-n}\, c_{k,k-n}(z^2\mu^2) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2 z^2\right)$$

✓ Perturbative matching matrix (lower-triangular)

$$c_{k,k-n} = \begin{pmatrix} c_{1,0} & 0 & 0 & 0 & 0 \\ 0 & c_{2,0} & 0 & 0 & 0 \\ c_{3,2} & 0 & c_{3,0} & 0 & 0 \\ 0 & c_{4,2} & 0 & c_{4,0} & 0 \\ c_{5,4} & 0 & c_{5,2} & 0 & c_{5,0} \end{pmatrix}$$

RGE: $\mu^2 \frac{d}{d\mu^2} C(z^2\mu^2) = \gamma(\alpha_s(\mu))\, C(z^2\mu^2),$

with $\gamma(\alpha_s) = \gamma_0\, \alpha_s + \gamma_1\, \alpha_s^2, \quad \beta(\alpha_s) \equiv \mu \frac{d\alpha_s}{d\mu} = -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{2(2\pi)^2}\alpha_s^3.$ NLL-RGR

Resummation order	$\alpha^n L^k$ log resummed	Anomalous dimension	β-function	$c_{n,n-k}$
LL	$n = k$	1-loop ✓	1-loop ✓	tree-level ✓
NLL	$n-1 \le k \le n$	2-loop ✓	2-loop ✓	1-loop ✓
NNLL	$n-2 \le k \le n$	3-loop ✗	3-loop ✓	2-loop ✗

✗ No solution for $\xi \neq 0$ but $\xi = 0$ exists.

Matching and renormalization-group (RG) resummation

$$\widetilde{H}^R(z,P_z,\xi,t) = \sum_{k=1}^{\infty}\frac{(-izP_z)^{k-1}}{(k-1)!}\sum_{n=1}^{k} H_n(\xi,t,\mu)\,\xi^{k-n}\,c_{k,k-n}(z^2\mu^2)+\mathcal{O}\left(\Lambda_{\rm QCD}^2 z^2\right)$$

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$$c_{k,k-n} = \begin{pmatrix} c_{1,0} & 0 & 0 & 0 & 0 \\ 0 & c_{2,0} & 0 & 0 & 0 \\ c_{3,2} & 0 & c_{3,0} & 0 & 0 \\ 0 & c_{4,2} & 0 & c_{4,0} & 0 \\ c_{5,4} & 0 & c_{5,2} & 0 & c_{5,0} \end{pmatrix}$$

$$\gamma_1^{ij} = \begin{pmatrix} 0.266 & 0 & 0 & 0 & 0 \\ 0 & 0.512 & 0 & 0 & 0 \\ -0.064 & 0 & 0.618 & 0 & 0 \\ 0 & -0.061 & 0 & 0.688 & 0 \\ -0.023 & 0 & -0.066 & 0 & 0.742 \end{pmatrix}$$

✓ Derived from Ref. [Y.Ji, **F.Yao** and J.H. Zhang, arXiv:2504.09367]

Lattice calculation

- **Asymmetric (non-Breit) approach**: computationally economical

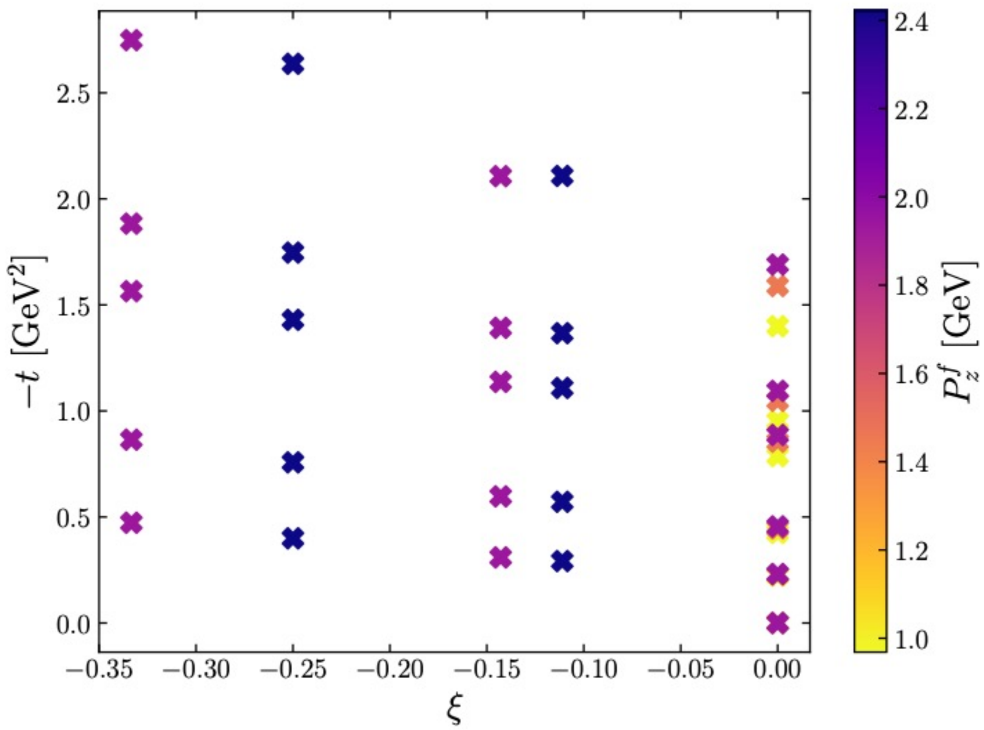
S. Bhattacharya et al., PRD 106 (2022)

$$\vec{p}_f = (0,0,P_z),\ \ \vec{p}_i = \vec{p}_f - \vec{\Delta} = \left(-\Delta_x, -\Delta_y, P_z - \Delta_z\right)$$

- **Lattice setup:** HISQ gauge ensembles

HotQCD collab., PRD 75 (2007)

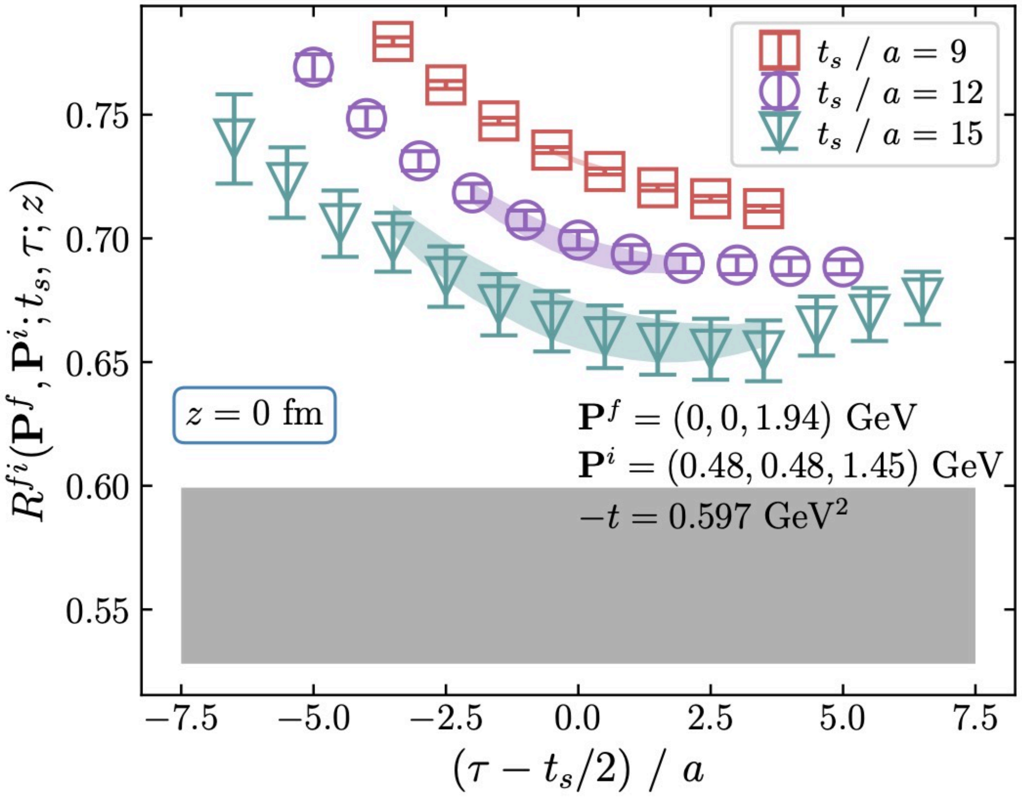
- $a = 0.04$ fm
- Pion mass: 300 MeV
- $P_z = \{0.968, 1.453, 1.937, 2.428\}$ GeV
- $\xi = \{-0.33, -0.25, -0.14, -0.11, 0\}$
- $-t$ up to 2.748 GeV^2



Extraction of bare matrix elements

- Bare matrix elements are extracted from simultaneous fits at multiple source–sink separations t_s.

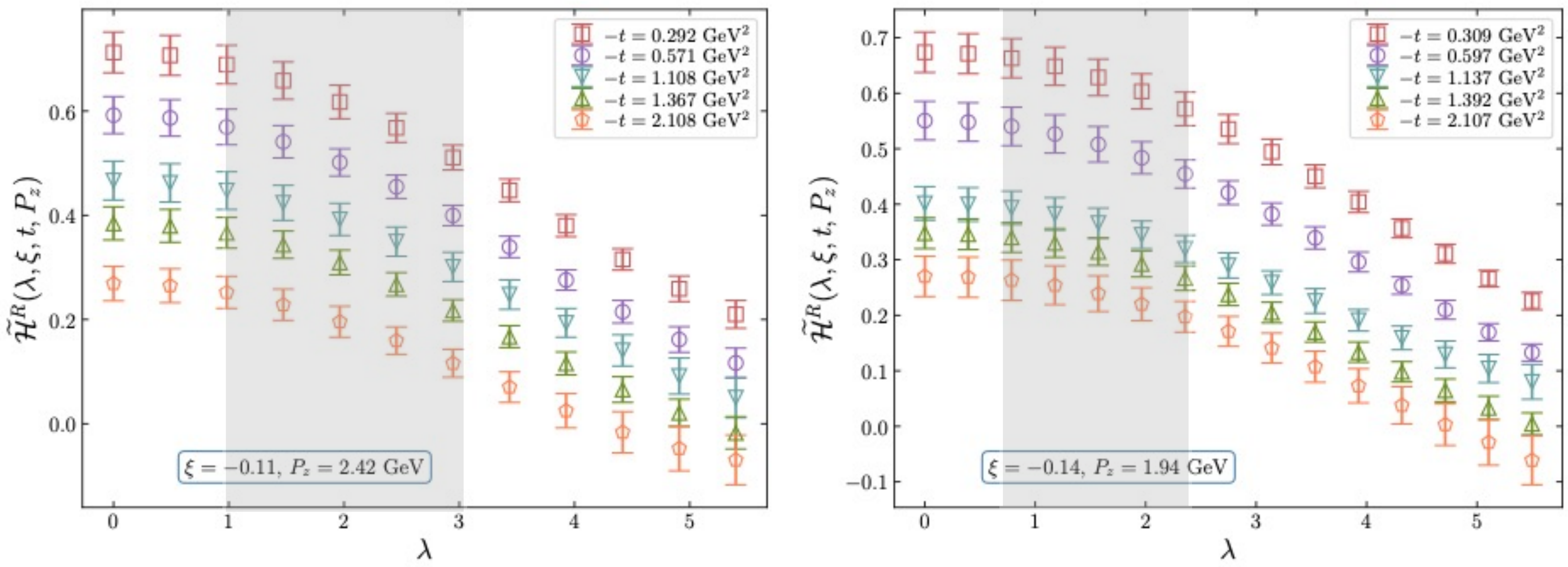
$$R^{fi}(\mathbf{P}^f, \mathbf{P}^i; t_s, \tau; z) \equiv \frac{2\sqrt{E_0^f E_0^i}}{E_0^f + E_0^i} \frac{C_{3\text{pt}}(\mathbf{P}^f, \mathbf{P}^i; t_s, \tau; z)}{C_{2\text{pt}}(\mathbf{P}^f, t_s)} \times \left[\frac{C_{2\text{pt}}(\mathbf{P}^i, t_s - \tau) C_{2\text{pt}}(\mathbf{P}^f, \tau) C_{2\text{pt}}(\mathbf{P}^f, t_s)}{C_{2\text{pt}}(\mathbf{P}^f, t_s - \tau) C_{2\text{pt}}(\mathbf{P}^i, \tau) C_{2\text{pt}}(\mathbf{P}^i, t_s)}\right]^{1/2}$$



(a) $\xi = -0.14$, $-t = 0.597$ GeV^2

Renormalized matrix elements

- Bare matrix elements are renormalized by ratio scheme: $\widetilde{H}^R(z, P_z, \xi, t) = \dfrac{\widetilde{H}(z, P_z, \xi, t)}{\widetilde{H}(z,0,0,0)}$



$\lambda = zP_z$

Fitting region used in this work: $z = \{2a, 6a\} = \{0.08, 0.24\}$ fm

Choice of fitting region

Resummation order	$\alpha^n L^k$ log resummed	Anomalous dimension	β-function	$c_{n,n-k}$
LL	$n = k$	1-loop ✓	1-loop ✓	tree-level ✓
NLL	$n-1 \le k \le n$	2-loop ✓	2-loop ✓	1-loop ✓
NNLL	$n-2 \le k \le n$	3-loop ✗	3-loop ✓	2-loop ✗

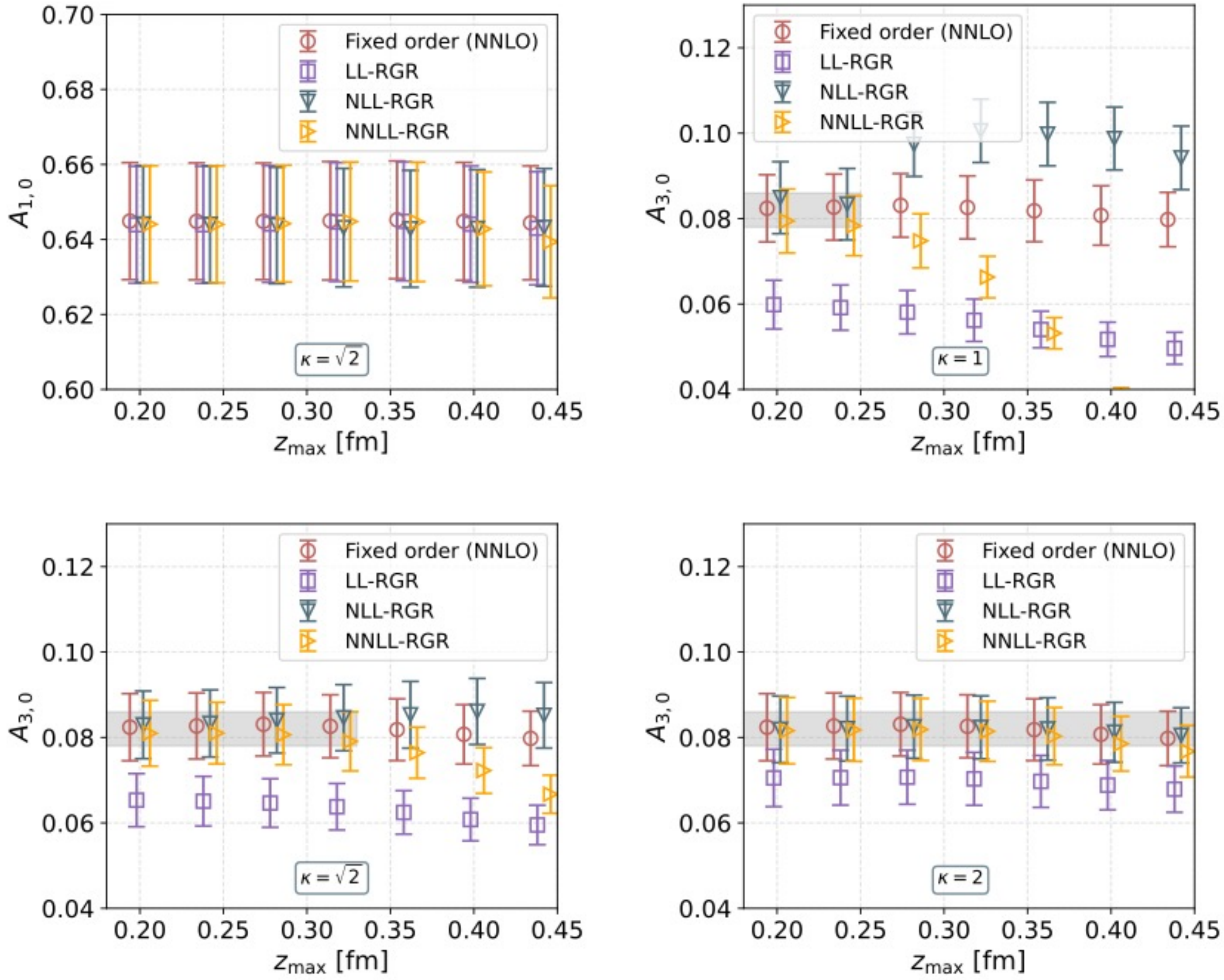
✗ No solution for $\xi \neq 0$ but $\xi = 0$ exists.

$$\widetilde{H}^R(z, P_z, \xi = 0, t) = A_{1,0}(t)\, c_{1,0} + \frac{(-izP_z)^2}{2!} A_{3,0}(t)\, c_{3,0} + \frac{(-izP_z)^4}{4!} A_{5,0}(t)\, c_{5,0}$$

- Fixed $z_{\mathrm{min}} = 0.08$ fm, then varied z_{max}.
- Considering **LL, NLL and NNLL resummation** effects.
- Evolve from $\mu_0 = (2\kappa e^{-\gamma_E})/z$ to $\mu = 2$ GeV, with $\kappa = \{1, \sqrt{2}, 2\}$.

Choice of fitting region

$$P_z = 1.937\ \mathrm{GeV},$$
$$\xi = 0,$$
$$-t = 0.446\ \mathrm{GeV}^2$$



Numerical results for GPD moments

- Extracting the moments through a joint fit of matrix elements over multiple ξ and $-t$,

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^{k} H_n(\xi, t, \mu^2)\, \xi^{k-n}\, c_{k,k-n}(z^2\mu^2)$$

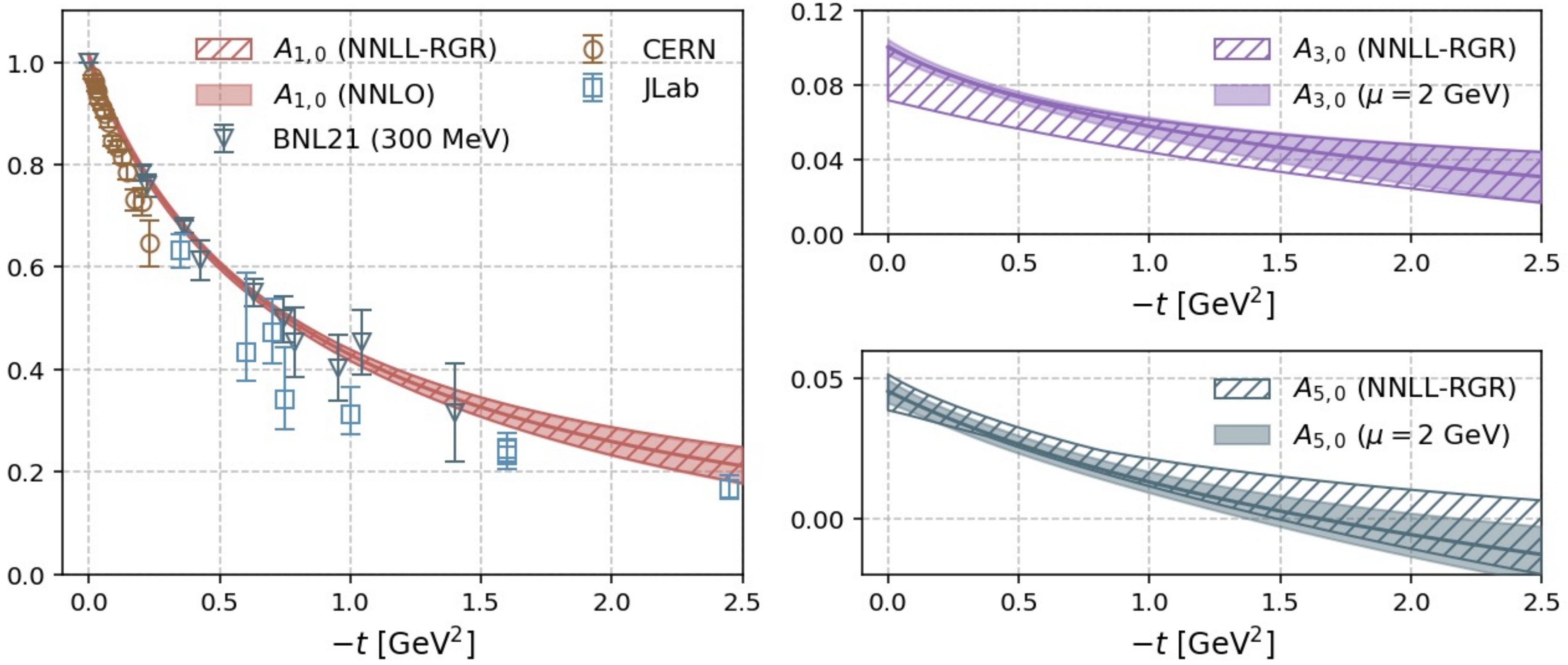
$$= H_1(\xi,t)\, c_{1,0} + \frac{(-izP_z)^2}{2!}\left(H_1(\xi,t)\,\xi^2\, c_{3,2} + H_3(\xi,t)\, c_{3,0}\right) + \frac{(-izP_z)^4}{4!}\left(H_1(\xi,t)\,\xi^4\, c_{5,4} + H_3(\xi,t)\,\xi^2\, c_{5,2} + H_5(\xi,t)\, c_{5,0}\right)$$

$$= A_{1,0}(t)\, c_{1,0} + \frac{(-izP_z)^2}{2!}\left[A_{1,0}(t)\,\xi^2\, c_{3,2} + \underbrace{\left(A_{3,0}(t) + A_{3,2}(t)(2\xi)^2\right)}_{H_3} c_{3,0}\right] + \frac{(-izP_z)^4}{4!}\Big[A_{1,0}(t)\,\xi^4\, c_{5,4} +$$

$$\left(A_{3,0}(t) + A_{3,2}(t)(2\xi)^2\right)\xi^2\, c_{5,2} + \underbrace{\left(A_{5,0}(t) + A_{5,2}(t)(2\xi)^2 + A_{5,4}(t)(2\xi)^4\right)}_{H_5} c_{5,0}\Big]$$

$$A_{n,k}(t) = \sum_{l=0}^{2} a_{n,k,l}\,\mathbf{z}^l, \quad \mathbf{z}(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

GFFs $A_{1,0}$, $A_{3,0}$ and $A_{5,0}$ ($\xi = 0$)



✓ The GFFs $A_{n,0}$ are extracted from **zero-skewness data**.

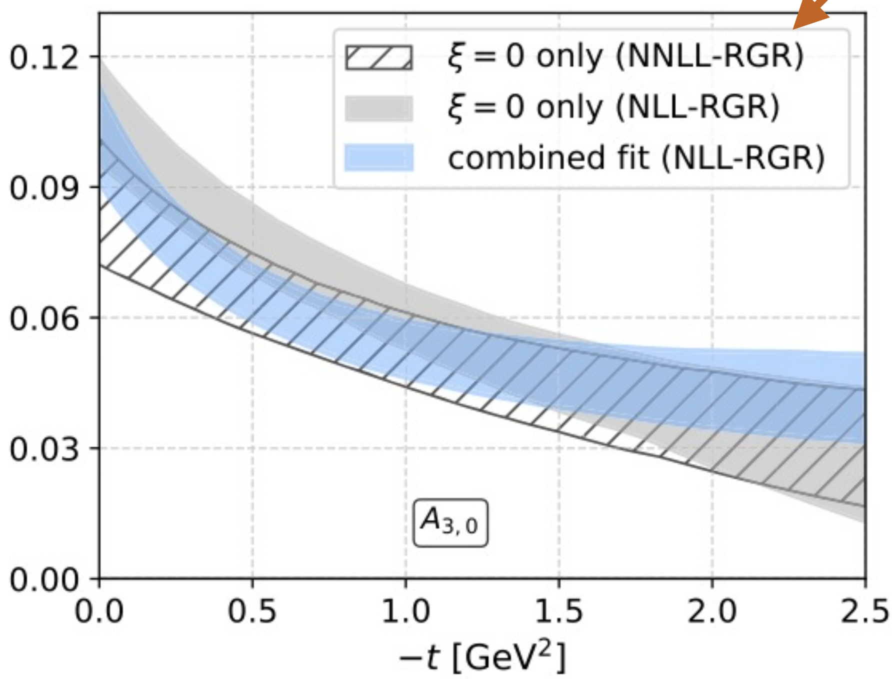
✓ $A_{1,0} = F_\pi$ agree with CERN/JLab data and BNL21 results.

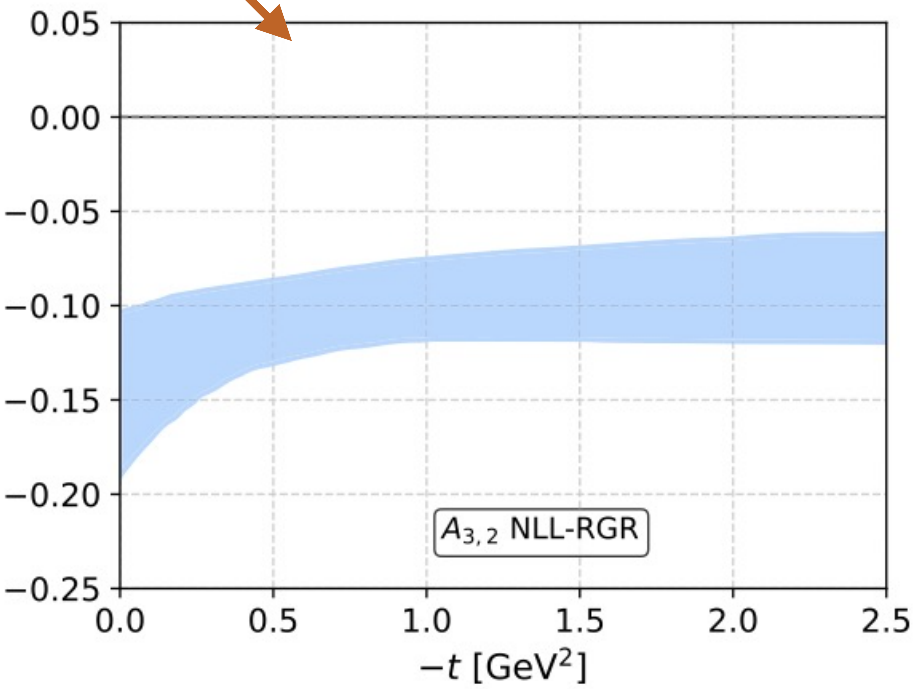
✓ NNLL-RGR uncertainty: varying κ between 1 and 2.

CERN: Amendolia et al., NPB 277 (1986)
JLab: Volmer et al., PRL 86 (2001);
Horn et al., PRL 97 (2006);
Huber et al., PRC 78 (2008);
BNL21: Gao et al., PRD 104 (2021)

GFFs $A_{3,i}$ and third Mellin moment $H_3(\xi, t)$

$$H_3(\xi,t) = A_{3,0}(t) + (2\xi)^2 A_{3,2}(t)$$



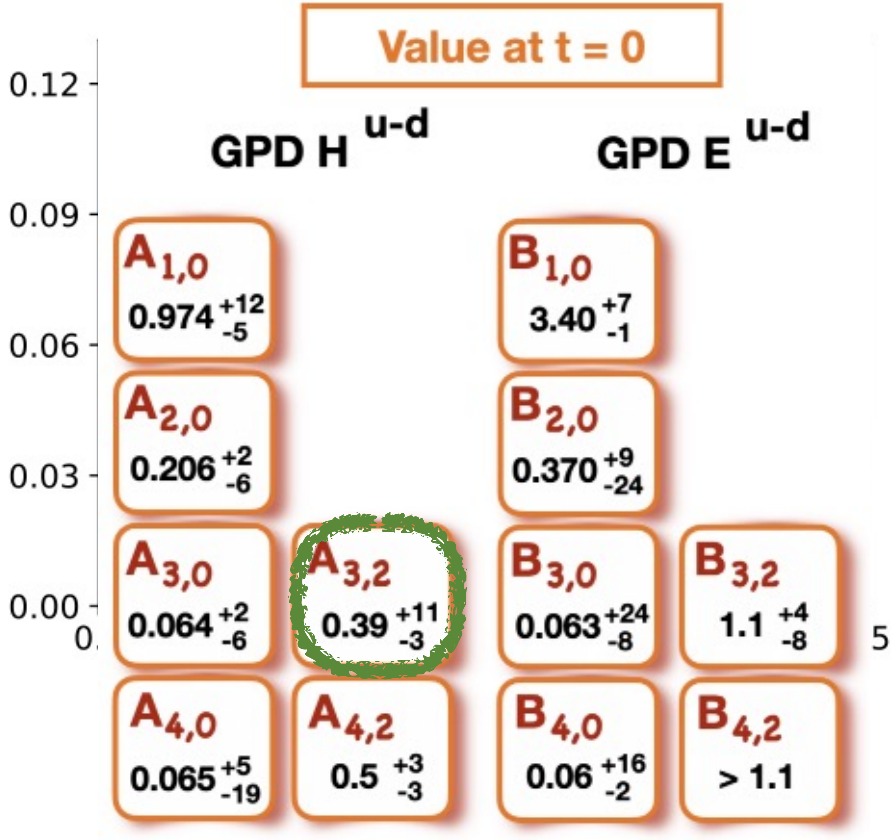


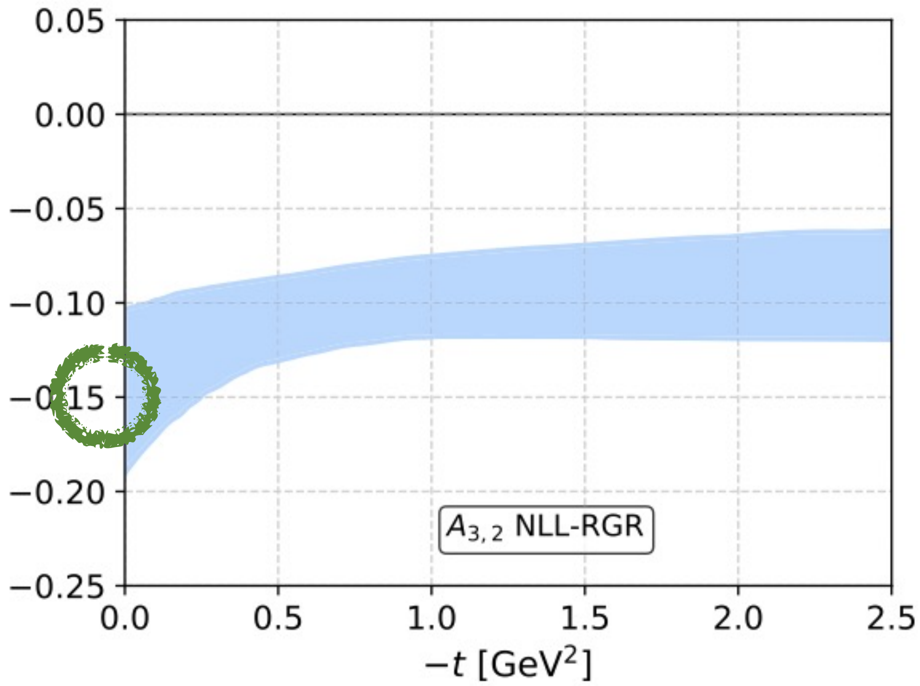
✓ Combined fit using both $\xi = 0$ and $\xi \neq 0$ data.

✓ $A_{3,2}$ is negative, giving visible skewness dependence.

GFFs $A_{3,i}$ and third Mellin moment $H_3(\xi, t)$

$$H_3(\xi, t) \;=\; A_{3,0}(t) + (2\xi)^2\, A_{3,2}(t)$$



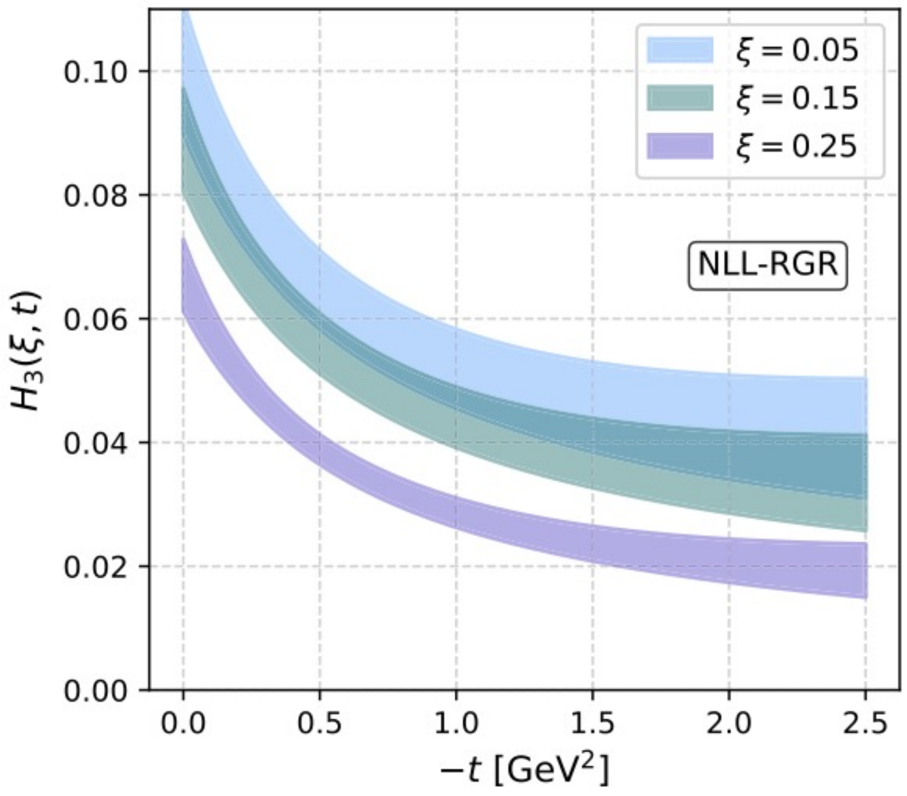


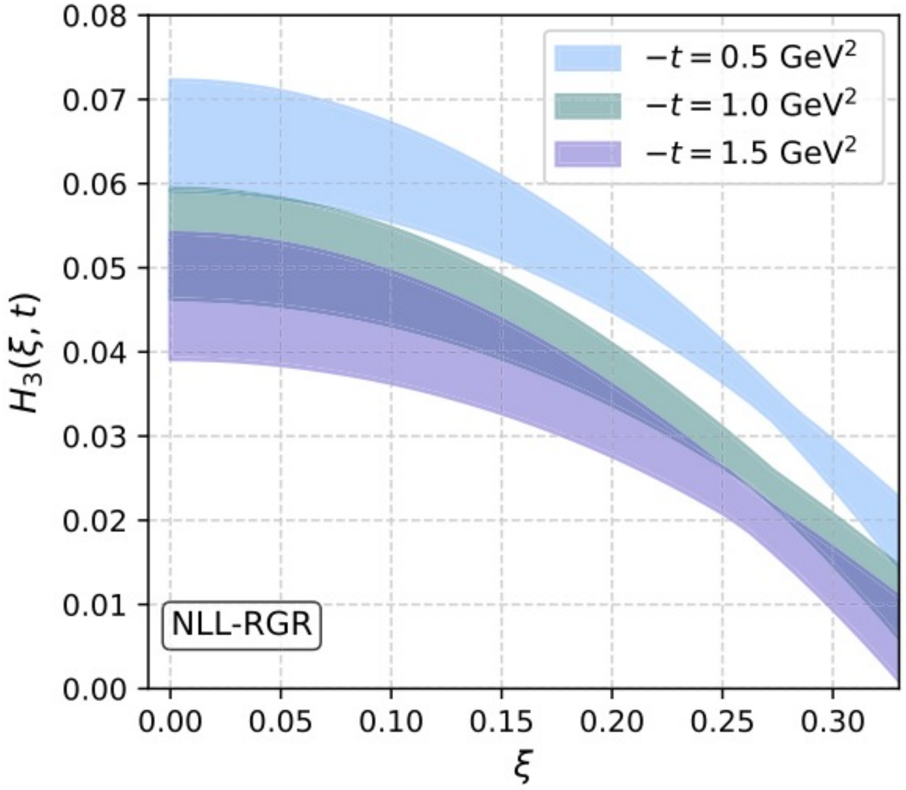
Opposite trend compared with the nucleon case.

HadStruc Collab., JHEP 08, 162 (2024) / arXiv:2604.21476

GFFs $A_{3,i}$ and third Mellin moment $H_3(\xi, t)$

$$H_3(\xi, t) = A_{3,0}(t) + (2\xi)^2 A_{3,2}(t)$$

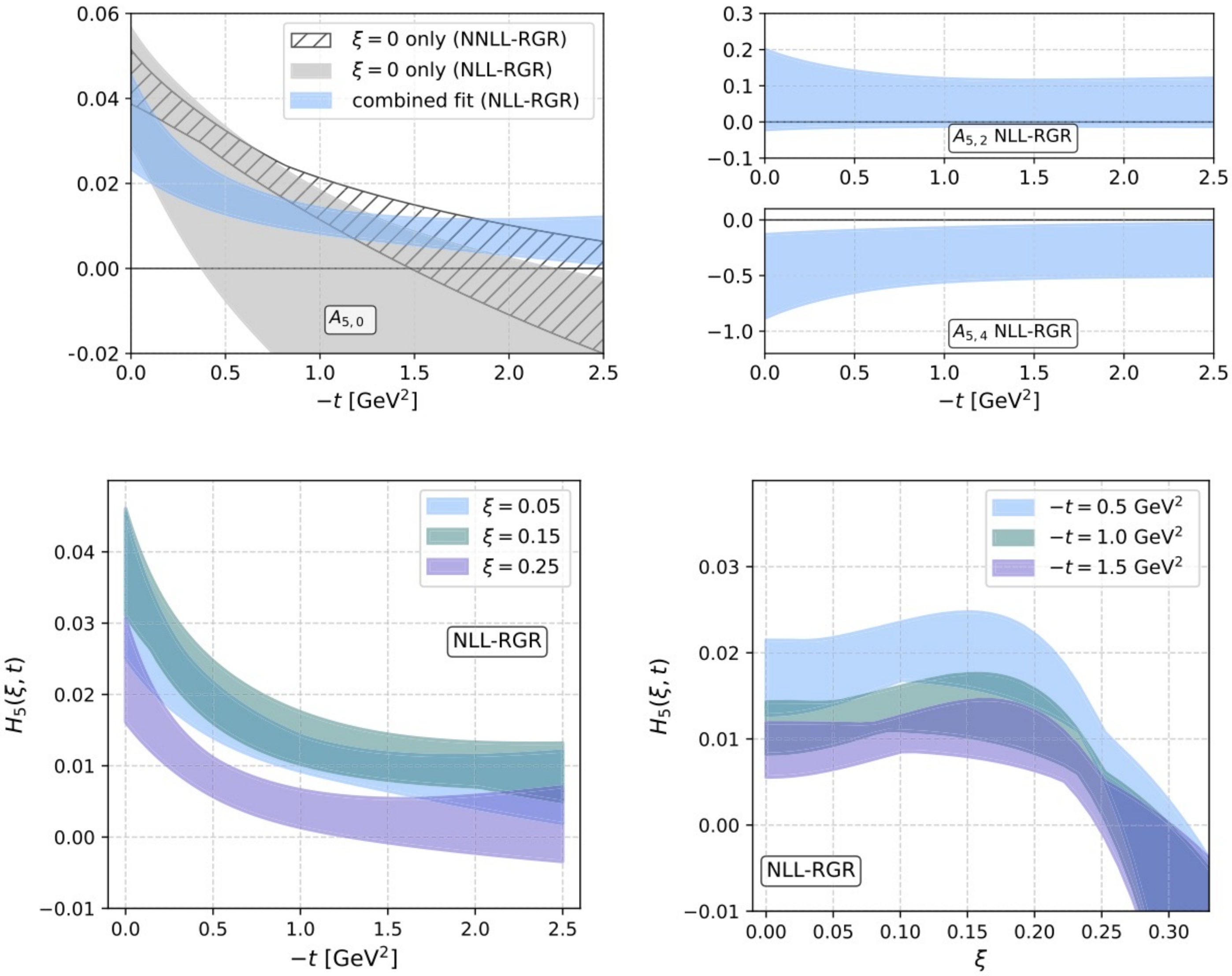




✓ $H_3(\xi, t)$ decreases as $|\xi|$ increases.

✓ It also decreases with increasing $-t$.

GFFs $A_{5,i}$ and Fifth Mellin moment $H_5(\xi, t)$



$$H_5(\xi, t) = A_{5,0} + (2\xi)^2 A_{5,2} + (2\xi)^4 A_{5,4}$$

✓ $\xi = 0$ and $\xi \neq 0$ data combined fit.

✓ The fifth-moment GFFs $A_{5,i}$ are strongly suppressed.

✓ $H_5(\xi, t)$ decreases with increasing $|\xi|$ and $-t$.

Summary and outlook

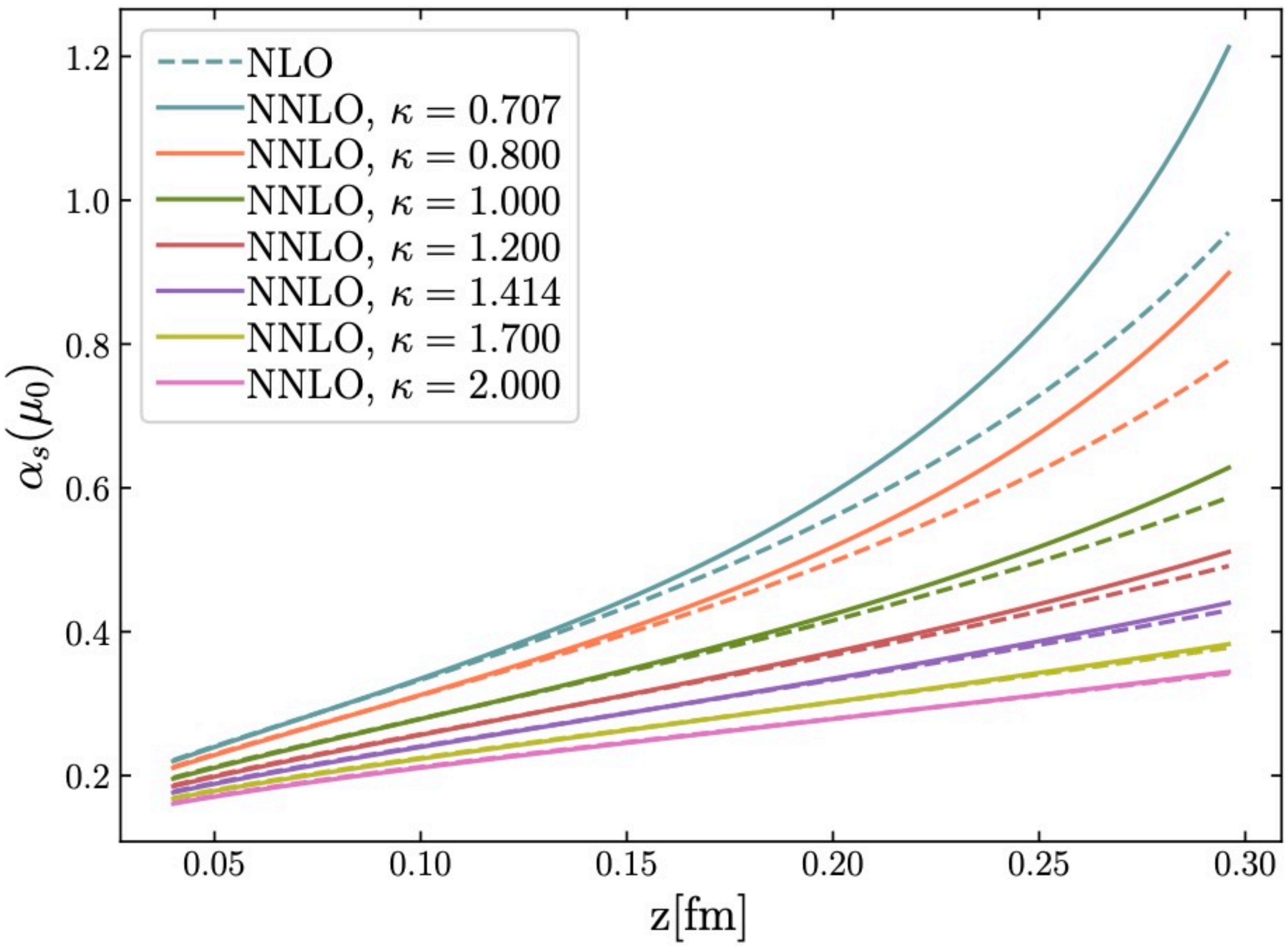
Summary

- ☑ First lattice QCD determination of pion GPD Mellin moments at **nonzero skewness.**
- ☑ Combined fits to $\xi = 0$ and $\xi \neq 0$ data $\rightarrow$ access to (ξ, t) dependence.
- ☑ Implemented **NLL resummation** to reduce perturbative matching uncertainties.

Outlook

- Extraction of x-dependent pion GPDs at nonzero skewness.
- D-term extraction from the imaginary part of matrix elements.
- Simulations at physical pion mass with higher statistics.

Thank you!


NLO
NNLO, $\kappa = 0.707$
NNLO, $\kappa = 0.800$
NNLO, $\kappa = 1.000$
NNLO, $\kappa = 1.200$
NNLO, $\kappa = 1.414$
NNLO, $\kappa = 1.700$
NNLO, $\kappa = 2.000$
$\alpha_s(\mu_0)$
1.2
1.0
0.8
0.6
0.4
0.2
0.05
0.10
0.15
0.20
0.25
0.30
z[fm]

$$\int_{-1}^{1} dx\, x^{n-1} \begin{pmatrix} H^{u-d} \\ E^{u-d} \end{pmatrix}(x,\xi,t) = \sum_{k\,=\,0,\,even}^{n-1} \begin{pmatrix} A_{n,k}(t) \\ B_{n,k}(t) \end{pmatrix} \xi^k \pm \mathrm{mod}(n+1,2)\, \xi^n \left(C_n(t)\right).$$

- Solution (γ_0, γ_i do not commute)

$$C(z^2\mu^2) = \mathscr{P} \exp\left[2\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}\gamma(\alpha)\right] C(z^2\mu_0^2),$$

- LL-resummation

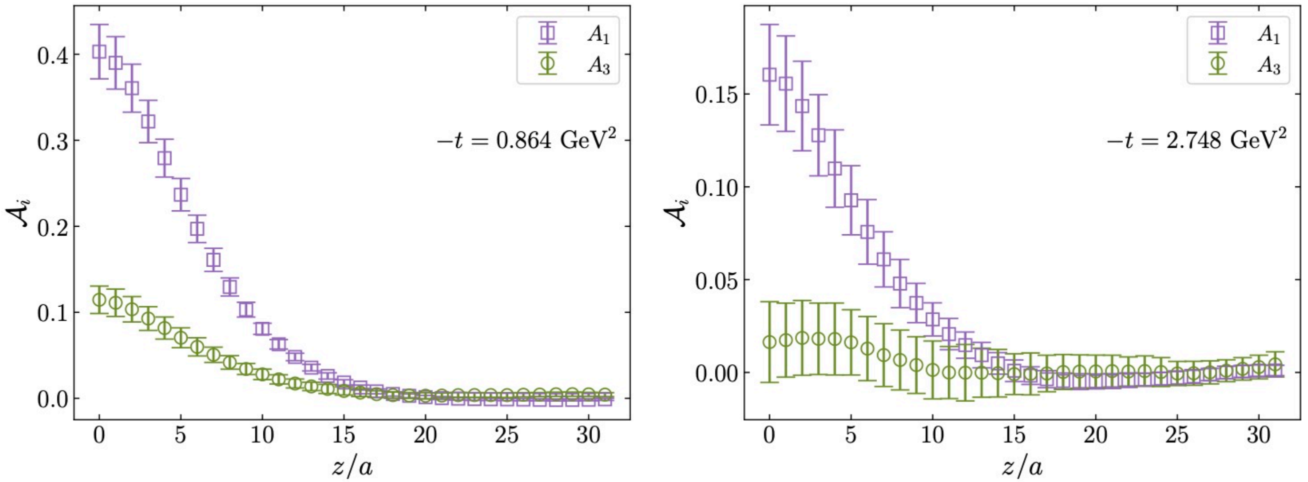
$$c_{i,j}(\alpha_s(\mu), z^2\mu^2) = c_{i,j}(\alpha_s(\mu_0), z^2\mu_0^2)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{\frac{\gamma_0}{\beta_0}},$$

✓NLL-resummation

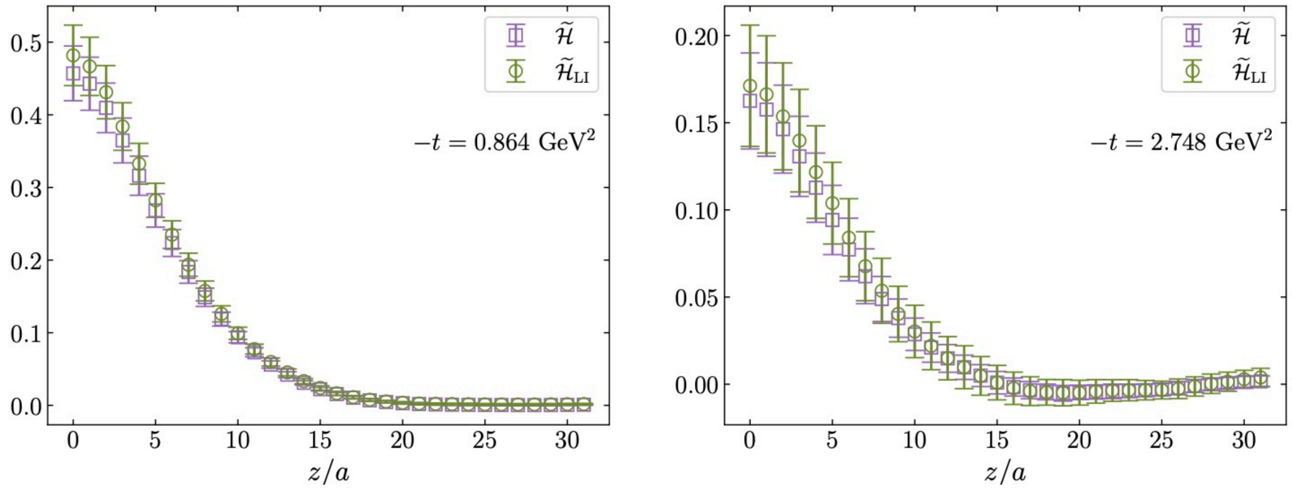
$$c_{i,j}(\alpha_s(\mu), z^2\mu^2) = c_{i,j}(\alpha_s(\mu_0), z^2\mu_0^2)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{\frac{\gamma_0}{\beta_0}} \exp\left\{\frac{(\beta_0\gamma_1 - \beta_1\gamma_0)}{\beta_0\beta_1}\ln\left(\frac{\alpha_s(\mu_0)\beta_1 + (4\pi)\beta_0}{\alpha_s(\mu)\beta_1 + (4\pi)\beta_0}\right)\right\}.$$

Comparison between $\widetilde{H}_{LI}$ and $\widetilde{H}$

$P_z = 1.937$ GeV,
$\xi = -0.33$,
$-t = \{0.864, 2.748\}$ GeV^2



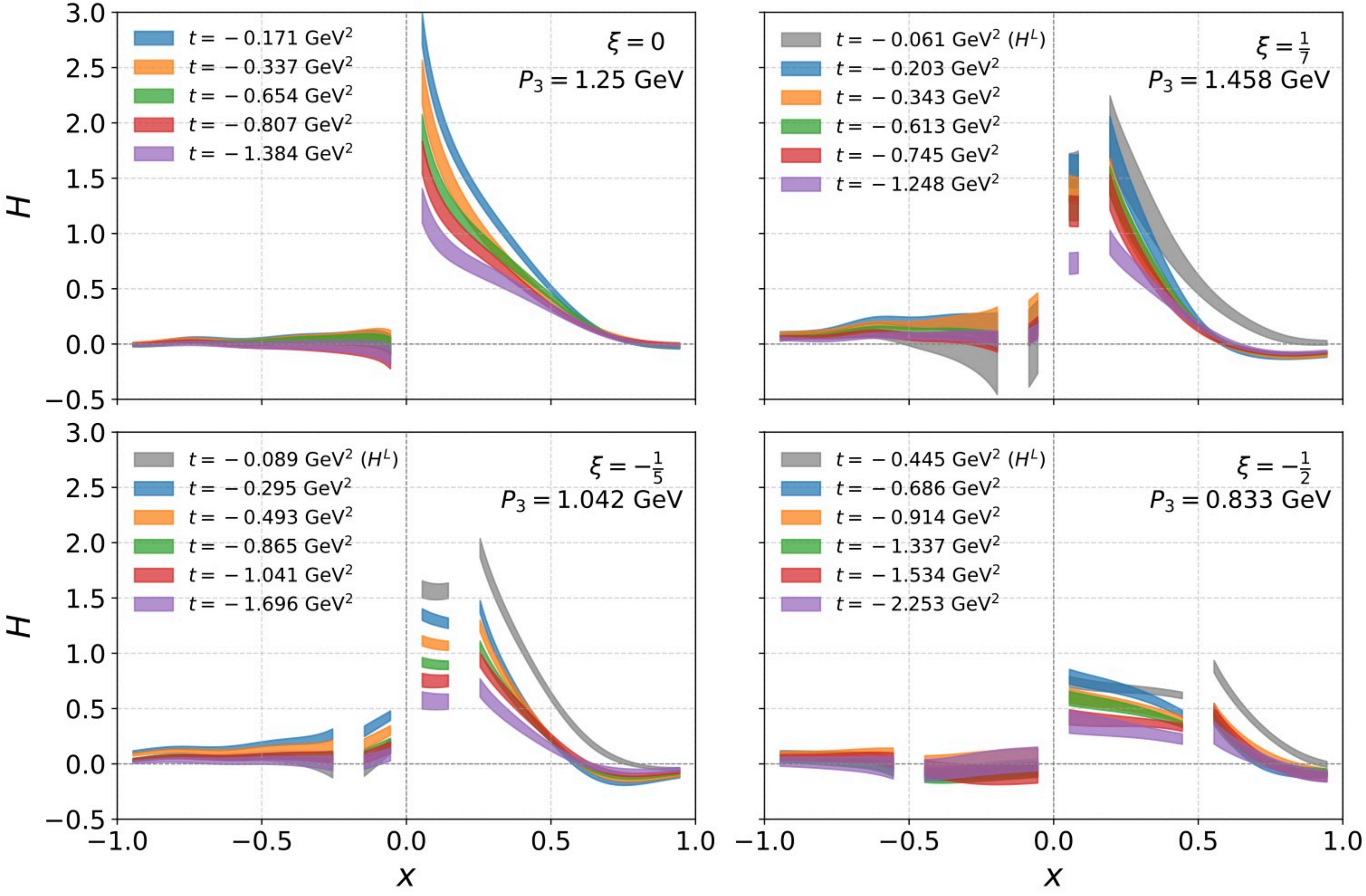
$$\widetilde{H}_{LI}(z\cdot P, z\cdot \Delta, \Delta^2, z^2) = A_1 + \frac{z\cdot\Delta}{z\cdot P} A_3$$



✓Original definition by γ^0

$$\widetilde{H}(z, P_z, \xi, t) = \frac{1}{P^0} M^0(z, P, \Delta)$$

✓**Adopted in this work.**



M.H. Chu et al., *Phys.Rev.D* 112 (2025) 9, 094510