Global CFFs and D-term fitting from DVCS data

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Towards improved hadron femtography with hard exclusive reactions
Charlottesville, University of Virginia
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Jefferson Lab

Outline

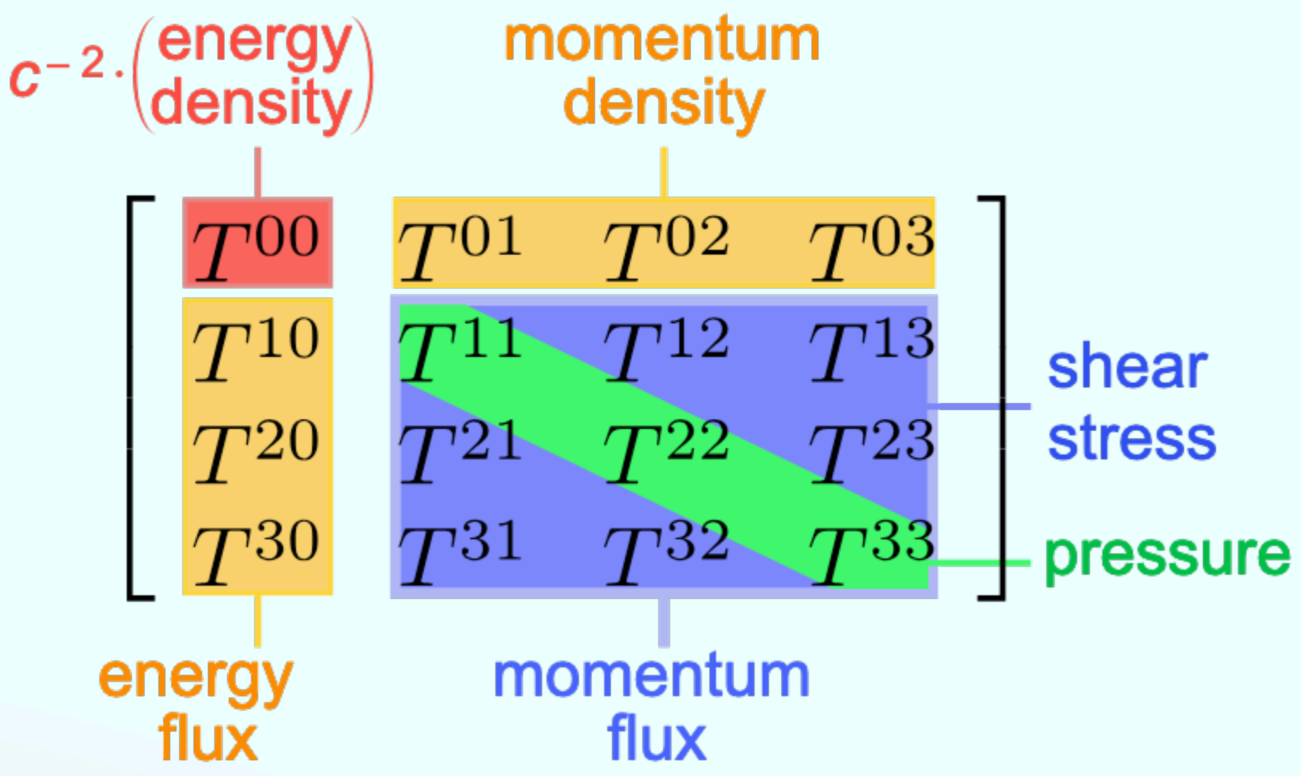
1. Motivation:
   - Mechanical properties of the proton.
   - From observables to Compton form factors.
2. Framework:
   - Gepard + Neural Networks
   - Extraction of CFFs from experimental data using NN.
3. Analysis process:
   - Model stability and optimization: Closure test and Optuna Study.
   - Update on global analysis of CFFs.
   - Dispersion relation and D-term extraction.
4. Deliverables.

Motivation and scope of the project:

- Mechanical properties of the proton.
- From DVCS observables to Compton form factors

Mechanical properties of the proton

The Matrix element of the QCD Energy-Momentum tensor encodes key information of the proton including the mass and spin, as the distributions of energy, angular momentum, and various mechanical properties such as, e.g., internal forces inside the system.



Momentum

$$\langle p', \vec{s}' | T_a^{\mu\nu} | p, \vec{s} \rangle = \bar{u}(p', \vec{s}') \Big[A_a(t) \frac{P^\mu P^\nu}{M_N} + D_a(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{4M_N} + \bar{C}_a(t) M_N g^{\mu\nu} + J_a(t) \frac{P^{\{\mu} i\sigma^{\nu\}\lambda}\Delta_\lambda}{M_N} - S_a(t) \frac{P^{[\mu} i\sigma^{\nu]\lambda}\Delta_\lambda}{M_N} \Big] u(p, \vec{s})$$

Pressure

Total angular momentum

Anisotropy Pressure

Intrinsic angular momentum

Symmetric kinematical variables:

$P = (p' + p)/2$ and $\Delta = p' - p$

Free Dirac spinor: $u(p, \vec{s})$

GFFs:

$A_a(t)$, $D_a(t)$, $\bar{C}_a(t)$, and $J_a(t)$

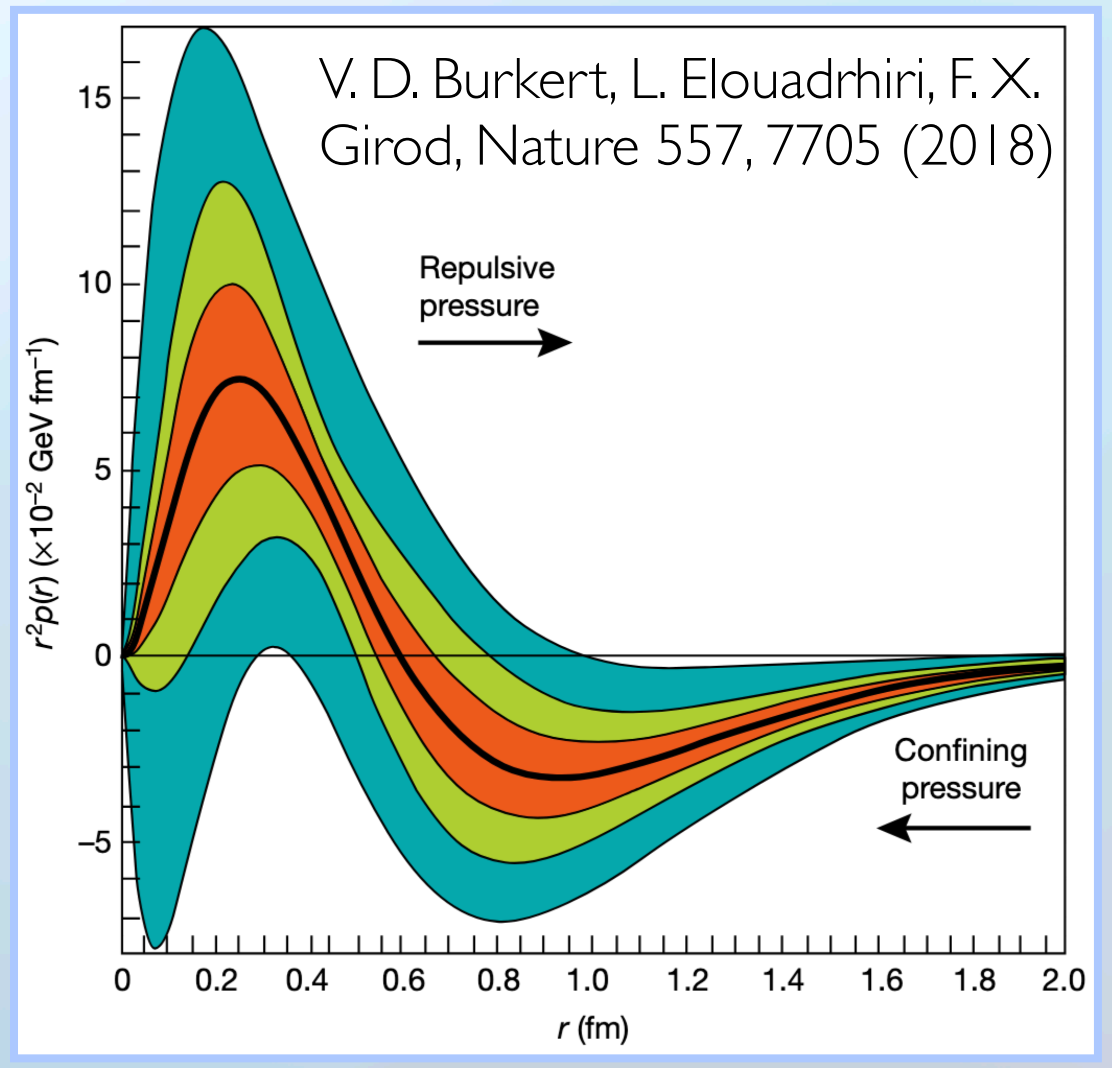
On top of restricting the number of GFFs, Poincaré symmetry imposes additional constraints:

$$\begin{aligned}
A(0) &= \sum_q A_q(0) + A_G(0) = 1, \\
J(0) &= \sum_q J_q(0) + J_G(0) = \tfrac{1}{2}, \\
\tfrac{1}{2}\Delta\Sigma &= \sum_q S_q(0), \\
\bar{C}(t) &= \sum_q \bar{C}_q(t) + \bar{C}_G(t) = 0,
\end{aligned}$$

D(t) is the lesser-known GFF. Its value is not fixed by spacetime symmetries and must be determined from experiment.

$$D(t) = \,? \quad \Longrightarrow \quad \boxed{\Delta(t) \propto D(t) \propto \int d^3\mathbf{r}\; p(r) \frac{j_0(r\sqrt{-t})}{t}}$$

The D-term is not related to "external properties" of a particle like mass and spin, but to the stress tensor and internal force.



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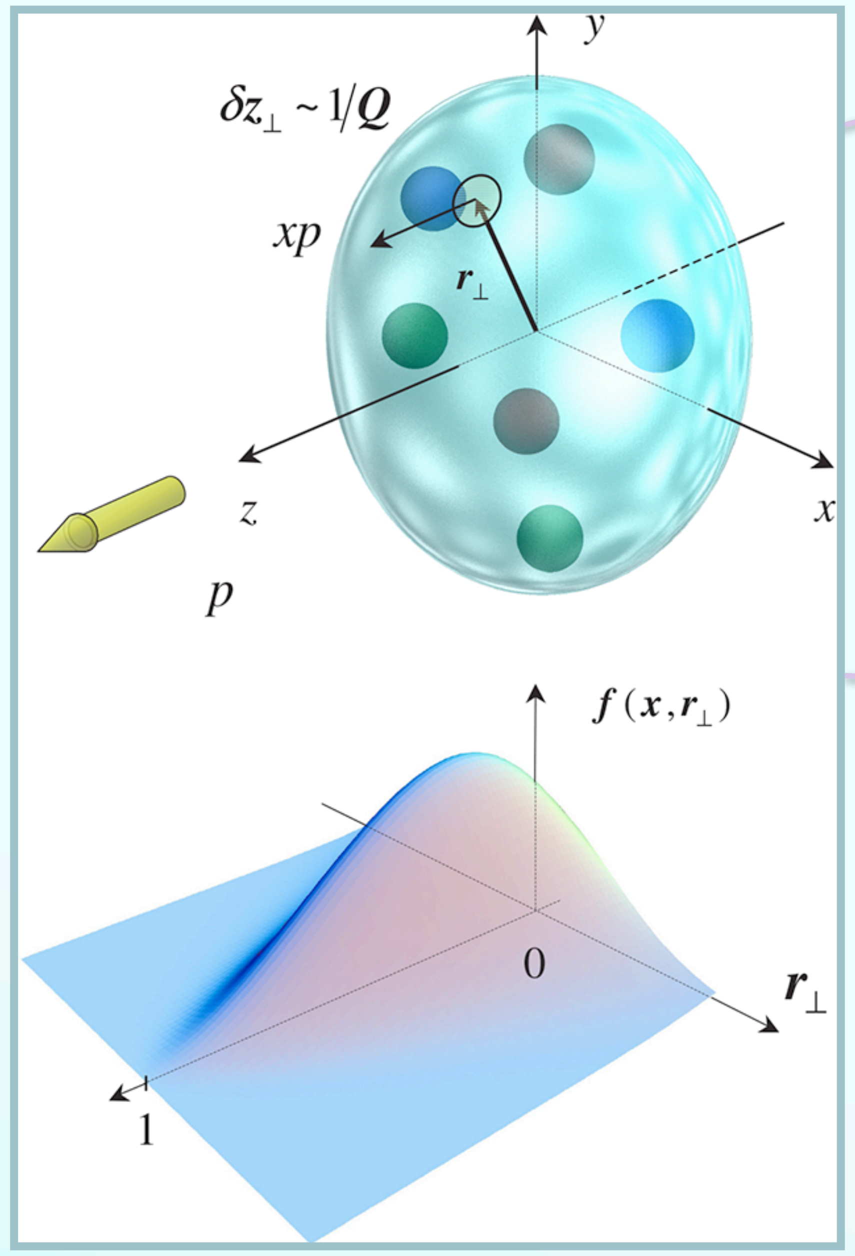
From DVCS observables to Compton form factors

The DVCS amplitude depends on the complex-valued convolution integrals of the GPDs: Compton form factors (CFFs)



We cannot access GPDs directly

$$\int_{-1}^{1} dx\; xH^q(x,\xi,t) = A^q(t) + \xi^2 D^q(t)$$

$$\int_{-1}^{1} dx\; xE^q(x,\xi,t) = B^q(t) - \xi^2 D^q(t)$$

twist 2 GPDs: $\{H, E, \tilde{H}, \tilde{E}\}$

$$\mathrm{Re}\mathcal{H}(\xi,t) + i\,\mathrm{Im}\mathcal{H}(\xi,t) = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H_q(x,\xi,t)$$

$\xi = x_B/(2 - x_B)$ is the skewness variable
x_B is the Bjorken scaling variable
t is the squared momentum transfer to the nucleon.

- The inverse problem is intrinsically ill-posed.

Thanks to the DVCS LO, it is possible to obtain the fixed-t dispersion relation (DR) where the subtraction constant is related to the so-called D-term.

- Dispersion integral computed via Gauss-Legendre quadrature (18 nodes), after regularizing the principal-value singularity.

$$\mathrm{Re}\mathcal{H}(\xi,t) = \Delta(t) + \frac{1}{\pi}\,\mathrm{P.V.}\int_0^1 d\xi' \left[\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right] \mathrm{Im}\mathcal{H}(\xi',t)$$

Subtraction constant

Where
$$\Delta(t) = 2\sum_q e_q^2 \int_{-1}^{1} dz\, \frac{D^q_{\mathrm{term}}(z,t)}{1-z},$$

$$D^q_{\mathrm{term}}(z,t) = (1 - z^2) \sum_{\text{odd } n} d_n^q(t)\, C_n^{3/2}(z)$$

1. At leading order, we assume the subtraction constant is dominantly coming from quarks.
2. In the asymptotic limit $d_n^q(t)$ vanish for n>1.
3. Dominance of the flavor singlet combination.

$$\rightarrow \quad D^Q(t) = \frac{18}{25}\Delta(t)\,.$$

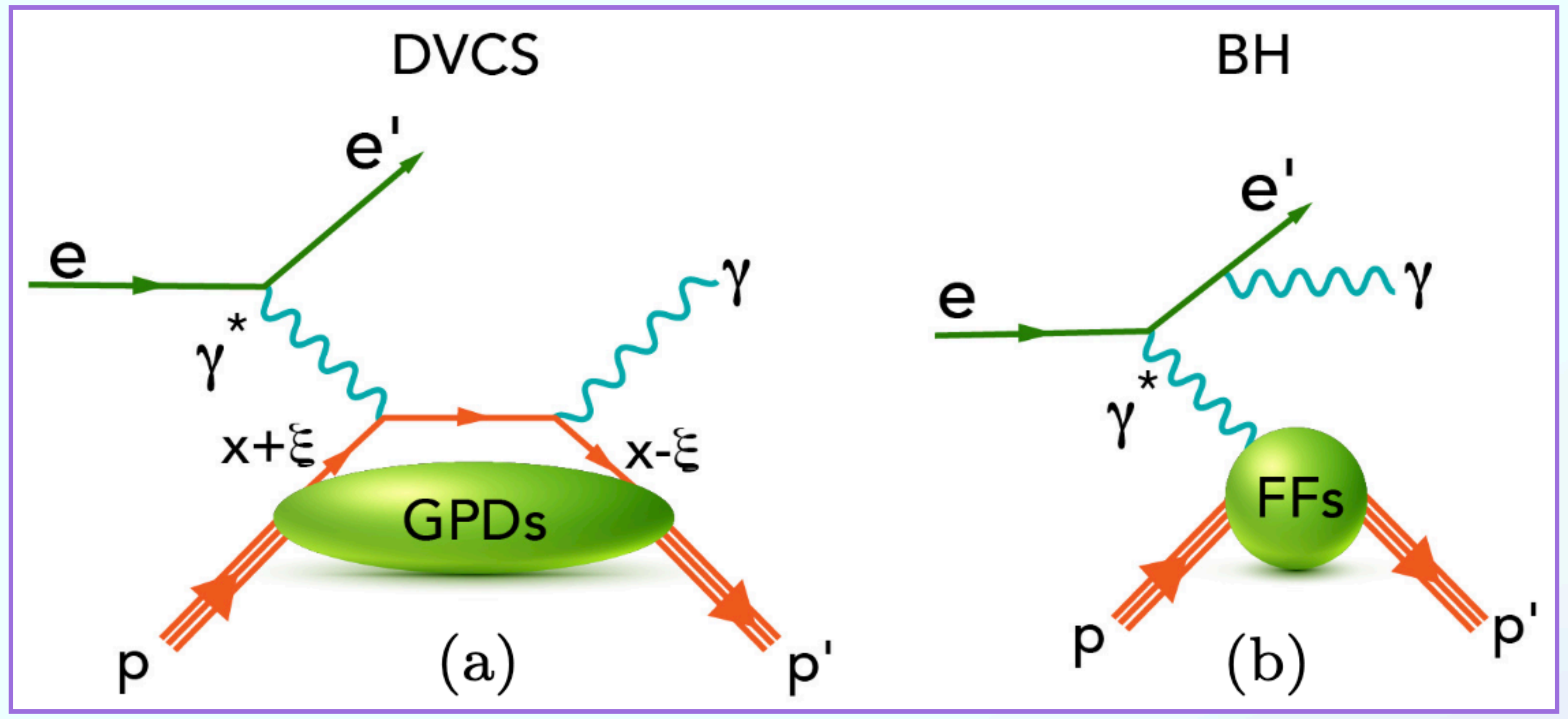
From DVCS observables to Compton form factors

Measurements of DVCS are mostly realized via the process of leptoproduction of a real photon, where also an interference with the Bethe-Heitler radiation also occurs

The BH process gives access to elastic form factors, it does not see the 3D structure of hadrons.

$$\frac{d^5\sigma}{dx_B dQ^2 d|t| d\phi d\phi_S} = \frac{\alpha^3 x_B}{16\pi^2 Q^4 \sqrt{1+\epsilon^2}} |\mathcal{T}|^2 ,$$

$$|\mathcal{T}|^2 = |\mathcal{T}_{\rm BH} + \mathcal{T}_{\rm DVCS}|^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I} .$$



The interference is actually what gives us strong sensitivity to the CFFs. The DVCS amplitude can be decomposed either in helicity amplitudes or, equivalently, in complex-valued CFFs.

$$|\mathcal{T}_{\rm DVCS}|^2 = \frac{2(2-2y+y^2)}{y^2 Q^2 (2-x_{\rm B})^2} \Big[4(1-x_{\rm B}) \left(|\mathcal{H}|^2 + |\widetilde{\mathcal{H}}|^2\right) - \left(x_{\rm B}^2 + (2-x_{\rm B})^2 \frac{\Delta^2}{4M^2}\right) |\mathcal{E}|^2 - x_{\rm B}^2 \left(\mathcal{H}\mathcal{E}^* + \mathcal{E}\mathcal{H}^* + \widetilde{\mathcal{H}}\widetilde{\mathcal{E}}^* + \widetilde{\mathcal{E}}\widetilde{\mathcal{H}}^*\right) - x_{\rm B}^2 \frac{\Delta^2}{4M^2} |\widetilde{\mathcal{E}}|^2 \Big], \quad y = \frac{Q^2}{xs}.$$

BMK equations relate the CFFs to measurable quantities such as $\Delta^4\sigma, d^4\sigma, A_{LU}, A_{UL}, A_C, A_{LL} \ldots$

The first sine harmonic of the beam spin asymmetry (BSA), as measured e.g. in Jefferson Lab is defined as:

- Beam spin asymmetry: $A_{LU}^{-,\sin\phi} \equiv \frac{1}{\pi} \int_{-\pi}^{\pi} d\phi \, \sin\phi \, A_{LU}^{-}(\phi)$

The CFFs are the direct bridge between theory and measurable data. → For truly Global analysis, the CFFs are usually indirectly modeled by GPDs models. However, we don't do that.

DVCS Experimental observables

Example of some of the observables used:

- Beam spin difference cross section:

$$\Delta^4\sigma = \frac{1}{2}\left[\frac{d^4\sigma(\lambda=+1)}{dQ^2 dx_B dt d\phi} - \frac{d^4\sigma(\lambda=-1)}{dQ^2 dx_B dt d\phi}\right]$$

- Beam spin sum cross section:

$$d^4\sigma = \frac{1}{2}\left[\frac{d^4\sigma(\lambda=+1)}{dQ^2 dx_B dt d\phi} + \frac{d^4\sigma(\lambda=-1)}{dQ^2 dx_B dt d\phi}\right]$$

- Beam charge asymmetry:

$$A_C = \frac{\sigma^+_{UU} - \sigma^-_{UU}}{\sigma^+_{UU} + \sigma^-_{UU}} \propto \Re e\left[F_1\mathscr{H} + \xi(F_1+F_2)\widetilde{\mathscr{H}} - \frac{\Delta^2}{4M^2}F_2\mathscr{E}\right]\cos\phi$$

- Beam spin asymmetry:

$$A_{LU}(\phi) = \frac{d\sigma^{\uparrow}(\phi) - d\sigma^{\downarrow}(\phi)}{d\sigma^{\uparrow}(\phi) + d\sigma^{\downarrow}(\phi)} \propto \Im m\left\{F_1\mathscr{H} + \xi(F_1+F_2)\widetilde{\mathscr{H}} - \frac{\Delta^2}{4M^2}F_2\mathscr{E}\right\}\sin(\phi)$$

- Target spin asymmetry:

$$A_{UL}(\phi) = \frac{d\sigma^{\Rightarrow}(\phi) - d\sigma^{\Leftarrow}(\phi)}{d\sigma^{\Rightarrow}(\phi) + d\sigma^{\Leftarrow}(\phi)} \propto \Im m\left[F_1\widetilde{\mathscr{H}} + \xi(F_1+F_2)\left(\mathscr{H} + \frac{x_B}{2}\mathscr{E}\right) - \xi\left(\frac{x_B}{2}F_1 + \frac{t}{4M^2}F_2\right)\widetilde{\mathscr{E}}\right]\sin(\phi)$$

- Double spin asymmetry:

$$A_{LL}(\phi) = \frac{d\sigma^{\uparrow\Uparrow}(\phi) - d\sigma^{\downarrow\Uparrow}(\phi) - d\sigma^{\uparrow\Downarrow}(\phi) + d\sigma^{\downarrow\Downarrow}(\phi)}{d\sigma^{\uparrow\Uparrow}(\phi) + d\sigma^{\downarrow\Uparrow}(\phi) + d\sigma^{\uparrow\Downarrow}(\phi) + d\sigma^{\downarrow\Downarrow}(\phi)} \propto \Re e\left[F_1\widetilde{\mathscr{H}} + \xi(F_1+F_2)\left(\mathscr{H} + \frac{x_B}{2}\mathscr{E}\right) - \xi\left(\frac{x_B}{2}F_1 + \frac{t}{4M^2}F_2\right)\widetilde{\mathscr{E}}\right]\cos\phi + BH$$

Collab	Year	x_B	$Q^2[GeV^2]$	$\|t\|[GeV^2]$	Har. No. After cut
CLAS	2007-2015	0.176 – 0.442	1.510 – 3.314	0.11 - 0.507	201/208
CLAS	20018	0.185 – 0.5	1.666 – 3.982	0.108 - 0.890	352/475
CLAS	2022	0.112 – 0.462	1.546 – 5.941	0.157 - 0.854	45/80
Hall A	2015	0.33-0.40	1.51-2.6	0.17-0.37	48/50
Hall A	2023	0.35-0.36	1.51-2	0.18-0.36	52/61
HERMES	2008-2012	0.069 – 0.123	1.720 – 3.730	0.019-0.463	30/30
COMPAS	2019	0.049 – 0.055	1.770 – 1.910	0.15-0.43	3/4
H1	2005-2009	0.0008 – 0.0124	4.000 – 25.000	0.1-0.8	80/80
ZEUS	2008	0.0003	3.200 – 3.200	0.14-0.47	4/4

Total points: 815 harmonics

We made cuts at t=0.51 GeV^2, and Q^2<1.5 GeV^2 or $-t/Q^2$<0.25

If cross-sections are measured, using weighted Fourier integrals can help to cancel the strongly oscillating Bethe-Heitler propagators and interference terms, improving the convergence of harmonic terms.

OpenGPD Database

opengpd / gpddatabase

Currently, there was no unified common database storing all available data, which is necessary to reduce the uncertainties.

We developed a publicly accessible database format capable of storing current and future data as well as pseudodata.

- Stores data and pseudodata coming from various elastic and exclusive measurements and lattice-QCD results.
- Interfaces are provided in the main programming languages: Python and C++.
- Interface with GEPARD and PARTONS.
- Accessible by anyone in the world using Github.
- Get the list of available data files.
- Load a given data file
- Get metadata.
- Upload your own data file for the use of the GPD community.

Database for studying Generalised Parton Distributions (GPDs)

This page:
about
installation
data format
basic usage
available data
adding new data

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acknowledgements
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View GitHub repository:
link

View PyPI repository:
link

View reference article:
arXiv inspire

Available datasets

uuid	collaboration	reference	type	pseudo	observables	comment
EqbtDRkv	HallA	https://arxiv.org/pdf/1703.09442.pdf	DVCS	False	CrossSectionDifferenceLU	None
PusMstKs	CLAS	https://arxiv.org/pdf/2211.11274	DVCS	False	ALL	None
vGAKAf7P	CLAS	https://arxiv.org/pdf/1501.07052.pdf	DVCS	False	ALU AUL ALL	None
75ueQoQw	COMPASS	https://arxiv.org/abs/1802.02739	DVCS	False	CrossSectionUUVirtualPhotoProduction TSlope	None
ob8hLTm2	CLAS	https://arxiv.org/pdf/1810.02110	DVCS	False	CrossSectionUU	Differential CrossSection for exclusive DVCS electroproduction
3gYp9R4P	HERMES	https://arxiv.org/pdf/hep-ex/0605108.pdf	DVCS	False	AcCos1Phi	None
bmTzHHvg	HallA	https://arxiv.org/pdf/2109.02076	DVCS	False	CrossSectionUU	Beam-helicity-independent cross section
RQncbKtk	CLAS	https://arxiv.org/pdf/1504.02009v1.pdf	DVCS	False	CrossSectionUU CrossSectionDifferenceLU	None
AtY8o7Ej	HallA	https://arxiv.org/pdf/1504.05453v1.pdf	DVCS	False	CrossSectionUU CrossSectionDifferenceLU	None

In collaboration with P. Sznajder and C. Mezrag

arxiv.org/abs/2503.18152

https://opengpd.github.io/gpddatabase

Framework:

- Gepard + Neural Networks
- Extraction of CFFs from experimental data using NN.

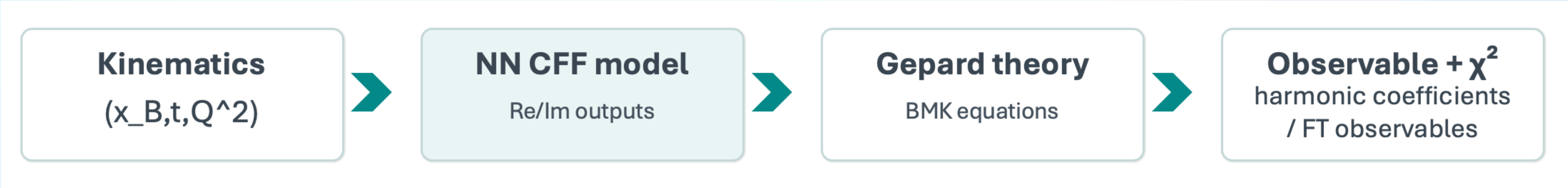
Framework: Gepard + Neural Networks

We are building upon existing tools: Gepard

- Python software framework dedicated to the study of GPDs.
- K. Kumerički, D. Müller, K. Passek-Kumerički, Nuclear Physics B, Volume 794, Issues 1–2, 2008

To obtain a more reliable estimate of uncertainties with reduced model dependence, we decided to utilize Artificial Neural Networks to parameterize the CFFs.

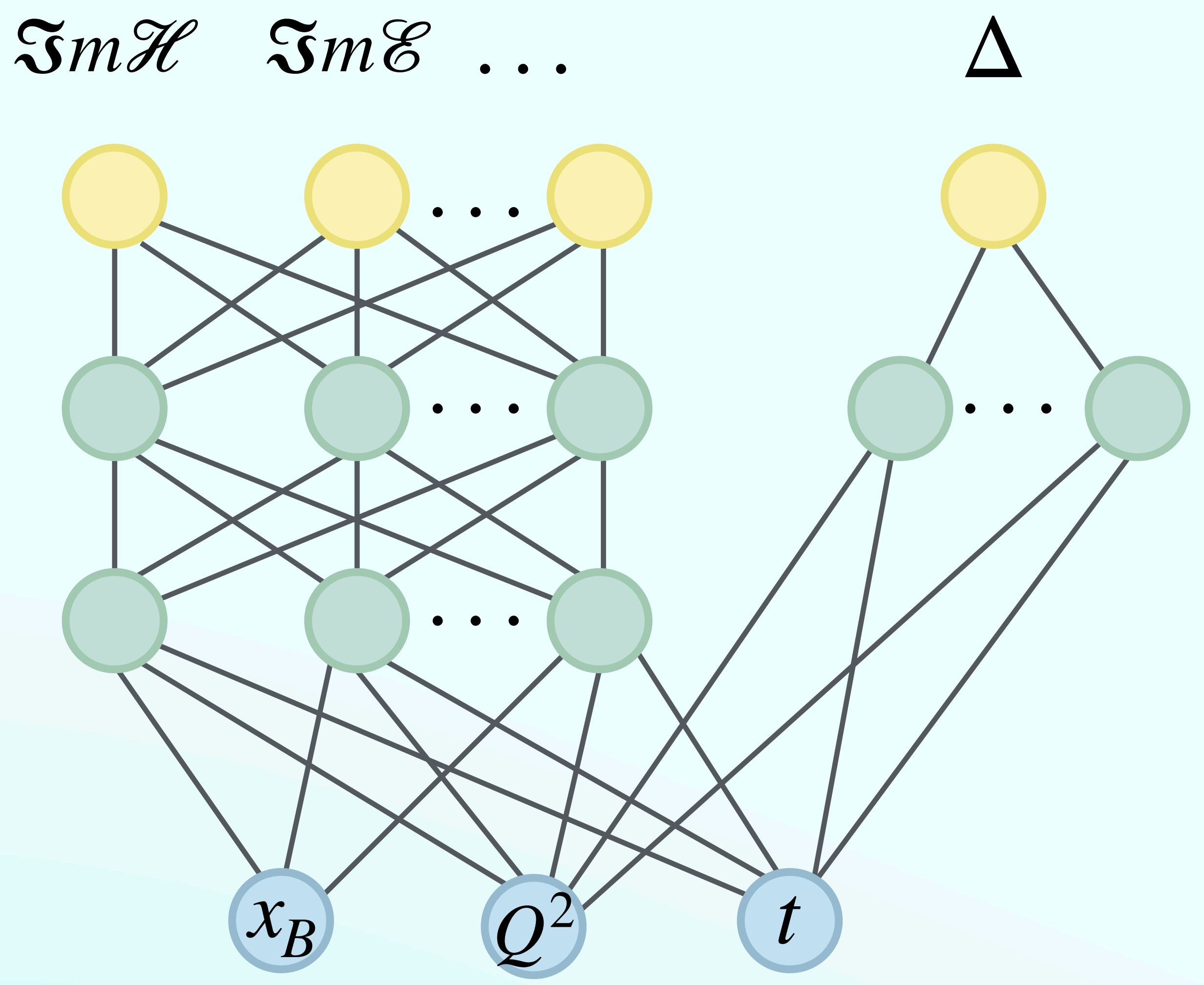
- Use State-of-the-art standard ML libraries. PyTorch
- A tool to do a Global fit to extract CFFs and GFFs to ensure the results are driven by data in a model indeper manner.

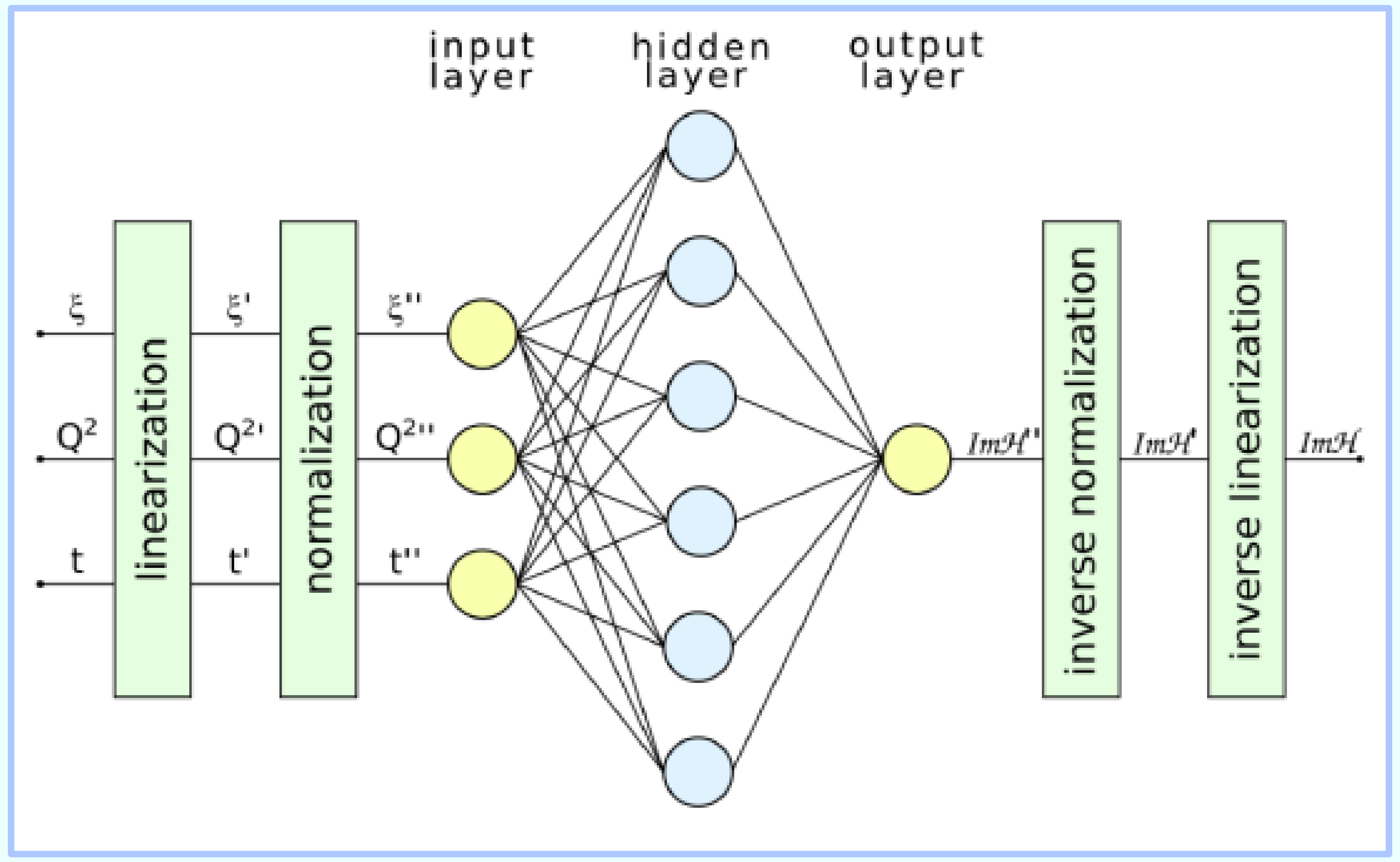


Extraction of CFFs from experimental data using NN

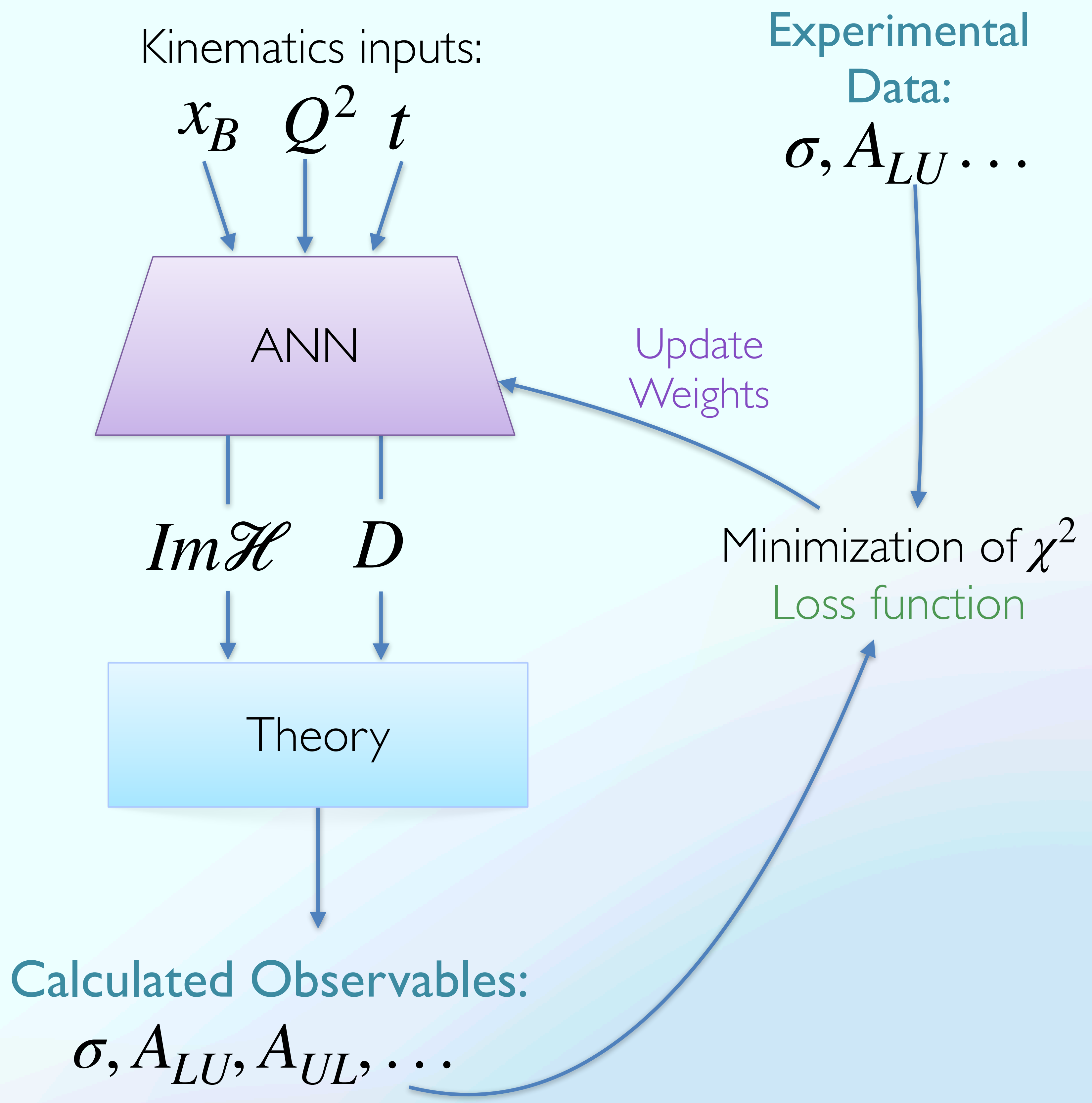
Artificial Neural Networks (ANN)

A machine learning (ML) algorithm that uses artificial intelligence (AI) to learn how to process data by analyzing training examples.





$$\mathrm{Re}\mathcal{H}(\xi,t) = \mathcal{C}_{\mathcal{H}}(t) + \frac{1}{\pi}\,\mathrm{P.V.}\int_0^1 \mathrm{d}\xi' \left[\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right] \mathrm{Im}\mathcal{H}(\xi',t)$$



$$\chi^2(\theta) = \frac{1}{N}\sum_i \left(\frac{\mathcal{O}^{\mathrm{Gepard}}(\mathrm{CFF}_\theta;\, kin_i) - \mathcal{O}_i^{\mathrm{data}}}{\sigma_i}\right)^2$$

➡The process is essentially a least-squares fit of a complex, multi-parameter function, implying no theoretical bias.

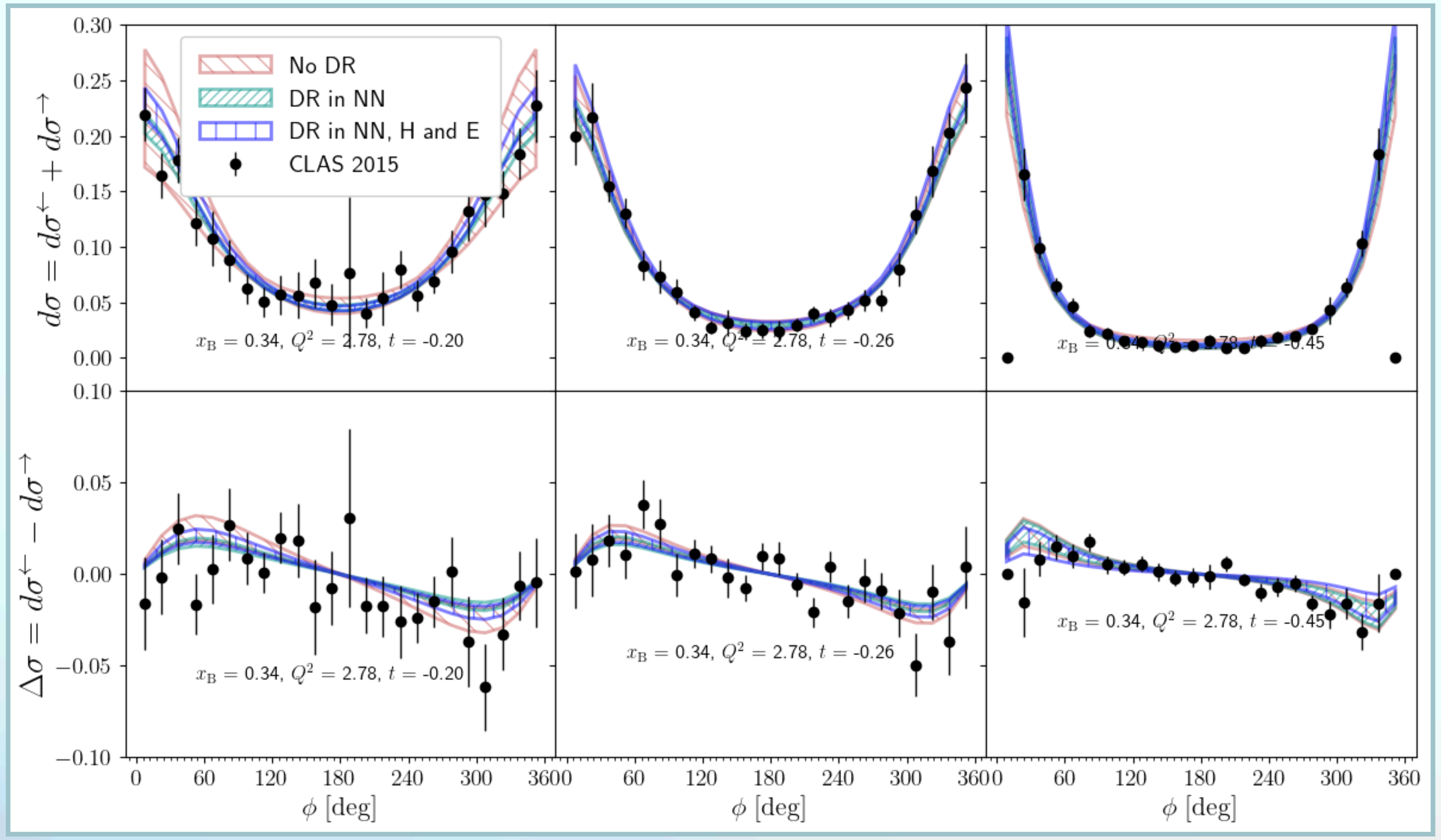
➡For a truly global coverage, it is necessary to add a Q^2 dependence, which was previously absent at Gepard.

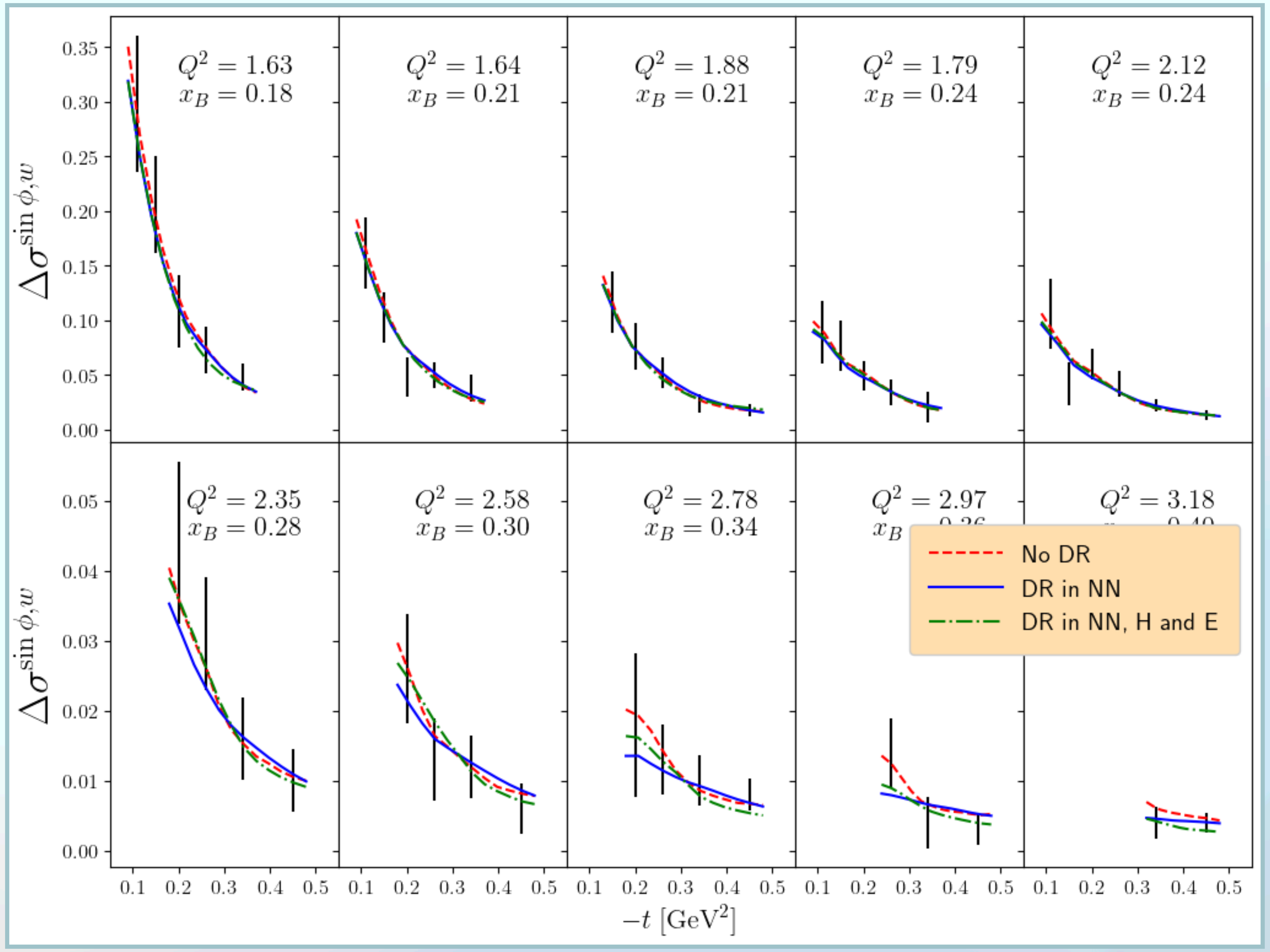
Analysis process:

- Model stability and the Closure test.
- Update on global analysis of CFFs.
- Dispersion relation and D-term extraction.

CLAS fit

Example of description of 2015 CLAS data through the NN



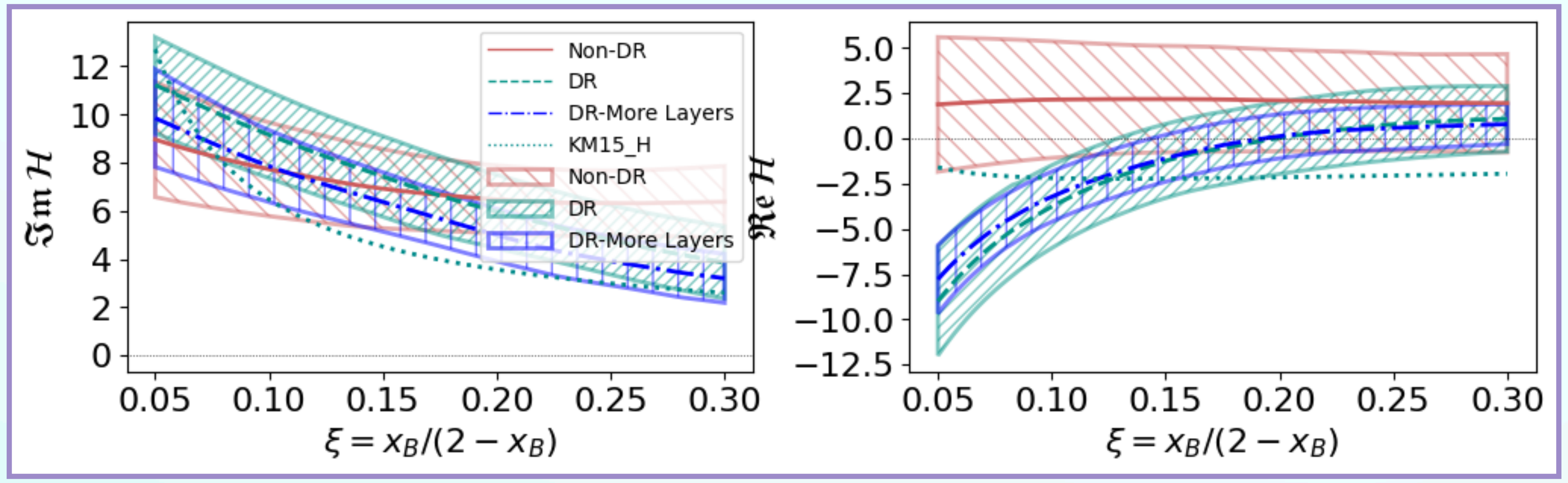


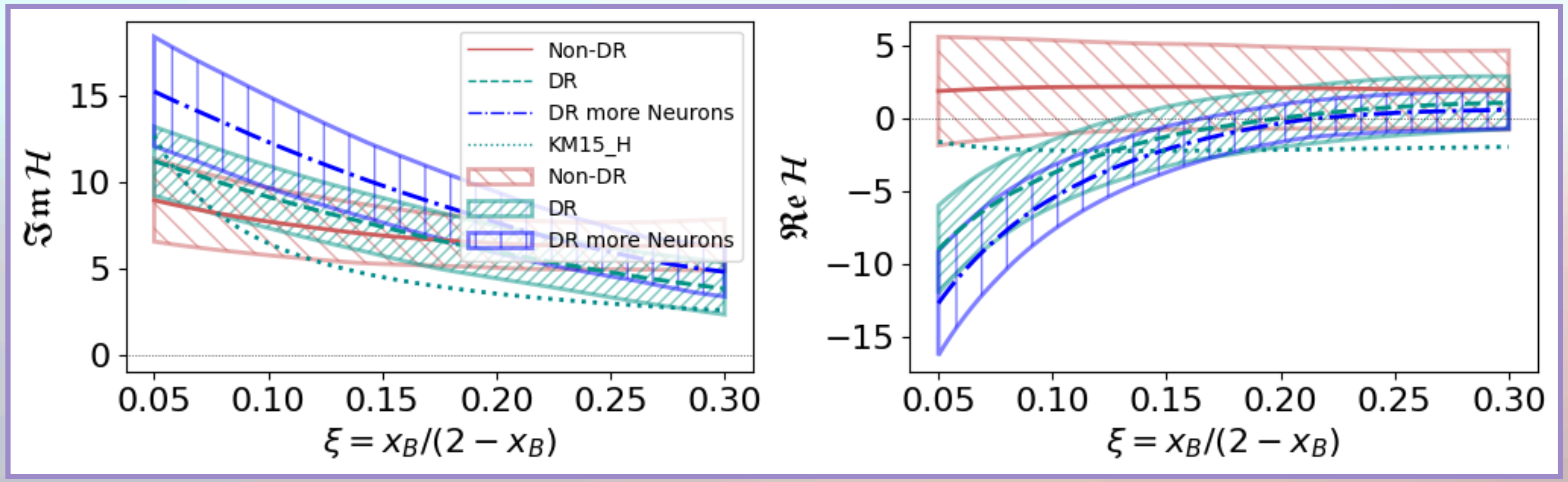
(CLAS Collaboration) Phys. Rev. Lett. 115, 212003,2015

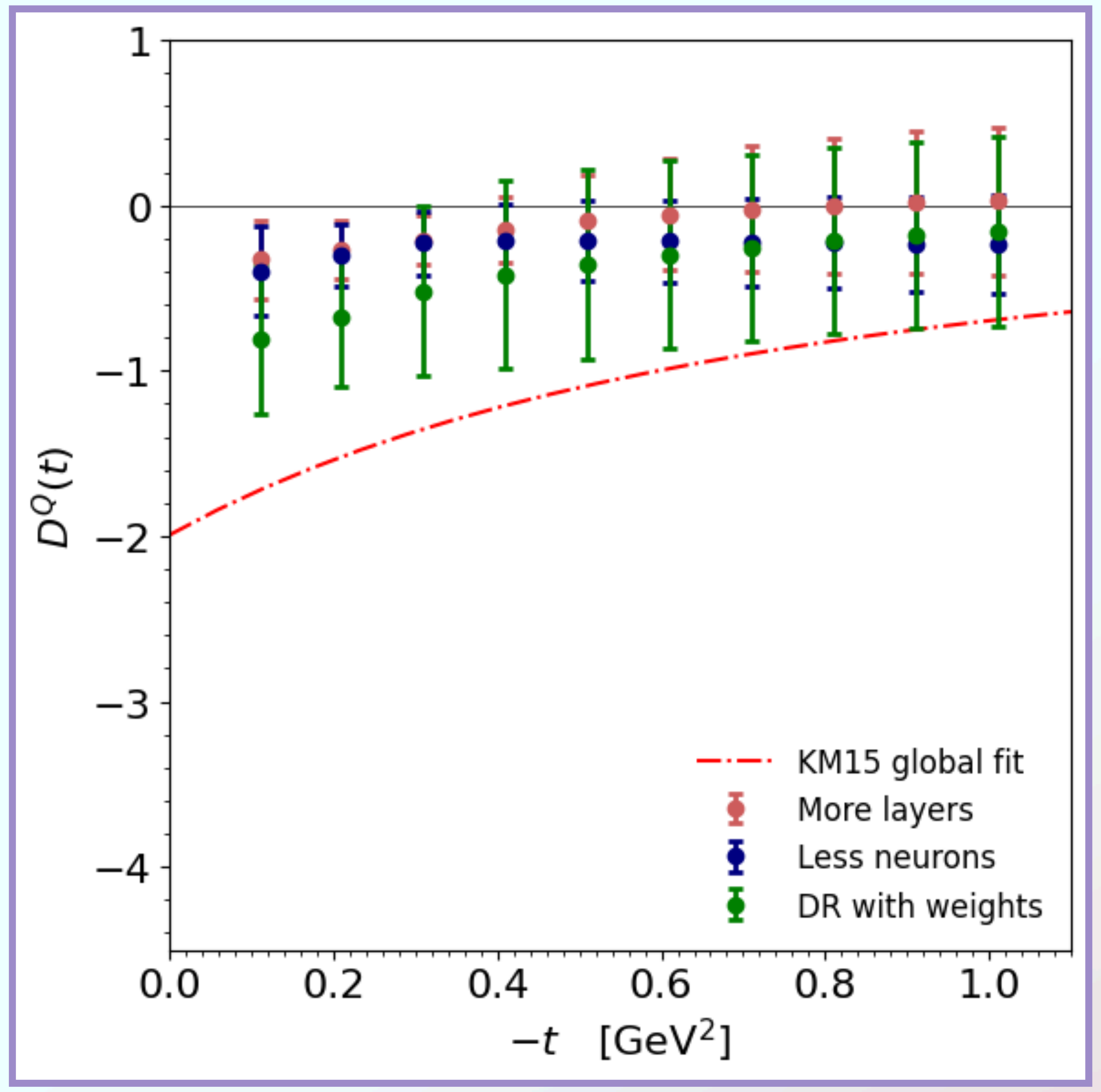
Model stability (architecture dependence)

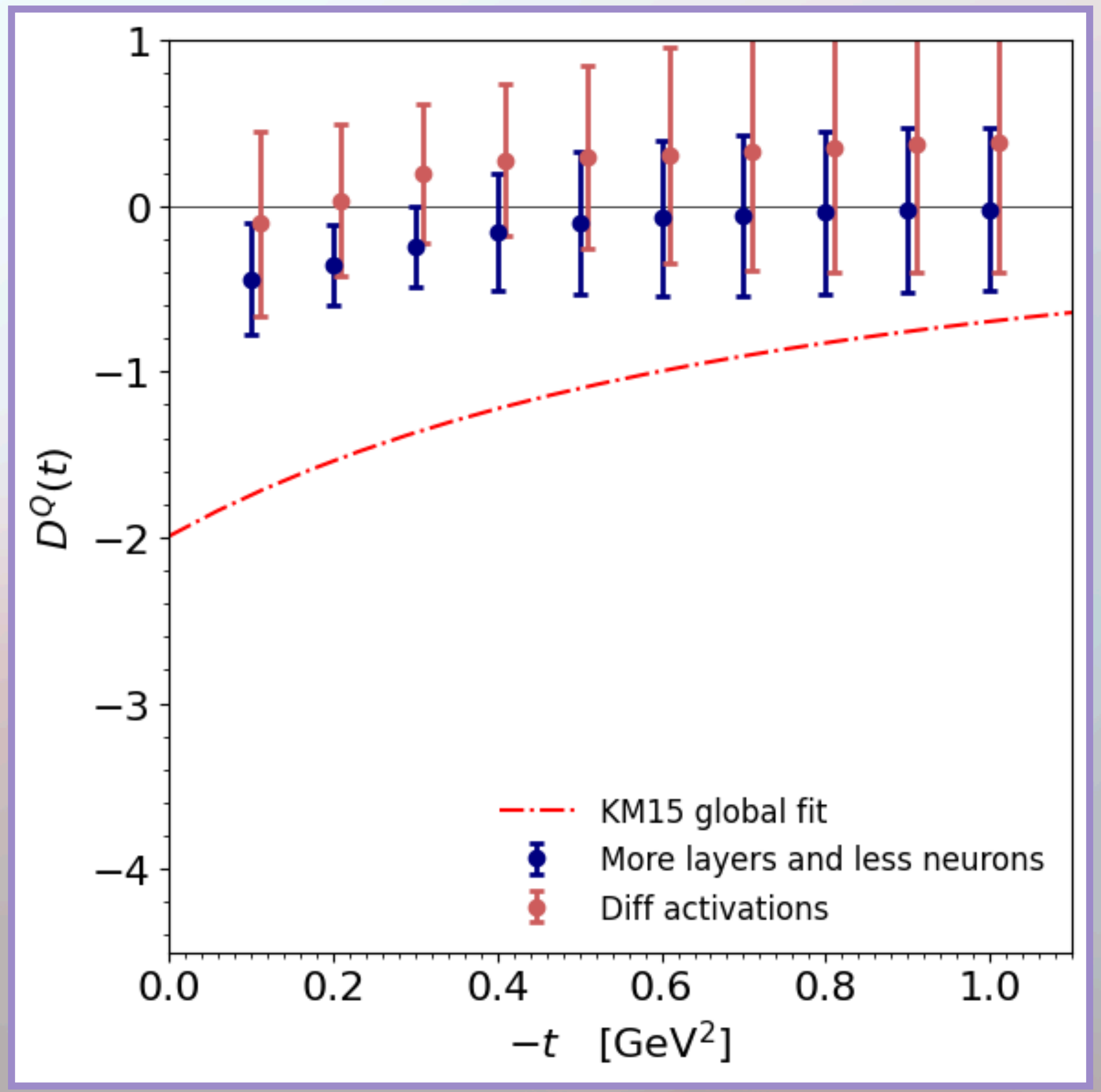
When fitting data:

- Same dataset + same theory → different architectures produce different D(t)
- The fit was found to be very sensitive to changes. Mainly, for ReH and D(t).









Network architecture analysis on CLAS data

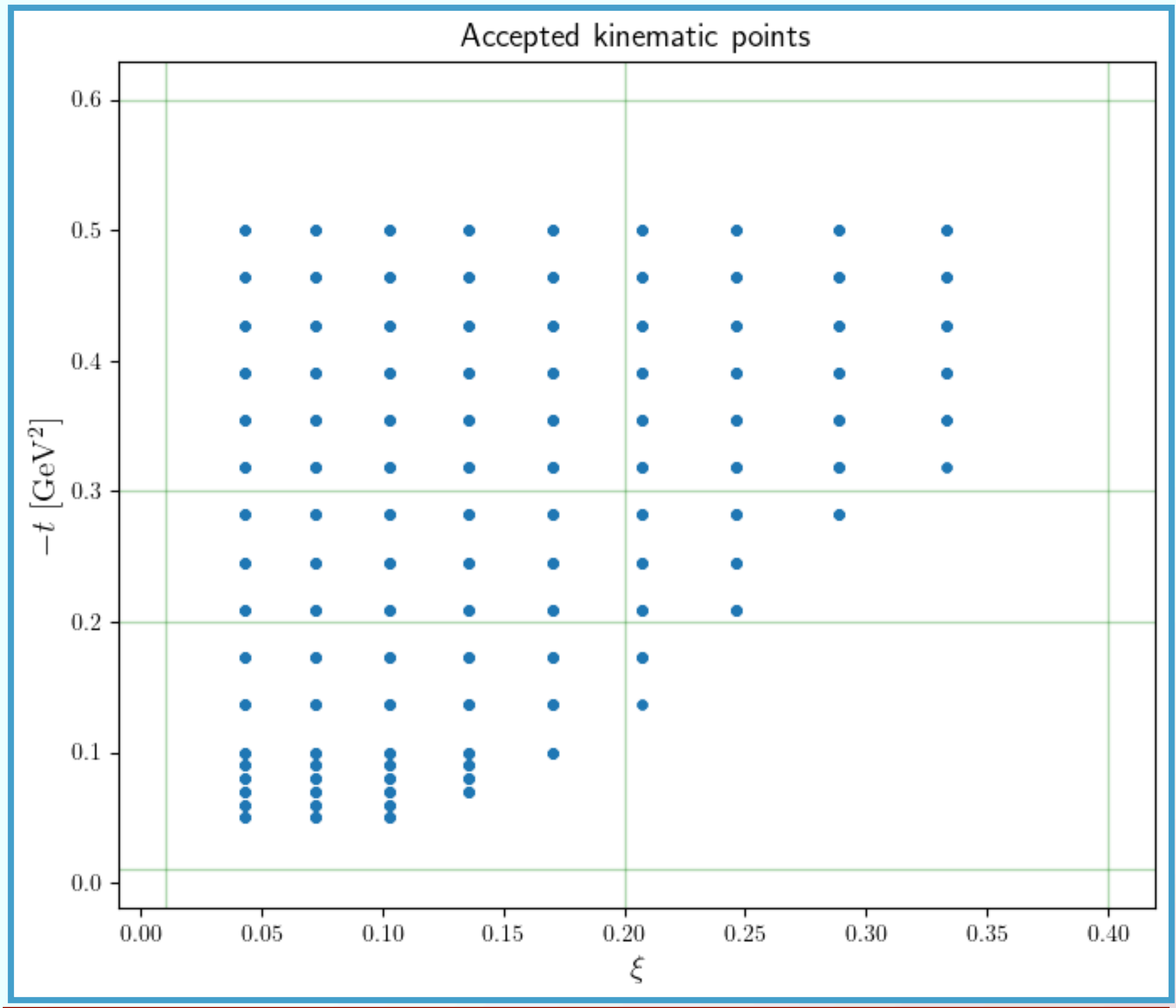
Model stability: Closure test

Our objective in the closure test is to generate pseudodata from theoretical models and verify that our neural networks can recover the original CFFs.

Pseudo-data must lie in the physical DVCS/BH region (BMK propagators must be real)

We fit Fourier coefficients / harmonics → inputs are (xB, Q^2, t).

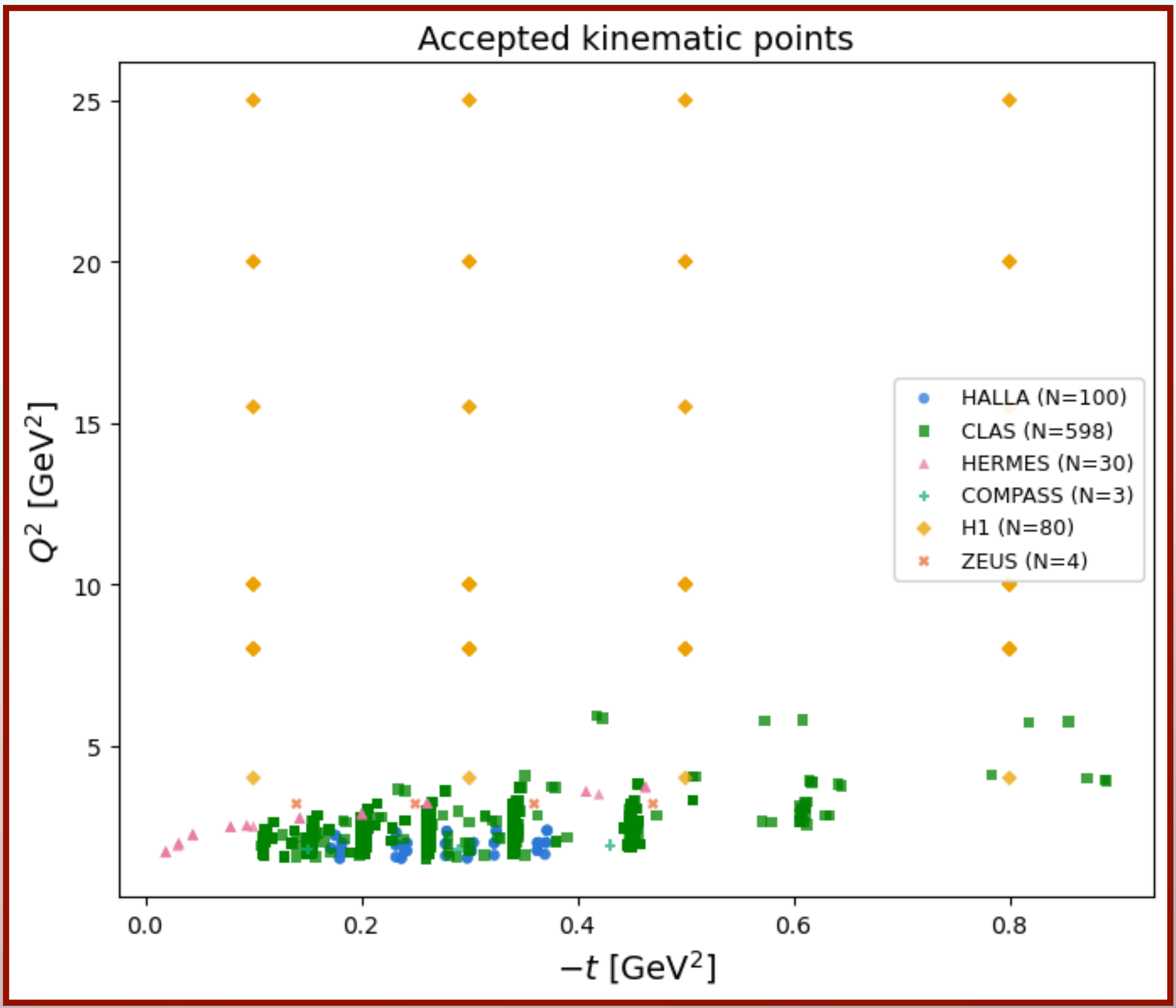
Pseudodata kinematic points

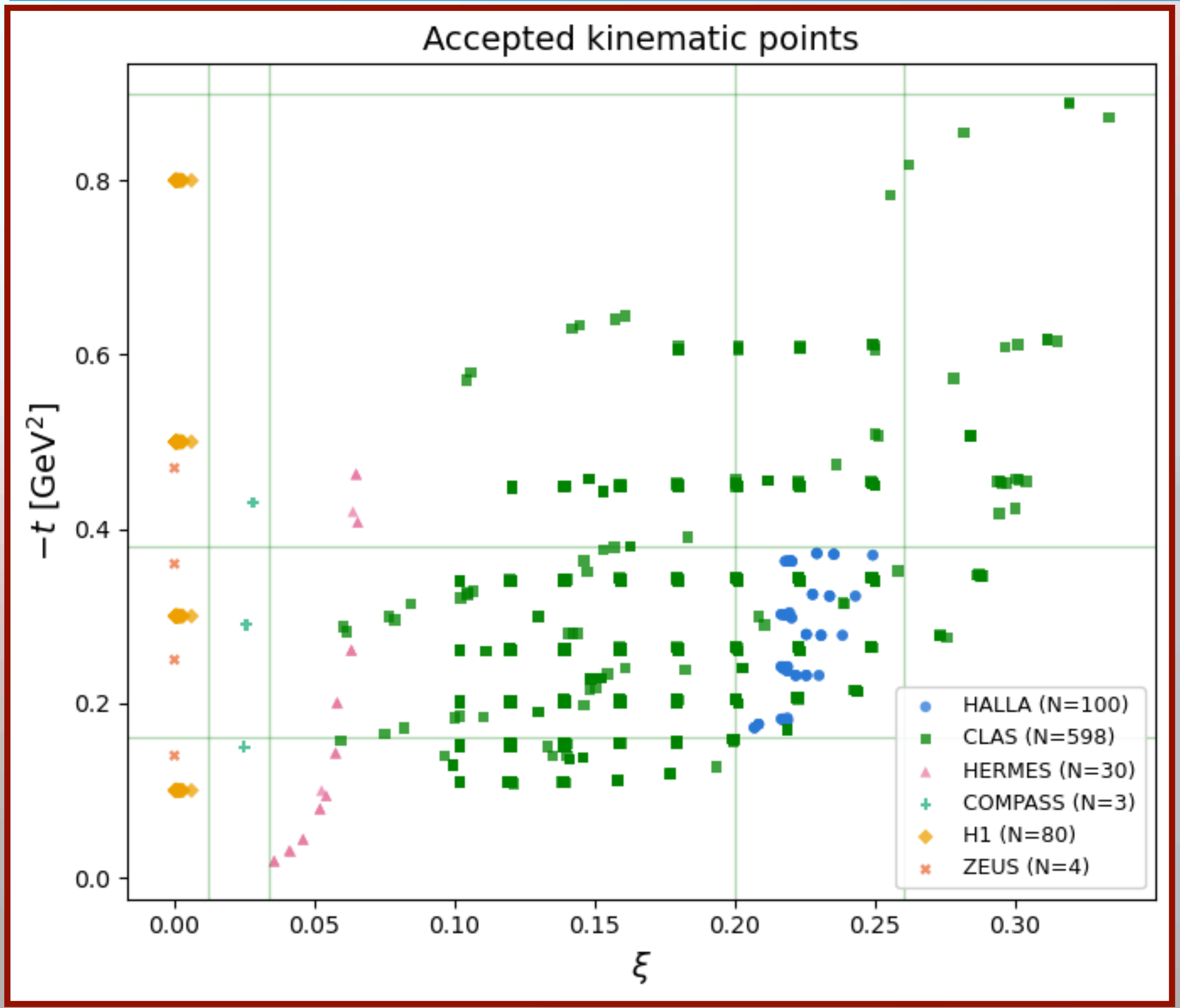


For fixed-target DVCS, not every combination of (xB, Q^2, t) is allowed. The constraints are:

- $W^2 = M^2 + Q^2(1/x_B - 1) > (M + m_\pi)^2$ (hadronic final state must exist)
- $0 < y < 1$ where $y = \dfrac{\nu}{E} = \dfrac{Q^2}{2Mx_BE}$ (lepton energy loss fraction)
- tmin (xB ,Q2) ≤ t ≤ 0 (momentum transfer bounds)

Experimental kinematic points





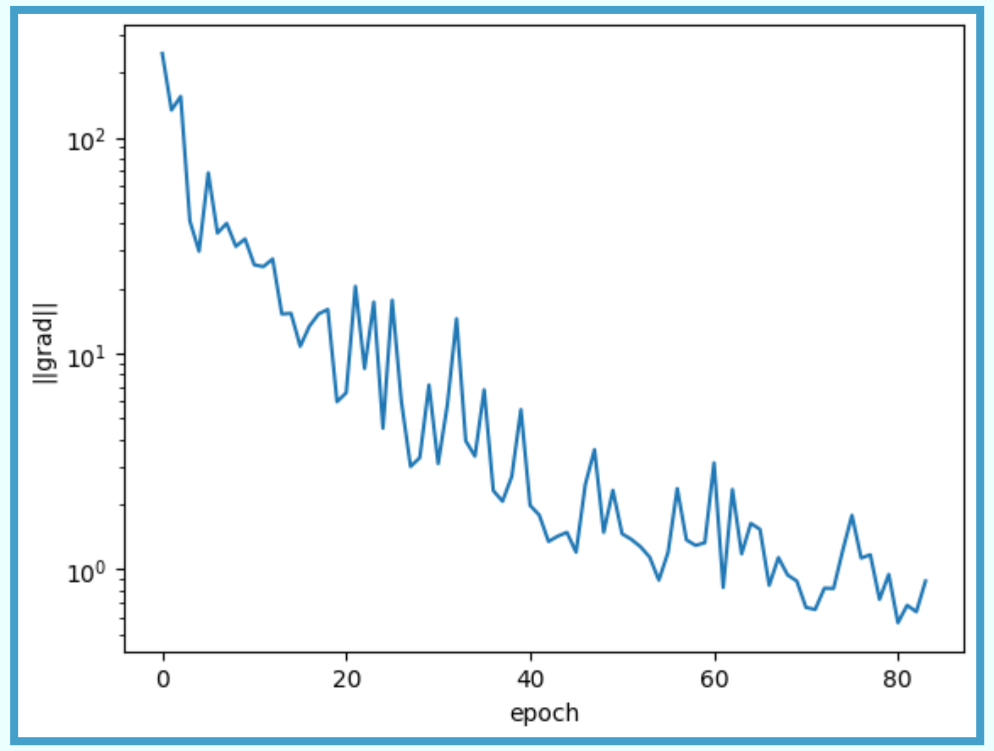
Experimental datasets: CLAS + Hall A + HERMES + COMPAS + ZEUS + H1 (twist-2 safe region cut: $-t/Q^2 < 0.25$)

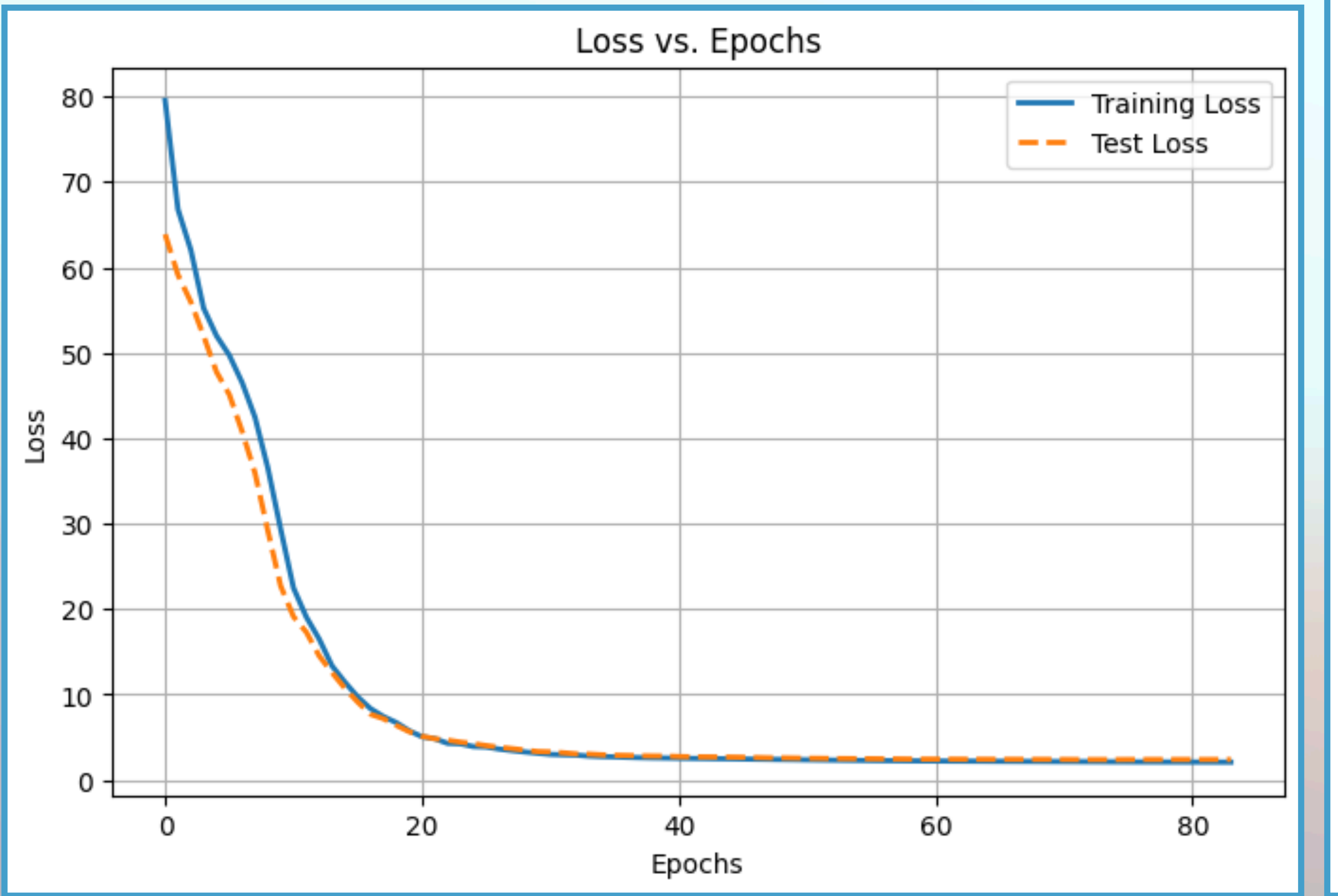
Model stability: Closure test

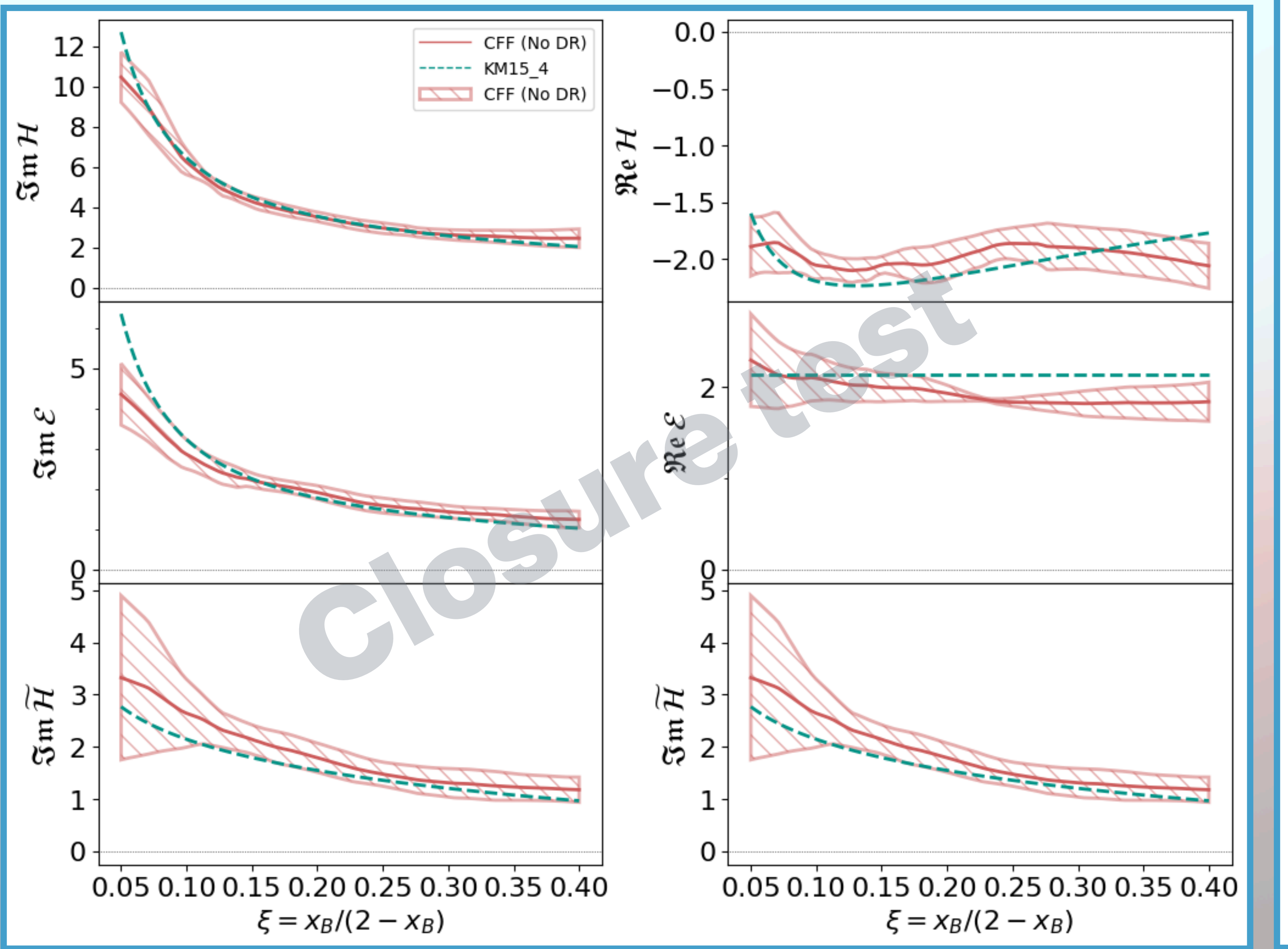
No dispersion relation (non-DR) neural model

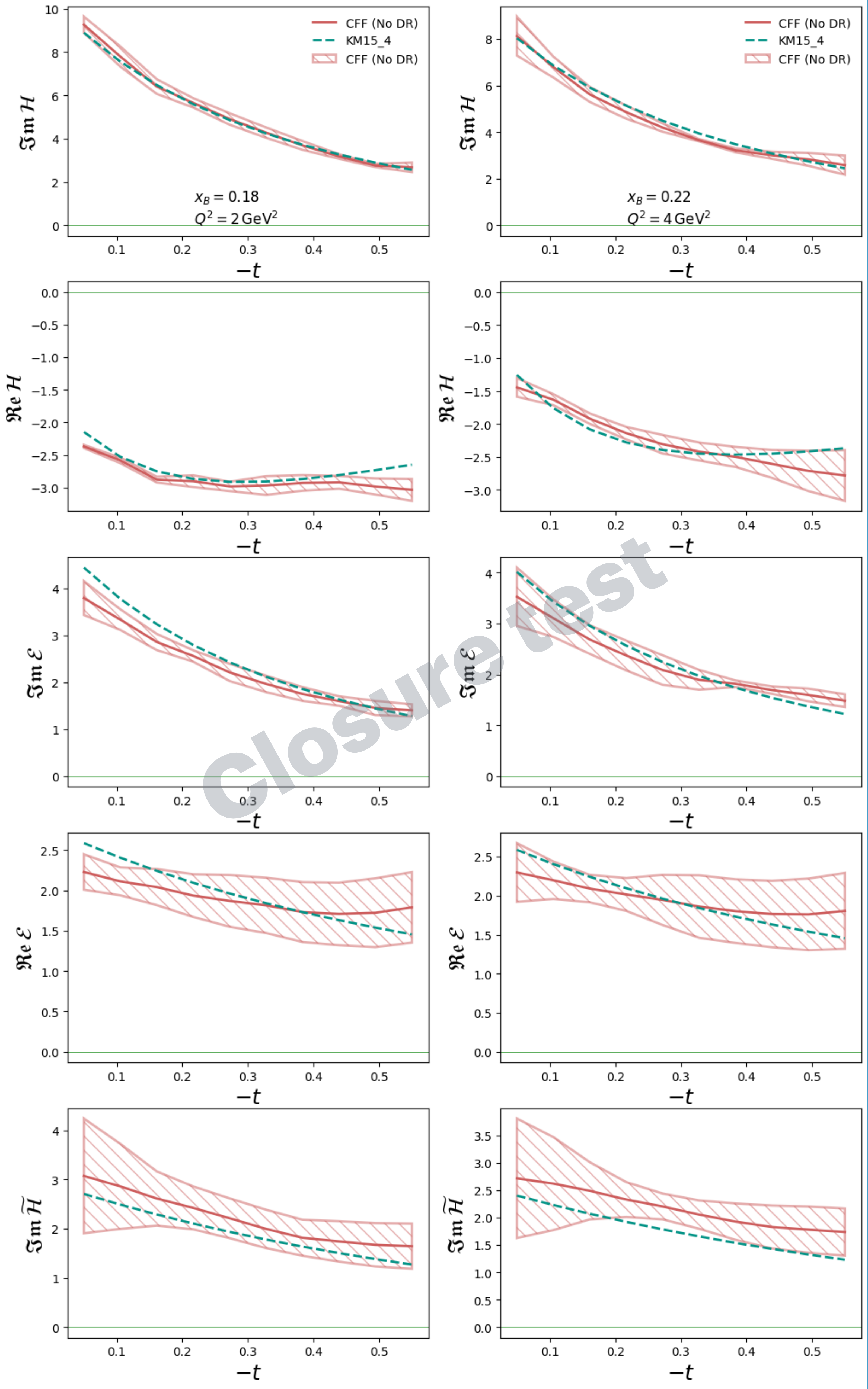
- Pseudo-data generated from KM09
- NN learns CFFs directly → predicts observables with $\chi^2 \sim 1$
- The NN can learn the CFFs that the observables are sensitive to.
- The training pipeline + Gepard theory evaluation + harmonic observable machinery is working.

Obs	χ^2
XLU	1.06
XUU	1.09
AC	1.08
ALU	0.96









Closure test: Dispersion relation (DR) neural model

We implemented a DR-constrained neural model in Gepard

$$\mathrm{Re}\mathcal{H}(\xi,t) = \mathcal{C}_{\mathcal{H}}(t) + \frac{1}{\pi}\,\mathrm{P.V.}\int_0^1 \mathrm{d}\xi' \left[\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right] \mathrm{Im}\mathcal{H}(\xi',t)$$

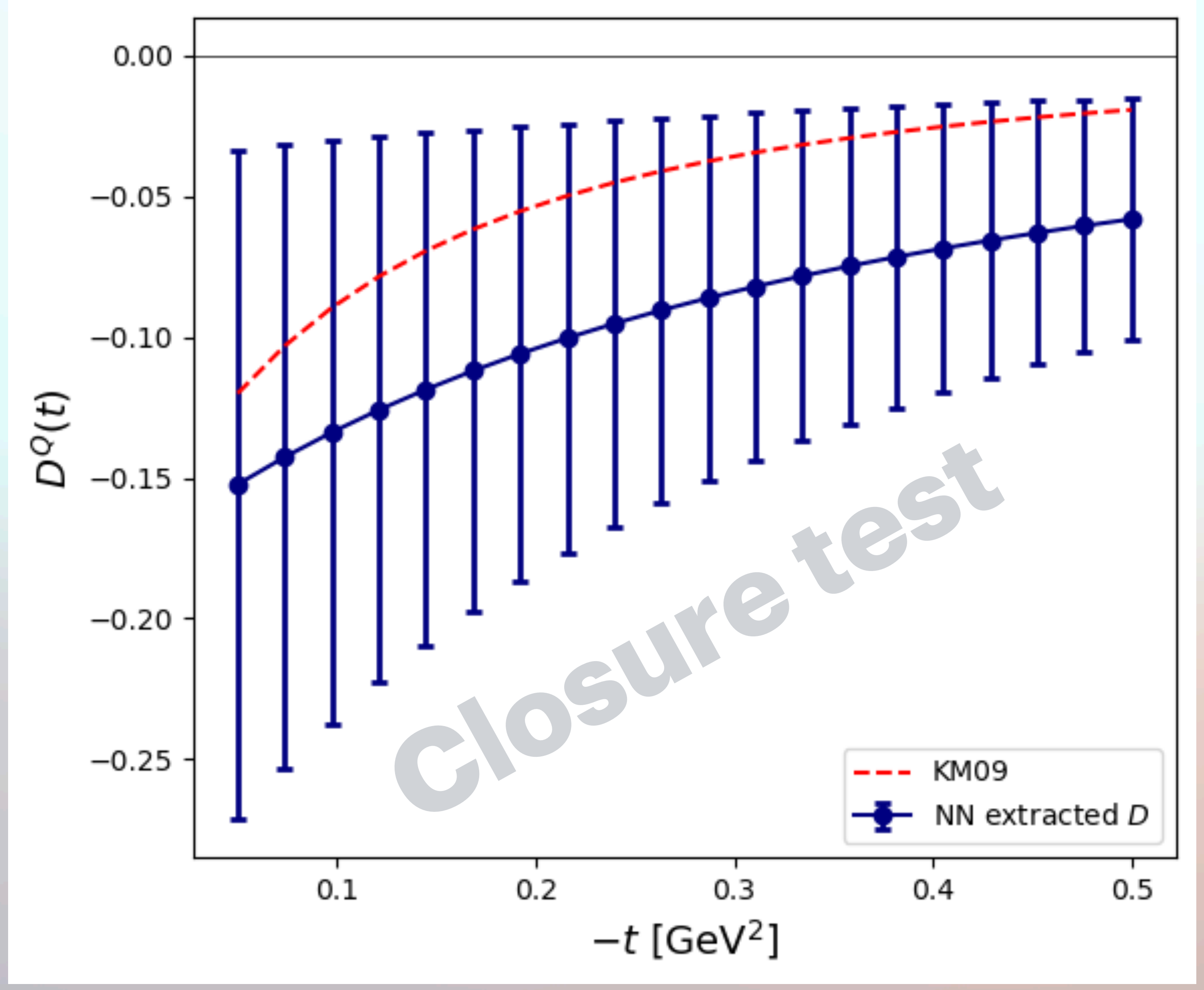
- PV integral + small-ξ correct extrapolation controls ReH stability.
- D-term is still weakly constrained to be left free unless ReH-sensitive observables are ensured.

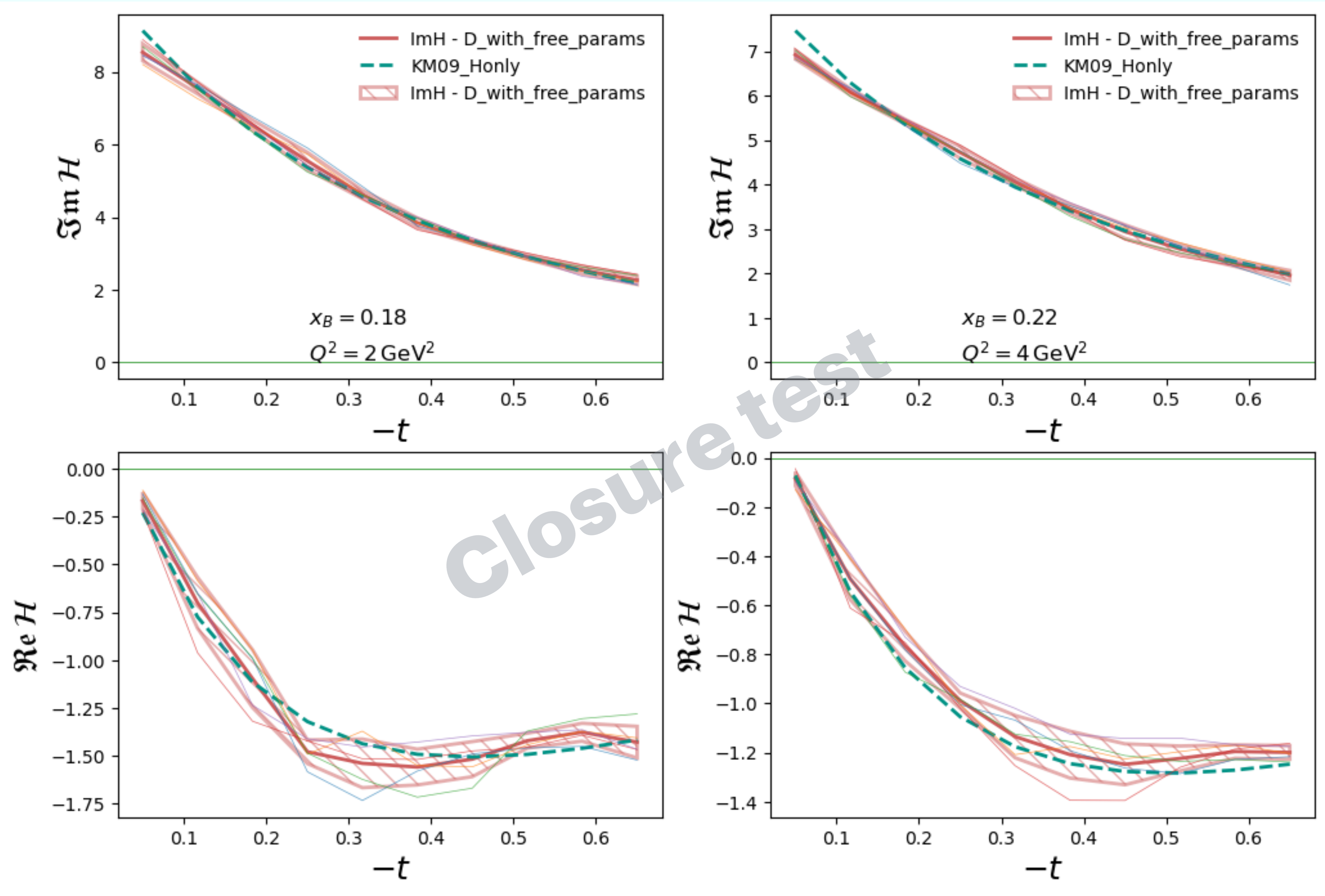
$$D(t) = \frac{D_0}{(1-\frac{t}{M^2})^{\alpha}}$$

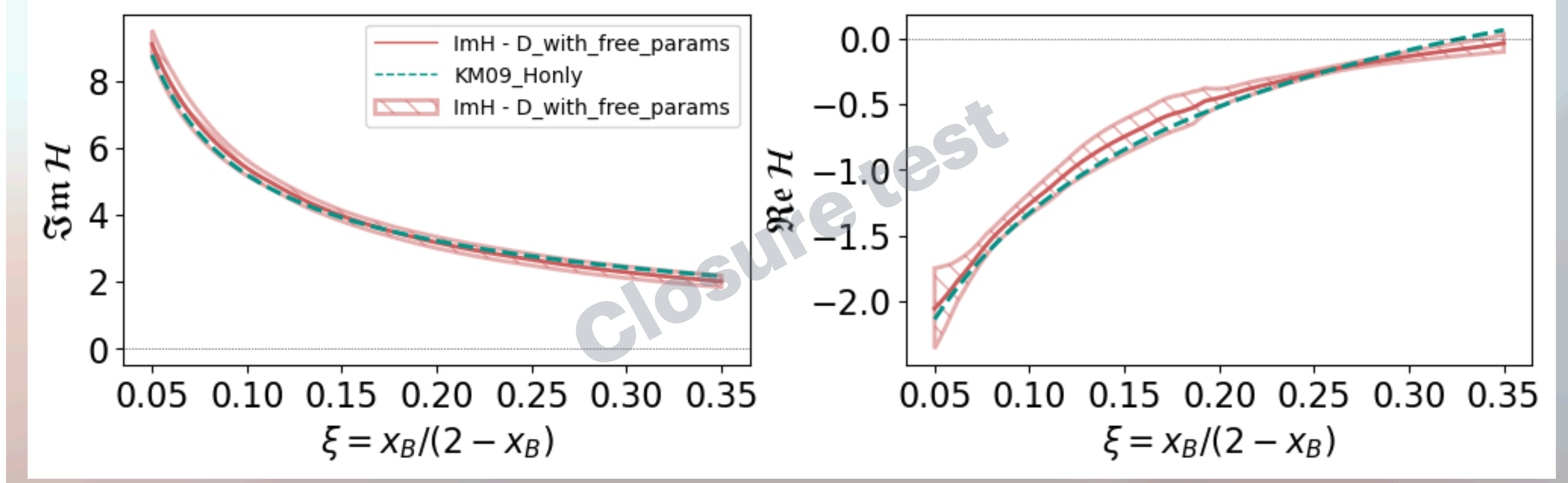
$D_0 = 0.245 \qquad \alpha = 1.378$

$M^2 = 0.693$

Obs	χ^2
XLU	1.06
XUU	1.03
ALU	1.51
AUT,DVCS	1.06





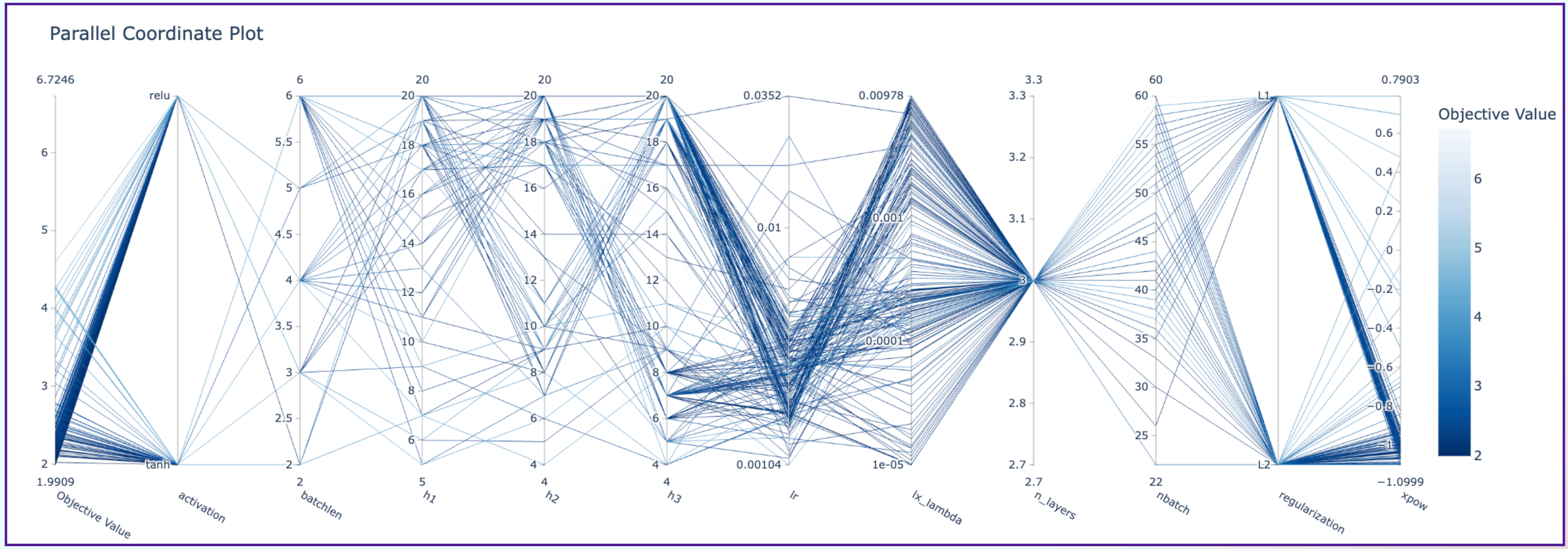


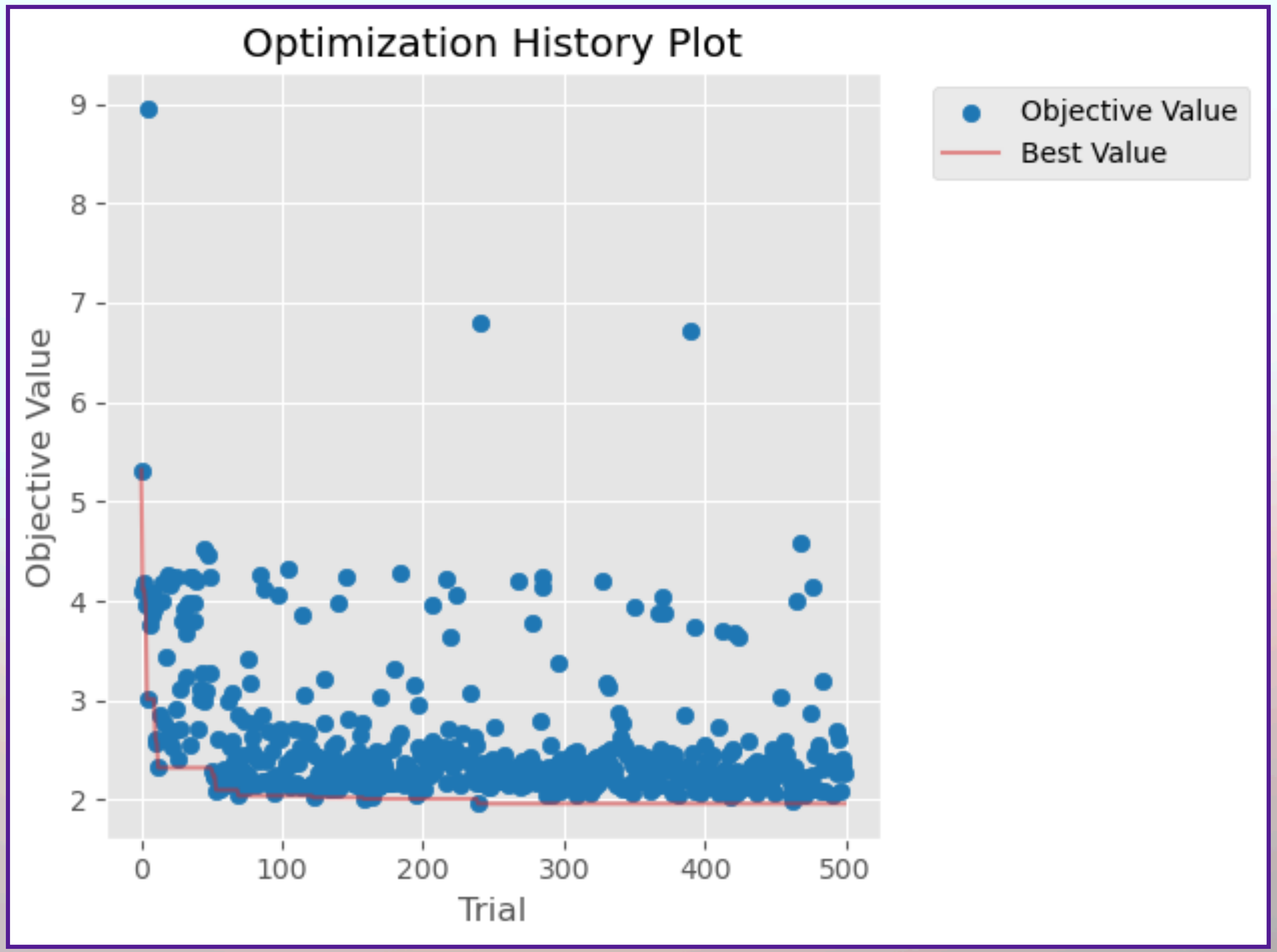
Optuna test with experimental data

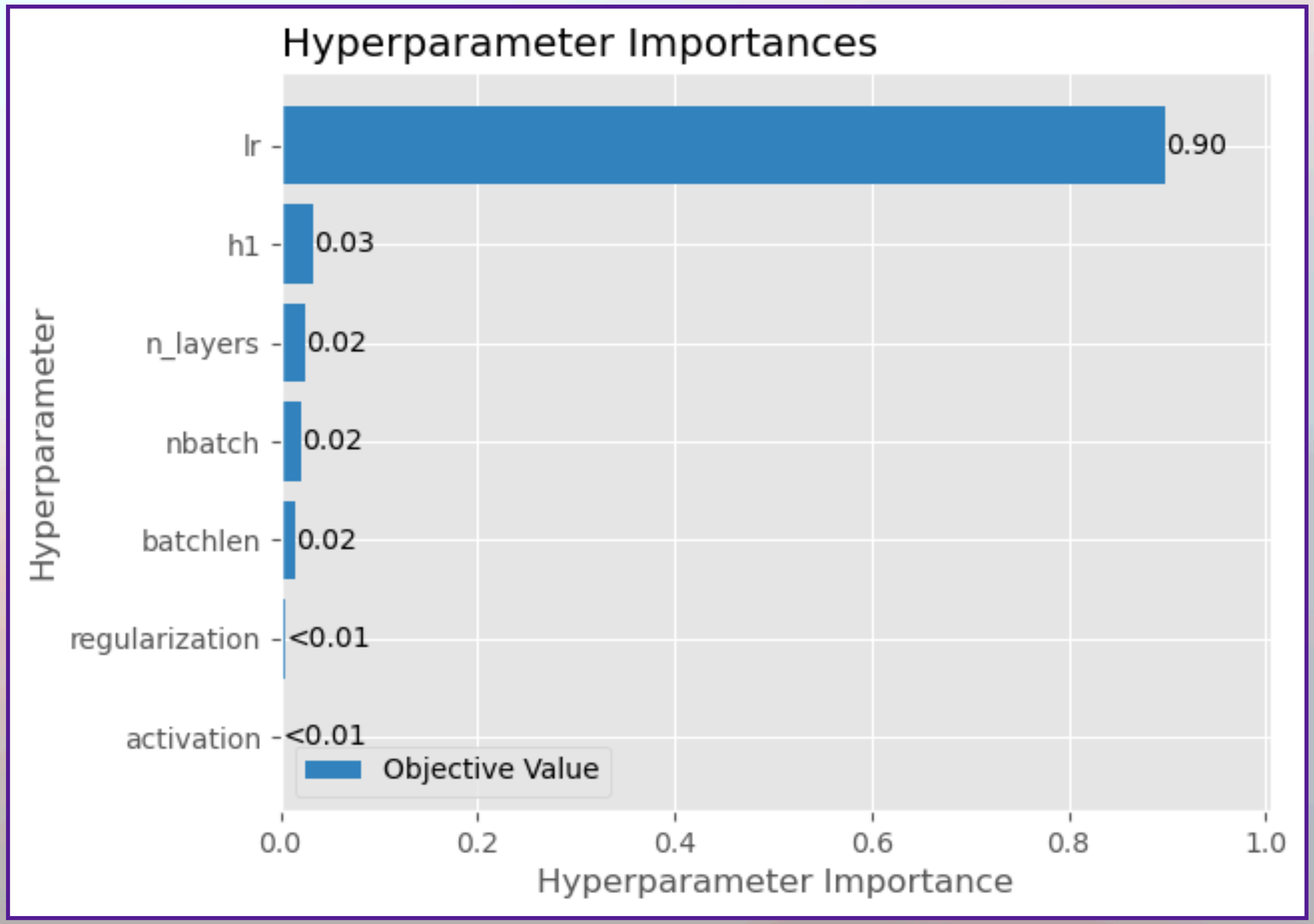
Hyperparameters (learning rate, layers, regularization, etc) aren't learned by gradient descent; they must be searched.

Optuna: an open-source Python framework for hyperparameter optimization.

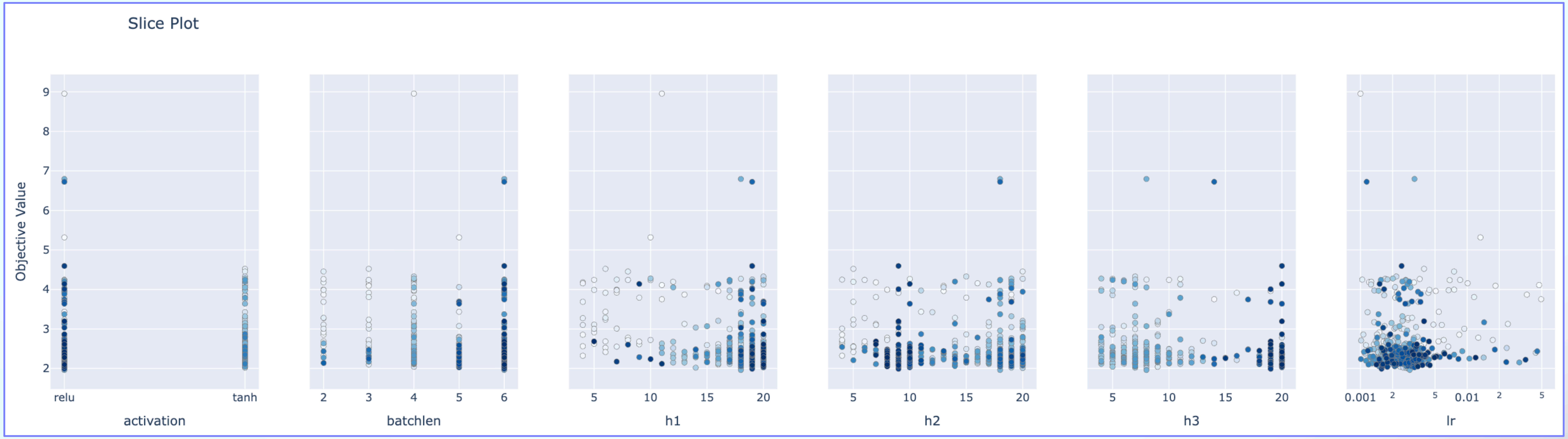
- Optuna automates this search using Bayesian optimization (TPE), not blind grid/random search.
- It learns from past trials — focuses on promising regions instead of testing blindly.
- Objective here: minimize the global χ^2 across all datasets.
- 500 trials were run for both DR and non-DR models.

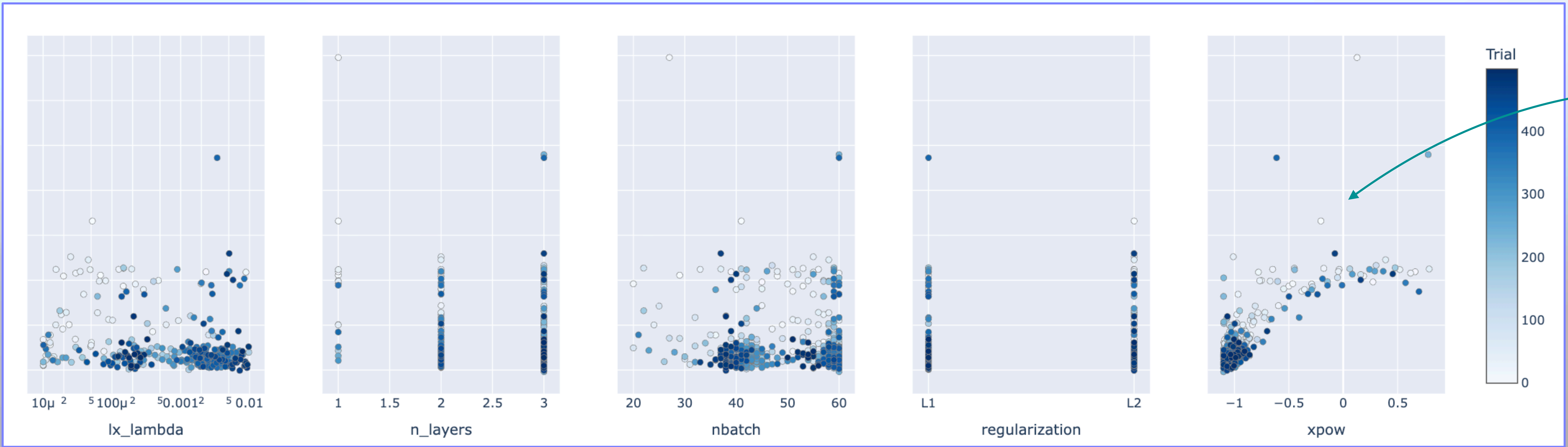






Non-DR Optima test with experimental data





A power-law factor:

$$F = F_{NN} * x_B^{xpow}$$

$$F = H, E, \tilde{H}, \tilde{E}$$

Controls extrapolation behavior at small x_B , critical for the DR integral's small-ξ tail

Update on global analysis of CFFs: No-DR NN model

- Global non-DR fit. The uncertainties from replicas spread.
- We use an Optuna study strategy: Compute best objective (chi^2) → Best hyper parameters

collab=CLAS	obs=ALU	chi2/npt=0.5861
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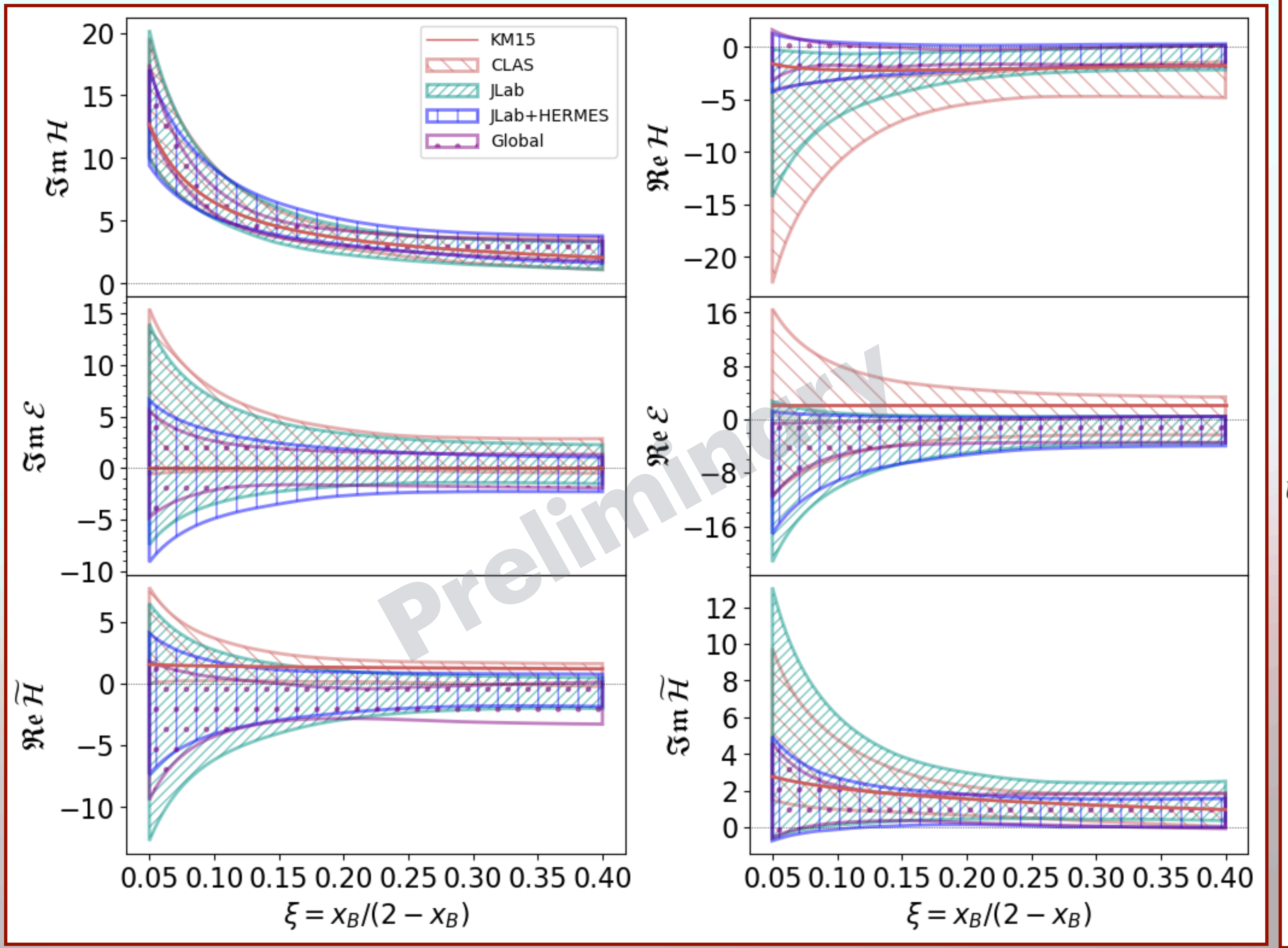
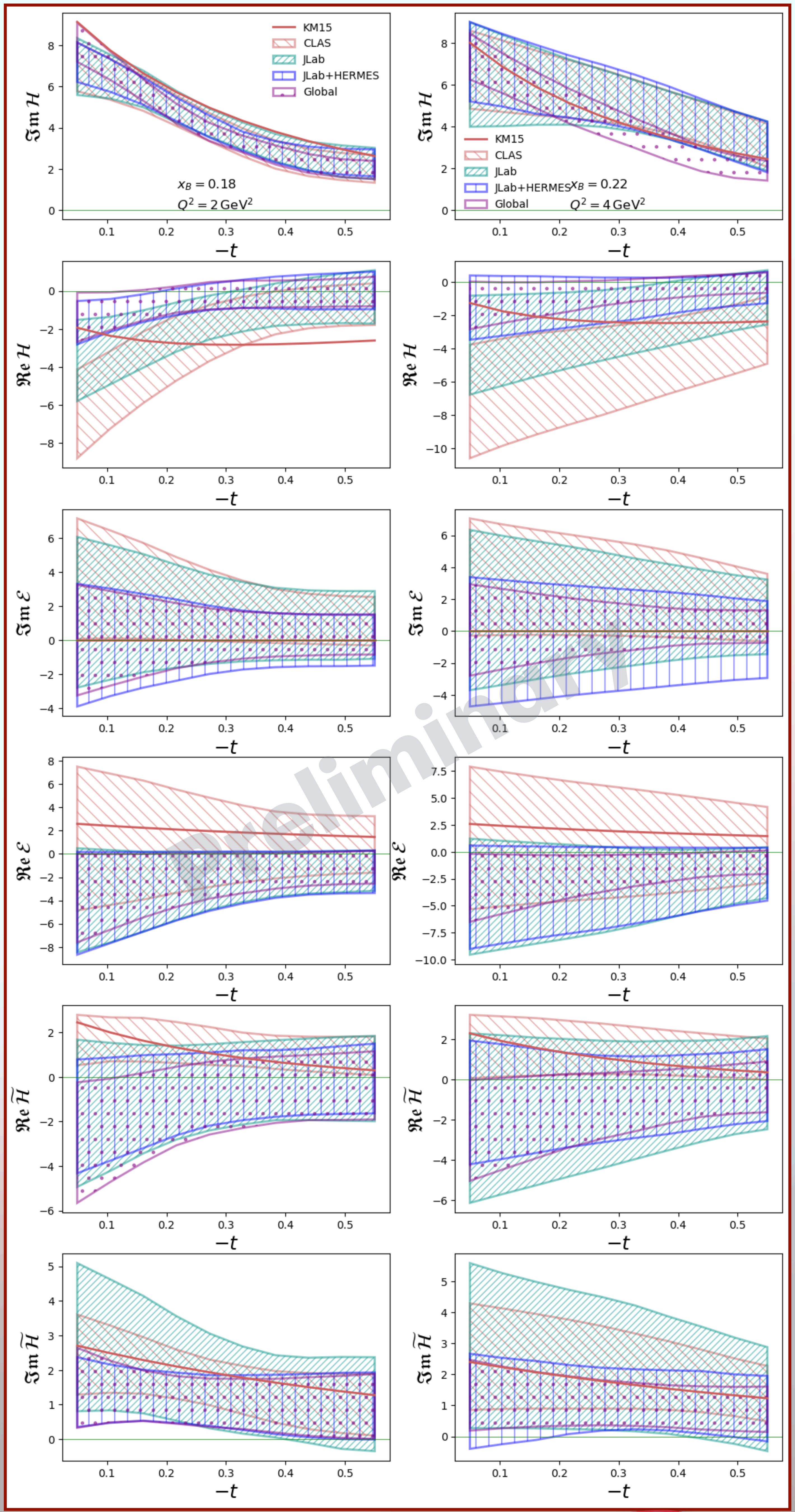
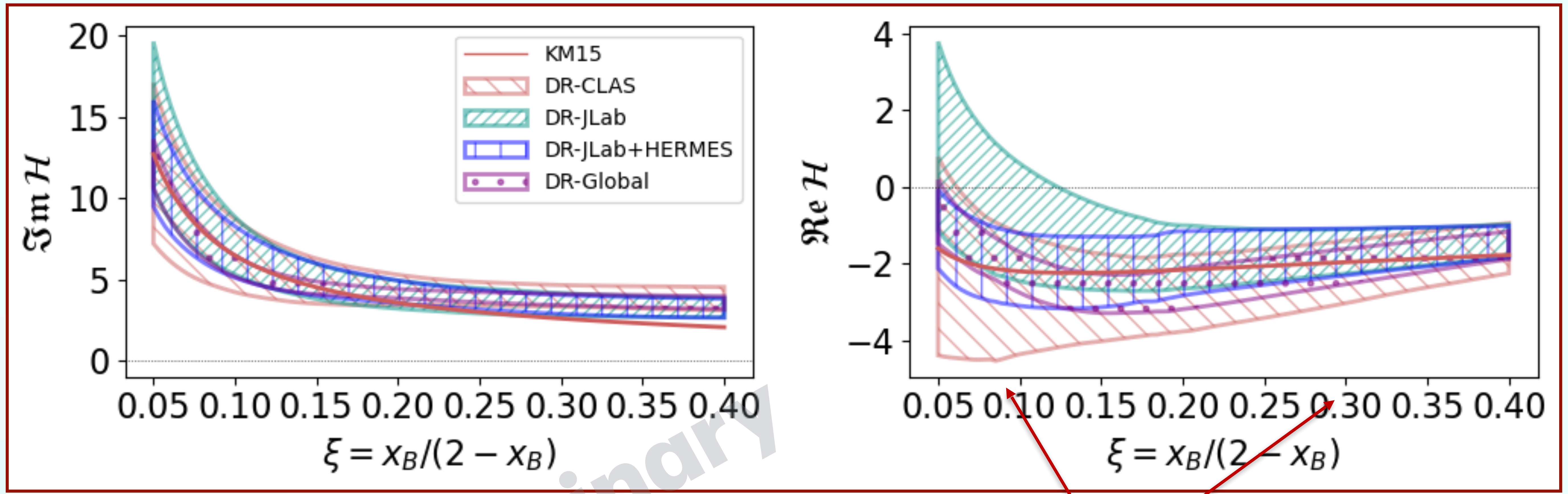


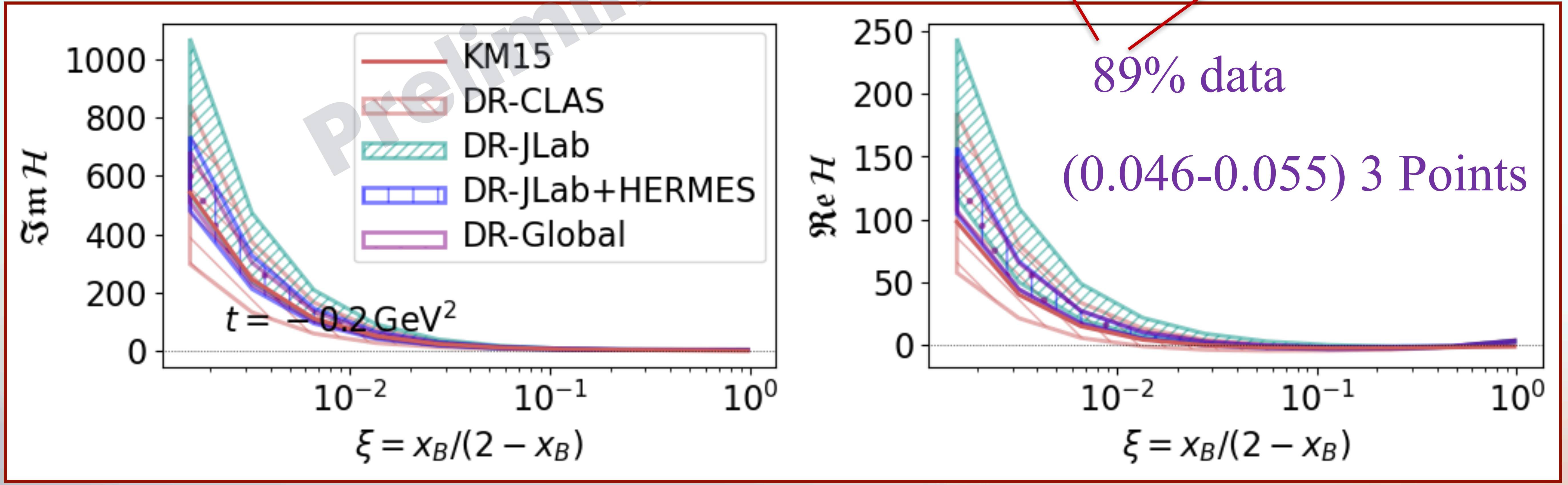


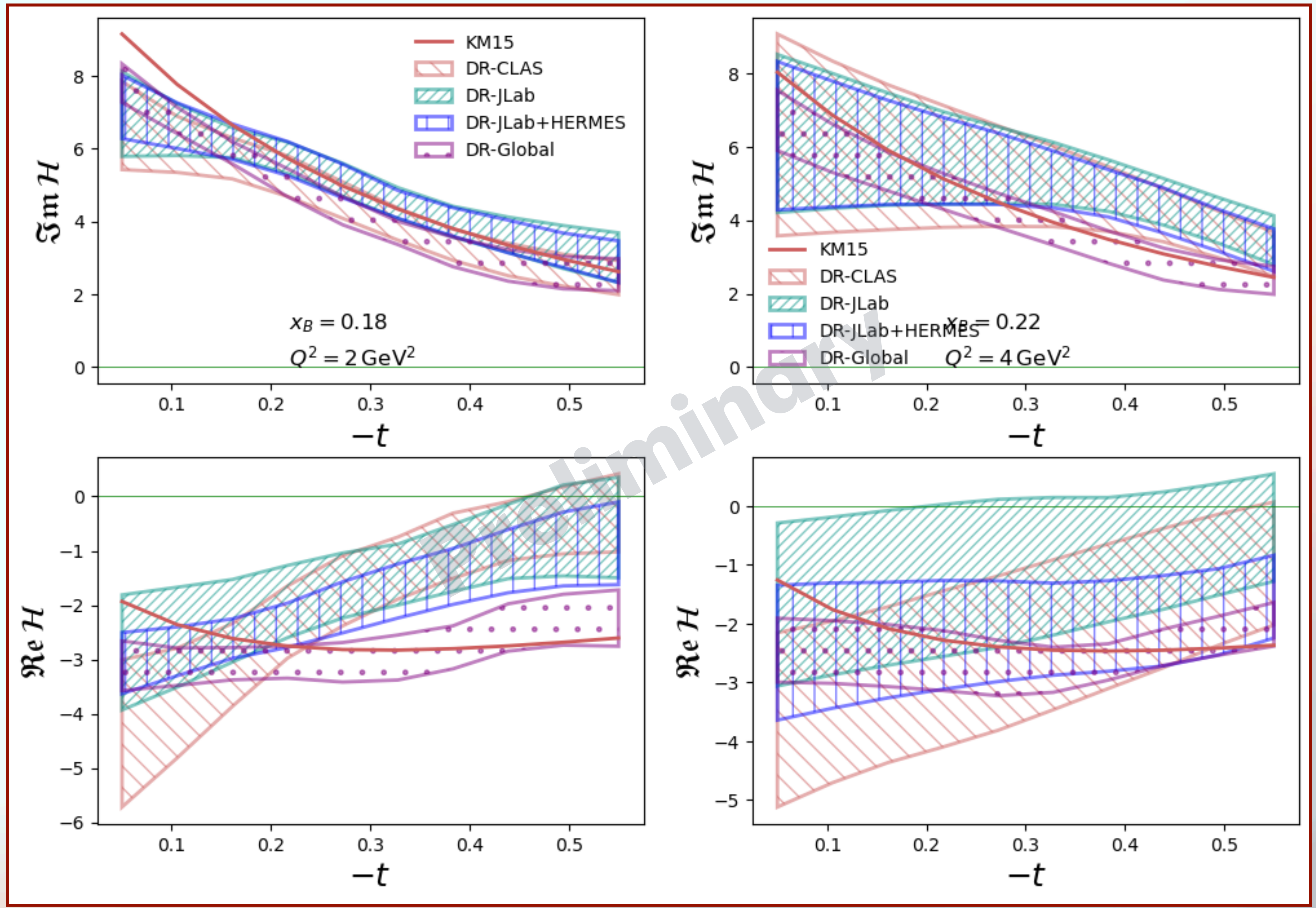
Update on global analysis of CFFs: DR NN model

The DR reconstruction is extremely sensitive to the behavior of ImH outside the region directly constrained by the observables — especially the small-ξ tail and the neighborhood of the principal-value point

xpow=-1.0







- Roadmap for a Global analysis: Extract harmonics for each observable, remove outliers, perform preliminary fit adjustments, make closure tests , tune hyperparameters for optimal performance, and conduct ensemble analyses from several NN models.

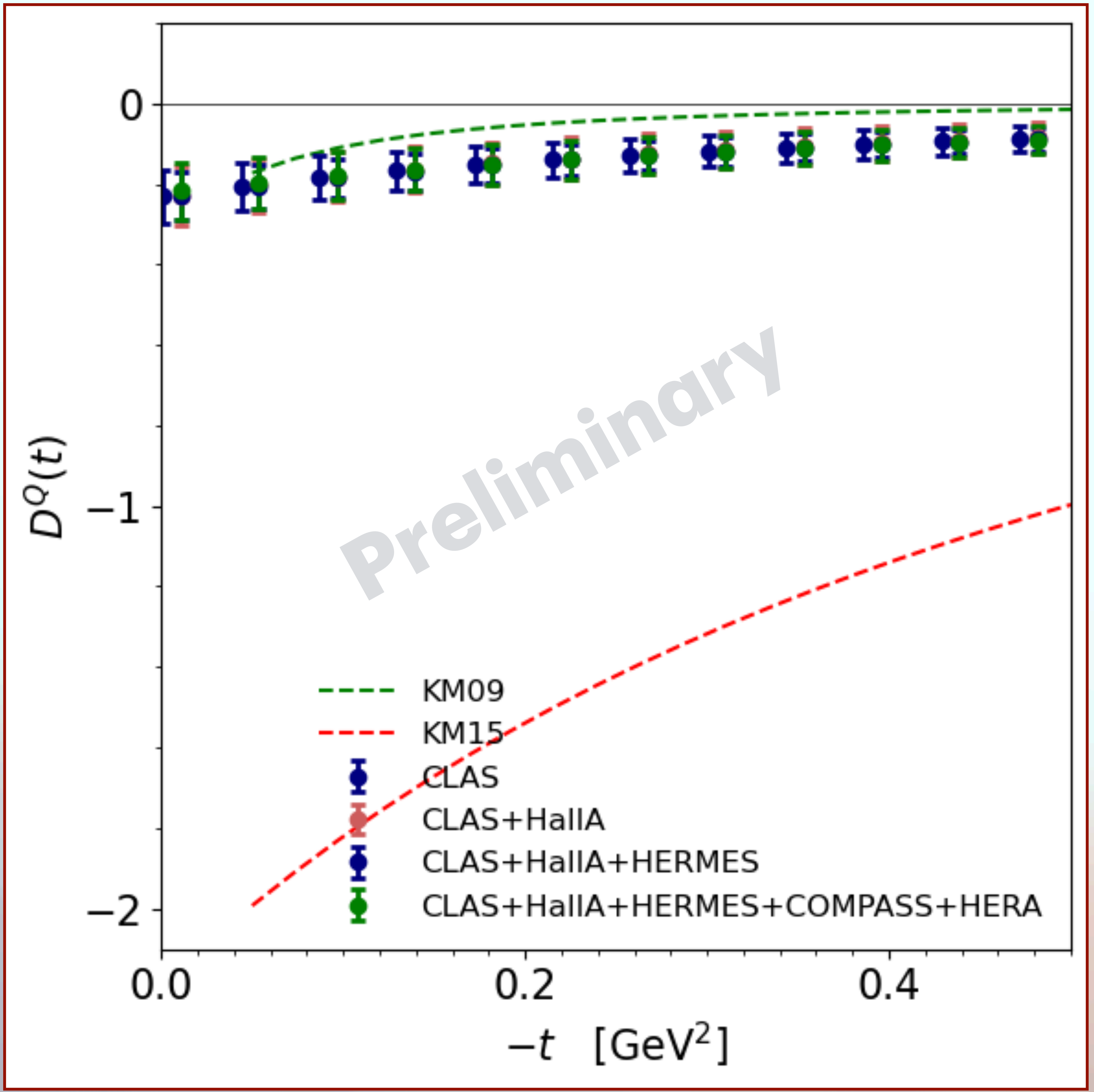
DR NN model - Experimental Data

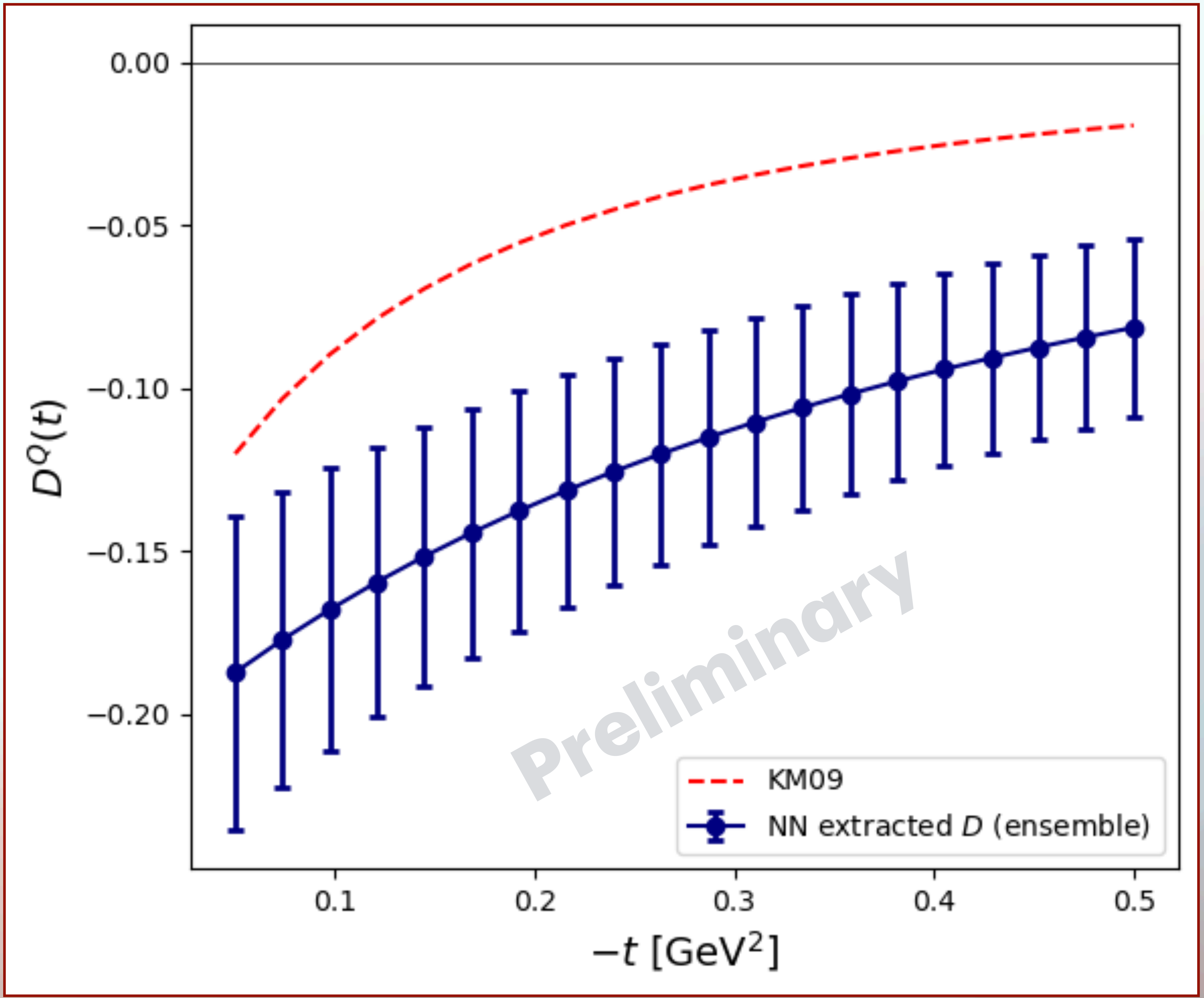
$$\mathrm{Re}\mathcal{H}(\xi,t) = \mathcal{C}_{\mathcal{H}}(t) + \frac{1}{\pi}\,\mathrm{P.V.}\int_0^1 d\xi' \left[\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right] \mathrm{Im}\mathcal{H}(\xi',t)$$

- PV integral + small-ξ correct extrapolation controls ReH stability.
- D-term is still weakly constrained to be left free unless ReH-sensitive observables are ensured.

$$D(t) = \frac{D_0}{(1-\frac{t}{M^2})^{\alpha}}$$

```
D0 = 0.2028837352991104
M2 = 0.9892549514770508
alpha = 2.1099915504455566
```





Deliverables and future directions

Deliverables, ongoing work and future directions

Deliverables:

- The DR reconstruction is extremely sensitive to the behavior of ImH outside the region directly constrained by the observables.
- We now have a validated DR-NN pipeline and can study D-term sensitivity in a controlled manner.
- Our approach to determining the D-term from experimental data resulted in a negative value. This is consistent with the expectation of a mechanically stable proton.

Ongoing work:

➢ Ongoing work on Global fit: Analyze the behavior change due to kinematics.

- Laboratories that study or have studied DVCS: Jefferson Lab Hall A [6→12 GeV], Jefferson Lab CLAS experiment [6→12 GeV], DESY with H1, ZEUS [320 GeV], and HERMES [27.6 GeV], CERN with COMPAS [160 GeV]

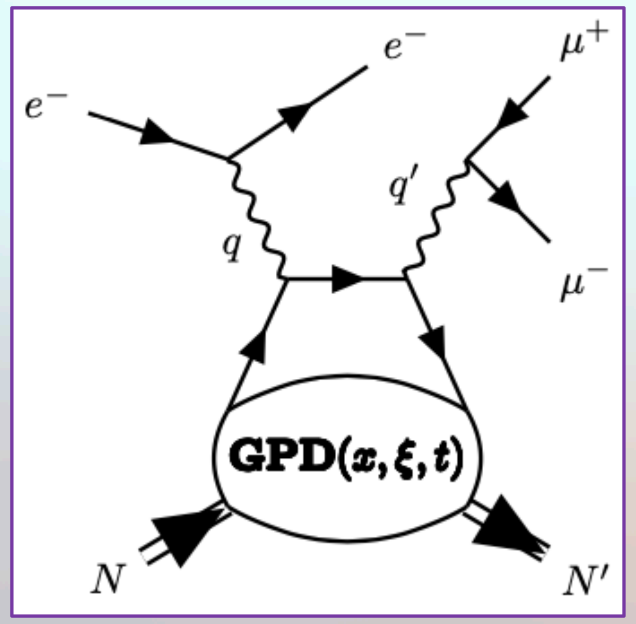
⟶ Studies of the impact on Q^2 ⟶ Ongoing work adding ERBL evolution equations.

Future directions:

➢ Create new modules on Gepard for TCS and DDVCS:

- Double DVCS:

⟶ Possibility of directly measuring GPDs for $x \neq \pm\xi$

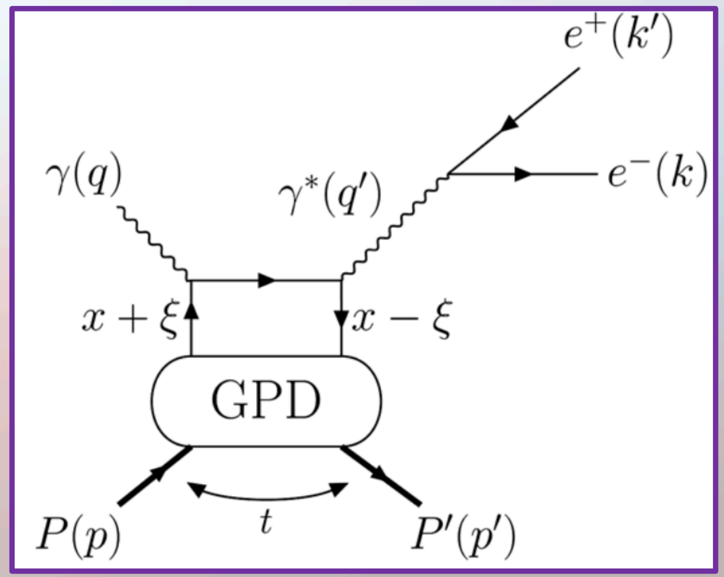


K. Deja, et. al, Acta Phys.Polon.Supp. 16 (2023) 7, 7-A24

- TCS:

⟶ Test universality

⟶ Data already available



Jefferson Lab CLAS Collaboration Phys. Rev. Lett. 127, 262501

➢ Take beyond LO in the D approximation

$$D^q_{\text{term}}(z,t) = (1-z^2) \sum_{\text{odd } n} d^q_n(t)\, C^{3/2}_n(z)$$

⟶ Lattice data as a constraint for the D-term

➢ Generate DDVCS/EIC pseudodata using EPIC and integrate into the analysis.

ePIC EIC Software: Simulation Toolkit

Thank you all for your attention!

Backup

Accessing GFFs

Considering some assumptions like:

1. At LO, gluons don't contribute to DVCS much, so one can assume that the subtraction constant is dominantly coming from quarks, but for the moment, we also neglect strange and heavier quarks.

2. To talk about "quark pressure," one needs to neglect energy-momentum flow from quarks to gluons described by the EMT form factor $\bar{C}(t)$.

2. In the asymptotic limit $d_n^q(t)$ vanish for n>1. Where $d_1^q(t)$ determine the asymptotic form of the GPDs.

3. Considerate the dominance of the flavor singlet combination.

$$\Delta(t) = 2\sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z \, \frac{D_{\mathrm{term}}^q(z,t)}{1-z},$$

$$d_1^u = d_1^d = d_1^Q/2$$

$$D_{\mathrm{term}}^q(z,t) = (1-z^2) \sum_{\mathrm{odd}\, n} d_n^q(t)\, C_n^{3/2}(z)$$

$$D_q(t) = \frac{4}{5} d_1^q(t) = \int_{-1}^{1} \mathrm{d}z\, z\, D_{\mathrm{term}}^q(z,t)$$

$$\Delta(t) = 4\left(\frac{4}{9} d_1^u(t) + \frac{1}{9} d_1^d(t)\right)$$

$$D^Q(t) = \frac{18}{25}\Delta(t)\,.$$

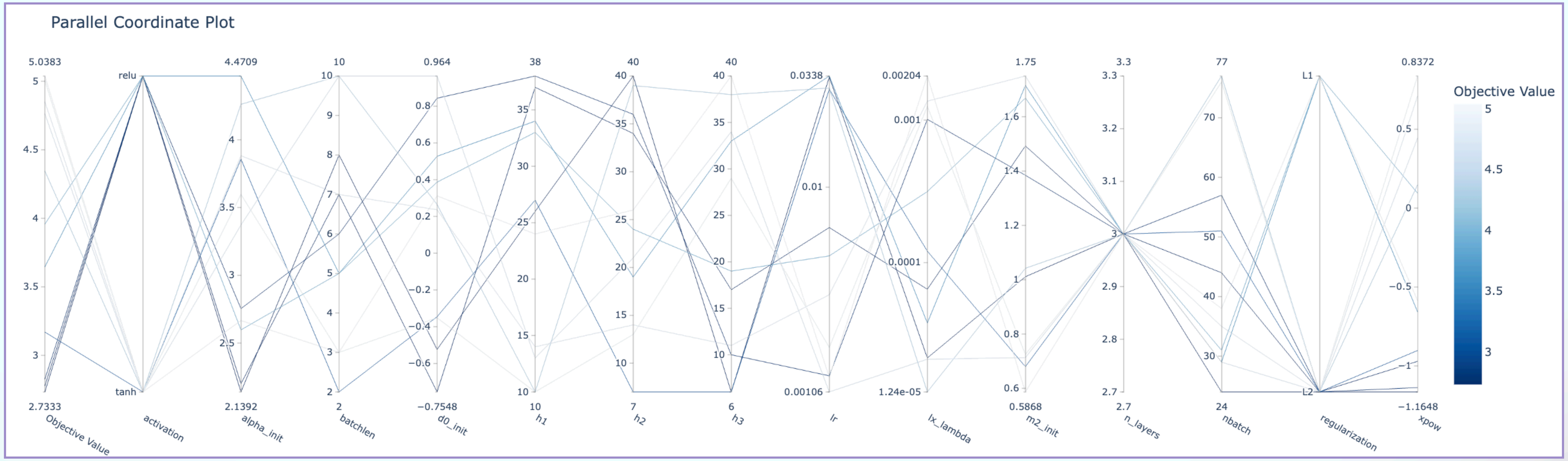
To date, a more conservative extraction of the subtraction constant from the currently available experimental data remains compatible with zero within large uncertainties.

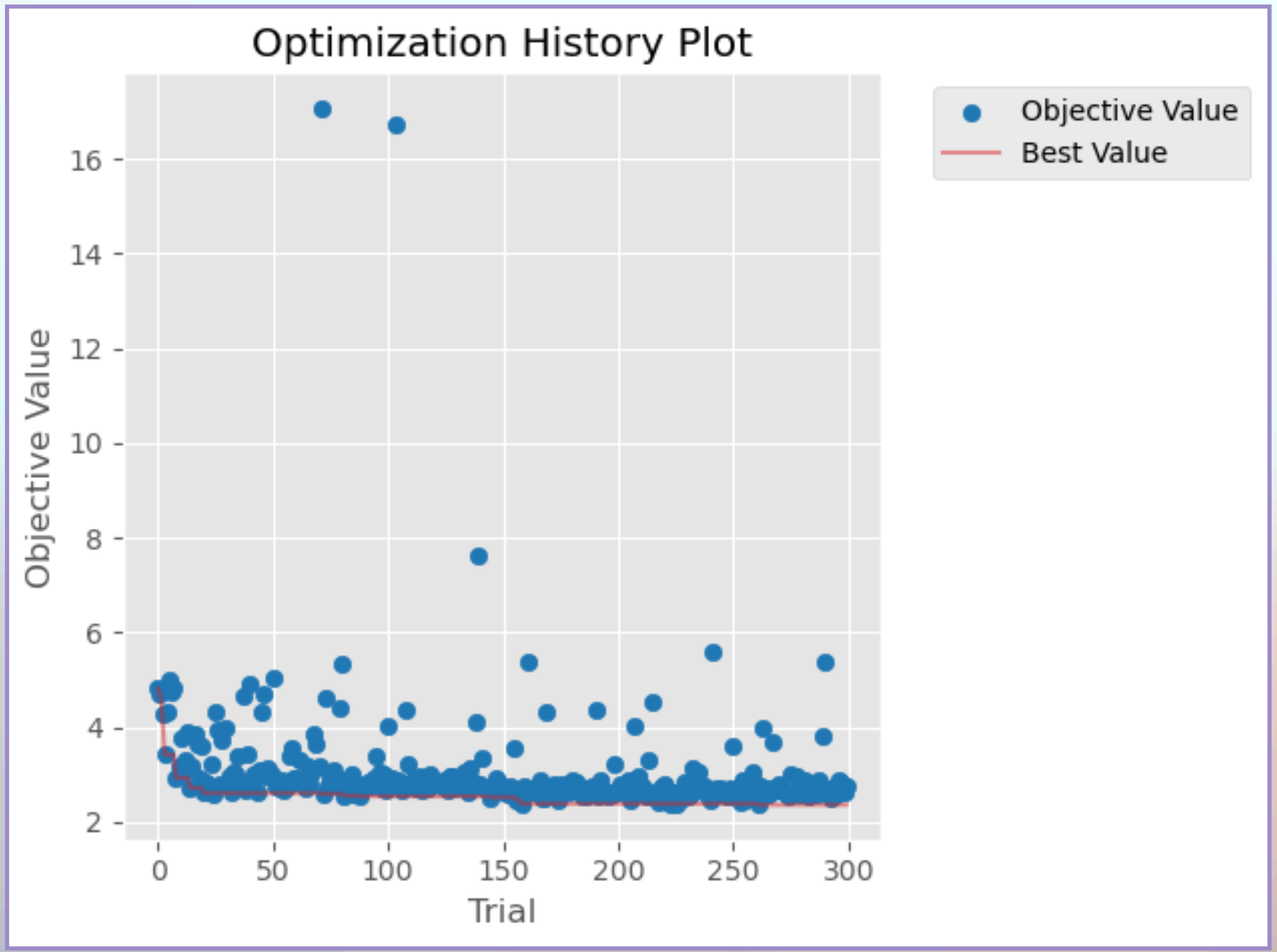
By utilizing this DR, along with other phenomenological assumptions, we can extract the CFFs and GFFs from experimental data and enhance the results with lattice calculations!

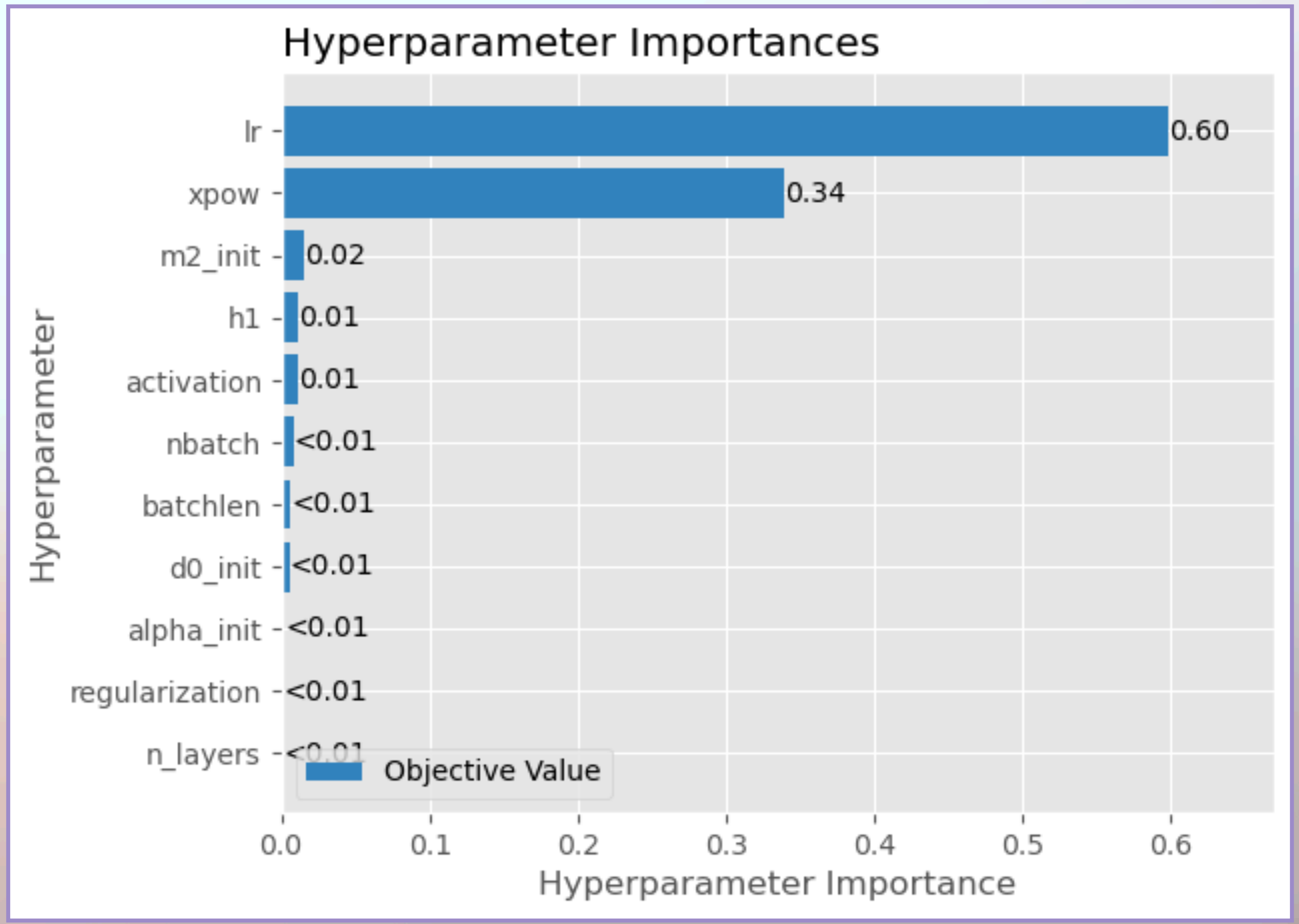
Jefferson Lab

DR Optuna study with experimental data

The Optuna study was run for 500 trials
The objective of the study is the global chi2







Jefferson Lab

DR Optima test with experimental data

The Optuna study was run for 500 trials
The objective of the study is the global chi2

Jefferson Lab

All Experimental data with best parameters from Optuna:

```
collab=CLAS      obs=ALU     chi2/npt=1.3292   npt=  9  prob=0.2154
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 collab=COMPASS   obs=XGAMMA  chi2/npt=2.1780  npt=  3  prob=0.08833
```
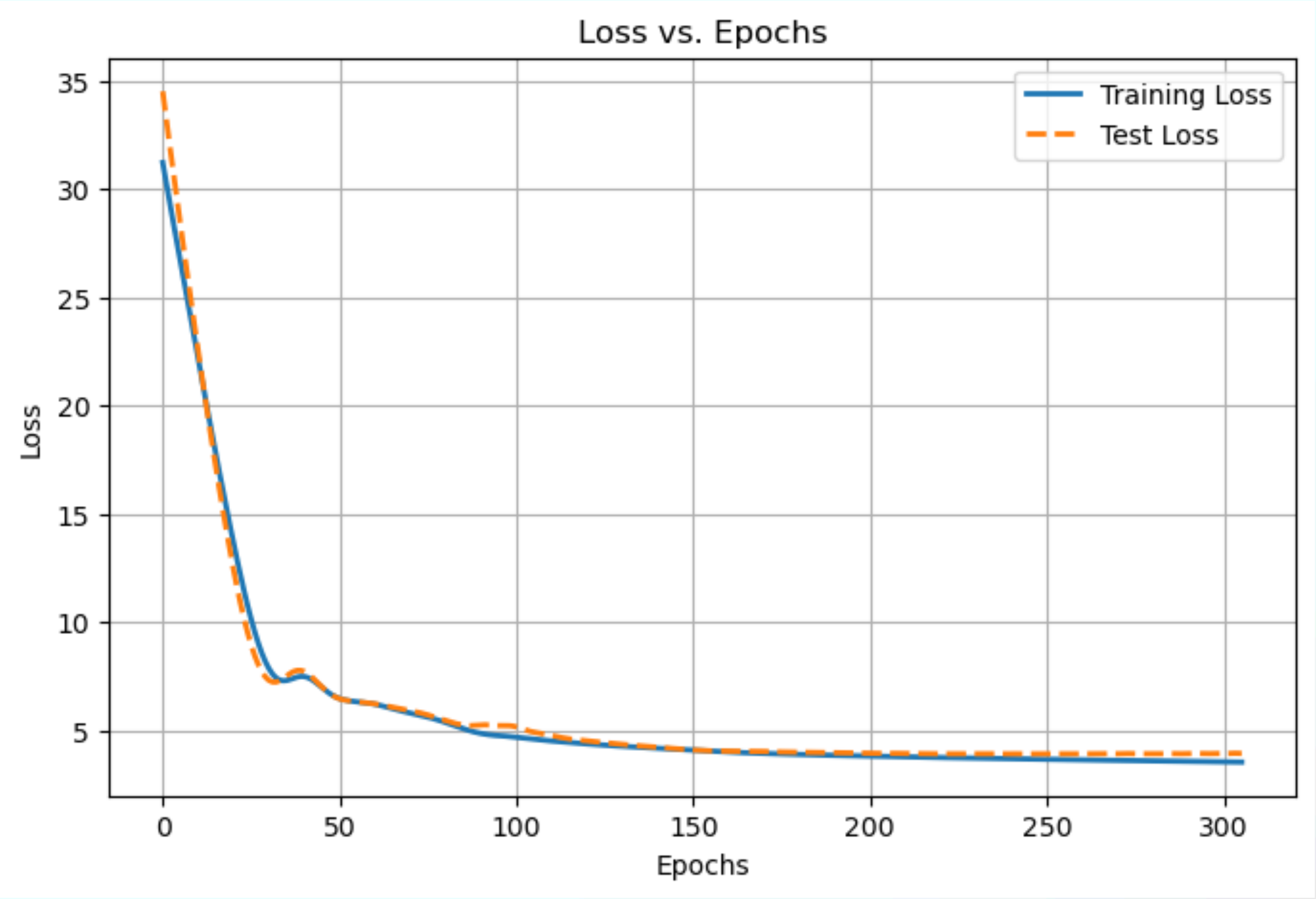
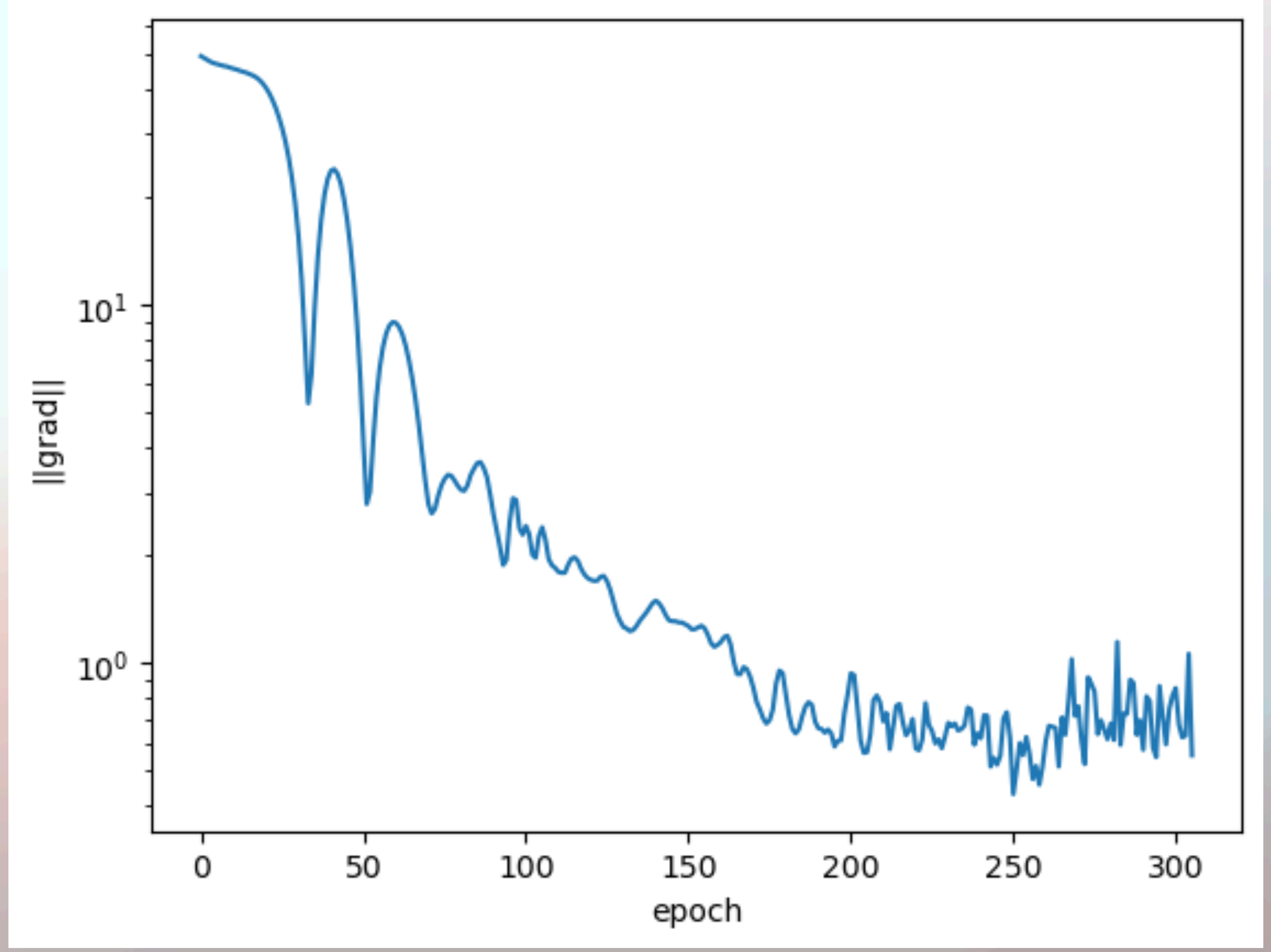



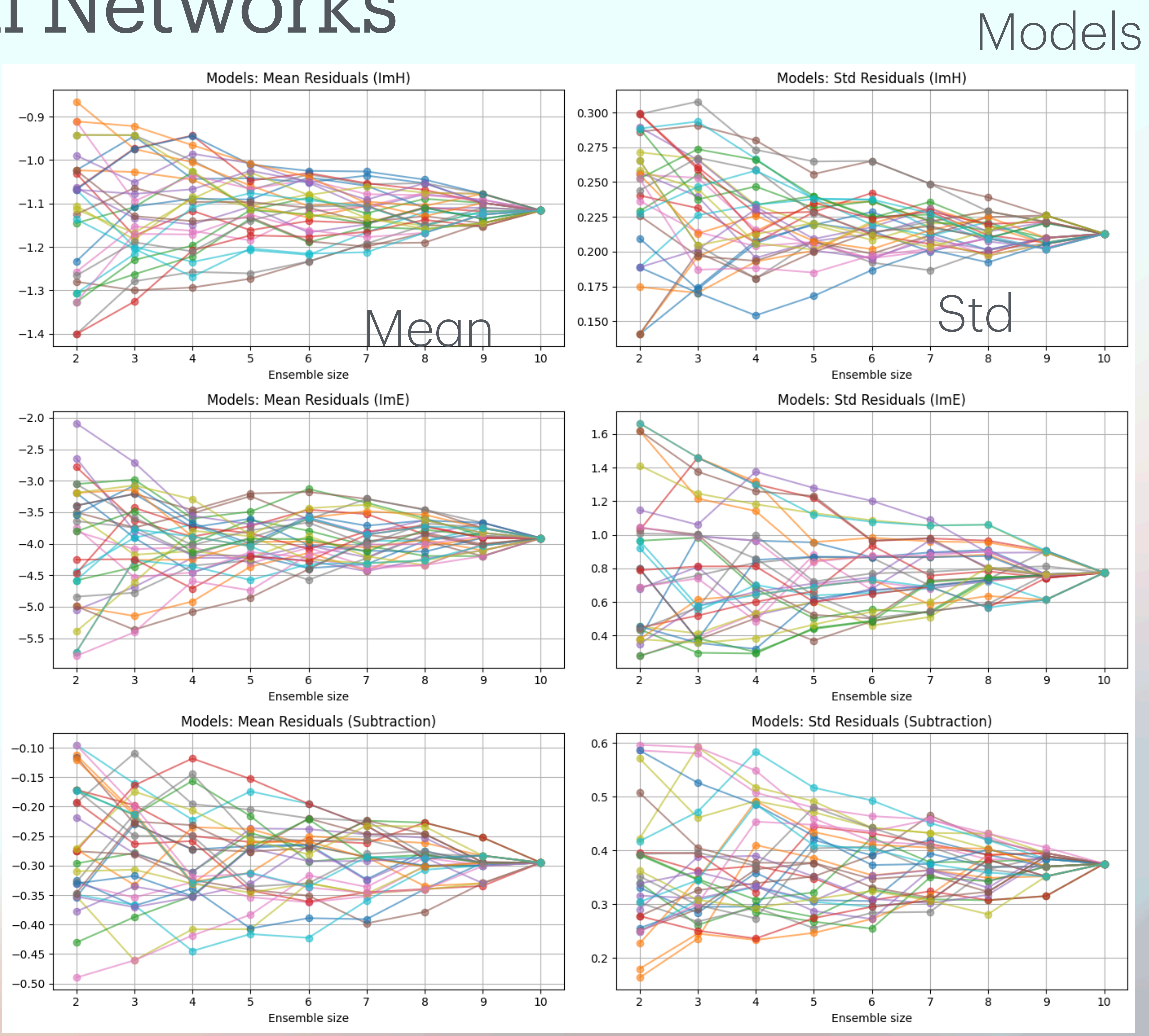


Ensemble analysis of the Neural Networks

To study real errors from NN, we started ensemble analysis by averaging over multiple learned representations.

- We use an ensemble approach, averaging over multiple trained models.
- As the number of models in the ensemble increases, the mean residual prediction stabilizes, reducing noise.
- The standard deviation plots demonstrate that uncertainty decreases as the ensemble size grows.
- We can prune the results by selecting the best nets with smaller test and train errors.

A larger ensemble tends to 'average out' individual model noise, improving the predictive stability results and uncertainty estimation of our model.



In addition, we can obtain reasonable behavior and reduce uncertainty according to the best hyperparameter selection.

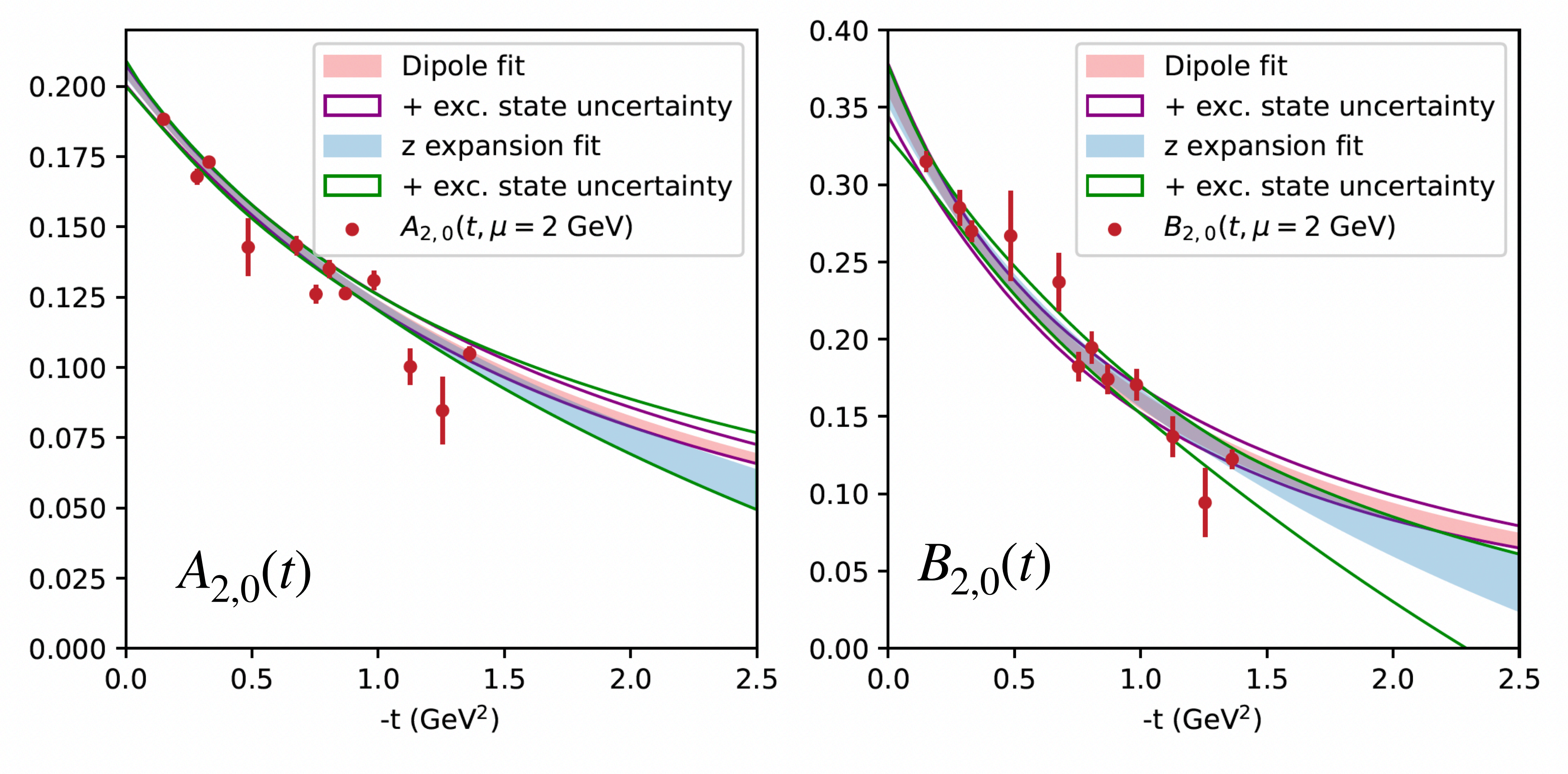
State of the project: Inputs from Lattice QCD

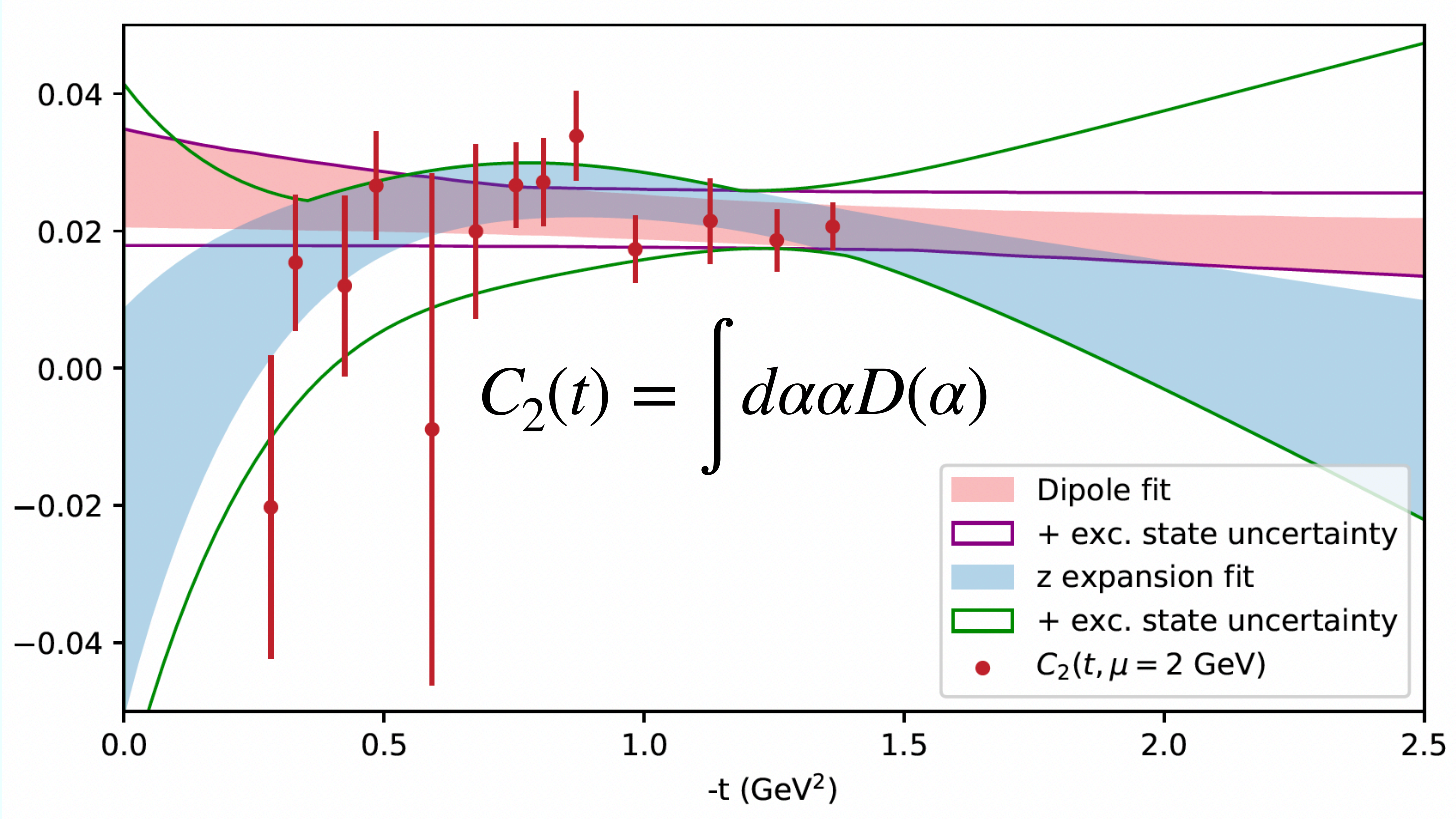
$$D^q_{\text{term}}(z,t) = (1-z^2)\sum_{\text{odd}\,n} d^q_n(t)\, C^{3/2}_n(z)$$

Lattice QCD (LQCD) constraints can help reduce the large systematic uncertainties in GFF extractions by accounting for higher-order terms in Gegenbauer expansions.

Extraction of the isovector GFFs from Lattice QCD

I.J. High Energy. Phys. 2024, 162 (2024)





- Calculation of the non-singlet "$u - d$" combination of GFF, which cannot be probed through DVCS, has been performed in Lattice across a wide range of t.
- To fully constrain the analysis, isoscalar GFFs (connected contributions) are also needed.

How can we use this data in a Global analysis?

- Lattice calculations can provide direct information about CFF.
- Plan to merge LQCD-derived CFFs into the neural network fitting framework, complementing experimental data and improving constraints in the loss function.